

Nucleon electromagnetic form factors in nonlocal chiral effective theory

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In this paper, we calculate the pion one-loop corrections to the electromagnetic form factors of the nucleon up to $\mathcal{O}(p^4)$ order in the nonlocal framework. To systematically investigate the role of vector-meson dominance in the model, we also include the leading-order nucleon–vector-meson coupling. Owing to the renormalization-scheme dependence of chiral low-energy coupling constants, we refit these constants to the nucleon magnetic moments, electromagnetic radii, and the Q^2 -dependence of the electromagnetic form factors. Compared with the previous local analysis, the model results show that, in the nonlocal framework, the Q^2 -dependence of the electromagnetic form factors in the large- Q^2 region is significantly improved and is consistent with experimental data.

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I. INTRODUCTION

The nucleon electromagnetic form factors are important observables in electron-nucleon elastic scattering and encode the fundamental electromagnetic structure of the nucleon. Over the past seven decades, as an important task in high-energy experiments, the extraction of nucleon electromagnetic form factors from electron-nucleon elastic scattering has made significant progress and revealed abundant data. Recently, attention has been drawn to the Lattice QCD simulations of the electromagnetic form factors of the nucleon [1–3]. This makes it possible to study the nucleon electromagnetic structure from the theoretical perspective. On the other hand, at the low-energy scale, the interaction between low-lying hadrons is described by chiral perturbation theory. As an important application, the electromagnetic properties of the nucleon have been investigated in chiral perturbation theory with various renormalization approaches, such as heavy-baryon, IR and extended-on-mass-shell (EOMS) regularization schemes [4–7]. Although these results successfully explained the chiral expansion of magnetic moments, electromagnetic radii, and Q^2 -dependence of the electromagnetic form factors in the small- Q^2 region, the discrepancy between experimental data and model results in the large- Q^2 region is not yet well understood, even when the vector-meson contributions are included [8–10].

As a novel approach for renormalizing the loop contributions, the nonlocal (or finite-range) renormalization

method has been successfully applied in gravity and other areas of phenomenological studies [11–20]. The main advantage of the nonlocal renormalization method is that the renormalization procedure is encoded into a modified nonlocal Lagrangian. Consequently, loop contributions from such a Lagrangian are UV convergent [21–23]. More importantly, the nonlocal Lagrangians still fulfill the underlying symmetries of the theory with the help of the corresponding gauge-link operators [24,25]. This feature is reflected in the conservation of currents, which holds even at the loop level. Recently, the nonlocal renormalization method has been extended to chiral perturbation theory, and within this model, pion-nucleon splitting functions as well as the nucleon electromagnetic form factors have been calculated [26–32]. However, we note that in the previous nonlocal analysis the higher chiral orders and vector-meson contributions were not included. In fact, vector mesons play an important role in improving the Q^2 -dependence of the electromagnetic form factors in the large momentum transfer region. On the other hand, the chiral symmetry should be satisfied within the nonlocal framework. As shown in Ref. [33], the nonlocal SU(2) theory is chiral invariant by introducing corresponding gauge link operators. This results in the significant improvement in the Q^2 -dependence of nucleon axial form factors in the large momentum transfer region when the pion-pole contribution is included. This sparked interest of studying the nucleon electromagnetic form factors within the same methodology. In this work, for completeness, we systematically investigate the pion-loop corrections to the nucleon electromagnetic form factors up to $\mathcal{O}(p^4)$ in the SU(2) nonlocal chiral framework and investigate the contributions of vector mesons.

This paper is organized as follows. In Sec. II, we review the basic formulation of the nucleon electromagnetic form factors. In Sec. III, we briefly review the construction of the

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nonlocal chiral Lagrangians that will be used to calculate loop corrections to the nucleon electromagnetic form factors. In Sec. IV, we explicitly present the loop corrections for all Feynman diagrams up to $\mathcal{O}(p^4)$. In Sec. V, we first refit the second-, third-, and fourth-order low-energy constants (LECs) as well as the vector-meson coupling constants, to the nucleon magnetic moments, electromagnetic radii, and the Dipole and Galster parametrizations of the form factors, then compare the model results for the Q^2 -dependence of the nucleon electromagnetic form factors with experimental data. Finally, we discuss the contributions of different chiral orders and the fourth moments of the electromagnetic form factors. In the final section, we briefly summarize our results.

II. FORMULATION

The electromagnetic form factors of a spin- $\frac{1}{2}$ particle are defined in terms of the matrix element of electromagnetic vector current as

$$\begin{aligned} \langle N(p', s') | J^\mu | N(p, s) \rangle &= \Gamma^\mu(p, q) \\ &= e \bar{u}(p + q) \left[\gamma^\mu F_1(Q^2) \right. \\ &\quad \left. + i \frac{\sigma^{\mu\nu} q^\nu}{2m} F_2(Q^2) \right] u(p), \end{aligned} \quad (1)$$

where $\Gamma^\mu(p, q)$ denotes the nucleon-photon vertex function, m is the physical mass of the nucleon, q is the 4-momentum transfer with $q^2 = -Q^2$, and the Lorentz-invariant scalar functions $F_1(Q^2)$ and $F_2(Q^2)$ are called the Dirac and Pauli form factors, respectively. For the proton and neutron, $F_1(Q^2)$ and $F_2(Q^2)$ are normalized as $F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.793$, $F_1^n(0) = 0$ and $F_2^n(0) = \kappa_n = -1.913$, where κ represents the nucleon anomalous magnetic moments. In the nonrelativistic framework, the matrix element of the electromagnetic vector current is parametrized in terms of the electric and magnetic Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$, which are related to $F_1(Q^2)$ and $F_2(Q^2)$ via

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) + \frac{Q^2}{4m^2} F_2(Q^2), \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2). \end{aligned} \quad (2)$$

To characterize the distributions of charge and magnetic moment, it is necessary to expand the nucleon

electromagnetic form factors in polynomial form [34,35],

$$\begin{aligned} G_E^N(Q^2) &= G_E^N(0) - \frac{\langle r^2 \rangle_E^N}{6} Q^2 + \frac{\langle r^4 \rangle_E^N}{120} Q^4 + \dots, \\ G_M^N(Q^2) &= G_M^N(0) \left(1 - \frac{\langle r^2 \rangle_M^N}{6} Q^2 + \frac{\langle r^4 \rangle_M^N}{120} Q^4 + \dots \right), \end{aligned} \quad (3)$$

where $\langle r^n \rangle$ represents the n th moments of the electromagnetic form factors. For example, the second moments describe the charge and magnetic radii and are defined in terms of the slope of the form factors at $Q^2 = 0$,

$$\begin{aligned} \langle (r_E^p)^2 \rangle &= -\frac{6}{G_E^p(0)} \frac{dG_E^p(Q^2)}{dQ^2}, \\ \langle (r_M^p)^2 \rangle &= -\frac{6}{G_M^p(0)} \frac{dG_M^p(Q^2)}{dQ^2}, \\ \langle (r_E^n)^2 \rangle &= -\frac{6dG_E^n(Q^2)}{dQ^2}, \\ \langle (r_M^n)^2 \rangle &= -\frac{6}{G_M^n(0)} \frac{dG_M^n(Q^2)}{dQ^2}. \end{aligned} \quad (4)$$

Furthermore, the fourth moments of the electromagnetic form factors define the curvatures of the form factors at $Q^2 = 0$,

$$\begin{aligned} \langle (r_E^p)^4 \rangle &= \frac{120}{G_E^p(0)} \frac{d^2 G_E^p(Q^2)}{dQ^4}, \quad \langle (r_M^p)^4 \rangle = \frac{120}{G_M^p(0)} \frac{d^2 G_M^p(Q^2)}{dQ^4}, \\ \langle (r_E^n)^4 \rangle &= \frac{120d^2 G_E^n(Q^2)}{dQ^4}, \quad \langle (r_M^n)^4 \rangle = \frac{120}{G_M^n(0)} \frac{d^2 G_M^n(Q^2)}{dQ^4}. \end{aligned} \quad (5)$$

In the loop calculation, it can be assumed that there are two projection operators $\Lambda_\mu^1(p, q)$ and $\Lambda_\mu^2(p, q)$, which project out the form factors F_1 and F_2 from the vertex operator $\bar{\Gamma}^\mu(p, q)$ [36–38],

$$\begin{aligned} F_1(q^2) &= \text{Tr}[\Lambda_\mu^1(p, q) \bar{\Gamma}^\mu(p, q)], \\ F_2(q^2) &= \text{Tr}[\Lambda_\mu^2(p, q) \bar{\Gamma}^\mu(p, q)], \end{aligned} \quad (6)$$

where the vertex operator $\bar{\Gamma}^\mu(p, q)$ is related to the $\Gamma^\mu(p, q)$ by $\Gamma^\mu(p, q) = \bar{u}(p') \bar{\Gamma}^\mu(p, q) u(p)$. From Eq. (6), one can readily derive these projection operators as

$$\begin{aligned} \Lambda_\mu^1(p, q) &= (\not{p} + m) \left[\frac{1}{4(q^2 - 4m^2)} \gamma_\mu + \frac{3m}{2(q^2 - 4m^2)^2} (2p_\mu + q_\mu) \right] (\not{p}' + m), \\ \Lambda_\mu^2(p, q) &= (\not{p} + m) \left[-\frac{m^2}{q^2(q^2 - 4m^2)} \gamma_\mu - \frac{m(q^2 + 2m^2)}{q^2(q^2 - 4m^2)^2} (2p_\mu + q_\mu) \right] (\not{p}' + m), \end{aligned} \quad (7)$$

with the on-shell condition $p^2 = m^2$. With the help of Eqs. (6) and (7), one is able to extract the form factors $F_1(q^2)$ and $F_2(q^2)$ without explicitly analyzing the gamma matrix structure of $\Gamma^\mu(p, q)$. Moreover, many high-energy physics program packages have been developed that include some useful tools such as trace calculations and one-loop integrals. For convenience, we will use *Mathematica* Package-X during the calculation [39].

III. NONLOCAL CHIRAL LAGRANGIANS

A. Nucleon-pion nonlocal chiral Lagrangians

Since the pion and nucleon are the basic building blocks of SU(2) chiral perturbation theory, we start by reviewing their chiral transformations. Under the SU(2) chiral transformation, they transform as

$$U'(x) = R(x)U(x)L^\dagger(x), \quad N'(x) = K(x)N(x), \quad (8)$$

where $N(x)$ represents the nucleon field and is defined as $N(x) = [n(x), p(x)]^T$ and $U(x)$ denotes the SU(2) representation of the pion field and is given by $U(x) = \exp(i\phi(x)/f)$ with the pion field matrix $\phi(x)$,

$$\phi(x) = \vec{\tau} \cdot \vec{\pi}(x) = \begin{pmatrix} \pi^0(x) & \sqrt{2}\pi^+(x) \\ \sqrt{2}\pi^-(x) & -\pi^0(x) \end{pmatrix}. \quad (9)$$

As discussed in Ref. [33], the nonlocal chiral Lagrangians for the pion-nucleon interaction can be readily obtained by replacing the pion field operator $U(x)$ with an extended nonlocal pion field $\int d^4a F_\pi(a)U(x+a)$, where $F_\pi(a)$ is a scalar regulator function for the pion. Nevertheless, the chiral transformation of the nonlocal pion fields does not obey the chiral transformation rules of the local pion field in Eq. (8). To recover the chiral transformation of the nonlocal fields, it needs to be modified by adding a gauge link operator. Thus, the total nonlocal operators can be defined as [33]

$$\hat{U}(x) = \exp \left\{ \frac{1}{c_0} \int d^4a F_\pi(a) \text{Log}[G_R(x, x+a)U(x+a) \times G_L^\dagger(x, x+a)] \right\}, \quad (10)$$

where c_0 is a normalization constant and satisfies $c_0 \equiv \int da F_\pi(a)$ and $G_R(x, x+a)$ and $G_L(x, x+a)$ represent the Wilson line operators for the left and right-handed gauge fields and are conventionally parametrized as

$$G_R(x, x+a) = \mathcal{P} \exp \left[-i \int_x^{x+a} dz^\mu r_\mu(z) \right], \\ G_L(x, x+a) = \mathcal{P} \exp \left[-i \int_x^{x+a} dz^\mu l_\mu(z) \right], \quad (11)$$

where $\mathcal{P}$ denotes the path-ordering operator. It can be proved that under the chiral transformation the left- and right-handed Wilson line operators transform as

$$G'_{L[R]}(x, y) = L(x)[R(x)]G_{L[R]}(x, y)L^\dagger(y)[R^\dagger(y)]. \quad (12)$$

Using these Wilson line operators, one can derive that the nonlocal pion field operator $\hat{U}(x)$ transforms approximately as in Eq. (8) [33]. In particular, under the SU_V(2) transformation [$L(x) = R(x) = V(x)$], the nonlocal pion field exactly transforms as $\hat{U}'(x) = V(x)\hat{U}(x)V^\dagger(x)$. This implies that the nonlocal Lagrangian for the pion-nucleon interactions can be constructed by replacing the original local pion field $U(x)$ with the nonlocal one $\hat{U}(x)$. For example, the lowest-order Lagrangian for pion-nucleon interactions is given by [40]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\tilde{\not{D}} - m_0 + \frac{g_A}{2} u_\mu \gamma^\mu \gamma_5 \right) \Psi, \quad (13)$$

where m_0 is the nucleon mass in the chiral limit; g_A is the nucleon axial charge in the chiral limit; the nucleon covariant derivative D_μ is defined as $D_\mu = \partial_\mu + \Gamma_\mu$ with the nonlocal vector and axial vector connection Γ_μ and u_μ ,

$$\Gamma_\mu = \frac{1}{2} (\hat{u}^\dagger \partial_\mu \tilde{u} + \tilde{u} \partial_\mu \hat{u}^\dagger) - \frac{i}{2} (\hat{u}^\dagger Q \hat{u} + \tilde{u} Q \tilde{u}^\dagger) A_\mu, \\ u_\mu = i(\hat{u}^\dagger \partial_\mu \hat{u} - \hat{u} \partial_\mu \hat{u}^\dagger) + (\hat{u}^\dagger Q \hat{u} - \tilde{u} Q \tilde{u}^\dagger) A_\mu, \quad (14)$$

where the nonlocal $\hat{u}$ is defined as $\hat{u} = \sqrt{\hat{U}}$, $\hat{u}^\dagger = \sqrt{\hat{U}^\dagger}$; A_μ represents the photon field; and Q denotes the nucleon charge matrix and takes the form $Q = -e \text{diag}(1, 0)$. At the second order, the pion-nucleon interactions include terms proportional to LECs c_i ($i = 1-7$), where only c_2 , c_4 , c_6 , and c_7 -related terms contribute to the electromagnetic form factors and are given by [40]

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi} \left\{ -\frac{c_2}{8m^2} (\langle u_\mu u_\nu \rangle \{D^\mu, D^\nu\} + \text{h.c.}) + \frac{ic_4}{4} \sigma^{\mu\nu} [u_\mu, u_\nu] \right. \\ \left. + \frac{c_6}{8m} \sigma^{\mu\nu} F_{\mu\nu}^+ + \frac{c_7}{8m} \sigma^{\mu\nu} \langle F_{\mu\nu}^+ \rangle \right\} \Psi, \quad (15)$$

where the chiral covariant field strength $F_{\mu\nu}^+ = \hat{u}^\dagger F_{\mu\nu} \hat{u} + \hat{u} F_{\mu\nu} \hat{u}^\dagger$, with the photon field strength tensor $F_{\mu\nu} = Q(\partial_\mu A_\nu - \partial_\nu A_\mu)$. As for the third- and fourth-order chiral Lagrangian, we only list the terms that contribute to the nucleon electromagnetic form factors [40],

$$\begin{aligned}\mathcal{L}_{\pi N}^{(3)} &= \bar{\Psi} \left\{ \frac{id_6}{2m} ([D^\mu, \hat{F}_{\mu\nu}^+] D_\nu + \text{h.c.}) + \frac{id_7}{2m} ([D^\mu, \langle F_{\mu\nu}^+ \rangle] D_\nu + \text{h.c.}) \right\} \Psi, \\ \mathcal{L}_{\pi N}^{(4)} &= \bar{\Psi} \left\{ -\frac{e_{54}}{2} [D^\lambda, [D_\lambda, \langle F_{\mu\nu}^+ \rangle]] \sigma^{\mu\nu} - \frac{e_{74}}{2} [D^\lambda, [D_\lambda, \hat{F}_{\mu\nu}^+]] \sigma^{\mu\nu} - \frac{e_{105}}{2} \langle F_{\mu\nu}^+ \rangle \langle \chi_+ \rangle \sigma^{\mu\nu} - \frac{e_{106}}{2} \hat{F}_{\mu\nu}^+ \langle \chi_+ \rangle \sigma^{\mu\nu} \right\} \Psi, \quad (16)\end{aligned}$$

where the traceless operator $\hat{A}$ is defined as $\hat{A} = A - \frac{1}{2} \langle A \rangle$ and the chiral covariant mass term $\chi_\pm$ is given by $\chi_\pm = \hat{u}^\dagger \chi \hat{u}^\dagger \pm \hat{u} \chi^\dagger \hat{u}$, where the mass field χ is expressed in terms of linear combination of the scalar and pseudoscalar fields via $\chi = 2B(s + ip)$. The constants d_6, d_7, e_{54} , and e_{74} are the third- and fourth-order LECs, which usually can be fitted to the electromagnetic charge radii of the proton and neutron.

B. Nucleon–vector-meson interaction

It is well established that the vector-meson pole contribution plays an important role in describing the Q^2 -dependence of the nucleon form factors, especially when one investigates the distributions of the nucleon

electromagnetic form factors in the large Q^2 -region. To this end, we need to include the vector mesons in the nonlocal chiral perturbation theory. To construct the nonlocal interaction between the vector meson and nucleon, by analogy with the pion-nucleon interaction, the vector-meson field operator $V_\mu(x)$ can be replaced with the nonlocal one $\int d^4a F_V(a) V_\mu(x+a)$. Nevertheless, the nonlocal vector field operator also breaks the rule of local chiral transformation. We omit the explicit details of the chiral transformation of the nonlocal vector meson and the corresponding gauge link operator, since they do not introduce additional contributions to the nucleon electromagnetic form factors. Following Refs. [41–43], we construct the leading-order nonlocal vector-meson–nucleon interaction as

$$\begin{aligned}\mathcal{L}_{NV}^{(1)} &= \int F_V(a) d^4a \left[g_{e\rho} \bar{N} \gamma^\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(x+a) N - \frac{g_{m\rho}}{2m} \bar{N} \sigma^{\mu\nu} \partial_\nu \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(x+a) N + g_{e\omega} \bar{N} \gamma^\mu \omega_\mu(x+a) N \right. \\ &\quad \left. - \frac{g_{m\omega}}{2m} \bar{N} \sigma^{\mu\nu} \partial_\nu \omega_\mu(x+a) N \right], \quad (17)\end{aligned}$$

where $g_{e\rho}, g_{e\omega}, g_{m\rho}$, and $g_{m\omega}$ represent the vector and tensor coupling constants between the nucleon and vector mesons, and the ρ vector-meson matrix is parametrized as

$$\boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu = \begin{pmatrix} \rho_\mu^0 & \sqrt{2} \rho_\mu^+ \\ \sqrt{2} \rho_\mu^- & -\rho_\mu^0 \end{pmatrix}. \quad (18)$$

Note that the nonlocal interaction in Eq. (17) naturally generates an extra regulator $\tilde{F}_V(k)$, whose role has been investigated in early works on nucleon-nucleon scattering [42,44–46]. In addition to this, the Lagrangian for leading-order vector-meson–photon interaction is given by [47]

$$\mathcal{L}_{V\gamma}^{(2)} = -\frac{eF_\rho}{2m_\rho} \rho_{\mu\nu}^0 f^{\mu\nu} - \frac{eF_\omega}{2m_\omega} f^{\mu\nu} \omega_{\mu\nu}, \quad (19)$$

where m_ρ and m_ω are the masses of the vector mesons and F_ρ and F_ω represent the vector-meson decay constants and can be determined from the dilepton decay widths via $\Gamma(V \rightarrow ee^-) = \frac{4\pi\alpha^2 F_V^2}{3m_V} [47–50]$. Using the vector-meson masses $m_\rho = 770$ MeV, $m_\omega = 782$ MeV and the latest data for the dilepton decay widths $\Gamma(\rho \rightarrow ee^-) = 6.96$ keV and $\Gamma(\omega \rightarrow ee^-) = 0.64$ keV [51], we obtain the

vector-meson decay constants as $F_\rho = 155$ MeV, $F_\omega = 47.37$ MeV, respectively. By contracting the vector mesons in Eqs. (17) and (19), one can obtain the vector-meson pole diagram, which contains a vector-meson propagator. In the effective field theory (EFT), the dynamics of the vector meson can be described in two distinct ways: the vector and the tensor representations. This, in turn, gives rise to two different forms of the vector-meson propagator. In this work, we make use of the vector field representation of the vector meson, and the propagator for the massive vector field is given by [52]

$$G^{\mu\nu}(q) = \frac{i(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_V^2})}{q^2 - m_V^2 + i\epsilon}. \quad (20)$$

IV. NUCLEON ELECTROMAGNETIC FROM FACTORS

In this section, we explicitly present the pion one-loop corrections to the electromagnetic form factor of the nucleon. The relevant Feynman diagrams are displayed in Fig. 1, where Figs. 1(a) to 1(j) represent the lowest chiral order contributions arising from the Lagrangian in Eq. (13), Figs. 1(k) to 1(n) are the second chiral order contributions from Eq. (15), Figs. 1(o) and 1(p) represent the third- and

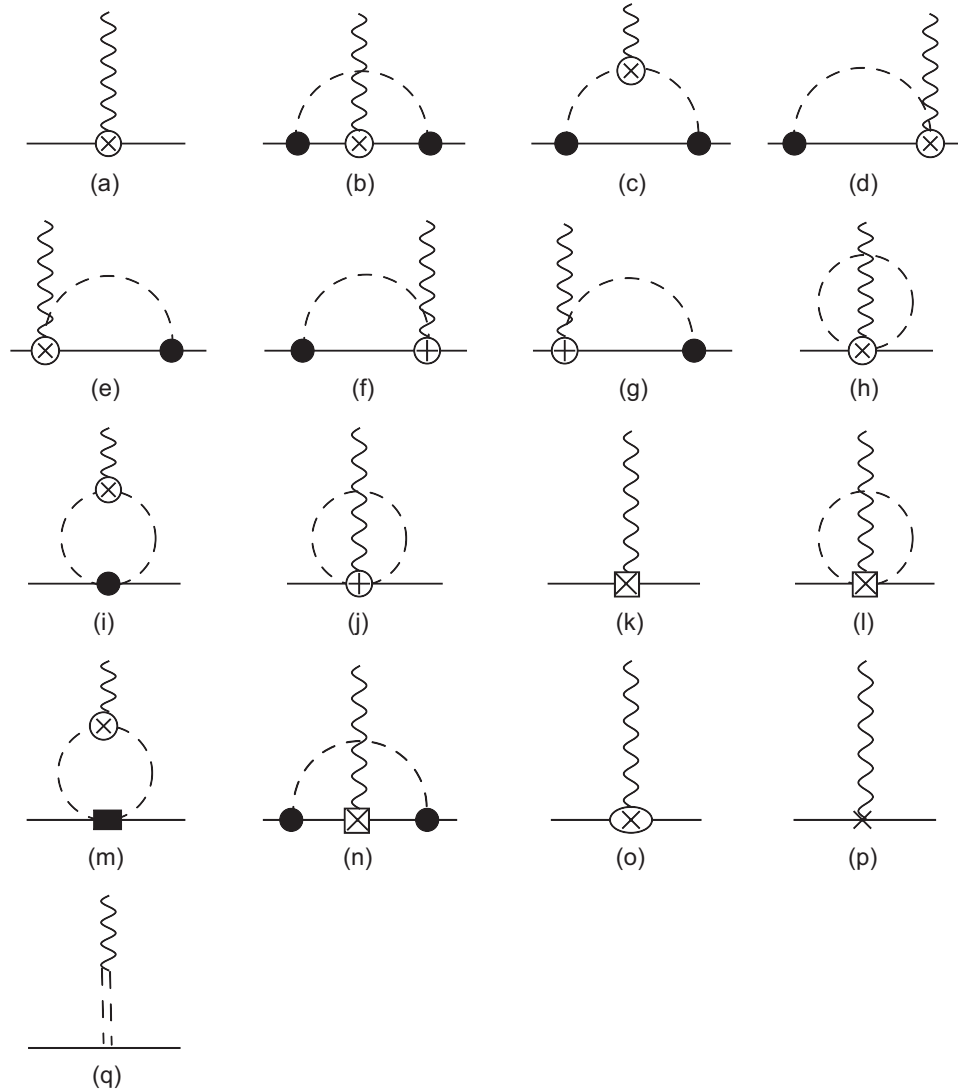


FIG. 1. The Feynman diagrams arise from the first-, second-, third-, and fourth-order chiral interactions of Eqs. (13), (15), and (16), where the solid, dashed, double-dashed, and wiggly lines represent the nucleon, pion, vector-meson, and photon fields; the solid dots and squares denote the first- and second-order pion-nucleon strong coupling vertices; the cross with circle and square are the lowest- and second-order photon-baryon interactions; the circled plus refers to the additional gauge link vertex; and the cross and cross with ellipse are the third- and fourth-order photon-nucleon interactions.

fourth-order contributions arising from the Lagrangian in Eq. (13), while Fig. 1(q) represents the vector-meson contribution. The explicit contributions of each Feynman diagram to the electromagnetic vertex function $\Gamma(p, q)$ are listed in the Appendix.

Note that at lowest order the tree level-diagram contribution is identical to that in the local case and only contributes to the electric form factor $G_E(Q^2)$ of the proton. As for the loop diagrams, they contain a nonlocal regulator function $\tilde{F}_\pi(k)$ due to the pion nonlocality. Consequently, the loop contributions are convergent by virtue of $\tilde{F}_\pi(k)$, which modifies the superficial degree of divergence of the loop integral. Moreover, as mentioned in the previous section, the gauge link operators of Eq. (11) give rise to additional loop contributions as shown in Figs. 1(f), 1(g),

and 1(j), which guarantee the gauge invariance of the electromagnetic interaction. Analytically, these contributions include the derivative of the nonlocal regulator $\tilde{F}_\pi(k)$, which vanishes in the local limit $\tilde{F}_\pi(k) \rightarrow 1$. As a result, the loop corrections reduce to those of the local case.

At the second order, the tree diagram contributes to the magnetic form factors of the neutron and proton, and the resulting the vertex function is identical to that of the local case. In addition to this, at the second chiral order, we also include pion loop corrections as displayed in Figs. 1(l)–1(n). Compared with the first-order contributions, the second-order nonlocal Lagrangian does not produce additional gauge-link diagrams, but the loop integrals are UV convergent as desired.

As for the third- and fourth-order contributions, we only take into account the tree diagrams, which contribute to the

both $F_1(Q^2)$ and $F_2(Q^2)$ form factors and are proportional to the 4-momentum transfer Q^2 . Therefore, the electromagnetic radii of the nucleon, which are defined in terms of the slopes of the electromagnetic form factors at $Q^2 = 0$, receive non-negligible contributions from these chiral orders. Finally, as displayed in Fig. 1(q), the photon couples to the nucleon via a vector-meson propagator. The corresponding the vector-meson vertex function, as shown in Eq. (A14), contains an additional regulator $\tilde{F}_V(k)$. As a result, the local vector-meson pole is rapidly suppressed and generates adequate curvatures for the electromagnetic form factors.

V. NUMERICAL RESULTS

A. Fitting of LECs and cutoff mass

As demonstrated in the previous section, within the nonlocal framework, the loop contributions to the electromagnetic form factors contain a nonlocal regulator. To calculate the loop integrals, it is necessary to define the explicit expression of the regulator function of the pion and vector meson $\tilde{F}_\phi(k)$ ($\phi = \pi, V$). In general, $\tilde{F}_\phi(k)$ is an analytic function of k^2 and can usually be parameterized in terms of Gaussian and dipole form. Following Refs. [53,54], we choose the regulator to have a simple dipole shape in k^2 ,

$$\tilde{F}_\phi(k) = \left(\frac{m_\phi^2 - \Lambda_\phi^2}{k^2 - \Lambda_\phi^2 + i\epsilon} \right)^2, \quad (21)$$

where m_ϕ^2 denotes the masses of the pion and vector mesons and Λ_ϕ is the cutoff mass of the dipole regulator. For the pion cutoff mass, following Refs. [31,32], Λ_π was determined as $\Lambda_\pi = (1.0 \pm 0.1)$ GeV. As for the vector meson, we assume that the vector mesons have a common cutoff mass $\Lambda_\rho = \Lambda_\omega$.

Apart from this, at the second chiral order, the loop corrections involve second-order LECs c_1 , c_2 , c_3 , and c_4 . However, since they depend on the renormalization scheme, their values in the local framework cannot be directly applied here. As shown in Ref. [33], in nonlocal framework, they can be fitted to the pion mass dependence of the nucleon mass m_N and axial charge g_A and take the values of $c_1 = (-0.211 \pm 0.020)$ GeV⁻¹, $c_2 = (-0.221 \pm 0.021)$ GeV⁻¹, $c_3 = (-0.055 \pm 0.084)$ GeV⁻¹, and $c_4 = (1.095 \pm 0.128)$ GeV⁻¹. As for the remaining second-order LECs c_6 and c_7 , we follow the standard procedure and fit them to the proton and neutron magnetic moments $\mu_p = 2.793$ and $\mu_n = -1.913$. The fit yields $c_6 = 6.263$ and $c_7 = -2.979$, respectively. By comparison, we find that the absolute values of second-order chiral coupling constants c_6 and c_7 are considerably larger than their local counterparts [8].

At the third and fourth chiral orders, we only take into account the tree diagrams, which are proportional to LECs d_6 , d_7 , e_{74} , and e_{54} . Indeed, without the vector-meson contribution, these parameters can be accurately determined from the electromagnetic radii of the proton and neutron. However, the inclusion of the vector meson introduces additional free parameters—nucleon-vector-meson strong coupling constants $g_{\rho p}$, $g_{\rho n}$, $g_{\omega p}$, and $g_{\omega n}$ and the vector-meson dipole cutoff mass Λ_ρ . To constrain the number of parameters, we adopt the latest experimental values of the nucleon electromagnetic radii $\langle r_E^2 \rangle^p = 0.8409^2$ (fm²), $\langle r_M^2 \rangle^p = 0.851^2$ (fm²), $\langle r_E^2 \rangle^n = -0.1155$ (fm²), and $\langle r_M^2 \rangle^n = 0.864^2$ (fm²) [51]. Nevertheless, the vector-meson coupling constants and the cutoff mass remain unfixed. In fact, they can be fixed from the nucleon-nucleon scattering and dispersion analysis. Unfortunately, the vector-meson coupling constants are poorly constrained across the different fitting procedures and exhibit strong model dependence [47]. For example, the meson-exchange model of the nucleon-nucleon scattering yields $1.8 \leq g_{\rho p} \leq 3.2$, $4.3 \leq g_{\rho n}/g_{\rho p} \leq 6.6$, $8 \leq g_{\omega p} \leq 20$, and $-1 \leq g_{\omega n}/g_{\omega p} \leq 0$, while dispersion analysis yields $g_{\rho p} = 2$, $g_{\rho n}/g_{\rho p} = 6.1$, $g_{\omega p} = 20.9$, and $g_{\omega n}/g_{\omega p} = -0.16$ even though the nonlocal effects were taken into account by including the nonlocal vertex regulator function $\tilde{F}_\phi(k)$ with the cutoff masses $\Lambda_\rho = 1.3$ and 2 GeV [42,44,55,56]. Although we attempt to extract the vector-meson coupling constants from the curvatures (fourth moments) of the electromagnetic form factors, the values of curvature are not accurately determined experimentally (for more details, see Sec. V C).

Therefore, in a straightforward manner, we fit them to the Q^2 -dependence of the nucleon electromagnetic form factors, such as dipole and Galster-type forms, rather than directly to the experimental data, since the parametrizations provide a smooth and analytic representation of the data and reduce the sensitivity to individual data fluctuations. Generally, the nucleon electromagnetic form factors can be parametrized in dipole and Galster-type forms as [57,58]

$$G_i^j(Q^2) = \frac{G_i^j(0)}{(1 + Q^2/M_{ij}^2)^2},$$

$$G_n^E(Q^2) = \frac{1}{(1 + Q^2/A)^2} \frac{B\tau}{1 + C\tau}, \quad (22)$$

where $i = E, M$ and $j = p, n$, M_{ij}^2 denotes the cutoff masses of nucleon electromagnetic form factors, and $\tau = \frac{Q^2}{4m^2}$ is the dimensionless variable, with the fitted parameters $A = (0.505 \pm 0.079)$ GeV², $B = 1.655 \pm 0.126$, and $C = 0.909 \pm 0.583$. In the traditional dipole parametrization, a universal cutoff mass $M_{ij}^2 = 0.71$ GeV² is used. However, such parametrization cannot produce accurate nucleon electromagnetic radii. To improve the description,

TABLE I. The cutoff masses of the nucleon electromagnetic form factors are fitted to the experimental data of the nucleon electromagnetic radii.

	G_E^p	G_M^p	G_E^n	G_M^n
EM radii $\langle r^2 \rangle_i$ (fm ²) [51]	0.8409 ²	0.851 ²	-0.110	0.864 ²
cut-off mass M_{ij}^2 (GeV ²)	0.813 ²	0.803 ²	...	0.791 ²

we refit them to the latest experimental data for the nucleon electromagnetic charge radii. The fitted values of cutoff mass M_{ij}^2 are listed in Table I. As expected, these values are different from the conventional cutoff mass $M_{ij}^2 = 0.71$ GeV². With these updated cutoff masses, and using the dipole and Galster-type parametrization in Eq. (22) as a data generator, we fit the vector-meson-nucleon coupling constants $g_{e\rho}$, $g_{e\omega}$, $g_{m\rho}$, and $g_{m\omega}$ and the vector-meson cutoff mass Λ_ρ to $G(Q^2)$ by performing χ^2 analysis.

The least-squares fit analysis yields $g_{e\rho} = 5.241$, $g_{e\omega} = 25.197$, $g_{m\rho}/g_{e\rho} = 6.183$, $g_{m\omega}/g_{e\omega} = 0.055$, and $\Lambda_\rho = 1.79$ GeV. Notably, the fitted values for the vector-to-tensor coupling constant ratios $g_{m\rho}/g_{e\rho}$ and $g_{m\omega}/g_{e\omega}$ as well as Λ_ρ are almost consistent with the ranges extracted from nucleon-nucleon scattering. In contrast, the vector coupling constants $g_{e\rho}$ and $g_{e\omega}$ are slightly larger than the

upper limits of the nucleon-nucleon scattering data ranges. This discrepancy indicates that the nucleon electromagnetic form factors may be affected by the massive vector mesons and complicated off-shell background effects. Correspondingly, $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ chiral low-energy coupling constants are obtained as $d_6 = -0.247$ GeV⁻², $d_7 = -0.092$ GeV⁻², $e_{54} = 0.041$ GeV⁻³, and $e_{74} = 0.438$ GeV⁻³. Clearly, these values totally differ in magnitude and sign from those obtained with the IR and EOMS renormalization methods [8,10]. This reflects the fact that chiral low-energy coupling constants are renormalization-scheme dependent.

B. Q^2 -dependence of nucleon electromagnetic form factors

With these best-fit coupling constants and cutoff masses, we proceed to compute the nucleon electromagnetic form factors. The nonlocal model predictions for the Q^2 -dependence of the electromagnetic form factors in the region $0 \leq Q^2 \leq 1$ GeV² are displayed in Fig. 2. The correspondence between the symbols and the data is given in the caption of Fig. 2. For the proton, the experimental data are taken from Refs. [59,60], while the lattice data are taken from Ref. [2]. Clearly, the model results for the proton electric form factor are in good agreement with the experimental data for $Q^2 < 0.8$ GeV²

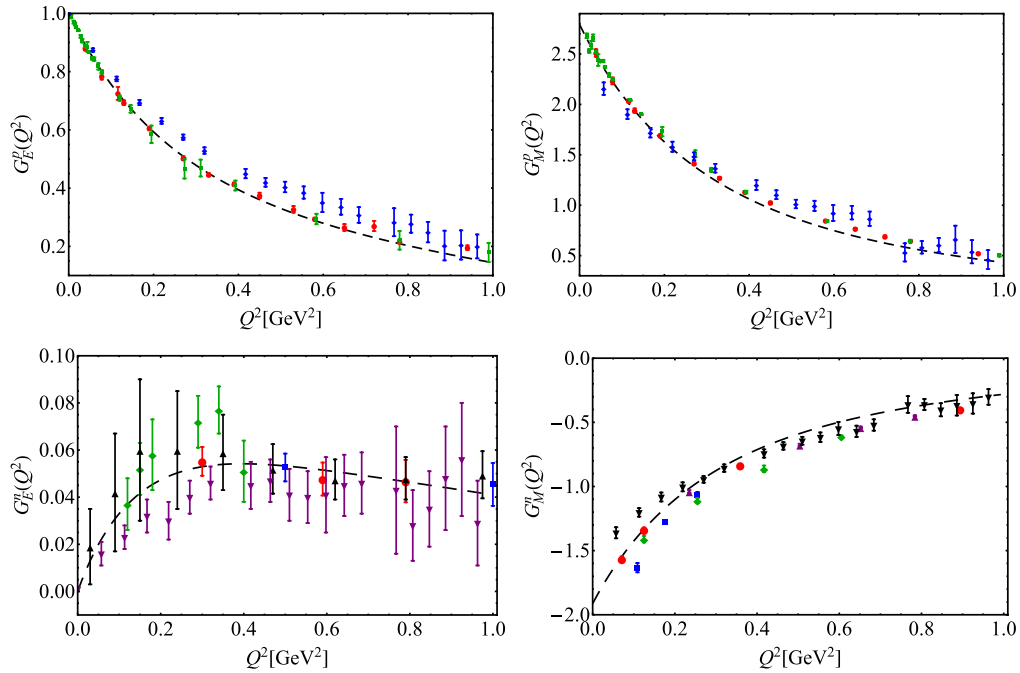


FIG. 2. The model results (short-dashed lines) for the proton (upper panel) and neutron (lower panels) electromagnetic form factors are compared with experimental and lattice data. The symbols in the figures correspond to the following references. For the proton electromagnetic form factors (upper panel): red solid circles [59], green squares [60], and blue diamonds [2]. For the neutron electric form factor (lower left panel): red solid circles [61], blue squares [62], green diamonds [63], black triangles [64], and purple inverted triangles [2]. For the neutron magnetic form factor (lower right panel): red solid circles [65], blue squares [66], green diamonds [67], black triangles [68], and purple inverted triangles [2].

but lie below the central values of the lattice data. In the large- Q^2 region, the theoretical result decreases more rapidly than the experimental data and is slightly lower than their central values. Nevertheless, it remains consistent with the lattice data within their error bars. For the proton magnetic form factor, the model result agrees with both the experimental and lattice data for $Q^2 < 0.4 \text{ GeV}^2$. However, at larger Q^2 , it deviates from the experimental values and lies below them.

For the neutron electric form factor, the experimental and lattice data are taken from Refs. [61–64] and Ref. [2], respectively. These data exhibit relatively large uncertainties, which is a typical feature of neutron electric form factor measurements. Our model prediction describes the overall trend of the data well, within the uncertainties. As for the neutron magnetic form factor, the experimental data are taken from Refs. [65–68], and the lattice data are taken from Ref. [2]. In the 4-momentum transfer region $0 \leq Q^2 \leq 1 \text{ GeV}^2$, the model prediction yields slightly larger values than the central values of the experimental data. A direct comparison with the lattice data is less straightforward because of their relatively large fluctuations. Overall, the Q^2 -dependence of the nucleon electromagnetic form factors in the large- Q^2 region is significantly improved and generally consistent with the experimental and lattice data. This is an important feature of the nonlocal formalism, since the higher-order contributions that govern the high-energy (short-distance) behavior of chiral loops in local perturbation theory are suppressed by the nonlocal regulator. Similar conclusions were reached in earlier analyses of pion and nucleon electromagnetic form factors within the nonlocal quark model [53,54,69–71].

Similarly, as shown in Fig. 3, the model predictions for the proton and neutron electromagnetic form factor ratios are compared with experimental and lattice data, with the proton data taken from Refs. [59,72–75] and the neutron data taken from Refs. [2,62,76–78]. The proton electromagnetic form factor ratio lies above the central values of

the experimental data but remains consistent with the lattice data within their error bars. The neutron electromagnetic form factor ratio likewise lies above the central values of the experimental data and tends to increase more rapidly. Overall, the model results for the proton and neutron form factor ratios are broadly consistent with the data over a wide range, $0 \leq Q^2 \leq 1 \text{ GeV}^2$. The remaining deviations at larger Q^2 may originate from higher-order chiral contributions and from differences in the extraction methods used in the experimental analyses.

C. Curvatures of nucleon electromagnetic form factors

As shown in Eq. (5), the fourth moments of the nucleon electromagnetic form factors $\langle r^4 \rangle$ describe the curvature of $G(Q^2)$ at $Q^2 = 0$. Mathematically, any perturbative variation of the $\langle r^4 \rangle$ will cause a drastic change of the Q^2 -distributions of the electromagnetic form factors. More importantly, this also leads to a wide range of uncertainty in the electromagnetic radii since the electron-nucleon scattering cannot reach $Q^2 = 0$ [35,79,80]. To handle this issue, it is necessary to accurately fix the $\langle r^4 \rangle$ as well as other higher order moments of form factors.

However, as listed in Table II, the curvatures of nucleon electromagnetic form factors explicitly depend on the functional form of $G(Q^2)$ and have a wide uncertainty range across the different parameterizations. For example, the values $\langle r^4 \rangle$ from the standard dipole-Galster parameterization in Eq. (22) are smaller than those from the polynomial and rational function parametrization [34,81–83]. In fact, the dipole type form of $G(Q^2)$ decreases more quickly than the experimental data [84]. Thus, the curvatures obtained from both dipole and nonlocal methods are close to each other since the higher-energy contributors of meson loops is suppressed due to the nonlocal regulator. This further highlights the model dependence of the extraction procedure.

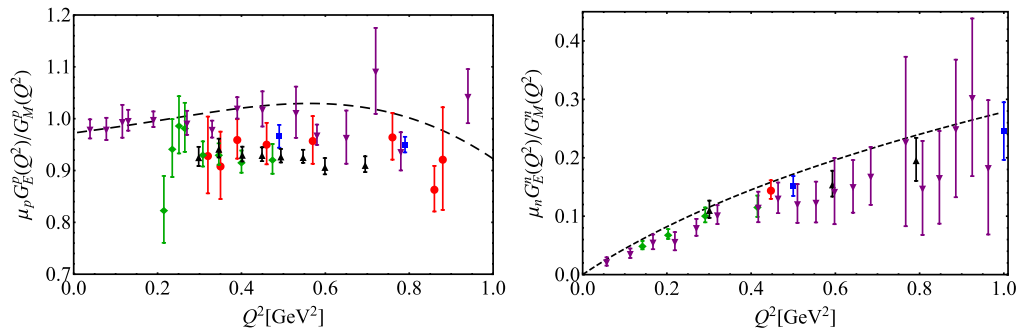


FIG. 3. The model results (short-dashed lines) for the ratio of the proton (left panel) and neutron (right panel) electromagnetic form factors are compared with experimental and lattice data. The symbols in the figures correspond to the following references. For the proton (left panel): red solid circles [72], blue squares [73], green diamonds [74], black triangles [75], and purple inverted triangles [59]. For the neutron (right panel): red solid circles [76], blue squares [62], green diamonds [77], black triangles [78], and purple inverted triangles [2].

TABLE II. The curvatures of the proton and neutron electromagnetic form factors from different parametrization groups.

Fit	$\langle r^4 \rangle_E^p$ (fm ⁴)	$\langle r^4 \rangle_M^p$ [fm ⁴]	$\langle r^4 \rangle_E^n$ (fm ⁴)	$\langle r^4 \rangle_M^n$ (fm ⁴)
Ref. [34]	1.08 ± 0.284	-2.0 ± 1.88	-0.33 ± 0.242	-2.3 ± 2.37
Ref. [81]	1.621 ± 0.064	1.471 ± 0.034	-0.330 ± 0.016	2.324 ± 0.748
Ref. [82]	1.620 ± 0.050	1.720 ± 0.025	-0.330 ± 0.140	2.920 ± 2.600
Ref. [83]	1.536 ± 0.065	1.729 ± 0.053	-0.571 ± 0.065	2.044 ± 0.002
Dipole and Galster	1.250	1.311	-0.361	1.393
This work	1.331	1.380	-0.422	1.516

D. Contributions from different chiral orders

In Fig. 4, we display the nucleon electromagnetic form factors at different chiral orders in the region $0 \leq Q^2 \leq 1 \text{ GeV}^2$. It is obvious that the $\mathcal{O}(p)$ -, $\mathcal{O}(p^2)$ -, and $\mathcal{O}(p^3)$ -order pionic contributions exhibit approximately linear behavior in Q^2 in this region at all chiral orders. In particular, the $\mathcal{O}(p)$ contributions depend weakly on Q^2 . These results are consistent with the conclusions from the local chiral perturbation theory [8,9] and imply that higher-chiral-order and vector-meson contributions are necessary for describing the electromagnetic radii and curvatures.

It is worthwhile to mention that at $\mathcal{O}(p^3)$ the proton and neutron Pauli form factors are equal in magnitude but opposite in sign. Consequently, the magnetic form factors vanish at this order. Nevertheless, the nucleon electric form factors acquire small but nonzero contributions. The inclusion of the vector-meson contribution, however,

significantly modifies the curvature of the form factors due to the vector-meson pole dominance. Consequently, the total electromagnetic form factors are reshaped and lead to better agreement with experimental data.

In Table III, we list the contributions of the different chiral orders to the electromagnetic radii. Note that the pion $\mathcal{O}(p)$, $\mathcal{O}(p^2)$, and $\mathcal{O}(p^3)$ contributions to the proton charge radius are 0.088, 0.110, and 0.068 fm², respectively, yielding a total pionic contribution is 0.266 fm². In addition, the vector-meson contribution is 0.441 fm², which accounts for about 62.4% of the proton charge radius. This result indicates that vector-meson contributions are important component of the nucleon electromagnetic structure. For the proton magnetic radius, the total pionic contribution is 0.154 fm², while the vector-meson contribution is 0.57 fm², which corresponds to about 78.6% of the total magnetic radius. These conclusions agree with the results from local chiral perturbation

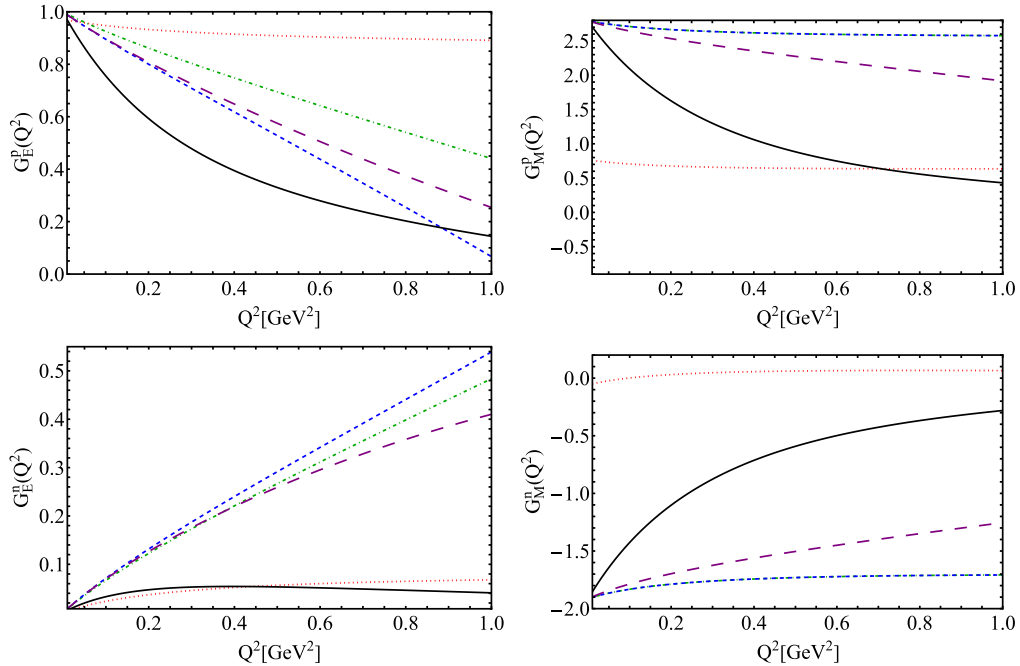


FIG. 4. The Q^2 -dependence of the nucleon electromagnetic form factors at $\mathcal{O}(p)$ (red dotted curve), $\mathcal{O}(p^2)$ (green dot-dashed curve), $\mathcal{O}(p^3)$ (blue short dashed curve), and $\mathcal{O}(p^4)$ (purple long dashed curves) and for the result with vector mesons (black solid curve).

TABLE III. Contribution of different chiral orders to the nucleon electromagnetic radii.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$	Vector meson	Total
$\langle r^2 \rangle_E^p$ (fm ²)	0.088	0.110	0.068	0	0.441	0.707
$\langle r^2 \rangle_M^p$ (fm ²)	0.101	-0.001	0	0.055	0.569	0.724
$\langle r^2 \rangle_E^n$ (fm ²)	-0.086	-0.094	-0.010	0	0.074	-0.116
$\langle r^2 \rangle_M^n$ (fm ²)	0.146	-0.001	0	0.055	0.547	0.747

theory [8]. In the neutron case, the vector-meson contribution partially cancels the negative pionic contributions to the neutron electric charge radius, resulting in the total value -0.115 fm². Conversely, the vector-meson contribution is responsible for approximately 73.2% of the total neutron magnetic radius. This confirms again the non-negligible role of vector mesons in shaping the neutron magnetic structures.

VI. CONCLUSIONS

In this work, to cure the UV divergences of pion loop integrals, we first introduce nonlocal meson fields and construct the nonlocal chiral Lagrangians for the pion-nucleon and vector-meson–nucleon interactions. Within this framework, we then analytically derive the pion one-loop corrections to the nucleon electromagnetic form factors up to $\mathcal{O}(p^4)$ order. Because of the nonlocality, the loop corrections contain a regulator function $\tilde{F}_\phi^2(k)$, which modifies the superficial degree of divergence of the loop integrals. As a result, the loop integrals are UV convergent with a cutoff mass Λ_ϕ .

Meanwhile, because the chiral low-energy coupling constants depend on the renormalization scheme, the low-energy coupling constants for the pion-nucleon and vector-meson–nucleon interaction are fitted to the Q^2 -dependence of the nucleon electromagnetic form factors within the nonlocal framework. The least-squares fit analysis yields the vector-meson coupling constants and the vector-meson cutoff mass as $g_{\pi p} = 5.241$, $g_{\pi n} = 25.197$, $g_{\rho p}/g_{\pi p} = 6.183$, $g_{\rho n}/g_{\pi n} = 0.055$, and $\Lambda_\rho = 1.79$ GeV. By comparing, we find that the fitted values for the vector-to-tensor coupling constant ratios $g_{\rho p}/g_{\pi p}$ and $g_{\rho n}/g_{\pi n}$ as well as the Λ_ρ are consistent with the ranges extracted from nucleon-nucleon scattering, while the vector coupling constants $g_{\rho p}$ and $g_{\rho n}$ are larger than the upper limits of those ranges. Correspondingly, the $\mathcal{O}(p^2)$, $\mathcal{O}(p^3)$, and $\mathcal{O}(p^4)$ chiral low-energy coupling constants are fixed as $c_6 = 6.263$, $c_7 = -2.979$, $d_6 = -0.247$ GeV⁻², $d_7 = -0.092$ GeV⁻², $e_{54} = 0.041$ GeV⁻³, and $e_{74} = 0.438$ GeV⁻³, respectively. We find that the third- and fourth-order LECs are totally different from those in the local case, whereas the absolute values of the second-order low-energy coupling constants are significantly larger than the local values.

With these fitted LECs and cutoff masses, we compute the Q^2 -dependence of the nucleon electromagnetic form factors in the $0 \leq Q^2 \leq 1$ GeV² region. The model results show that the nonlocal method greatly improves the Q^2 -dependence of the nucleon electromagnetic form factors at the large- Q^2 region. In particular, the vector-meson contributions are essential for describing the electromagnetic properties in the large- Q^2 region. Moreover, the model calculation yields the fourth moments of the nucleon electromagnetic form factors as $\langle r^4 \rangle_E^p = 1.331$ (fm⁴), $\langle r^4 \rangle_M^p = 1.380$ (fm⁴), $\langle r^4 \rangle_E^n = -0.422$ (fm⁴), $\langle r^4 \rangle_M^n = 1.516$ (fm⁴), which are smaller than the central values of the experimental data. We therefore expect that these results for the fourth moments will provide a prediction for future experimental observations.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: VERTEX FUNCTIONS FROM INDIVIDUAL FEYNMAN DIAGRAMS

1. $\mathcal{O}(p)$ contributions

$$\Gamma_a^\mu(p, q) = \frac{1 + \tau^3}{2} \bar{u}(p') \gamma^\mu u(p), \quad (\text{A1})$$

where the isospin index τ^3 takes $\tau^3 = \pm 1$ for the proton and neutron.

$$\begin{aligned} \Gamma_b^\mu(p, q) = & \frac{-(3 - \tau^3)g_A^2}{8f^2} \bar{u}(p') \int \frac{d^4k}{(2\pi)^4} (-ik\gamma^5) \frac{i}{D_N(p' - k)} \\ & \times \gamma^\mu \frac{i}{D_N(p - k)} (ik\gamma^5) \frac{i}{D_\pi(k)} u(p) \tilde{F}_\pi^2(k), \end{aligned} \quad (\text{A2})$$

where $D_N(k) = k - m_N + i\epsilon$, $D_\pi(k) = k^2 - m_\pi^2 + i\epsilon$, $\tilde{F}_\pi(k)$ is the Fourier transform of the nonlocal regulator function $\tilde{F}_\pi(k) = \int^4 \frac{d^4a}{(2\pi)^4} F_\pi(a) e^{-ik \cdot a}$.

$$\begin{aligned} \Gamma_c^\mu(p, q) = & \frac{g_A^2}{2f^2} \bar{u}(p') \int \frac{d^4k}{(2\pi)^4} [-i(k + q)\gamma^5] \frac{i}{D_N(p - k)} \\ & \times \frac{i}{D_\pi(k + q)} (2k + q)^\mu \frac{i}{D_\pi(k)} (ik\gamma^5) u(p) \\ & \times \tilde{F}_\pi(k) \tilde{F}_\pi(k + q). \end{aligned} \quad (\text{A3})$$

$$\begin{aligned}\Gamma_{d+e}^\mu(p, q) = & \frac{-\tau^3 g_A^2}{2f^2} \bar{u}(p') \int \frac{d^4 k}{(2\pi)^4} \left[(-ik\gamma^5) \frac{i}{D_N(p'-k)} \right. \\ & \times \frac{i}{D_\pi(k)} \gamma^\mu \gamma^5 \tilde{F}_\pi(k-q) \tilde{F}_\pi(k) + \gamma^\mu \gamma^5 \frac{i}{D_N(p-k)} \\ & \left. \times \frac{i}{D_\pi(k)} (-ik\gamma^5) \tilde{F}_\pi(k+q) \tilde{F}_\pi(k) \right] u(p). \quad (\text{A4})\end{aligned}$$

$$\begin{aligned}\Gamma_{f+g}^\mu(p, q) = & \tau^3 \frac{g_A^2}{2f^2} \int_0^1 dt \int \frac{d^4 k}{(2\pi)^4} \bar{u}(p') \left\{ -(-ik\gamma^5) \right. \\ & \times \frac{i}{D_N(p'-k)} \frac{i}{D_\pi(k)} (k\gamma^5) \frac{\partial \tilde{F}_\pi(k-qt)}{\partial k_\mu} \tilde{F}_\pi(k) \\ & + (k\gamma^5) \frac{\partial \tilde{F}_\pi(k+qt)}{\partial k_\mu} \tilde{F}_\pi(k) \\ & \left. \times \frac{i}{D_N(p-k)} \frac{i}{D_\pi(p-k)} (-ik\gamma^5) \right\} u(p). \quad (\text{A5})\end{aligned}$$

$$\begin{aligned}\Gamma_h^\mu(p, q) = & \frac{-\tau^3}{4f^2} \bar{u}(p') \int \frac{d^4 k}{(2\pi)^4} \frac{i}{D_\pi(k)} \gamma^\mu u(p) [\tilde{F}_\pi(k-q) \\ & + \tilde{F}_\pi(k+q)] \tilde{F}_\pi(k). \quad (\text{A6})\end{aligned}$$

$$\begin{aligned}\Gamma_i^\mu(p, q) = & \frac{-i\tau^3}{4f^2} \bar{u}(p') \int \frac{d^4 k}{(2\pi)^4} \frac{i}{D_\pi(k+q)} \frac{i}{D_\pi(k)} \\ & \times (\not{q} + 2k)(2k+q)^\mu u(p) \tilde{F}_\pi(k+q) \tilde{F}_\pi(k). \quad (\text{A7})\end{aligned}$$

$$\begin{aligned}\Gamma_j^\mu(p, q) = & \frac{i\tau^3}{4f^2} \bar{u}(p') \int_0^1 dt \int \frac{d^4 k}{(2\pi)^4} \frac{i}{D_\pi(k)} 2ik \left\{ \frac{\partial \tilde{F}_\pi(k+ tq)}{\partial k_\mu} \right. \\ & \left. \times \tilde{F}_\pi(k) + \frac{\partial \tilde{F}_\pi(k- tq)}{\partial k_\mu} \tilde{F}_\pi(k) \right\} u(p). \quad (\text{A8})\end{aligned}$$

2. $\mathcal{O}(p^2)$ contributions

$$\Gamma_k^\mu(p, q) = \left[\frac{(1+\tau^3)}{2} c_6 + c_7 \right] \bar{u}(p') \frac{i\sigma^{\mu\nu} q^\nu}{2m} u(p). \quad (\text{A9})$$

$$\begin{aligned}\Gamma_l^\mu(p, q) = & \frac{1}{f^2} \bar{u}(p') \int \frac{d^4 k}{(2\pi)^4} \frac{i}{D_\pi(k)} \left[-\tau^3 \frac{c_6}{2} \frac{i\sigma^{\nu\mu} q^\nu}{2m} \right. \\ & \left. + 3(\tau^3 + 1) \frac{c_2}{2m^2} (p \cdot k + p' \cdot k) k_\mu \right] u(p) \tilde{F}_\pi^2(k). \quad (\text{A10})\end{aligned}$$

$$\begin{aligned}\Gamma_m^\mu(p, q) = & \frac{i\tau^3}{f^2} \bar{u}(p') \int \frac{d^4 k}{(2\pi)^4} \frac{i}{D_\pi(k+q)} \frac{i}{D_\pi(k)} \\ & \times \left\{ \frac{c_2}{m^2} [(p \cdot k)(p \cdot k') + (p' \cdot k)(p' \cdot k')] \right. \\ & \left. + c_4 [k, k'] \right\} (2k+q)^\mu \\ & \times \tilde{F}_\pi(k+q) \tilde{F}_\pi(k) u(p). \quad (\text{A11})\end{aligned}$$

$$\begin{aligned}\Gamma_n^\mu(p, q) = & \frac{g_A^2 [c_6(3-\tau^3) + 6c_7]}{32f^2} \bar{u}(p) \int \frac{d^4 k}{(2\pi)^4} (-ik\gamma^5) \\ & \times \frac{i}{D_N(p'-k)} (-i2q^\nu \sigma^{\nu\mu}) \frac{i}{D_N(p-k)} (ik\gamma^5) \\ & \times \frac{i}{D_\pi(k)} u(p) \tilde{F}_\pi^2(k). \quad (\text{A12})\end{aligned}$$

3. $\mathcal{O}(p^3)$ contribution

$$\Gamma_o^\mu(p, q) = -t(\tau^3 d_6 + 2d_7) u(p') \frac{(p+p')_\mu}{2m} u(p). \quad (\text{A13})$$

4. $\mathcal{O}(p^4)$ contribution

$$\Gamma_p^\mu(p, q) = 2m_N t(\tau^3 e_{54} + 2e_{74}) \bar{u}(p') \frac{i\sigma^{\mu\nu} q^\nu}{2m} u(p).$$

5. Vector-meson contribution

$$\begin{aligned}\Gamma_q^\mu(p, q) = & - \left[\tau^3 \frac{q^2 F_\rho g_{\rho\pi}}{q^2 - m_V^2} \tilde{F}_\rho^2(k) + \frac{q^2 F_\omega g_{\omega\pi}}{(q^2 - m_V^2)} \tilde{F}_\omega^2(k) \right] \\ & \times \bar{u}(p') \gamma^\mu u(p) \\ & - \left[\tau^3 \frac{q^2 F_\rho g_{\rho\pi}}{q^2 - m_V^2} \tilde{F}_\rho^2(k) + \frac{q^2 F_\omega g_{\omega\pi}}{(q^2 - m_V^2)} \tilde{F}_\omega^2(k) \right] \\ & \times \frac{i\sigma^{\mu\nu} q^\nu}{2m} u(p). \quad (\text{A14})\end{aligned}$$

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