

Positively identifying Higgs effective field theory or standard model effective field theory

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We establish the bounds on Wilson coefficients of the Higgs effective field theory (HEFT) mandated by unitarity and analyticity. These positivity constraints can be projected into the space of the standard model effective field theory (SMEFT) as $\text{HEFT} \supset \text{SMEFT}$. Doing so reveals a subspace allowed by the HEFT but forbidden by SMEFT positivity, thereby identifying a region that could herald the use of the wrong EFT rather than a pathological UV. Restricting to custodial symmetric dimension-eight Higgs operators, there is a unique pair within the SMEFT where this concept can be sharply realized and is already being probed at colliders.

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I. INTRODUCTION

In the absence of a discovery of novel particles at the energy frontier, the possibility has sharpened that new physics may first appear indirectly through precision tests of the standard model (SM). But how can we characterize the impact of states that exist at scales our experiments cannot yet reach? The answer is provided by effective field theory (EFT), a broad set of tools developed in the middle decades of the twentieth century through which the effects of new physics can be parametrized even before it is discovered. With EFT, the influence of ultraviolet (UV) states is consistently encoded in operators of higher mass dimension that modify the interactions of the particles accessible at low energies.

Remarkably, the full space of couplings of these higher-dimension operators—dubbed Wilson coefficients—is not completely spanned by healthy UV theories. If we assume the UV respects the axioms of our most successful theories—unitarity, locality, and causality—then its shadow in the IR only allows couplings with certain signs and magnitudes [1–6]. Such positivity bounds have found application to a stunning array of EFTs [7], the most important of which for our present discussion is that of the SM (SMEFT) [22–36]. This success begs the question: If a detection is made of a Wilson coefficient violating positivity bounds, what does

this mean? The apparent implications would be profound, suggesting the breakdown of a basic field theory axiom at energy scales near experimental reach. However, an apparent violation could also be generated by the use of the wrong EFT.

An EFT is constructed from both the low-energy field content and assumed symmetries. However, which symmetries to infer from the *relevant* (mass dimension four and lower) operators in constructing the *irrelevant* (mass dimension five or greater) ones can be ambiguous. In the SM, a natural choice is $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$, and imposing this symmetry generates the SMEFT. Below the electroweak scale, however, what is realized is $\text{SU}(3)_C \times \text{U}(1)_{\text{EM}}$, and imposing this less restrictive symmetry gives rise to the Higgs EFT (HEFT). That is, in the HEFT, electroweak symmetry is nonlinearly realized; while this represents a mere field redefinition in the SM, leaving the S-matrix invariant, it has significant consequences for the EFT, as the four real scalars associated with the Higgs field H are no longer required to transform together as an electroweak doublet. Although an old topic [37–39], the HEFT is being actively developed: from its operator counting [40–42], geometric interpretation [43–45], and phenomenological necessity [44–49]. (For a more comprehensive review of the HEFT, see Refs. [50–52].)

What have not been carefully studied are positivity bounds within the HEFT. In particular, how should one interpret SMEFT positivity bounds if the UV in fact generates the HEFT? These issues are the focus of this work, and an answer is provided already in Fig. 1. There, we depict the region consistent with SMEFT positivity in the parameter space of the two custodial symmetric dimension-eight Higgs

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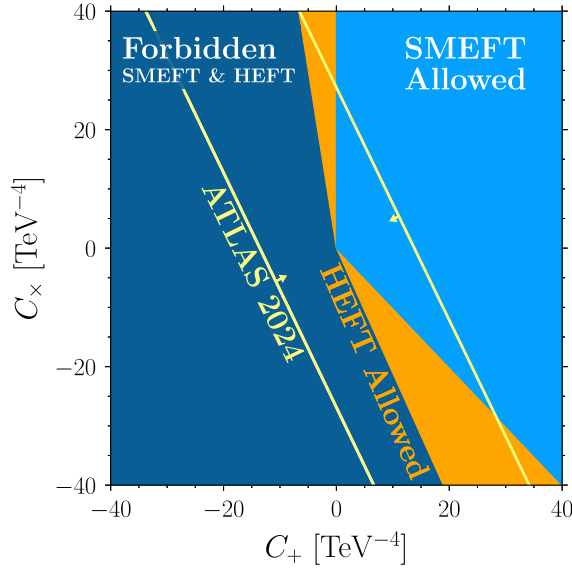


FIG. 1. Positivity bounds for the HEFT projected into the two-dimensional space spanned by the custodial symmetric SMEFT. Applying positivity to the SMEFT directly restricts the allowable EFTs to live in the light blue region. If the Higgs sector is instead described by the HEFT, this region is enlarged to include the region shown in orange: a unique region within which new physics could emerge as indicating a breakdown of the SMEFT rather than violation of analyticity or unitarity. ATLAS measurements constrain these coefficients to within the yellow lines [54]. We justify these partitions in the present paper.

operators in the theory, both of the parametric form $(\partial H)^4$, which we define in detail shortly. We further show how this region is extended from the application of positivity to the HEFT. In this Letter, we construct the pertinent HEFT operators—there are five—and derive the associated positivity bounds resulting from analyticity and unitarity. We construct these bounds using both elastic forward scattering and the generalized optical theorem techniques of Refs. [8,30–33,53]. Building example UV completions, we find that the full allowed space of Wilson coefficients can be spanned by simple one-particle extensions, so these operators are of realistic phenomenological interest. These HEFT bounds are then projected down into the SMEFT subspace, obtaining the region shown in Fig. 1 that would indicate new physics associated with the HEFT rather than a perverse UV. Although the HEFT and SMEFT subspaces are not disjoint, if the unique HEFT region were not included, discoveries could be erroneously associated with a violation of positivity.

Our observation is not purely esoteric: the depicted parameter space is already experimentally accessible through collider searches for anomalous quartic gauge couplings (aQGCs) [55]—the complete basis of which was recently constructed in Ref. [56]—and is being actively constrained at the Large Hadron Collider. Indeed, Fig. 1 includes ATLAS constraints derived from a search for final

states involving a pair of same-sign W bosons and two jets [54]. Assuming a linear representation of the SM gauge symmetry (i.e., SMEFT), the ATLAS measurements constrain certain $(DH)^4$ operators, which we detail shortly; moreover, if one instead assumes a nonlinear realization (i.e., HEFT) and then projects down to the SMEFT subspace, regardless of the UV physics [57], one finds that the various regions allowed or forbidden in HEFT or SMEFT—detailed in our analysis below—intersects nontrivially with the space permitted by the ATLAS measurement.

II. CUSTODIAL SMEFT AND HEFT

We begin by establishing the operator basis. In order to maximize any distinction between the SMEFT and HEFT, we focus on the Higgs sectors of both theories. Further, as reviewed below, sharp positivity bounds arise most directly from dimension-eight operators that support two-to-two scattering amplitudes. Accordingly, the operators of interest take the schematic form $(DH)^4$. Operators at this mass dimension but lower order in derivatives, such as $XH^2(DH)^2$, are higher order in field insertions and therefore do not contribute to quartic Higgs scattering at leading order in the EFT coefficients, which is our focus.

Within the SMEFT, there are three operators of this type [55,58]. For our purposes, it is convenient to write these operators as follows,

$$\begin{aligned}\mathcal{L} &\supset C_+ \mathcal{O}_+ + C_- \mathcal{O}_- + C_\times \mathcal{O}_\times \\ \mathcal{O}_+ &= (\partial_\mu H^\dagger \partial_\nu H)(\partial^\mu H^\dagger \partial^\nu H) \\ \mathcal{O}_- &= (\partial_\mu H^\dagger \partial_\nu H)(\partial^\mu H^\dagger \partial^\nu H) \\ \mathcal{O}_\times &= (\partial^\mu H^\dagger \partial_\mu H)(\partial^\nu H^\dagger \partial_\nu H).\end{aligned}\quad (1)$$

Regarding notation, we write $D_\mu \rightarrow \partial_\mu$, as the distinction is irrelevant in constructing positivity bounds. Higgs $SU(2)_L$ indices are contracted among terms within parentheses. For Lorentz indices, we use round or square brackets to denote normalized symmetrization or antisymmetrization, respectively, i.e., $T_{(\mu\nu)} = (T_{\mu\nu} + T_{\nu\mu})/2$.

To isolate the essential physics, we demand an additional symmetry of the UV: custodial invariance. Custodial symmetry is the $O(4)$ invariance of the SM Higgs sector as one transforms among the four scalar degrees of freedom in H ; after electroweak symmetry breaking, the symmetry is spontaneously broken down to $O(3)$ by the Higgs vacuum expectation value (vev). Imposing custodial invariance mandates $C_- = 0$ and thereby reduces the SMEFT to the following space of two operators,

$$\mathcal{L} \supset C_+ \mathcal{O}_+ + C_\times \mathcal{O}_\times. \quad (2)$$

The coefficients of these operators form the axes in Fig. 1.

Turning to the HEFT, in general, there need be no relation among the four scalar degrees of freedom that combine to

form H in the SMEFT. There are then 55 dimension-eight four-scalar operators to consider [59]. Custodial invariance provides a dramatic simplification. Following Ref. [45], we decompose H into four scalars h and π_i that transform as a singlet and fundamental under custodial $O(3)$ symmetry. With this restriction, explicit calculation reveals five remaining independent HEFT operators, up to total derivatives and field redefinitions,

$$\begin{aligned}\mathcal{L} &\supset c_1^h \mathcal{O}_1^h + c_1^{h\pi} \mathcal{O}_1^{h\pi} + c_2^{h\pi} \mathcal{O}_2^{h\pi} + c_1^\pi \mathcal{O}_1^\pi + c_2^\pi \mathcal{O}_2^\pi \\ \mathcal{O}_1^h &= (\partial h)^4 \\ \mathcal{O}_1^{h\pi} &= (\partial h)^2 (\partial_\mu \pi_i \partial^\mu \pi_i) \\ \mathcal{O}_2^{h\pi} &= (\partial_\mu h \partial_\nu h) (\partial^\mu \pi_i \partial^\nu \pi_i) \\ \mathcal{O}_1^\pi &= (\partial^\mu \pi_i \partial_\mu \pi_i) (\partial^\nu \pi_j \partial_\nu \pi_j) \\ \mathcal{O}_2^\pi &= (\partial^\mu \pi_i \partial^\nu \pi_i) (\partial_\mu \pi_j \partial_\nu \pi_j).\end{aligned}\quad (3)$$

While power counting is subtle in the HEFT [44,60], in our case, where we are interested in the four-derivative operators that appear in subtracted dispersion relations [1], the operators in Eq. (3) are precisely those that contribute. HEFT operators are more commonly defined from a Coleman-Callan-Wess-Zumino construction [61,62], $\exp[\pi_i \tau_i / v]$, with τ_i the three generators of $O(4)$ broken by the Higgs vev. Our operator basis can be written in an unbroken, $O(4)$ invariant way [63], but as positivity bounds are computed from scattering amplitudes, the above basis in terms of asymptotic states is more convenient. In principle, operators of lower order in field multiplicity or derivatives could also contribute to the amplitudes we study. At present, since for dispersion relations we are sensitive to the part of the IR amplitude quartic in momenta, we ignore such operators by assuming a weakly coupled completion in which loops or multiple insertions of EFT operators are suppressed, though this would be interesting to generalize. While it is possible that running of couplings from loops of light states can eventually dominate over tree-level insertions in the deep IR, the assumption of weak coupling by definition implies the existence of some kinematic region—below the UV scale but not in the exponentially far IR [8]—in which the contact insertions of Wilson coefficient computed at tree level will be dominant, which we take to be our regime of interest.

Further, HEFT operators in principle need only be suppressed by the Higgs vev v , although given the lack of clear new physics signals, we take our UV scale $\Lambda_{\text{UV}} \gg v$, which ensures that our theory satisfies perturbative unitarity [43] and allows us to treat the Higgs fields as effectively massless throughout. More generally, in differentiating HEFT from SMEFT, we are making the distinction at fixed order in the expansion in derivatives and multiplicities; of course, at all orders in the expansion, HEFT and SMEFT should coincide. As a result, certain higher-order SMEFT terms could lead to apparent violation

of fixed order SMEFT positivity bounds. Definitive conclusions about the suitability of HEFT versus SMEFT as the proper low-energy description will of course require multiple measurements to probe the validity of the EFT expansion. In this work, we restrict ourselves to quartic order in derivatives and fields, to illustrate how, even at fixed order, positivity bounds illustrate a difference between HEFT and SMEFT. Therefore, at fixed order $\text{HEFT} \supset \text{SMEFT}$ [64], and a particular two-dimensional slice of Eq. (3) reduces to Eq. (2) (cf. Ref. [65]). Since

$$\begin{aligned}\mathcal{O}_+ &= \frac{1}{4} (\mathcal{O}_1^h + 2\mathcal{O}_2^{h\pi} + \mathcal{O}_2^\pi) \\ \mathcal{O}_\times &= \frac{1}{4} (\mathcal{O}_1^h + 2\mathcal{O}_1^{h\pi} + \mathcal{O}_1^\pi),\end{aligned}\quad (4)$$

the slice is defined by

$$\begin{aligned}4c_1^h &= C_+ + C_\times, & 2c_1^{h\pi} &= 4c_1^\pi = C_\times, \\ 2c_2^{h\pi} &= 4c_2^\pi = C_+.\end{aligned}\quad (5)$$

III. POSITIVITY BOUNDS

We next determine the bounds on the SMEFT and HEFT operators that result from demanding that the UV be unitary, local, and causal. For the SMEFT, the required bounds have already been established [22,30]. Restricting to the custodial sector in Eq. (2), those results become [66]

$$C_+ > 0 \quad \text{and} \quad C_+ + C_\times > 0. \quad (6)$$

These restrictions are shown in Fig. 1.

The equivalent HEFT bounds have not been constructed. As a first step to doing so, we construct the two-to-two scattering amplitudes mediated by the operators in Eq. (3). We consider the most general elastic scattering processes with the incident states constructed from arbitrary superpositions of the π_i and h . In detail, we take one initial state to be $|1\rangle = \alpha_i |\pi_i\rangle + \alpha_h |h\rangle$, with the four coefficients normalized by $\alpha^2 + \alpha_h^2 = 1$, writing α for the vector α_i . Without loss of generality, we can choose the overall sign so that $\alpha_h > 0$. As we have real scalars, the four coefficients can be taken to be real. The second initial state is defined similarly, but with (β, β_h) .

It is convenient to arrange the fields into a multiplet $\Phi_I = (\pi_i, h)$, so that $\Phi_4 = h$. Doing so, the HEFT operators can be combined into $c_{IJKL} \partial_\mu \Phi_I \partial^\mu \Phi_J \partial_\nu \Phi_K \partial^\nu \Phi_L$, where by definition $c_{IJKL} = c_{JIKL} = c_{IJLK} = c_{KLIJ}$. In terms of the Wilson coefficients defined in Eq. (3),

$$\begin{aligned}c_{IJKL} &= c_1^h \delta_{I4} \delta_{J4} \delta_{K4} \delta_{L4} + c_2^{h\pi} \delta_{4(I} \bar{\delta}_{J)(K} \delta_{L)4} \\ &\quad + \frac{1}{2} c_1^{h\pi} (\delta_{I4} \delta_{J4} \bar{\delta}_{KL} + \bar{\delta}_{IJ} \delta_{K4} \delta_{L4}) \\ &\quad + c_1^\pi \bar{\delta}_{IJ} \bar{\delta}_{KL} + c_2^\pi \bar{\delta}_{I(K} \bar{\delta}_{L)J},\end{aligned}\quad (7)$$

where for brevity we write $\bar{\delta}_{IJ} = \delta_{IJ} - \delta_{I4} \delta_{J4}$.

We next compute the forward elastic scattering amplitude, $A(s)$, which is a function purely of the center-of-mass energy squared (Mandelstam s), as in the forward limit we have vanishing exchanged momentum, so that Mandelstam $t \rightarrow 0$. As we review shortly, positivity can be related to the s^2 coefficient of the forward amplitude, which in terms of the general notation above is given by $A''(s) = A''_{IJKL} \alpha_I \beta_J \alpha_K \beta_L$, where

$$A''_{IJKL} = 4(c_{IJKL} + c_{ILKJ}). \quad (8)$$

Using Eq. (7) and writing $\alpha = |\alpha|$ and $\beta = |\beta|$,

$$\begin{aligned} A''(s) = & 8c_1^h(1 - \alpha^2)(1 - \beta^2) + 2c_2^{h\pi}(\alpha^2 + \beta^2 - 2\alpha^2\beta^2) \\ & + 4(2c_1^{h\pi} + c_2^{h\pi})\sqrt{(1 - \alpha^2)(1 - \beta^2)}(\alpha \cdot \beta) \\ & + 8(c_1^\pi + c_2^\pi)(\alpha \cdot \beta)^2 + 4c_2^\pi(\alpha \times \beta)^2. \end{aligned} \quad (9)$$

This result is primed for positivity. In particular, when analytically continued to complex s , $A(s)$ is an analytic function up to discontinuities along the real axis associated with single- or multiparticle exchanges in the s - and u -channels [68–72]. One can therefore use contour integration relate the EFT amplitude $A''(s)$ to the UV cross section by connecting a contour at small $|s|$ to one at large $|s|$. In detail, one finds the classic result [1]

$$A''(s) = \frac{4}{\pi} \int_0^\infty \frac{ds}{s^2} \sigma(s) > 0. \quad (10)$$

See Ref. [22] for further review. Here we simply emphasize that the result only holds if the UV is unitary and local, using locality to invoke analyticity of the amplitude off of the real axis and thereby deform the contour and unitarity to invoke the optical theorem connecting the discontinuity across the axis to the cross section.

Once positivity in Eq. (10) is established, Eq. (9) implies a set of constraints on the HEFT coefficients. Positivity must hold for *any* α and β satisfying $\alpha, \beta \leq 1$. Marginalizing over all possible choices, we find the following succinct set of positivity bounds,

$$\begin{aligned} c_1^h &> 0, & c_2^{h\pi} &> 0, \\ c_1^\pi + c_2^\pi &> 0, & c_2^\pi &> 0, \\ -c_2^{h\pi} - \sqrt{4c_1^h(c_1^\pi + c_2^\pi)} &< c_1^{h\pi} < \sqrt{4c_1^h(c_1^\pi + c_2^\pi)}. \end{aligned} \quad (11)$$

A careful derivation of these bounds is provided in the Appendix. There we further show that the constraints are actually more general than they naively appear. In particular, the results in Eq. (11) are derived from the optical theorem applied to elastic scattering. However, for general EFTs it is possible to obtain even stronger bounds using the *generalized* optical theorem, as discussed in Refs. [8,30–33,53]. For

the HEFT, however, we show that the generalized optical theorem yields *precisely* the same bounds as in Eq. (11).

IV. POSITIVITY AND PROJECTION

We next consider the implications of the newly derived bounds for the SMEFT parameter space in Fig. 1. If we simply take the SMEFT slice of the HEFT using Eq. (5), then Eq. (11) collapses exactly to the SMEFT constraints of Eq. (6). But this conclusion would be too quick: the shadow cast by the HEFT onto this plane could be larger. We instead need to project the positive HEFT onto the SMEFT subspace.

To perform this projection, we first observe that the SMEFT subspace is defined by three constraints, $c_1^{h\pi} - 2c_1^\pi = 0$, $c_2^{h\pi} - 2c_2^\pi = 0$, and $2c_1^h - c_1^{h\pi} - c_2^{h\pi} = 0$, which we write in matrix form as $V_{ij}c_j = 0$, where

$$V_{ij} = \frac{1}{\sqrt{35}} \begin{pmatrix} 0 & -\sqrt{7} & 0 & 2\sqrt{7} & 0 \\ 0 & 0 & -\sqrt{7} & 0 & 2\sqrt{7} \\ -5 & 2 & 2 & 1 & 1 \end{pmatrix}, \quad (12)$$

defining the labels $c_i = (c_1^h, c_1^{h\pi}, c_2^{h\pi}, c_1^\pi, c_2^\pi)$. The constraints are invariant under adding any linear combinations of the rows of V , and we have used this freedom to ensure $V_{ij}V_{kj} = \delta_{ik}$. That is, V_{1j} , V_{2j} , and V_{3j} define a basis of orthonormal vectors perpendicular to the SMEFT plane. The projection of HEFT coefficients c_k onto the SMEFT plane is then given by $\hat{c}_k = c_k - V_{ij}c_jV_{ik}$. We can thus decompose completely general HEFT coefficients c_k into a contribution within and perpendicular to the SMEFT plane, as

$$c_k = \hat{c}_k + d_i V_{ik} \quad (13)$$

for some $d_{1,2,3}$. Using Eq. (5), we write the $\hat{c}_k$ in terms of the SMEFT coefficients $C_{+, \times}$,

$$\hat{c}_k = \frac{1}{4}(C_+ + C_\times, 2C_\times, 2C_+, C_\times, C_+). \quad (14)$$

On the slice of the HEFT that corresponds to the SMEFT, we have $d_{1,2,3} = 0$, and as noted, the general bounds reduce to those of the SMEFT. In other words, if low-energy physics is described by the SMEFT, and we parametrize physics in terms of the HEFT and then postselect down to the SMEFT, the constraints are the same as if we had used the SMEFT all along, as expected.

What about the converse? That is, what if the low-energy physics of our universe is described by the HEFT, but we instead naively use the SMEFT in defining our positivity bounds and experimentally measuring deviations from the SM? In that case, the allowed space of bounds in the SMEFT plane is that for which there *exist* some $d_{1,2,3}$ for which the point in the SMEFT plane satisfies the HEFT bounds in Eq. (11). In other words, using the SMEFT in

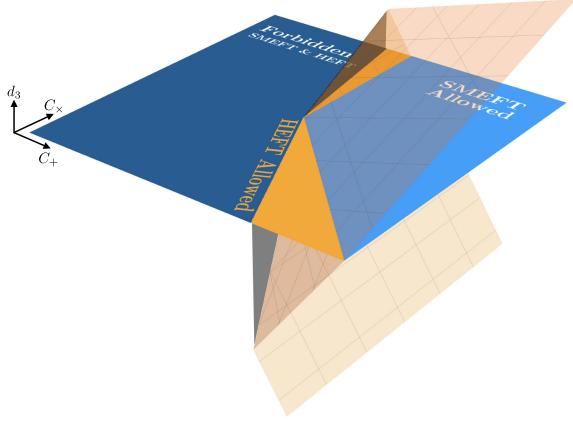


FIG. 2. A perspective on the projection of the HEFT to the SMEFT. Beyond the SMEFT plane shown in Fig. 1, we have included the additional dimension d_3 of the HEFT; we continue to marginalize over $d_{1,2}$. The space permitted by HEFT positivity bounds—that is, where there exist some $d_{1,2}$ such that a given point (C_+, C_+, d_3) complies with the HEFT positivity bounds—is to the right of the hatched light orange contour. The contour provides an indication of how the HEFT can enlarge the allowed parameters to the orange region, and in dark gray we illustrate the projection to the SMEFT.

a world defined by the HEFT, the space of $C_{+,x}$ consistent with unitarity and locality is the *projection* of the five-dimensional HEFT cone onto the SMEFT plane, rather than the slice of the cone through the plane. Inputting the parametrization of the c_k in Eq. (13) into the HEFT bounds and marginalizing over the $d_{1,2,3}$, we find that the projected bounds become

$$6C_+ + C_x > 0 \quad \text{and} \quad 19C_+ + 9C_x > 0. \quad (15)$$

The region allowed by the bounds for the SMEFT itself in Eq. (6) forms a *strict subset* of the region permitted by Eq. (15). If the universe is described by the HEFT and not the SMEFT, but one erroneously parametrizes new physics in terms of the SMEFT anyway, then one could see an apparent violation of positivity in the SMEFT coefficients simply due to this projection. This justifies the HEFT region depicted in Fig. 1; see Fig. 2 for a perspective on how the additional parameter space emerges.

V. ULTRAVIOLET EXTENSIONS

As an illustration that our bounds capture realistic scenarios for new physics, let us write down UV extensions of the HEFT under which the bounds in Eq. (11) are saturated. That is, we consider tree-level models involving a single massive state coupled to a bilinear of the light Higgs fields, where integrating out the new heavy field gives the operators in Eq. (3). The phrasing “UV extensions” indicates that the theories need not be explicitly UV complete so long as they parametrically raise the cutoff of

the theory. For example, if we have a scalar X of mass m that couples to h via the dimension-five operator $(X/\Lambda_X)(\partial h)^2$, for some scale $\Lambda_X \gg m$, then integrating out X generates the dimension-eight HEFT operator $(\partial h)^4$ with Wilson coefficient $c_1^h = 1/2\Lambda_X^2 m^2$. That is, integrating in X raises the HEFT cutoff from $\Lambda_{UV} = \sqrt{2\Lambda_X m}$ up to $\Lambda_X \gg \Lambda_{UV}$, so this model is a UV extension of the HEFT [73].

Let us consider a theory containing the following massive particles: scalars X and Y transforming as singlets under $O(3)$, a scalar Z_i transforming as a $\mathbf{3}$ (vector) of $O(3)$, and two massive spacetime vectors A_i^μ and B_i^μ also transforming as $\mathbf{3}$ (vectors) of $O(3)$. We couple these states to the Higgs in a custodial invariant Lagrangian,

$$\begin{aligned} \mathcal{L} \supset & g_1 X \partial_\mu h \partial^\mu h + g_2 X \partial_\mu \pi_i \partial^\mu \pi_i + g_3 Y \partial_\mu h \partial^\mu h \\ & + g_4 Z_i \partial_\mu \pi_i \partial^\mu h + m g_5 h A_i^\mu \partial_\mu \pi_i + m g_6 \epsilon_{ijk} B_i^\mu \pi_j \partial_\mu \pi_k, \end{aligned} \quad (16)$$

taking all heavy particles to have mass m . We have introduced factors of m such that all of the couplings g_i have mass dimension -1 . Just as for the HEFT basis, the interactions in Eq. (16) can be straightforwardly written with explicit $O(4)$ invariance [74]. Integrating out the UV states, we have the following Wilson coefficients for the HEFT operators in Eq. (3),

$$\begin{aligned} c_1 &= \hat{g}_1^2 + \hat{g}_3^2, & c_2 &= 2\hat{g}_1\hat{g}_2 - \hat{g}_5^2, & c_3 &= \hat{g}_4^2 + \hat{g}_5^2 \\ c_4 &= \hat{g}_2^2 - 2\hat{g}_6^2, & c_5 &= 2\hat{g}_6^2, \end{aligned} \quad (17)$$

for brevity defining $\hat{g}_i = g_i/\sqrt{2}m$, in addition to the dimension-six terms,

$$\mathcal{L} \supset \frac{1}{2} g_5^2 h^2 (\partial_\mu \pi_i \partial^\mu \pi_i) + g_6^2 g_{\mu\nu} (\pi_i \partial^\mu \pi_j) (\pi_i \partial^\nu \pi_j). \quad (18)$$

The region spanned by the couplings $\hat{g}_i$ within the space of Wilson coefficients is precisely that given by the positivity bounds in Eq. (11). That is, we have found tree-level UV extensions of the HEFT that span the full space of coefficients consistent with unitarity.

A simple UV extension of the HEFT that violates SMEFT positivity is the interaction $gX\partial_\mu \pi_i \partial^\mu \pi_i$, that is, Eq. (16) in the case where the couplings satisfy $g_i = g \times (0, 1, 0, 0, 0, 0)$. This UV extension of the HEFT via a single scalar field, reminiscent of a sigma model, generates Wilson coefficients that, when projected down to the SMEFT plane, live on the line $6C_+ + C_x = 0$, with $C_+ < 0$, violating SMEFT positivity (6) [but satisfying $19C_+ + 9C_x > 0$ from HEFT (15)]. Similarly, the interaction $gX(\partial_\mu h \partial^\mu h - 2\partial_\mu \pi_i \partial^\mu \pi_i)$, i.e., $g_i = g \times (1, -2, 0, 0, 0, 0)$, also reminiscent of a sigma model, generates Wilson coefficients after projection living on the line $19C_+ + 9C_x = 0$, but with $C_+ > 0$, thereby violating the SMEFT bounds (6) with $C_+ + C_x < 0$ [but satisfying $6C_+ + C_x > 0$ from HEFT (15)]. That is, these

two simple scalar extensions of the HEFT generate the two rays in Fig. 1 separating the orange HEFT-only region from the dark blue forbidden region. We reiterate that the scenarios studied here are simple UV extensions; we leave open the important question of how generic it is for UV complete models to populate the HEFT window.

VI. DISCUSSION

The discovery of a nonvanishing SMEFT Wilson coefficient would herald the first clear sign at a collider of the breakdown of the SM. The promise of positivity is that it can enhance any such discovery into a probe of the principles governing the emerging UV: is the theory unitary, local, and causal? However, as we have shown in this paper, these searches also probe the symmetry structure of the UV, and a discovery in the orange region in Fig. 1 would be a strong indication that the new physics is best described by the HEFT.

By assuming custodial symmetry and performing a purely tree-level analysis, the present work represents a first glimpse into positivity applied to the HEFT. Moving forward, it would be important to understand the impact of multiple insertions of operators of lower mass dimension that arise when considering loops; there is a growing appreciation that such contributions can play a key role in dispersive analyses [6,75–78]. Further, it will be important to identify and analyze complete UV models that can populate the HEFT allowed region in Fig. 1 to provide context for future experimental searches. Such models will provide a testing ground for understanding the importance of various simplifying assumptions adopted in our work (such as neglecting loop effects) and on to what extent a discovery is within reach when constraints from other searches are incorporated (see, e.g., Ref. [79]).

One of the challenges in studying EFTs is identifying interesting regions of the vast parameter space; the SMEFT alone has 44,807 operators at dimension eight [80]. Although simplifying assumptions have been adopted, this paper demonstrates that the two-dimensional space of Fig. 1 appears rich. It is also a space well poised to be further tested at the high luminosity run at the Large Hadron Collider. Both ATLAS [54,81–85] and CMS [86–96] have performed analyses searching for aQGCs induced by the SMEFT at dimension eight, including explicit studies of two-dimensional slices of this parameter space [54,85,86]. The space of Fig. 1 is therefore ready to be explored and may yet reveal the first hints of SMEFT or even HEFT.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: ALL POSSIBLE SUPERPOSITIONS

Starting from the expression for $A''(s)$ in Eq. (9), let us see how the positivity bound $A''(s) > 0$ from the optical theorem reduces to the set of conditions in Eq. (11) after marginalizing over all possible superpositions of scattering states. Let us define parameters $\mu, \nu \geq 0$ and $\rho \in [-1, 1]$ via $\mu = \alpha/(1 - \alpha^2)^{1/2}$, $\nu = \beta/(1 - \beta^2)^{1/2}$, and $\rho = \alpha \cdot \beta / \alpha\beta$, as well as a reparametrization of the Wilson coefficients,

$$\begin{aligned} x &= \frac{c_1^{h\pi}}{\sqrt{4c_1^h(c_1^\pi + c_2^\pi)}}, & y &= \frac{c_2^{h\pi}}{\sqrt{4c_1^h(c_1^\pi + c_2^\pi)}}, \\ z &= \sqrt{\frac{c_1^h}{c_1^\pi + c_2^\pi}}, & w &= \sqrt{\frac{c_2^\pi}{c_1^\pi + c_2^\pi}}. \end{aligned} \quad (\text{A1})$$

The bounds on the Wilson coefficients then become the requirement that (x, y, z, w) satisfy

$$\begin{aligned} 0 < z^2 + \mu^2 \nu^2 \rho^2 + \frac{1}{2} yz(\mu^2 + 2\mu\nu\rho + \nu^2) \\ + 2xz\mu\nu\rho + \frac{1}{2} w^2 \mu^2 \nu^2 (1 - \rho^2) \end{aligned} \quad (\text{A2})$$

for all $\mu, \nu \geq 0$ and $-1 \leq \rho \leq 1$. Imposing $A''(s) > 0$ on Eq. (9) for $\alpha = \beta = 0$, we have $c_1^h > 0$, while from $\alpha = \beta = 1$ and $\alpha = \beta$, we have $c_1^\pi + c_2^\pi > 0$. Meanwhile, with $\alpha = \beta = 1$ and choosing $\alpha \perp \beta$, we find $c_2^\pi > 0$, while with $\beta = 1$ and $\alpha = 0$, we obtain $c_2^{h\pi} > 0$. In terms of Eq. (A1), we therefore have

$$y, z, w > 0 \quad \text{and} \quad x \in \mathbb{R}. \quad (\text{A3})$$

First marginalizing Eq. (A2) over all $\mu, \nu \geq 0$, we find the condition

$$y + (2x + y)\rho + \sqrt{4\rho^2 + 2w^2(1 - \rho^2)} > 0. \quad (\text{A4})$$

Marginalizing Eq. (A4) over all $\rho \in [-1, 1]$ while imposing Eq. (A3), we arrive at the conditions

$$y, z, w > 0, \quad x < 1, \quad \text{and} \quad 1 + x + y > 0. \quad (\text{A5})$$

Putting the definitions in Eq. (A1) together with Eq. (A5) and the positivity bounds in the previous paragraph, we obtain the final set of conditions on the Wilson coefficients given in Eq. (11).

APPENDIX B: GENERALIZED OPTICAL THEOREM

Elastic positivity bounds result from the standard optical theorem, $\text{Im} A(s) = s\sigma(s)$, where A is a forward amplitude with the two-particle ingoing state matching the outgoing one. However, this is a special case of the *generalized* optical theorem, which allows us to consider cases where the two states are not identical. Specifically, scattering $\Phi_I \Phi_J \rightarrow \Phi_K \Phi_L$ with zero momentum transfer ($t = 0$) and center-of-mass energy squared s , described by an amplitude $A_{IJKL}(s)$, the generalized optical theorem states that $\text{Disc} A_{IJKL}(s) = i \sum_X M_{IJ \rightarrow X}(s) M_{KL \rightarrow X}^*(s)$, where $M_{IJ \rightarrow X}$ is the amplitude for $\Phi_I \Phi_J \rightarrow X$ for any intermediate state X , and $\sum_X$ denotes the Källén-Lehmann-like sum over all such states [97,98], including multi-particle loops treated as integrals over on-shell configurations [5,99]. From a dispersion relation construction analogous to Eq. (10) for $A_{IJKL}'' = \lim_{s \rightarrow 0} \partial_s^2 A_{IJKL}(s)$, one finds [8,53] that A_{IJKL}'' in Eq. (8) must satisfy the relation

$$A_{IJKL}'' = \sum_X \left(M_{IJ}^{(X)} M_{KL}^{(X)} + M_{IL}^{(X)} M_{JK}^{(X)} \right) \quad (\text{B1})$$

for some collection of real matrices $M_{IJ}^{(X)}$ that run over the real and imaginary parts of $M_{IJ \rightarrow X}$. Finding the space of Wilson coefficients parametrized by A_{IJKL}'' and swept out by all choices of $M_{IJ}^{(X)}$ is equivalent to marginalizing over all UV completions of the EFT and is in general a highly complex problem, which can nonetheless be solved explicitly in certain special cases [53].

However, we can decompose the right-hand side of Eq. (B1) into irreducible representations of the symmetry group, and in doing so we find a set of candidate so-called “extremal rays” [30]. One can show that the cone of allowed A_{IJKL}'' is spanned by the convex hull of all such extremal rays. In physical terms, these rays correspond to one-particle UV extensions of the EFT operators. In the case at hand, each of our external states comprises a multiplet $\Phi_I = (\pi_i, h)$, with π_i and h transforming as a vector and singlet of the $O(3)$ custodial symmetry, respectively. Hence, we must construct the irreducible representations of $\mathbf{R} \otimes \mathbf{R}$, where $\mathbf{R} = \mathbf{3} \oplus \mathbf{1}$. Our analysis here complements that for elastic bounds in Eq. (11), which were necessary but in principle not sufficient for unitarity. We find that the bounds constructed with these two methods precisely match, so the constraints in Eq. (11) are indeed both necessary and sufficient. Expanding the product, we have $\mathbf{R} \otimes \mathbf{R} = (\mathbf{3} \otimes \mathbf{3}) \oplus (\mathbf{3} \otimes \mathbf{1}) \oplus (\mathbf{1} \otimes \mathbf{3}) \oplus (\mathbf{1} \otimes \mathbf{1})$. The final three parenthetical terms are already irreducible to two

vectors and a singlet. Viewing the intermediate state X in $\Phi_I \Phi_J \rightarrow X_{IJ}$ as a matrix,

$$X = \begin{pmatrix} X_{ij} & X_{i4} \\ X_{4i} & X_{44} \end{pmatrix}, \quad (\text{B2})$$

X_{44} is the singlet $\mathbf{1} \otimes \mathbf{1} = \mathbf{1}$, X_{4i} and X_{i4} are the vectors $\mathbf{1} \otimes \mathbf{3} = \mathbf{3}$ and $\mathbf{3} \otimes \mathbf{1} = \mathbf{3}$, and the $\mathbf{3} \otimes \mathbf{3} = \mathbf{9}$ term, corresponding to the matrix X_{ij} , decomposes as $\mathbf{9} = \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{5}$, where $\mathbf{1}$ corresponds to the trace, $\mathbf{3}$ to the antisymmetric part (the dual of a vector), and $\mathbf{5}$ to the symmetric traceless matrices.

If there is no degeneracy in the group structure of our exchanged states—i.e., if we only have at most one of each type of irreducible representation—then we can construct projectors onto each of those states, and these projectors define candidate extremal rays that form the cone giving us the optimal bounds. However, in the case above, we have two singlets in $\mathbf{R} \otimes \mathbf{R}$: one from the product of the $\mathbf{1}$ factors in $\mathbf{R}$, and the $\mathbf{1}$ in the decomposition of the $\mathbf{9}$. In more physical terms, if we scatter two π_i states, they can form a state in the UV that is a singlet under custodial symmetry, which could then decay to two h particles, or alternatively into two π_i . Similarly, we have three triplets: the X_{4i} and X_{i4} components, as well as the $\mathbf{3}$ from the $\mathbf{9}$. We only lack degeneracy in the $\mathbf{5}$. When there is degeneracy, the extremal cone acquires curved facets, associated with the continuous families of candidate extremal rays parametrized by the couplings allowing the degenerate states to transition into each other [100].

We are interested in constructing the projection operators P_{IJKL} onto the above irreducible representations. That is, for an arbitrary matrix X_{KL} , $P_{IJKL} X_{KL}$ is the projection onto the components of the desired representation. In the presence of degeneracy, the P tensors contain free parameters in order to account for the freedom of the couplings mixing the representations. Conveniently for our present purposes, the system of W and B fields in the SM has a group theoretic structure that maps neatly onto $\mathbf{R}$: there is a singlet B boson and an $SU(2)$ triplet of W bosons (which, up to the $\mathbb{Z}_2$ of $\pi_i \leftrightarrow -\pi_i$, is locally equivalent to our $O(3)$ custodial group). The projectors for this system were determined explicitly in Sec. 5 of Ref. [31]. We write $\bar{\delta}_{IJ}(r) = \delta_{IJ} + (r-1)\delta_{I4}\delta_{J4}$, so that the $\bar{\delta}_{IJ}$ defined in the main text is $\bar{\delta}_{IJ}(0)$. The projector onto the singlet is

$$P_{IJKL}^1(r) = \frac{1}{3} \bar{\delta}_{IJ}(r) \bar{\delta}_{KL}(r). \quad (\text{B3})$$

For the triplets, we first define the following matrices,

$$f_{IJ}^{(1)}(r_1, r_2) = \begin{pmatrix} 0 & 0 & 0 & r_1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ r_2 & 0 & 0 & 0 \end{pmatrix} \quad f_{IJ}^{(2)}(r_1, r_2) = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & r_1 \\ 1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \end{pmatrix} \quad f_{IJ}^{(3)}(r_1, r_2) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_1 \\ 0 & 0 & r_2 & 0 \end{pmatrix}. \quad (\text{B4})$$

As one can explicitly verify, we have two projectors onto the triplet states,

$$P_{IJKL}^{3+}(r_1, r_2) = \frac{1}{2} \sum_{i=0}^3 f_{[IJ]}^{(i)}(r_1, r_2) f_{[KL]}^{(i)}(r_1, r_2)$$

$$P_{IJKL}^{3-}(r_1, r_2) = \frac{1}{2} \sum_{i=0}^3 f_{[IJ]}^{(i)}(r_1, r_2) f_{[KL]}^{(i)}(r_1, r_2). \quad (\text{B5})$$

Noting that $\bar{\delta}_{IJ}(0)$ defines a projector onto the subspace where $I, J \neq 4$, we can define the projector of the 5,

$$P_{IJKL}^5 = \bar{\delta}_{I(K}(0) \bar{\delta}_{L)J}(0) - \frac{1}{3} \bar{\delta}_{IJ}(0) \bar{\delta}_{KL}(0). \quad (\text{B6})$$

At last, we define the candidate extremal rays by symmetrizing on $J \leftrightarrow L$, which we write as $\hat{P}_{IJKL} = (P_{IJKL} + P_{ILKJ})/2$. We find that $\hat{P}_{IJKL}^{3+}$ is proportional to $(r_1 + r_2)^2$. Since we are concerned with rays, we can divide out by this combination in $\hat{P}_{IJKL}^{3+}$. Similarly, $\hat{P}_{IJKL}^{3-}$ depends only on $r_1 - r_2$, which we will denote by q . We find that $\hat{P}_{IJKL}^5$ is redundant, since it can be written as a positive linear combination of the others, $\hat{P}_{IJKL}^5 = 2\hat{P}_{IJKL}^1(0) + \hat{P}_{IJKL}^3(0)$, so we discard it. Thus, our candidate extremal rays are

$$\hat{P}_{IJKL}^1(r), \quad \hat{P}_{IJKL}^{3-}(q), \quad \text{and} \quad \hat{P}_{IJKL}^{3+}. \quad (\text{B7})$$

Two depend on parameters, and one does not.

The power of the generalized optical theorem is that we can replace Eq. (B1) with $A''_{IJKL} = \sum_{\alpha} N_{\alpha} \hat{P}_{IJKL}^{\alpha}$, where α runs over all of the labels of the projectors, including an integral over all possible parameters q and r . Matching with $A''_{IJKL} = 4(c_{IJKL} + c_{ILKJ})$ from Eq. (8), where the Wilson coefficient tensor is given in Eq. (7) in terms of the HEFT coefficients defined in Eq. (3), we find

$$\int_{-\infty}^{\infty} dr N_1(r) = 24(c_1^{\pi} + c_2^{\pi})$$

$$\int_{-\infty}^{\infty} dq N_{3-}(q) = 8c_2^{\pi}$$

$$3N_{3+} + 4 \int_{-\infty}^{\infty} dr r N_1(r) = 48(c_1^{h\pi} + c_2^{h\pi})$$

$$N_{3+} + \int_{-\infty}^{\infty} dq q^2 N_{3-}(q) = 16c_2^{h\pi}$$

$$\int_{-\infty}^{\infty} dr r^2 N_1(r) = 24c_1^h. \quad (\text{B8})$$

Unitarity bounds the HEFT coefficients to be such that there exists a positive constant N_{3+} and positive functions $N_1(r)$ and $N_{3-}(q)$ such that the constraints in Eq. (B8) are satisfied. To turn this statement into a bound on the HEFT coefficients alone, let us first suppose that $N_1(r) = n_1 \delta(r - r_0)$ and $N_{3-}(q) = n_{3-} \delta(q - q_0)$. Then we require that there exist some real values of r_0 and q_0 and some positive n_1, n_{3-} , and N_{3+} such that Eq. (B8) is satisfied. This marginalization can be done in closed form, and in terms of Eq. (A1) we find

$$y, z, w > 0 \quad \text{and} \quad (x < -1 < x + y \quad \text{or} \quad x < 1 < x + y). \quad (\text{B9})$$

In Eq. (B9), we required fixed values of q and r . In actuality, each value of q and r defines a new extremal ray. To find the true bounds, we must therefore take the convex hull over the bounds implied by all possible choices of (q, r) . The convex hull in (x, y) from Eq. (B9) gives precisely the set of conditions in Eq. (A5) that we found from the elastic scattering dispersion relation. Thus, the bounds on the HEFT that we obtain from the generalized optical theorem are precisely those in Eq. (11), giving the necessary and sufficient conditions for unitarity.

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$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$$

and associate the vev with $\langle \phi_I \rangle = v \delta_{I4}$. In the notation of Ref. [45], we can then decompose H into polar coordinates determined by a radius, $v + h$, and unit direction $\mathbf{n}$ on the 3-sphere, constructed such that $\phi_I = (v + h)n_I$. Being a

unit vector, we have $\mathbf{n} \cdot \mathbf{n} = 1$, and it is parametrized by the three angular coordinates π_i/v , in detail

$$\mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \exp[\pi_i \tau_i / v],$$

where again τ_i are the generators of the rotations broken when the Higgs vev reduces custodial symmetry from $O(4) \rightarrow O(3)$. Using this language, the operators in the HEFT can be explicitly written in terms of $\mathbf{n}$ rather than π_i . As a single example, we have $\mathcal{O}_1^{h\pi} = v^2 (\partial h)^2 (\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n})$, up to terms of higher multiplicity in fields.

- [64] More explicitly, the Wilson coefficients of SMEFT form a self-contained vector subspace within the space of HEFT coefficients. While a violation of the SMEFT bounds, and the observation of nonzero Wilson coefficients in the HEFT-allowed region of Fig. 1, would arguably correspond to a smoking gun for the HEFT, the converse is not true; it is possible for the correct EFT to be the HEFT, but for its Wilson coefficients to be such that they do not allow the theory to be distinguished from the SMEFT via positivity bounds, e.g., if they land in the SMEFT-allowed region of Fig. 1.
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