

Probing high-frequency gravitational waves with entangled vibrational qubits in linear Paul traps

Ryoto Takai 

KEK Theory Center, Tsukuba 305-0801, Japan; The Graduate University for Advanced Studies, SOKENDAI, Tsukuba 305-0801, Japan; and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan



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This work investigates the use of linear Paul traps as quantum sensors for detecting megahertz gravitational waves. Single-ion configurations exploit graviton-photon conversion in the presence of external magnetic fields, while two-ion systems use relative-motion excitations, which do not require magnets, to distinguish gravitational waves from axion dark matter. Furthermore, we show that entanglement of N vibrational qubits enhances the signal probability by a factor of N^2 , improving sensitivity beyond the standard quantum limit.

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I. INTRODUCTION

High-frequency gravitational waves are compelling probes of the early Universe, which remain inaccessible to electromagnetic observations. Such signals may reveal imprints of phenomena such as phase transitions, cosmic strings, and preheating after inflation [1]. While stochastic gravitational wave backgrounds at megahertz frequencies are strongly constrained by big bang nucleosynthesis bound [2], nonstochastic gravitational waves, such as those potentially generated by light primordial black hole mergers [3], exotic compact objects [4], and black hole super-radiance [5], may also exist.¹ Several proposals have been made for detecting megahertz gravitational waves [6–10], but a large region of parameter space remains unexplored. The detection of high-frequency gravitational waves requires the development of entirely new experimental strategies [11].

Quantum technologies have advanced rapidly across diverse architectures, including superconducting circuits, trapped atoms, and photonic platforms, enabling precise control of quantum states. These developments are expected to have a profound impact in various domains. For example, quantum computers can solve certain problems more efficiently than classical computers by

exploiting superposition and entanglement of qubits, the quantum analogs of classical bits [12]. Another particularly promising direction for high-energy physics is quantum sensing. Quantum techniques can surpass the precision and resolution of classical approaches, providing unprecedented sensitivity to weak signals such as wavelike dark matter and high-frequency gravitational waves [13].

In this work, we focus on the use of linear Paul traps for the detection of gravitational waves in the megahertz range, extending the framework previously applied to searches for axion and dark photon dark matter [14]. A natural extension of these ideas involves graviton-photon conversion, in which a gravitational wave is converted into an electric field resonant with the center-of-mass oscillation of trapped ions. Considering a trap with two ions, the relative motion is useful even in the absence of magnetic fields, as gravitational waves periodically expand and contract the ion separation, while axion dark matter would not induce such an effect. Furthermore, excitations induced by gravitational waves can surpass the standard quantum limit when maximal entanglement is employed.

II. LINEAR PAUL TRAPS

The linear Paul trap is one of the most promising platforms for both quantum computation and quantum sensing [15]. Ions are confined in an ultrahigh-vacuum environment by a combination of static and radio-frequency electric fields, forming a chain of coupled harmonic oscillators aligned along the axis of the alternating-current voltages, conventionally referred to as the z axis. Singly ionized atoms such as beryllium, calcium, barium, and ytterbium are widely employed owing to their favorable ionization and level-structure properties. The two-level

¹They are typically expected to produce strain amplitudes in the range $\sim 10^{-20} - 10^{-30}$, depending on the source parameters and distance.

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electronic states of these ions, arising from optical, fine-structure, hyperfine, or Zeeman splittings, form the basis of computational qubits. We collectively refer to these two-level systems as spin qubits, with $|g\rangle$ and $|e\rangle$ denoting the ground and excited states, respectively.

A distinctive feature of Paul traps is their use of quantized vibrational motion of trapped ions to mediate control of internal states [16]. The ions arrange themselves at intervals of the order of 1–10 m, determined by the balance between Coulomb repulsion and external trapping fields. After laser cooling, the system exhibits quantized motional degrees of freedom, whose collective oscillations are described as quantum harmonic modes. In what follows, we focus on oscillations along the z direction.

To illustrate the mechanism, we first review the single-ion case. We restrict attention to the lowest two motional states: the ground state ($|n=0\rangle$) and the first excited state ($|n=1\rangle$). These states constitute what we call a vibrational qubit. The Hamiltonian is

$$H_0 = \frac{\omega_0}{2} \hat{\sigma}_z + \omega_z \hat{a}^\dagger \hat{a}, \quad (2.1)$$

where ω_z ($\sim$ MHz) is the vibrational frequency, ω_0 ($\sim$ GHz) is the energy splitting between $|g\rangle$ and $|e\rangle$, and $\hat{\sigma}_z$ is the third Pauli operator acting on the spin qubit. Here, $\hat{a}^\dagger$ and $\hat{a}$ denote creation and annihilation operators of the vibrational mode, while $\hat{\sigma}_\pm$ act as spin raising and lowering operators (e.g., $\hat{a}^\dagger \hat{\sigma}_- |e, 0\rangle = |g, 1\rangle$).

Lasers tuned to specific frequencies allow precise control of ion states. In the interaction picture, the effective Hamiltonian describing the coupling between the ion and a laser of frequency ω and phase ϕ is [17]

$$\begin{aligned} H_{\text{laser}} = & \frac{\Omega}{2} \hat{\sigma}_+ e^{-i(\omega-\omega_0)t+i\phi} \\ & + \frac{i}{2} \eta \Omega \hat{a}^\dagger (\hat{\sigma}_+ e^{-i(\omega-\omega_0-\omega_z)t+i\phi} - \hat{\sigma}_- e^{i(\omega-\omega_0+\omega_z)t-i\phi}) \\ & + \text{H.c.}, \end{aligned} \quad (2.2)$$

where Ω is the Rabi frequency and η is the Lamb-Dicke parameter, typically small (~ 0.01 – 0.1), which justifies truncating higher-order terms. Depending on the laser frequency, three resonances can be selectively addressed: the red sideband transition ($|g, 1\rangle \leftrightarrow |e, 0\rangle$) at $\omega = \omega_0 - \omega_z$, the blue sideband transition ($|g, 0\rangle \leftrightarrow |e, 1\rangle$) at $\omega = \omega_0 + \omega_z$, and the carrier transition ($|g, 0\rangle \leftrightarrow |e, 0\rangle$) at $\omega = \omega_0$. Together, these three resonances enable arbitrary single-qubit gates for quantum computation, such as the Hadamard gate, $|g\rangle \rightarrow (|g\rangle + |e\rangle)/\sqrt{2}$, $|e\rangle \rightarrow (|g\rangle - |e\rangle)/\sqrt{2}$.

With two ions in the trap, motion along the z axis decomposes into center-of-mass and stretch modes. The center-of-mass mode is a quantum harmonic oscillation with effective mass $2m_{\text{ion}}$ and frequency $\omega_{\text{cm}} = \omega_z$, allowing information transfer between ions and enabling

two-qubit gates such as the controlled-NOT (CNOT) gate: $|g, g\rangle \rightarrow |g, g\rangle$, $|g, e\rangle \rightarrow |g, e\rangle$, $|e, g\rangle \rightarrow |e, e\rangle$, and $|e, e\rangle \rightarrow |e, g\rangle$. The equilibrium separation between the two ions is given by

$$d \equiv \left(\frac{2\alpha_{\text{EM}}}{m_{\text{ion}}\omega_{\text{cm}}^2} \right)^{1/3}, \quad (2.3)$$

where $\alpha_{\text{EM}} = e^2/4\pi$ is the fine-structure constant. For reference, $d = (16, 3.5, 0.74)$ m at $\omega_{\text{cm}} = 2\pi \times (0.1, 1, 10)$ MHz with $^{171}\text{Yb}^+$ ions. The small deviation around this stationary point, z , can be described as the harmonic oscillation with effective mass $m_{\text{ion}}/2$ and frequency $\omega_{\text{rel}} = \sqrt{3}\omega_{\text{cm}}$, neglecting $\mathcal{O}(z^3)$ terms. In Secs. III B and III C, we denote the relative-motion states $|n=0\rangle$ and $|n=1\rangle$ as a vibrational qubit, again neglecting higher motional levels.

III. SIGNALS AND SENSITIVITIES

In this section, we investigate detection schemes for gravitational waves and estimate their corresponding sensitivities. First, we utilize the center-of-mass mode to probe effective electric fields induced by gravitational waves via graviton-photon conversion, following the approach previously employed for axionlike particles [14]. Next, we exploit the relative motion of the two-ion system, which enables detection without requiring a strong external magnetic field, in contrast to the center-of-mass method. Finally, we show that entanglement of vibrational qubits can be used to enhance the signal beyond the standard quantum limit.

A. Graviton-photon conversion and the center-of-mass mode

In our analysis, we treat the gravitational wave as a weak perturbation around the background Minkowski metric and expand relevant quantities in the metric fluctuation $h_{\mu\nu}$ up to linear order. Under the transverse-traceless gauge, the metric takes the form

$$\begin{aligned} h_{ij}^{\text{TT}}(\mathbf{x}, t) = & h^+ e_{ij}^+ \cos(\omega t - \mathbf{k} \cdot \mathbf{x} + \phi^+) \\ & + h^\times e_{ij}^\times \cos(\omega t - \mathbf{k} \cdot \mathbf{x} + \phi^\times), \end{aligned} \quad (3.1)$$

where ω is the angular frequency, $\mathbf{k} = (\omega \sin \theta, 0, \omega \cos \theta)$ is the wave vector, and

$$\begin{aligned} e_{ij}^+ = & \begin{pmatrix} \cos^2 \theta & 0 & -\cos \theta \sin \theta \\ 0 & -1 & 0 \\ -\cos \theta \sin \theta & 0 & \sin^2 \theta \end{pmatrix}, \\ e_{ij}^\times = & \begin{pmatrix} 0 & \cos \theta & 0 \\ \cos \theta & 0 & -\sin \theta \\ 0 & -\sin \theta & 0 \end{pmatrix} \end{aligned} \quad (3.2)$$

represent the polarization tensors. We consider monochromatic gravitational waves for simplicity.

Graviton-photon conversion is a phenomenon in which gravitational waves convert into photons in the presence of strong external magnetic fields. Under a electromagnetic background $\bar{A}^\mu$, the effective Lagrangian describing the interaction between the graviton field $h_{\mu\nu}$ and the induced electromagnetic field A^μ is $\mathcal{L}_{\text{int}} = j_{\text{eff}}^\mu A_\mu$, where

$$j_{\text{eff}}^\mu = \partial_\nu (h_\lambda^\mu \bar{F}^{\lambda\nu} + h_\lambda^\nu \bar{F}^{\mu\lambda}) - \frac{1}{2} (\partial_\lambda h_\nu^\lambda) \bar{F}^{\mu\lambda} \quad (3.3)$$

is the effective current with the field strength tensor $\bar{F}^{\mu\nu} = \partial^\mu \bar{A}^\nu - \partial^\nu \bar{A}^\mu$ [18]. To describe the observable electromagnetic fields in the detector, we adopt the proper detector frame, i.e., Fermi normal coordinates constructed along the detector's worldline, which provide a natural description in the long-wavelength regime.

We focus on a static and uniform magnetic field $\mathbf{B} = (0, 0, B_z)$ inside a cylinder of radius R and height L . To define the quantization axis of spins, a weak magnetic field (~ 1 mT) along z axis are used in real Paul traps [19]. We expect that the region in which the external magnetic field is applied can be extended independently of the Paul trap system by employing, for example, superconducting magnets. In this case, the parameter R is typically on the order of meters. The wavelength of a megahertz gravitational wave is about 300 m, much longer than the experimental length scale, and thereby the proper detector frame is well suited. At the leading order of ωR , the induced electric field along z axis is

$$E_z = -h^\times B_z (\omega R)^2 f(L/2R) \sin^2 \theta \cos(\omega t + \phi^\times), \quad (3.4)$$

where the logarithmic factor $f(x) = \log(x + \sqrt{1 + x^2})$, determined by the geometry of the magnet, contributes an $\mathcal{O}(1)$ correction around $x \sim 1$.

The interaction Hamiltonian of the single vibrational qubit with the electric field induced by a gravitational wave is expressed as

$$H_{\text{int}} = \frac{eE_z}{\sqrt{2m_{\text{ion}}\omega_z}} (\hat{a}^\dagger e^{i\omega_z t} + \hat{a} e^{-i\omega_z t}), \quad (3.5)$$

which becomes resonant when $\omega = \omega_z$. Then, using the rotating-wave approximation, the time evolution of the vibrational qubit is governed by the displacement operator $D(\beta) = \exp(\beta \hat{a}^\dagger - \beta^* \hat{a})$ with

$$\beta = \alpha T, \quad \alpha = \frac{h^\times e^{i\phi}}{2} \frac{eB_z}{\sqrt{2m_{\text{ion}}\omega_z}} (\omega_z R)^2 f(L/2R) \sin^2 \theta. \quad (3.6)$$

Here, T is the observation time, and ϕ absorbs all complex phases.

Starting from the initial state $|\text{init}\rangle = |g, 0\rangle$, the action of this operator evolves the vibrational state into

$$\begin{aligned} D(\beta)|g, 0\rangle &= \cos|\beta||g, 0\rangle + e^{i\phi} \sin|\beta||g, 1\rangle \\ &\simeq |g, 0\rangle + \beta|g, 1\rangle, \end{aligned} \quad (3.7)$$

yielding an excitation probability of $|\beta|^2 = |\alpha|^2 T^2$. The vibrational state is then mapped onto the spin state by applying a red-sideband π -pulse, $|g, 1\rangle \rightarrow |e, 0\rangle$. Subsequently, the spin state is measured using standard protocols, for example, by observing fluorescence after applying a laser that excites one of the spin states into a short-lived auxiliary level [20]. The measurement is performed after an interrogation time T and repeated N_{shot} times, such that the total duration of the experiment is approximately $T_{\text{total}} \simeq N_{\text{shot}} T$. The expected number of signal events induced by gravitational waves is therefore $S = N_{\text{shot}} |\alpha|^2 T^2$.

In actual configurations, the vibrational qubit is perturbed primarily by thermal photons originating from the trap electrodes [21]. The contribution of the noise increases linearly over T , characterized by the heating rate $\dot{n}$. The number of noise events is given by $B = N_{\text{shot}} \dot{n} T$, as discussed in Ref. [14]. Defining the sensitivity through the criterion $S/\sqrt{B} > 1.645$ (corresponding to the 95% confidence level), we obtain the sensitivity to the amplitude of electric field with angular frequency $\omega = \omega_z$ as

$$\begin{aligned} E_0 &= 1.5 \text{ nV/m} \times \left(\frac{\dot{n}}{0.1 \text{ Hz}} \right)^{1/4} \left(\frac{T_{\text{total}}}{1 \text{ day}} \right)^{-1/4} \left(\frac{T}{1 \text{ s}} \right)^{-1/2} \\ &\times \left(\frac{m_{\text{ion}}}{37 \text{ GeV}} \right)^{1/2} \left(\frac{\omega_z/2\pi}{1 \text{ MHz}} \right)^{1/2}, \end{aligned} \quad (3.8)$$

where we assume the use of a $^{40}\text{Ca}^+$ ion. The reference heating rate of 0.1 Hz is several times lower than the best values reported in Ref. [21]. Such a low rate is necessary to satisfy the condition $\dot{n} T \ll 1$ for interrogation times of order 1 sec. In practice, the heating rate decreases with larger trap sizes and lower temperatures. Therefore, both the trap size and cryogenic operation should be optimized to maximize sensitivity.

This detection scheme can thus be employed to probe gravitational waves, which induce weak electric fields as described in Eq. (3.4). The corresponding strain sensitivity for a single-ion system is obtained as

$$\begin{aligned} h^\times &= 1.7 \times 10^{-12} \times \left(\frac{\dot{n}}{0.1 \text{ Hz}} \right)^{1/4} \left(\frac{T_{\text{total}}}{1 \text{ day}} \right)^{-1/4} \left(\frac{T}{1 \text{ s}} \right)^{-1/2} \\ &\times \left(\frac{m_{\text{ion}}}{37 \text{ GeV}} \right)^{1/2} \left(\frac{\omega_z/2\pi}{1 \text{ MHz}} \right)^{-3/2} \left(\frac{B_z}{1 \text{ mT}} \right)^{-1} \left(\frac{R}{3 \text{ m}} \right)^{-2} \\ &\times \left(\frac{f(L/2R)}{1} \right)^{-1} \end{aligned} \quad (3.9)$$

at the 95% confidence level. In deriving Eq. (3.9), we have averaged over the polarization angle θ . Furthermore, by

tuning the resonance frequency of the vibrational mode, one can scan over a broad frequency range of gravitational waves. In practice, the resonance frequency may be tunable within the range 100 kHz to 10 MHz.

Paul traps also provide a versatile platform for generating entanglement among qubits, making them suitable for quantum computing and quantum-enhanced sensing. As shown in Ref. [14], a maximally entangled state of N vibrational qubits, prepared through a sequence of quantum operations, such as the red-sideband interaction and the Hadamard and CNOT gates, in a Paul trap, enhances the excitation rate by a factor of N^2 compared to the single-ion case.

There are two principal approaches to preparing multiple vibrational qubits and enabling their interaction. The first involves transporting ions and performing gate operations on individual vibrational qubits after generating entanglement among spin qubits. In this case, one must carefully account for gate fidelities and the heating rate in an N -ion detector, which generally degrade relative to the single-ion case [15]. The second approach employs a network of interconnected Paul traps, each containing a single ion [22]. In this article, we assume that the latter technology is available. Under this assumption, thermal noise photons act incoherently on different ions, and the strain sensitivity is expected to improve as $N^{3/4}$.

B. Stretch mode of a pair of ions

In this and the following subsections, we consider two-ion detectors. The center-of-mass mode provides a feasible means of searching for gravitational waves. However, the resulting signal is indistinguishable from that produced by axion dark matter [14]. To discriminate between the two cases, an alternative method for gravitational wave detection is required. As discussed in Sec. II, the relative motion of the ions offers such a possibility. After laser cooling [23], this mode is quantized and described as a quantum simple harmonic oscillator with an effective mass of $m_{\text{ion}}/2$ and an angular frequency $\omega_{\text{rel}} = \sqrt{3}\omega_{\text{cm}}$. Importantly, this strategy does not require the presence of external magnetic fields.

The nonrelativistic Hamiltonian, expanded up to order m_{ion} , is [24]²

$$H = \frac{1}{2}m_{\text{ion}}R_{0k0l}x^kx^l \supset \frac{h^+}{4}m_{\text{ion}}\omega^2 \sin^2\theta e^{i(\omega t + \phi^+)}dz + \text{H.c.}, \quad (3.10)$$

with $R_{\mu\nu\rho\sigma}$ being the Riemann tensor evaluated at the origin $\mathbf{x} = 0$, corresponding to the center of mass of the ions. The

indices k and l run over spatial directions, and z denotes the displacement of the distance between two ions. Terms of order lower than m_{ion} are neglected. A resonance arises when $\omega = \omega_{\text{rel}}$, yielding the effective Hamiltonian

$$H_{\text{int}} = \left[\frac{h^+}{2} e^{i\phi} \left(\frac{9}{16} \alpha_{\text{EM}}^2 m_{\text{ion}} \omega_{\text{rel}}^5 \right)^{1/6} \sin^2\theta \right] \hat{a}^\dagger + \text{H.c.} \\ \equiv \epsilon \hat{a}^\dagger + \text{H.c.} \quad (3.11)$$

under the rotating-wave approximation. In this expression, $\hat{a}^{(\dagger)}$ are the ladder operators of the stretch mode, which we refer to as the vibrational qubit henceforth. A real parameter ϕ , encapsulating all complex phases, is assumed to be randomly distributed. The time evolution is then described by the displacement operator $D(\beta) = \exp(\beta \hat{a}^\dagger - \beta^* \hat{a})$ with $\beta = -i\epsilon T$.

The signal corresponds to an excitation of the vibrational mode from the ground state $|n=0\rangle$ to the first excited state $|n=1\rangle$, as described in Eq. (3.7). This vibrational excitation is mapped onto the spin state by applying a red-sideband π -pulse. The spin state is subsequently measured using standard detection protocols, for instance, fluorescence observation after driving one of the spin levels to a short-lived auxiliary state with a laser. The probability of observing the signal is given by $|\epsilon|^2 T^2$, where T denotes the observation time per single shot. As in the previous subsection, the measurement is repeated $N_{\text{shot}} = T_{\text{total}}/T$ times. The number of signal events is therefore $S = N_{\text{shot}} |\epsilon|^2 T^2$, while the number of the noise events is $B = N_{\text{shot}} \dot{n} T$, with $\dot{n}$ being the heating rate of the stretch mode. It has been reported that the heating rate of the stretch mode is lower than that of the center-of-mass mode in other Paul trap architectures [23,25]. The sensitivity, defined by the condition $S/\sqrt{B} > 1.645$ corresponding to the 95% confidence level, is obtained as

$$h^+ = 3.6 \times 10^{-11} \times \left(\frac{\dot{n}}{0.1 \text{ Hz}} \right)^{1/4} \left(\frac{T_{\text{total}}}{1 \text{ day}} \right)^{-1/4} \left(\frac{T}{1 \text{ s}} \right)^{-1/2} \\ \times \left(\frac{m_{\text{ion}}}{159 \text{ GeV}} \right)^{-1/6} \left(\frac{\omega_{\text{cm}}/2\pi}{1 \text{ MHz}} \right)^{-5/6}, \quad (3.12)$$

where we consider $^{171}\text{Yb}^+$ ions and average over the polarization angle θ . Notably, heavier ions are advantageous for this method, in contrast to detection schemes based on graviton-photon conversion.

C. Quantum enhancement with multiple ion pairs

In this subsection, we demonstrate that a maximally entangled state of N vibrational qubits, generated through a sequence of quantum operations in Paul traps, enhances the excitation rate by a factor of N^2 relative to the single-ion case. Assuming the availability of technology that permits state manipulation of qubits distributed across separate traps, we consider a configuration in which N qubits,

²Here, we work in the proper detector frame, where the leading coupling between the relative displacement of two ions and spacetime curvature is described by the Riemann tensor. It remains valid for gravitational waves in the long-wavelength limit, where the wavelength is much larger than the ion separation.

corresponding to $2N$ ions, are arranged in N two-ion detectors. We label the ions in the j th two-ion detector as A_j and B_j . The state of the j th detector is characterized by three qubits: the spin qubits of A_j and B_j and the vibrational qubit.

The time evolution of the ions induced by gravitational waves, which resonantly couple to the vibrational qubits, is described by the displacement operator $D(\beta) = \prod_j D_j(\beta)$, $D_j(\beta) = \exp(\beta \hat{a}_j^\dagger - \beta^* \hat{a}_j)$, where $\hat{a}_j^{(\dagger)}$ are the ladder operators acting on the j th vibrational qubit. The coupling $\beta = \beta_r + i\beta_i$ is assumed to be sufficiently small so that quadratic terms can be neglected. In this limit, the operator can be rewritten as

$$D_j(\beta) = (1 + i\beta_i \hat{\sigma}_j^1) e^{-i\beta_r \hat{\sigma}_j^2} = (1 - i\beta_r \hat{\sigma}_j^2) e^{i\beta_i \hat{\sigma}_j^1}, \quad (3.13)$$

with $\hat{\sigma}_j^1 = \hat{a}_j^\dagger + \hat{a}_j$ and $\hat{\sigma}_j^2 = i\hat{a}_j^\dagger - i\hat{a}_j$ representing the first and second Pauli matrices acting on the j th vibrational qubit.

We describe a protocol for measuring the imaginary part β_i . The procedure begins with the ground state $|\text{init}\rangle \equiv |g, g, 0\rangle^{\otimes N}$. A Hadamard gate is first applied to ion B_1 , followed by CNOT gates on every pair (B_1, B_j) , $j = 1, 2, \dots, N$. This sequence prepares the state

$$\frac{1}{\sqrt{2}} [|g, g, 0\rangle^{\otimes N} + |g, e, 0\rangle^{\otimes N}]. \quad (3.14)$$

At this stage, the B ions are maximally entangled in the state commonly referred to as the Greenberger-Horne-Zeilinger state. By subsequently applying Hadamard gates to the B ions and the red-sideband operators $(|e, 0\rangle \rightarrow |g, 1\rangle)$ on all ion pairs, the system is transformed into

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} [|+\rangle^{\otimes N} + |-\rangle^{\otimes N}], \quad (3.15)$$

where $|\pm\rangle = (|g, g, 0\rangle \pm |g, g, 1\rangle)/\sqrt{2}$ are eigenstates of $\hat{\sigma}_j^1$ with eigenvalues ± 1 .

The gravitational wave acts on the ions, yielding

$$|\psi(T)\rangle = D(\beta)|\psi(0)\rangle = \frac{1}{\sqrt{2}} [e^{iN\beta_i}(|+\rangle - \beta_r|-\rangle)^{\otimes N} + e^{-iN\beta_i}(|-\rangle + \beta_r|+\rangle)^{\otimes N}] \quad (3.16)$$

with $\beta = -ieT$. By applying the inverse of the state-preparation operators, we obtain

$$|\text{fin}\rangle = (|g, g, 0\rangle + iN\beta_i |g, e, 0\rangle) \otimes |g, g, 0\rangle^{\otimes N-1} + \dots, \quad (3.17)$$

where the omitted terms are orthogonal to the first term. Since these orthogonal components do not contribute to the measurement of the target state, they can be neglected in the

following analysis. The sequence of operations returns the system to its initial state, $|\text{fin}\rangle = |\text{init}\rangle$, if $\beta = 0$.

The signal event is the excitation of the B_1 spin state, and its probability is enhanced by a factor of N^2 :

$$|(\langle g, e, 0| \otimes \langle g, g, 0|^{\otimes N-1})|\text{fin}\rangle|^2 = N^2 \beta_i^2.$$

This enhancement arises from quantum interference in the maximally entangled state, where the collective contribution of N qubits coherently amplifies the excitation probability compared to the single-ion case. To measure the real part β_r , one should prepare the state

$$|\psi'(0)\rangle = \frac{1}{\sqrt{2}} \left[\left(\frac{|g, g, 0\rangle + i|g, g, 1\rangle}{\sqrt{2}} \right)^{\otimes N} + \left(\frac{|g, g, 0\rangle - i|g, g, 1\rangle}{\sqrt{2}} \right)^{\otimes N} \right], \quad (3.18)$$

composed of the eigenstates of $\hat{\sigma}_j^2$, instead of $|\psi(0)\rangle$. In both measurements, the procedures are repeated N_{shot} times. Since the phase of gravitational wave is unknown, we take an average: $\overline{\beta_r^2} = \overline{\beta_i^2} = |\epsilon|^2 T^2 / 2$.

If heating noise excites the ion oscillations incoherently, the excitation rate scales linearly with N . Consequently, the significance of the signal scales as

$$\frac{S}{\sqrt{B}} \propto N^{3/2}. \quad (3.19)$$

The sensitivity is improved accordingly, yielding $h^+ = 5.1 \times 10^{-11} \times N^{-3/4}$ under the same parameters as in Sec. III B. We implicitly assume that the system coherence time, including the lifetimes of the excited states and the decoherence time of the entanglement, is much longer than the single-shot observation time.³ While the entanglement-enhanced scheme is experimentally challenging with current technology [19,27], ongoing progress in trapped-ion coherence times, gate fidelities, and large-scale entanglement suggests that the required conditions could be explored in future experiments.

IV. CONCLUSIONS

We investigate the use of linear Paul traps as quantum sensors for detecting high-frequency gravitational waves, which are probes of the early Universe. A linear Paul trap confines ions in an ultrahigh vacuum using a combination of static and radio-frequency electric fields, forming a chain along the trap axis. The quantized vibrational modes of ions and internal two-level electronic states serve as vibrational and spin qubits, enabling precise quantum control for computation and sensing applications.

³A coherence time of about 50 s has been demonstrated in a trapped-ion experiment [26].

Single-ion configurations exploit graviton-photon conversion in the presence of strong external magnetic fields, where a gravitational wave induces an effective electric field that couples to the vibrational qubit of the ion. The vibrational excitation is mapped onto the spin state with a laser, and fluorescence measurements allow extraction of the signal. Two-ion systems utilize not only the center-of-mass mode but also the relative motion between ions, which can be affected by gravitational waves but not by axion dark matter, allowing discrimination between the two. By tuning the resonance frequency of the vibrational mode, a range of gravitational wave frequencies, from hundreds of kilohertz to several megahertz, can be scanned.

The use of multiple entangled vibrational qubits across separate Paul traps enables quantum enhancement of the signal. By preparing maximally entangled states, the excitation probability of the vibrational modes scales as N^2 for

N qubits, leading to a signal-to-noise improvement proportional to $N^{3/2}$. This method allows the system to probe strains of gravitational waves beyond the standard quantum limit. It is required that the decoherence time and the lifetime of qubit excitations exceed the observation time and that high-fidelity operations capable of connecting a large number of ions are anticipated to become available in the near future.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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- [1] C. Caprini and D. G. Figueroa, Cosmological backgrounds of gravitational waves, *Classical Quantum Gravity* **35**, 163001 (2018).
 - [2] T.-H. Yeh, J. Shelton, K. A. Olive, and B. D. Fields, Probing physics beyond the standard model: Limits from BBN and the CMB independently and combined, *J. Cosmol. Astropart. Phys.* **10** (2022) 046.
 - [3] G. Franciolini, A. Maharana, and F. Muia, Hunt for light primordial black hole dark matter with ultrahigh-frequency gravitational waves, *Phys. Rev. D* **106**, 103520 (2022).
 - [4] G. F. Giudice, M. McCullough, and A. Urbano, Hunting for dark particles with gravitational waves, *J. Cosmol. Astropart. Phys.* **10** (2016) 001.
 - [5] R. Brito, V. Cardoso, and P. Pani, Superradiance: New frontiers in black hole physics, *Lect. Notes Phys.* **906**, 1 (2015).
 - [6] M. Goryachev and M. E. Tobar, Gravitational wave detection with high frequency phonon trapping acoustic cavities, *Phys. Rev. D* **90**, 102005 (2014); **108**, 129901(E) (2023).
 - [7] A. S. Chou, R. Gustafson, C. Hogan, B. Kamai, O. Kwon, R. Lanza *et al.*, MHz gravitational wave constraints with decameter Michelson interferometers, *Phys. Rev. D* **95**, 063002 (2017).
 - [8] N. Aggarwal, G. P. Winstone, M. Teo, M. Baryakhtar, S. L. Larson, V. Kalogera, and A. A. Geraci, Searching for new physics with a levitated-sensor-based gravitational-wave detector, *Phys. Rev. Lett.* **128**, 111101 (2022).
 - [9] V. Domcke, S. A. R. Ellis, and N. L. Rodd, Magnets are weber bar gravitational wave detectors, *Phys. Rev. Lett.* **134**, 231401 (2025).
 - [10] A. Patra, L. Aiello, A. Ejlli, W. L. Griffiths, A. L. James, N. Kuntimaddi *et al.*, Broadband limits on stochastic length fluctuations from a pair of table-top interferometers, *Phys. Rev. Lett.* **135**, 101402 (2025).
 - [11] N. Aggarwal, O. D. Aguiar, D. Blas, A. Bauswein, G. Cella, S. Clesse *et al.*, Challenges and opportunities of gravitational-wave searches above 10 kHz, *Living Rev. Relativity* **28**, 10 (2025).
 - [12] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O'Brien, Quantum computers, *Nature (London)* **464**, 45 (2010).
 - [13] C. L. Degen, F. Reinhard, and P. Cappellaro, Quantum sensing, *Rev. Mod. Phys.* **89**, 035002 (2017).
 - [14] A. Ito, R. Kitano, W. Nakano, and R. Takai, Quantum entanglement of ions for light dark matter detection, *J. High Energy Phys.* **02** (2024) 124.
 - [15] C. D. Bruzewicz, J. Chiaverini, R. McConnell, and J. M. Sage, Trapped-ion quantum computing: Progress and challenges, *Appl. Phys. Rev.* **6**, 021314 (2019).
 - [16] J. I. Cirac and P. Zoller, Quantum computations with cold trapped ions, *Phys. Rev. Lett.* **74**, 4091 (1995).
 - [17] D. Walls and G. Milburn, *Quantum Optics*, Springer Study Edition (Springer, New York, 1994), 10.1007/978-3-642-79504-6.
 - [18] W. Ratzinger, S. Schenk, and P. Schwaller, A coordinate-independent formalism for detecting high-frequency gravitational waves, *J. High Energy Phys.* **08** (2024) 195.
 - [19] I. Pogorelov, T. Feldker, C. D. Marciniak, L. Postler, G. Jacob, O. Kriegelsteiner *et al.*, Compact ion-trap quantum computing demonstrator, *PRX Quantum* **2**, 020343 (2021).
 - [20] F. Bernardini, A. Chakraborty, and C. R. Ordóñez, Quantum computing with trapped ions: A beginner's guide, *Eur. J. Phys.* **45**, 013001 (2024).
 - [21] M. Brownnutt, M. Kumph, P. Rabl, and R. Blatt, Ion-trap measurements of electric-field noise near surfaces, *Rev. Mod. Phys.* **87**, 1419 (2015).
 - [22] D. Kielpinski, C. Monroe, and D. J. Wineland, Architecture for a large-scale ion-trap quantum computer, *Nature (London)* **417**, 709 (2002).

- [23] R. Lechner, C. Maier, C. Hempel, P. Jurcevic, B. P. Lanyon, T. Monz, M. Brownnutt, R. Blatt, and C. F. Roos, Electromagnetically-induced-transparency ground-state cooling of long ion strings, *Phys. Rev. A* **93**, 053401 (2016).
- [24] A. Ito, Inertial and gravitational effects on a geonium atom, *Classical Quantum Gravity* **38**, 195015 (2021).
- [25] B. E. King, C. S. Wood, C. J. Myatt, Q. A. Turchette, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Cooling the collective motion of trapped ions to initialize a quantum register, *Phys. Rev. Lett.* **81**, 1525 (1998).
- [26] T. P. Harty, D. T. C. Allcock, C. J. Ballance, L. Guidoni, H. A. Janacek, N. M. Linke, D. N. Stacey, and D. M. Lucas, High-fidelity preparation, gates, memory, and readout of a trapped-ion quantum bit, *Phys. Rev. Lett.* **113**, 220501 (2014).
- [27] S. A. Moses *et al.*, A race-track trapped-ion quantum processor, *Phys. Rev. X* **13**, 041052 (2023).