

Species quantum mechanics

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(Received 7 November 2025; accepted 2 January 2026; published 23 January 2026)

In this paper we take the initial steps toward a formulation of species quantum mechanics. Specifically, we consider quantum operators that correspond to the species number N_s and the tower mass scale m_t in the context of the swampland distance conjecture. We discuss the commutation relations, a possible wave function, and symplectic duality transformations on the conjugate variables. Furthermore, we argue that the Castellano-Ruiz-Valenzuela (CRV) pattern is a consequence of the canonical commutation rules of moduli space quantum mechanics. We also connect the canonical quantization to the periods of $\mathcal{N} = 2$ Calabi-Yau compactifications to explore other aspects of the CRV pattern, including its possible connection to the Ooguri-Vafa-Verlinde black hole quantization procedure.

DOI: [10.1103/vwjx-lrhl](https://doi.org/10.1103/vwjx-lrhl)

I. INTRODUCTION—GENERAL IDEA

In quantum mechanics noncommuting operators correspond to conjugate variables. The most famous example is the pair of position $\hat{x}$ and momentum $\hat{p}$ operators, which are noncommutative and form the basis of Heisenberg's uncertainty principle [1]. In its modern form, the Robertson uncertainty relation for two noncommuting observables A and B is defined by

$$\Delta A \cdot \Delta B \geq \frac{1}{2} | \langle [\hat{A}, \hat{B}] \rangle |, \quad (1)$$

where $(\Delta A)^2 = \langle A^2 \rangle - \langle A \rangle^2$ denotes the variance of the operator $\hat{A}$ in a given quantum state ρ , and $\langle \cdot \rangle = \text{Tr}\{\rho \cdot\}$ indicates the expectation value [2]. The commutator $[\hat{A}, \hat{B}]$ captures the fundamental incompatibility of the operators representing observables A and B . Then for canonical conjugate pairs (e.g., $\hat{x}$ and $\hat{p}$) their nonzero commutator $[\hat{x}, \hat{p}] = i\hbar$ guarantees that a fundamental minimum uncertainty exists.

Applying the quantum rules to gravity is still a big challenge in theoretical physics. Normally, one expects that effects from quantum gravity become important at the Planck scale $M_p \simeq 10^{19}$ GeV. However, it has been conjectured that quantum gravity effects could manifest at scales much lower than the Planck scale [3–6]. In fact, in the last two decades this conjecture has seen an intense Swamplandish revival [7]. The essence of this resurgence can be encapsulated as follows. The principle of naturalness has historically guided progress in high energy physics. Indeed, the effective field theory (EFT) approach provides a powerful framework for describing physical phenomena at a particular energy scale. A key characteristic of an EFT is its finite domain of validity, defined by a cutoff energy. The concept of naturalness implicitly assumes that heavy, integrated-out fields do not contribute significantly to low-energy physics. While naturalness arguments have proven over time useful in high energy physics, they are now being challenged by the significant fine-tuning problems posed by the Higgs and the cosmological constant. In this vein, recent developments in theoretical physics advocate for replacing the naturalness logic with guiding principles derived from the demand for ultraviolet (UV) consistency of EFTs. This is particularly motivated by the requirement that such EFTs must be consistent with a valid UV completion in a theory of quantum gravity, an essential consideration due to the presence of gravitational

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interactions in our universe. The swampland conjectures, which outline the conditions for an EFT to be compatible with quantum gravity, provide concrete examples of how UV consistency dictates the parameters and structures of low-energy physics [8–10].

In particular, the swampland *distance conjecture* (DC) [11] together with the *emergent string conjecture* (ESC) [12] provide a concrete mechanism by which an EFT drastically breaks down. The DC predicts the emergence of an infinite tower of states, with their masses decreasing exponentially as the geodesic distance traveled in the field space grows. In addition, the ESC is stating that the towers of species that become light in some infinite distance limit can be either light Kaluza-Klein (KK) modes, corresponding to a decompactification limit, or light string states, belonging to a weakly coupled string limit. The large number of states in the tower implies the breakdown of semiclassical Einstein gravity at a cutoff scale, the quantum gravity scale commonly also dubbed the species scale Λ_s [13–15].¹ This scale can be expressed in terms on the number N_s of species below Λ_s in the following way in Planck units:

$$\Lambda_s = N_s^{\frac{1}{d-2}}. \quad (2)$$

Another important aspect of the distance conjecture is the existence of duality symmetries, which connect different infinite distance limits to each other. In fact, the emergence of duality symmetries together with finiteness and compactifiability of the moduli space can be seen from certain bottom-up arguments [23]. Because of the emergence of duality symmetries the DC is also sometimes called the *duality conjecture*. Further aspects of the DC in relation to information theory were discussed in [24,25]. Finally, a complementary discussion of the DC in terms of flow equations can be found in [26].

Another generic and important feature of quantum gravity and string theory is that the moduli that describe families of quantum gravity vacua are given in terms of vacuum expectation values of scalar fields ϕ . Actually, in the EFT these scalar fields ϕ should be treated as quantum fields, denoted by $\hat{\phi}$. Together with the conjugate momentum operator $\hat{\pi}$ they act on the quantum mechanical Hilbert space. Furthermore, the fields $\hat{\phi}$ and $\hat{\pi}$ satisfy some canonical commutation relations of the form

$$[\hat{\phi}, \hat{\pi}] = i\hbar. \quad (3)$$

Looking at the EFT of quantum gravity, it turns out that most of the couplings, operators, and quantities therein, in general denoted by $\mathcal{O}$, are field dependent, i.e., $\mathcal{O} = \mathcal{O}(\phi)$. Therefore, they also should be treated as quantum

mechanical operators $\hat{\mathcal{O}}(\hat{\phi})$. More generally, operators that depend both on $\hat{\phi}$ and on $\hat{\pi}$ should obey some quantum mechanical commutation relations with commutators of the form $[\hat{\mathcal{O}}_1, \hat{\mathcal{O}}_2] \simeq \hat{\mathcal{O}}_3$. For example, the KK mass scale, or more generally the mass scale m_t appearing in the DC, are scalar field dependent, as they are functions of the scalars that correspond to geometric moduli and the string coupling constant, which we will collectively denote by t_i . The same holds also for the field dependent species number $N_s(t)$ [27]. Hence, the tower mass scale m_t as well as N_s correspond to some quantum operators $\hat{m}_t(\hat{t})$ and $\hat{N}_s(\hat{t})$.

Species thermodynamics states that species enjoy thermodynamic properties [28–32]. The species entropy $\mathcal{S}_s$ is determined by an area law, it is given in terms of the number of species in a given tower, i.e., $\mathcal{S}_s = N_s$, and it can be derived from the entropy of corresponding minimal black holes: $\mathcal{S}_s = \mathcal{S}_{\text{BH}}$ [33]. Species thermodynamics also provides bottom-up arguments for the emergence distance conjecture. Furthermore, species thermodynamics provides information about the classical dynamics of N_s and m_t , as for example, the second law of species thermodynamics states that the number of species must not decrease, i.e., $\delta N_s \geq 0$.

In this note we take the next steps after the introduction of species thermodynamics to lay a foundation of species quantum mechanics. We will discuss some aspects of the quantum properties of $\hat{m}_t$ and $\hat{N}_s$, together with some related commutation and uncertainty relations. In this way, species quantum mechanics will constitute a kind of minisuperspace version of quantum gravity by considering the quantum Hilbert space of the species.

Recently, Castellano, Ruiz, and Valenzuela (CRV) pointed out that in infinite-distance limits there is a universal pattern linking the (logarithmic) gradients of the mass gap m_t of the lightest tower to the species scale Λ_s ,

$$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla \Lambda_s}{\Lambda_s} = \frac{1}{d-2}, \quad (4)$$

where d is the number of dimensions of the lower dimensional EFT [34,35]. In terms of the number of species, using eq. (2) the CRV pattern can be recast as

$$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla N_s}{N_s} = -1. \quad (5)$$

Note that this relation is independent from the number of space-time dimensions.

In our paper we first identify the pair of conjugate variables of the EFT with moduli space $\mathcal{M}_\phi$. Then, we will show that N_s and m_t are themselves closely related to conjugate variables and we will discuss the commutation relations involving these operators. This will allow us to relate the pattern (4) or (5) to a particular commutation

¹Other aspects of species were discussed in [16–22].

relation of species quantum mechanics. We will discuss the commutation relations involving the operators N_s and m_t , a possible wave function, and symplectic duality transformations on the conjugate variables. Finally, using the periods of $\mathcal{N} = 2$ Calabi-Yau (CY) compactifications we make contact with the quantization rules set up by Ooguri, Vafa, and Verlinde (OVV) in [36]. A synopsis of our arguments as well as some important caveats are discussed at the end of the summary section.

II. SPECIES QUANTUM MECHANICS

Let us consider a d -dimensional EFT of the (schematic) form

$$S_{\text{EFT}} \sim \int d^d x \sqrt{-h} \left(M_p^{d-2} \left(R + \frac{1}{2} g_{ij}(t) \partial_\mu t^i \partial_\mu t^j + V(t) \right) + \sum_{n \geq 1} c_n N_s^{\frac{n-2}{d-2}}(t) \mathcal{O}(R^{2n}) \right) + \sum_{n > d} \tilde{c}_n m_t^{d-n}(t) \tilde{\mathcal{O}}(R^{2n}) + \dots \quad (6)$$

where h refers to the space-time metric, the scalar fields t^i correspond to the moduli of the string compactification, $g_{ij}(t)$ is the moduli space metric and $V(t)$ a possible moduli dependent scalar potential. In addition, this action is a double EFT expansion [37] with respect to the two scales that are relevant for our discussion: there is a first set of higher derivate operators $\mathcal{O}(R^{2n})$ that are determined by the moduli dependent species number $N_s(t)$ [27]. On the other hand, the second set of higher derivative operators $\tilde{\mathcal{O}}(R^{2n})$ is suppressed by the tower mass scale $m_t(t)$.

A. Canonical variables and commutation relations

We will now treat the scalar fields t^i as quantum fields, and we first want to determine their conjugate momenta π_t . Following the standard canonical quantization procedure, the conjugate momenta in quantum field theory are given by $\pi_{i,\mu} = \frac{\delta \mathcal{L}_{\text{EFT}}}{\delta(\partial_\mu t^i)} = g_{ij} \partial_\mu t^j$. But to obtain a sensible quantization, i.e., in order to describe a quantum mechanical system, the canonical momenta should only be defined with respect to the time derivative. Therefore, as also considered in the context of finiteness [23], we assume that all spatial coordinates are compactified and only the time coordinate τ is left.² Then following the standard canonical quantization procedure, the canonical momenta are just given by the time-derivative of the moduli fields

²Actually, in a way we do not really compactified to one dimension, but we just ignore the three spacelike dimensions. A subtle point of this truncation is that we work in four-dimensional Planck units and so the relations between the tower masses and the species scale derived in the four-dimensional EFT do not change.

$$\pi_{t,i} = \frac{\delta \mathcal{L}_{\text{EFT}}}{\delta(\partial_\tau t^i)} = g_{ij} \partial_\tau t^j = g_{ij} \dot{t}^j. \quad (7)$$

This leads to an Hamiltonian of the following form:

$$\mathcal{H} = \frac{1}{2} g^{ij}(t) \pi_{t,i} \pi_{t,j} + V(t). \quad (8)$$

The canonical quantization of the quantum mechanical moduli space is now pursued by imposing the following canonical equal-time commutation relation (we have set $\hbar = 1$ in all the following commutators) on the pair $(t^i, \pi_{t,j})$ of conjugate variables³

$$[t^i, \pi_{t,j}] = [t^i, g_{jk} \dot{t}^k] = i \delta_j^i. \quad (9)$$

Let us now proceed by computing the commutation relations between some functions of the operators t^i and $\pi_{t,i}$. Specifically, we are interested in the species number operator N_s , in the tower scale operator m_t and in the commutation relations that involve these operators. Bearing this in mind, we relate N_s and m_t to the canonical variables t^i and π_j in a concrete way. To do that we assume that we are close to the boundary of the moduli space, where the infinite distance conjecture holds [11]. This conjecture relates the tower mass scale m_t to the moduli space metric g_{ij} in the following way:

$$m_t(t) = e^{-ad(t)}, \quad (10)$$

where $d(t)$ is the geodesic distance from an arbitrary reference point t_0 to the point labeled by t , for asymptotically large t . It is computed from the moduli space metric as (here we assume that $V(t) = 0$)

$$d(t) = \int_0^1 d\lambda \sqrt{\frac{dt^i}{d\lambda} g_{ij}(\lambda) \frac{dt^j}{d\lambda}}, \quad (11)$$

where λ is a parameter of the path, with $\lambda = 0$ corresponding to t_0 and $\lambda = 1$ to t . For simplicity, we now assume that we are dealing with a tower of species with a tower mass scale

$$m_t = 1/t^\alpha. \quad (12)$$

For the case that the species are given in terms of KK modes corresponding to p compact dimensions, the parameter α is given as [38]

$$\alpha = \frac{p+2}{2p} \quad (13)$$

³We could also consider nonequal-time commutators of the form $[t^i(\tau), t^j(\tau')] = i G^{ij}(\tau, \tau')$ and $[t^i(\tau), \pi_j(\tau')] = i g_{jk} \partial_{\tau'} G^{ik}(\tau, \tau')$ for a suitable commutator function G^{ij} .

The corresponding metric g_{tt} is given by

$$g_{tt} = \alpha/t^2. \quad (14)$$

The case $p = \infty$ corresponds to the case of a tower of light string excitations. In the next section on Calabi-Yau periods we will discuss three cases in more detail: the emergent string limit with $\alpha = 1/2$, the decompactification to six dimensions with $\alpha = 1$, and the decompactification to five dimensions with $\alpha = 3/2$.

Using the metric (14) we can easily determine the canonically normalized scalar field ϕ

$$\phi = \sqrt{\alpha} \log t. \quad (15)$$

The canonical momentum is then simply given by

$$\pi_\phi = \dot{\phi} \quad (16)$$

and satisfies the commutation relation

$$[\phi, \dot{\phi}] = i. \quad (17)$$

Next, as we discuss below, for KK compactifications, the number of species is proportional to the volume of the compact space in string units, and thus just given by t itself, i.e.,

$$N_s = t. \quad (18)$$

Thus, together with (14), the canonical momentum π_{N_s} , conjugate to N_s , can be written as

$$\pi_{N_s} = \frac{\alpha}{N_s^2} \dot{N}_s. \quad (19)$$

This is of course consistent with the time evolution of N_s in quantum mechanics, which is given by the following commutator:

$$\dot{N}_s = i[\mathcal{H}, N_s] = g_{tt}^{-1} \pi_{N_s}. \quad (20)$$

Next, we like to express π_{N_s} by the tower mass scale m_t and after a short computation we obtain

$$\tilde{m}_t := \pi_{N_s} = -m_t^{\frac{1-\alpha}{\alpha}} \dot{m}_t. \quad (21)$$

Note that for $\alpha = 1$, $\tilde{m}_t = -\dot{m}_t$. Hence, assuming that the swampland distance conjecture together with the given form of the species towers holds, the pair of operators $(N_s, \tilde{m}_t)$ are canonically conjugate to each other

$$[N_s(t), \tilde{m}_t(t)] = -m_t^{\frac{1-\alpha}{\alpha}} [N_s(t), \dot{m}_t] = i. \quad (22)$$

We see that for $\alpha = 1$ the pair $(N_s, -\dot{m}_t)$ are conjugate to each other

$$[N_s, -\dot{m}_t] = i. \quad (23)$$

The commutator (22) leads to a Robertson uncertainty relation of the form

$$\Delta N_s \Delta \tilde{m}_t \geq \frac{1}{2}. \quad (24)$$

The interpretation of this Robertson uncertainty is very interesting: it means that if one does a precise measurement of the number of species, or respectively the species scale, quantum mechanics forbids to measure the time variation of the tower mass scale with high precision. Of course the reverse statement also is true.

B. The CRV pattern

Let us now investigate the CRV pattern in more detail. First we note that commutator between the two functions $f(t)$ and $\dot{h}(t)$ can be evaluated as follows:

$$\begin{aligned} \left[f(t), \frac{d}{d\tau} h(t) \right] &= [f(t), \partial_i h(t) \dot{t}^i] \\ &= [f(t), \partial_i h(t) g^{ij}(t) \pi_j] \\ &= \partial_i h(t) g^{ij}(t) [f(t), \pi_j] \\ &= \partial_i f(t) g^{ij}(t) \partial_k h(t) [t^k, \pi_j] \\ &= i \partial_i f(t) g^{ij}(t) \partial_j h(t). \end{aligned} \quad (25)$$

So in short we get

$$[f(t), \dot{h}(t)] = i \nabla f(t) \cdot \nabla h(t). \quad (26)$$

It corresponds to a Robertson uncertainty relation of the following form

$$\Delta f(t) \Delta \dot{h}(t) \geq \frac{1}{2} |\langle \nabla f(t) \cdot \nabla h(t) \rangle|. \quad (27)$$

Now, let us apply this to the functions, which correspond to the tower mass scale and number of species in the CRV pattern. Specifically, we set $f(t) = \log N_s(t)$ and $h(t) = \log m_t(t)$. Then the commutator (26) becomes

$$\left[\log N_s(t), \frac{d}{d\tau} \log m_t(t) \right] = i \frac{\nabla N_s(t)}{N_s(t)} \cdot \frac{\nabla m_t(t)}{m_t(t)}. \quad (28)$$

We see that the r.h.s. of this equation is just the expression as given by the CRV pattern. So assuming that the CRV pattern holds we can conclude that the commutator between these two operators is constant

$$\left[\log N_s(t), \frac{d}{d\tau} \log m_t(t) \right] = -i. \quad (29)$$

In this case N_s and the time derivative of m_t are related to canonically conjugate operators.

One can also reverse the argument by imposing that $\log N_s(t)$ and $\frac{d}{d\tau} \log m_t(t)$ are conjugate operators. Then the CRV pattern, written as an operator product, is a consequence of the canonical commutation rules of moduli space quantum mechanics. Let us demonstrate the equivalence relation between the CRV pattern and the considered pair of conjugate operators in an explicit way. For that we use the relations between N_s and m_t to the canonical variables t^i and π_j , which we have introduced before. Then, we can directly evaluate the left-hand side (lhs) of (28)

$$\begin{aligned} \left[\log N_s(t), \frac{d}{d\tau} \log m_t(t) \right] &= \frac{1}{t} \left[t, \frac{d}{d\tau} \log m_t(t) \right] \\ &= \frac{1}{t} \left[t, \frac{\partial_t m_t(t)}{m_t(t)} t \right] \\ &= -\frac{\alpha}{t^2} [t, g_t^{-1} \pi_t] \\ &= -[t, \pi_t] = -i. \end{aligned} \quad (30)$$

So we see that the CRV pattern (5) is indeed a consequence of the basic commutation rule.

The CRV pattern looks particularly simple in terms of ϕ and $\dot{\phi}$. Specifically, one has that

$$\log N_s(t) = \frac{\phi}{\sqrt{\alpha}}, \quad \frac{d}{d\tau} \log m_t = -\sqrt{\alpha} \dot{\phi}. \quad (31)$$

So in the canonical field basis $\log N_s$ and $\log m_t$ are directly related to the conjugate variables and the CRV pattern can be directly expressed as

$$\left[\log N_s(t), \frac{d}{d\tau} \log m_t(t) \right] = -[\phi, \dot{\phi}] = -i. \quad (32)$$

Finally, we see that $\frac{d}{d\tau} \log m_t$ and $\log N_s$ obey a quantum mechanical uncertainty relation of the form

$$\Delta(\log N_s) \cdot \Delta\left(\frac{d}{d\tau} \log m_t\right) \geq \frac{1}{2}. \quad (33)$$

All the above relations were shown to hold at the asymptotic boundaries of the moduli space. Inside the moduli space the moduli dependence of N_s , m_t as well as the CRV pattern is expected to receive corrections. For example, the tower mass scale m_t may be replaced by the so-called black hole scale Λ_{BH} [39], where certain black hole phase transitions take place. In addition, some explicit corrections were computed in [27,40,41]. Going to string compactifications with a higher number of unbroken supersymmetry, these corrections can be computed or are under better control. For example, this is the case in nine or in ten dimensional type string vacua, or also for

$\mathcal{N} = 2$ string compactifications, which we will consider in the next section. An alternative method to obtain some information about the interior of the moduli space from its boundary was discussed in [42].

C. Symplectic duality transformations

Within the swampland approach, duality transformations typically relate different, but often physically equivalent infinite distance limits to each other. Swampland arguments, as well as the related finiteness and compactifiability of moduli space, how duality symmetries lead to bottom-up predictions of duality symmetries, were recently given in [23]. Examples are the well-known T-duality transformations of the form $t \rightarrow 1/t$, which exchange the KK tower with the stringy winding tower, or the S-duality that relates weak and strong coupling to each other.

On the other hand, in quantum mechanics symplectic transformations on a pair of conjugate variable (q, π) of the form

$$q \rightarrow \pi, \quad \pi \rightarrow -q \quad (34)$$

are linear canonical transformations that preserve the commutation relations and act as unitary transformations on the Hilbert space.

Now, we want to argue that the duality transformations in the string moduli space have their origin in the symplectic transformation of the canonical variables of the species quantum mechanics. In particular T-duality should correspond to a symplectic transformation of the form

$$t \rightarrow \pi_t, \quad \pi_t \rightarrow -t, \quad (35)$$

with $\pi_t = \frac{\alpha}{t^2} \dot{t}$, or, respectively, to

$$N_s \rightarrow \pi_{N_s} = \tilde{m}_t, \quad \pi_{N_s} = \tilde{m}_t \rightarrow -N_s \quad (36)$$

with $\tilde{m}_t = -m_t^{\frac{1-\alpha}{\alpha}} \dot{m}_t$. This can be best seen for the case $\alpha = 1$, where $N_s = t$ and $m_t = 1/t$, and $[N_s, \dot{m}_t] = -i$. Here, the T-duality transformation $t \rightarrow 1/t$ directly exchanges N_s and m_t . Hence, it is equivalent to the symplectic transformation in (36). Actually, these are the cases in which the symplectic transformation acts as a symmetry transformation on the Hilbert space, and T-duality corresponds to a self-duality on the string spectrum. An analogous behavior is for example expected for the $\text{SL}(2, \mathbb{Z})$ duality transformation of type IIB string theory.

We will also mention the connection between T-duality and symplectic transformations on the Calabi-Yau periods in the next section.

D. Species wave function

So far we only discussed some operators and their commutation relations of species quantum mechanics,

but not the Hilbert space, respectively the quantum mechanical wave functions, on which the operators act. This is of course of importance for the dynamics of the tower of species, i.e., how the number of species and the tower mass scale evolves in time. This should be compared with the classical dynamics of the species, as, e.g., described by some flow equations or also arguments coming from species thermodynamics.

It has long been known that the relevant field equation in quantum gravity is the Wheeler-DeWitt equation (WDW) [43,44]. More concretely, in the Arnowitt-Deser-Misner (ADM) formalism [45] the WDW equation takes the form

$$\mathcal{H}\Psi[h] = 0, \quad (37)$$

where h is the space-time metric and the Hamiltonian is given by

$$\mathcal{H} = \frac{1}{\sqrt{h}} \left(\pi^{ij} \pi_{ij} - \frac{1}{2} \pi^2 \right) - \sqrt{h} R. \quad (38)$$

Note that time and time derivatives do not appear in the WDW equation, i.e., no time evolution emerges in terms of the WDW wave function dependence.⁴

However, herein we are not discussing the quantum evolution of the d -dimensional space-time metric, but only the part from the internal geometry in the form of the matter moduli fields ϕ . This amounts to replace $\mathcal{H}$ by $\mathcal{H}_h + \mathcal{H}_\phi$ and the wave function will also depend of ϕ , viz., $\Psi[h, \phi]$. Moreover, in the presence of the extra field ϕ , one need to define time and a corresponding energy for ϕ . Then, in the WKB approximation, the WDW equation turns into an Hamilton-Jacobi equation for ϕ , see, e.g., [47]. This Hamilton-Jacobi equation can be regarded as coming from a corresponding Schrödinger equation. Therefore, we assume that the dynamics of the matter is determined by the wave function $\psi_s(t)$ of species quantum mechanics, which should correspond to some “wave function of the universe” satisfying a Schrödinger equation of the form

$$\mathcal{H}\psi_s = E\psi_s, \quad (39)$$

with the Hamiltonian as given in (8). The solutions of the Schrödinger equation will provide the expectation values for observables $\hat{\mathcal{O}}(t)$

$$\langle \hat{\mathcal{O}}(t) \rangle = \langle \psi_s | \hat{\mathcal{O}}(t) | \psi_s \rangle. \quad (40)$$

For example the expectation values of the number of species, $\langle N_s \rangle = \langle \psi_s | N_s | \psi_s \rangle$, or of the species scale, $\langle \Lambda_s \rangle = \langle \psi_s | \Lambda_s | \psi_s \rangle$, are important observables that can be measured in principle by experiments.

⁴Some ideas about the wave function of the universe in brane world models are given in [46].

The kinetic term of this Hamiltonian is just the Laplace operator on the moduli space $\mathcal{M}$.⁵ It can be most conveniently expressed in terms of the canonical field basis

$$\mathcal{H} = \frac{1}{2} \pi_\phi \pi_\phi. \quad (41)$$

Using that the operator π_ϕ can be expressed as $\pi_\phi = -i \frac{\partial}{\partial \phi}$ the Hamiltonian becomes

$$\mathcal{H} = -\frac{1}{2} \frac{\partial^2}{\partial \phi^2}. \quad (42)$$

For the case of a vanishing potential, $V(t) = 0$, the wave function has a similar form as the one of a free particle, namely

$$\psi_s(t) \sim \exp(ik_\phi \phi), \quad (43)$$

where k_ϕ is the momentum eigenvalue of π_ϕ . With $\phi = \sqrt{\alpha} \log t = \sqrt{\alpha} \log N_s$, the species wave function is proportional to

$$\psi_s(t) \sim \exp[i\sqrt{\alpha} k_\phi \log N_s(t)]. \quad (44)$$

This expression provides an interesting connection to the species thermodynamics: since N_s corresponds to the species entropy, namely to the entropy of a minimal black hole, i.e. $N_s = \mathcal{S}_{\text{BH}}$, we can write the wave function as

$$\psi_s(t) \sim \exp(i\sqrt{\alpha} k_\phi \log \mathcal{S}_{\text{BH}}). \quad (45)$$

For supersymmetric black holes, which are following the attractor mechanism, the black hole entropy as well as the scalar fields at the horizon are fully determined by a set of electric and magnetic charges (Q, P) : $t = t(Q, P)$ and $\mathcal{S}_{\text{BH}} = \mathcal{S}_{\text{BH}}(Q, P)$. So the species wave function will eventually be proportional to an expression of the form

$$\psi_s(P, Q) \sim \exp[i\sqrt{\alpha} k_\phi \log \mathcal{S}_{\text{BH}}(Q, P)]. \quad (46)$$

This wave function resembles some similarity with the OVV wave function of the universe, which we will mention in the next section. Note also that the charges (P, Q) undergo symplectic transformations, like N_s and π_t do.

Before proceeding, a brief word of caution is necessary. While the plane wave given in (43) is a definite-momentum solution to the Schrödinger equation, it is not a localized solution. Actually, ψ_s as given in (43) is not normalizable over infinite moduli space. Thus, to determine, e.g., $\langle N_s \rangle$ the wave function ψ_s must be a wave packet with a finite extent in moduli space (e.g., a Gaussian wave packet),

⁵Eigenfunctions of the Laplacian operator were discussed in [48].

which is a superposition of different momentum (or wave number k_ϕ) eigenstates,

$$\begin{aligned}\psi_s(t) &\sim \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \varphi(k_\phi) e^{ik_\phi \phi(t)} dk_\phi \\ &\sim e^{-\frac{[\phi(t)-\phi_0]^2}{2\sigma^2}} e^{ik_{\phi_0} \phi(t)},\end{aligned}\quad (47)$$

where σ determines the width of the packet and the expectation value $\langle \phi \rangle \sim \phi_0$ aligns with the classical trajectory. The expectation value then evolves over time according to Ehrenfest's theorem, where its velocity is related to the average momentum $\langle \pi_\phi \rangle = k_{\phi_0}$. Using (15) and (18) it is now straightforward to obtain $\langle N_s \rangle$.

We can also consider the case with a nonvanishing potential $V(t)$. At the classical level, the distance conjecture and the classical trajectories were recently investigated in several works [26,49–57]. E.g., a refined distance measure [54]

$$d(t) = \int d\lambda \sqrt{\frac{dt^i}{d\lambda} g_{ij}(\lambda) \frac{dt^j}{d\lambda} + 2V(t(\lambda))}. \quad (48)$$

can be used to determine the geodesics distance in parameter space in the presence of a potential. It would be interesting to compare the classical trajectories $t_{cl}(\tau)$ with the quantum trajectories $t_{qm}(\tau)$, which we will discuss in the following.

For that let us assume that the potential follows the anti-de Sitter conjecture (ADC) [49], which states that the value of the potential at its minimum is proportional to some power of the tower mass scale m_t . Here let us assume the entire function $V(t)$ follows the ADC, which leads to

$$V(t) = c[m_t(t)]^n, \quad (49)$$

where n is a positive parameter of order one, and c is negative for anti-de Sitter spaces and positive for de Sitter spaces. E.g., in many nonscale separated supersymmetric AdS vacua $n = 2$. With (12) the ADC looks like

$$V(t) = ct^{-m} \quad (m = n\alpha). \quad (50)$$

In terms of the canonical field basis this is identical to an exponential potential of the form

$$V(\phi) = c \exp(-n\sqrt{\alpha}\phi). \quad (51)$$

Let us briefly discuss what kind of wave functions are solutions of the Schrödinger equation (39). Actually, this potential is known as the Liouville potential and the Schrödinger equation with this potential is exactly solvable. The solutions for the wave function are given in terms of modified Bessel functions. For $\phi \rightarrow \infty$, they behave like plane waves: the form discussed before. For the AdS case,

the potential is attractive, whereas for the de Sitter case, the potential is repulsive. One relevant example is the dark dimension scenario [6] with $n = 4$ and $c^{1/4} \simeq 10^{-1} - 10^{-3}$; the size $R = 1/m_t$ of the dark dimension is in the micron range. For one dark dimension ($\alpha = 3/2$), one then gets that

$$V(t) = ct^{-6} = c \exp(-2\sqrt{6}\phi), \quad (52)$$

and for two dark dimensions ($\alpha = 1$) [58], the potential becomes

$$V(t) = c \exp(-4\phi). \quad (53)$$

Both potentials are repulsive and do not lead to bound states. It will be interesting to study the solutions of the Schrödinger equation for these kind of potentials in more detail.

III. $\mathcal{N} = 2$ CALABI-YAU STRING VACUA

In this section, we look at a specific class of $\mathcal{N} = 2$ models and also present their relation to the CRV pattern.

A. Type IIA Calabi-Yau compactifications and cubic prepotential

Type IIA compactifications on CY manifolds in the large Kähler moduli regime can be described by $\mathcal{N} = 2$ supergravity, with a classical prepotential (or F_0 term) that is a cubic polynomial in the vector multiplet scalars,

$$F_0(X^\Lambda) = -\frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0}, \quad (54)$$

where $X^\Lambda = (X^0, X^i)$, the indices i, j, k run over $1, \dots, h^{1,1} \equiv n_V$, and where C_{ijk} are the triple intersection numbers of the CY manifold. F_0 determines the classical two-derivative action, which includes the Einstein-Hilbert term and the kinetic energy of the scalar fields at quadratic level. The higher derivative corrections to the classical prepotential are organized in a series expansion,

$$F = F_0 + \sum_{g=1}^{\infty} F_g, \quad (55)$$

where F_g represents the contribution at genus g , corresponding to g -loop effects in the supergravity theory. Each F_g term contributes to the effective action by generating higher-derivative corrections, beyond the two-derivative level.

Defining $\mathcal{F}_0(z) = F_0(X)/(X^0)^2$, we can write the classical prepotential as

$$\mathcal{F}_0(z) = -\frac{1}{6} C_{ijk} z^i z^j z^k, \quad (56)$$

where the physical scalars of the vector multiplets can be parametrized as

$$z^i = \frac{X^i}{X^0}. \quad (57)$$

We recall that $(X^\Lambda, F_{0,\Lambda})$ are symplectic sections, with

$$F_{0,\Lambda} = \frac{\partial F_0}{\partial X^\Lambda} = X^0 \left(\frac{1}{6} C_{ijk} z^i z^j z^k, -\frac{1}{2} C_{ijk} z^j z^k \right). \quad (58)$$

Hyper multiplets are not described by the prepotential.

The Kähler potential is given by

$$\begin{aligned} K &= -\log [i(\bar{X}^\Lambda F_{0,\Lambda} - X^\Lambda \bar{F}_{0,\Lambda})] \\ &= -\log \left[\frac{i}{6} C_{ijk} (z^i - \bar{z}^i)(z^j - \bar{z}^j)(z^k - \bar{z}^k) X^0 \bar{X}^0 \right] \\ &= -\log \left[\frac{4}{3} C_{ijk} t^i t^j t^k X^0 \bar{X}^0 \right] \\ &= -\log [8\mathcal{V} X^0 \bar{X}^0], \end{aligned} \quad (59)$$

where we split the complex scalars into real and imaginary parts,

$$z^i = b^i + it^i, \quad (60)$$

and

$$\mathcal{V} = \frac{1}{6} \int_{CY} J \wedge J \wedge J = \frac{1}{6} C_{ijk} t^i t^j t^k \quad (61)$$

is the overall classical volume of the CY with Kähler 2-form J . The associate KK mass scale, related to the overall 6-dimensional volume is given by

$$m_{KK} = \mathcal{V}^{-1/6}. \quad (62)$$

Later, we will consider the case of anisotropic internal CY manifolds and their associated KK tower mass scales m_t in more detail, as well the so-called emergent string limit.

But before that, one can calculate

$$\begin{aligned} K_i &= \partial_i K = -\frac{i}{2} e^K C_{ijk} (z^j - \bar{z}^j)(z^k - \bar{z}^k) X^0 \bar{X}^0 \\ &= 2ie^K C_{ijk} t^j t^k X^0 \bar{X}^0, \end{aligned} \quad (63)$$

$$\begin{aligned} K_{\bar{i}} &= \partial_{\bar{i}} K = \frac{i}{2} e^K C_{ijk} (z^j - \bar{z}^j)(z^k - \bar{z}^k) X^0 \bar{X}^0 \\ &= -2ie^K C_{ijk} t^j t^k X^0 \bar{X}^0, \end{aligned} \quad (64)$$

and then the Kähler metric is explicitly

$$g_{i\bar{j}} = K_i K_{\bar{j}} + ie^K C_{ijk} (z^k - \bar{z}^k) X^0 \bar{X}^0. \quad (65)$$

Asking for the volumes of all complex submanifolds of the CY to be positive defines a region of the moduli space known as Kähler cone. For the supergravity approximation to be reliable, one should remain within the bulk of this region. It is convenient to pass to the real basis of fields t^i , since their axionic partners b^i do not enter the Kähler potential. Hence, throughout we set $b^i = 0$.

B. Three infinite distance limits and their associated mass scales

While remaining in the large volume regime, infinite distance limits in the vector multiplet moduli space of these theories correspond to sending a subset of the saxions to infinity, i.e., $t^i \rightarrow \infty$ with $i = 1, \dots, \mathbf{n}$ and $\mathbf{n} \leq h_{1,1}$, while all the axions are kept fixed. In general, the other saxions can be kept fixed at very large values—so as to stay in the large volume regime—or sent to infinity at slower rates. We will further specify the prepotential and we will consider three particular infinite distance limits of going to the asymptotic boundary of the moduli space. Concretely, we will consider the simple case of three Kähler moduli z^i ($i = 1, 2, 3$) and a classical prepotential with nonvanishing intersection number $C_{123} = 1$ of the form

$$F_0(X) = -z^1 z^2 z^3 (X^0)^2. \quad (66)$$

The infinite-distance limits have been carefully classified in [12,59,60] and in the context of the CRV pattern in [34,35,61,62]. In addition, the relation to classical black holes as probes of three infinite distance limits was discussed in [63]. Following the notation in [59], we will denote them as type IV, III and II. Let us summarize the three asymptotic limits in turn.

1. Type IV: Decompactification to five dimensions

Geometrically, this type of limit arises when the CY volume blows up uniformly. That is by identifying all three moduli, and we have $t^i \sim t \rightarrow \infty$ such that

$$\mathcal{V} \sim t^3. \quad (67)$$

In this limit, the prepotential simply becomes

$$\mathcal{F}_0(t) = it^3. \quad (68)$$

Furthermore, the mass scale of the related tower in 4D Planck units is now given as

$$\frac{(m_t)_{IV}}{M_p} \sim t^{-3/2}. \quad (69)$$

Comparing with the prepotential we see that

$$(m_t)_{\text{IV}} \sim (\Im \mathcal{F}_0)^{-1/2}. \quad (70)$$

2. Type III: Decompactification to six dimensions

Type III limits are allowed when the CY admits an elliptic fibration. In particular, the infinite distance limit corresponds to blowing up the four-dimensional base while the volume of the elliptic fiber remains constant in string units. Hence, the CY volume satisfies

$$\mathcal{V} \sim t^2. \quad (71)$$

This can be achieved by setting one modulus to a constant, e.g. $t_1 = 1$, and identifying the remaining two: $t_2 = t_3 = t$. Then the prepotential in (56) is *quadratic* in the moduli whose saxions are being sent to infinity

$$\mathcal{F}_0(t) = it^2. \quad (72)$$

The leading tower is composed by D0-branes and D2-branes wrapping the elliptic fiber, such that this limit corresponds to a decompactification to the 6D theory obtained by compactifying F-theory on the same CY. The mass scale of this KK tower is given by

$$\frac{(m_t)_{\text{III}}}{M_p} \sim t^{-1}. \quad (73)$$

Comparing with the prepotential we again see that

$$(m_t)_{\text{III}} \sim (\Im \mathcal{F}_0)^{-1/2}. \quad (74)$$

3. Type II: Emergent string limit

Finally, type II limits can happen when the CY admits a K3 or an Abelian (i.e., $\mathbb{T}^4$) fibration. In this case, the infinite distance limit is reached when the volume of the two-dimensional base blows up while that of the fibration remains fixed in string units. That is,

$$\mathcal{V} \sim t. \quad (75)$$

Something similar holds for the prepotential in (56), where we now fix $t_1 = t_2 = 1$, and sent $t_3 = t \rightarrow \infty$. Then the prepotential is found to be *linear* in the modulus whose saxion is being sent to infinity

$$\mathcal{F}_0(t) = it. \quad (76)$$

In this limit, the lightest object is given by an NS5-brane wrapping the fiber. The tower scale satisfies

$$\frac{(m_t)_{\text{II}}}{M_p} \sim t^{-1/2}. \quad (77)$$

One more time we recover the relation

$$(m_t)_{\text{II}} \sim (\Im \mathcal{F}_0)^{-1/2}. \quad (78)$$

In summary, as one can observe, the different types of limits are nicely distinguished from the EFT perspective by having a cubic, quadratic or linear behavior of the prepotential with the moduli whose saxions are being sent to infinity. On the other hand, we like to highlight that the associated tower mass scale m_t enjoys a universal relation to the prepotential in any of the three limits of the form

$$m_t(t) \sim (\Im \mathcal{F}_0(t))^{-1/2}. \quad (79)$$

The species scale shows up suppressing the curvature-squared corrections with respect to the Einstein-Hilbert term [27,40,64]. In four-dimensional $\mathcal{N} = 2$ supergravity, the Wilson coefficient of this higher-curvature correction is given by the genus-one topological string free energy, $F_1(X) = \mathcal{F}_1(z)$, such that

$$\Lambda_s^{-2} = N_s \sim \mathcal{F}_1(t^i). \quad (80)$$

For any asymptotic limit, $\mathcal{F}_1$ behaves linearly with t^i , and is given in terms of the second Chern number of the CY manifold as

$$N_s \sim \Im \mathcal{F}_1(t^i) \sim c_{2,i} t^i. \quad (81)$$

With respect to the large field limit of the leading field (omitting the index i) we see that there is a universal behavior of the species number like

$$N_s \sim c_2 t, \quad (82)$$

regardless of the type of limit that is being explored.

Using this result, we can now express the species scale in terms of the tower mass scale, first for the type IV limit as

$$\Lambda_s \sim (m_t)_{\text{IV}}^{1/3} M_p^{2/3}. \quad (83)$$

Consequently, the species scale is given by the 5D Planck scale, i.e., $\Lambda_s \sim M_{p,5}$.

Similarly, for the case III limit, we obtain

$$\Lambda_s \sim (m_t)_{\text{III}}^{1/2} M_p^{1/2}, \quad (84)$$

implying that the species scale corresponds to the 6D Planck scale, and thus is determined by $\Lambda_s \sim M_{p,6}$.

Lastly, for the type II limit, we obtain that

$$\Lambda_s \sim (m_t)_{\text{II}}, \quad (85)$$

confirming that this limit corresponds to an emergent string limit with $\Lambda_s \sim M_{\text{string}}$.

C. The CRV pattern and commutators in terms of Calabi-Yau periods

We have now prepared the ground to express the CRV relation, as given in (5), in terms of the relevant CY quantities. Recall that for the three different asymptotic limits IV, III, and II, we found the following two universal relations

$$\begin{aligned} m_t(t) &\sim (\Im \mathcal{F}_0(t))^{-1/2} \sim t^{-\alpha}, \\ N_s(t) &\sim \Im \mathcal{F}_1(t) \sim t. \end{aligned} \quad (86)$$

Then, we can rewrite the left-hand side of (5) as

$$\begin{aligned} \frac{\nabla m_t}{m_t} \cdot \frac{\nabla N_s}{N_s} &= -\frac{1}{2} \frac{\nabla \mathcal{F}_0(t)}{\mathcal{F}_0(t)} \cdot \frac{\nabla t}{t} \\ &= -\frac{1}{2} \frac{\partial t^\alpha}{t^\alpha} \frac{\partial t}{t} g^{tt}. \end{aligned} \quad (87)$$

Using the inverse Kähler metrics $g^{tt} = t^2/\alpha$ one easily confirms the CRV relation for all three limits IV, III, and II.

Let us now connect the Calabi-Yau periods to the basic commutation relations of the previous section. Duplicating our course of action, we define for a particular tower with label α the following pair of conjugate variables in terms of the periods $(t, \tilde{\mathcal{F}}_{0,t})$

$$N_s \simeq t \quad (88)$$

and

$$\begin{aligned} \tilde{m}_t &\simeq \Im \tilde{\mathcal{F}}_{0,t} = \frac{1}{2} t^{-1-2\alpha} \frac{d\Im \mathcal{F}_0}{d\tau} \\ &= \frac{1}{2} t^{-1-2\alpha} \Im \mathcal{F}_{0,t} \dot{t}. \end{aligned} \quad (89)$$

Then one can show that indeed

$$[t, \Im \tilde{\mathcal{F}}_{0,t}] = i. \quad (90)$$

We can try to generalize this for several towers with labels α_i to obtain the following canonical commutation relations for $(t^i, \Im \tilde{\mathcal{F}}_{0,i})$

$$[t^i, \Im \tilde{\mathcal{F}}_{0,j}] = i\delta_j^i. \quad (91)$$

In terms of the periods $(\Im X^\Lambda, \Re F_{0,\Lambda})$ one has the following quantization condition

$$[\Im X^\Lambda, \Re \tilde{F}_{0,\Lambda'}] = i\delta_{\Lambda'}^\Lambda. \quad (92)$$

Actually note that in this equation, the indices span over the following range: $\Lambda, \Lambda' = 0, \dots, n_V$. The corresponding periods $(X^\Lambda, \tilde{F}_{0,\Lambda})$ contain one nonpropagating scalar field, which we conveniently can set to one, e.g., $X^0 = 1$.

It follows that its conjugate momentum is zero: $\tilde{F}_{0,0} \simeq \dot{X}^0 = 0$. So with this choice the commutators in Eq. (92) reduce to the ones in Eq. (91). One can also see that T- and S-duality transformations act on the periods $(X^\Lambda, \tilde{F}_{0,\Lambda})$ as $\text{Sp}(2n_V + 2)$ symplectic transformations [65–67]. These symplectic transformation also act in natural way on the $2n_V + 2$ electric and magnetic field strengths, which include also the graviphoton and its magnetic dual.

Finally, concerning possible corrections to the number of species N_s and the tower mass scale m_t inside the $\mathcal{N} = 2$ moduli space we now like to suggest that inside the moduli space, the tower mass scale m_t and the species number N_s are still given as in (86), however, with $\mathcal{F}_0$ and $\mathcal{F}_1$ being the full nonperturbative expressions, which contain all exponentially suppressed instanton corrections. Hence we propose for the general CRV pattern including instanton corrections the following relation:

$$\frac{\nabla m_t}{m_t} \cdot \frac{\nabla N_s}{N_s} = -\frac{1}{2} \frac{\nabla \mathcal{F}_0(t)}{\mathcal{F}_0(t)} \cdot \frac{\nabla \mathcal{F}_1(t)}{\mathcal{F}_1(t)}. \quad (93)$$

It is tempting to speculate about a possible connection or generalization of this relation to the Bershadsky-Cecotti-Ooguri-Vafa holomorphic anomaly equations [68,69].

D. Connection to the OVV quantization

At the end of this section, we want to make contact with the quantization rules set up in OVV in [36]. In this paper it is argued that in the context of black hole quantization the topological string partition function can be interpreted as a “wave function of the universe in the minisuperspace sector of physical superstring theory.” Furthermore, a corresponding black hole entropy can be viewed as the square of this topological string wave function.

Actually, in [36] a Dirac bracket among the CY periods of the following form arises [see Eq. (5.11)⁶]:

$$[\Im X^\Lambda, \Im F_{0,\Lambda'}]_{\text{Dirac}} \sim i\delta_{\Lambda'}^\Lambda. \quad (94)$$

Clearly, this OVV commutator exhibits a very similar structure as the commutator in (92), which we have derived from the canonical commutation procedure.

Note that, unlike (92) and the other commutator relations discussed above, the OVV commutators do not involve time derivatives of the fields. This is because they are restricted to BPS configurations that live on a reduced phase space constrained by the BPS equations.

We can also briefly mention the mini-superspace wave function Ψ_{OVV} of OVV, which has the following schematic form

⁶Note that in our paper we are using a different convention compared to OVV, namely the real and imaginary parts of the periods are swapped.

$$|\Psi_{\text{OVV}}(Q, P)|^2 \sim \exp(\mathcal{S}_{\text{BH}}(Q, P)). \quad (95)$$

Here $\mathcal{S}_{\text{BH}} = F_{\text{top}} + \bar{F}_{\text{top}} + \dots$ is the entropy of an $\mathcal{N} = 2$ black hole that follows the attractor flow. Furthermore $F_{\text{top}} = \sum_{g=0}^{\infty} F_g$ is the full topological string partition function. We see that this wave function is quite similar to the species wave function (46), which we have discussed before.

IV. SUMMARY AND CONCLUSIONS

We have taken the first steps for laying a foundation of species quantum mechanics. Our main arguments can be summarized as follows.

First we defined canonical conjugate pairs of operators in the following way:

Consider a tower of species that satisfies the swampland distance conjecture or respectively the emergent string conjecture and also the CRV pattern relation. Then (asymptotically), this tower equivalently defines a pair of conjugate variables N_s and $\tilde{m}_t$ with $[N_s, \tilde{m}_t] = i$.

This leads to an uncertainty relation basically saying that a large species uncertainty implies a small uncertainty in the time variation of the tower mass and vice versa. As a consequence, we also demonstrated that the CRV pattern is closely related to this pair of conjugate variables.⁷ Furthermore we also investigated the corresponding canonical commutation relations of Calabi-Yau period vectors in $\mathcal{N} = 2$ string compactifications. Here we highlighted a universal relation of the species number N_s and the tower mass scale m_t in terms of the genus zero and the genus one prepotentials $\mathcal{F}_0$ and $\mathcal{F}_1$, which led us to a proposal for the CRV pattern including instanton corrections.

Second, we identified a species wave function of the universe that determines the quantum evolution of number of species and the tower mass scale in the following way:

The species wave function satisfies a Schrödinger equation $\mathcal{H}\psi_s = E\psi_s$, and particularly if the potential vanishes everywhere is proportional to $\psi_s(t) \sim \exp[ik \log N_s(t)] \sim \exp(ik \log \mathcal{S}_{\text{BH}})$.

This wave function corresponds to a kind of mini-superspace wave function of quantum gravity. It is important to stress that herein we do not consider the quantum

gravity over the four-dimensional space-time, but of the scalars that come from the compact part of the metric. It would be interested to see, which expectation values for N_s and m_t have high or low probability (see also [70,71]). Furthermore, we noted that the species wave function ψ_s is very similar to the OVV wave function in $\mathcal{N} = 2$ string compactifications. We also discussed some cases with nonvanishing potentials like those characterizing the dark dimension scenario.

Third we defined duality transformations in the following way:

Symplectic transformations on the pairs of conjugate variables $(N_s, \tilde{m}_t)$ lead to T-duality or S-duality transformations in the moduli space.

These duality transformations also correspond to symplectic transformations on Calabi-Yau periods in $\mathcal{N} = 2$ string compactifications.

Ultimately, the reader should be aware of a few caveats and questions in our arguments: (i) In order to define the species quantum mechanics, we simplified the discussion by considering only one dimension. It should be possible to extend the discussion to a more field theoretic framework in higher dimensions. (ii) There are corrections to the moduli dependence of the species tower as well as to the CRV pattern in the interior of the moduli space. So, how do these corrections change the commutator relations? (iii) We would like to have a better understanding about the corresponding species Hilbert space and the species wave functions. It would be interesting to use the species wave function to compute the probability density in moduli space. For example, does this probability density adhere to the (macroscopic) second law of species thermodynamics, being high at the boundary of the moduli space and low inside the moduli space?

Finally, it would be also interesting to consider other swampland conjectures and bounds, like the V'/V -bound [72] of the de Sitter conjecture [72–78]. These and other questions would be worthwhile to pursue in the future.

ACKNOWLEDGMENTS

We very much thank Carmine Montella for many enlightening discussions. We also thank Ivano Basile for valuable comments on the manuscript. The work of L. A. A. is supported by the U.S. National Science Foundation (NSF Grant No. PHY-2412679). The work of D. L. is supported by the Origins Excellence Cluster and by the German-Israel-Project (DIP) on Holography and the Swampland.

DATA AVAILABILITY

No data were created or analyzed in this study.

⁷As a side note, it is worth mentioning that the commutator in (28) always equates to (i times) the CRV pattern, because this relation is a proven property of quantum mechanical operators, as demonstrated in (26). Now, assuming that the CRV pattern holds, the operators within the commutator may be treated as conjugates, but of course we can explicitly use the pattern in the asymptotic regime. We have then examined the pattern for three possible types of light towers in infinite-distance limits, where the pattern has been shown to be satisfied.

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