

Probing Lorentz invariance violation in Z boson mass measurements at high-energy colliders

J. Jejelava^{1,2,*} and Z. Kepuladze^{1,2,†}

¹*Institute of Theoretical Physics, Ilia State University, 0179 Tbilisi, Georgia*

²*Andronikashvili Institute of Physics, TSU, 0186 Tbilisi, Georgia*



(Received 16 April 2025; accepted 9 March 2026; published 9 April 2026)

We propose a minimal extension of the Standard Model by introducing a Lorentz invariance violation (LIV) term into the Z boson's dispersion relation, expressed as $p_\mu p^\mu = M_Z^2 + \delta_{\text{LIV}}(p_\mu n^\mu)^2$, where δ_{LIV} defines the violation scale and n^μ is a unit Lorentz vector specifying the direction of the violation. This perturbative modification alters the Z boson propagator and decay rate, impacting the Drell-Yan process cross section at high-energy colliders. Observable effects are most pronounced near the resonance region at high rapidities ($|Y| > 4$), potentially shifting the perceived Z boson mass in general LIV cases and inducing additional sidereal-time modulations for spacelike and lightlike LIV due to the Earth's rotation relative to distant stars. We outline a targeted search strategy for ATLAS and CMS, achieving sensitivity to LIV signatures down to $|\delta_{\text{LIV}}| \approx 10^{-8}$ (or even 10^{-9} in an optimistic scenario), offering a fresh perspective on historical and future collider data. Our model predicts that, in experiments with higher collision energies (and thus greater rapidities), the conventionally reconstructed weak boson mass exhibits a systematic shift. This could be particularly intriguing in the historical context of Tevatron and LHC weak boson mass measurements, where the discrepancy in W mass measurements aligns with our LIV scenario.

DOI: [10.1103/vfjv-7v9n](https://doi.org/10.1103/vfjv-7v9n)

I. INTRODUCTION

Lorentz invariance (LI) is intertwined with almost every aspect of particle physics and cosmology, and for many researchers, the possibility of Lorentz invariance violation (LIV) is considered speculative and highly unlikely. There is valid motivation for such beliefs; the special theory of relativity dictates severe constraints on the nature of our space-time. The high-energy experiments and cosmic ray observations do not offer much evidence in this regard either. Specific restrictions could be found in Refs. [1,2].

High-energy experiments, such as those at the LHC, have largely exhausted significant improvements to the Standard Model (SM). Further improvements are likely to be incremental, as any breakthrough would necessitate higher energies that are unavailable with the current generation of colliders. Given the typical time frame required to construct large accelerators, we do not

anticipate a breakthrough in this regard for the next 10–20 years. Thus, the pursuit of exotic physics is becoming increasingly appealing, as it offers the potential to explore beyond the conventional methodologies of standard physics. Unlike standard approaches, which follow established procedures, exotic physics could leverage new, heuristic search strategies to significantly enhance the probability of uncovering new phenomena. LIV represents a prime example of such a topic, making it imperative to seize the opportunity to investigate it using the highest-energy experiments available on Earth, such as those at the LHC.

Cosmic ray observations impose stringent constraints on stable particles traversing interstellar space; however, evidence from cosmic ray experiments suggests potential violations of LI. For example, the AGASA experiment detected ultra-high-energy cosmic protons [3] that, in a Lorentz-invariant framework, should be absent due to the Greisen-Zatsepin-Kuzmin (GZK) cutoff [4]. Additionally, observations of neutrinos from SN 1987A indicate possible inconsistencies with LI [5]. Numerous studies explore this possibility, including a recent analysis [6]. In contrast, constraints on unstable particles are less stringent due to their inherent instability.

Dark matter experiments such as the DAMA/LIBRA also report an annual modulation of their signal, offering LIV as a potential explanation.

The primary strength of the study presented in this paper lies in its feasibility using current-generation accelerators.

*Contact author: juansher.jejelava@iliauni.edu.ge, juansher.jejelava@cern.ch

†Contact author: zurab.kepuladze.1@iliauni.edu.ge, zkepuladze@yahoo.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

This feasibility stems from significantly relaxed constraints on neutrinos, as detailed in the following discussion.¹ Whether this relaxation is a feature of neutrinos and/or weak interactions remains to be seen. Notably, if these relaxed constraints in neutrino physics extend to the broader weak interaction sector, they could yield effects detectable at the threshold of current LHC energies. At the LHC, the Drell-Yan processes are studied, during which the interaction of the partons of protons produces intermediate bosons that then decay into final states. One important aspect of this process is the physics associated with intermediate weak bosons, which becomes dominant due to the resonance effect near the weak boson mass value. Analyzing this region could provide valuable insights into the possibility of LIV in the weak sector. Therefore, we intend to examine specific aspects of LIV physics related to the Z boson, as the signal and analysis for the neutral boson are cleaner and more tractable than those for the charged W bosons.

II. PHENOMENOLOGICAL LIV EXPANSION OF Z BOSON PHYSICS

If we imagine that LI is broken and there is a distinct direction in the 4D space-time given by the unit vector n_μ , then anything that interacts with it in any way should experience LIV. In this case, there are only so many ways we can minimally modify the physics of a vector particle in a non-Lorentz-invariant manner.

One obvious way is to introduce an interaction with matter, which breaks Lorentz invariance (and perhaps gauge and CPT invariance as well),

$$L_{\text{LIV}} = \delta_{\text{int}}(n^\mu Z_\mu) \bar{\psi} n^\nu \gamma_\nu \psi. \quad (1)$$

This interaction influences all weak processes and modifies the cross section of physical processes in this manner:

$$\sigma_{\text{new}} = \sigma_{\text{LI}}(1 + \delta_{\text{int}} \cdot f(\Omega)), \quad (2)$$

where $f(\Omega)$ is only a function of the special orientation of the process. Such an effect can only be detected by an experiment with very high sensitivity to special orientation, requiring the Lorentz breaking parameter to be no smaller than $10^{-4} - 10^{-5}$ to be visible at the current LHC [7,8], which might be too large a value for realistic LIV, considering restrictions from cosmic ray physics. An important feature of such an effect is that at all energies, it will always be of the same order of magnitude, comparatively.²

¹LIV effects are generally not expected to be similar across different interactions and particles. Even if they are comparable at some high energies, they are likely to diverge at lower energies due to different running.

²It is assumed that δ_{int} is constant or scales slowly with energy.

Additionally, the kinematics of the particles involved in this interaction will be modified through loop diagrams, transferring LIV to other particles as well. However, these effects are further suppressed as higher-order processes. This will further push away Lorentz breaking parameters from the detectable range due to the cosmological restrictions [1,2]. We want to highlight that, despite the LIV interaction parameter potentially being larger than the parameters it induces, such as those influencing the dispersion relation or propagator, surprisingly, searching for some of these induced effects experimentally might be more feasible.

In contrast to the LIV interaction, modification to the dispersion relation may prove to be more impactful and easier to discover, as the threshold energy of the reactions is significantly influenced through kinematics as well as cross sections in resonance regions at high energies. Therefore, we look for possible LIV in the kinematic sector, specifically in the context of the LHC experiment, and hope to observe some LIV effect in the resonance region of the Z boson.

LIV modifications in the quark sector are studied in detail in the context of LHC [8]. For the Z boson, we can invoke some published extensions of the Standard Model (SM) for possible Lorentz-violating operators in the kinetic sector [9,10]. However, to explain in a general sense, we can only have these three terms with the LIV vector n_μ and without introducing higher-order derivatives:

$$\begin{aligned} \Delta L_{\text{LIV}} = & \frac{\delta_{\text{LIV}}}{2} (\partial_n Z^\mu) (\partial_n Z_\mu) + \frac{\delta_{\text{LIV}}}{2} (\partial_\mu Z_n) (\partial^\mu Z_n) \\ & + \delta_{2\text{LIV}} (\partial_\mu Z^\mu) (\partial_n Z_n), \end{aligned} \quad (3)$$

where we have denoted

$$\partial_n \equiv n_\mu \partial^\mu, \quad Z_n \equiv n_\mu Z^\mu. \quad (4)$$

Within a specific scenario or with any additional condition, we can only connect δ_{LIV} parameters with each other. If we require gauge invariance, then we must set

$$\delta_{\text{LIV}} = \delta_{\text{1LIV}} = -\delta_{2\text{LIV}}. \quad (5)$$

This condition ensures gauge invariance within the kinetic sector alone. To achieve full gauge invariance, the corresponding interaction terms must also be introduced in accordance with the non-Abelian gauge structure.

Similar kinetic-sector modifications are usually parametrized in the Standard Model extension framework by the coefficients $(k_w)_{\alpha\beta\mu\nu}$ that appear in the gauge-boson sector, as discussed, for instance, in [2,11]. Assuming gauge invariance, our single-parameter deformation can be viewed as a simplified subset of this structure, corresponding approximately to $(k_w)_{\alpha\beta\mu\nu} \sim \delta_{\text{LIV}} g_{\alpha\mu} n_\beta n_\nu$. The strongest current constraints on combinations of the k_w coefficients—particularly those entering modified Møller scattering

$g^{\alpha\mu}(k_w)_{\alpha 0\mu x}$, $g^{\alpha\mu}(k_w)_{\alpha 0\mu y}$ —are of order 10^{-7} , mainly arising from photon- Z mixing effects. Even if such mixing were included in an extended phenomenological scenario consistent with gauge invariance, the correspondence to our δ_{LIV} remains only qualitative, as restricted k_w coefficients represent a richer tensorial pattern of Lorentz violation.

$$\mathcal{K}_{\mu\nu} = i(k_\lambda k^\lambda - \delta_{\text{LIV}}(k_n)^2 - M_Z^2) \left[g_{\mu\nu} - \frac{k_\mu k_\nu + \delta_{\text{LIV}}(k_\lambda k^\lambda) n_\mu n_\nu + \delta_{\text{LIV}} k_n (n_\mu k_\nu + n_\nu k_\mu)}{k_\lambda k^\lambda - \delta_{\text{LIV}}(k_n)^2 - M_Z^2} \right]. \quad (6)$$

Here, k_μ denotes the four-momentum of the particle, and $k_n \equiv n_\mu k^\mu$, similar to (4), while M_Z is the Z boson mass. From the form of (6), we see that the dispersion relation is affected only by the δ_{LIV} term, which, consequently, produces a distinct signature in scattering processes. Therefore, in this paper, we focus solely on the first term and treat our model as the Standard Model in unitary gauge, supplemented by the additional term ΔL_{LIV} ,³

$$\Delta L_{\text{LIV}} = \frac{\delta_{\text{LIV}}}{2} (\partial_n Z^\mu) (\partial_n Z_\mu). \quad (7)$$

Thus, Eq. (7) will modify the dispersion relation and propagator of the vector field. The dispersion relation will take the form

$$k_\mu k^\mu = M_{\text{eff}}^2 = M_Z^2 + \delta_{\text{LIV}}(k_n)^2. \quad (8)$$

Here, we have introduced the notion of effective mass since threshold energy, resonance, and generally all the kinematics and momentum conservation are now determined by M_{eff} instead of M_Z [1,10,12]. As for the propagator, we find a fairly simple expression resembling the massive vector field propagator in the unitary gauge,

$$\frac{-i}{k_\lambda^2 - M_{\text{eff}}^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_{\text{eff}}^2} \right). \quad (9)$$

A. LIV propagator of the unstable particle

We can not accept (9) as a full propagator for our vector boson because near the mass shell (8) it fails. We must understand the reason for this and account for it. Usually, the propagator of the particle is corrected by self-energy graphs. For the scalar particle, it looks like

$$\begin{aligned} D_{\text{corrected}} &= \frac{i}{k_\mu^2 - m_0^2} \left(1 + \frac{\Sigma(k)}{k_\mu^2 - m_0^2} + \dots \right) \\ &= \frac{i}{k_\mu^2 - m_0^2 - \Sigma(k)}, \end{aligned} \quad (10)$$

³We should also clarify that (7) is the final effective form in the electroweak broken phase and it does not affect the Higgs sector.

Accordingly, the quoted limits cannot be directly reinterpreted as limits on our δ_{LIV} , and the experiments establishing them are not sensitive to the same level.

The terms in Eq. (3) play different roles, and their effect becomes apparent when examining the corresponding modified kinetic operator in momentum space:

where m_0^2 is the bare mass and $\Sigma(k)$ is the loop diagram. Then, one fixes the mass of the particle on the regularization momentum p_r , $\bar{m}^2 = m_0^2 + \Sigma(p_r)$. However, in the case of an unstable particle, $\Sigma(k)$ has an imaginary part, which, using the optical theorem, is calculated as follows:

$$\text{Im } \Sigma(k) = k_0 \Gamma(k), \quad (11)$$

where $\Gamma(k)$ is the decay width of the scalar particle. Thus, the corrected propagator takes the form

$$D_{\text{corrected}} = \frac{i}{k_\mu^2 - \bar{m}^2 + ik_0 \Gamma(k)}. \quad (12)$$

Various forms for $\bar{m}^2$ are used with different success and accuracy, but in our opinion, the approach of [13,14] appears the most coherent for our purpose,

$$D_{\text{corrected}} = \frac{i}{k_\mu^2 - (m - \frac{i}{2} k_0 \Gamma(k)/m)^2}. \quad (13)$$

Additionally, we can observe

$$k_0 \Gamma(k) = m \Gamma_0, \quad (14)$$

where Γ_0 is the decay rate in the comoving reference frame. This gives the textbook form of $D_{\text{corrected}}$.

However, in order not to fool ourselves, we should clearly acknowledge that the imaginary part originates from the on-shell discontinuity of the self-energy and is physically meaningful only near the mass shell. Away from the pole, the analytic continuation of the propagator is not unique, which explains the variety of prescriptions (constant width, running width, complex mass). Some are more accurate than others or exhibit less dependence on gauge parameters; for instance, the complex-mass scheme provides a formally gauge-invariant continuation but conceptually treats the boson mass and related quantities as intrinsically complex. It is therefore important to recognize the limitations and inherent drawbacks associated with each scheme. Many of these aspects are discussed in [13]. Despite such possible limitations, the present near-resonance analysis—like other approaches—is well equipped to capture the relevant physics with accuracy far exceeding that required for our phenomenological discussion.

To repeat the same steps for the vector gauge boson without changing the tensor structure of the vector field propagator, we should naturally obtain the following:

$$D_{\text{corrected}} = \frac{-iD_{\mu\nu}}{k_\mu^2 - M_0^2} \left[g_{\nu\lambda} + \frac{\Sigma_{\nu\rho}(k)D_{\rho\lambda}(k)}{k_\mu^2 - M_0^2} + \dots \right] \quad (15)$$

and

$$D_{\mu\nu}(k)\Sigma_{\nu\rho}(k)D_{\rho\lambda}(k) \sim D_{\nu\lambda}(k). \quad (16)$$

This happens in the gauge-invariant setup, but when it is broken by LIV, one has to carefully check the outcome. Without delving deep into renormalization and loop calculations for the SM modified by the supposed Lorentz violation, from (9), in analogy to the derivation of (13), we can conclude that

$$D_{\text{corrected}} = -i \frac{g_{\mu\nu} - (1 + \bar{\delta}_{\text{LIV}})k_\mu k_\nu / M_{\text{eff}}^2}{k_\mu^2 - (M_{\text{eff}} - ik_0\Gamma_{\text{eff}}(k)/2M_{\text{eff}})^2}, \quad (17)$$

where Γ_{eff} is the decay rate of the unstable vector particle and $\bar{\delta}_{\text{LIV}}$ is the modified Lorentz breaking parameter from the loop calculation. The possible appearance of $\bar{\delta}_{\text{LIV}}$ can only be caused by LIV in the loop diagrams; therefore, it should be at least suppressed as $\bar{\delta}_{\text{LIV}} \sim \alpha \cdot \delta_{\text{LIV}}$ (α being a fine-structure constant) and consequently negligible.⁴ Thus, we can safely accept

$$D_{\text{corrected}} = -i \frac{g_{\mu\nu} - k_\mu k_\nu / M_{\text{eff}}^2}{k_\mu^2 - (M_{\text{eff}} - ik_0\Gamma_{\text{eff}}(k)/2M_{\text{eff}})^2}. \quad (18)$$

We should be mindful that this is a phenomenological choice suited to our purpose. The exact expression would require evaluating the full set of vacuum-polarization loop diagrams to determine the total effect of Lorentz violation. For example, within the complex-mass framework—where the couplings are also treated as complex quantities—additional complexity would appear in the definition of $\bar{\delta}_{\text{LIV}}$ itself. In any case, as noted above, any possible deviation from (18) is expected to be further suppressed and therefore inconsequential for the present analysis.

III. LORENTZ VIOLATION FOR THE Z BOSON

In the previous section, we calculated how propagator (18) and dispersion relation (8) are modified by (7). This will directly affect the decay rate (decay width) of the weak boson as well as the corresponding reactions' cross sections.

⁴Additionally, this $\bar{\delta}_{\text{LIV}}$ term is proportional to the momentum. Therefore, when the Z boson interacts with a neutral current, due to its conservation, it may only produce contributions further suppressed by the ratio $\frac{m_{\text{lepton}}^2}{M_Z^2}$, which is already way below the LHC sensitivity.

In order to calculate the process of our primary interest—the cross section of the Drell-Yan process via the Z boson and neutral current—we need to derive an expression for its decay rate since it is needed in (18).

A. Modified Z decay rate

Let us consider the decay of the Z boson into a lepton-antilepton pair to understand how its decay rate is affected by the LIV. If we denote the Z boson's 4-momentum k_μ and it decays into a lepton-antilepton pair with momentum p_μ and q_μ , then the square of the matrix element for the process is

$$|\mathcal{M}_{el}|^2 = \frac{e^2}{4\sin^2 2\theta_w} \xi_\mu \xi_\nu \text{Tr}[(\hat{p} + m_l)\gamma^\mu (g_l - \gamma^5) \times (\hat{q} - m_l)\gamma^\nu (g_l - \gamma^5)]. \quad (19)$$

Here, ξ_μ is a weak boson polarization operator, e is the electric charge, θ_w is the Weinberg angle, and m_l and g_l are the mass and coupling to the specific lepton ($m_l = 0$ and $g_l = 1$ for the neutrino; $g_l = 1 - 4\sin^2 \theta_w$ for the electron, muon, and tauon). After we sum over the polarization of the vector field using (9) and take into account mass shell conditions and momentum conservation $k_\lambda = p_\lambda + q_\lambda$, we obtain the following expression,

$$|\mathcal{M}_{el}|^2 = \frac{e^2}{\sin^2 2\theta_w} [(g_l^2 + 1)M_{\text{eff}}^2 + (g_l^2 - 2)m^2], \quad (20)$$

where M_{eff} is defined in (8).

After accounting for all averaging and normalization factors, the decay rate takes the form

$$\begin{aligned} \Gamma_{\text{eff}} &= \frac{1}{96\pi^2 k_0} \int \frac{d^3 q}{2q_0} \frac{d^3 p}{2p_0} \delta^4(k - q - p) |\mathcal{M}_{el}|^2 \\ &= \frac{e^2(g_l^2 + 1)}{96\pi^2 \sin^2 2\theta_w} \frac{1}{k_0} \left(M_{\text{eff}}^2 + 2 \frac{g_l^2 - 2}{g_l^2 + 1} m^2 \right) \\ &\quad \times \int d\Omega_p \frac{p^2 dp}{q_0 p_0} \delta(k_0 - q_0 - p_0). \end{aligned} \quad (21)$$

The phase-space integral is Lorentz invariant in the sense that it can only depend on k_λ^2 , but k_λ^2 is not Lorentz covariant itself on the mass shell (8). Taking into account that $p_0 = \sqrt{\mathbf{p}^2 + m^2}$ and $q_0 = \sqrt{\mathbf{k}^2 + \mathbf{p}^2 - 2\mathbf{k}\mathbf{p}\cos\beta + m^2}$ (β is an angle between k_μ and p_μ and $\mathbf{k}, \mathbf{p}$ are their spatial parts), we solve $k_0 - q_0 - p_0 = 0$ and obtain

$$\begin{aligned} p &= \frac{\mathbf{k}M_{\text{eff}}^2 \cos\beta + k_0 \sqrt{M_{\text{eff}}^4 - 4m^2(k_0^2 - \mathbf{k}^2 \cos^2\beta)}}{2(k_0^2 - \mathbf{k}^2 \cos^2\beta)}, \\ p_0 &= \frac{k_0 M_{\text{eff}}^2 + \mathbf{k} \cos\beta \sqrt{M_{\text{eff}}^4 - 4m^2(k_0^2 - \mathbf{k}^2 \cos^2\beta)}}{2(k_0^2 - \mathbf{k}^2 \cos^2\beta)}, \end{aligned}$$

which, in the integration over phase space, gives

$$\int d\Omega_p \frac{p^2 dp}{q_0 p_0} \delta(k_0 - q_0 - p_0) = \int d\Omega_p \frac{p^2}{|pk_0 - p_0 k \cos \beta|} \\ = 2\pi \sqrt{1 - 4 \frac{m^2}{M_{\text{eff}}^2}}. \quad (22)$$

Finally, we find that, as expected, the integral takes a somewhat Lorentz-invariant form. Thus, we obtain

$$\Gamma_{\text{eff}} = \frac{\alpha(g_l^2 + 1)}{12 \sin^2 2\theta_w k_0} \left(M_{\text{eff}}^2 + 2 \frac{g_l^2 - 2}{g_l^2 + 1} m^2 \right) \sqrt{1 - 4 \frac{m^2}{M_{\text{eff}}^2}}, \quad (23)$$

where α is the famous fine-structure constant. If we isolate the first order of the LIV effect, we obtain

$$\Gamma_{\text{eff}} \approx \Gamma_{\text{SM}} \left[1 + \frac{\delta k_n^2}{M_Z^2} \right] = \Gamma_{\text{SM}} \frac{M_{\text{eff}}^2}{M_Z^2}, \quad (24)$$

$$\Gamma_{\text{SM}} = \frac{\alpha(g_l^2 + 1)}{12 \sin^2 2\theta_w k_0} \left(M_Z^2 + 2 \frac{g_l^2 - 2}{g_l^2 + 1} m^2 \right) \sqrt{1 - 4 \frac{m^2}{M_Z^2}}, \quad (25)$$

where Γ_{SM} denotes the pure SM value. In the comoving frame, during spacelike violation $k_0 = M_Z$, even for the general violation case, $\Gamma_{0\text{eff}} \approx \Gamma_{0\text{SM}}$,

$$\Gamma_{0\text{eff}} \approx \Gamma_{0\text{SM}} = \frac{\alpha(g_l^2 + 1)}{12 \sin^2 2\theta_w} \frac{1}{M_Z} \left(M_Z^2 + 2 \frac{g_l^2 - 2}{g_l^2 + 1} m^2 \right) \\ \sqrt{1 - 4 \frac{m^2}{M_Z^2}} \approx \frac{\alpha(g_l^2 + 1)}{12 \sin^2 2\theta_w} M_Z. \quad (26)$$

The decay rate depends on the scalar product k_n ; the bigger the momentum carried by the boson, the bigger the effect of the LIV.

The leading-order result can be summarized as

$$\Gamma_{\text{eff}} \approx \Gamma_{\text{SM}} \frac{M_{\text{eff}}^2}{M_Z^2}, \quad (27)$$

where $M_{\text{eff}}^2 = M_Z^2 + \delta_{\text{LIV}}(k_n)^2$ and $k_n \equiv n_\mu k^\mu$.

If LIV is timelike, modification is isotropic, and the decay rate should vary abnormally with initial energy. In the case of spacelike LIV, the spatial asymmetry appears in the decay rate, and the modification is maximal when the initial 3-momentum is parallel to the preferred direction. While we only explicitly calculated the lepton channel decay rate, the general form of (24) will be maintained and can be generalized for the total decay rate.

B. Drell-Yan process

To adapt the effects of (7) for the experimental testing at the LHC, we need to calculate the cross section of the proton-proton collision, which produces a lepton-antilepton pair through intermediate neutral bosons (the neutral

current Drell-Yan process). According to the parton model, when quark-antiquark partons annihilate into some final state X , the cross section in the leading order is

$$\sigma(pr(P_1) + pr(P_2) \rightarrow X) \\ = \sum_f \int dx_1 dx_2 f_{q_f}(x_1) f_{\bar{q}_f}(x_2) \sigma(q_f(x_1 P_1) \\ + \bar{q}_f(x_2 P_2) \rightarrow X),$$

where $P_{1,2}$ are initial momenta of the protons, $x_{1,2}$ are portions of the protons' momentum carried by quark and antiquark with a probability $f_{q_f}(x_1)$, $f_{\bar{q}_f}(x_2)$. Additionally, $f_{q_f}(x_1)$, $f_{\bar{q}_f}(x_2)$ are parton distribution functions, and f denotes the quark type.

In our case, final state X is the lepton-antilepton. Thus, the proton-proton scattering cross section, in this channel, is proportional to the cross section of quark-antiquark annihilation. Therefore, we need $\sigma(u(x_1 P_1) + \bar{u}(x_2 P_2) \rightarrow l + \bar{l})$, $\sigma(d(x_1 P_1) + \bar{d}(x_2 P_2) \rightarrow l + \bar{l})$. This process is carried out by producing the virtual photon, the Z boson, and the Higgs after quark-antiquark annihilation, which decays into a lepton-antilepton pair afterward. The diagram with an intermediate Higgs is extremely suppressed and can be safely neglected here.

First, we discuss some kinematic aspects of the process. We denote by Q_μ the momentum carried by the intermediate boson; it is determined by the sum of the initial momenta,

$$Q = x_1 P_1 + x_2 P_2. \quad (28)$$

When the experiment is prepared in the laboratory frame, we can take $P_{1,2}$ to be

$$P_1 = (E, P\vec{r}), \quad P_2 = (E, -P\vec{r}), \quad (29)$$

where $\vec{r}$ is the unit vector, which fixes the collision axis and defines the direction of the beam (collinear with the detector axis). Since protons have very high energies, one can neglect the proton mass with certain accuracy and assume $E = P$. Thus,

$$Q_\mu = ((x_1 + x_2)E, (x_1 - x_2)P\vec{r}), \quad (30)$$

$$Q^2 = Q_\mu Q^\mu = 4x_1 x_2 E^2 + (x_1 - x_2)^2 m_p^2, \quad (31)$$

where m_p is the proton mass. If we neglect it, then we obtain the familiar form

$$Q_\mu = E((x_1 + x_2), (x_1 - x_2)\vec{r}), \quad (32)$$

$$Q^2 = 4x_1 x_2 E^2. \quad (33)$$

In the literature, it is accepted to parametrize Q via the following observables: invariant mass M^2 and rapidity Y ,

$$Q_\mu = M(\cosh Y, \vec{r} \sinh Y), \quad Q^2 = M^2, \quad (34)$$

where we note that higher rapidity corresponds to the higher transferred 3-momentum.

We can derive the relation for $x_1(M, Y)$, $x_2(M, Y)$ at energies greater than the mass of the proton,

$$x_{1,2} = \frac{M}{2E} \exp(\pm Y), \quad (35)$$

and calculate the Jacobian,

$$\frac{\partial(M^2, Y)}{\partial(x_1, x_2)} = 4E^2. \quad (36)$$

This allows us to rewrite

$$\frac{d^2\sigma_p}{dM^2 dY} = \sum_f \frac{f_{qf}(x_1) f_{\bar{q}f}(x_2)}{4E^2} \sigma_f, \quad (37)$$

where we have denoted

$$\sigma_p = \sigma(pr(P_1) + pr(P_2) \rightarrow l + \bar{l}), \quad (38)$$

$$\sigma_f = \sigma(q_f(x_1 P_1) + \bar{q}_f(x_2 P_2) \rightarrow l + \bar{l}). \quad (39)$$

1. Cross section of $q + \bar{q} \rightarrow l + \bar{l}$

For brevity, let us denote

$$q_{1,2}^\mu = x_{1,2} P_{1,2}^\mu,$$

and if $k_{1,2}^\mu = (E_{1,2}, \vec{k}_{1,2})$ are the momenta of a lepton-antilepton pair, we can write

$$\begin{aligned} \sigma_f &= \frac{1}{4x_1 x_2 E^2 |\mathbf{v}| (2\pi)^2} \int \frac{d^3 k_1 d^3 k_2}{4E_1 E_2} \delta^4(Q_\mu - k_{1\mu} - k_{2\mu}) |\mathcal{M}_f|^2 \\ &= \frac{1}{(2\pi)^2 M^2 |\mathbf{v}|} \int \frac{d^3 k_1}{4E_1 E_2} \delta^0(Q_0 - E_1 - E_2) |\mathcal{M}_f|^2, \end{aligned} \quad (40)$$

where $\mathcal{M}_f$ is the matrix element for the process and $|\mathbf{v}|$ is the relative speed of the partons, which in the first approximation is 2.

Since we neglect the mass of the proton for the same reason, we can also neglect the mass of lighter particles. Therefore, let us also mention that

$$q_1^\mu q_{2\mu} = q_1^\mu Q_\mu = q_2^\mu Q_\mu = \frac{M^2}{2}, \quad (q_{1,2}^\mu)^2 = 0. \quad (41)$$

Using (22), for integration over phase space, we find

$$\sigma_f = \frac{|\mathcal{M}_f|^2}{16\pi M^2}. \quad (42)$$

For the explicit form of the cross section, we need to calculate the matrix element (including averaging over fermion helicities).

Matrix element. In our analysis of the Drell-Yan process, we adopt the following Z boson propagator:

$$\begin{aligned} D_{\mu\nu} &= \frac{i}{Q^2 - (M_{\text{eff}} - iQ_0 \Gamma_{\text{eff}}(Q_0)/2M_{\text{eff}})^2} \\ &\times \left[g_{\mu\nu} - \frac{Q_\mu Q_\nu}{M_{\text{eff}}^2} \right], \end{aligned} \quad (43)$$

which can be rewritten using (24) and (26),

$$\frac{Q_0 \Gamma_{\text{eff}}(k)}{M_{\text{eff}}} \approx Q_0 \Gamma_{\text{SM}}(Q_0) \frac{M_{\text{eff}}}{M_Z^2} \approx \frac{M_{\text{eff}}}{M_Z} \Gamma_{\text{OSM}}, \quad (44)$$

even if $Q_0 \gg M_Z$. The $Q_\mu Q_\nu$ term in the expression (43) will not contribute in the limit of massless fermions since

$$Q_\alpha J^\alpha = Q_\alpha J_A^\alpha = 0, \quad (45)$$

where J^α and J_A^α are vector and axial currents. Therefore, effectively, we can use this propagator,

$$D_{\mu\nu} = \frac{i g_{\mu\nu}}{Q^2 - M_{\text{eff}}^2 (1 - i \Gamma_{\text{OSM}}/2M_Z)^2}. \quad (46)$$

Diagrams involving both the photon and the Z boson are of the same order of magnitude everywhere except in the resonance region, which is the primary focus of our study, where the Z boson diagram becomes dominant. By omitting only the Higgs diagram, the resulting matrix element can be expressed as follows:

$$\mathcal{M}_f = \frac{i J_{e\mu}^q J_{e\nu}^{l\mu}}{Q^2} + J_\mu^q J_\nu^l D^{\mu\nu},$$

where $J_{e\mu}^q$ and $J_{e\mu}^l$ are electromagnetic currents and J_μ^q and J_μ^l are quark and lepton neutral currents, which are

$$J_\mu^l = \frac{-e}{2 \sin 2\theta_w} \bar{\psi}_l \gamma_\mu (g_l - \gamma_5) \psi_l, \quad g_l = 1 - 4 \sin^2 \theta_w, \quad (47)$$

$$J_\mu^u = \frac{e}{2 \sin 2\theta_w} \bar{u} \gamma_\mu (g_u - \gamma_5) u, \quad g_u = 1 - \frac{8}{3} \sin^2 \theta_w, \quad (48)$$

$$J_\mu^d = \frac{-e}{2 \sin 2\theta_w} \bar{d} \gamma_\mu (g_d - \gamma_5) d, \quad g_d = 1 - \frac{4}{3} \sin^2 \theta_w, \quad (49)$$

with e being the electric charge and θ_w the Weinberg angle.

By introducing the Mandelstam variable $\hat{s}$ at the partonic level,

$$\hat{s} = M^2 = Q^2 = (q_1 + q_2)_\mu^2 = (p_1 + p_2)_\mu^2, \quad (50)$$

the matrix element takes the familiar form

$$|\mathcal{M}_f|^2 = \frac{16e^4 e_f^2}{3} + \frac{8e^4 |e_f| g_q g_l (S - M_{\text{eff}}^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2))}{3\hat{s} \sin^2 2\theta_w} R(\hat{s}) + \frac{e^4 (1 + g_q^2)(1 + g_l^2)}{3\sin^4 2\theta_w} R(\hat{s}), \quad (51)$$

$$R(\hat{s}) = \frac{\hat{s}^2}{(\hat{s} - M_{\text{eff}}^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2))^2 + M_{\text{eff}}^4 \Gamma_{0\text{SM}}^2 / M_Z^2}, \quad (52)$$

where $|e_f|$ equals 2/3 or 1/3, respectively, for u and d quarks. In addition, for brevity, we have introduced the resonance factor $R(S)$. Consequently, for the cross

section (42), we obtain

$$\sigma_f = \sigma_{\text{EM}} \left[1 + \frac{g_q g_l (\hat{s} - M_{\text{eff}}^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2))}{2|e_f| \hat{s} \sin^2 2\theta_w} R(\hat{s}) + \frac{(1 + g_q^2)(1 + g_l^2)}{16e_f^2 \sin^4 2\theta_w} R(\hat{s}) \right], \quad (53)$$

$$\sigma_{\text{EM}} = \frac{4\pi\alpha^2}{9\hat{s}} e_f^2. \quad (54)$$

Finally, the differential cross section for the proton-proton collision is

$$\frac{d^2\sigma_p}{d\hat{s}dY} = \sum_f \frac{f_{qf}(x_1) f_{\bar{q}f}(x_2)}{4E^2} \sigma_{\text{EM}} \left[1 + \frac{g_q g_l (\hat{s} - M_{\text{eff}}^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2))}{2|e_f| \hat{s} \sin^2 2\theta_w} R(\hat{s}) + \frac{(1 + g_q^2)(1 + g_l^2)}{16e_f^2 \sin^4 2\theta_w} R(\hat{s}) \right]. \quad (55)$$

Now, we make some conclusions and predictions. In the standard LI setup, this cross section experiences resonance growth and peaks at the resonance value $\hat{s}_r = M_Z^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2)$. The mass and the decay rate define the resonance value, and the maximum of the cross section is defined by $1/\Gamma_{0\text{SM}}^2$. However, during LIV, the resonance mechanism of the differential cross section (55) is modified, becoming strongly dependent on the transferred momentum. The new resonance maximum is located at

$$\hat{s}_r = M_{\text{eff}}^2 (1 - \Gamma_{0\text{SM}}^2 / 4M_Z^2), \quad (56)$$

and the peak value for the cross section, where we can neglect the electromagnetic part, becomes

$$\sigma_{f\text{max}} \approx \sigma_{f\text{max}}^{\text{LI}} \left(1 - \frac{\delta_{\text{LIV}} (n \cdot Q_r)^2}{M_Z^2} \right), \quad (57)$$

where Q_r stands for the resonance value of the momentum.

Thus, basically, by measuring $\hat{s}_r$, we measure the effective mass, which is a function of energy, in the case of $\delta_{\text{LIV}} > 0$, with the higher transferred momentum peak moving right and decreasing in height.

Since the LIV effect depends on n_μ , three distinct cases can be identified—timelike, spacelike, and lightlike—defined as follows:

$$\text{timelike: } n_\mu = (1, \vec{0}), \quad (58)$$

$$\text{spacelike: } n_\mu = (0, \vec{n}), \quad \text{with } \vec{n}^2 = 1, \quad (59)$$

$$\text{lightlike: } n_\mu = (1, \vec{n}). \quad (60)$$

These cases are analyzed separately below.

2. Timelike violation case

For the timelike case, Eq. (34) leads to

$$n \cdot Q = M \cosh Y = \sqrt{\hat{s}} \cosh Y, \quad (61)$$

giving the expression for the effective mass,

$$M_{\text{eff}}^2 = M_Z^2 + \delta_{\text{LIV}} \cdot \hat{s} \cosh^2 Y, \quad (62)$$

which is ready to apply to (55). The expression (56) will change accordingly,

$$\hat{s}_r \approx M_Z^2 (1 + \delta_{\text{LIV}} \cosh^2 Y) - \Gamma_{0\text{SM}}^2 / 4, \quad (63)$$

$$\sigma_{f\text{max}} \approx \sigma_{f\text{max}}^{\text{LI}} (1 - \delta_{\text{LIV}} \cosh^2 Y). \quad (64)$$

We observe that LIV should be amplified by the rapidity Y of the Z boson while staying independent of the spatial orientation of the experiment. If the data are sorted by the rapidity Y , then low-rapidity data should allow better determination of the M_Z^2 through the resonance value $\hat{s}_r$, while with larger Y , the LIV effect kicks in, offering the possibility of observing it in the experiment. If one is to constrain δ_{LIV} from the Z mass uncertainty ΔM_Z (Atlas value), we can say that

$$\delta_{\text{LIV}} \leq \frac{2\Delta M_Z}{M_Z \cosh^2 Y}, \quad (65)$$

which, for $Y = 5, 6$, gives $10^{-8} (10^{-9})$.

We observe that LIV should be amplified by the rapidity Y of the Z boson while staying independent of the spatial orientation of the experiment. Sorted by the rapidity, data allow better determination of M_Z^2 through the resonance value

$\hat{s}_r$ for low values of Y . The LIV effects fully kick in only for high values of rapidity. To constrain δ_{LIV} within the precision of the experiment, we can use the following relation:

$$\delta_{\text{LIV}} \leq \frac{2\Delta M_Z}{M_Z \cosh^2 Y}, \quad (66)$$

which, for $Y = 5$, gives 10^{-8} (10^{-9}).

3. Spacelike violation case

For the spacelike violation, we have

$$M_{\text{eff}}^2 = M_Z^2 + \delta_{\text{LIV}} \cdot \hat{s} \sinh^2 Y \cos^2 \beta, \quad (67)$$

where β denotes the angle between the preferred direction of the Lorentz violation and the collision axis (i.e., $\vec{n}$ and $\vec{r}$). Then, for the resonance, we have

$$\hat{s}_r \approx M_Z^2(1 + \delta_{\text{LIV}} \sinh^2 Y \cos^2 \beta) - \Gamma_{\text{OSM}}^2/4, \quad (68)$$

$$\sigma_{f \text{ max}} \approx \sigma_{f \text{ max}}^{\text{LI}}(1 - \delta_{\text{LIV}} \sinh^2 Y \cos^2 \beta). \quad (69)$$

During spacelike violation, like the timelike case, dependence on the rapidity Y is strong, but now we have variation with respect to the orientation of the experiment; since it rotates with the Earth, hourly (sidereal) modulation is expected in the experimental data.

4. Lightlike violation case

For the lightlike violation, we have

$$M_{\text{eff}}^2 = M_Z^2 + \delta_{\text{LIV}} \cdot \hat{s}(\cosh Y - \sinh Y \cos \beta)^2, \quad (70)$$

which is somewhat of a hybrid case but with a slightly more complicated behavior,

$$\hat{s}_r \approx M_Z^2(1 + \delta_{\text{LIV}}(\cosh Y - \sinh Y \cos \beta)^2) - \Gamma_{\text{OSM}}^2/4, \quad (71)$$

$$\sigma_{f \text{ max}} \approx \sigma_{f \text{ max}}^{\text{LI}}(1 - \delta_{\text{LIV}}(\cosh Y - \sinh Y \cos \beta)^2). \quad (72)$$

Unlike the spacelike case, the LIV effect peaks when n_μ is perpendicular to the colliding beam while still exhibiting time modulation.⁵

⁵In principle, lightlike behavior is similar to the general case scenario. For $n_\mu = (n_0, \vec{n})$, with $|n_0^2 - \vec{n}^2| = 1$,

$$M_{\text{eff}}^2 = M_Z^2(1 + \delta_{\text{LIV}}(n_0 \cosh Y - |\vec{n}| \sinh Y \cos \beta)^2),$$

$$\sigma_{f \text{ max}} \approx \sigma_{f \text{ max}}^{\text{LI}}(1 - \delta_{\text{LIV}}(n_0 \cosh Y - |\vec{n}| \sinh Y \cos \beta)^2),$$

with strong dependence both on rapidity and spatial orientation. Here, $n_0 \gg 1$ gives the lightlike limit, and $n_0 \approx 1$ and $|\vec{n}| \approx 1$ give timelike and spacelike limits.

IV. EXPERIMENTAL SIGNATURE OF LIV

As seen from the form of the cross section (53), possible LIV effects depend strongly on two factors—the rapidity of the Z boson and the orientation of the experiment—since the momentum of the intermediate Z boson is nearly parallel to the detector axis. In both spacelike and lightlike violation scenarios, anisotropy remains a key feature, leading to a modulation of the signal with sidereal time. This arises because the spatial component of LIV is defined relative to a global frame of reference (the distant stars); thus, the Earth’s rotation naturally imprints a sidereal modulation on any terrestrial experiment.

However, the essential prerequisite for detecting LIV is the isolation of high-rapidity events. The LIV effects are almost exponentially amplified with rapidity, and for low-rapidity events, even existing spatial anisotropy will be undetectable due to its smallness. Unfortunately, high-rapidity events are relatively rare compared to the bulk of low-rapidity events [15]. Without separating them, the LIV signal is diluted, producing only a marginal, undetectable shift in the cross section at current energies. Therefore, the correct strategy is to sort the data by rapidity, which increases the sensitivity to LIV signatures.

The search strategy. For timelike violation, rapidity binning alone is sufficient. In this case, anisotropy searches are unnecessary, and each bin benefits from better statistics. Examining the resonance peak’s shape, size, and position then provides adequate constraints on LIV parameters. The spacelike and lightlike violations require additional binning by sidereal time. We propose segmenting the data into N discrete time bins with a width $\Delta t = D_s/N$, where $D_s \approx 86164.0905$ s is the sidereal day. This ensures that the bins are phase locked to the global frame, consistently mapping the experiment’s orientation relative to the stars.⁶ In practice, smaller bin sizes improve the resolution but reduce statistics, especially at high rapidity, where events are scarce. Thus, the analysis must balance bin width against event count to find the optimal sensitivity. Our search strategy is summarized in Table I.

Because different types of violations share the same general feature—crucial dependence on rapidity—we focus on the timelike violation case as a simpler case study to highlight LIV characteristic behavior.

LIV in the cross section. Until now, we have not discussed the sign of δ_{LIV} , as it has not been necessary. However, from Eqs. (8), (56), and (57), we understand that it defines several features differently. We can isolate the

⁶While mass redistribution (e.g., ice cap melting or large dam construction) can subtly alter the Earth’s rotation on a millisecond scale over 30 years, a more significant concern is the leap seconds added in 1996–1998, 2005, 2008, 2012, 2015, and 2016. However, the angular drift caused by a 1 second time offset is approximately 15 arcseconds, a negligible value in the context of collision axis positioning.

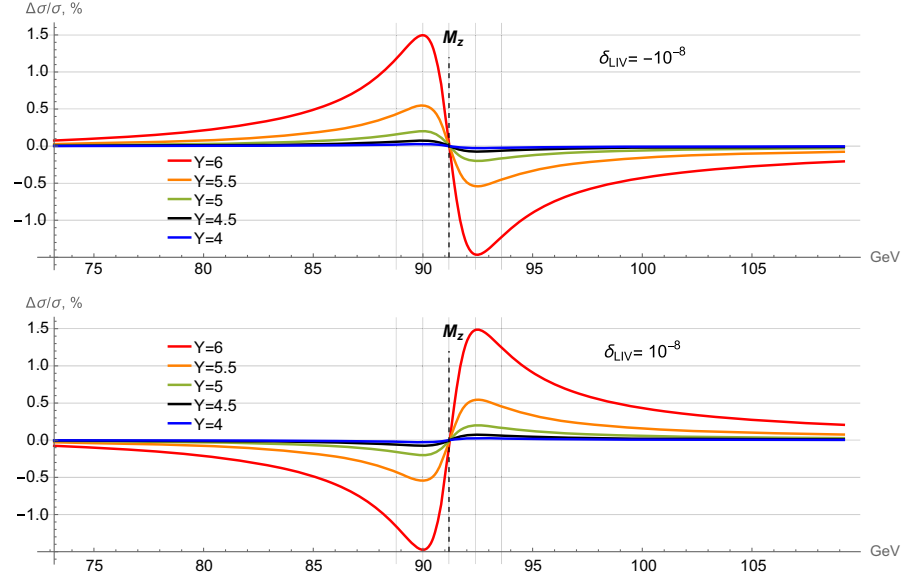


FIG. 1. $(\sigma_{\text{LIV}} - \sigma_{\text{LI}})/\sigma_{\text{LI}}$. The plot illustrates the relative difference between σ_{LIV} (the LIV cross section) and σ_{LI} (the LI cross section, equivalent to the Standard Model cross section) at the parton level for $|\delta_{\text{LIV}}| = 10^{-8}$. Plotting the relative difference helps to clarify the structure of LIV. Despite its visual appearance, the LIV effect is not exactly zero at the resonance point, as becomes evident in the exaggerated representation shown in Fig. 2. LIV reaches its maximum value at a point approximately 1.2 GeV away from the true mass ($M_Z = 91.1876$ GeV). This should not be confused with the perceived mass shift defined by the resonance value $\hat{s}_r$, which is expected to be of order a few MeV; the maximum LIV deviation does not coincide with the resonance value. The mass shift arises during the standard fitting procedure if the distorted shape of the LIV cross section is not properly accounted for. The sign of δ_{LIV} determines whether the shift leads to an overestimation or underestimation of the resonance mass relative to M_Z .

effects of violation by plotting the relative difference between the LIV (53) and the standard (LI) cross sections at the parton level, for $|\delta_{\text{LIV}}| = 10^{-8}$ (Fig. 1).

Figure 1 is constructed for the parton-level cross section; however, the same effect applies to the proton differential cross section since the difference between the proton and parton cross sections arises only from Lorentz-invariant factors defined by the parton distribution functions. This figure also illustrates how the sign of δ_{LIV} influences the outcome. In both cases, the magnitude of the LIV effect is nearly identical, but the sign differs. LIV effects show maximal deviations from Lorentz invariance in the resonance region, rapidly diminishing farther from it. For $|\delta_{\text{LIV}}| = 10^{-8}$, used in the graph, the LIV effect increases by a few tenths of a percent at $Y = 5$ and up to 1.5% at $Y = 6$, which is relevant for future experimental energies. Even small LIV effects, such as 0.2%, remain significant.

TABLE I. Data binning requirements for different LIV scenarios. While the timelike case only requires separation by Z boson rapidity, the spacelike and lightlike cases necessitate sorting data by both rapidity and collision time.

LIV case	Y binning	Time binning
Timelike: $n_\mu = (1, \vec{0})$	Yes	No
Spacelike: $n_\mu = (0, \vec{n})$	Yes	Yes
Lightlike: $n_\mu = (1, \vec{n})$	Yes	Yes

Although these differences are too subtle to discern here, for demonstration purposes, we provide an exaggerated plot to highlight the LIV effect's structure in contrast to the Lorentz-invariant cross section at the parton level (53) (Fig. 2).

As mentioned earlier, despite the fact that LIV becomes significant for high rapidities, the number of such events is relatively small; during the typical data collection, all events are gathered together, regardless of LIV effects. Therefore, the majority of events contain an unnoticeable amount of LIV data. Imagine a scenario where an analysis team attempts to find the resonance mass without knowing the LIV nature of the data. The result would be an attempt to fit the affected data to the standard cross section. Because of the perturbative nature of the Lorentz-violating scenario, drastic changes are precluded, and the most likely result would be a change in the reconstructed mass and possibly decay rate since they define the peak and general shape of the resonance region. To better understand this behavior, we compare perceived ΔM_Z for different mixtures of LI and LIV data, with results presented in Table II.

The table clearly shows what we have discussed in this article: The low-rapidity cases provide no detectable results, while for high rapidities, results may be detectable. When the data are a mixture of 90% LI and 10% LIV events (for the experimentally feasible $Y = 5$), the estimated mass shift is $|\Delta M_Z| \approx 0.25$ MeV. However, if one isolates only LIV

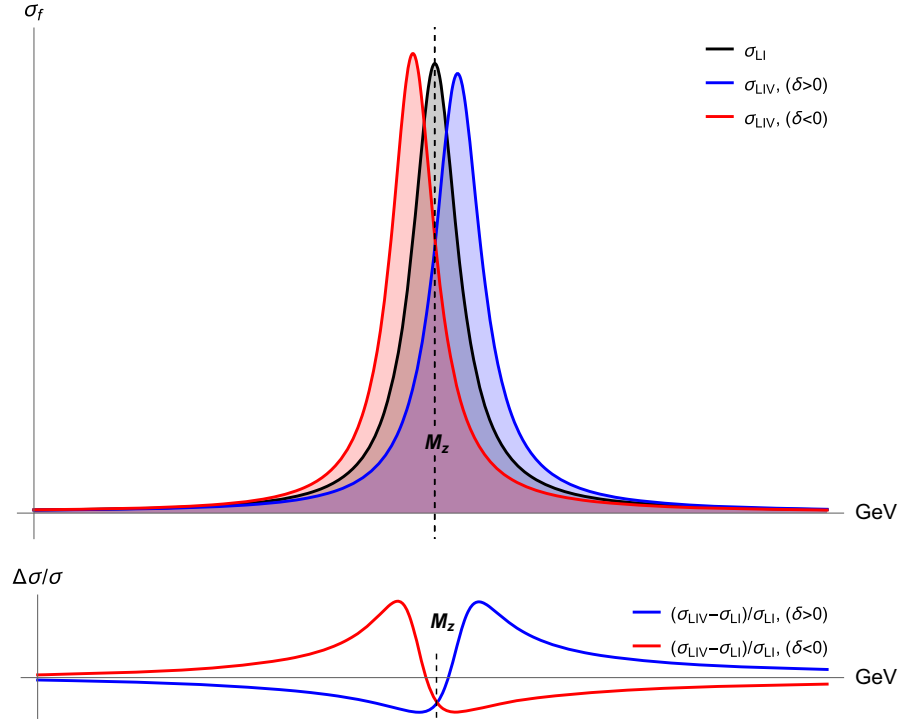


FIG. 2. Highly exaggerated comparison of LIV and LI cross sections for $Y = 8$. This $Y = 8$ plot is for demonstrative purposes only. For the LHC data, the expected difference is limited to about 0.1% for the rapidity $Y = 4.5$, rendering it practically indistinguishable by visual inspection alone.

data (for the same $Y = 5$), the mass shift increases to $|\Delta M_z| \approx 2.5$ MeV, too big to be accommodated within experimental uncertainty since the declared accuracy is 2.1 MeV, for both the Tevatron and LHC. The deviation for the rapidities $Y \geq 6$ (unfortunately unavailable for the current generation of accelerators) is naturally more pronounced and easier to identify. This mass deviation for any single rapidity value is well approximated by the relation (65),

$$\Delta M_z = \frac{\delta_{\text{LIV}}}{2} M_Z \cosh^2 Y. \quad (73)$$

Table II serves as a proxy intended to mimic the realistic effect of PDFs. Even within this simplified approximation, the nature of the LIV effect remains clearly illustrative. Exact calculations using PDFs, or including lower order processes (quantum chromodynamics or electroweak vertex corrections, different treatment of the pole in the propagator by using running width or complex mass schemes), would not alter the general behavior or can be confused with LIV, though they would certainly yield a more quantitatively accurate picture. In standard data analysis, the limited amount of LIV data typically results in only marginal effects. However, when $Y = 4, 5$ cases are analyzed separately, the LIV effect is distinctly evident in the data for $|\delta_{\text{LIV}}| = 10^{-8}$, and possibly for $|\delta_{\text{LIV}}| = 10^{-9}$. If LIV occurs within this range, next-generation accelerators

would likely detect it, as the effect becomes highly pronounced at $Y \geq 6$. Nevertheless, our analysis remains valuable for current accelerators, as the predicted mass shifts, lying at the edge of measurement precision, open new possibilities for comparing data from accelerators with differing collision energies, such as the Tevatron and LHC.

Realistic experimental implementations should account for detector acceptances, which influence observable rapidity distributions. The ATLAS and CMS detectors reliably measure Z boson events up to $|Y| \approx 3.5$ with high efficiency ($> 90\%$ in central regions, $|\eta| < 2.5$), but efficiencies drop in forward regions ($|\eta| > 2.5$) due to spectrometer coverage and trigger thresholds. PDF uncertainties also increase at higher parton energies, though these hurdles are well known and standard [16]. For LIV analyses requiring high rapidities ($|Y| > 4$) to enhance sensitivity to δ_{LIV} , data from forward-placed detectors (e.g., ATLAS Forward Proton or CMS-TOTEM) are essential; alternatively, so is the adaptation of LHCb data, which specializes in the forward pseudorapidity range ($2 < \eta < 5$) with dedicated Z boson measurements [17]. The efficiency for such high- Y data remains comparatively low, potentially reducing event yields and lepton momentum measurement accuracy. Reduced yields primarily rescale normalizations and can be mitigated in binned differential analyses, though inaccuracies in lepton momentum reconstruction at high $|\eta|$ are

TABLE II. Absolute mass shift $|\Delta M_Z|$ as a function of rapidity Y and the fractional composition of LI and LIV contributions of cross sections in the data.

LI:	100%	90%	80%	70%	60%	50%	40%	30%	20%	10%	0%
LIV:	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
$Y = 0.5$	0 eV	0.1 keV	0.1 keV	0.2 keV	0.2 keV	0.3 keV	0.3 keV	0.4 keV	0.5 keV	0.5 keV	0.6 keV
$Y = 1.0$	0 eV	0.1 keV	0.2 keV	0.3 keV	0.4 keV	0.5 keV	0.7 keV	0.8 keV	0.9 keV	1.0 keV	1.1 keV
$Y = 1.5$	0 eV	0.3 keV	0.5 keV	0.8 keV	1.0 keV	1.3 keV	1.5 keV	1.8 keV	2. keV	2.3 keV	2.5 keV
$Y = 2.0$	0 eV	0.6 keV	1.3 keV	1.9 keV	2.6 keV	3.2 keV	3.9 keV	4.5 keV	5.2 keV	5.8 keV	6.5 keV
$Y = 2.5$	0 eV	1.7 keV	3.4 keV	5.1 keV	6.9 keV	8.6 keV	10.3 keV	12 keV	13.7 keV	15.4 keV	17.1 keV
$Y = 3.0$	0 eV	4.6 keV	9.2 keV	13.9 keV	18.5 keV	23.1 keV	27.7 keV	32.4 keV	37. keV	41.6 keV	46.2 keV
$Y = 3.5$	0 eV	12.5 keV	25.1 keV	37.6 keV	50.1 keV	62.6 keV	75.2 keV	87.7 keV	100.2 keV	112.7 keV	125.3 keV
$Y = 4.0$	0 eV	34 keV	68 keV	102 keV	136 keV	170 keV	204 keV	238 keV	272 keV	306 keV	340 keV
$Y = 4.5$	0 eV	92 keV	185 keV	277 keV	370 keV	462 keV	0.6 MeV	0.6 MeV	0.7 MeV	0.8 MeV	0.9 MeV
$Y = 5.0$	0 eV	251 keV	0.5 MeV	0.8 MeV	1.0 MeV	1.3 MeV	1.5 MeV	1.8 MeV	2.0 MeV	2.3 MeV	2.5 MeV
$Y = 5.5$	0 eV	0.7 MeV	1.4 MeV	2.0 MeV	2.7 MeV	3.4 MeV	4.1 MeV	4.8 MeV	5.5 MeV	6.1 MeV	6.8 MeV
$Y = 6.0$	0 eV	1.9 MeV	3.7 MeV	5.6 MeV	7.4 MeV	9.3 MeV	11.1 MeV	13.0 MeV	14.8 MeV	16.7 MeV	18.6 MeV
$Y = 6.5$	0 eV	5 MeV	10 MeV	15 MeV	20 MeV	25 MeV	30 MeV	35 MeV	40 MeV	45 MeV	51 MeV
$Y = 7.0$	0 eV	14 MeV	27 MeV	41 MeV	55 MeV	69 MeV	82 MeV	96 MeV	110 MeV	124 MeV	137 MeV

This table presents the resonance mass shifts obtained when the σ_{LIV} cross section is reconstructed using a Standard Model fit. The notation “LI” refers to the Standard Model cross section (without Lorentz violation), while “LIV” denotes the cross section affected by LIV effects. Cross sections are assumed at parton level, according to formula (53), where the photon contribution, while subdominant, is still taken into account. The total experimental data are assumed to be a mixture of low-rapidity events (containing undetectable perturbations from the Standard Model) and high-rapidity events (exhibiting detectable LIV effects, manifested as a resonance mass shift from the true mass). Although real data will consist of a distribution of events across all possible rapidities, for demonstrative purposes, we simplify the data composition to a mixture of LI and LIV data. As expected, the magnitude of the mass shift depends on the level of LIV contamination within the overall data. The sign of δ_{LIV} leads to an overestimation ($\delta_{\text{LIV}} > 0$) or underestimation ($\delta_{\text{LIV}} < 0$) of the resonance mass, but the difference remains within the margin of displayed precision, which is why, for the sake of simplification, we combine both cases into one chart. Once again, we emphasize that these resonance mass shifts would occur within a standard analysis framework due to the fitting of an LI cross-section function to an LIV-distorted distribution. Consequently, the LIV contamination affects the fitting parameters of conventional analysis methods.

harder to mitigate. In any case, provided that reconstruction accuracies at high Y do not degrade far beyond current expectations, the estimated LIV-induced effects should remain within reach of experimental scrutiny.

LIV search on proton-proton colliders through the Z decay. Our analysis would be incomplete without addressing a specific aspect of Z boson research that inevitably arises at colliders like ATLAS or CDF. Historically, measuring the mass of the Z boson was not the primary goal of these experiments; rather, the Z signal served as a standard candle to calibrate detectors, using high-precision data from the Large Electron-Positron Collider as the reference standard.

This may create a significant challenge: using the Z boson to search for new physics (LIV) while simultaneously using it for calibration. Fortunately, partitioning the data by rapidity and potentially by time remains a viable strategy, provided the analysis pipeline is modified. We must first investigate the resonance across different rapidities, potentially isolating low-rapidity events to serve as a “null” control group for calibration. Following this, the high-rapidity events can be scrutinized for LIV-induced shifts in the cross section. These steps constitute the prerequisites for a further investigation of LIV and the expansion of this search into the W boson sector.

V. CONCLUSIONS

In this work, we have explored the potential for detecting LIV in the weak sector through modifications in the dispersion relation of the neutral Z boson. Our analysis shows that LIV can affect the decay rate (24) and propagator (43), consequently impacting the cross section of Drell-Yan processes (55) observed at proton-proton colliders (LHC, Tevatron). We identified that the effect is most pronounced near the resonance region at higher rapidities, where it alters both the resonance mass and value of the decay rate.

We discussed three distinct cases of LIV: timelike, spacelike, and lightlike, each exhibiting unique signatures in experimental data. Our findings indicate that timelike violation is amplified with the rapidity of the Z boson, while spacelike violation is not only amplified with rapidity but also introduces variations dependent on the experiment’s orientation, leading to potential daily modulations due to the Earth’s rotation. Lightlike violation presents a hybrid case with more complex behavior influenced by both rapidity and spatial orientation.

To detect LIV, we propose dividing the experimental data by rapidity and sidereal time, thereby selecting events based on transferred momentum and spatial orientation. This approach enhances the likelihood of identifying LIV

effects, which may otherwise be lost in averaging procedures of traditional analysis. For pure timelike violation, our proposal offers a simplified scenario without anisotropy, eliminating the need for sidereal time division and yielding more robust statistics for each rapidity bin.

Our estimates for timelike LIV suggest that a perceived deviation $\Delta M_Z \approx -2.5$ MeV could be observed for $\delta_{\text{LIV}} = -10^{-8}$, and the corresponding value may remain noticeable even for $\delta_{\text{LIV}} = -10^{-9}$, especially on the future upgraded LHC. This indicates that LIV effects of this magnitude are within reach of current experimental capabilities at the LHC, providing a promising avenue for future investigations into the fundamental nature of Lorentz invariance and its potential violations.

The primary result of this paper is particularly compelling in light of the cross-collider discrepancies and W mass “tension.” In the LIV framework considered here, higher collision energies can cause a systematic underestimation of the reconstructed Z boson mass for $\delta_{\text{LIV}} < 0$. This arises because low-energy collisions are dominated by low-rapidity events, rendering LIV effects practically undetectable, whereas higher-energy collisions increase the fraction of high-rapidity events (where effects are strongly amplified), potentially producing a detectable shift around the resonance mass peak. The initial choice of the Z boson was driven by the higher precision attainable in its mass measurements. However, we expect a similar effect for the W boson since our approach should apply to the entire weak vector multiplet. This is particularly intriguing in light of the recent tensions between Tevatron and LHC measurements of M_W , which show a discrepancy of about 75 MeV ($M_{W\text{CDF}(2022)} = 80,433.5 \pm 9.4$ MeV [18], $M_{W\text{CMS}(2024)} = 80,360.2 \pm 9.9$ MeV [19], and

$M_{W\text{ATLAS}(2023)} = 80,360 \pm 16$ MeV [20]). LIV offers a compelling explanation for these discrepancies among the CDF, ATLAS, and CMS experiments.

Indeed, for negative δ_{LIV} , higher-energy colliders would experience greater LIV contamination in the data, leading to a more pronounced systematic underestimation of M_W . It remains unclear whether this discrepancy is genuine or originates from experimental data handling issues, and this does not appear likely to be resolved in the near future. Nevertheless, if there is even partial merit to this measurement discrepancy, it should warrant serious screening for LIV effects in resonance mass measurements.

Taken together, the weak boson cases highlight how collider observables provide a unique and complementary window on LIV—one that cannot be accessed through astrophysical probes alone.

ACKNOWLEDGMENTS

This research was supported by a grant from the Shota Rustaveli National Science Foundation (SRNSF) under Project No. STEM-22-2604. We would like to express our sincere gratitude to Professor Jon Chkareuli for his invaluable spiritual guidance and for instilling in us a passion for LIV from the very beginning of our careers. We also want to extend our appreciation to Uta Klein, Enrico Lunghi, and Nathaniel Sherrill for their interesting discussions and shared interest for Lorentz invariance violation.

DATA AVAILABILITY

The data supporting this study’s findings are available within the article.

-
- [1] S. Coleman and S. L. Glashow, High-energy tests of Lorentz invariance, *Phys. Rev. D* **59**, 116008 (1999), [arXiv:hep-ph/9812418](#).
 - [2] A. Kostelecky and N. Russell, Data tables for Lorentz and CPT violation, *Rev. Mod. Phys.* **83**, 11 (2011), [arXiv:0801.0287](#).
 - [3] M. Takeda *et al.* (AGASA Collaboration), Extension of the cosmic-ray energy spectrum beyond the predicted Greisen-Zatsepin-Kuz’mín cutoff, *Phys. Rev. Lett.* **81**, 1163 (1998), [arXiv:astro-ph/9807193](#); Energy determination in the Akeno giant air shower array experiment, *Astropart. Phys.* **19**, 447 (2003), [arXiv:astro-ph/0209422](#).
 - [4] K. Greisen, End to the cosmic-ray spectrum?, *Phys. Rev. Lett.* **16**, 748 (1966); G. T. Zatsepin and V. A. Kuz’mín, Upper limit of the spectrum of cosmic rays, *JETP Lett.* **41**, 78 (1966).
 - [5] S. Desai, Astrophysical and cosmological searches for Lorentz invariance violation, edited by C. Bambi and A. Cárdenas-Avendaño, *Recent Progress on Gravity Tests*, Springer Series in Astrophysics and Cosmology (Springer, Singapore, 2024).
 - [6] C. A. Moura, L. Quintino, and F. Rossi-Torres, Analyzing the time spectrum of supernova neutrinos to constrain their effective mass or Lorentz invariance violation, *Universe* **9**, 259 (2023).
 - [7] CMS Collaboration, Searches for violation of Lorentz invariance in top quark pair production using dilepton events in 13 TeV proton-proton collisions, *Phys. Lett. B* **857**, 138979 (2024), [arXiv:2405.14757](#).
 - [8] E. Lunghi, N. Sherrill, A. Szczepaniak, and A. Vieira, Quark-sector Lorentz violation in Z -boson production, *J. High Energy Phys.* **04** (2021) 228; **07** (2024) 167(E), [arXiv:2011.02632](#).
 - [9] D. Colladay and A. Kostelecky, Lorentz-violating extension of the Standard Model, *Phys. Rev. D* **58**, 116002 (1998), [arXiv:hep-ph/9809521](#).

- [10] J.L. Chkareuli and Z. Kepuladze, Standard Model with partial gauge invariance, *Eur. Phys. J. C* **72**, 1954 (2012), [arXiv:1108.0399](#).
- [11] Hao Fua and Ralf Lehnert, Møller scattering and Lorentz-violating Z bosons, *Phys. Lett. B* **762**, 33 (2016).
- [12] J.L. Chkareuli, Z. Kepuladze, and G. Tatishvili, Spontaneous Lorentz violation via QED with non-exact gauge invariance, *Eur. Phys. J. C* **55**, 309 (2008), [arXiv:0802.1950](#).
- [13] S. Willenbrock, Mass and width of an unstable particle, *Eur. Phys. J. Plus* **139**, 523 (2024), [arXiv:2203.11056](#).
- [14] W. Greiner and B. Müller, *Gauge Theory of Weak Interactions*, 4th ed. (Springer-Verlag, Berlin, Heidelberg, 2009).
- [15] P. Banerjee, G. Das, P. K. Dhani, and V. Ravindran, Threshold resummation of the rapidity distribution for Drell-Yan production at NNLO + NNLL, *Phys. Rev. D* **98**, 054018 (2018), [arXiv:1805.01186](#).
- [16] CMS Collaboration, Measurements of differential Z boson production cross sections in proton-proton collisions at $\sqrt{s} = 13$ TeV, *J. High Energy Phys.* **12** (2019) 061, [arXiv:1909.04133](#).
- [17] LHCb Collaboration, Precision measurement of forward Z boson production in proton-proton collisions at $\sqrt{s} = 13$ TeV, *J. High Energy Phys.* **07** (2022) 026, [arXiv:2112.07458](#).
- [18] C. Hays (CDF Collaboration), High precision measurement of the W-boson mass with the CDF II detector, *Proc. Sci. ICHEP2022* (2022) 898.
- [19] CMS Collaboration, Measurement of the W boson mass in proton-proton collisions at $\sqrt{s} = 13$ TeV, CMS-PAS-SMP-23-002.
- [20] ATLAS Collaboration, Improved W boson mass measurement using 7 TeV proton-proton collisions with the ATLAS detector, Report No. ATLAS-CONF-2023-004, CERN, Geneva, 2023.