

Dynamical Casimir effect in the worldline formulation

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We evaluate the effective action for the Dynamical Casimir Effect (DCE) for a real scalar field in $d + 1$ dimensions within the worldline formulation of quantum field theory. The scalar field is coupled to a spacetime-dependent mass term, which here plays the role of the moving medium and imposes imperfect boundary conditions on time-dependent surfaces. Expanding in powers of the departure of the geometry from a planar configuration, the worldline path integral factorizes into simpler, lower-dimensional ones. In the limit of a strong coupling to the surface, we recover the Dirichlet result and derive the systematic corrections in inverse powers of the coupling, calculating the imaginary part of the effective action to arbitrary order in those inverse powers. Finally, we also apply the method to a two-surface configuration.

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I. INTRODUCTION

The dynamical Casimir effect (DCE) refers to the production of particles from the vacuum, due to a time dependence of the boundary conditions or even of background fields [1–3]. While traditionally approached via canonical or functional integral quantization, the worldline formalism offers a powerful alternative. This has indeed been the case for the static Casimir effect, where in the approach popularized by Gies *et al.* [4] the interaction with the medium is introduced through a background potential. This, in turn, allows for the evaluation of the one-loop effective action in the context of the worldline [5]. Since the background potentials are assumed there to be time independent, the corresponding effective action is proportional to the Casimir energy. We show here how the worldline approach may also be useful for the analysis of the DCE. In particular, it allows us to evaluate the effective action as a measure of the amplitude for the creation of field quanta out of the vacuum.

This paper is organized as follows. In Sec. II we define the system under consideration and introduce the notation and conventions. In Sec. III we apply the worldline formalism to the effective action, develop its perturbative expansion, and analyze its dependence on λ , the coupling to the moving medium, calculating explicitly the imaginary part of the effective action to arbitrarily high order in $1/\lambda$.

We also identify the general structure of the higher-order terms.

Section IV extends the formalism to a two-surface configuration. Finally, Sec. V presents our conclusions.

II. THE MODEL

We consider a real scalar field $\varphi(x)$ in $D = d + 1$ dimensions, coupled to dynamical media, in such a way that its Euclidean action $\mathcal{S}(\varphi, V)$ may be written, after an integration by parts, as follows:

$$\mathcal{S}(\varphi, V) = \frac{1}{2} \int d^D x \varphi(x) (-\partial^2 + V) \varphi(x), \quad (1)$$

where x has been used to denote the $d + 1$ spacetime coordinates. We use the notation $x \equiv (x_{\parallel}, x_d)$, $x_{\parallel} \equiv (x_0, x_1, \dots, x_{d-1})$ being the spacetime coordinates which parametrize the world-volume of the dynamical medium. Space-time indices are denoted by letters from the middle of the Greek alphabet ($\mu, \nu, \dots$), taking values $0, 1, \dots, d$, with 0 reserved for the imaginary time (natural units are used: $\hbar \equiv 1$ and $c \equiv 1$). The metric reduces to the $(d + 1) \times (d + 1)$ identity matrix, so no distinction is implied by placing an index upstairs or downstairs. Unless stated otherwise, we also adopt the Einstein summation convention over repeated indices in monomial expressions.

The role of the potential is straightforward: it suppresses the scalar field wherever it becomes large and positive, and in the strong-coupling limit it enforces Dirichlet boundary conditions. This occurs even though the form of the action places the system within the regime of linear response. Moreover, when more than one object is coupled to the

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vacuum field φ , the full potential V is obtained by adding the corresponding contributions from each object, as dictated by the assumed linear response of the media. The additivity of potentials does not, however, imply any linear superposition principle for the physical effects: energies, probabilities, and related observables are not obtained by summing the contributions of the individual parts of the media, but are instead nontrivial functionals of V .

The potential $V(x)$, for the case of a single surface Σ , is assumed to have the form

$$V(x) = v[F(x)], \quad F(x) \equiv x_d - \psi(x_{\parallel}) \quad (2)$$

where Σ is defined by $x_d = \psi(x_{\parallel})$, and ψ is treated as a small perturbation around a flat configuration. On the other hand, v is a real non-negative function of a single variable, highly concentrated around 0. In fact, the first example we consider corresponds to a δ function (zero-width surface). We find it convenient, at this point, to mention that the choice of F is of course not unique. Let $F: \mathbb{R}^D \rightarrow \mathbb{R}$ be a smooth function with $\nabla F \neq 0$ on the level set $\Sigma = F^{-1}(0)$, which defines Σ . By the coarea formula,

$$\delta(F(x)) = \frac{1}{|\nabla F(x)|} \delta_{\Sigma}(x), \quad (3)$$

where δ_{Σ} denotes the delta distribution intrinsic to Σ . It follows that the combination $|\nabla F(x)|\delta(F(x))$ defines a measure on Σ that is manifestly independent of the choice of defining function F . For the Monge parametrization $F(x) = x_d - \psi(x_{\parallel})$, one has $|\nabla F| = \sqrt{1 + (\nabla_{\parallel}\psi)^2} = 1 + O(\psi^2)$, so the discrepancy arising from the $|\nabla F|$ factor enters only at second order in derivatives of ψ . Moreover, as explained in [6], this factor contributes only to the real (nondissipative) part of the effective action and is therefore irrelevant for the evaluation of pair-creation effects.

III. EFFECTIVE ACTION

The one-loop effective action $\Gamma(V)$ is defined in terms of the vacuum-to-vacuum Euclidean transition amplitude, $\mathcal{Z}$, via

$$e^{-\Gamma(V)} \equiv \frac{\mathcal{Z}(V)}{\mathcal{Z}(0)}, \quad \mathcal{Z}(V) \equiv \int \mathcal{D}\varphi e^{-S(\varphi, V)}, \quad (4)$$

where the $\mathcal{Z}(0)$ factor has been introduced in order to subtract the usual zero-point contribution of free space (namely, in the absence of any object). Note, however, that we perform further subtractions to the effective action, corresponding to terms that do not contribute to pair creation, which is the main focus of this work.

Since the functional integrals are Gaussian, we can safely apply the worldline approach [4,5], whereby we may write

$$\Gamma(V) = -\frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int d^{d+1}x \left[\left\langle e^{-\int_0^T d\tau V(x(\tau))} \right\rangle_x - 1 \right] \quad (5)$$

where

$$V(x(\tau)) \equiv v(x_d(\tau) - \psi(x_{\parallel}(\tau))), \quad (6)$$

and $\langle \dots \rangle_x$ denotes a functional average over closed loops $x_{\mu}(\tau)$ starting and ending at the same point, $x_{\mu}(0) = x_{\mu}(T) \equiv x$,

$$\langle \dots \rangle_x \equiv \frac{\int_{x(0)=x}^{x(T)=x} \mathcal{D}x \dots e^{-\frac{1}{4} \int_0^T d\tau \dot{x}_{\mu}(\tau) \dot{x}_{\mu}(\tau)}}{\int_{x(0)=x}^{x(T)=x} \mathcal{D}x e^{-\frac{1}{4} \int_0^T d\tau \dot{x}_{\mu}(\tau) \dot{x}_{\mu}(\tau)}} \quad (7)$$

Expression (5) for the effective action, when rotated back to Minkowski spacetime in its dependence on ψ , will allow us to extract the dissipative effects due to pair creation from the evaluation of its imaginary part.

A. Perturbative expansion

Let us expand the potential in powers of ψ

$$V(x) \equiv v(x_d - \psi(x_{\parallel})) = v(x_d) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} v^{(n)}(x_d) (\psi(x_{\parallel}))^n, \quad (8)$$

and introduce it into the worldline, to get the respective expansion for the effective action,

$$\Gamma(V) = \Gamma_0 + \Gamma_1(\psi) + \Gamma_2(\psi) + \dots \quad (9)$$

A key point is that, in the perturbative expansion above, all required worldline path integrals factorize into parallel and perpendicular averages. Let us display this explicitly for the first few terms.

At zeroth order one obtains

$$\Gamma_0 = -\frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int d^{d+1}x \left[\left\langle e^{-\int_0^T d\tau v(x_d(\tau))} \right\rangle_{x_d} - 1 \right], \quad (10)$$

with

$$\langle \dots \rangle_{x_d} \equiv \frac{\int_{x_d(0)=x_d}^{x_d(T)=x_d} \mathcal{D}x_d \dots e^{-\frac{1}{4} \int_0^T d\tau (\dot{x}_d(\tau))^2}}{\int_{x_d(0)=x_d}^{x_d(T)=x_d} \mathcal{D}x_d e^{-\frac{1}{4} \int_0^T d\tau (\dot{x}_d(\tau))^2}}. \quad (11)$$

It is evident from (10) that this contribution to the effective action is divergent, since it is proportional to the spacetime volume swept by the surface over an infinite time interval.

We therefore introduce a spacetime box with total evolution time T and a $(d-1)$ -dimensional “area” L^{d-1} for the flat surface. It is then natural to evaluate $\mathcal{E}_0$, the vacuum energy per unit area, in the limit when the size of the box tends to infinity,

$$\mathcal{E}_0 \equiv \left[\frac{\Gamma_0}{T L^{d-1}} \right]_{T, L \rightarrow \infty} = -\frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int dx_d \left[\langle e^{-\int_0^T d\tau v(x_d(\tau))} \rangle_{x_d} - 1 \right], \quad (12)$$

which, depending on d and v , is usually divergent in the UV. On the other hand, note that it does not contribute to the imaginary part of the effective action.

We now consider the first-order term, which after standard manipulations may be written as

$$\Gamma_1(\psi) = \frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau \left[\int d^d x_{\parallel} \langle \psi(x_{\parallel}(\tau)) \rangle_{x_{\parallel}} \int dx_d \langle v'(x_d(\tau)) e^{-\int_0^T d\tau' v(x_d(\tau'))} \rangle_{x_d} \right]. \quad (13)$$

To tackle this term, it is convenient to deal first with the factor involving a functional average of ψ . In terms of the Fourier transform of ψ , $\tilde{\psi}(k_{\parallel}) \equiv \int d^d x_{\parallel} e^{-ik_{\parallel} \cdot x_{\parallel}} \psi(x_{\parallel})$, and taking into account the fact that the worldline correlation function is,

$$\langle x_{\mu}(\tau_1) x_{\nu}(\tau_2) \rangle = \delta_{\mu\nu} \Delta(\tau_1 - \tau_2), \quad \Delta(\tau_1 - \tau_2) = \frac{(\tau_1 - \tau_2)^2}{T} - |\tau_1 - \tau_2|, \quad (14)$$

we find,

$$\int d^d x_{\parallel} \langle \psi(x_{\parallel}(\tau)) \rangle_{x_{\parallel}} = \tilde{\psi}(0) = \int d^d x_{\parallel} \psi(x_{\parallel}). \quad (15)$$

This term may therefore be set to zero by a shift of origin, chosen so that the mean value of ψ vanishes. We assume that such a choice has been made, and therefore $\Gamma_1(\psi) = 0$.

For the second-order term $\Gamma_2(\psi)$ one finds a natural decomposition into two contributions, $\Gamma_2 = \Gamma_{2,1} + \Gamma_{2,2}$, namely

$$\begin{aligned} \Gamma_{2,1}(\psi) &= \frac{1}{4(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau \int dx_d \langle v''(x_d(\tau)) \rangle_{x_d} \int d^d x_{\parallel} \langle (\psi(x_{\parallel}(\tau)))^2 \rangle_{x_{\parallel}}, \\ \Gamma_{2,2}(\psi) &= -\frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int d^d x_{\parallel} \langle \psi(x_{\parallel}(\tau_1)) \psi(x_{\parallel}(\tau_2)) \rangle_{x_{\parallel}} \\ &\quad \times \int dx_d \langle v'(x_d(\tau_1)) v'(x_d(\tau_2)) e^{-\int_0^T d\tau v(x_d(\tau))} \rangle_{x_d}. \end{aligned} \quad (16)$$

In both terms we need the average of two ψ insertions, one with coincident proper-time arguments. Using Fourier transforms once again, we obtain

$$\int d^d x_{\parallel} \langle \psi(x_{\parallel}(\tau_1)) \psi(x_{\parallel}(\tau_2)) \rangle_{x_{\parallel}} = \int \frac{d^d k_{\parallel}}{(2\pi)^d} |\tilde{\psi}(k_{\parallel})|^2 e^{\Delta(\tau_1 - \tau_2) k_{\parallel}^2}. \quad (17)$$

It then follows that $\Gamma_{2,1}$, independently of the detailed form of v , yields a term that cannot generate an imaginary part in the effective action; it may be interpreted as a mass term for ψ ,

$$\Gamma_{2,1}(\psi) = \frac{1}{2} m^2 \int d^d x_{\parallel} (\psi(x_{\parallel}))^2, \quad (18)$$

with

$$m^2 = \frac{1}{2(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau \int dx_d \langle v''(x_d(\tau)) \rangle_{x_d}. \quad (19)$$

For the evaluation of $\Gamma_{2,2}(\psi)$, we need to be more explicit about the form of v . We first assume that $v(x_d) = \lambda\delta(x_d)$. Inserting the explicit form of the worldline average into (16) and using the normalization of the free path integral, the relevant object may be written in terms of the propagator of a one-dimensional system,

$$\begin{aligned} I(\tau_1, \tau_2; T) &\equiv \int dx_d \left\langle v'(x_d(\tau_1)) v'(x_d(\tau_2)) e^{-\int_0^T d\tau v(x_d(\tau))} \right\rangle_{x_d} \\ &= (4\pi T)^{\frac{1}{2}} \int_{x_d(T)=x_d(0)} \mathcal{D}x_d v'(x_d(\tau_1)) v'(x_d(\tau_2)) e^{-\int_0^T \left(\frac{1}{4} \dot{x}_d^2(\tau) + \lambda\delta(x_d(\tau)) \right) d\tau}, \end{aligned} \quad (20)$$

so that $\Gamma_{2,2}$ becomes

$$\Gamma_{2,2}(\psi) = \frac{1}{2} \int \frac{d^d k_{\parallel}}{(2\pi)^d} \gamma(k_{\parallel}) |\tilde{\psi}(k_{\parallel})|^2 \quad (21)$$

with

$$\gamma(k_{\parallel}) = -\frac{1}{(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 I(\tau_1, \tau_2; T) e^{\Delta(\tau_1 - \tau_2) k_{\parallel}^2}. \quad (22)$$

The value of γ at zero momentum corresponds to a redefinition of m^2 . We shall use this fact to control the UV behavior of the integrals defining γ .

The path integral (20) may be written in terms of K , the propagator in the presence of a δ potential [7], as follows:

$$I(\tau_1, \tau_2; T) = (4\pi T)^{\frac{1}{2}} \lambda^2 \int_{-\infty}^{+\infty} dx_d \int_{-\infty}^{+\infty} dy_d \int_{-\infty}^{+\infty} dz_d [K_a(x_d, y_d) \delta'(y_d) K_b(y_d, z_d) \delta'(z_d) K_c(z_d, x_d)] \quad (23)$$

where

$$K_a \equiv K(T_a; \cdot, \cdot), \quad K_b \equiv K(T_b; \cdot, \cdot), \quad K_c \equiv K(T_c; \cdot, \cdot), \quad (24)$$

with proper-time intervals

$$T_a = T - \tau_1, \quad T_b = \tau_1 - \tau_2, \quad T_c = \tau_2, \quad T_a + T_c = T - T_b \quad (25)$$

and

$$K(t; x, y) = \frac{1}{2\sqrt{\pi t}} e^{-(x-y)^2/(4t)} - \frac{\lambda}{4} \exp \left[\frac{\lambda}{2} (|x| + |y|) + \frac{\lambda^2}{4} t \right] \operatorname{erfc} \left[\frac{|x| + |y| + \lambda t}{2\sqrt{t}} \right], \quad (26)$$

where $t = \tau'' - \tau'$ denotes the elapsed proper time.

Using $\int f(x) \delta'(x) dx = -f'(0)$ twice in succession, the three spatial integrals reduce to four terms involving the middle kernel K_b and its derivatives evaluated at the origin,

$$\begin{aligned} I &= (4\pi T)^{\frac{1}{2}} \lambda^2 \int dx_d [\partial_2 K_a(x_d, 0) \partial_2 K_b(0, 0) K_c(0, x_d) + K_a(x_d, 0) \partial_1 \partial_2 K_b(0, 0) K_c(0, x_d) \\ &\quad + \partial_2 K_a(x_d, 0) K_b(0, 0) \partial_1 K_c(0, x_d) + K_a(x_d, 0) \partial_1 K_b(0, 0) \partial_1 K_c(0, x_d)]. \end{aligned} \quad (27)$$

where ∂_1, ∂_2 denote derivatives with respect to the first and second spatial arguments.

Evaluating the derivatives of K at the origin using (26), performing the x_d integral via the semigroup property of the heat kernel, and combining the four terms, we find,

$$I(\tau_1, \tau_2; T) = \frac{\lambda^2 \sqrt{T}}{2} \left[\frac{K(T - T_b; 0, 0)}{T_b^{3/2}} + \frac{K(T_b; 0, 0)}{(T - T_b)^{3/2}} \right] \quad (28)$$

where $T_b = \tau_1 - \tau_2$. The expression is manifestly symmetric under $T_b \leftrightarrow T - T_b$.

Consider now the large- λ expansion of I . Introducing $S \equiv T - T_b$,

$$I(\tau_1, \tau_2; T) = \frac{\sqrt{T}}{\sqrt{4\pi}(T_b S)^{3/2}} \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!! 2^n}{\lambda^{2n}} \times \left[\frac{1}{S^n} + \frac{1}{T_b^n} \right] \quad (29)$$

of which the Dirichlet limit ($\lambda \rightarrow \infty$) is,

$$I_{\infty}(\tau_1, \tau_2; T) = \frac{\sqrt{T}}{\sqrt{\pi}[T_b(T - T_b)]^{3/2}}. \quad (30)$$

$$\begin{aligned} \gamma_{\infty}(k_{\parallel}) &= -\frac{1}{2\pi(4\pi)^{\frac{d}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T d\tau_1 \\ &\times \int_0^{\tau_1} d\tau_2 \frac{e^{\left[\frac{(\tau_1 - \tau_2)^2}{T} - |\tau_1 - \tau_2|\right]k_{\parallel}^2}}{[(\tau_1 - \tau_2)(T - \tau_1 + \tau_2)]^{3/2}} \\ &= -\frac{1}{2\pi(4\pi)^{\frac{d}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T d\tau \frac{e^{\left[\frac{\tau^2}{T} - |\tau|\right]k_{\parallel}^2}}{\tau^{3/2}(T - \tau)^{1/2}}. \end{aligned} \quad (31)$$

Subtracting its value at $k_{\parallel} = 0$,

$$\begin{aligned} \gamma_D(k_{\parallel}) &\equiv \gamma_{\infty}(k_{\parallel}) - \gamma_{\infty}(0) \\ &= -\frac{1}{2\pi(4\pi)^{\frac{d}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T d\tau \frac{e^{\left[\frac{\tau^2}{T} - |\tau|\right]k_{\parallel}^2} - 1}{\tau^{3/2}(T - \tau)^{1/2}}. \end{aligned} \quad (32)$$

Under a change of variables,

$$\gamma_D(k_{\parallel}) = -\frac{1}{2\pi(4\pi)^{\frac{d}{2}}} |k_{\parallel}|^{d+2} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+4}{2}}} \int_0^1 d\tau \frac{e^{T(\tau^2 - \tau)} - 1}{\tau^{3/2}(1 - \tau)^{1/2}}. \quad (33)$$

The double integral can be evaluated in closed form. Setting $v = \tau(1 - \tau)$, the proper-time integral is performed by dimensional regularization,

$$\int_0^{\infty} \frac{dT}{T^{(d+4)/2}} (e^{-vT} - 1) = v^{(d+2)/2} \Gamma\left(-\frac{d+2}{2}\right), \quad (34)$$

where the subtraction is automatically handled by the analytic continuation in d . The remaining τ integral yields a beta function,

$$\begin{aligned} \int_0^1 d\tau \frac{[\tau(1 - \tau)]^{(d+2)/2}}{\tau^{3/2}(1 - \tau)^{1/2}} &= \int_0^1 d\tau \tau^{(d-1)/2} (1 - \tau)^{(d+1)/2} \\ &= \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{d+3}{2})}{\Gamma(d+2)}. \end{aligned} \quad (35)$$

Combining, one obtains the closed-form expression

$$\gamma_D(k_{\parallel}) = -\frac{|k_{\parallel}|^{d+2}}{2\pi(4\pi)^{d/2}} \frac{\Gamma(-\frac{d+2}{2})\Gamma(\frac{d+1}{2})\Gamma(\frac{d+3}{2})}{\Gamma(d+2)}, \quad (36)$$

which agrees with the result for the Euclidean form factors obtained in Refs. [6,8], where the Dirichlet limit was treated as a starting point. For odd $d = 2q + 1$, all Gamma functions are finite and the coefficient simplifies to $(-1)^{q+1}(q!)^2/[(2\pi)^{q+1}(2q+1)!(2q+3)!!]$ [cf. Eq. (39) with $n = 0$]. The first few explicit values are,

$$\gamma_D = \begin{cases} -\frac{1}{6\pi} |k_{\parallel}|^3 & (d = 1), \\ +\frac{1}{360\pi^2} |k_{\parallel}|^5 & (d = 3), \\ -\frac{1}{25200\pi^3} |k_{\parallel}|^7 & (d = 5). \end{cases} \quad (37)$$

B. Large- λ corrections to γ

Let us now derive the corrections to γ_D that arise from keeping subleading terms in the $1/\lambda$ expansion of I . Substituting the asymptotic expansion (29) into the expression for γ , and performing the center-of-mass integration over the double proper-time integral (which produces a factor of $T - T_b$), we find that the n th order contribution is,

$$\begin{aligned} \gamma_n(k_{\parallel}) &= -\frac{(-1)^n (2n+1)!! 2^{n-1}}{\lambda^{2n} 2\pi(4\pi)^{\frac{d}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T dT_b e^{\Delta(T_b)k_{\parallel}^2} \\ &\times \left[\frac{1}{T_b^{3/2}(T - T_b)^{n+\frac{1}{2}}} + \frac{1}{T_b^{n+\frac{3}{2}}(T - T_b)^{1/2}} \right]. \end{aligned} \quad (38)$$

The subtracted form $\gamma_n^{\text{sub}} \equiv \gamma_n(k_{\parallel}) - \gamma_n(0)$, after rescaling $T_b = uT$ and $T \rightarrow T/k_{\parallel}^2$, reads,

$$\gamma_n^{\text{sub}}(k_{\parallel}) = \frac{(-1)^{n+1} (2n+1)!! 2^{n-1}}{\lambda^{2n} 2\pi(4\pi)^{\frac{d}{2}}} |k_{\parallel}|^{d+2n+2} C_n(d), \quad (39)$$

where

$$C_n(d) = \int_0^{\infty} \frac{dT}{T^{\frac{d+2n+4}{2}}} \int_0^1 du \frac{e^{-u(1-u)T} - 1}{u^{3/2}(1-u)^{1/2}} \left[\frac{1}{(1-u)^n} + \frac{1}{u^n} \right]. \quad (40)$$

The T integral is evaluated by dimensional regularization, $\int_0^{\infty} T^{-\alpha} e^{-vT} dT = v^{\alpha-1} \Gamma(1 - \alpha)$, and the resulting u integral factorizes into Beta functions. One obtains,

$$\begin{aligned} C_n(d) &= \frac{\Gamma(-\frac{d+2n+2}{2})}{\Gamma(d+n+2)} \left[\Gamma\left(\frac{d+2n+1}{2}\right) \Gamma\left(\frac{d+3}{2}\right) \right. \\ &\quad \left. + \Gamma\left(\frac{d+1}{2}\right) \Gamma\left(\frac{d+2n+3}{2}\right) \right]. \end{aligned} \quad (41)$$

The leading (Dirichlet, $n = 0$) contribution recovers γ_D .

For d odd, the gamma functions are finite for all n . For instance, setting $d = 2q + 1$,

$$\gamma_0^{\text{sub}} = \frac{(-1)^{q+1}(q!)^2}{(2\pi)^{q+1}(2q+1)!(2q+3)!!} |k_{\parallel}|^{2q+3}, \quad (42)$$

in agreement with Ref. [6]. The first two corrections, for $d = 1$, read,

$$\gamma_1^{\text{sub}} = -\frac{1}{5\pi\lambda^2} |k_{\parallel}|^5, \quad \gamma_2^{\text{sub}} = -\frac{8}{21\pi\lambda^4} |k_{\parallel}|^7. \quad (43)$$

For even d , the factor $\Gamma[-(d+2n+2)/2]$ develops poles. These singularities are treated by dimensional regularization and give rise to logarithmic contributions. As discussed below, these logarithms are the terms that generate the imaginary part of the effective action when d is even.

C. Imaginary part and pair creation

To extract the imaginary part of the effective action, which encodes the dissipative effects of the system under consideration, we perform a Wick rotation to Minkowski spacetime. More explicitly, writing $k_{\parallel} = (k_0, \mathbf{k})$, the Wick rotation is implemented through the continuation $k_0 \rightarrow -i(\omega + i0)$, so that $k_{\parallel}^2 \rightarrow \mathbf{k}^2 - \omega^2 - i0 = -p_{\parallel}^2 - i0$, with $p_{\parallel}^2 \equiv \omega^2 - \mathbf{k}^2$. Together with the corresponding continuation of the effective action, $-\Gamma_2 \rightarrow i\Gamma_2^{(M)}$, this prescription fixes the branch of the nonanalytic terms and determines the imaginary part associated with time-like modes.

For odd values of the spatial dimension d , the timelike modes of the surface give an imaginary contribution because of the half-integer power in the factor $(-p_{\parallel}^2)^{\frac{d+2n+2}{2}}$. Setting $d = 2q + 1$, this can be written as

$$\begin{aligned} \text{Im}(\Gamma_2^{(M)}) &= \frac{1}{2} \int \frac{d^{2q+1} p_{\parallel}}{(2\pi)^{2q+1}} \Theta(p_{\parallel}^2) |\tilde{\psi}(p_{\parallel})|^2 \\ &\times \sum_{n=0}^{\infty} \zeta(n, q) (p_{\parallel}^2)^{\frac{2q+2n+3}{2}} \end{aligned} \quad (44)$$

$$\zeta(n, q) = \frac{(-1)^n 2^{2n}}{\lambda^{2n} (2\pi)^{q+1}} \frac{(2n+1)!!}{(2q+2n+3)!!} \frac{q!(q+n)!}{(2q+n+1)!} \quad (45)$$

where $\zeta(0, q)$ corresponds to the dissipation coefficient in the Dirichlet case, which agrees with Ref. [6].

For even values of d , we use dimensional regularization $d = 2q - \epsilon$ and set $\epsilon \rightarrow 0$, introducing imaginary contributions through $\log(-\frac{p_{\parallel}^2}{\mu^2})$, where μ^2 is introduced for dimensional consistency. This allows us to write

$$\text{Im}(\Gamma_2^{(M)}) = \frac{1}{2} \int \frac{d^{2q} p_{\parallel}}{(2\pi)^{2q}} \Theta(p_{\parallel}^2) |\tilde{\psi}(p_{\parallel})|^2 \sum_{n=0}^{\infty} \xi(n, q) (p_{\parallel}^2)^{q+n+1} \quad (46)$$

$$\xi(n, q) = \frac{(-1)^n \pi^{1-q} (2n+1)!(2q+2n)!(2q)!}{4^{3q+n+1} n! \lambda^{2n} (q+n)! q! (q+n+1)!(2q+n)!} \quad (47)$$

where $\xi(0, q)$ also gives the correct Dirichlet factor of Ref. [6].

Several interesting features emerge from these expressions. First, the coefficients alternate in sign with n , with even values of n giving positive contributions. This is consistent with the positivity of the leading Dirichlet term, which controls the dominant contribution to the imaginary part of the effective action and, consequently, to the pair-creation probability. The higher- n terms should be understood as finite-coupling corrections to this result: their alternating signs indicate that successive corrections may either enhance or reduce the leading contribution within the regime of validity of the expansion. Second, each higher- n correction has the same momentum-power dependence as the Dirichlet contribution in the shifted dimension $d + 2n$. Finally, the factor $\Theta(p_{\parallel}^2)$ shows that only timelike Fourier components of the surface deformation contribute, as expected for the production of real quanta. In particular, the kinematic threshold for pair creation is the same for all n .

D. Exact form factor at finite coupling

In this subsection we evaluate (22) using (28), obtaining the exact form factor $\gamma(k_{\parallel})$ valid for arbitrary coupling strength λ . Substituting the full propagator (26) into (28) and then into (22), we arrive at one of the central results of this work,

$$\begin{aligned} \gamma(k_{\parallel}) &= -\frac{\lambda^2}{2(4\pi)^{(d+1)/2}} \int_0^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 e^{k_{\parallel}^2 \left(\frac{(\tau_1 - \tau_2)^2}{T} - (\tau_1 - \tau_2) \right)} \sqrt{T} \left[\frac{1}{2\sqrt{\pi}} \left(\frac{1}{(T - T_b)^{\frac{1}{2}} T_b^{\frac{3}{2}}} + \frac{1}{(T - T_b)^{\frac{3}{2}} T_b^{\frac{1}{2}}} \right) \right. \\ &\quad \left. - \frac{\lambda}{4} \left(\frac{e^{\frac{\lambda^2}{4}(T - T_b)}}{T_b^{\frac{3}{2}}} \text{erfc} \left(\frac{\lambda}{2} \sqrt{T - T_b} \right) + \frac{e^{\frac{\lambda^2}{4} T_b}}{(T - T_b)^{\frac{3}{2}}} \text{erfc} \left(\frac{\lambda}{2} \sqrt{T_b} \right) \right) \right]. \end{aligned} \quad (48)$$

This function has UV divergences, as expected, and all of them can be shown to have the form of a polynomial in $k_{\parallel}^2$. This is indeed what one could expect since those divergences should correspond to local counterterms, that could be introduced

as part of an action for the surface. Being then a polynomial, they cannot contribute to the imaginary part of the effective action, since the latter is always nonanalytic in momenta, and finite.

The integral is regularized using the standard BPHZ prescription, named after Bogoliubov, Parasiuk, Hepp, and Zimmermann, by subtracting a polynomial in $k_{\parallel}^2$ of degree $N = \lfloor d/2 \rfloor$. Indeed, setting $T_b = \tau_1 - \tau_2$, this subtraction is achieved by replacing the momentum-dependent exponential in the integrand with its Taylor remainder up to order N ,

$$e^{-k_{\parallel}^2 \frac{T_b(T-T_b)}{T}} \rightarrow e^{-k_{\parallel}^2 \frac{T_b(T-T_b)}{T}} - \sum_{n=0}^{\lfloor d/2 \rfloor} \frac{(-1)^n}{n!} \left(k_{\parallel}^2 \frac{T_b(T-T_b)}{T} \right)^n. \quad (49)$$

This suppresses both ($T_b \rightarrow 0$) and small- T singularities. It shifts the proper-time integration scaling from $T^{-\frac{d}{2}-1}$ to $T^{\lfloor d/2 \rfloor - \frac{d}{2}}$, securing both UV and IR convergence. The degree of the subtraction is, as always, *sufficient* to render the expression finite.

The subtraction of this polynomial in $k_{\parallel}^2$ to render the expression finite can be rephrased as follows: the divergent part is local in momentum space and has the form,

$$\gamma_{\text{div}}(k_{\parallel}) = \sum_{n=0}^{\lfloor d/2 \rfloor} c_n(d, \lambda) (k_{\parallel}^2)^n, \quad (50)$$

(where some of the coefficients may be absent). Hence it does not contribute to the DCE.

Let us now study the finite part of $\gamma(k_{\parallel})$, which we denote by $\gamma_{\text{sub}}(k_{\parallel})$, obtained by performing the subtraction above.¹ We start by decomposing (48) as $\gamma(k_{\parallel}) = \gamma_1(k_{\parallel}) + \gamma_2(k_{\parallel})$, where γ_1 collects the free-propagator contribution and γ_2 contains the *erfc* terms.

We begin with $\gamma_1(k_{\parallel})$. After integrating over τ_1 and renaming $T_b = \tau$, we obtain

$$\gamma_1(k_{\parallel}) = -\frac{\lambda^2}{2(4\pi)^{(d+1)/2}} \int_0^\infty \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T d\tau (T-\tau) e^{k_{\parallel}^2 \left[\frac{\tau^2}{T} - \tau \right]} \times \frac{1}{2\sqrt{\pi}} \left(\frac{1}{(T-\tau)^{\frac{1}{2}} \tau^{3/2}} + \frac{1}{(T-\tau)^{3/2} \tau^{\frac{1}{2}}} \right), \quad (51)$$

which, after the change of variable $\tau = T\mu$, leaves us with

$$\gamma_1(k_{\parallel}) = -\frac{\lambda^2}{2(4\pi)^{(d+2)/2}} \int_0^\infty \frac{dT}{T^{\frac{d+2}{2}}} \int_0^1 d\mu e^{-T k_{\parallel}^2 \mu(1-\mu)} \times \left(\frac{(1-\mu)^{1/2}}{\mu^{3/2}} + \frac{1}{(1-\mu)^{1/2} \mu^{1/2}} \right) \quad (52)$$

$$= -\lambda^2 (k_{\parallel}^2)^{\frac{d}{2}} \frac{\Gamma(-\frac{d}{2}) \Gamma(\frac{d-1}{2}) \Gamma(\frac{d+1}{2})}{2(4\pi)^{\frac{d+2}{2}} \Gamma(d)}. \quad (53)$$

Equation (52) is well defined for all odd $d > 1$, while for even d it must be understood through dimensional regularization, as expected.

For $\gamma_2(k_{\parallel})$ one finds

$$\gamma_2(k_{\parallel}) = \frac{\lambda^3}{8(4\pi)^{(d+1)/2}} \int_0^\infty \frac{dT}{T^{\frac{d+2}{2}}} \int_0^T d\tau (T-\tau) e^{k_{\parallel}^2 \left[\frac{\tau^2}{T} - \tau \right]} \left(\frac{e^{\frac{k_{\parallel}^2}{4}(T-\tau)}}{\tau^{\frac{3}{2}}} \text{erfc}\left(\frac{\lambda}{2} \sqrt{T-\tau}\right) + \frac{e^{\frac{k_{\parallel}^2}{4}\tau}}{(T-\tau)^{\frac{3}{2}}} \text{erfc}\left(\frac{\lambda}{2} \sqrt{\tau}\right) \right), \quad (54)$$

where we use $\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-x^2} dx$, and we do the change of variables $\tau = \mu T$, and for the *erfc* expressions we use $x = \sqrt{T}y$ and $x = \sqrt{T}z$, leading to

$$\gamma_2(k_{\parallel}) = \frac{\lambda^3}{2(4\pi)^{(d+2)/2}} \int_0^\infty \frac{dT}{T^{\frac{d+2}{2}}} T \int_0^1 d\mu e^{-k_{\parallel}^2 T \mu(1-\mu)} \left[\frac{T(1-\mu)}{T^{\frac{3}{2}} \mu^{\frac{3}{2}}} e^{\frac{k_{\parallel}^2}{4} T(1-\mu)} \sqrt{T} \int_{y_{\min}}^\infty e^{-Ty^2} dy + \frac{1}{T^{\frac{1}{2}} (1-\mu)^{\frac{1}{2}}} e^{\frac{k_{\parallel}^2}{4} T \mu} \sqrt{T} \int_{z_{\min}}^\infty e^{-Tz^2} dz \right], \quad (55)$$

where $y_{\min} = \frac{\lambda}{2} \sqrt{1-\mu}$ and $z_{\min} = \frac{\lambda}{2} \sqrt{\mu}$.

Using the symmetry under $\mu \rightarrow (1-\mu)$, we obtain

$$\begin{aligned} \gamma_2(k_{\parallel}) &= \frac{\lambda^3}{2(4\pi)^{(d+2)/2}} \int_0^\infty \frac{dT}{T^{\frac{d}{2}}} \int_0^1 \frac{d\mu}{\mu^{3/2}} e^{-T[k_{\parallel}^2 \mu(1-\mu) - \frac{\lambda^2}{4}(1-\mu)]} \int_{y_{\min}}^\infty dy e^{-Ty^2} \\ &= \frac{\lambda^3 \Gamma(1-\frac{d}{2})}{2(4\pi)^{(d+2)/2}} \int_0^1 d\mu \mu^{-\frac{3}{2}} \eta(k_{\parallel}^2, \mu, \lambda), \end{aligned} \quad (56)$$

¹Note that, since we are interested here in nonanalytic terms in momentum, the different renormalization conditions one could have used will not affect our results.

where we integrated T with analytic continuation, and we define

$$\begin{aligned}\eta(k_{\parallel}^2, \mu, \lambda) &= \int_{y_{\min}}^{\infty} dy [k_{\parallel}^2 \mu (1 - \mu) + y^2 - y_{\min}^2]^{\frac{d}{2}-1} \\ &= \frac{1}{2} \int_0^{\infty} dt (k_{\parallel}^2 \mu (1 - \mu) + t)^{\frac{d}{2}-1} \left(\frac{\lambda^2}{4} (1 - \mu) + t \right)^{-\frac{1}{2}},\end{aligned}\quad (57)$$

after the change of variables $t = y^2 - y_{\min}^2$.

The integral (57) is of the form covered by formula 3.197.1 of Ref. [9],

$$\begin{aligned}\int_0^{\infty} x^{\nu-1} (x+a)^{-\mu} (x+b)^{-\rho} dx \\ = a^{-\mu} b^{\nu-\rho} B(\nu, \mu - \nu + \rho) {}_2F_1 \left[\mu, \nu; \mu + \rho; 1 - \frac{b}{a} \right],\end{aligned}\quad (58)$$

valid for $\text{Re} \nu > 0$ and $\text{Re}(\mu + \rho - \nu) > 0$.

Identifying $x = t$, $a = A \equiv k_{\parallel}^2 \mu (1 - \mu)$, $b = B \equiv \frac{\lambda^2}{4} (1 - \mu)$, with exponents $\nu = 1$, $\mu = 1 - \frac{d}{2}$, $\rho = \frac{1}{2}$, the convergence condition $\text{Re}(\mu + \rho - \nu) = \frac{1}{2} - \frac{d}{2} > 0$ restricts us to $d < 1$; for $d \geq 1$ the result is defined by analytic continuation in d , as is standard in dimensional regularization. With these identifications, and using the simplification $B(1, \frac{1}{2} - \frac{d}{2}) = \Gamma(\frac{1}{2} - \frac{d}{2}) / \Gamma(\frac{3}{2} - \frac{d}{2}) = (\frac{1}{2} - \frac{d}{2})^{-1}$, we obtain,

$$\begin{aligned}\eta(k_{\parallel}^2, \mu, \lambda) &= \frac{\lambda}{2(1-d)} [k_{\parallel}^2 \mu (1 - \mu)]^{\frac{d}{2}-1} \sqrt{1 - \mu} \\ &\times {}_2F_1 \left(1 - \frac{d}{2}, 1; \frac{3}{2} - \frac{d}{2}; 1 - \frac{\lambda^2}{4k_{\parallel}^2 \mu} \right).\end{aligned}\quad (59)$$

An alternative representation, useful in the regime $\lambda \gg |k_{\parallel}|$, follows from Pfaff's transformation ${}_2F_1(a, b; c; z) = (1-z)^{-a} {}_2F_1(a, c-b; c; \frac{z}{z-1})$,

$$\eta = \frac{\lambda^{d-1}}{2^d(1-d)} \frac{(1-\mu)^{\frac{d}{2}-1}}{\mu} {}_2F_1 \left(1 - \frac{d}{2}, \frac{1}{2} - \frac{d}{2}; \frac{3}{2} - \frac{d}{2}; 1 - \frac{4k_{\parallel}^2 \mu}{\lambda^2} \right).\quad (60)$$

In this form the argument of the hypergeometric function is $1 - 4k_{\parallel}^2 \mu / \lambda^2$, which lies close to 1 when λ is large, making it well suited for expansions in that regime.

Substituting (59) into the expression for γ_2 , we arrive at

$$\begin{aligned}\gamma_2(k_{\parallel}) &= \frac{\lambda^4 \Gamma(1 - \frac{d}{2})}{8(4\pi)^{\frac{d+2}{2}}} (k_{\parallel}^2)^{\frac{d}{2}-1} \int_0^1 d\mu \mu^{\frac{d}{2}-\frac{5}{2}} (1-\mu)^{\frac{d}{2}-\frac{1}{2}} \\ &\times {}_2F_1 \left(1 - \frac{d}{2}, 1; \frac{3}{2} - \frac{d}{2}; 1 - \frac{\lambda^2}{4k_{\parallel}^2 \mu} \right),\end{aligned}\quad (61)$$

where we have used $\frac{1}{2}(\frac{1}{2} - \frac{d}{2}) / (1-d) = \frac{1}{4}$ to simplify the overall prefactor.

The integral (61) requires the subtraction introduced previously to render it UV finite. The apparent poles for odd values of d are artifacts of the decomposition $\gamma = \gamma_1 + \gamma_2$, which generates purely real counterterms and therefore does not affect the dissipative contributions. For even values of d (or more precisely, for d treated as a continuous parameter via dimensional regularization), both γ_1 and γ_2 are individually well defined.

For general d and arbitrary ratio $\zeta \equiv \lambda^2 / (4k_{\parallel}^2)$, the μ integral in (61) is most efficiently evaluated numerically. We have carried out this computation, obtaining the full λ interpolation of the form factor γ^{sub} between the weak-coupling ($\lambda \rightarrow 0$) and Dirichlet ($\lambda \rightarrow \infty$) limits. The results are displayed in Fig. 1.

E. Higher-order terms

The third-order contribution to the effective action involves the term ψ^3 from the Taylor expansion of V . After factorization, its parallel part requires the worldline average of three ψ insertions,

$$\int d^d x_{\parallel} \langle \psi(x_{\parallel}(\tau_1)) \psi(x_{\parallel}(\tau_2)) \psi(x_{\parallel}(\tau_3)) \rangle_{x_{\parallel}}.\quad (62)$$

After performing the $x_{\parallel}$ integration and the Gaussian worldline average, this evaluates to,

$$\int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \tilde{\psi}(k_1) \tilde{\psi}(k_2) \tilde{\psi}(-k_1 - k_2) e^{\frac{i}{2} \sum_{i,j} k_i \cdot k_j \Delta(\tau_i - \tau_j)}.\quad (63)$$

However, the third-order terms in the expansion of $e^{-\int V}$ involve an odd number of $v^{(n)}$ insertions in the perpendicular sector. For the δ -potential case, $v(x_d) = \lambda \delta(x_d)$, the perpendicular path integral is governed by a Hamiltonian that is even under $x_d \rightarrow -x_d$. Therefore, the integrand for each perpendicular average contains an odd power of $\delta'(x_d)$ (which is odd under reflection), rendering the integral zero. This is not, however, something that is restricted to the δ -potential case: we only need an even potential, namely $v(-x_d) = v(x_d)$.

More explicitly, the three contributions at third order in ψ are: (i) a single v''' insertion, (ii) a $v'' \cdot v'$ cross term, and (iii) a cubic v'^3 term. In all three cases, the total number of derivatives of v acting on the perpendicular coordinate is odd. Since $K(t; x, y)$ depends on $|x|$ and $|y|$ (and is therefore even under $x_d \rightarrow -x_d$), and since the derivatives of odd order of the potential are odd functions, each perpendicular integral vanishes. This argument extends to all odd orders in ψ ,

$$\Gamma_{2n+1}(\psi) = 0, \quad n = 0, 1, 2, \dots\quad (64)$$

for any even potential.

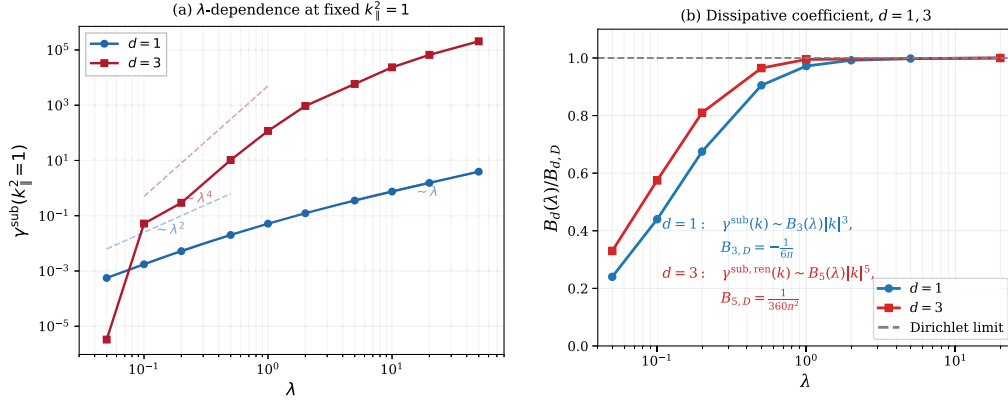


FIG. 1. Subtracted form factor $\gamma^{\text{sub}}(k_{||}) = \gamma(k_{||}) - \gamma(0)$ obtained by numerical evaluation of the full expression (48). (a) Dependence on the coupling λ at fixed $k_{||}^2 = 1$ for $d = 1$ (circles) and $d = 3$ (squares). At weak coupling, $\gamma^{\text{sub}} \sim \lambda^2$ for $d = 1$ and $\gamma^{\text{sub}} \sim \lambda^4$ for $d = 3$ (dashed guides); at strong coupling, the $d = 1$ curve crosses over to approximately linear growth, reflecting the accumulation of subleading $1/\lambda$ corrections at fixed momentum. (b) Ratio of the dissipative coefficient $B(\lambda)$ to its Dirichlet-limit value $B_{d,D}$, for $d = 1$ and $d = 3$. Here $B(\lambda)$ denotes the $|k_{||}|^m$ coefficient extracted from γ^{sub} (for $d = 1$, $m = 3$, and for $d = 3$, $m = 5$) by fitting the low-momentum expansion $\gamma^{\text{sub}} = A(\lambda)k_{||}^2 + \dots + B(\lambda)|k_{||}|^m + \dots$. As $\lambda \rightarrow \infty$, the ratio $B(\lambda)/B_{d,D} \rightarrow 1$, recovering the Dirichlet result [6] with $B_{1,D} = -1/(6\pi)$ and $B_{3,D} = \frac{1}{360\pi^2}$.

The fourth-order contribution $\Gamma_4(\psi)$ receives five types of terms, arising from the expansion of $\langle e^{-\int_0^T d\tau V} \rangle$ to fourth order in ψ . We write them schematically as,

$$\Gamma_4 = \Gamma_{4,1} + \Gamma_{4,2} + \Gamma_{4,3} + \Gamma_{4,4} + \Gamma_{4,5}, \quad (65)$$

where the subscripts indicate the partition of four ψ insertions into groups associated with different $v^{(n)}$ vertices:

- (a) $\Gamma_{4,1}$: a single $v^{(4)}$ insertion ($n = 4$ in the Taylor expansion, one proper-time variable);
- (b) $\Gamma_{4,2}$: a $v''' \cdot v'$ cross term ($n = 3$ and $n = 1$, two proper-time variables);
- (c) $\Gamma_{4,3}$: a $(v'')^2$ term ($n = 2$ twice, two proper-time variables);
- (d) $\Gamma_{4,4}$: a $v'' \cdot (v')^2$ term ($n = 2$ and two $n = 1$ insertions, three proper-time variables); and
- (e) $\Gamma_{4,5}$: a $(v')^4$ term ($n = 1$ four times, four proper-time variables).

Each contribution factorizes into a parallel part (involving products of ψ in Fourier space) and a perpendicular part (a worldline average of derivatives of v with the full $e^{-\int v}$ insertion).

IV. TWO SURFACES

In this section we analyze the case of two surfaces, one of them flat, located at $x_d = a$ (we leave the sign of a free), and one curved surface located at $x_d = \psi(x_{||})$. This setup leaves us with $V(x) = \lambda_1 \delta(x_d - \psi(x_{||})) + \lambda_2 \delta(x_d - a)$. The two-surface dynamical Casimir effect has been studied in a functional-determinant framework for imperfect mirrors with nonlocal couplings [10,11]; here we show how the

worldline approach naturally incorporates the second surface through the two- δ propagator.

Following the procedure of Sec. III A, we focus on the first term in the power expansion that can generate dissipative effects, namely

$$\begin{aligned} \Gamma_{2,2}(\psi) = & -\frac{\lambda_1^2}{4(4\pi)^{\frac{d+1}{2}}} \int_{0^+}^{\infty} \frac{dT}{T^{\frac{d+3}{2}}} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \\ & \times \left[\int d^d x_{||} \langle \psi(x_{||}(\tau_1)) \psi(x_{||}(\tau_2)) \rangle_{x_{||}} \right. \\ & \times \left. \int dx_d \langle \delta'(x_d(\tau_1)) \delta'(x_d(\tau_2)) e^{-\int_0^T d\tau V(x_d(\tau))} \rangle_{x_d} \right], \end{aligned} \quad (66)$$

where $V(x_d(\tau)) = \lambda_1 \delta(x_d) + \lambda_2 \delta(x_d - a)$.

The difference between (66) and the single-surface case (16) is captured entirely by the difference between the Euclidean propagator for a particle in the presence of one δ potential at the origin, Eq. (26), and the propagator $K^{(\delta_2)}$ corresponding to two δ potentials, one at the origin and one at $x_d = a$. The latter may be represented via its resolvent as follows:

$$K^{(\delta_2)}(t, x, y) = -i \int_{-\infty}^{\infty} \frac{dE}{2\pi} e^{-Et} G^{(\delta_2)}(E, x, y), \quad (67)$$

where, following [7], the two- δ Green function is expressed in terms of the single- δ one,

$$G^{(\delta_2)}(E, x, y) = G^{(\delta_1)}(E, x, y) - \frac{G^{(\delta_1)}(E, x, a)G^{(\delta_1)}(E, a, y)}{G^{(\delta_1)}(E, a, a) - \frac{1}{\lambda_2}}, \quad (68)$$

with

$$G^{(\delta_1)}(E, x, y) = \frac{e^{-\sqrt{-E}|x-y|}}{2\sqrt{-E}} + \frac{\lambda_1 e^{-\sqrt{-E}(|x|+|y|)}}{2\sqrt{-E}(2\sqrt{-E}-\lambda_1)}. \quad (69)$$

The inverse Laplace transform of $G^{(\delta_1)}$ via (67) reproduces the single- δ kernel K of Eq. (26) (with $\lambda = \lambda_1$).

We decompose the two- δ propagator as

$$K^{(\delta_2)}(t, x, y) = K(t, x, y) + \delta K(t, x, y), \quad (70)$$

where the correction δK due to the second wall is

$$\delta K(t, x, y) = -i \int_{-\infty}^{\infty} \frac{dE}{2\pi} e^{-Et} \frac{\lambda_2}{2\sqrt{-E}(2\sqrt{-E}-\lambda_1)} \times \frac{A(E, x)A(E, y)}{D(E)}, \quad (71)$$

with

$$A(E, z) = (2\sqrt{-E}-\lambda_1)e^{-\sqrt{-E}|z-a|} + \lambda_1 e^{-\sqrt{-E}(|z|+|a|)}, \quad (72)$$

$$D(E) = (2\sqrt{-E}-\lambda_1)(2\sqrt{-E}-\lambda_2) - \lambda_1\lambda_2 e^{-2\sqrt{-E}|a|}. \quad (73)$$

Since the parallel sector of the worldline average is identical to the single-surface case, the full result adopts the same quadratic form,

$$\Gamma_{2,2}(\psi) = \frac{1}{2} \int \frac{d^d k_{\parallel}}{(2\pi)^d} \chi(k_{\parallel}) |\tilde{\psi}(k_{\parallel})|^2, \quad (74)$$

with

$$\chi(k_{\parallel}) = -\frac{\lambda_1^2}{2(4\pi)^{d/2}} \int_0^{\infty} \frac{dT}{T^{(d+2)/2}} \int_0^T d\tau_1 \times \int_0^{\tau_1} d\tau_2 e^{k_{\parallel}^2 \Delta(\tau_1-\tau_2)} I^{(\delta_2)}(T, \tau_1, \tau_2). \quad (75)$$

Here $I^{(\delta_2)}$ is obtained by replacing $K \rightarrow K^{(\delta_2)}$ in the four-term structure of Eq. (27). The factor $(4\pi T)^{1/2} \lambda^2$ that appears in the single-surface definition of I [Eq. (27)] is absorbed into the prefactor and the power of T in Eq. (75), exactly as in the passage from Eqs. (27) to (22). Keeping the same definitions $T_b = \tau_1 - \tau_2$ and $S = T - T_b$ as before, the result is,

$$I^{(\delta_2)} = \partial_1 \partial_2 K_b^{(\delta_2)}(0,0) K_S^{(\delta_2)}(0,0) + K_b^{(\delta_2)}(0,0) \partial_1 \partial_2 K_S^{(\delta_2)}(0,0) + \partial_1 K_b^{(\delta_2)}(0,0) \partial_2 K_S^{(\delta_2)}(0,0) + \partial_2 K_b^{(\delta_2)}(0,0) \partial_1 K_S^{(\delta_2)}(0,0), \quad (76)$$

where $K_b^{(\delta_2)} \equiv K^{(\delta_2)}(T_b; \cdot, \cdot)$ and similarly for $K_S^{(\delta_2)}$.

Introducing (70) into (76), we write $I^{(\delta_2)} = I^{(\delta_1)} + \delta I$, where $I^{(\delta_1)}$ is the single-surface contribution (setting $\lambda_1 = \lambda$), and δI collects all terms involving at least one factor of δK . Setting $\chi = \gamma + \delta\gamma$, we need only compute $\delta\gamma$,

$$\delta\gamma(k_{\parallel}) = -\frac{\lambda_1^2}{2(4\pi)^{d/2}} \int_0^{\infty} \frac{dT}{T^{(d+2)/2}} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 e^{k_{\parallel}^2 \Delta(\tau_1-\tau_2)} \delta I(T, \tau_1, \tau_2). \quad (77)$$

We organize $\delta I = \delta I_1 + \delta I_2$ by the order in δK . The terms linear in δK are,

$$\delta I_1 = \partial_1 \partial_2 \delta K(T_b; 0, 0) K(S; 0, 0) + \partial_1 \partial_2 K(T_b; 0, 0) \delta K(S; 0, 0) + \delta K(T_b; 0, 0) \partial_1 \partial_2 K(S; 0, 0) + K(T_b; 0, 0) \partial_1 \partial_2 \delta K(S; 0, 0), \quad (78)$$

where we used the single- δ identity $\partial_i K(t; 0, 0) = 0$. The terms quadratic in δK are,

$$\delta I_2 = \partial_1 \partial_2 \delta K(T_b; 0, 0) \delta K(S; 0, 0) + \delta K(T_b; 0, 0) \partial_1 \partial_2 \delta K(S; 0, 0) + 2\partial_1 \delta K(T_b; 0, 0) \partial_1 \delta K(S; 0, 0), \quad (79)$$

where the factor of 2 in the last term arises because δK is symmetric in its spatial arguments, so $\partial_1 \delta K(t; 0, 0) = \partial_2 \delta K(t; 0, 0)$, and the two cross terms in (76) are equal.

To proceed, we need explicit expressions for δK and its derivatives evaluated at $x = y = 0$. The key simplification comes from the function $A(E, z)$, which at $z = 0$ reduces to

$$A(E, 0) = (2p - \lambda_1) e^{-p|a|} + \lambda_1 e^{-p|a|} = 2p e^{-p|a|}, \quad (80)$$

where $p \equiv \sqrt{-E}$. For the derivative with respect to z , we note that $\partial_z |z - a|_{z=0} = -\text{sgn}(a)$ (well defined for $a \neq 0$), while $\partial_z |z|_{z=0} = 0$ in the symmetric-derivative sense

appropriate to $\int f \delta' dx = -f'(0)$. This gives

$$A'(E, 0) \equiv \partial_z A(E, z)|_{z=0} = p(2p - \lambda_1) \text{sgn}(a) e^{-p|a|}. \quad (81)$$

Since $\delta K(t; x, y)$ factorizes as $A(E, x)A(E, y)$ (times scalar functions of E), the derivatives of δK at the origin are determined by $A(E, 0)$ and $A'(E, 0)$ alone. Substituting (80)–(81) into the spectral representation (71) and writing $\tilde{D}(p) \equiv D(-p^2)$, one obtains the three independent building blocks,

$$\delta K(t; 0, 0) = \frac{1}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{2\lambda_2 p e^{-2p|a|}}{(2p - \lambda_1) \tilde{D}(p)}, \quad (82)$$

$$\partial_t \delta K(t; 0, 0) = \frac{\text{sgn}(a)}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{\lambda_2 p e^{-2p|a|}}{\tilde{D}(p)}, \quad (83)$$

$$\partial_1 \partial_2 \delta K(t; 0, 0) = \frac{1}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{\lambda_2 p (2p - \lambda_1) e^{-2p|a|}}{2\tilde{D}(p)}, \quad (84)$$

with

$$\tilde{D}(p) = (2p - \lambda_1)(2p - \lambda_2) - \lambda_1 \lambda_2 e^{-2p|a|}. \quad (85)$$

The integrals above are spectral (inverse-Laplace) representations, obtained by collapsing the energy contour onto the continuum branch cut $E \leq 0$ (real $p = \sqrt{-E}$). Two remarks regarding their validity are in order. First, in the regime of interest here—repulsive couplings $\lambda_1, \lambda_2 > 0$, and in particular the Dirichlet limit $\lambda_i \rightarrow \infty$ —the resolvent $\tilde{D}^{-1}$ has no poles off the cut, so the representations are exact as written. Second, bound states can occur only when at least one coupling is attractive; each such state sits at a fixed $p = p_b$ with energy $E_b = -p_b^2 < 0$ (a zero of $\tilde{D}$), and adds to the kernels a discrete contribution $\propto e^{-E_b t} = e^{p_b^2 t}$. Being entire (hence analytic) in the proper time t , such a term integrates to a purely analytic, *local* dependence on $k_\parallel^2$; it does not affect the large- p (UV, $T \rightarrow 0$) behavior of $\delta\gamma$, and, being local, it drops out of the imaginary part and is therefore immaterial for the DCE. We may consequently ignore these terms throughout.

A useful identity is

$$\partial_1 \partial_2 K(t; 0, 0) = \frac{1}{4\sqrt{\pi} t^{3/2}}, \quad (86)$$

valid for all values of λ_1 . This follows from the fact that, at

$x = y = 0$, the symmetric derivative $\partial_x |x|_{x=0} = 0$ kills the λ -dependent correction in (26), leaving only the free-propagator contribution.

Substituting the building blocks into δI_1 and using $\partial_i K(t; 0, 0) = 0$ together with (86), the four terms become

$$\begin{aligned} \delta I_1 &= \partial_1 \partial_2 \delta K(T_b) K(S) + \frac{\delta K(S)}{4\sqrt{\pi} T_b^{3/2}} \\ &\quad + \frac{\delta K(T_b)}{4\sqrt{\pi} S^{3/2}} + K(T_b) \partial_1 \partial_2 \delta K(S), \end{aligned} \quad (87)$$

where we have suppressed the arguments $(0, 0)$ on all kernels.

For δI_2 , which is quadratic in δK , all three terms in (79) are retained as written. Since every factor of δK or its derivatives carries a suppression factor $e^{-2p|a|}$ from the spectral integrals, $\delta I_2 \sim e^{-4p|a|}$ in the large- $|a|$ regime, while $\delta I_1 \sim e^{-2p|a|}$. The leading correction to $\delta\gamma$ thus comes entirely from δI_1 .

When $|a|$ is large compared to the typical worldline scale $\sim 1/\sqrt{k_\parallel^2}$, the exponential factor $e^{-2p|a|}$ in the spectral integrals suppresses δK and its derivatives. At leading order, we expand $\tilde{D}^{-1}$ by neglecting the exponential term in (85),

$$\frac{1}{\tilde{D}(p)} \approx \frac{1}{(2p - \lambda_1)(2p - \lambda_2)} \sum_{n=0}^\infty \left[\frac{\lambda_1 \lambda_2 e^{-2p|a|}}{(2p - \lambda_1)(2p - \lambda_2)} \right]^n. \quad (88)$$

At leading ($n = 0$) order, the mixed derivative (84) simplifies by a cancellation of $(2p - \lambda_1)$,

$$\partial_1 \partial_2 \delta K(t; 0, 0) \approx \frac{1}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{\lambda_2 p e^{-2p|a|}}{2(2p - \lambda_2)}, \quad (89)$$

while δK and $\partial_i \delta K$ retain both couplings,

$$\delta K(t; 0, 0) \approx \frac{1}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{2\lambda_2 p e^{-2p|a|}}{(2p - \lambda_1)^2 (2p - \lambda_2)}, \quad (90)$$

$$\partial_i \delta K(t; 0, 0) \approx \frac{\text{sgn}(a)}{\pi} \int_0^\infty dp e^{-p^2 t} \frac{\lambda_2 p e^{-2p|a|}}{(2p - \lambda_1)(2p - \lambda_2)}. \quad (91)$$

The correction to the form factor at this order is

$$\delta\gamma(k_\parallel) = -\frac{\lambda_1^2}{2(4\pi)^{d/2}} \int_0^\infty \frac{dT}{T^{(d+2)/2}} \int_0^T dT_b (T - T_b) e^{\Delta(T_b) k_\parallel^2} \delta I_1(T_b, T - T_b) + O(e^{-4p|a|}), \quad (92)$$

where δI_1 is given by (87) with the leading-order spectral functions.

The physical content of (92) is that the second surface modifies the dissipative form factor through an interference between the single-wall propagator $K(t; 0, 0)$ and the wall-to-wall correction $\delta K(t; 0, 0)$. This interference is exponentially suppressed for large wall separations, reflecting the fact that the worldline must traverse the distance $|a|$ to “feel” the second surface, at a cost $\sim e^{-2p|a|}$ in the spectral integral.

A. Dirichlet limit

In the limit $\lambda_1, \lambda_2 \rightarrow \infty$ at fixed $|a|$, the building blocks exhibit a hierarchy,

$$\begin{aligned} K(t; 0, 0) &\sim \frac{1}{\lambda_1^2 \sqrt{\pi} t^{3/2}}, \quad \delta K(t; 0, 0) \sim O(1/\lambda_1^2), \\ \partial_i \delta K(t; 0, 0) &\sim O(1/\lambda_1), \quad \partial_1 \partial_2 \delta K(t; 0, 0) \rightarrow \partial_1 \partial_2 \delta K^{\text{Dir}}(t), \end{aligned} \quad (93)$$

where only the mixed derivative remains $O(1)$. Its limiting form is

$$\partial_1 \partial_2 \delta K^{\text{Dir}}(t) = -\frac{1}{2\pi} \int_0^\infty dp e^{-p^2 t} \frac{p e^{-2p|a|}}{1 - e^{-2p|a|}}. \quad (94)$$

The integrand admits a geometric-series expansion

$$\frac{p e^{-2p|a|}}{1 - e^{-2p|a|}} = \sum_{n=1}^\infty p e^{-2np|a|}, \quad (95)$$

interpretable as a sum over image charges at distances $2n|a|$. This is the standard method-of-images result for the correction to the Dirichlet propagator due to a second wall [7,11], providing an independent verification of the two-surface formalism.

A further simplification occurs in δI_1 : in the Dirichlet limit, $\delta K(t; 0, 0)$ is proportional to $\partial_1 \partial_2 \delta K(t; 0, 0)$ with coefficient $4/\lambda_1^2$. Substituting into (87) and factoring out $1/\lambda_1^2$, each of the four terms contributes with the same $1/t^{3/2}$ structure from either K or δK , yielding

$$\delta I_1^{\text{Dir}} = \frac{2}{\lambda_1^2 \sqrt{\pi}} \left[\frac{\partial_1 \partial_2 \delta K^{\text{Dir}}(T_b)}{S^{3/2}} + \frac{\partial_1 \partial_2 \delta K^{\text{Dir}}(S)}{T_b^{3/2}} \right]. \quad (96)$$

After multiplication by λ_1^2 in the prefactor of (77), the coupling cancels and the result is

$$\delta \gamma^{\text{Dir}}(k_{\parallel}) = -\frac{1}{(4\pi)^{d/2} \sqrt{\pi}} \int_0^\infty \frac{dT}{T^{(d+2)/2}} \int_0^T dT_b (T - T_b) e^{\Delta(T_b) k_{\parallel}^2} \left[\frac{\partial_1 \partial_2 \delta K^{\text{Dir}}(T_b)}{S^{3/2}} + \frac{\partial_1 \partial_2 \delta K^{\text{Dir}}(S)}{T_b^{3/2}} \right]. \quad (97)$$

We have verified numerically that the exact δI_1 (with finite $\lambda_1 = \lambda_2 = \lambda$) converges to (96) as $\lambda \rightarrow \infty$, with the ratio approaching unity.

To obtain a more explicit expression, we substitute the image-charge expansion (95) into (97) and rescale $T_b = uT$. The proper-time integral over T is then evaluated by dimensional regularization (exactly as in the single-surface derivation of Sec. III A), with the $k_{\parallel} = 0$ subtraction handled automatically by analytic continuation. The result is

$$\begin{aligned} \delta \gamma^{\text{Dir,sub}}(k_{\parallel}) &= \frac{\Gamma(-\frac{d}{2})}{2\pi^{3/2} (4\pi)^{d/2}} \sum_{n=1}^\infty \int_0^\infty dp p e^{-2np|a|} \int_0^1 du (1-u)^{-1/2} [(p^2 u)^{d/2} - (u(1-u)k_{\parallel}^2 + p^2 u)^{d/2}] \\ &\quad + \frac{\Gamma(-\frac{d}{2})}{2\pi^{3/2} (4\pi)^{d/2}} \sum_{n=1}^\infty \int_0^\infty dp p e^{-2np|a|} \int_0^1 du (1-u) u^{-3/2} [(p^2 (1-u))^{d/2} - (u(1-u)k_{\parallel}^2 + p^2 (1-u))^{d/2}]. \end{aligned} \quad (98)$$

For odd $d = 2q + 1$, $\Gamma(-d/2)$ is finite and the u integrals can be evaluated in closed form. For $d = 1$, both integrals reduce to logarithms and inverse tangents, yielding,

$$\delta \gamma^{\text{Dir,sub}}|_{d=1} = -\frac{1}{2\pi^2} \sum_{n=1}^\infty \int_0^\infty dp p e^{-2np|a|} \left[\frac{2|k_{\parallel}|}{p} \arctan \frac{|k_{\parallel}|}{p} - \log \left(1 + \frac{k_{\parallel}^2}{p^2} \right) \right]. \quad (99)$$

The p integral in (99) can be expressed in terms of exponential integrals; in practice it is most efficiently evaluated by direct quadrature. For $|k_{\parallel}| \ll 1/|a|$ the bracket behaves as $\sim k_{\parallel}^2/p^2$, and for $|k_{\parallel}| \gg 1/|a|$ as $\sim \pi |k_{\parallel}|/p$, interpolating between a $k_{\parallel}^2$ (nondissipative) and a $|k_{\parallel}|^3$ (dissipative) regime. In particular, at large separation, the leading image ($n = 1$) dominates and

the dissipative piece scales as $|k_{\parallel}|^3 e^{-2|k_{\parallel}||a|}$, exponentially suppressed relative to the single-surface result $\gamma_D = -|k_{\parallel}|^3/(6\pi)$. This, together with the exact agreement of the single-surface Dirichlet limit with the direct calculation of Ref. [6], provides an independent verification of the two-surface formalism.

The Dirichlet-limit correction (98) has a structure parallel to the single-surface result: comparing with the Dirichlet form factor γ_D from (36), the factor $1/(4\sqrt{\pi}t^{3/2})$ associated with the single-wall Dirichlet propagator is replaced by $\partial_1 \partial_2 \delta K^{\text{Dir}}(t)$, which encodes the effect of the second wall through the image-charge sum (95). For large $|a|$, $\delta\gamma^{\text{Dir}}$ is exponentially suppressed, recovering the single-surface Dirichlet result. In the opposite limit $|a| \rightarrow 0$, the image sum diverges and the perturbative expansion in δK breaks down, signaling the coalescence of the two surfaces.

V. CONCLUSIONS

We have developed a worldline approach to the dynamical Casimir effect for a real scalar field coupled to a time-dependent surface modeled by a background potential. The key advantage of this formulation is the natural factorization of the worldline path integral into parallel and perpendicular sectors, which reduces the computation of the effective action to one-dimensional quantum mechanical problems.

For the case of a δ -function potential of strength λ , we have obtained an exact expression for the dissipative form factor $\gamma(k_{\parallel})$, valid for arbitrary coupling strength. In the strong-coupling limit $\lambda \rightarrow \infty$, we recover the Dirichlet result of Ref. [6], and derive the systematic corrections in inverse powers of λ : the n th order contribution scales as $|k_{\parallel}|^{d+2n+2}/\lambda^{2n}$, with a closed-form coefficient $C_n(d)$ expressed through gamma functions.

The imaginary part of the effective action was computed for arbitrary values of d , including corrections up to arbitrary order n in $1/\lambda$. At leading order, the imaginary part is positive, as expected, whereas the signs of the higher-order contributions alternate as $(-1)^n$, for both odd and even spatial dimensions d . The momentum-power

dependence of the n th order correction matches that of the Dirichlet case in the shifted dimension $d + 2n$.

The full λ -dependent form factor has been obtained in closed form via hypergeometric functions, by splitting γ into a free-propagator part γ_1 and a δ -potential correction γ_2 . The latter is expressed through a ${}_2F_1$ function whose argument interpolates continuously between the weak-coupling and Dirichlet regimes. We have verified the cancellation of spurious poles between γ_1 and γ_2 at odd spatial dimensions.

Using a parity argument based on the evenness of the perpendicular Hamiltonian under $x_d \rightarrow -x_d$, we have shown that all odd-order contributions in ψ vanish identically for any even potential, and have classified the five distinct types of terms arising at fourth order.

The formalism has been extended to a two-surface configuration, where the second surface introduces corrections to the propagator that can be treated perturbatively through the Green's function of the two- δ potential system. In the Dirichlet limit, the correction to the form factor reduces to an image-charge sum, recovering the standard method-of-images structure and connecting with the functional-determinant results of Refs. [10,11].

Several directions remain open. The explicit computation of the fourth-order term $\Gamma_4(\psi)$, together with the Bern-Kosower representation of the resulting amplitude, would provide a diagrammatic interpretation analogous to that of the string-inspired formalism and will be addressed elsewhere. Extensions to the electromagnetic field and to finite temperature are also natural next steps within this framework.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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