Lorentz violating quadratic gravity

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In this paper, we explore the perturbative renormalization and study the classical dynamics of the bumblebee model coupled to quadratic gravity, a theoretical setting that allows the violation of Lorentz symmetry. Such a violation arises from a vector field whose potential is engineered to induce a nonzero vacuum expectation value, thereby leading to the emergence of a preferred direction in spacetime and, consequently, to the spontaneous breaking of Lorentz symmetry. Working in dimensional regularization and expanding the metric around flat space, we compute the one-loop divergent parts of the two-point functions of the bumblebee and graviton fields, with special emphasis on the role of Lorentz violating insertions in internal lines. These results determine the counterterms required to renormalize the gravitational and bumblebee sectors in the presence of a preferred background direction, and make explicit how Lorentz violating interactions feed back into the UV structure of quadratic gravity. On the classical side, we derive the field equations and identify exact solutions supported by bumblebee backgrounds. In particular, we show that the Schwarzschild geometry remains an exact solution for an appropriate bumblebee configuration, even in the presence of nonminimal couplings. We close with a discussion of the operator content suggested by the one-loop structure and of prospective extensions to cosmological and less symmetric backgrounds.

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I. INTRODUCTION

The idea that gravity could be consistently formulated as a quantum field theory has been explored from several complementary perspectives. One line of investigation has focused on constructing renormalizable extensions of general relativity by introducing curvature-squared terms into the action [1–7]. Such higher-derivative formulations are known to improve the UV behavior of the theory and have been repeatedly proposed as potential frameworks for quantum gravity (see also reviews [8,9]). Nevertheless, Einstein's original theory without these modifications remains nonrenormalizable [10–12], and its predictive power is restricted to that one of an effective field theory, valid below the Planck scale [13–15].

At the same time, higher-derivative theories are not free from conceptual difficulties. Their particle spectra often include nonphysical states, such as ghosts or tachyons, which threaten unitarity or stability. A variety of studies have addressed these issues from different angles [9,16–20], suggesting that while ghostlike modes might be tamed within certain formulations [5], tachyonic instabilities usually signal more severe inconsistencies. Even so, as emphasized by Donoghue and Menezes [9], there is no definitive consensus regarding the presence or absence of such pathologies, leaving the fundamental status of quadratic gravity an open question that requires further theoretical clarification.

A particularly intriguing scenario emerges when quadratic curvature invariants are treated as the sole building blocks of the gravitational sector [21–23]. In this context, the absence of a fundamental mass scale leads to a renormalizable theory characterized by a graviton kinetic term with four derivatives and a propagator scaling as $1/p^4$ [24], the so-called agravity. A notable outcome of this framework is the potential dynamical generation of the Planck scale at the quantum level [5,25]. This occurs through a nonminimal coupling between a scalar field

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and gravity, $\xi\phi^2 R$, which induces a nonzero vacuum expectation value (VEV) $\langle\phi\rangle \neq 0$ via radiative corrections, thereby setting the gravitational scale.

A compelling line of research in modified theories of gravity concerns the extension of general relativity to scenarios in which Lorentz symmetry is not preserved. As first emphasized in Ref. [26], the breaking of Lorentz invariance may naturally emerge from a spontaneous symmetry breaking mechanism operating in the low-energy limit of a more fundamental framework, the string theory being a prominent example. Within this context, nontrivial vacuum configurations of vector or tensor fields can arise as the minima of a potential, thereby providing a dynamical origin for Lorentz violating (LV) background fields. The spontaneous realization of Lorentz symmetry violation (LSV) in curved spacetimes is particularly attractive: not only does it furnish a consistent theoretical mechanism, but it also removes the usual constraint that LV vectors or tensors must be constant, a condition often imposed in flat spacetime treatments but creating fundamental difficulties in curved spacetimes (see the discussion in Ref. [27] and references therein).

One of the most widely studied realizations of this idea is the so-called bumblebee model [27–29], in which a vector field acquires a nonzero vacuum expectation value due to the presence of a potential term, while its dynamics are governed by a Maxwell-type kinetic structure. When coupled to gravity—possibly together with other matter fields—the resulting theory is known as bumblebee gravity. This framework has been extensively investigated in a variety of settings, including the analysis of modified black hole solutions [30–33], cosmological dynamics [34,35], Gödel and Gödel-type universes [36,37], and even wormhole geometries [38]. Linearized versions of the theory have also been used to study modified dispersion relations [39]. Beyond classical solutions, perturbative aspects of bumblebee gravity have also been explored, with explicit computations of quantum corrections performed in the metric-affine formalism [40–43]. These results provide further insights into the structure and consistency of LV gravitational dynamics.

Given that the bumblebee field provides a natural framework for introducing LSV into gravitational models, it is a natural progression to extend the scale-invariant constructions discussed in Refs. [5,6] by incorporating operators built from the bumblebee sector. A particularly noteworthy development in this direction was presented in Ref. [44], where it was demonstrated that coupling the bumblebee field to agravity can trigger the Coleman-Weinberg mechanism [45], leading to dynamical Lorentz symmetry breaking within a scale-free framework.

It is important to emphasize that the present work is a different extension of the bumblebee-gravity framework. In Ref. [44], the gravitational sector was restricted to the scale-free agravity regime, involving only marginal curvature-squared operators, and Lorentz symmetry breaking was generated dynamically through radiative corrections via the Coleman-Weinberg mechanism. In contrast, the theory considered here also contains relevant gravitational operators, in particular the Einstein-Hilbert term, in addition to the marginal quadratic-curvature terms. As a consequence, the graviton propagator displays a richer pole structure, including the massless spin-2 pole together with the massive spin-2 and spin-0 poles characteristic of quadratic gravity. Moreover, in the bumblebee sector, Lorentz symmetry is already spontaneously broken at tree level by the choice of a potential whose minimum occurs at $\mathcal{B}_\mu\mathcal{B}^\mu = \pm b^2$. Therefore, while the two models share the same general motivation of coupling a bumblebee field to a renormalizable gravitational framework, the present analysis addresses a different perturbative regime, with a different propagator structure, a different pattern of Lorentz symmetry breaking, and a different set of counterterms required for the renormalization of the effective theory.

One further motivation for studying this setup is the existence of nontrivial *exact* solutions in bumblebee gravity. It is therefore natural to ask which geometries continue to be valid once the bumblebee sector is embedded into quadratic gravity and, in particular, how the solution space is affected by the nonminimal couplings introduced in Sec. II. This extends earlier investigations of exact solutions in related LV gravitational models [30,31,36,40,41,46] to the present renormalizable framework.

Guided by the motivations discussed above, in this paper we carry out a twofold analysis of the bumblebee extension of quadratic gravity. First, we establish the perturbative framework by expanding around flat spacetime and determining the propagators and interaction vertices in the presence of the leading nonminimal couplings. We then compute the one-loop divergent parts of the bumblebee and graviton two-point functions, identifying the LV structures induced by gravitational fluctuations and extracting the counterterms required for renormalization when a preferred direction b_μ is present. Second, we address the classical sector by deriving the field equations and searching for exact solutions supported by bumblebee backgrounds; in particular, we show that the Schwarzschild geometry remains an exact solution for a suitable bumblebee configuration even at nonzero nonminimal couplings.

The structure of the paper is as follows: in Sec. II, we introduce the Lagrangian of our theory and write down the propagators and the relevant vertices. In Sec. III, we calculate the radiative corrections to two-point functions of the bumblebee and graviton fields, and in Sec. IV, we discuss exact solutions in our model. In Sec. V, we summarize our results.

Throughout this paper, we use natural units $c = \hbar = 1$.

II. THE BUMBLEBEE-QUADRATIC GRAVITY LAGRANGIAN

The bumblebee model was proposed to investigate the possible spontaneous violation of Lorentz symmetry [28].

It is defined by the action

$$S_B = \int d^4x \left\{ -\frac{1}{4} \mathcal{B}^{\mu\nu} \mathcal{B}_{\mu\nu} - V(\mathcal{B}^\mu \mathcal{B}_\mu \mp b^2) \right\}, \quad (1)$$

where $\mathcal{B}_{\mu\nu} = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu$, and the potential $V(\mathcal{B}^\mu \mathcal{B}_\mu \mp b^2)$ is chosen to induce a nonzero VEV for the bumblebee field. This leads to the emergence of a preferred direction in spacetime, resulting in spontaneous Lorentz symmetry breaking. It is worth mentioning that, first, the Maxwell-like kinetic term is necessary to avoid propagating ghost modes [47]; second, the bumblebee model is characterized by the presence of a potential displaying an infinite set of minima, being a consistent alternative to Einstein-aether gravity [48] where the Lorentz symmetry breaking is introduced through the constraint instead of the potential term.

A common choice for the potential is $V = \lambda(\mathcal{B}^\mu \mathcal{B}_\mu \mp b^2)^2$, where the sign $\mp$ allows for both spacelike and timelike configurations of $\mathcal{B}^\mu$, with $b^2 > 0$. The potential is responsible for Lorentz symmetry breaking, and its minimum occurs when $b^\mu b_\mu = \pm b^2$, implying that the VEV of the bumblebee field is given by $\langle \mathcal{B}_\mu \rangle = b_\mu$.

The coupling of the bumblebee field to the gravitational field, including quadratic curvature terms, is described by the action

$$\begin{aligned} \mathcal{S} = \int d^4x \sqrt{-g} \Big\{ & \beta R^2 - \alpha R^{\mu\nu} R_{\mu\nu} - \frac{\gamma}{\kappa^2} R - \frac{1}{4} \mathcal{B}^{\mu\nu} \mathcal{B}_{\mu\nu} \\ & - \frac{1}{2g_l} (\nabla^\mu \mathcal{B}_\mu)^2 - \lambda(\mathcal{B}^\mu \mathcal{B}_\mu - b^2)^2 \\ & + \xi_1 \left(\mathcal{B}^\mu \mathcal{B}^\nu - \frac{g^{\mu\nu}}{4} \mathcal{B}^2 \right) R_{\mu\nu} + \xi_2 \mathcal{B}^2 R \\ & + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}} + \mathcal{L}_{\mathrm{CT}} \Big\}, \end{aligned} \quad (2)$$

where R denotes the Ricci curvature scalar, $R_{\mu\nu}$ represents the Ricci tensor, and ∇_μ is the covariant derivative. The parameters α, β and γ characterize the gravitational sector of the model. The constants ξ_1 and ξ_2 are the coupling coefficients associated with the traceless and trace components of the interaction between the bumblebee field and the gravitational field, respectively. Furthermore, $\mathcal{L}_{\mathrm{GF}}$ and $\mathcal{L}_{\mathrm{FP}}$ denote the gauge-fixing term and the corresponding Faddeev-Popov ghosts Lagrangian of the gravitational sector, respectively, while $\mathcal{L}_{\mathrm{CT}}$ represents the Lagrangian of counterterms.

Let us consider a perturbative expansion of the metric tensor around the flat Minkowski background, given by

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad (3)$$

where $\eta_{\mu\nu}$ denotes the Minkowski metric and $h_{\mu\nu}$ represents the perturbation, assumed to be small.

At the same time, the full bumblebee field is expanded around the LV vacuum as

$$B_\mu = b_\mu + B_\mu, \qquad b_\mu b^\mu = \pm b^2, \quad (4)$$

where B_μ denotes the full field, b_μ is the vacuum expectation value, and B_μ is the quantum fluctuation, with $\langle B_\mu \rangle = 0$. Thus, the fixed vector b_μ is not introduced *a posteriori*, but defines the vacuum about which the perturbative expansion is performed. In the following, the covariant action is written in terms of B_μ, while the weak-field expansion and the loop calculation are expressed in terms of the fluctuation B_μ.

By adopting this approach, one can formulate the tree-level Lagrangian before gauge fixing, represented as

$$\mathcal{L} = \mathcal{L}_h + \mathcal{L}_b + \mathcal{L}_{nm} + \mathcal{L}_V, \quad (5)$$

where $\mathcal{L}_h$ denotes the quadratic kinetic term of the gravitational Lagrangian

$$\begin{aligned} \mathcal{L}_h = & -\gamma \left[h \partial_\mu \partial_\nu h^{\mu\nu} + 2 \partial_\nu h^{\mu\nu} \partial_\rho h_\mu{}^\rho + \frac{1}{4} h \Box h - \partial_\mu h_\nu{}^\rho \partial_\rho h^{\mu\nu} + \partial_\rho h \partial^\nu h_\nu{}^\rho + \frac{1}{2} \partial_\rho \partial_\nu h h^{\nu\rho} + \frac{1}{4} h^{\mu\nu} \Box h_{\mu\nu} - \frac{1}{2} \partial^\rho h_{\mu\rho} \partial_\nu h^{\mu\nu} \right] \\ & + \alpha\kappa^2 \left[\left(\partial^\rho \partial_\mu h^{\mu\nu} \partial_\nu \partial_\rho h + \partial^\rho \partial_\mu h^{\mu\nu} \Box h_{\nu\rho} - \frac{1}{2} (\partial_\mu \partial_\nu h^{\mu\nu})^2 - \frac{1}{2} \partial^\rho \partial_\mu h^{\mu\nu} \partial_\rho \partial_\sigma h_\nu{}^\sigma \right) \right. \\ & \left. - \frac{1}{4} (\Box h)^2 - \frac{1}{4} \Box h^{\mu\nu} \Box h_{\mu\nu} + \beta (\Box h - \partial^\mu \partial_\rho h_\mu{}^\rho)^2 \right] - \frac{\kappa^2}{2\zeta_g} \partial_\mu h^{\rho\mu} \Box \partial^\nu h_{\rho\nu} + \mathcal{O}(h^3), \end{aligned} \quad (6)$$

$\mathcal{L}_{nm}$ comprises terms involving the couplings ξ_1 and ξ_2, as follows:

$$\mathcal{L}_{nm} = \xi_1\kappa\mathcal{B}^\alpha\mathcal{B}^\beta\left(\partial^\gamma\partial_\beta h_{\alpha\gamma} - \partial_\alpha\partial_\beta h - \frac{1}{2}\Box h_{\alpha\beta}\right) + \kappa\left(\xi_2 - \frac{\xi_1}{4}\right)\mathcal{B}^2(\partial^\gamma\partial^\beta h_{\gamma\beta} - \Box h)$$
$$+ \xi_1\kappa^2\mathcal{B}^\alpha\mathcal{B}^\beta\left(\frac{1}{2}\partial^\nu h_\alpha{}^\mu\partial_\nu h_{\beta\mu} - \frac{1}{2}\partial^\nu h_{\alpha\mu}\partial^\mu h_{\beta\nu} + \frac{1}{4}\partial^\alpha h^{\mu\nu}\partial_\beta h_{\mu\nu} + \frac{1}{2}\partial^\nu h_{\alpha\beta}\partial^\mu h_{\nu\mu}\right.$$
$$- \partial_\beta h_\alpha{}^\mu\partial^\nu h_{\mu\nu} - \frac{1}{4}\partial^\nu h_{\alpha\beta}\partial_\nu h + \frac{1}{2}\partial_\beta h_{\alpha\mu}\partial^\mu h + \Box h_\beta{}^\mu h_{\alpha\mu} - \partial^\nu\partial^\mu h_{\beta\mu}h_{\alpha\nu} - \frac{1}{4}\Box h_{\alpha\beta}h$$
$$- \partial_\beta\partial^\nu h_\nu{}^\mu h_{\alpha\mu} + \partial^\mu\partial_\beta hh_{\alpha\mu} + \frac{1}{2}\partial^\nu\partial_\beta h_{\alpha\nu}h - \frac{1}{4}\partial_\alpha\partial_\beta hh + \frac{1}{2}\partial^\nu\partial^\mu h_{\alpha\beta}h_{\mu\nu}$$
$$\left. - \partial^\nu\partial_\beta h_\alpha{}^\mu h_{\mu\nu} + \frac{1}{2}\partial_\beta\partial^\alpha h^{\mu\nu}h_{\mu\nu}\right) + \kappa^2\left(\xi_2 - \frac{\xi_1}{4}\right)\mathcal{B}^\alpha\mathcal{B}^\beta(\Box hh_{\alpha\beta} - \partial^\mu\partial^\nu h_{\mu\nu}h_{\alpha\beta})$$
$$+ \kappa^2\left(\xi_2 - \frac{\xi_1}{4}\right)\mathcal{B}^2\left(\frac{3}{4}\partial^\alpha h^{\beta\mu}\partial_\alpha h_{\beta\mu} - \frac{1}{2}\partial^\alpha h^{\beta\mu}\partial_\mu h_{\beta\alpha} + \partial^\alpha h\partial^\mu h_{\mu\alpha} - \partial^\alpha h^{\alpha\beta}\partial^\mu h_{\beta\mu}\right.$$
$$\left. - \frac{1}{2}\Box hh - \frac{1}{4}\partial^\alpha h\partial_\alpha h + \frac{1}{2}\partial^\alpha\partial^\beta h_{\alpha\beta}h + \Box h^{\mu\nu}h_{\mu\nu} - 2\partial^\alpha\partial_\beta h_{\alpha\mu}h^{\mu\beta} + \partial^\alpha\partial^\beta hh_{\alpha\beta}\right) + \mathcal{O}(h^3), \tag{7}$$

$\mathcal{L}_b$ encompasses contributions of the second order in the bumblebee field, generated by its kinetic term, as follows:

$$\mathcal{L}_b = -\frac{1}{4}\mathcal{B}^{\mu\nu}\mathcal{B}_{\mu\nu} - \frac{1}{2}\partial^\mu\mathcal{B}_\mu\partial^\nu\mathcal{B}_\nu + \frac{\kappa}{2}\mathcal{B}^{\mu\nu}\mathcal{B}^\tau_\mu\left(h_{\nu\tau} - \frac{1}{4}\eta_{\nu\tau}h\right)$$
$$- \frac{\kappa}{4}(\partial^\mu\mathcal{B}_\mu\partial^\nu\mathcal{B}_\nu h + 2\mathcal{B}^\mu\partial^\nu\mathcal{B}_\nu\partial_\mu h - 4\mathcal{B}^\mu\partial^\nu\mathcal{B}_\nu\partial^\alpha h_{\mu\alpha} - 4\partial^\mu\mathcal{B}_\mu\partial^\nu\mathcal{B}^\alpha h_{\nu\alpha})$$
$$- \frac{\kappa^2}{32}(h^2 - 2h^{\mu\nu}h_{\mu\nu})\mathcal{B}^{\alpha\beta}\mathcal{B}_{\alpha\beta} - \frac{\kappa^2}{2}\mathcal{B}^\mu\mathcal{B}^\nu\left(\partial^\alpha h_{\mu\alpha}\partial^\beta h_{\nu\beta} - \partial^\alpha h_{\mu\alpha}\partial^\nu h + \frac{1}{4}\partial_\mu h\partial_\nu h\right)$$
$$- \frac{\kappa^2}{2}\mathcal{B}^\mu\partial^\beta\mathcal{B}_\beta\left(2\partial_\nu h^{\alpha\nu}h_{\mu\alpha} - \partial^\nu hh_{\mu\nu} - \partial^\nu h_{\mu\nu}h + \frac{1}{2}\partial_\mu hh + 2\partial_\alpha h_{\mu\nu}h^{\nu\alpha} - \partial_\mu h^{\alpha\nu}h_{\alpha\nu}\right)$$
$$- \frac{\kappa^2}{2}\partial^\alpha\mathcal{B}_\alpha\left(2\partial^\mu\mathcal{B}_\nu h^{\nu\beta}h_{\mu\beta} - \partial^\mu\mathcal{B}_\nu h^{\mu\nu}h - \frac{1}{4}\partial^\mu\mathcal{B}_\mu h^{\nu\beta}h_{\nu\beta} + \frac{1}{8}\partial^\mu\mathcal{B}_\mu h^2\right)$$
$$- \frac{\kappa^2}{2}\mathcal{B}^\mu\partial^\alpha\mathcal{B}^\nu(2\partial^\beta h_{\mu\beta}h_{\nu\alpha} - \partial_\mu hh_{\nu\alpha}) - \frac{\kappa^2}{2}\partial^\mu\mathcal{B}^\nu\partial^\alpha\mathcal{B}^\beta h_{\mu\nu}h_{\alpha\beta} + \mathcal{O}(h^3), \tag{8}$$

and $\mathcal{L}_V$ stands for the bumblebee potential and its coupling to gravity, as follows:

$$\mathcal{L}_V = -\lambda(\mathcal{B}_\mu\mathcal{B}^\mu - b^2)^2 - \kappa\lambda\left\{2\left[\mathcal{B}^\mu\mathcal{B}^\nu b^2h_{\mu\nu} - (\mathcal{B}_\mu\mathcal{B}^\mu(\mathcal{B}^\nu\mathcal{B}^\alpha - b^\nu b^\alpha) - b^2b^\nu b^\alpha]h_{\nu\alpha} + \frac{1}{4}(\mathcal{B}_\mu\mathcal{B}^\mu - b^2)^2h\right]\right\}$$
$$- \kappa^2\lambda\left\{\frac{1}{8}(\mathcal{B}_\mu\mathcal{B}^\mu - b^2)^2 + \mathcal{B}^\mu\mathcal{B}^\nu b^2h_{\mu\nu}h - \frac{1}{4}(\mathcal{B}_\mu\mathcal{B}^\mu - b^2)^2h_{\alpha\beta}h^{\alpha\beta} - [\mathcal{B}_\mu\mathcal{B}^\mu(\mathcal{B}^\nu\mathcal{B}^\alpha - b^\nu b^\alpha) + b^2b^\nu b^\alpha]h_{\nu\alpha}h\right.$$
$$\left. + (\mathcal{B}^\mu\mathcal{B}^\nu - b^\mu b^\nu)(\mathcal{B}^\alpha\mathcal{B}^\beta - b^\alpha b^\beta)h_{\mu\nu}h_{\alpha\beta} + 2(\mathcal{B}_\mu\mathcal{B}^\mu(\mathcal{B}^\nu\mathcal{B}^\alpha - b^\nu b^\alpha) + b_\mu b^\mu b^\nu b^\alpha)h^\beta_\nu h_{\alpha\beta}\right\} + \mathcal{O}(h^3). \tag{9}$$

We omit the weak-field expansion of the Faddeev-Popov Lagrangian $\mathcal{L}_{\rm FP}$, since within the one-graviton exchange approximation adopted in this work, the gravitational ghost fields do not contribute to the loop calculations.

In a certain sense, our study can be regarded as an analog of the one conducted in Ref. [49], where the weak-field limit was examined in the standard Einstein gravity coupled to the bumblebee field at the tree level.

The quadratic part of our Lagrangian (5) corresponds to the following propagators:

$$\Delta^{\mu\nu}(p) = -\frac{i}{p^2}[T^{\mu\nu} + L^{\mu\nu}];$$
$$\Delta_{\mu\nu\rho\sigma}(p) = i\left[\frac{2}{\gamma}\left(\frac{P^{(2)}_{\mu\nu\rho\sigma} - 2P^{(0)}_{\mu\nu\rho\sigma}}{p^2} - \frac{P^{(2)}_{\mu\nu\rho\sigma}}{p^2 - M_2^2} + \frac{2P^{(0)}_{\mu\nu\rho\sigma}}{p^2 - M_0^2}\right)\right.$$
$$+ \frac{\zeta_g}{\kappa^2p^4}(3P^{(0)}_{\mu\nu\rho\sigma} - \sqrt{3}(P^{(0-sw)}_{\mu\nu\rho\sigma} + P^{(0-ws)}_{\mu\nu\rho\sigma})$$
$$\left. + P^{(0w)}_{\mu\nu\rho\sigma})\right], \tag{10}$$

where ζ_g is the gravitational gauge-fixing parameter, which we set to zero for simplicity. The quantities $\Delta^{\mu\nu}(p)$ and $\Delta_{\mu\nu\rho\sigma}(p)$ denote the propagators of the bumblebee and graviton fields, respectively. The mass parameters M_0^2 and M_2^2 are defined as

$$M_0^2 = \frac{\gamma}{2\kappa^2}(3\beta - \alpha), \qquad M_2^2 = \frac{\gamma}{\alpha\kappa^2}. \tag{11}$$

Here, M_0 corresponds to the effective mass associated with the scalar (spin-0) excitation of the metric, while M_2 is the effective mass of the tensor (spin-2) excitation. Both these constants arise from the higher-derivative terms in the quadratic action and characterize the propagating modes of the gravitational sector.

The projectors are defined as follows:

$$\begin{aligned}
P^{(2)}_{\mu\nu\rho\sigma} &= \frac{1}{2}T_{\mu\rho}T_{\nu\sigma} + \frac{1}{2}T_{\mu\sigma}T_{\nu\rho} - \frac{1}{D-1}T_{\mu\nu}T_{\sigma\rho};\\
P^{(1)}_{\mu\nu\rho\sigma} &= \frac{1}{2}(T_{\mu\rho}L_{\nu\sigma} + T_{\mu\sigma}L_{\nu\rho} + L_{\mu\rho}T_{\nu\sigma} + L_{\mu\sigma}T_{\nu\rho});\\
P^{(0)}_{\mu\nu\rho\sigma} &= \frac{1}{D-1}T_{\mu\nu}T_{\sigma\rho};\\
P^{(0w)}_{\mu\nu\rho\sigma} &= L_{\mu\nu}L_{\sigma\rho};\\
P^{(0-sw)}_{\mu\nu\rho\sigma} &= \frac{1}{\sqrt{3}}T_{\mu\nu}L_{\sigma\rho};\\
P^{(0-ws)}_{\mu\nu\rho\sigma} &= \frac{1}{\sqrt{3}}L_{\mu\nu}T_{\sigma\rho};
\end{aligned} \tag{12}$$

where $T_{\mu\nu}$ and $L_{\mu\nu}$ are given by

$$\begin{aligned}
T_{\mu\nu} &= \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2},\\
L_{\mu\nu} &= \frac{p_\mu p_\nu}{p^2}.
\end{aligned} \tag{13}$$

These projectors will be used in our calculations.

Before proceeding to the loop calculation, let us clarify the field content around the LV vacuum. The full bumblebee field is expanded as in Eq. (4). Fluctuations tangent to the vacuum manifold are identified with the Nambu-Goldstone sector associated with the spontaneous breaking of local Lorentz symmetry [29,50], whereas the fluctuation normal to the constraint surface corresponds to the potential, or massive, mode. In the perturbative calculation performed below, the LV structures proportional to b_μ are treated as insertions around this vacuum configuration.

The gravitational sector is also richer than that of Einstein-Maxwell theory. Indeed, because the action contains quadratic-curvature operators, the graviton propagator decomposes into the usual spin projectors and contains, besides the massless spin-2 pole, the massive spin-2 and spin-0 poles characterized by M_2 and M_0, respectively. The usual caveats associated with the higher-derivative spin-2 pole are inherited from quadratic gravity and are not the focus of the present work. The nonminimal couplings ξ_1 and ξ_2 then generate Lorentz violating mixing structures and local insertions in the quadratic action. Our purpose is therefore not to establish an equivalence with Einstein-Maxwell theory, but rather to determine how the bumblebee vacuum modifies the UV counterterm structure of quadratic gravity.

III. RADIATIVE CORRECTIONS IN LV QUADRATIC GRAVITY

In this section, we present the UV renormalization of the quadratic part of the effective action of the model. We begin by computing the radiative corrections to the quadratic sector of the effective action for the bumblebee field, and then proceed with evaluating the corresponding corrections for the gravitational field.

A. The bumblebee field self-energy

We begin by examining the renormalization of the quadratic sector of the effective action for the bumblebee field. To proceed, we introduce the field redefinitions $B_\mu \to Z_B^{1/2}B_\mu$ and $b \to Z_b^{1/2}b$, where the Zs denote the renormalization constants, each expanded perturbatively. Thus, the Lagrangian takes the form

$$\begin{aligned}
\mathcal{L}_B = &-\frac{Z_B}{4}\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} - \frac{Z_B}{2g_l}(\partial_\mu\mathcal{B}^\mu)(\partial_\nu\mathcal{B}^\nu)\\
&- \lambda(Z_B\mathcal{B}_\mu\mathcal{B}^\mu - Z_b b^2)^2.
\end{aligned} \tag{14}$$

The classical potential attains its minimum at $B^\mu = b^\mu$. Therefore, we expand the Lagrangian around this vacuum configuration by performing the shift $\mathcal{B}^\mu \to B^\mu + b^\mu$. After doing so, and defining $Z_B = 1 + \delta_B$, $\lambda Z_B^2 = \lambda + \delta_\lambda$, and $2\lambda(Z_b - Z_B)Z_B = \delta_{m^2}$, the renormalized Lagrangian can be written as

$$\begin{aligned}
\mathcal{L}_B = &-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2g_l}(\partial_\mu B^\mu)(\partial_\nu B^\nu)\\
&- \lambda(4b^\mu b^\nu B_\mu B_\nu - 2b^2B^2) - \frac{\delta_B}{4}B^{\mu\nu}B_{\mu\nu}\\
&- \frac{\delta_{g_l}}{2g_l}(\partial_\mu B^\mu)(\partial_\nu B^\nu) - 4\delta_\lambda b^\mu b^\nu B_\mu B_\nu\\
&+ \delta_{m^2}b^2B^2 + \cdots,
\end{aligned} \tag{15}$$

where the ellipsis $(\cdots)$ denotes higher-order interaction terms.

We compute the one-loop corrections to the self-energy of the bumblebee field, represented in Fig. 1, as well as the corresponding LV contributions shown in Fig. 2. By summing all these diagrams, we obtain the total one-loop contribution to the bumblebee field effective action, which can be expressed in the form

$$S_B = \int d^4x \left\{ -\frac{1}{4}(\delta_T - \Gamma_T) B^{\mu\nu} B_{\mu\nu} - \frac{1}{2}(\delta_L - \Gamma_L)\partial_\mu B^\mu \partial_\nu B^\nu + (\delta_{m^2} b^2 - \Gamma_M) B^2 - 4(\delta_\lambda - \Gamma_\lambda) b^\mu b^\nu B_\mu B_\nu \right\}, \tag{16}$$

where

$$\begin{aligned}
\Gamma_T &= \frac{\kappa^2[(\xi_1(48\xi_2(2g_l-1)+4\xi_1(5g_l-4)-5)-132\xi_2)M_0^2+5\xi_1(\xi_1(2g_l-1)+4)M_2^2]}{288\pi^2\gamma\epsilon};\\
\Gamma_L &= \frac{\kappa^2}{96\pi^2\gamma g_l\epsilon}[(36(1-\xi_2)+4\xi_1(5+\xi_1)g_l-48(1+2\xi_1)\xi_2 g_l-8\xi_1(\xi_1-6\xi_2)g_l^2-3\xi_1)M_0^2\\
&\quad +5(9+\xi_1(6+(2+\xi_1(g_l-2))g_l))M_2^2];\\
\Gamma_M &= \frac{\kappa^2}{256\pi^2\gamma\epsilon}[(16\xi_2(3\xi_2(3+g_l)-2)+4\xi_1(6\xi_2(g_l-1)-1)+\xi_1^2(1+3g_l))M_0^4+10(\xi_1-\xi_1^2+2\xi_2)M_2^4]-\frac{3\lambda^2 b^2}{\pi^2\epsilon}\\
&\quad -\frac{\lambda\kappa^2 b^2}{288\pi^2\gamma\epsilon}[10(32-21\xi_1)\xi_1 M_2^2+(3\xi_1^2(29+3g_l^2-4g_l)+2\xi_1(12\xi_2(g_l-1)(7+3g_l)+38-9g_l)\\
&\quad +72\xi_2(2\xi_2(55+g_l(4+g_l))-2-g_l)+9)M_0^2];\\
\Gamma_\lambda &= \frac{3\lambda^2}{\pi^2\epsilon}+\frac{\lambda\kappa^2}{1152\epsilon\pi^2\gamma}[40(22-15\xi_1)\xi_1 M_2^2+(45+182\xi_1+201\xi_1^2-72\xi_2-408\xi_1\xi_2+19728\xi_2^2\\
&\quad -6(15+\xi_1-12\xi_2)(\xi_1+4\xi_2)g_l+45(\xi_1+4\xi_2)^2 g_l^2)M_0^2].
\end{aligned} \tag{17}$$

By imposing the finiteness requirement through the minimal subtraction (MS) scheme, we find the counterterms to be $\delta_M = b^2\Gamma_M$, $\delta_T = \Gamma_T$ and $\delta_L = \Gamma_L$.

A noteworthy structural outcome of the quadratic expansion, Eq. (16), is that the bumblebee sector is not closed under renormalization if one restricts the action to the Maxwell-type kinetic term alone. Indeed, the one-loop divergences contain a purely longitudinal structure proportional to $(\partial_\mu B^\mu)^2$, which cannot be absorbed by renormalizing the transverse Maxwell operator $-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}$. Therefore, perturbative renormalizability requires the inclusion of the longitudinal counterterm $\Delta\mathcal{L}_{\rm ct} = -\frac{\delta_L}{2}(\partial_\mu B^\mu)^2$ (or an equivalent operator basis), ensuring that the set of local operators is stable under radiative corrections. In the absence of this term (or in singular parameter limits where the longitudinal sector becomes degenerate), the UV divergences cannot be removed by a finite renormalization of the original couplings, signaling the loss of perturbative renormalizability.

Although a counterterm of the cosmological-term type, δ_Λ, is physically admissible, in this work we restrict our analysis to counterterms proportional to b^2, neglecting higher-order contributions such as b^4.

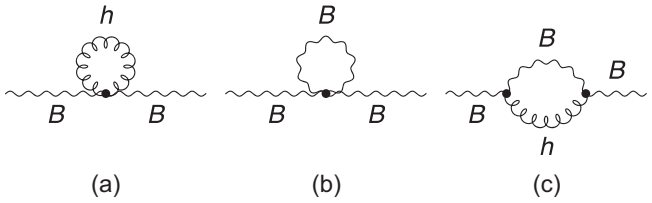


FIG. 1. Bumblebee self-energy. Wavy and wiggly lines represent the bumblebee and graviton propagators, respectively.

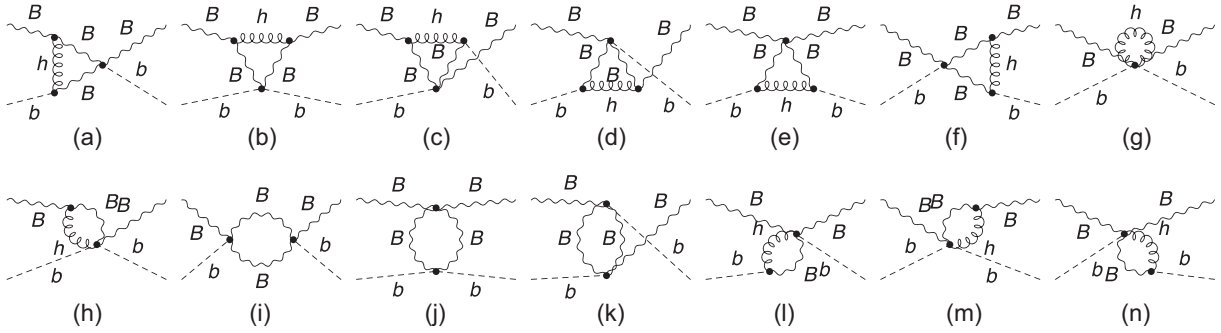


FIG. 2. Bumblebee self-energy diagram with a Lorentz violating vertex insertion, indicated by dashed lines.

B. Graviton-bumblebee mixing at tree level

We proceed to compute the tree-level amplitude for the graviton-bumblebee interaction by evaluating the contributions from the LV vertices. The corresponding Feynman diagram is shown in Fig. 3. The resulting expression for $|\mathcal{M}|^2$ is given by

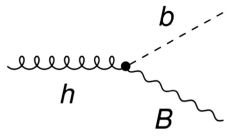


FIG. 3. Graviton-bumblebee transmutation induced by the LV background.

$$
\begin{aligned}
|\mathcal{M}|^2 = \frac{\kappa^2}{64 g_l^2}\{&\varepsilon^{(g)}_{\mu\nu}(p,\sigma)[8(\xi_1 g_l - 1)(p\cdot\epsilon(p,\rho))b^\mu p^\nu - 4(\xi_1 - 4\xi_2)g_l(b\cdot\epsilon(p,\rho))p^\mu p^\nu \\
&+ 4(b\cdot p)((1-2\xi_1 g_l)(p\cdot\epsilon(p,\rho))\eta^{\mu\nu} + 2\xi_1 g_l \epsilon^\mu(p,\rho)p^\nu)] \\
&\times \varepsilon^{(g)*}_{\alpha\beta}(p,\sigma)[8(\xi_1 g_l - 1)(p\cdot\epsilon^*(p,\rho))b^\alpha p^\beta - 4(\xi_1 - 4\xi_2)g_l(b\cdot\epsilon^*(p,\rho))p^\alpha p^\beta \\
&+ 4(b\cdot p)((1-2\xi_1 g_l)(p\cdot\epsilon^*(p,\rho))\eta^{\alpha\beta} + 2\xi_1 g_l \epsilon^{*\alpha}(p,\rho)p^\beta)]\},
\end{aligned}
\tag{18}
$$

where $\varepsilon^{(g)}(p,\sigma)$ and $\epsilon(p,\rho)$ are the graviton polarization tensor and bumblebee polarization vector, respectively.

It is straightforward to verify that the squared amplitude in Eq. (18) vanishes identically for an external graviton on shell. In fact, every term in $|\mathcal{M}|^2$ involves either a contraction of the graviton polarization tensor with the external momentum, $\varepsilon_{\mu\nu}(p)p^\nu$, or its trace, $\varepsilon^\mu{}_\mu(p)$, always multiplied by factors proportional to $p\cdot\varepsilon_\pm(p)$. Since physical graviton polarizations satisfy the transversality and tracelessness conditions, $p^\mu\varepsilon_{\mu\nu}(p)=0$ and $\varepsilon^\mu{}_\mu(p)=0$, while the bumblebee polarizations obey $p\cdot\varepsilon_\pm(p)=0$, each contribution vanishes separately. Therefore, the squared amplitude reduces to zero once the on-shell conditions are imposed.

In contrast, when this vertex is embedded in a loop, the amplitude (see Fig. 4) does not vanish. The reason is that, in this case, the internal bumblebee lines are off shell and must be accounted for through the full propagator rather than through the physical polarization conditions. Since the bumblebee field is treated as massless and all LV vertices are considered perturbative insertions, the completeness relation for the polarization sum introduces the axial-gauge type projector $\pi_{\mu\nu}(p,n)=\eta_{\mu\nu}-(p_\mu n_\nu + p_\nu n_\mu)/(p\cdot n)$. This projector does not annihilate momentum dependence, and contractions with the background vector b^μ generate nontrivial structures such as $(b\cdot p)^2$ and b^2p^2 that survive after integration. Consequently, although $|\mathcal{M}|^2$ vanishes for external on-shell gravitons, loop diagrams involving internal bumblebee propagators give rise to nonzero contributions.

C. The graviton self-energy

Our next step is to compute the quadratic part of the graviton effective action, that is, to evaluate the one-particle-irreducible diagrams contributing to the graviton self-energy. The corresponding Feynman diagrams are shown in Fig. 4. It is immediate to see that the contribution from Fig. 4(a) vanishes identically due to the massless nature of the B field. The expression associated with Fig. 4(b) yields the contribution of the bumblebee fluctuations to the quadratic part of the graviton effective action, which can be written as

$$
\begin{aligned}
S_g^{(2)} = \int d^4x\sqrt{-g}\Big[&-(\delta_\alpha-\Gamma_\alpha)R^{\mu\nu}R_{\mu\nu} \\
&+(\delta_\beta-\Gamma_\beta)R^2 - \frac{\delta_\gamma-\Gamma_\gamma}{\kappa^2}R\Big],
\end{aligned}
\tag{19}
$$

where the superscript $^{(2)}$ denotes the quadratic part in the graviton field $h^{\mu\nu}$, and

$$
\begin{aligned}
\Gamma_\alpha &= \frac{7-10\xi_1(5+g_l)+5\xi_1^2(7+4g_l+g_l^2)}{960\pi^2\epsilon}; \\
\Gamma_\beta &= \frac{6-10\xi_1(5+g_l)-40\xi_2(g_l-3)-240\xi_2^2(3+g_l^2)+5\xi_1^2(7+4g_l+g_l^2)}{3840\pi^2\epsilon}; \\
\Gamma_\gamma &= 0.
\end{aligned}
\tag{20}
$$

Notice that the counterterms δ_α, δ_β, and δ_γ are generated from Eq. (6) through the field redefinition $h^{\mu\nu}\to\sqrt{Z_h}h^{\mu\nu}$, and by identifying $\alpha+\delta_\alpha = Z_h\alpha$, $\beta+\delta_\beta = Z_h\beta$ and $\gamma+\delta_\gamma = Z_h\gamma$.

By imposing finiteness within the framework of the MS renormalization scheme, we obtain the counterterms $\delta_\alpha=\Gamma_\alpha$, $\delta_\beta=\Gamma_\beta$, and $\delta_\gamma=0$. It is worth emphasizing that since the LV terms are treated as perturbations and the

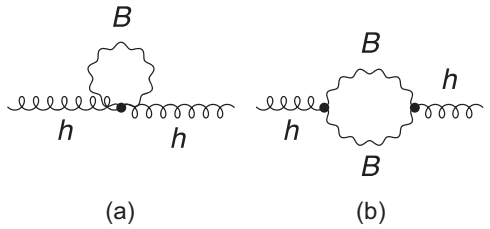


FIG. 4. Bumblebee corrections to the graviton propagation.

bumblebee field is massless at zeroth order in the LV parameters, there is no renormalization of the Einstein-Hilbert term.

We now focus our attention on the LV corrections to the graviton self-energy. The relevant contributions are encoded in the Feynman diagrams displayed in Fig. 5. The LV correction to the quadratic part of the graviton effective action can be expressed as

$$S_g^{(2)} = \int d^4x\sqrt{-g}\Bigg[\left(\delta_{\xi_1} - \frac{\lambda[\xi_1(7+4g_l+g_l^2)-(5+g_l)]}{24\pi^2\epsilon}\right) \times \left(b^\mu b^\nu - \frac{g^{\mu\nu}}{4}b^2\right)R_{\mu\nu} + \left(\delta_{\xi_2} - \frac{\lambda[12\xi_2(3+g_l^2)+g_l-3]}{96\pi^2\epsilon}\right)b^2R\Bigg], \tag{21}$$

where $\delta_{\xi_i} = Z_h Z_B \xi_i - \xi_i$, with $i = 1, 2$. It is worth emphasizing that the insertions of LV vertices induce the renormalization of ξ_1, which is associated with the traceless part of the nonminimal coupling between the bumblebee and gravitational fields. In contrast, the trace part, proportional to ξ_2, corresponds to the LV correction to the Einstein-Hilbert term and appears only at the order b^2. We note also that the gravitational aether term $b^\mu b^\nu R_{\mu\nu}$ proposed in Ref. [51] naturally arises within our calculations.

IV. CLASSICAL SOLUTIONS

Our bumblebee-gravity model (2) can also be studied at the tree level. In this case, we disregard the gauge-fixing, bumblebee longitudinal term, ghosts, and counterterms to obtain the equations of motion.

Let our Lagrangian look like

$$\mathcal{L} = \sqrt{-g}\Bigg(-\frac{1}{4}\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} - \frac{\lambda}{4}(\mathcal{B}^\mu\mathcal{B}_\mu \mp b^2)^2 + \alpha R + \beta R^2 + \gamma R_{\mu\nu}R^{\mu\nu} + \xi_1\mathcal{B}^\mu\mathcal{B}^\nu R_{\mu\nu} + \xi_2\mathcal{B}^2R\Bigg), \tag{22}$$

that is, Eq. (2) where the gauge-fixing and ghost terms are omitted as will as counterterms since all these contributions are relevant only within the perturbative framework. The constant parameters are relabeled as α, β, γ for convenience.

The equation of motion for the bumblebee field has the simple form, being a generalization of those given in Ref. [36], as follows:

$$\nabla_\mu\mathcal{B}^{\mu\nu} - \lambda(\mathcal{B}^\rho\mathcal{B}_\rho \mp b^2)\mathcal{B}^\nu + 2\xi_1\mathcal{B}_\mu R^{\mu\nu} + 2\xi_2\mathcal{B}^\nu R = 0. \tag{23}$$

The following equations of motion for the gravitational field display some similarity with Ref. [31], where, however, terms proportional to β and γ were absent:

$$\begin{aligned}&\alpha\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 4\beta\left[RR_{\mu\nu} - \frac{1}{4}R^2g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)R\right] \\ &+ \gamma\left[-\frac{1}{2}R_{\rho\sigma}R^{\rho\sigma}g_{\mu\nu} + 2R^\rho_\mu R_{\nu\rho} + 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} + \Box R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\Box R - \nabla_\mu\nabla_\nu R\right] \\ &+ \xi_1\left[\frac{1}{2}g_{\mu\nu}\mathcal{B}^\alpha\mathcal{B}^\beta R_{\alpha\beta} - 2\mathcal{B}_{(\mu}\mathcal{B}^\rho R_{\nu)\rho} - \nabla^\rho\nabla_{(\mu}(\mathcal{B}_{\nu)}\mathcal{B}_\rho) + \frac{1}{2}g_{\mu\nu}\nabla^\rho\nabla^\sigma(\mathcal{B}_\rho\mathcal{B}_\sigma) + \frac{1}{2}\Box(\mathcal{B}_\mu\mathcal{B}_\nu) - \nabla^\rho\nabla_{(\mu}\mathcal{B}_{\nu)}\mathcal{B}_\rho\right] \\ &+ \xi_2[-\mathcal{B}^\rho\mathcal{B}_\rho G_{\mu\nu} + \nabla_\mu\nabla_\nu(\mathcal{B}^\rho\mathcal{B}_\rho) - g_{\mu\nu}\Box(\mathcal{B}^\rho\mathcal{B}_\rho)] = T^B_{\mu\nu},\end{aligned} \tag{24}$$

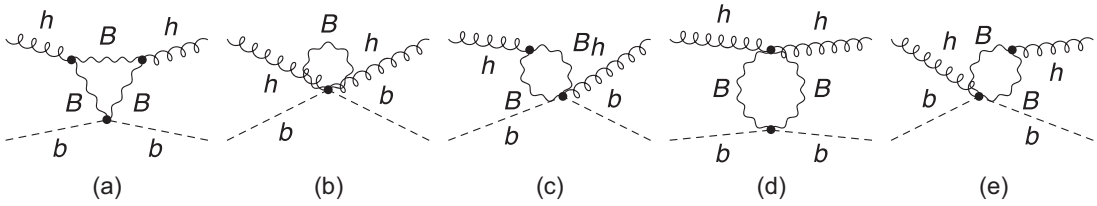


FIG. 5. LV corrections to the graviton propagation.

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the standard Einstein tensor, and

$$T^{B}_{\mu\nu} = \mathcal{B}_{\mu\alpha}\mathcal{B}^{\alpha}_{\nu} - \frac{1}{4}g_{\mu\nu}\mathcal{B}_{\lambda\rho}\mathcal{B}^{\lambda\rho} - Vg_{\mu\nu} + 2V'\mathcal{B}_{\mu}\mathcal{B}_{\nu}, \quad (25)$$

is the standard (that is, ξ-independent) part of the energy-momentum tensor of the bumblebee field (cf., e.g., Ref. [36]).

Let us solve these equations. We note that here, in the bumblebee sector, our aim consists in finding the minimum of the potential compatible with the equation of motion of the bumblebee field. As a first attempt, we can choose the purely radial $\mathcal{B}_{\mu}(r) = (0, \mathcal{B}_{r}(r), 0, 0)$, similar to [30]. In this case, one easily arrives at $\mathcal{B}_{\mu\nu} = 0$, moreover, it is shown in Ref. [30] that for the static spherically symmetric metric corresponding to the following line element

$$ds^2 = e^{2\phi(r)}dt^2 - e^{2\rho(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (26)$$

and for a certain form of $\mathcal{B}_{r}(r)$, that is, $\mathcal{B}_{r}(r) = be^{\rho(r)}$, where b is constant, one has $\mathcal{B}^{\mu}\mathcal{B}_{\mu} = -b^2$ with $\mathcal{B}_{\mu}(r)$ being the vacuum. So, for this choice, $T^{B}_{\mu\nu}$ (25) vanishes. Then, let us for the sake of simplicity, consider the case $\xi_1 = 0$. In this case, our modified Einstein equation (24) reduces to

$$\begin{aligned}
&\alpha\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 4\beta\left[RR_{\mu\nu} - \frac{1}{4}R^2 g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu})R\right] \\
&+ \gamma\left[-\frac{1}{2}R_{\rho\sigma}R^{\rho\sigma}g_{\mu\nu} + 2R_{\mu}{}^{\rho}R_{\nu\rho} + 2R^{\rho\sigma}R_{\mu\rho\nu\sigma}\right. \\
&\left. + \Box R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\Box R - \nabla_{\mu}\nabla_{\nu}R\right] \\
&- \xi_2\mathcal{B}^{\rho}\mathcal{B}_{\rho}G_{\mu\nu} = 0. \quad (27)
\end{aligned}$$

This equation (where $\mathcal{B}_{\mu}$ is chosen to be in the radial form we discussed above) certainly admits various solutions. The simplest one is the Schwarzschild metric—indeed, for this metric $R_{\mu\nu} = R = 0$, and the bumblebee equation of motion (23) is also satisfied. So, we succeeded in demonstrating that our modified Einstein equations, besides the trivial Minkowski solution, are solved at least by the Schwarzschild metric. Another interesting solution for the case $\xi_1 = 0$ is the static de Sitter metric for which one has $R_{\mu\nu\lambda\rho} = \frac{1}{a^2}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$, where a is a constant, with $R_{\mu\nu} = \frac{3}{a^2}g_{\mu\nu}$ and $R = \frac{12}{a^2}$; in this case, at $\xi_1 = 0$, Eq. (27) is satisfied if the following algebraic relationship between constants of the theory holds: $-\frac{3\alpha}{a^2} + \frac{18\gamma}{a^4} - 3\frac{\xi_2 b^2}{a^2} = 0$. In a certain sense, the nonminimal coupling, together with γ, in this case, is related to the cosmological constant $\Lambda = \frac{1}{a^2}$.

The natural question is whether one can have a nontrivial solution when both ξ_1 and ξ_2 are nonzero. There is a nontrivial solution in this case, that is, the *cylindrically* symmetric metric known as the Levi-Civita metric (see, e.g., Ref. [52] and references therein):

$$ds^2 = \rho^{4m}dt^2 - \rho^{4m(m-1)}(d\rho^2 + \rho^2 d\phi^2) - dz^2, \quad (28)$$

where m is a real dimensionless parameter. This metric satisfies the vacuum Einstein equations $R_{\mu\nu} = 0$. At the same time, if we choose the vacuum vector along the z axis, $\mathcal{B}_{\mu} = (0, 0, 0, b)$ (so, $\mathcal{B}^{\mu}\mathcal{B}_{\mu} = -b^2$ as it must be in the spacelike case), we see that, as for the Levi-Civita metric, one easily finds $\Gamma^{3}_{\mu\nu} = \Gamma^{\mu}_{3\nu} = 0$ and the identity $\nabla_{\mu}\mathcal{B}_{\nu} = 0$ is valid for our vacuum. Hence, all terms proportional to ξ_1 vanish as well, and we proved that this metric satisfies the modified Einstein equation for both ξ_1, $\xi_2 \neq 0$.

Perhaps some other metrics can be shown to solve these equations perturbatively, taking into account that solutions of Eqs. (23) and (24) at $\xi_{1,2} = 0$ can be modified by small perturbations proportional to $\xi_{1,2}$. The cosmological issues related to this theory certainly deserve a separate study. Also, possible nonspherically-symmetric solutions (for example, Kerr metric or pp waves) in our theory can be tested.

V. FINAL REMARKS

In this work, we have formulated a model displaying spontaneous Lorentz symmetry breaking within the framework of quadratic gravity. This model includes the terms of first and second orders in curvature, therefore being an extension of the model considered in Ref. [44], by incorporating the most general renormalizable curvature sector.

Within the classical context, we derived the field equations and analyzed stationary, spherically symmetric configurations supported by bumblebee backgrounds. In particular, we showed that the Schwarzschild geometry remains an *exact* solution of the full theory for an appropriate bumblebee configuration, even in the presence of nonminimal couplings.

Within the quantum context, we expanded the action to second order in fluctuations around flat spacetime and computed the one-loop two-point functions of the bumblebee and graviton fields. A notable structural outcome concerns the mixed graviton-bumblebee two-point function induced by a LV vertex insertion: for on-shell external legs, it vanishes, consistently with the transversality/tracelessness properties of the physical graviton polarization tensor. However, this kinematical decoupling does not prevent the corresponding mixed structures from contributing inside loop diagrams, since the internal lines are generically off shell and the same cancellations no longer apply. At the same time, LV insertions generate, in the graviton sector, the aether-type operators advocated in Ref. [51]. Importantly, we find that these aetherlike contributions

arise with UV divergences, implying that they must be included as counterterms in a renormalized treatment and providing a concrete illustration of how spontaneous Lorentz violation feeds back into the UV operator content of quadratic gravity.

A further continuation of this study could consist, first, in testing more involved gravitational solutions within our model; second, in studying the homogeneous and isotropic cosmological backgrounds and the resulting modifications of the Friedmann dynamics (in particular, generalizing the results obtained in Ref. [53]); and third, in computing the effective potential for the bumblebee field, in close analogy with Ref. [41], thereby connecting the UV counterterm structure found here with the dynamical generation of LV vacua. We expect to perform these studies in forthcoming papers.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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