

Holography for stress-energy tensor flows

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We study the holographic description for general stress-energy tensor deformations in arbitrary dimensions using the metric flow approach. Mixed boundary conditions corresponding to these deformations emerge from solutions to the metric flow equations. To test this proposal, we analyze planar anti-de Sitter black holes with such boundary conditions and find that the deformed energies satisfy flow equations consistent with the field theory interpretation. We further derive the commuting condition for stress-energy tensor deformations and extend the mixed boundary condition description to accommodate families of commuting deformations.

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Introduction. Extending the AdS/CFT correspondence [1–3] via controlled deformations can offer new insights into quantum gravity and holography. One notable example is the two-dimensional $T\bar{T}$ deformation [4–6], which has attracted considerable attention due to its remarkable properties. For comprehensive reviews, see Refs. [7,8]. It was proposed in Ref. [9] that turning on the $T\bar{T}$ deformation is holographically dual to introducing a finite radial cutoff surface in the bulk. However, this cutoff prescription is only applicable to one sign of the deformation parameter. For the opposite sign, a glue-on proposal was introduced in Ref. [10]. Both the cutoff and glue-on approaches are restricted to the pure gravity sector. A more flexible proposal arises from imposing mixed boundary conditions at infinity [11], which works for both signs of the deformation parameter and accommodates matter fields in the bulk. Beyond these proposals, the random geometry formulation of $T\bar{T}$ [12] has inspired an alternative holographic interpretation [13].

Considering general dimensions, the $T\bar{T}$ deformation can be generalized to stress-energy tensor deformations driven by composite operators built from the stress-energy tensor [12,14–25]. These deformations retain some of the geometric features of the $T\bar{T}$ deformation [12,26], and their effects can be reformulated as an auxiliary flow in the background metric [19,27,28]. This metric flow picture provides a powerful tool for establishing connections

between quadratic stress-energy tensor deformations and Ricci flows [29,30].

In this Letter, we intend to explore the holographic description for general stress-energy tensor deformations in arbitrary dimensions. While the cutoff prescription has been generalized to higher-dimensional $T\bar{T}$ -like deformations [14,31–36], we instead employ the mixed boundary condition approach, applicable to more general deformations. Motivated by a key observation that the mixed boundary condition for the $T\bar{T}$ deformation is closely related to the metric flow approach in Ref. [19], we propose that this connection extends to general stress-energy tensor deformations. The metric flow equations provide a systematic method for identifying the corresponding mixed boundary conditions. As a concrete example, we analyze planar anti-de Sitter (AdS) black holes. We find that the resulting spectrum of deformed energies satisfies flow equations consistent with those derived from large- N deformed field theories.

As a related topic, we explore the commutativity of stress-energy tensor deformations. The commutativity between the $T\bar{T}$ and root- $T\bar{T}$ [19,37–41] deformations has been used in Ref. [42] to determine the holographic mixed boundary condition for the root- $T\bar{T}$ deformation. In this work, we derive a general commuting condition for stress-energy tensor deformations using the metric flow approach. Since both the mixed boundary conditions and commuting condition can be derived from the metric flow equations, commuting deformations naturally lead to compatible mixed boundary conditions, opening new possibilities to construct richer and more flexible holographic models.

Metric flows. We begin by reviewing the metric flow prescription for stress-energy tensor deformations and clarifying its relation to the variational principle method for the mixed boundary conditions. The action of a d -dimensional field theory deformed by an operator constructed from the

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stress-energy tensor obeys the differential equation

$$\frac{\partial S_\tau(g_{\mu\nu}, \phi)}{\partial \tau} = \int d^d x \sqrt{g} O, \quad g = |\det g_{\mu\nu}|, \quad (1)$$

where ϕ denotes the collection of matter fields, τ is the deformation parameter, and the deformation operator O is a Lorentz-invariant function of the stress-energy tensor $T^\mu{}_\nu = T^{\mu\sigma} g_{\sigma\nu}$ defined by

$$T^{\mu\nu} = \frac{-2}{\sqrt{g}} \frac{\delta S_\tau(g_{\mu\nu}, \phi)}{\delta g_{\mu\nu}}. \quad (2)$$

The operator O may also have explicit dependence on τ or other dimensionful constants, as in $T\bar{T} + \Lambda_2$ [43,44] and in the deformations studied in Refs. [25,45]. We call a deformation stationary if O has no explicit dependence on τ and homogeneous if O is a homogeneous function of $T^\mu{}_\nu$ so that

$$T^\mu{}_\nu \frac{\partial O}{\partial T^\mu{}_\nu} = mO, \quad (3)$$

where m is the degree of O .

As noted in Refs. [19,27,28], a stress-energy tensor flow can alternatively be interpreted as an auxiliary flow in the space of the background metrics

$$\begin{aligned} \frac{dg_{\mu\nu}}{d\tau} &= 2 \frac{\partial O}{\partial T^{\mu\nu}}, \\ \frac{dT^{\mu\nu}}{d\tau} &= (T^{\rho\sigma} g^{\mu\nu} - T^{\mu\nu} g^{\rho\sigma}) \frac{\partial O}{\partial T^{\rho\sigma}} - 2 \frac{\partial O}{\partial g_{\mu\nu}} - O g^{\mu\nu}, \end{aligned} \quad (4)$$

arising from a classical dynamical equivalence between the deformed theory and the undeformed theory with a modified background metric. Consequently, the deformed action for a stationary homogeneous deformation can be obtained as

$$S_\tau(g_{\mu\nu}, \phi) = S_0(g_{\mu\nu,0}, \phi) + (1-m)\tau \int d^d x \sqrt{g_0} O(T^\mu{}_{\nu,0}), \quad (5)$$

where $g_{\mu\nu,0}$ and $T^\mu{}_{\nu,0}$ are, respectively, the initial values of $g_{\mu\nu}$ and $T^\mu{}_\nu$ at $\tau = 0$. A derivation of (6), along with its generalization to broader classes of deformations, is given in Ref. [30]. Alternatively, the flow equations (4) and (5) can be understood as the flows along the characteristic curves in the procedure of solving the classical deformed Lagrangian with the method of characteristics [20].

Within the AdS/CFT correspondence, multitrace deformations of the boundary theory correspond to modified boundary conditions for the dual bulk fields in AdS [46–53]. Boundary conditions for the $J\bar{T}$ and $T\bar{T}$ deformations were analyzed via the variational principle in Refs. [54,11], respectively. This approach was later applied to more general stress tensor deformations in AdS₃/CFT₂ in Ref. [42]. Here, we relate it to the flow equations (4) and (5). We consider a large- N field theory deformed by a multitrace operator $f(\mathcal{O})$ where $\mathcal{O}$ is a single-trace operator

dual to a fundamental bulk field. In the large- N limit, the deformed generating functional $W_\tau[J]$ is related to the undeformed one as

$$W_\tau[J_\tau] = W_0[J_0] + \tau \int d^d x (f - \sigma f'(\sigma)), \quad (6)$$

where J_τ and J_0 are the sources coupled to $\mathcal{O}$ in the deformed and undeformed theories, respectively, related by $J_\tau = J_0 - \tau f'(\sigma)$. Here, $\sigma = \delta W_0 / \delta J_0$ is the undeformed expectation value of $\mathcal{O}$. The details of derivation can be found in Ref. [53] (see also Ref. [42]). Taking the limit $\tau \rightarrow 0$, we get the differential equations

$$\partial_\tau W_\tau[J_\tau] = \int d^d x (f - \sigma f'(\sigma)), \quad (7)$$

$$\partial_\tau J_\tau = -f'(\sigma). \quad (8)$$

In stress-energy tensor deformations, the source J_τ coupled to the deformed stress tensor $T^{\mu\nu}$ is the deformed metric $g_{\mu\nu}$. As discussed in Ref. [42], the function f depends on both single-trace operators and their sources, leading to more complicated behavior. Therefore, it is more convenient to use the varied version of (8) for a stress tensor deformation $O(T^\mu{}_\nu)$,

$$\begin{aligned} \frac{1}{2} \frac{\partial}{\partial \tau} \left(\int d^d x \sqrt{g} T_{\mu\nu} \delta g^{\mu\nu} \right) \\ = \delta \left(\int d^d x \sqrt{g} \left(O - T^\mu{}_\nu \frac{\partial O}{\partial T^\mu{}_\nu} \right) \right). \end{aligned} \quad (9)$$

With a slight abuse of notation, here we use the same symbols for the operators $T^\mu{}_\nu$ and their expectation values $\langle T^\mu{}_\nu \rangle$, justified by large- N factorization. By matching varied terms and their corresponding coefficients, this yields

$$\frac{1}{2} \partial_\tau (\sqrt{g} T^\mu{}_\nu) g^{\nu\sigma} \delta g_{\sigma\mu} = \left(T^\mu{}_\nu \frac{\partial O}{\partial T^\mu{}_\nu} - O \right) \delta \sqrt{g}, \quad (10)$$

$$\frac{1}{2} \sqrt{g} T^\mu{}_\nu \partial_\tau (g^{\nu\sigma} \delta g_{\sigma\mu}) = \sqrt{g} \delta \left(T^\mu{}_\nu \frac{\partial O}{\partial T^\mu{}_\nu} - O \right), \quad (11)$$

from which one can derive the same flow equations (4) and (5) obtained from the classical analysis. This is expected, since the computation of the generating functional reduces to a classical saddle point in the large- N limit. As a nontrivial check, the solution of the flow equations (4) and (5) for the root- $T\bar{T}$ operator gives the mixed boundary condition proposed in Ref. [42] (see also Refs. [24,55,56]).

Solution to the flow equations. Equations (4) and (5) can be solved perturbatively in small τ , with initial values given by $g_{\mu\nu,0}$ and $T^\mu{}_{\nu,0}$. Both $g^{\mu\alpha,0} g_{\alpha\nu}$ and $T^\mu{}_\nu$ admit formal power series expansions in τ , and the coefficients are functions of $T^\mu{}_{\nu,0}$. As a result, $g^{\mu\sigma,0} g_{\sigma\nu}$, $T^\mu{}_\nu$, and $T^\mu{}_{\nu,0}$ can be diagonalized simultaneously by a τ -independent matrix. We denote the eigenvalues of $g^{\mu\sigma,0} g_{\sigma\nu}$ and $T^\mu{}_\nu$ as ω_α and t_α ,

respectively. In terms of these eigenvalues, O can be treated as a function of t_α , and the flow equations take the form

$$\frac{d\omega_\alpha}{d\tau} = 2 \frac{\partial O}{\partial t_\alpha} \omega_\alpha, \quad (13)$$

$$\frac{dt_\alpha}{d\tau} = -O + \sum_\beta t_\beta \frac{\partial O}{\partial t_\beta} - t_\alpha \sum_\beta \frac{\partial O}{\partial t_\beta}. \quad (14)$$

Throughout the paper, repeated indices of eigenvalues are not implicitly summed over, while repeated indices of tensors are implicitly summed over their respective ranges.

For a stationary homogeneous deformation, the solution to the flow equation for t_α (14) with initial condition $t_{\alpha,0}$ is given by

$$t_\alpha = \frac{t_{\alpha,0} + s}{\sqrt{g/g_0}}, \quad \sqrt{g/g_0} = \left(\frac{O|_{t_\alpha \rightarrow t_{\alpha,0}+s}}{O|_{t_\alpha \rightarrow t_{\alpha,0}}} \right)^{\frac{1}{m-1}}, \quad (15)$$

where $s = \tau(m-1)O|_{t_\alpha \rightarrow t_{\alpha,0}}$. For marginal deformations, the solution can be obtained by taking the limit $m \rightarrow 1$. This solution can be obtained by following a similar strategy used in Ref. [20]. First, one can show that the quantity $\sqrt{g}O(t_\alpha)$ is invariant along the flow by using the flow equations [30]. Then, one finds that

$$\frac{d}{d\tau}(\sqrt{g}t_\alpha) = \sqrt{g} \left(\sum_\beta t_\beta \partial_{t_\beta} - 1 \right) O = (m-1)\sqrt{g}O \quad (16)$$

remains invariant along the flow, allowing $\sqrt{g}t_\alpha$ to be determined. Finally, $\sqrt{g}$ can be determined from the relation $\sqrt{g}O(t_\alpha) = \sqrt{g_0}O(t_{\alpha,0})$ and the homogeneity of O . Once t_α are determined, we can integrate equations (13) to get ω_α .

While explicit solutions for general deformations are not available in closed form, the formula

$$\frac{d}{d\tau}(\sqrt{g}(t_\alpha - t_\beta)) = 0, \quad (17)$$

for any two t -eigenvalues, will be useful for our analysis.

Holography. We now turn to mixed boundary conditions associated with the stress-energy tensor in holography. The metric of an asymptotically AdS spacetime can be written in the Graham-Fefferman coordinate system

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{1}{\rho} \gamma_{\mu\nu}(\rho) dx^\mu dx^\nu, \quad (18)$$

where

$$\gamma_{\mu\nu}(\rho) = \gamma_{\mu\nu}^{(0)} + \dots + \gamma_{\mu\nu}^{(d)} \rho^{d/2} + \rho^{d/2} h_{\mu\nu} \log \rho + \dots \quad (19)$$

From now on, we use $\mu, \nu, \dots = 0, \dots, d-1$ to label spacetime coordinates and $i, j, \dots = 1, \dots, d-1$ to label spatial coordinates. According to the AdS/CFT dictionary [57,58], the boundary metric and stress-energy tensor in the

undeformed field theory are related to the coefficients $\gamma_{\mu\nu}^{(n)}$ as

$$g_{\mu\nu,0} = \gamma_{\mu\nu}^{(0)}, \quad T_{\mu\nu,0} = \frac{d\ell^{d-3}}{16\pi G} \gamma_{\mu\nu}^{(d)} + X_{\mu\nu}(\gamma_{\mu\nu}^{(n)}), \quad (20)$$

where $X_{\mu\nu}$ is a function of $\gamma_{\mu\nu}^{(n)}$ with $n < d$ encoding the conformal anomalies of the boundary conformal field theory. After turning on the stress-energy deformation, the boundary metric and stress-energy tensor can be obtained by solving the flow equations with (20) as the initial values. In this way, the solution to the flow equation provides a holographic prescription for the mixed boundary condition of the stress-energy tensor.

The mixed boundary condition proposals of $T\bar{T}$ and root- $T\bar{T}$ deformation were verified respectively in Refs. [11,42] by matching the energy of a deformed Bañados-Teitelboim-Zanelli black hole with the field theory answer. We now verify the mixed boundary condition proposal for general stress-energy tensor deformations in planar AdS black holes, following similar steps as in Refs. [11,42]. The metric of a planar AdS black hole in $d+1$ dimensions is

$$ds^2 = \frac{\ell^2}{z^2} \left[- \left(1 - \frac{z^d}{z_0^d} \right) dt^2 + \left(1 - \frac{z^d}{z_0^d} \right)^{-1} dz^2 + \sum_{i=1}^{d-1} dx^i dx^i \right]. \quad (21)$$

The transformation $z = \ell \sqrt{\rho} (\mathcal{L} \rho^{d/2} + 1)^{-2/d}$ with $z_0 = 4^{-1/d} \ell \mathcal{L}^{-1/d}$ brings the metric to the Fefferman-Graham form

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} - \frac{(\mathcal{L} \rho^{d/2} - 1)^2 (\mathcal{L} \rho^{d/2} + 1)^{\frac{4}{d}-2}}{\rho} dt^2 + \frac{(\mathcal{L} \rho^{d/2} + 1)^{4/d}}{\rho} \sum_{i=1}^{d-1} dx^i dx^i. \quad (22)$$

Expanding the metric near the boundary $\rho \rightarrow 0$ and comparing with the expansion (19), we find that

$$\gamma_{\mu\nu}^{(0)} = \text{diag}(-1, 1, \dots, 1), \quad \gamma_{\mu\nu}^{(n)} = 0, \quad 0 < n < d, \quad (23)$$

$$\gamma_{\mu\nu}^{(d)} = \frac{4\mathcal{L}}{d} \text{diag}(d-1, 1, \dots, 1).$$

From the AdS/CFT dictionary, the undeformed boundary metric and stress-energy tensor can be read off as

$$g_{\mu\nu,0} = \gamma_{\mu\nu}^{(0)}, \quad (24)$$

$$T_{\mu\nu,0} = \frac{d\ell^{d-3}}{16\pi G} \gamma_{\mu\nu}^{(d)} = \text{diag}((d-1)h, h, \dots, h), \quad (25)$$

where we introduce $h = \ell^{d-3} (4\pi G)^{-1} \mathcal{L}$ to simplify the expression. In this case, the boundary metric is flat, and there are no gravitational conformal anomalies, so $X_{\mu\nu} = 0$. The deformed boundary metric and stress tensor can be obtained as the solution to the flow equations

$$g^{\mu\alpha,0}g_{\alpha\nu} = \text{diag}(\omega_0, \dots, \omega_{d-1}), \quad (26)$$

$$T^\mu_\nu = \text{diag}(t_0, \dots, t_{d-1}). \quad (27)$$

It follows from (17) that the spatial t -eigenvalues are equal, so we denote $t_i = t_x$ for $i = 1, \dots, d-1$. For a stationary homogeneous deformation, we have

$$t_0 = \frac{1}{\sqrt{g/g_0}}((1-d)h + \tau(m-1)O(t_{\alpha,0})), \quad (28)$$

$$t_x = \frac{1}{\sqrt{g/g_0}}(h + \tau(m-1)O(t_{\alpha,0})). \quad (29)$$

To obtain the spectrum, we need to work with coordinates such that the boundary metric takes the standard form:

$$g_{\mu\nu}dx^\mu dx^\nu = -dT^2 + \sum_{i=1}^{d-1} d\phi^i d\phi^i. \quad (30)$$

This can be achieved by $t \rightarrow T/\sqrt{\omega_0}$ and $\phi_i \rightarrow x_i/\sqrt{\omega_i}$. To introduce nontrivial angular momenta, one can further make a Lorentz boost. Then, the full coordinate transformation is

$$t = \frac{1}{\sqrt{\omega_0}}\left(\Xi T + \sum_{i=1}^{d-1} a_i \phi^i\right), \quad (31)$$

$$x^i = \frac{1}{\sqrt{\omega_i}}\left(a_i T + \phi^i + \sum_{j=1}^{d-1} \frac{a_i a_j}{\Xi + 1} \phi^j\right), \quad (32)$$

where $\Xi = (1 + \sum_{i=1}^{d-1} a_i^2)^{1/2}$. To investigate off-diagonal components of the stress tensor, we compactify the ϕ -coordinates on a nonorthogonal torus defined by identifying

$$\phi^i \sim \phi^i + R^i_a, \quad i, a = 1, \dots, d-1. \quad (33)$$

The volume of the torus is $V = \det(R^i_a)$. The components of the stress-energy tensor in the new coordinate are

$$\begin{aligned} T^0_0 &= \Xi^2 t_0 - (\Xi^2 - 1)t_x, & T^0_i &= a_i \Xi(t_0 - t_x), \\ T^i_j &= a_i a_j(t_x - t_0) + \delta^i_j t_x. \end{aligned} \quad (34)$$

The deformed energy and angular momentum are

$$E = -VT^0_0, \quad J_i = -VT^0_i. \quad (35)$$

To obtain the deformed energy for a given deformation, one should fix $\mathcal{L}$ (or h) and a_i by identifying quantities that are not deformed. One such quantity is the area of the event horizon at $\rho = \mathcal{L}^{-d/2}$,

$$A = 4^{\frac{d-1}{d}} V \mathcal{L}^{\frac{d-1}{d}} \Xi \prod_{i=1}^{d-1} \omega_i^{-1/2}. \quad (36)$$

The event horizon area corresponds to the degeneracy of states, which should be unchanged under variation of τ and R^i_a . The bulk angular momenta correspond to the momenta

of the states in the field theory. The periodicity condition of the wave function requires that $J_i R^i_a$ should be integer multiples of 2π and thus invariant under continuous change of τ and R^i_a . Now, we have

$$V \mathcal{L}^{\frac{d-1}{d}} \Xi \prod_{i=1}^{d-1} \omega_i^{-1/2} = \text{constant}, \quad (37)$$

$$V \Xi(t_0 - t_x) a_i R^i_a = \text{constant}. \quad (38)$$

The constants are fixed by the undeformed energy and momenta. For a specific deformation, one can obtain the deformed energy by using the explicit expressions of t_α and ω_α to solve the constraints. For generic deformations, we instead derive flow equations for the deformed energy. By taking derivatives of these constraints with respect to τ and R^i_a , one can express the derivatives of $\mathcal{L}$ and a_i as functions of $\mathcal{L}$ and a_i . We show in Supplemental Material [59] that

$$\partial_\tau E = V O(t_\alpha), \quad (39)$$

$$T^i_j = -\frac{1}{V} R^i_a \frac{\partial E}{\partial R^j_a}. \quad (40)$$

Equation (40) is consistent with the field theory interpretation of the stress-energy tensor.

In a large- N field theory, we assume that the expectation value of O factorizes as $\langle O(T^\mu_\nu) \rangle = O(\langle T^\mu_\nu \rangle)$. Then, the energy of an eigenstate of the energy and momentum operators satisfies the flow equation

$$\partial_\tau E = V O(\langle T^\mu_\nu \rangle). \quad (41)$$

Therefore, Eq. (39) agrees with the flow equation of the energy obtained from the field theory in the large- N limit.

Commuting flows. Now, we investigate the holographic mixed boundary condition for commuting stress-energy tensor deformations. We need to derive the condition that two deformations commute with each other. We consider two deformations driven by O_1 and O_2 . When two deformations commute with each other, we have

$$\partial_{\tau_1} \partial_{\tau_2} \omega_\alpha = \partial_{\tau_2} \partial_{\tau_1} \omega_\alpha, \quad \partial_{\tau_1} \partial_{\tau_2} t_\alpha = \partial_{\tau_2} \partial_{\tau_1} t_\alpha. \quad (42)$$

Using the flow equations, we show in the Supplemental Material [59] that the commuting condition is equivalent to

$$\begin{aligned} C(O_1, O_2) &\equiv \sum_\alpha \left(O_1 \frac{\partial O_2}{\partial t_\alpha} - O_2 \frac{\partial O_1}{\partial t_\alpha} \right) - \sum_{\alpha, \beta} (t_\beta - t_\alpha) \\ &\quad \times \frac{\partial O_1}{\partial t_\beta} \frac{\partial O_2}{\partial t_\alpha} + \tilde{\partial}_{\tau_2} O_1 - \tilde{\partial}_{\tau_1} O_2 = 0. \end{aligned} \quad (43)$$

Here, the symbol $\tilde{\partial}_{\tau_p} O_q$ refers only to the explicit dependence of O_q on τ_p . To simplify the analysis and uncover

more interesting structures, we now restrict our discussion to stationary deformations. We observe that:

- (i) A deformation is called traceless if $\sum_a \partial_{t_a} O = 0$. Traceless deformations mutually commute.
- (ii) Marginal deformations mutually commute.
- (iii) Traceless marginal deformations commute with arbitrary deformations, which is consistent with results in Ref. [24].

For a given deformation with a generic operator O_1 , one can obtain commuting deformations by solving $C(O_1, O_2) = 0$, which is a linear partial differential equation for O_2 . Using the method of characteristics, a general solution can be written as

$$O_2(t_a) = O_1(t_a) G\left(\frac{t_1 - t_2}{O_1(t_a)}, \frac{t_2 - t_3}{O_1(t_a)}, \dots, \frac{t_{d-1} - t_d}{O_1(t_a)}\right), \quad (44)$$

where G is an arbitrary function. Interestingly, deformations generated by operators of the form given in Eq. (44), associated with different G functions, mutually commute. This allows the construction of an infinite commuting family of deformations. For example, in two dimensions, the commuting family containing the $T\bar{T}$ deformation can be generated by the operators

$$O_p = t_1 t_2 \left(\frac{t_1 - t_2}{t_1 t_2}\right)^p, \quad p \in \mathbb{R}, \quad (45)$$

where O_0 is the $T\bar{T}$ operator and O_1 is the root- $T\bar{T}$ operator.

For a commuting family of stationary deformations, one can choose a basis of homogeneous operators $\{O_p\}$. One can show that

$$\partial_{\tau_p}(\sqrt{g} O_q(t_{\alpha,0})) = 0, \quad (46)$$

for any p and q . Therefore, the solution of the flow equations is given by

$$t_\alpha = \frac{t_{\alpha,0} + s}{\sqrt{g/g_0}}, \quad s = \sum_p \tau_p (m_p - 1) O_p(t_{\alpha,0}), \quad (47)$$

where $\sqrt{g}$ can be determined by choosing one of the operators and using its homogeneity property. In the holographic context, this solution enables a generalization of the mixed boundary conditions proposal to commuting deformations. In the example of the planar AdS black hole, a similar holographic analysis can be performed. The deformed energy satisfies a set of commuting flow equations

$$\partial_{\tau_p} E = V O_p(t_\alpha), \quad (48)$$

and Eq. (40) remains valid in the presence of commuting deformations, in agreement with the field theory interpretation.

Conclusions. We proposed a holographic prescription for general stress-energy tensor deformations through mixed boundary conditions. The metric flow equation solutions encode these conditions, as verified for planar AdS black holes. We derived a commuting condition for such deformations and identify infinite commuting families. These results extend previous studies of the holographic $T\bar{T}$ deformation and suggest new routes to engineering holographic dualities.

Several promising directions remain for future work. Extending our analysis beyond the large- N limit is a challenging but important step. One possible approach is to define the deformations by requiring the flow equations of the generating functional to hold exactly, even at finite N . Recently, the holographic prescription for $T\bar{T}$ -deformed conformal field theories with gravitational anomalies was studied in Ref. [60]. It would be interesting to apply mixed boundary conditions to more general bulk geometries to include holographic conformal anomalies.

Holography has played a central role in computing many observables in $T\bar{T}$ -deformed theories [61–68]. A natural direction is to generalize these calculations to arbitrary stress-energy tensor deformations. Recent holographic studies have connected $T\bar{T}$ -like deformations to diverse topics, including black hole interiors [69], bulk reconstruction [70], and emergent gravity [71]. Our results provide a pathway to extend these ideas to general stress-energy tensor deformations in arbitrary dimensions. Finally, the rich nonperturbative effects of $T\bar{T}$ deformations [72–77] raise the question of whether analogous effects occur for general stress-energy tensor deformations and, if so, whether they are related to nonperturbative bulk geometries subject to mixed boundary conditions.

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- [1] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* **428**, 105 (1998).
- [3] E. Witten, Anti de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [4] A. B. Zamolodchikov, Expectation value of composite field T anti- T in two-dimensional quantum field theory, [arXiv:hep-th/0401146](#).
- [5] A. Cavaglià, S. Negro, I. M. Szécsényi, and R. Tateo, $T\bar{T}$ -deformed 2D quantum field theories, *J. High Energy Phys.* **10** (2016) 112.
- [6] F. A. Smirnov and A. B. Zamolodchikov, On space of integrable quantum field theories, *Nucl. Phys. B* **915**, 363 (2017).
- [7] Y. Jiang, A pedagogical review on solvable irrelevant deformations of 2D quantum field theory, *Commun. Theor. Phys.* **73**, 057201 (2021).
- [8] S. He, Y. Li, H. Ouyang, and Y. Sun, $T\bar{T}$ deformation: Introduction and some recent advances, *Sci. China Phys. Mech. Astron.* **68**, 101001 (2025).
- [9] L. McGough, M. Mezei, and H. Verlinde, Moving the CFT into the bulk with $T\bar{T}$, *J. High Energy Phys.* **04** (2018) 010.
- [10] L. Apolo, P.-X. Hao, W.-X. Lai, and W. Song, Glue-on AdS holography for $T\bar{T}$ -deformed CFTs, *J. High Energy Phys.* **06** (2023) 117.
- [11] M. Guica and R. Monten, $T\bar{T}$ and the mirage of a bulk cutoff, *SciPost Phys.* **10**, 024 (2021).
- [12] J. Cardy, The $T\bar{T}$ deformation of quantum field theory as random geometry, *J. High Energy Phys.* **10** (2018) 186.
- [13] S. Hirano and M. Shigemori, Random boundary geometry and gravity dual of $T\bar{T}$ deformation, *J. High Energy Phys.* **11** (2020) 108.
- [14] M. Taylor, $T\bar{T}$ deformations in general dimensions, *Adv. Theor. Math. Phys.* **27**, 37 (2023).
- [15] G. Bonelli, N. Doroud, and M. Zhu, $T\bar{T}$ -deformations in closed form, *J. High Energy Phys.* **06** (2018) 149.
- [16] R. Conti, L. Iannella, S. Negro, and R. Tateo, Generalised Born-Infeld models, Lax operators and the $T\bar{T}$ perturbation, *J. High Energy Phys.* **11** (2018) 007.
- [17] C. Ferko, H. Jiang, S. Sethi, and G. Tartaglino-Mazzucchelli, Non-linear supersymmetry and $T\bar{T}$ -like flows, *J. High Energy Phys.* **02** (2020) 016.
- [18] H. Babaei-Aghbolagh, K. Babaei Velni, D. M. Yekta, and H. Mohammadzadeh, $T\bar{T}$ -like flows in non-linear electrodynamic theories and S-duality, *J. High Energy Phys.* **04** (2021) 187.
- [19] R. Conti, J. Romano, and R. Tateo, Metric approach to a $T\bar{T}$ -like deformation in arbitrary dimensions, *J. High Energy Phys.* **09** (2022) 085.
- [20] J. Hou, $T\bar{T}$ flow as characteristic flows, *J. High Energy Phys.* **03** (2023) 243.
- [21] C. Ferko, Y. Hu, Z. Huang, K. Koutrolikos, and G. Tartaglino-Mazzucchelli, $T\bar{T}$ -like flows and 3d nonlinear supersymmetry, *SciPost Phys.* **16**, 038 (2024).
- [22] C. Ferko, S. M. Kuzenko, L. Smith, and G. Tartaglino-Mazzucchelli, Duality-invariant nonlinear electrodynamics and stress tensor flows, *Phys. Rev. D* **108**, 106021 (2023).
- [23] C. Ferko, S. M. Kuzenko, K. Lechner, D. P. Sorokin, and G. Tartaglino-Mazzucchelli, Interacting chiral form field theories and $T\bar{T}$ -like flows in six and higher dimensions, *J. High Energy Phys.* **05** (2024) 320.
- [24] H. Babaei-Aghbolagh, S. He, T. Morone, H. Ouyang, and R. Tateo, Geometric formulation of generalized root- $T\bar{T}$ deformations, *Phys. Rev. Lett.* **133**, 111602 (2024).
- [25] E. Tsolakidis, Massive gravity generalization of $T\bar{T}$ deformations, *J. High Energy Phys.* **09** (2024) 167.
- [26] R. Conti, S. Negro, and R. Tateo, The $T\bar{T}$ perturbation and its geometric interpretation, *J. High Energy Phys.* **02** (2019) 085.
- [27] T. Morone, S. Negro, and R. Tateo, Gravity and $T\bar{T}$ flows in higher dimensions, *Nucl. Phys. B* **1005**, 116605 (2024).
- [28] X.-Y. Ran, F. Hao, and M. Yamada, Geometric realization via irrelevant deformations induced by the stress-energy tensor, *Phys. Rev. D* **111**, 085033 (2025).
- [29] N. Brizio, T. Morone, and R. Tateo, Stress-energy tensor deformations, Ricci flows and black holes, [arXiv:2408.06031](#).
- [30] T. Morone and R. Tateo, Solutions to the Ricci flow via Einstein field equations, [arXiv:2411.10265](#).
- [31] T. Hartman, J. Kruthoff, E. Shaghoulian, and A. Tajdini, Holography at finite cutoff with a T^2 deformation, *J. High Energy Phys.* **03** (2019) 004.
- [32] V. Shyam, Finite cutoff AdS₅ holography and the generalized gradient flow, *J. High Energy Phys.* **12** (2018) 086.
- [33] P. Caputa, S. Datta, and V. Shyam, Sphere partition functions & cut-off AdS, *J. High Energy Phys.* **05** (2019) 112.
- [34] A. Belin, A. Lewkowycz, and G. Sarosi, Gravitational path integral from the T^2 deformation, *J. High Energy Phys.* **09** (2020) 156.
- [35] A. Parvizi, M. M. Sheikh-Jabbari, and V. Taghiloo, Free-lance holography, Part I: Setting boundary conditions free in gauge/gravity correspondence, *SciPost Phys.* **19**, 043 (2025).
- [36] A. Parvizi, M. M. Sheikh-Jabbari, and V. Taghiloo, Free-lance holography, Part II: Moving boundary in gauge/gravity correspondence, [arXiv:2503.09372](#).
- [37] P. Rodríguez, D. Tempo, and R. Troncoso, Mapping relativistic to ultra/non-relativistic conformal symmetries in 2D and finite $\sqrt{T\bar{T}}$ deformations, *J. High Energy Phys.* **11** (2021) 133.
- [38] H. Babaei-Aghbolagh, K. B. Velni, D. M. Yekta, and H. Mohammadzadeh, Emergence of non-linear electrodynamic theories from $T\bar{T}$ -like deformations, *Phys. Lett. B* **829**, 137079 (2022).
- [39] C. Ferko, L. Smith, and G. Tartaglino-Mazzucchelli, On current-squared flows and ModMax theories, *SciPost Phys.* **13**, 012 (2022).
- [40] C. Ferko, A. Sfondrini, L. Smith, and G. Tartaglino-Mazzucchelli, Root- $T\bar{T}$ deformations in two-dimensional quantum field theories, *Phys. Rev. Lett.* **129**, 201604 (2022).
- [41] H. Babaei-Aghbolagh, K. Babaei Velni, D. Mahdavian Yekta, and H. Mohammadzadeh, Marginal $T\bar{T}$ -like deformation and modified Maxwell theories in two dimensions, *Phys. Rev. D* **106**, 086022 (2022).
- [42] S. Ebert, C. Ferko, and Z. Sun, Root- $T\bar{T}$ deformed boundary conditions in holography, *Phys. Rev. D* **107**, 126022 (2023).

- [43] A. Lewkowycz, J. Liu, E. Silverstein, and G. Torroba, $T\bar{T}$ and EE, with implications for (A)dS subregion encodings, *J. High Energy Phys.* **04** (2020) 152.
- [44] V. Gorbenko, E. Silverstein, and G. Torroba, dS/dS and $T\bar{T}$, *J. High Energy Phys.* **03** (2019) 085.
- [45] C. D. A. Blair, J. Lahnsteiner, N. A. Obers, and Z. Yan, Matrix theory reloaded: A BPS road to holography, *J. High Energy Phys.* **02** (2025) 024.
- [46] I. R. Klebanov and E. Witten, AdS/CFT correspondence and symmetry breaking, *Nucl. Phys.* **B556**, 89 (1999).
- [47] E. Witten, Multitrace operators, boundary conditions, and AdS/CFT correspondence, [arXiv:hep-th/0112258](https://arxiv.org/abs/hep-th/0112258).
- [48] M. Berkooz, A. Sever, and A. Shomer, ‘Double trace’ deformations, boundary conditions and space-time singularities, *J. High Energy Phys.* **05** (2002) 034.
- [49] W. Mueck, An improved correspondence formula for AdS/CFT with multitrace operators, *Phys. Lett. B* **531**, 301 (2002).
- [50] S. S. Gubser and I. Mitra, Double trace operators and one loop vacuum energy in AdS/CFT, *Phys. Rev. D* **67**, 064018 (2003).
- [51] T. Hartman and L. Rastelli, Double-trace deformations, mixed boundary conditions and functional determinants in AdS/CFT, *J. High Energy Phys.* **01** (2008) 019.
- [52] D. E. Diaz and H. Dorn, Partition functions and double-trace deformations in AdS/CFT, *J. High Energy Phys.* **05** (2007) 046.
- [53] I. Papadimitriou, Multi-trace deformations in AdS/CFT: Exploring the vacuum structure of the deformed CFT, *J. High Energy Phys.* **05** (2007) 075.
- [54] A. Bzowski and M. Guica, The holographic interpretation of $J\bar{T}$ -deformed CFTs, *J. High Energy Phys.* **01** (2019) 198.
- [55] S. Ebert, Holographic renormalization group and stress tensor operators, Ph.D. thesis, UCLA, Los Angeles (main), UCLA, 2024, [arXiv:2404.10190](https://arxiv.org/abs/2404.10190).
- [56] J. Tian, T. Lai, and F. Omid, Modular transformations of on-shell actions of (root-) $T\bar{T}$ deformed holographic CFTs, *Nucl. Phys.* **B1007**, 116675 (2024).
- [57] V. Balasubramanian and P. Kraus, A stress tensor for Anti-de Sitter gravity, *Commun. Math. Phys.* **208**, 413 (1999).
- [58] S. de Haro, S. N. Solodukhin, and K. Skenderis, Holographic reconstruction of space-time and renormalization in the AdS/CFT correspondence, *Commun. Math. Phys.* **217**, 595 (2001).
- [59] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/v2g6-1983> for (1) Holographic derivation of flow equations of the energy; (2) Derivation of the commutativity condition.
- [60] D. Basu, Q. Wen, and M. Xu, The holographic $T\bar{T}$ deformation of the CFT₂ with gravitational anomalies, [arXiv:2507.20292](https://arxiv.org/abs/2507.20292).
- [61] J. Tian, On-shell action of $T\bar{T}$ -deformed Holographic CFTs, [arXiv:2306.01258](https://arxiv.org/abs/2306.01258).
- [62] M. He and Y. Sun, Holographic entanglement entropy in $T\bar{T}$ -deformed AdS₃, *Nucl. Phys.* **B990**, 116190 (2023).
- [63] S. He, Y. Li, Y.-Z. Li, and Y. Zhang, Holographic torus correlators of stress tensor in AdS₃/CFT₂, *J. High Energy Phys.* **06** (2023) 116.
- [64] M. He, J. Hou, and Y. Jiang, $T\bar{T}$ -deformed entanglement entropy for IQFT, *J. High Energy Phys.* **03** (2024) 056.
- [65] M. He, One-loop partition functions in $T\bar{T}$ -deformed AdS₃, *J. High Energy Phys.* **05** (2024) 067.
- [66] S. He, Y. Li, Y.-Z. Li, and Y. Zhang, Note on holographic torus stress tensor correlators in AdS₃ gravity, *J. High Energy Phys.* **09** (2024) 125.
- [67] J.-C. Chang, S. He, Y.-X. Liu, and L. Zhao, Holographic $T\bar{T}$ deformation of the entanglement entropy in (A)dS₃/CFT₂, *Phys. Rev. D* **112**, 026013 (2025).
- [68] Y.-Z. Li, Y. Xie, and S. He, Holographic correlators of boundary/crosscap CFTs in two dimensions, *J. High Energy Phys.* **07** (2025) 010.
- [69] S. Ali Ahmad, A. Almheiri, and S. Lin, $T\bar{T}$ and the black hole interior, [arXiv:2503.19854](https://arxiv.org/abs/2503.19854).
- [70] H.-C. Liang, J.-R. Sun, and Y. Sun, Surface growth scheme for bulk reconstruction and $T\bar{T}$ deformation, [arXiv:2507.18435](https://arxiv.org/abs/2507.18435).
- [71] H. Adami, M. M. Sheikh-Jabbari, and V. Taghiloo, Gravity is induced by renormalization group flow, [arXiv:2508.09633](https://arxiv.org/abs/2508.09633).
- [72] O. Aharony, S. Datta, A. Giveon, Y. Jiang, and D. Kutasov, Modular invariance and uniqueness of $T\bar{T}$ deformed CFT, *J. High Energy Phys.* **01** (2019) 086.
- [73] L. Griguolo, R. Panerai, J. Papalini, and D. Seminara, Exact $T\bar{T}$ deformation of two-dimensional Maxwell theory, *Phys. Rev. Lett.* **128**, 221601 (2022).
- [74] L. Griguolo, R. Panerai, J. Papalini, and D. Seminara, Exact $T\bar{T}$ deformation of two-dimensional Yang-Mills theory on the sphere, *J. High Energy Phys.* **10** (2022) 134.
- [75] J. Gu, Y. Jiang, and H. Wang, Resurgence of $T\bar{T}$ -deformed partition function, *Phys. Rev. Lett.* **135**, 081601 (2025).
- [76] J. Gu, Y. Jiang, and H. Wang, Resurgent properties of $T\bar{T}$ -deformed conformal field theories, *Phys. Rev. D* **112**, 045007 (2025).
- [77] S. Hirano and V. Raj, Nonperturbative effects in $T\bar{T}$ -deformed conformal field theories: A toy model for Planckian physics, [arXiv:2507.16262](https://arxiv.org/abs/2507.16262).