

Universal structure in the interactions of massless fields on the light cone

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We consider cubic interactions of massless fields of arbitrary spin in four spacetime dimensions, within the light cone formalism. We extend two key results from flat to (anti-)de Sitter spacetime. First, we present a simple explicit expression for all the cubic vertices. Second, we prove that the various self-dual/chiral theories, such as self-dual Yang-Mills, self-dual general relativity, and chiral higher-spin gravity, are fully consistent with no need for vertices beyond cubic. The key observation behind our results is that all cubic vertices in the light cone formalism can be expressed as a direct generalization of the “Abelian” vertices formed by multiplying linearized curvatures. The simple relationship between flat and (anti-)de Sitter then follows essentially from the conformal invariance of such linearized curvatures. The price for such simplicity is that our vertex expressions are nonlocal. It is, however, easy to bring them into a local form, which we also present.

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I. INTRODUCTION

In this paper, we discuss the structure of cubic interactions of spinning massless fields in four-dimensional (4D) spacetime (restricting to bosons for simplicity). This includes the cubic interactions of Yang-Mills and general relativity (GR), within themselves, with each other, and with massless matter. In the light cone formalism, all such fields and interactions can be viewed as pieces within a “master theory” of interacting massless fields with all spins: higher-spin (HS) gravity [1–4]. The latter is, in its own right, an important subject within mathematical physics—as the 4D holographic dual of three-dimensional (3D) vector models [5] and more general Chern-Simons-matter theories [6], as a model of holography in de Sitter space [7], as a cousin of supergravity with its own version of Bogomol’nyi-Prasad-Sommerfield objects [8–10], as a subsector of tensionless strings [11], and as a dual description of string theory itself [12].

Every spin- s degree of freedom with helicity $h = \pm s$ is described in the light cone formalism by a “scalar” field Φ_h , with no gauge redundancy (the fields might carry color indices, which we suppress). Real fields imply a complex-conjugation relation $\Phi_{-h} = \bar{\Phi}_h$. We work in Poincaré coordinates $x^\mu = (t, x, y, z)$. The spacetime

metric is either flat $\eta_{\mu\nu}$, de Sitter (dS) $\eta_{\mu\nu}/t^2$, or Anti-de Sitter (AdS) $\eta_{\mu\nu}/z^2$. In all these cases, the free-field quadratic Lagrangian reads simply

$$\mathcal{L}_{[2]} = \frac{1}{2} \sum_h \Phi_{-h} \square \Phi_h, \quad (1)$$

where both the d’Alembertian $\square$ and the implicit integration measure d^4x are defined with respect to the flat $\eta_{\mu\nu}$. HS Gravity contains one field of each integer (or even) helicity, while realistic lower-spin theories are restricted to $-2 \leq h \leq 2$.

The allowed cubic vertices and their detailed form are fixed by *consistency at the cubic order* of the spacetime symmetries, which are nonmanifest in the light cone formalism. There is a unique consistent cubic vertex for every triple of helicities with $h_{\text{tot}} \equiv \sum_{i=1}^3 h_i \neq 0$ [13–15], all of which appear in HS gravity [16–18]; the only other allowed vertex is the purely scalar $h_i = (0, 0, 0)$, which we will not consider. Yang-Mills interactions are covered by the vertices with $h_{\text{tot}} = \pm 1$, and gravitational ones—by those with $h_{\text{tot}} = \pm 2$. It is helpful to separate the cubic vertices into “chiral” ones with $h_{\text{tot}} > 0$, denoted collectively as $V_{[3]}$, and their “antichiral” complex conjugates with $h_{\text{tot}} < 0$, denoted as $\bar{V}_{[3]}$. The theory then takes the form

$$\mathcal{L} = \mathcal{L}_{[2]} - cV_{[3]} - \bar{c}\bar{V}_{[3]} + O(c\bar{c}); \quad (2)$$

$$V_{[3]} = \sum_{h_{\text{tot}} > 0} V_{h_1 h_2 h_3}, \quad (3)$$

where we introduced an overall complex coupling constant c for the chiral interactions. The $O(c\bar{c})$ term denotes quartic

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and higher-order vertices. The notation (2) expresses a *powerful, nontrivial statement* about massless theories: if we complexify and set $c \neq \bar{c} = 0$, then the resulting “self-dual” or “chiral” theory $\mathcal{L}_{[2]} + cV_{[3]}$ is *consistent to all orders*, with no need for higher vertices. For short, we will refer to this property as *higher-order consistency*. The “master” theory in this class is chiral HS gravity, with all helicities h and all vertices $h_{\text{tot}} > 0$ (this theory was originally constructed in the light cone formalism [19] and studied further in Refs. [16–18]; in the covariant unfolded approach, it is proposed in Ref. [2] as self-dual HS theory, its vertices are studied in Refs. [20–22], and the full theory is constructed in Refs. [23–25]). Smaller self-contained theories include self-dual Yang-Mills ($h = \pm 1$, $h_{\text{tot}} = 1$) and self-dual GR ($h = \pm 2$, $h_{\text{tot}} = 2$), as well as intermediate theories [26–28] that include HS fields but only vertices with $h_{\text{tot}} = 1, 2$. Such self-dual/chiral theories are important for describing instantons [29,30] (Euclidean solutions that govern tunneling) as well as maximally helicity violating scattering amplitudes, which are the starting point for all amplitude calculations in flat spacetime [31–34]. Note that for HS gravity, the chiral theory exists in both flat and (A)dS versions, while the full theory ($c, \bar{c}$ both nonzero) likely requires a cosmological constant to tame its nonlocality.

The vertices $V_{[3]}$ and their higher-order consistency are known in flat spacetime, but only piecemeal in (A)dS. Our aim is to fill this gap. In the process, we will uncover a universal structure that relates all cubic vertices, both flat and (A)dS, to the “Abelian” ones constructed from curvature tensors. We build on several previous works. In Refs. [35,36], an AdS light cone formalism was developed, and the vertices $V_{[3]}$ were found “in principle” [15], as a formal solution to a complicated differential equation. In Ref. [37], the results of Ref. [15] were extended from AdS to de Sitter (where the warp factor t cannot be orthogonal to the preferred lightlike direction). For self-dual GR and some cousins with $h_{\text{tot}} = 1, 2$, higher-order consistency in (A)dS can be inferred from the light cone-gauge results of Refs. [38,39], which inspired the present work.

II. CUBIC VERTICES

In the light cone formalism, the price of avoiding gauge redundancy is the loss of manifest Lorentz invariance. We single out a lightlike direction (equivalently, directional derivative) ∂_- , which defines a left-handed complex plane $\partial_- \wedge \partial_\xi$, and a right-handed plane $\partial_- \wedge \partial_{\bar{\xi}}$. These can be realized in terms of null coordinates $(x^+, x^-, \xi, \bar{\xi})$, defined as

$$x^\pm = \frac{x \pm t}{2}; \quad \xi = \frac{z - iy}{2}; \quad \bar{\xi} = \frac{z + iy}{2}. \quad (4)$$

The chiral vertices take their simplest form if we redefine the fields Φ_h through lightlike derivatives:

$$\phi_h \equiv (\partial_-)^{-h} \Phi_h. \quad (5)$$

The potentially negative power of ∂_- is a normal feature of the light cone formalism and does not imply nonlocality. The significance of the shifted fields (5) is that they are closely related to *linearized curvatures*: the electromagnetic field strength for $s = 1$, the Weyl tensor for $s = 2$, and their HS generalizations. In particular, for non-negative helicities $h = +s$, all the components of the right-handed curvature can be expressed as (see, e.g., Ref. [40])

$$C_{\dot{\alpha}_1 \dots \dot{\alpha}_{2h}} = z^{h+1} q^{\alpha_1} \dots q^{\alpha_{2h}} \partial_{\alpha_1 \dot{\alpha}_1} \dots \partial_{\alpha_{2h} \dot{\alpha}_{2h}} \phi_h. \quad (6)$$

Here, the α ’s and $\dot{\alpha}$ ’s are left-handed and right-handed spinor indices, q^α is the spinor square root of the lightlike direction ∂_- , and we used the AdS warp factor z for concreteness.

Now, consider three fields ϕ_{h_i} ($i = 1, 2, 3$) with helicities $h_i \geq 0$, satisfying the “triangle inequalities” $n_i \equiv h_{\text{tot}} - 2h_i \geq 0$. One can then construct a cubic vertex by contracting the spinor indices of the three linearized curvatures (6) (contracting n_3 index pairs between the first and second curvatures, and so on cyclically). To express the result in terms of the fields ϕ_{h_i} , let $\partial_\mu^{(i)}$ denote a spacetime derivative acting on the i th field. We then denote derivatives along the light ray ∂_- and the left-handed plane $q^\alpha q^\beta$ (namely, $\partial_- \wedge \partial_\xi$) as

$$\beta_i \equiv \partial_-^{(i)}; \quad \mathbb{P}_{ij} \equiv q^\alpha q^\beta \partial_{\alpha\dot{\alpha}}^{(i)} \partial_{\beta\dot{\beta}}^{(j)} = \partial_-^{(i)} \partial_\xi^{(j)} - \partial_-^{(j)} \partial_\xi^{(i)} = -\mathbb{P}_{ji}. \quad (7)$$

The cubic vertex in AdS then reads very simply

$$V_{h_1 h_2 h_3} = \frac{z^{h_{\text{tot}}-1}}{\Gamma(h_{\text{tot}})} \mathbb{P}_{23}^{n_1} \mathbb{P}_{31}^{n_2} \mathbb{P}_{12}^{n_3} \phi_{h_1} \phi_{h_2} \phi_{h_3}, \quad (8)$$

where the numerical factors $\Gamma(h_{\text{tot}})$ are the ones that will guarantee higher-order consistency of chiral HS gravity [16,18,19,41] as well as agreement with HS gravity’s holographic dual [37]. The vertices (8) are well known and are regarded in the literature as the “easy” ones. In particular, they are normally understood *not* to include the cubic vertices of Yang-Mills and GR, whose helicity configurations $(+1, +1, -1)$ and $(+2, +2, -2)$ violate both sets of inequalities $h_i \geq 0$ and $n_i \geq 0$.

The key insight of our paper is this: the vertex formula (8) is valid for all helicities with $h_{\text{tot}} > 0$, regardless of the inequalities $h_i \geq 0$ and $n_i \geq 0$ that enable its interpretation in terms of linearized curvatures. This idea has been overlooked for decades, likely because when $n_i \geq 0$ is violated the formula (8) becomes *nonlocal*: the spatial derivatives ∂_ξ , contained in the $\mathbb{P}_{ij}$ ’s, enter with negative powers. However, it is easy to bring (8) to a local form, using integration by parts. In general, in the presence of nonlocalities, integration by parts is subtle; one must be careful to never divide by a total derivative. A well-defined way to proceed is to systematically remove all derivatives

from, e.g., the third field ϕ_{h_3} (this is equivalent to taking a variational derivative, which computes the vertex's contribution to this field's equation of motion). The prescription reads

$$\mathbb{P}_{23} \rightarrow \mathbb{P}_{12} - \beta_2 \partial_z; \quad \mathbb{P}_{31} \rightarrow \mathbb{P}_{12} + \beta_1 \partial_z, \quad (9)$$

where the ∂_z derivatives are understood to act on the explicit position dependence $z^{h_{\text{tot}}-1}$ in (8). The vertex (8) then becomes

$$V_{h_1 h_2 h_3}^{\text{local}} = \phi_{h_3} \sum_{k_1, k_2=0}^{\infty} \frac{(-1)^{k_2}}{\Gamma(h_{\text{tot}} - k_1 - k_2)} \binom{n_1}{k_2} \binom{n_2}{k_1} \times z^{h_{\text{tot}}-k_1-k_2-1} \beta_1^{k_1} \beta_2^{k_2} \mathbb{P}_{12}^{h_{\text{tot}}-k_1-k_2} \phi_{h_1} \phi_{h_2}, \quad (10)$$

where $\binom{n}{k}$ are the Taylor coefficients of $(1+x)^n$. The series automatically truncates once the argument of $\Gamma(h_{\text{tot}} - k_1 - k_2)$ vanishes (i.e., when the ∂_z derivatives deplete all the factors of z). Thus, only positive powers of $\mathbb{P}_{12}$ appear, and locality is restored. While slightly more complicated than the original (8), the local form (10) is still much simpler and more explicit than the previous attempt in the literature [15]. There, the vertices were sought in terms of the symmetric combination $\mathbb{P} \equiv \frac{1}{3}(\mathbb{P}_{12} + \mathbb{P}_{23} + \mathbb{P}_{31})$, which did not readily reveal the simple structure of (8), (10).

To obtain the flat limit of (8)–(10), one replaces the z factors by a large constant. The ∂_z derivatives in (9) are then negligible, so the $\mathbb{P}_{ij}$'s are all equal up to integration by parts. The local formula (10) reduces to just the $k_1 = k_2 = 0$ term, which is the familiar flat vertex $\sim \mathbb{P}_{12}^{h_{\text{tot}}}$. The de Sitter vertices can be obtained from the AdS vertices (8), (10) by replacing the z factors with t , as established in Ref. [37] for vertices constructed out of the derivatives (7). All the results below can be extended to de Sitter following the analytic-continuation procedure of Ref. [37]. With this understanding, we will focus on the AdS vertices (8), (10) and their flat limit.

We have not yet proven that the vertex formula (8) is correct, i.e., that it is consistent at cubic order for arbitrary helicities with $h_{\text{tot}} > 0$. Also, we must prove the claim (2) that the theory with only chiral cubic vertices is consistent to all orders. Those are tasks of the next two sections.

III. CONSISTENCY AT CUBIC ORDER

Let us derive the cubic consistency of the vertices (8). At this level, each triple of helicities (h_1, h_2, h_3) can be considered separately. In the light cone formalism of Ref. [15], consistency refers to the closure of the AdS algebra, which we express as the conformal algebra (P^a, J^{ab}, D, K^a) of the 3D spacetime $x^a = (t, x, y)$, or equivalently $x^a = (x^+, x^-, y)$. The algebra's generators are constructed out of the fields ϕ_h on the $x^+ = 0$ null

hyperplane (x^-, y, z) , or equivalently $(x^-, \xi, \bar{\xi})$. The fields' canonical commutator reads

$$[\phi_h(x), \phi_{h'}(x')] = \frac{(-1)^h \delta_{-h, h'}}{2i\partial_-} \delta^3(x - x'), \quad (11)$$

where the delta function is with respect to the measure

$$d^3x \equiv dx^- dy dz = -\frac{i}{2} dx^- d\xi d\bar{\xi}. \quad (12)$$

Each AdS symmetry generator G can be expanded as $G = G_{[2]} + G_{[3]} + \dots$, where $G_{[2]}$ is the free-field quadratic piece and $G_{[3]}$ are the cubic interacting corrections. For any operator $\mathcal{O}$ constructed from, e.g., three fields ϕ_{h_i} , the action of a quadratic generator $G_{[2]}$ reduces to some linear differential operator G_{lin} acting on the ϕ_{h_i} :

$$\begin{aligned} i[G_{[2]}, \mathcal{O}(\phi_{h_1}, \phi_{h_2}, \phi_{h_3})] &= -G_{\text{lin}} \triangleright \mathcal{O} \\ &= \mathcal{O}(G_{\text{lin}} \phi_{h_1}, \phi_{h_2}, \phi_{h_3}) + \mathcal{O}(\phi_{h_1}, G_{\text{lin}} \phi_{h_2}, \phi_{h_3}) \\ &\quad + \mathcal{O}(\phi_{h_1}, \phi_{h_2}, G_{\text{lin}} \phi_{h_3}). \end{aligned} \quad (13)$$

For our analysis, we will need the linear form G_{lin} of three generators—the “Hamiltonian” P^- , the boost J^{-y} , and the special conformal generator K^y :

$$P_{\text{lin}}^- = -\frac{\partial_y^2 + \partial_z^2}{2\partial_-} = -\frac{\partial_\xi \partial_{\bar{\xi}}}{2\partial_-}; \quad (14)$$

$$J_{\text{lin}}^{-y} = x^- \partial_y - y P_{\text{lin}}^- - i h \frac{\partial_{\bar{\xi}}}{\partial_-}; \quad (15)$$

$$K_{\text{lin}}^y = 2\xi \bar{\xi} \partial_y - y(x^- \partial_- + y \partial_y + z \partial_z + 1) + 2ih\xi. \quad (16)$$

There are only three generators that receive interacting corrections: P^- , J^{-y} , and K^- . These are called “dynamical,” while the others, such as K^y , are called “kinematical.” The cubic corrections can all be constructed from the vertex $V_{[3]}$ and the kinematical generator K^y as

$$P_{[3]}^- = c \int d^3x V_{[3]}; \quad (17)$$

$$J_{[3]}^{-y} = i[K_{[2]}^y, P_{[3]}^-] = K_{\text{lin}}^y \triangleright P_{[3]}^-; \quad (18)$$

$$K_{[3]}^- = -i[K_{[2]}^y, J_{[3]}^{-y}] = -K_{\text{lin}}^y \triangleright J_{[3]}^{-y}. \quad (19)$$

Plugging (16) and (17) and our vertex formula (8) into (18) and (19), a straightforward calculation gives (neglecting total derivatives in the d^3x integrals throughout)

$$J_{[3]}^{-y} = -c \int d^3x y V_{[3]}; \quad (20)$$

$$K_{[3]}^- = \frac{c}{2} \int d^3x (y^2 + z^2) V_{[3]} = 2c \int d^3x \xi \bar{\xi} V_{[3]}. \quad (21)$$

This is in fact the simplest possible result, containing only the “orbital” terms that follow from the Hamiltonian density $cV_{[3]}$. This is an advantage of the specific vertex formula (8): adding total derivatives, e.g., to reach the local form (10), generally adds “intrinsic” correction terms to (20) and (21). Such terms appear in previous treatments [15], even in the flat limit [16].

Finally, we are ready to check the vertices’ consistency at cubic order. We build on the general analysis of Ref. [15], which shows that, for vertices constructed from the derivatives (7), the only nontrivial commutator to check is $[P^-, J^{-y}] = 0$. At cubic order, this requires

$$P_{\text{lin}}^- \triangleright J_{[3]}^{-y} - J_{\text{lin}}^{-y} \triangleright P_{[3]}^- = 0. \quad (22)$$

Plugging in (8), (14), (15), (17), and (20), this can indeed be verified by straightforward calculation.

As a cross-check, one can perform the same analysis starting from the local form (10) of the vertex. The computation is slightly longer but consists of the same ingredients. Instead of (17), we start with

$$P_{[3]}^- = c \int d^3x \sum_{h_{\text{tot}} > 0} V_{h_1 h_2 h_3}^{\text{local}}. \quad (23)$$

Computing $J_{[3]}^{-y}$ via (18) gives a result similar to (20), but with an additional intrinsic term,

$$J_{[3]}^{-y} = -c \int d^3x \sum_{h_{\text{tot}} > 0} (y V_{h_1 h_2 h_3}^{\text{local}} + i M_{h_1 h_2 h_3}^{\text{local}}), \quad (24)$$

which reads

$$M_{h_1 h_2 h_3}^{\text{local}} = \phi_{h_3} \sum_{k_1, k_2=0}^{\infty} \frac{(-1)^{k_2} (k_1 + k_2)}{\Gamma(h_{\text{tot}} + 1 - k_1 - k_2)} \binom{n_1}{k_2} \binom{n_2}{k_1} \times z^{h_{\text{tot}} - k_1 - k_2} \beta_1^{k_1} \beta_2^{k_2} \mathbb{P}_{12}^{h_{\text{tot}} - k_1 - k_2} \phi_{h_1} \phi_{h_2}. \quad (25)$$

One can now plug (23)–(25) into the cubic consistency condition (22) and verify straightforwardly that it holds.

IV. CONSISTENCY AT HIGHER ORDERS

Let us now prove the higher-order consistency of the chiral theory with just the cubic vertices (3), (8). Again, this requires checking the key commutator $[P^-, J^{-y}] = 0$, this time without stopping at cubic order. The only higher order that appears is quartic, where we must demonstrate

$$[P_{[3]}^-, J_{[3]}^{-y}] = -c^2 \left[\int d^3x V_{[3]}, \int d^3x y V_{[3]} \right] = 0, \quad (26)$$

where in the first equality we used (17) and (20). Let us express the vertices (8) in terms of their flat limit:

$$V_{h_1 h_2 h_3} = z^{h_{\text{tot}} - 1} V_{h_1 h_2 h_3}^{\text{flat}}; \quad (27)$$

$$V_{h_1 h_2 h_3}^{\text{flat}} = \frac{1}{\Gamma(h_{\text{tot}})} \mathbb{P}_{23}^{n_1} \mathbb{P}_{31}^{n_2} \mathbb{P}_{12}^{n_3} \phi_{h_1} \phi_{h_2} \phi_{h_3}. \quad (28)$$

Here, in the flat vertices, we drop the dimensionful constant that should replace the z factors; this will not affect the derivation below.

Now, note that the vertex (8) and canonical commutator (11) contain derivatives only within the left-handed $\partial_- \wedge \partial_\xi$ plane. Thus, in the d^3x integrals (12), the $d\bar{\xi}$ integration just factors out, and it is sufficient to focus on the plane $x^+ = \bar{\xi} = 0$, where we have $z = \xi$ and $y = i\xi$. Thus, the desired relation (26) takes the form

$$\left[\int d^2x \sum \xi^{h_{\text{tot}} - 1} V^{\text{flat}}, \int d^2x \sum \xi^{h_{\text{tot}}} V^{\text{flat}} \right] = 0, \quad (29)$$

where the measure is $d^2x \equiv dx^- d\xi$, the sum is over helicity triples with $h_{\text{tot}} > 0$, and we drop the helicity labels on the vertices for brevity. Now, we know that the quartic consistency (26) holds in the flat limit [16, 26, 28, 31, 42], where it reads

$$\left[\int d^2x \sum V^{\text{flat}}, \int d^2x \sum \xi V^{\text{flat}} \right] = 0. \quad (30)$$

We will now demonstrate that the AdS relation (29) can be derived from its flat counterpart (30), via a conformal transformation. It is not obvious that this should work, since the interacting theory (outside of the Yang-Mills sector $h_{\text{tot}} = 1$) is *not* conformally invariant. However, it is also not completely surprising, since our vertex formula (8) can be traced back to the curvatures (6), which *are* conformal (for a discussion of conformal symmetry in HS theory; see Ref. [43]).

Consider, then, the conformal generator $K^\xi = \frac{1}{2}(K^z - iK^y)$. Again, it is not part of the AdS algebra and thus not a symmetry of the interacting theory. However, it *is* a symmetry of the *free* theory and thus of the canonical commutator (11). On the $x^+ = \bar{\xi} = 0$ plane, its linear action [cf. (13)] reads simply

$$K_{\text{lin}}^\xi = -\xi(x^- \partial_- + \xi \partial_\xi + 1 - 2h). \quad (31)$$

A straightforward calculation yields, for any power m ,

$$K_{\text{lin}}^\xi \triangleright \int d^2x \xi^m V^{\text{flat}} = (m - h_{\text{tot}}) \int d^2x \xi^{m+1} V^{\text{flat}}.$$

Iterating this, we construct the exponentiated action of K_{lin}^ξ on both sides of the flat consistency relation (30):

$$e^{-aK_{\text{lin}}^\xi} \triangleright \int d^2x V^{\text{flat}} = \int d^2x (1 + a\xi)^{h_{\text{tot}}} V^{\text{flat}},$$

$$e^{-aK_{\text{lin}}^\xi} \triangleright \int d^2x \xi V^{\text{flat}} = \int d^2x \xi (1 + a\xi)^{h_{\text{tot}}-1} V^{\text{flat}}. \quad (32)$$

Since K^ξ is a symmetry of the commutators, these must still commute with each other as in (30), for any value of a :

$$\left[\int d^2x \sum (1 + a\xi)^{h_{\text{tot}}} V^{\text{flat}}, \right. \\ \left. \int d^2x \sum \xi (1 + a\xi)^{h_{\text{tot}}-1} V^{\text{flat}} \right] = 0. \quad (33)$$

We now wish to derive the AdS consistency relation (29) from the conformally transformed flat relation (33). To do this, let us isolate the coefficient of a particular quadruple of fields $\prod_{i=1}^4 \phi_{h_i}$ in the quartic expressions (29) and (33). There are six contributions to this coefficient, with different triples of helicities inside each factor in the commutator [recall that the first factor in (29) and (33) comes from $P_{[3]}^-$ and the second comes from $J_{[3]}^y$]. We denote these contributions as

$$(\{h_1, h_2, -h_5\}, \{h_3, h_4, h_5\}); (\{h_3, h_4, h_5\}, \{h_1, h_2, -h_5\}); \quad (34)$$

$$(\{h_3, h_1, -h_5\}, \{h_2, h_4, h_5\}); (\{h_2, h_4, h_5\}, \{h_3, h_1, -h_5\}); \quad (35)$$

$$(\{h_2, h_3, -h_5\}, \{h_1, h_4, h_5\}); (\{h_1, h_4, h_5\}, \{h_2, h_3, -h_5\}), \quad (36)$$

where in each case we list the helicities in the two factors. $\pm h_5$ denotes the “exchanged” helicity, i.e., the helicity of the fields that are contracted through the fundamental commutator (11). h_5 is summed over, with the sums cut off by the requirement $h_{\text{tot}} > 0$ in each triple. Note that the sum of the h_{tot} ’s in the two factors is always just the sum of external helicities $\sum_{i=1}^4 h_i$. Now, consider the expansion of (33) in powers of a . The lowest (i.e., zeroth) power just reproduces the flat consistency relation (30). The *highest* power of a for given external helicities h_i is naively $\sum_{i=1}^4 h_i - 1$, but its coefficients cancel between the two terms in each row of (34)–(36). The highest *nonvanishing* power of a in (33) is thus $\sum_{i=1}^4 h_i - 2$. Again summing the terms in each row of (34)–(36), it is easy to see that the coefficient of this power of a gives the desired AdS consistency relation (29).

This concludes our proof that the AdS chiral theory with only cubic vertices (3), (8) inherits all-orders consistency from its flat counterpart. In the Appendix, we supplement this somewhat formal proof with a more hands-on

derivation for the simplest nontrivial theory, namely, self-dual GR.

V. CONCLUSIONS

In this paper, we presented remarkably simple vertices (8) and dynamical symmetry generators (20) and (21) for the cubic interactions of massless fields of any spin in (A)dS in the light cone formalism. Our vertices are a straightforward, but nonlocal, generalization of the Abelian ones derived from linearized curvatures. We produced a manifestly local version in (10). We also showed that any chiral theory that is consistent to all orders in flat spacetime with only cubic vertices, such as self-dual GR or chiral HS gravity, enjoys the same property in (A)dS. It would be interesting to extend these results to half-integer spins, potentially with supersymmetry.

We are excited to apply the simple cubic vertices (8), (10) to bulk computations in (chiral) HS gravity. One application would be to (A)dS/CFT boundary correlators, especially as they pertain to constructing a holographic dual for the chiral theory [44,45]. Another application would be to scattering amplitudes in the static patch of de Sitter space [37,46,47]. Specifically, the vertex formula (10) can be directly plugged into the kinematical framework of Ref. [37], and one can start studying the resulting amplitudes for higher-spin symmetry patterns and holographic interpretations.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: QUARTIC CONSISTENCY DERIVATION FOR SELF-DUAL GR

In this appendix, we present an explicit derivation of higher-order consistency (26) for self-dual GR in AdS, without employing a conformal transformation from the flat case. As in the main text, we work on the plane $x^+ = \bar{\xi} = 0$, where we can substitute $z = \xi$ and $y = i\xi$, and the relation to prove is (29).

In self-dual GR, the only fields are $\phi_{\pm 2}$, i.e., the positive and negative helicities of the graviton, and the only vertex is the one with helicities $h_i = (+2, +2, -2)$. This vertex has $h_{\text{tot}} = 2$ and $n_i = (-2, -2, +6)$. Thus, the desired AdS consistency relation (29) reads

$$\left[\int d^2x \xi V^{\text{flat}}, \int d^2x \xi^2 V^{\text{flat}} \right] = 0, \quad (\text{A1})$$

where the flat vertex V^{flat} is given by (28). The only contribution to the quartic consistency relation (A1) has external helicities $(h_1, h_2, h_3, h_4) = (+2, +2, +2, -2)$. In the notation of (34)–(36), the exchanged helicity is then $h_5 = +2$.

To evaluate (A1), we must sum over the helicity assignments (34)–(36). For each helicity assignment, we apply the fundamental commutator (11) to the contracted fields $\phi_{\pm h_5}$, after removing all derivatives from $\phi_{\pm h_5}$ within each

cubic factor $\xi V^{\text{flat}}, \xi^2 V^{\text{flat}}$. For this, we integrate by parts, just as in (9), except now the explicit coordinate factors are ξ rather than z , and we denote the derivatives acting on them as ∂_ξ . Thus, in a cubic vertex with field labels $(i, j, 5)$, the prescription for integrating by parts to remove derivatives from field number 5 reads

$$\mathbb{P}_{i5} \longrightarrow -\mathbb{P}_{ij} - \beta_i \partial_\xi. \quad (\text{A2})$$

Applying this to the cubic factors in the first helicity assignment of (34), we get (omitting the fields factors ϕ_{h_i} and overall numerical factors)

$$\xi V^{\text{flat}} \sim \xi \mathbb{P}_{12}^6 \mathbb{P}_{25}^{-2} \mathbb{P}_{51}^{-2} \longrightarrow (\xi \mathbb{P}_{12}^2 + 2(\beta_2 - \beta_1) \mathbb{P}_{12}); \quad (\text{A3})$$

$$\xi^2 V^{\text{flat}} \sim \xi^2 \mathbb{P}_{34}^{-2} \mathbb{P}_{45}^{-2} \mathbb{P}_{53}^6 \longrightarrow (\xi^2 \mathbb{P}_{34}^2 + 4\xi(3\beta_3 + \beta_4) \mathbb{P}_{34} + 6(5\beta_3^2 + 4\beta_3\beta_4 + \beta_4^2)), \quad (\text{A4})$$

and similarly for the second helicity assignment of (34):

$$\xi V^{\text{flat}} \sim \xi \mathbb{P}_{34}^{-2} \mathbb{P}_{45}^{-2} \mathbb{P}_{53}^6 \longrightarrow (\xi \mathbb{P}_{34}^2 + 2(3\beta_3 + \beta_4) \mathbb{P}_{34}); \quad (\text{A5})$$

$$\xi^2 V^{\text{flat}} \sim \xi^2 \mathbb{P}_{12}^6 \mathbb{P}_{25}^{-2} \mathbb{P}_{51}^{-2} \longrightarrow (\xi^2 \mathbb{P}_{12}^2 + 4\xi(\beta_2 - \beta_1) \mathbb{P}_{12} + 2(3\beta_1^2 - 4\beta_1\beta_2 + 3\beta_2^2)). \quad (\text{A6})$$

Note that (A3) and (A5) are nothing but special cases of the local form (10) of the AdS vertex.

Let us now apply the fundamental commutator (11) to $\phi_{\pm h_5}$. This means multiplying the two lines (A3) and (A4) along with a factor of $1/(\beta_3 + \beta_4)$ [coming from the inverse derivative $1/\partial_-$ in (11)] and similarly the two lines (A5) and (A6) with a factor of $1/(\beta_1 + \beta_2)$. Because of momentum conservation $\beta_3 + \beta_4 = -(\beta_1 + \beta_2)$, some of the terms immediately cancel, and the remainder can be collected as (discarding an overall sign)

$$\begin{aligned} & \frac{2}{\beta_1 + \beta_2} (\xi^2 \{ (3\beta_3 + \beta_4) \mathbb{P}_{12}^2 \mathbb{P}_{34} + (\beta_1 - \beta_2) \mathbb{P}_{12} \mathbb{P}_{34}^2 \} + \xi \{ 3(5\beta_3^2 + 4\beta_3\beta_4 + \beta_4^2) \mathbb{P}_{12}^2 - (3\beta_1^2 - 4\beta_1\beta_2 + 3\beta_2^2) \mathbb{P}_{34}^2 \} \\ & + \{ 6(\beta_2 - \beta_1)(5\beta_3^2 + 4\beta_3\beta_4 + \beta_4^2) \mathbb{P}_{12} - 2(3\beta_3 + \beta_4)(3\beta_1^2 - 4\beta_1\beta_2 + 3\beta_2^2) \mathbb{P}_{34} \}). \end{aligned} \quad (\text{A7})$$

This is the contribution to the desired commutator (A1) from the helicity assignments (34). The other helicity assignments (35) and (36) are given by cyclic permutations on the labels (1, 2, 3) of the external positive-helicity fields. Thus, to prove the relation (A1), we must show that (A7) vanishes when symmetrized over these permutations. To do this, we manipulate (A7) using the identity

$$\beta_{[i} \mathbb{P}_{jk]} = 0, \quad (\text{A8})$$

along with the integration-by-parts (or momentum conservation) relations:

$$\sum_{i=1}^4 \beta_i = 0; \quad \sum_{j \neq i} \mathbb{P}_{ij} + \beta_i \partial_\xi = 0. \quad (\text{A9})$$

We start from higher orders in ξ and work toward the lower ones, picking up lower-order contributions along the way from the ∂_ξ derivative in (A9). Eventually, we manage to cancel the $\beta_1 + \beta_2$ denominator from all the terms in (A7), bringing (A7) to the form

$$\begin{aligned} & \xi^2 (\mathbb{P}_{12} + \mathbb{P}_{23} + \mathbb{P}_{31}) \mathbb{P}_{12} \mathbb{P}_{34} + \xi \{ (\mathbb{P}_{12} + \mathbb{P}_{23} + \mathbb{P}_{31}) (5\beta_3 \mathbb{P}_{12} + 3(\beta_1 \mathbb{P}_{14} - \beta_2 \mathbb{P}_{24})) + 2(\beta_3 \mathbb{P}_{12} \mathbb{P}_{14} - \beta_2 \mathbb{P}_{31} \mathbb{P}_{34}) \} \\ & - (\beta_1 + \beta_2 + \beta_3) (5\beta_3 \mathbb{P}_{12} + 6(\beta_1 \mathbb{P}_{14} - \beta_2 \mathbb{P}_{24})) - \mathbb{P}_{12} (12\beta_1\beta_2 + 12\beta_2^2) + \mathbb{P}_{23} (15\beta_2\beta_3 + 8\beta_1^2 - 12\beta_3\beta_1) \\ & - \mathbb{P}_{31} (3\beta_3\beta_1 - 12\beta_1^2 + 8\beta_2^2 - 12\beta_1\beta_2), \end{aligned} \quad (\text{A10})$$

where in some of the terms we sacrificed manifest symmetry under $1 \leftrightarrow 2$ in favor of a shorter expression.

Now, when symmetrized over the cyclic permutations of the (1, 2, 3) labels, Eq. (A10) indeed vanishes. The vanishing of the $\sim \xi^2$ term (which corresponds to the flat limit) is due to the identity

$$\mathbb{P}_{[ij}\mathbb{P}_{k]l} = 0. \quad (\text{A11})$$

The vanishing upon symmetrization of the $\beta_3\mathbb{P}_{12}$ terms in (A10) is due to the identity (A8). Finally, the $\beta_1\mathbb{P}_{14} - \beta_2\mathbb{P}_{24}$ and $\beta_3\mathbb{P}_{12}\mathbb{P}_{14} - \beta_2\mathbb{P}_{31}\mathbb{P}_{34}$ terms, as well as the last line of (A10), vanish upon symmetrization manifestly. This concludes our explicit derivation of higher-order consistency for self-dual GR, with only the cubic vertex (8) with helicities (+2, +2, -2).

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