

Generalized susceptibilities and the properties of charm degrees of freedom across the QCD crossover temperature

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We study the generalized charm susceptibilities in $2 + 1$ flavor QCD on the lattice at several lattice spacings. We show that, below the chiral crossover, these susceptibilities are well described by the hadron resonance gas (HRG) model if charmed hadrons not listed in tables of the Particle Data Group are included. However, the HRG description abruptly breaks down just above the chiral crossover. To understand this, we use a model for the charm pressure in which it is expressed as the sum of partial pressures from charmed baryons, charmed mesons, and charm quarks. We present continuum estimates of these partial pressures and find that, while the partial pressures of charmed mesons and baryons drop below their respective HRG predictions, the charm quark pressure becomes nonzero above the chiral crossover.

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I. INTRODUCTION

Strong interaction matter undergoes a transition at $T_{pc} = (156.5 \pm 1.5)$ MeV to a new state characterized by restoration of the chiral symmetry [1]. This state of matter is studied experimentally in heavy ion collisions; see Refs. [2,3] for recent reviews. Owing to their large masses, charm and bottom quarks are produced only during the earliest stages of a heavy-ion collision. They are affected by the hot-dense medium during its entire evolution, and ultimately hadronize into open and hidden heavy-flavor hadrons. From the yields of these hadrons, it is possible to gain valuable insights into the nature of the produced medium, in particular the strong collective behavior of the produced medium, where even heavy quarks/hadrons are involved in the collective behavior. It has been suggested early on that the heavy-flavor hadrons do provide an excellent probe for the deconfining nature of the quark-hadron transition in strong interaction matter [4]. This led to a large experimental and phenomenological program [5,6] dedicated to the study of properties of charmed hadrons in the QCD transition region [7–9]. To interpret

the results of the experiments, however, it is important to know the nature of the charm and bottom degrees of freedom across the QCD transition.

In order to study the nature of the charm degrees of freedom, one can calculate in the framework of lattice QCD, the so-called generalized charm susceptibilities, i.e., derivatives of the QCD pressure (P),

$$P = \frac{T}{V} \ln Z(T, V, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C), \quad (1)$$

with respect to chemical potentials coupling to the conserved currents of net baryon number (B), electric charge (Q), strangeness (S) and charm (C),

$$\chi_{klmn}^{BQSC} = \frac{\partial^{(k+l+m+n)} [P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \partial \hat{\mu}_S^m \partial \hat{\mu}_C^n} \Big|_{\vec{\mu}=0}. \quad (2)$$

Here we introduced a dimensionless notation for chemical potentials, $\hat{\mu}_X = \mu_X/T$, with $X \in \{B, Q, S, C\}$. The first study of generalized charm susceptibilities was performed about a decade ago [10,11]. These calculations have been performed on rather coarser lattices corresponding to temporal extent $N_\tau = 6$ and 8. We recall that the lattice spacing, a , is related to the temperature, T , and temporal extent, N_τ , as $a = 1/(TN_\tau)$. The statistics was also very limited in the above studies. Despite these limitations, the study of generalized charm susceptibilities provided

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important insights into the nature of the charm degrees of freedom at different temperatures. It showed that below the chiral crossover temperature, the generalized charm susceptibilities are well described by a gas of uncorrelated open-charm hadrons, while new degrees of freedom appear above the chiral crossover temperature [10,11].

Thermodynamics below T_{pc} can be understood in terms of hadron resonance gas (HRG) models. The main idea behind these models is that the interaction between hadrons in hot matter can be accounted for by including hadronic resonances, which are treated as stable particles when calculating the partition function. Below the crossover temperature, a HRG model based on resonances listed in tables of the Particle Data Group (PDG) gives a good description of the QCD equation of state [12–14] and fluctuations of conserved charges (BQS) [15–19]. We call this HRG model PDG-HRG. However, in some cases it is necessary to include missing hadronic resonances that are not listed in PDG tables but are predicted by the quark model (QM) calculations [17,19,20]. This HRG model with an extended list of states usually is called QM-HRG. The study of the ratios of various generalized charm susceptibilities in the previous lattice QCD studies also indicated that there are missing charmed meson and baryon states [10]. Very recently the need for additional charmed hadron states has been discussed in the context of the statistical hadronization model extended to include charm effects (SHMc) [21].

Cutoff effects in the generalized charm susceptibilities are large because of the large charm quark mass. At the same time, the large charm quark mass leads to poor signal to noise ratio in the calculation of these quantities. To overcome the problem of poor signal to noise ratios, a high statistics lattice QCD study of the generalized charm susceptibilities has been performed recently using $N_\tau = 8$ lattices [22]. This study significantly improved the lattice QCD results on the generalized susceptibilities at low temperatures and confirmed the assertion that additional charmed hadrons have to be included in HRG model calculations to arrive at quantitative agreement with lattice QCD results. On the other hand, the behavior of the generalized susceptibilities above the chiral transition can be well understood in a model, where the partial pressure is written as the sum of charm quark partial pressure, charmed meson partial pressure and charmed baryon partial pressure. Within this model charm quark appears as a new charm degree of freedom above the crossover temperature, while charmed hadronlike excitations still give the dominant contribution to the charm pressure up to $T = 176$ MeV [22]. In our previous work, we showed that this model successfully passes numerous validity tests and satisfies various constraints [22]. We will come back to one of these constraints in Sec. VI. The existence of such hadronlike excitations above T_{pc} is also supported by a recent analysis of charmed meson and

baryon correlation functions at nonzero temperature [23,24]. Furthermore, the generalized charm susceptibilities have been studied in the T-matrix microscopic model [25], and it was shown that charmed hadron resonances are needed in order to describe some of these susceptibilities.

The aim of this paper is to extend our previous study of the generalized charm susceptibilities by considering finer lattice spacings, corresponding to lattices with temporal extents, $N_\tau = 12$ and 16, and also by considering various combinations of generalized charm susceptibilities that will help us better understand the nature of charm degrees of freedom across the chiral crossover temperature. In addition to studying generalized charm susceptibilities normalized by the quartic charm fluctuation at $N_\tau = 8$, we also show a few $N_\tau = 12$ counterparts of these ratios. These results corroborate our previous deduction [22] that the cutoff effects get largely canceled in such ratios calculated on relatively coarser lattices. Therefore, it is reasonable to use continuum estimated quartic charm fluctuation to convert normalized generalized charm susceptibilities calculated on $N_\tau = 8$ lattices into continuum estimates of physical observables relevant to open charm physics. Moreover, we use $N_\tau = 8, 12$ and 16 lattices to obtain a continuum estimate of the quartic charm fluctuation, $\chi_4^C \equiv \chi_{0004}^{BQSC}$.

This paper is organized as follows. In Sec. II, we discuss various HRG models in the charmed hadron sector and the pressure of an ideal charm quark gas. In Sec. III, we present the details of the lattice setup used for our lattice QCD calculations, and discuss cutoff effects in charm susceptibilities and the continuum extrapolation of the quartic charm fluctuation. In Sec. IV, we present our results on various ratios of generalized susceptibilities and discuss how these support the idea of missing charmed hadrons. In Sec. V, we decompose the charm pressure in terms of partial pressure of charmed mesons, charmed baryons and charm quarks. Here we also show how these can be understood in terms of QM-HRG model calculations below the chiral crossover and discuss the implications of these findings on the nature of charm degrees of freedom above T_{pc} . Preliminary results from these studies have been presented in conference proceedings [26–30].

II. HADRON RESONANCE GAS MODELS

In a noninteracting HRG model, the QCD pressure can be written as the sum of the partial pressure of hadrons carrying open charm degrees of freedom and the partial pressure of hadrons with zero charm quantum number. Furthermore, the partial pressure of charmed hadrons, $P_C(T, \vec{\mu})$, can be written as the sum of partial pressures of charmed mesons, P_M^C , and charmed baryons, P_B^C ,

$$P_C(T, \vec{\mu}) = P_M^C(T, \vec{\mu}) + P_B^C(T, \vec{\mu}). \quad (3)$$

As the masses of charmed mesons and baryons are much larger than the temperatures in the temperature range of interest to us, one can use Boltzmann statistics and write P_M^C and P_B^C in the following form [10]:

$$P_M^C(T, \vec{\mu}) = \frac{1}{\pi^2} \sum_{i \in \text{C-mesons}} g_i \left(\frac{m_i}{T} \right)^2 K_2(m_i/T);$$

$$\cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C),$$

$$P_B^C(T, \vec{\mu}) = \frac{1}{\pi^2} \sum_{i \in \text{C-baryons}} g_i \left(\frac{m_i}{T} \right)^2 K_2(m_i/T);$$

$$\cosh(B_i \hat{\mu}_B + Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C). \quad (4)$$

Here each term in the sums takes care of a charmed hadron with mass m_i and its antiparticle. The summation is over all charmed mesons or baryons, respectively; g_i denotes the degeneracy factors of states with identical mass and quantum numbers; $K_2(x)$ is a modified Bessel function of the second kind, which for large arguments is given by $K_2(x) \sim \sqrt{\pi/2x} e^{-x} [1 + \mathcal{O}(x^{-1})]$.

While the sum over charmed baryons does contain contributions from $|C| = 1, 2$ and 3 sectors, the contributions of the $|C| \neq 1$ sectors are strongly suppressed as the mass of the only experimentally established (3-star) doubly charmed baryon (Ξ_{cc}^{++}) is roughly about 1.5 GeV larger than that of the lightest charmed baryon (Λ_c^+). In the temperature range of interest to us, i.e., $T \lesssim T_{pc}$, the ratio $(m_{\Xi_{cc}} - m_{\Lambda_c})/T \sim \mathcal{O}(10)$ and consequently the contribution of doubly charmed baryons to P_B^C are suppressed by a factor of $\sim e^{-10}$, i.e., this contribution is less than 0.01% of the charmed baryon pressure. Similarly, also possibly existing charmed baryonic molecules with baryon number $|B| > 1$ [31,32] would be strongly Boltzmann suppressed. We thus consider in the following only HRG models for charmed baryons with $|C| = \pm 1$. Note that our HRG lists contain baryons with $|C| \neq 1$.

We will use in the following two different HRG particle lists PDG-HRG2024c and QM-HRG2024c [33], which are similar in structure to our earlier lists PDG-HRG and QMHRG2020 containing only noncharmed states [34]. PDG-HRG2024c contains the recent compilation of all charmed hadrons listed by the Particle Data Group [35], and we augmented it in some cases by not observed isospin partners of experimentally observed states. For instance, we added the unobserved D_+^* in addition to the observed D_0^* . To construct the QM-HRG2024c list we used PDG-HRG2024c and added charmed hadron resonances not observed so far but obtained in quark model calculations [36–40]. More specifically, for not yet observed charmed states we used the relativistic quark model of Refs. [37,38].

In the HRG phase, generalized susceptibilities introduced in Eq. (2) are calculated by making use of the partial pressure expressions introduced above in Eq. (4). In

particular, for the calculation of generalized charm susceptibilities at vanishing chemical potentials, it suffices to replace the QCD pressure, P , with the partial charm pressure, P_C . The resulting expression for χ_{klmn}^{BQSC} takes the following form:

$$\chi_{klmn}^{BQSC} = \frac{1}{2\pi^2} \sum_{i \in \text{C-hadrons}} g_i \left(\frac{m_i}{T} \right)^2 K_2(m_i/T) B^k Q^l S^m C^n, \quad (5)$$

where the sum is over all charmed hadrons. As discussed above, the contribution of multiple charmed baryons is exponentially small, and effectively only the $|C| = 1$ sector contributes to the pressure. This means that $\chi_2^C = \chi_n^C = P_C(T, \vec{\mu})$, for n even, and $\chi_{11}^{BC} = \chi_{1m}^{BC} = P_B^C$, for m odd. Note that zero subscripts and their corresponding superscripts, e.g., QS in χ_{1m}^{BC} , are suppressed, and we will use the same notation in the following. The above observation motivates our construction for partial charm pressures in various quantum number channels. In particular, in the context of noninteracting HRG models, we use simple cumulants as proxies for the partial pressures of charmed baryon and meson sectors,

$$P_B^C(T, \vec{\mu}) = \chi_{1n}^{BC}, \quad (6)$$

$$P_M^C(T, \vec{\mu}) = \chi_{n+1}^C - \chi_{1n}^{BC}, \quad n \text{ odd}. \quad (7)$$

For definiteness and in order to be consistent with the lattice QCD calculations discussed in the following sections, we will use fourth order cumulants, i.e., we set $n = 3$.

In Fig. 1, we use these proxies of charmed baryon and meson pressures in the HRG phase, to show the contribution of experimentally observed hadrons to charm pressure in relation to the QM-HRG states. We introduced a

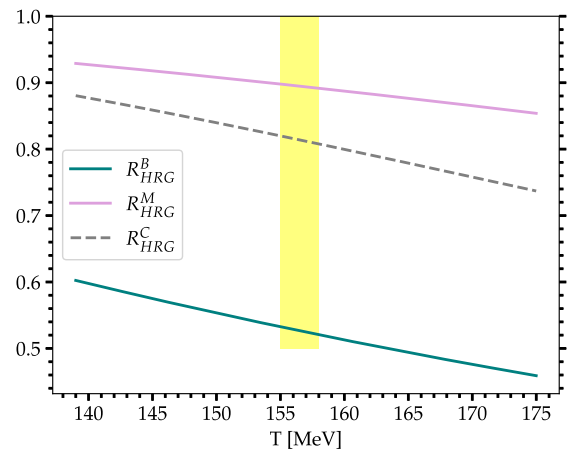


FIG. 1. Ratios of PDG-HRG to QM-HRG partial pressure predictions of charmed baryons and mesons, and the total charm pressure as functions of temperature. The yellow band represents T_{pc} with its uncertainty. See Eqs. (8) and (9) for the definitions of $R_{\text{HRG}}^{B/M/C}$.

temperature dependent ratio, $R_{\text{HRG}}^{B/M}$, separately for baryons and mesons, as well as the partial charm pressure ratio R_{HRG}^C ,

$$R_{\text{HRG}}^X(T) = \frac{P_X^C(T)_{\text{PDG-HRG}}}{P_X^C(T)_{\text{QM-HRG}}}, \quad X = B, M \quad (8)$$

$$R_{\text{HRG}}^C(T) = \frac{P_C(T)_{\text{PDG-HRG}}}{P_C(T)_{\text{QM-HRG}}}. \quad (9)$$

The ratio $R_{\text{HRG}}^B(T)$ is around 0.6 at the lowest temperature, and decreases as the temperature increases. This is because with increasing temperature, the partial pressure contribution of the experimentally undetected heavier resonances, which contribute in QM-HRG calculations, gain in importance at higher temperatures. At T_{pc} , R_{HRG}^B is around 0.52. As we will show in the next sections QM-HRG calculations are, in fact, in good agreement with lattice QCD results at and below T_{pc} . This implies that at T_{pc} , the experimentally known charmed baryons contribute only 52% to the total charmed baryon pressure of QCD, and a contribution of 48% arises from the not-yet-discovered charmed baryonic states. On the other hand, for the charmed mesonic sector, at T_{pc} , experimentally unobserved states contribute around 12%. As can be seen in Fig. 1, the difference between QM-HRG and PDG-HRG partial pressure predictions is much larger in the charmed baryon than in the charmed meson sector. This is due to the fact that there are many more missing charmed baryonic states than missing charmed mesonic states as pointed out already in a previous lattice QCD study [10]. In recent years, the experimental discovery of various Λ_C and Ω_C resonances, in particular by LHCb and Belle Collaborations [41–43] have confirmed this. As can be seen also in Fig. 1, the ratio for the partial charm pressure, R_{HRG}^C , is closer to R_{HRG}^M reflecting that the partial charm pressure is dominated by the charmed mesonic contributions. At T_{pc} we find $P_B^C(T_{pc}) = 0.2843 P_M^C(T_{pc})$.

Recently, the analysis of experimentally measured open charmed hadron yields using the extended SHMc [44], showed enhanced yields in the charmed baryon sector [21]. This enhancement is relative to the experimentally known charmed hadronic states tabulated in the PDG records, and therefore corroborates the existence of missing resonances in the open charm sector.

The fact that differences between QM-HRG and PDG-HRG show up predominantly in the charmed baryon sector also is apparent in cumulants for correlations between electric-charge and charm fluctuations. As states with $|Q| > 1$ show up only in the baryon sector, cumulants involving higher order electric charge fluctuations show larger differences between QM-HRG and PDG-HRG calculations. This is evident from Fig. 2 where we show HRG model calculations for the ratios χ_{mn}^{QC}/χ_4^C , with

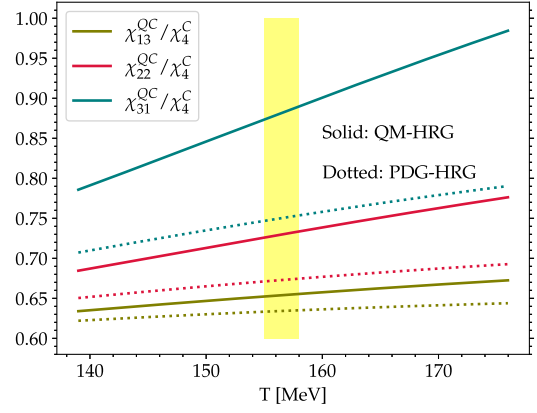


FIG. 2. Comparison of QM-HRG and PDG-HRG predictions for the ratio of generalized susceptibilities χ_{mn}^{QC}/χ_4^C . The solid (dotted) lines represent QM-HRG (PDG-HRG) prediction. The yellow band represents T_{pc} with its uncertainty.

$m = 1, 2, 3$ and $n = 4 - m$. While the ratio for $m = 1$ is dominated by contributions from charmed mesons, the baryon contribution is enhanced by a factor of 2^m for larger m leading to larger differences between QM-HRG and PDG-HRG calculations.

In Fig. 3 we show PDG-HRG and QM-HRG predictions for two strangeness-charm correlations normalized by χ_4^C . The effect of the missing states appears to be very small for χ_{22}^{SC}/χ_4^C , while it is significant for χ_{13}^{SC}/χ_4^C . The reason for this is that the missing charmed mesons and baryons contribute to both the numerator and denominator of χ_{22}^{SC}/χ_4^C , and the effect of the missing states cancels out in the ratio. However, this cancellation is coincidental for the QM-HRG and PDG-HRG lists used in this work. In other words, if in nature the spectrum of strange charmed hadrons, including the spectrum of $|S| = 2$ charmed baryons, is different from that tabulated in the current QM-HRG list, χ_{22}^{SC}/χ_4^C will be sensitive to those states and show deviations from the PDG-HRG prediction. On the other

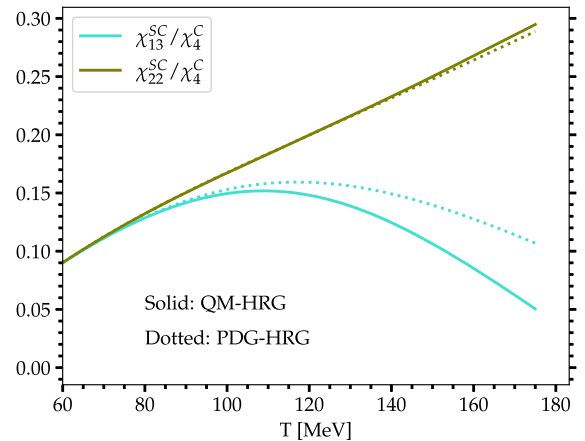


FIG. 3. Shown are the QM-HRG predictions for the two SC correlations normalized by χ_4^C .

TABLE I. The lattice gauge coupling $\beta = 10/g_0^2$, the corresponding temperature values and the bare charm quark masses for LCP_[a] and LCP_[b].

N_τ	β	T [MeV]	LCP _[a]		LCP _[b]	
			am_c	Statistics	am_c	Statistics
8	6.315	145.1	1.04112	449,689	0.892231	448,894
8	6.354	151.1	0.97025	519,953	0.857304	519,812
8	6.390	156.9	0.91534	406,878	0.816144	1,038,000
8	6.423	162.4	0.87069	661,254	0.787450	662,607
8	6.445	166.1	0.84320	505,573	0.765223	522,688
8	6.474	171.2	0.80920	229,013	0.742996	232,114
8	6.500	175.8	0.78059	...	0.723946	151,478
12	6.712	145.40	0.59316	49,589	0.574711	49,591
12	6.754	151.62	0.56328	43,367	0.549310	43,368
12	6.794	157.75	0.53656	50,352	0.530258	50,353
12	6.825	162.65	0.51694	58,547	0.511207	58,547
12	6.850	166.69	0.50176	36,801	0.498506	36,803
12	6.880	171.65	0.48426	36,076	0.485805	36,078
12	6.910	176.73	0.46751	39,079	0.469929	39,080
16	7.010	145.95	0.41657	13,546	0.419126	13,554
16	7.054	152.19	0.39631	13,391	0.409601	13,390
16	7.095	158.21	0.37849	14,050	0.393725	14,048
16	7.130	163.50	0.36405	6779	0.377849	6807
16	7.156	167.53	0.35375	7282	0.368323	7308
16	7.188	172.60	0.34155	7192	0.358798	7192
16	7.220	177.80	0.32985	3515	0.349272	...

hand, charmed mesons and baryons contribute to χ_{13}^{SC} with opposite signs; therefore, no cancellation between the numerator and denominator takes place for χ_{13}^{SC}/χ_4^C . At low temperature values, only charmed mesons contribute to χ_{nm}^{SC} , and therefore, at these temperatures all strangeness-charm correlations should be the same. We see from Fig. 3 that this is indeed the case.

We close this section by noting that the Boltzmann approximation not only holds for charmed hadrons but also is a good approximation for the pressure of charm quarks even at temperatures a few times T_{pc} . The partial pressure, P_q^C , of a gas of charm quarks and antiquarks with mass m_q^C thus may be written as

$$P_q^C(T, \vec{\mu}) = \frac{6}{\pi^2} \left(\frac{m_q^C}{T} \right)^2 K_2(m_q^C/T);$$

$$\cosh \left(\frac{2}{3} \hat{\mu}_Q + \frac{1}{3} \hat{\mu}_B + \hat{\mu}_C \right). \quad (10)$$

III. LATTICE SETUP AND LINES OF CONSTANT PHYSICS

A. Calculation of charmed susceptibilities on lines of constant physics

For our calculation of generalized susceptibilities involving charm quantum numbers we make use of gauge field

configurations generated by the HotQCD Collaboration in $(2+1)$ -flavor QCD with a physical strange to light quark mass ratio, $m_s/m_l = 27$ [17]. These datasets have been generated using the $(2+1)$ -flavor QCD HISQ (highly improved staggered quark) action and a Symanzik-improved gauge action. The temperature scale was set using the f_K -scale [17].

In our calculations the charm quark sector is treated in the quenched approximation. This approximation is well justified within the temperature range explored in this work, as a comparison between the $(2+1)$ -flavor QCD equation of state (EOS) with quenched charm and the $(2+1+1)$ -flavor QCD EOS including dynamical charm shows that dynamical charm effects become significant only for $T > 300$ MeV [45]. We use the HISQ action with the so-called epsilon term for charm quarks to remove $\mathcal{O}((am_c)^4)$ tree-level lattice artifacts [46]. This setup is known to have small discretization effects for charm quark related observables [46,47]. Table I shows the bare gauge coupling $\beta = 10/g_0^2$ and the number of gauge field configurations utilized for each temperature at three different temporal lattice extents, $N_\tau = 8, 12$, and 16. The analyzed gauge configurations have a fixed aspect ratio, $N_\sigma/N_\tau = 4$, where N_σ is the spatial lattice extent. This setup has been used by us previously for the calculation of partial pressures in the charmed sector of QCD [22].

To calculate the generalized charm susceptibilities we need to fix the input charm quark mass in lattice units, am_c ,

for each lattice spacing, such that some physical quantities depending on charm quark mass and expressed in appropriate units of f_K are fixed to their physical values even at finite values of the lattice cutoff. This defines the lines of constant physics (LCPs). In our previous analysis of charmed susceptibilities we used two different LCPs. On the one hand we used a LCP obtained by keeping the spin-averaged charmonium mass, $(3m_{J/\psi} + m_{\eta_{cc}})/4$, fixed to its physical value [26]. Furthermore, we considered also a LCP defined by the physical (PDG) charm to strange quark mass ratio, $m_c/m_s = 11.76$ [35], where bare strange mass values are taken from Ref. [17]. In the following, the results based on the above two LCPs will henceforth contain subscripts “a” and “b,” respectively. The values of the bare charm quark masses corresponding to these two LCPs are also given in Table I.

Comparing the results obtained using different LCPs for the charm quark mass helps one to understand the cutoff effects in the calculations of the generalized charm susceptibilities. For example, it is expected that cutoff effects are large in calculations of thermodynamic observables dominated by charm quark contributions as the quark mass is 1 order of magnitude larger than the temperatures of interest. Thus even small changes in the quark masses may lead to large changes in the relevant Boltzmann weights controlling the contribution of charmed hadrons in thermodynamic calculations of, e.g., absolute values of generalized susceptibilities. This will, in fact, become clear from our discussion in subsequent sections. For this reason previous analyses of charmed susceptibilities focused on the calculation of ratios of susceptibilities, where such cutoff effects are canceled to a large extent.

As we want to arrive here at quantitative results for absolute values of continuum extrapolated charmed susceptibilities which at low temperatures are dominated by contributions from open charmed mesons, we constructed a third LCP, which aims at reducing cutoff effects in these charmed hadron degrees of freedom. We thus will discuss in the next subsection the construction of a LCP obtained by keeping the lowest D -meson mass constant.

B. Line of constant physics from D -meson mass calculations

In order to minimize cutoff effects that can arise in thermodynamics of charmed hadrons from taste breaking lattice artifacts at low temperature—the hadronic phase of QCD—we construct a LCP obtained by demanding that the mass of the lightest charmed hadron, i.e., D -meson, attains its physical value already at nonzero values of the lattice spacing. We calculated the lightest D -meson mass at a few β values, with at least one β value belonging to the parameter range of relevance for thermodynamic calculations at each of the three N_τ values used in this work, $N_\tau = 8, 12$ and 16 . We performed calculations with bare charm quark mass values am_c used in previous calculations

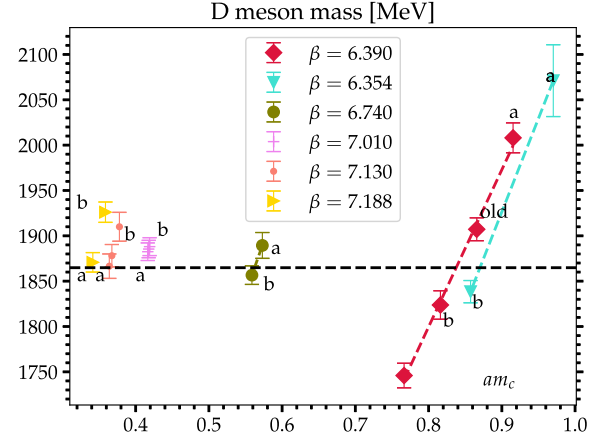


FIG. 4. The mass of the D -meson calculated at several values of the gauge coupling β and various values of the bare charm quark mass am_c on LCP_[a], LCP_[b] and an older LCP introduced in Ref. [10]. The dashed black line represents the physical D -meson mass = 1864.84 MeV.

for the construction of LCP_[a] and LCP_[b] in the range $\beta \leq 6.5$ [22] and constructed these LCPs also for larger β -values relevant for calculations on $N_\tau = 12$ and 16 . The resulting bare charm quark masses are given in Table I. We also used the charm quark mass values used for the LCP in Ref. [10]. We selected six β -values in the range $\beta \in [6.39 : 7.188]$ at which we calculated the D -meson mass. For this purpose we used $(2 + 1)$ -flavor zero temperature lattices of size 32^4 for $\beta = 6.354$ and 6.390 , 48^4 lattices for $\beta = 6.740$, and 64^4 lattices for $\beta \geq 7.010$. At $\beta = 6.39$, $\beta = 7.010$ and $\beta = 7.13$ we also performed calculations with an additional choice for am_c . Results for the D -meson mass obtained at these six different β -values and the various charm quark masses are shown in Fig. 4. The lowest two β values shown in this figure are relevant for $N_\tau = 8$, whereas the highest three are relevant for $N_\tau = 16$, and $\beta = 6.740$ is used in the $N_\tau = 12$ calculation.

For each β value shown in Fig. 4, we fitted the available D -meson masses as linear functions of am_c , and obtained am_c values corresponding to the physical D -meson mass. The resulting six am_c/af_K values, corresponding to physical D -meson mass, are shown for a comparison with other LCPs in Fig. 5, and are labeled as LCP_[D]. The errors on these six am_c values represent the standard deviation of three different linear fits: 1) all D -meson masses are moved up 1 sigma, 2) all D -meson masses are moved down 1 sigma, 3) fitting the original (nondisplaced) D -meson masses. Finally, we fitted these six am_c values using a renormalization group inspired ansatz,

$$am_c(\beta) = af_K(\beta) \frac{M_c^{\text{RGI}}}{f_K} \left(\frac{20b_0}{\beta} \right)^{\frac{4}{3}} \left[\frac{1 + \frac{10}{\beta} m_{1c} f^2(\beta)}{1 + \frac{10}{\beta} dm_{1c} f^2(\beta)} \right], \quad (11)$$

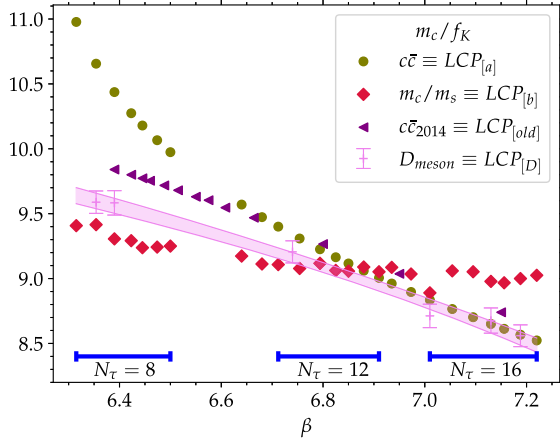


FIG. 5. Shown are the bare charm quark mass values normalized by af_K as a function of the inverse gauge coupling, β , tuned using four different criteria to define lines of constant physics: $LCP_{[a]}$, $LCP_{[b]}$ (see text), the LCP prescription used in Ref. [10], and $LCP_{[D]}$ constructed from Fig. 4 (see text). The band shows the bootstrap error of $LCP_{[D]}$ values. The blue lines explicitly show the ranges of gauge couplings β relevant for three different temporal lattice extents used in this work.

where $af_K(\beta)$ denotes the parametrization of the kaon decay constant times lattice spacing (af_K) given in [17] and $f(\beta)$ is the two-loop QCD β -function with $b_0 = 9/(16\pi^2)$ being its one-loop perturbative expansion coefficient in three-flavor QCD. We use $M_c^{\text{RG}} = 1.528$ GeV and $f_K = (155.7/\sqrt{2})$ MeV taken from Eqs. (58) and (80) of Ref. [48], respectively. The fit resulted in

$$\begin{aligned} m_{1c} &= 140585.579 \pm 16567.799, \\ dm_{1c} &= 92506.328 \pm 11434.443, \end{aligned} \quad (12)$$

with a $\chi^2/\text{dof} = 0.414$. In Fig. 5, the band represents the bootstrap error of fits performed on 100 Gaussian distributed fake samples using Eq. (11).

In addition to the $LCP_{[D]}$, shown in Fig. 5 as band, we show there also the values for m_c/f_K for $LCP_{[a]}$ and $LCP_{[b]}$ and the “old” LCP taken from Ref. [10], which is also based on the spin-averaged charmonium mass calculations. We note that the LCPs based on charmonium mass calculations and D -meson mass calculations start agreeing with each other in the continuum limit, i.e., for large values of β . For $\beta \geq 6.74$ differences between charm quark masses on $LCP_{[a]}$ and $LCP_{[D]}$ are less than 2% leading also to deviations of the D -meson mass on $LCP_{[a]}$ from the physical value that are smaller than 2%. The difference between $LCP_{[a]}$ and $LCP_{[D]}$ in the β range relevant for $N_\tau = 8$ lattices is due to the taste symmetry breaking associated with the staggered fermion formulation. Therefore, if the charm quark mass is fixed through charmonium mass at a given lattice spacing then the

D -meson mass will be larger than in the continuum limit and vice versa; see Ref. [12]. There are also discretization errors due to relatively large values of am_c ; however, these are under control, due to the use of HISQ action, for which these effects start at order $(am_c)^4$. The masses of χ_c and D_s mesons, that are not affected by taste symmetry breaking a lot, turned out to agree with the experimental values within errors (see, e.g., Ref. [49]), indicating that the discretization effects proportional to powers of am_c are small.

As can be seen in Fig. 5, $LCP_{[b]}$ also approaches $LCP_{[D]}$ with increasing β . However, for $\beta \geq 6.9$ $LCP_{[b]}$ results in significantly larger values of the charm quark mass. This is due to the fact the LCP used in $(2+1)$ -flavor QCD calculations with the HISQ action [12,17] had been obtained by keeping the strange $\eta_{s\bar{s}}$ meson mass fixed to a value, which is (2–3)% larger than its physical value. This is discussed in detail in Ref. [12] and resulted in a correspondingly larger strange quark mass, which thus is not suitable for fixing the charm quark mass at large values of β by demanding a physical value for the ratio m_c/m_s . The resulting charm quark mass is too large in this region resulting in too large values for the D -meson mass. This is reflected in the ordering of the D -meson masses calculated with am_c values corresponding to $LCP_{[a]}$ and $LCP_{[b]}$. As can be seen in Fig. 4, for $\beta \gtrsim 7.0$ the D -meson masses using as input the am_c values of $LCP_{[b]}$ start being larger than those based on input from $LCP_{[a]}$.

In the β range relevant for $N_\tau > 12$, $LCP_{[a]}$ is effectively the $LCP_{[D]}$. Since the generalized susceptibilities receive the most dominant contribution from the lightest charmed hadron, we use $LCP_{[D]}$ results to take the continuum limit.

IV. LATTICE QCD CALCULATIONS OF GENERALIZED CHARM SUSCEPTIBILITIES

A. Quartic charm fluctuations on $LCP_{[D]}$

In the previous section we have obtained four different LCPs. On three of them various charm susceptibilities have been calculated directly. On $LCP_{[D]}$ this is not the case as its construction is based on the calculation of charmed meson masses at only six values of the gauge couplings and subsequent fits to the resulting $am_c(\beta)$ values given in Eqs. (11) and (12). We now could go ahead and perform a direct calculation of charm cumulants at these bare charm quark mass values. However, in order to gain further insight into the sensitivity of quark mass and cutoff effects on the charmed cumulants we discuss here their determination through interpolation of data obtained on the other three LCPs.

At each temperature value given in Table I, we made use of the available bare charm masses to interpolate $\ln(\chi_4^C)$ as a linear function of am_c . This linear interpolation enabled us to eventually obtain χ_4^C as a function of temperature on $LCP_{[D]}$ using am_c parametrization defined in Eq. (11).

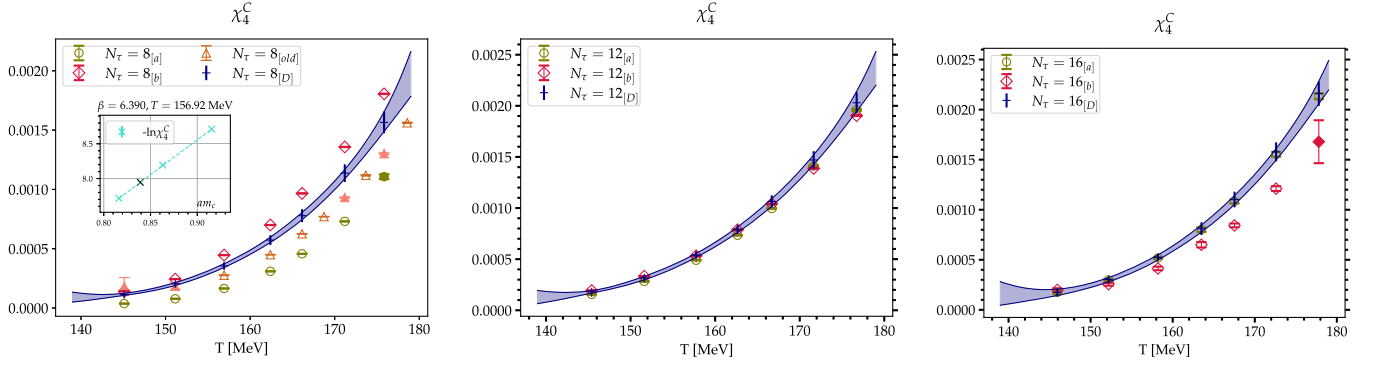


FIG. 6. Shown are the charm pressures constructed for $\text{LCP}_{[D]}$ as functions of temperature for $N_\tau = 8, 12, 16$ (see text). The solid points and the bands shown are results of $[2, 2]$ Padé interpolations. The inset in the (left) figure shows linear interpolation of $-\ln \chi_4^C$ in am_c at $\beta = 6.390$, and the black cross represents $-\ln \chi_4^C$ on $\text{LCP}_{[D]}$.

For $N_\tau = 8$, we have at each temperature three values of $am_c(\beta)$ corresponding to $\text{LCP}_{[a]}$, $\text{LCP}_{[b]}$ and $\text{LCP}_{[\text{old}]}$, respectively. The quartic charm fluctuation cumulant, χ_4^C , for these three values of am_c are shown in Fig. 6 (left). The inset in this figure shows $-\ln(\chi_4^C)$ for one value of T . As can be seen a straight line interpolation is very well justified. As expected χ_4^C receives its dominant contribution from the lightest charmed hadron or charm quark, respectively. In both cases these thermal excitations are exponentially suppressed, and the exponent is given in terms of a thermal mass that is linearly dependent on am_c , as shown in Fig. 4. We thus use linear interpolations of results for $-\ln(\chi_4^C)$ to determine χ_4^C values on $\text{LCP}_{[D]}$. Results for χ_4^C are shown in Fig. 6 (left) as blue plus markers. The errors shown for χ_4^C on $\text{LCP}_{[D]}$, incorporate not only the bootstrap error arising from the linear fitting process but also the propagated error due to the uncertainties in the am_c values of $\text{LCP}_{[D]}$. The propagated error at a given temperature is calculated as $\Delta am_{c[D]}(\chi_{4[D]}^C - \chi_{4[a]}^C)/(am_{c[a]} - am_{c[b]})$, where $\Delta am_{c[D]}$ is obtained from the band shown in Fig. 5. In Fig. 6 (left), the band for $\text{LCP}_{[D]}$ is constructed by bootstrapping over 100 fake Gaussian samples, and for each sample a $[2, 2]$ Padé was used as fit function. We also used a $[2, 2]$ Padé to interpolate in temperature and calculate the bootstrap error of χ_4^C at temperatures where data were missing for any of the three LCPs. We repeated the same procedure for $N_\tau = 12$ and 16, where data for only two LCPs are available, as is shown in Fig. 6 (middle) and (right). However, as can be seen in Fig. 5, at the lowest temperature of $N_\tau = 16$, for both $\text{LCP}_{[a]}$ and $\text{LCP}_{[b]}$ the charmed quark masses determined for these two LCPs are almost identical, which made a meaningful interpolation difficult. At this lowest temperature we used the $\text{LCP}_{[a]}$ result to estimate χ_4^C corresponding to $\text{LCP}_{[D]}$ in Fig. 6 (right).

In Fig. 7, we present χ_4^C calculated on $\text{LCP}_{[D]}$ for $N_\tau = 8, 12$ and 16. These values, denoted as $\chi_4^{C,\text{lat}}$, are normalized

by the QM-HRG value, $\chi_4^{C,\text{QM-HRG}}$. As can be seen, $\text{LCP}_{[D]}$ bands for $N_\tau = 12$ and 16 overlap. Therefore, we use $N_\tau = 16_{[D]}$ as our continuum estimate. We also see from the figure that our continuum estimate for χ_4^C agrees within errors with the QM-HRG result for this cumulant in the entire temperature range considered in this study.

The fact that HRG model calculations seem to describe the lattice results above T_{pc} may appear somewhat unexpected. We think this is accidental. Both quark gas and hadron gas show a qualitatively similar exponential increase of χ_4^C with increasing temperature since charm quark mass is not too different from the hadron masses. Given that we only have a crossover and not a phase transition, χ_4^C may not be very sensitive to the change in the degrees of freedom. In the next section, we consider

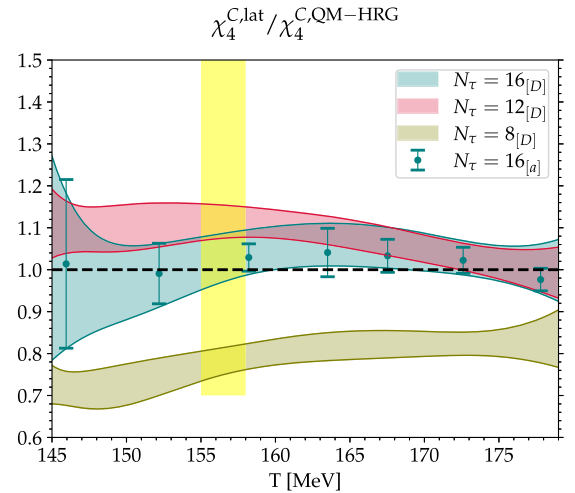


FIG. 7. Shown are χ_4^C values for $\text{LCP}_{[D]}$ normalized by the respective QM-HRG predictions as functions of temperature for three different temporal lattice extents. χ_4^C data points for $N_\tau = 16$ on $\text{LCP}_{[a]}$ are also shown. The dashed black line represents unity. The yellow band represents T_{pc} with its uncertainty. For the description of χ_4^C bands, see text.

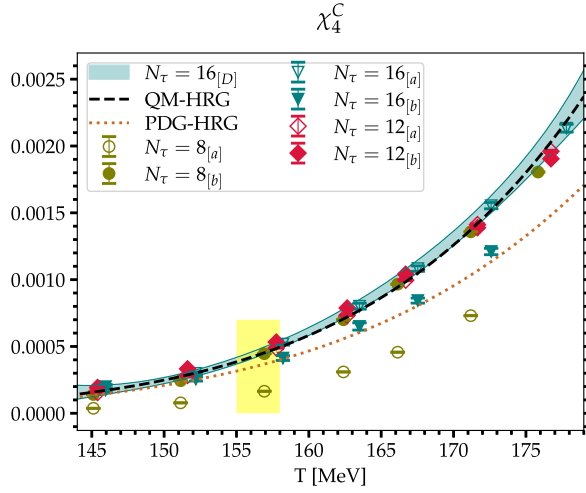


FIG. 8. Lattice QCD results for χ_4^C for different values of the temporal lattice extents calculated on the two different LCPs, i.e., $LCP_{[a]}$ and $LCP_{[b]}$ respectively. Also shown are the QM-HRG and the PDG-HRG predictions. The teal band shows χ_4^C on $LCP_{[D]}$ for $N_\tau = 16$, which also represents our continuum estimate of χ_4^C . The yellow band represents T_{pc} with its uncertainty.

different ratios of generalized susceptibilities, some of which are very sensitive to the change in charm degrees of freedom across T_{pc} . We will show there that sensitivity to the change in the charm degrees of freedom depends on the value of these ratios in the high temperature limit.

B. Lattice cutoff effects in generalized charm susceptibilities

In Fig. 8 we show results for χ_4^C obtained by us at all bare charm quark mass values corresponding to $LCP_{[a]}$, $LCP_{[b]}$ and the continuum estimate for χ_4^C on $LCP_{[D]}$. As can be seen the results obtained with quark masses $am_c(\beta)$, corresponding to $LCP_{[b]}$ and $LCP_{[D]}$ in the parameter range relevant on $N_\tau = 8$ lattices agree well, while the results on $LCP_{[a]}$ differ quite significantly. This discrepancy just reflects the large difference in quark mass values corresponding to $LCP_{[a]}$ and $LCP_{[D]}$, respectively. This clearly reflects the importance of cutoff effects arising from different physical criteria used to define LCPs on such coarse lattice, i.e., fixing the physical value of the charmonium mass ($LCP_{[a]}$) versus fixing the lightest D -meson mass ($LCP_{[D]}$). As can be seen, these differences are gone for values of $am_c(\beta)$ in the parameter range relevant on $N_\tau = 12$ lattices. All resulting χ_4^C values agree within errors despite the fact that the input values for $am_c(\beta)$ at a given temperature value differ slightly, i.e., by about 3%.

Larger differences between results obtained at quark mass values corresponding to $LCP_{[b]}$ and the two other LCPs show up again on $N_\tau = 16$ lattices. These however, can be traced back to discrepancies arising from the strange

quark mass tuning for $\beta \geq 6.9$ as discussed in the previous section.

From the comparison of results obtained with different values of the bare charm quark masses shown in Fig. 8 we conclude that input quark masses corresponding to $LCP_{[b]}$ yield results for χ_4^C that agree with results obtained on $LCP_{[D]}$ in the parameter range relevant for calculations on $N_\tau = 8$ and 12 lattices. In the discussion of other quartic charm cumulants we thus will use results for these cumulants directly obtained with bare quark masses corresponding to $LCP_{[b]}$, as given in Table I, rather than performing interpolations to obtain results on $LCP_{[D]}$ as we have done for χ_4^C .

V. RATIO OF GENERALIZED SUSCEPTIBILITIES ACROSS THE CHIRAL CROSSOVER

In this section, we discuss ratios of off diagonal charm susceptibilities to χ_4^C as functions of temperature across the chiral crossover using results from calculations on $N_\tau = 8$ and $N_\tau = 12$ lattices. For $T < 177$ MeV we will use $am_c(\beta)$ parameters corresponding to $LCP_{[b]}$, while for larger temperatures, we will use $N_\tau = 8$ results for these quantities calculated on $LCP_{[old]}$ [10].

Analyzing the ratios of generalized susceptibilities offers the advantage of significantly reducing sensitivity to the choice of LCP [22], and as we will see later, discretization errors become insignificant. The latter is important because for many generalized susceptibilities the lattice QCD results for $N_\tau = 12$ and $N_\tau = 16$ are too noisy.

In our previous study we only considered χ_{13}^{BC}/χ_4^C [22]. We will extend this study here by investigating further cross-correlations which allows us to become more sensitive to different quantum number channels correlated with charm degrees of freedom. We start our discussion with the ratios of baryon number charm correlations to the quartic charm fluctuation, χ_{mn}^{BC}/χ_4^C . The corresponding lattice QCD results are shown in Fig. 9. For $T < T_{pc}$, the ratios χ_{13}^{BC}/χ_4^C and χ_{22}^{BC}/χ_4^C agree with each other as expected based on Eq. (6). Furthermore, these ratios agree with HRG model calculations based on the QM-HRG particle list but clearly disagree with calculations based on the PDG-HRG list. The ratio χ_{13}^{BC}/χ_4^C agrees with the QM-HRG prediction even for $T > T_{pc}$. This agreement is somewhat accidental and is related to the fact that the high temperature ideal quark gas limit of this ratio is 1/3. The ratio χ_{22}^{BC}/χ_4^C which is expected to approach 1/9 at high temperature (ideal quark gas value) shows clear disagreement with the QM-HRG above T_{pc} . For χ_{13}^{BC}/χ_4^C we also show $N_\tau = 12$ results for a smaller temperature range, [T_{pc} : 177 MeV], calculated on $LCP_{[b]}$. These results agree well with the $N_\tau = 8$ results, showing that the cutoff effects cancel out in the ratios of generalized susceptibilities.

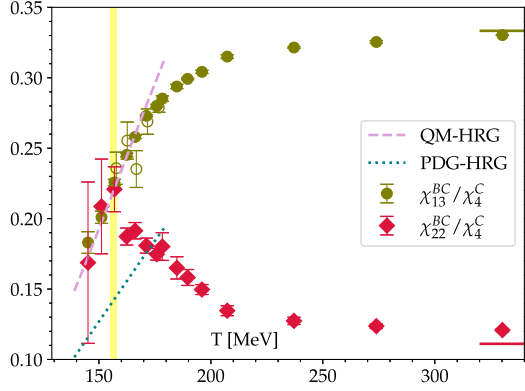


FIG. 9. The ratios χ_{mn}^{BC}/χ_4^C as functions of the temperature compared to QM-HRG and PDG-HRG as well as to the ideal quark gas predictions at high temperatures shown as horizontal lines. Solid markers represent $N_\tau = 8_{[b]}$ results, whereas unfilled markers of the same color represent the respective $N_\tau = 12_{[b]}$ results. Solid markers for $T > 176$ MeV represent $N_\tau = 8_{[old]}$ results taken from Ref. [10]. Solid lines represent noninteracting charm quark gas limits. The vertical yellow band indicates the chiral crossover temperature with the corresponding uncertainty.

We also show ratios of electric charge charm correlations to the quartic charm fluctuation, χ_{mn}^{QC}/χ_4^C , in Fig. 10. As can be seen from Table I, these quantities calculated on the high statistics datasets, enable disentangling contributions to partial charm pressure from different electrically charged charm subsectors i.e., $|Q| = 0, 1, 2$ and $1/3$ in the hadronic and Quark Gluon Plasma (QGP) phases, respectively. The fourth-order QC correlations normalized by the total charm pressure show agreement with the QM-HRG model calculations below T_{pc} for $N_\tau = 8$ lattices. This indicates the existence of experimentally unobserved charmed hadrons carrying $|Q| = 0, 1$ and 2 . We also show $N_\tau = 12$ results for ratios χ_{22}^{QC}/χ_4^C and χ_{13}^{QC}/χ_4^C , and despite being relatively noisy and based on lower statistics, these agree with their $N_\tau = 8$ counterparts calculated on the same LCP. Therefore, similar to the BC sector, the cutoff effects also cancel for ratios of generalized susceptibilities in the QC sector. We also see that χ_{13}^{QC} follows the QM-HRG model even above crossover temperature—again somewhat accidental. The ideal quark gas limit of this quantity is not very different from the corresponding QM-HRG value close to T_{pc} . The other correlations normalized by χ_4^C , i.e., χ_{31}^{QC}/χ_4^C and χ_{22}^{QC}/χ_4^C , have much smaller ideal quark gas limits, and therefore show significant deviations from QM-HRG just above T_{pc} .

Finally let us discuss the ratios involving the strangeness charm correlation. In Fig. 11 we show our results for χ_{13}^{SC}/χ_4^C and χ_{22}^{SC}/χ_4^C . The former ratio agrees well with the QM-HRG but not with PDG-HRG, while the latter is about 10% above the QM-HRG predictions in the vicinity of T_{pc} . As already argued in Sec. II this latter ratio is sensitive to the spectrum of $|S| = 2$ charmed hadrons. The deviations

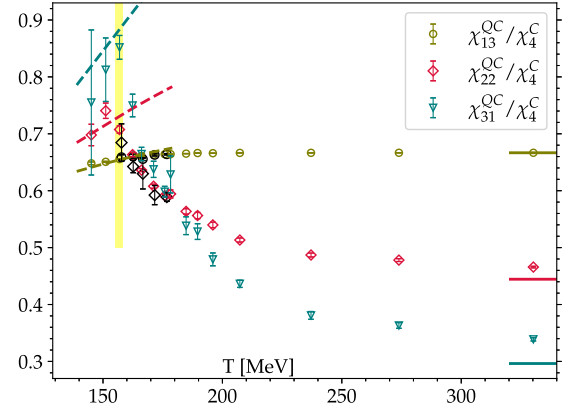


FIG. 10. Lattice QCD results for three different fourth order cumulants of net electric charge and net charm correlations. The cumulants have been normalized using the proxy for the total charm pressure, χ_4^C . The unfilled colored markers represent $N_\tau = 8_{[b]}$ results, whereas the black markers of the same style represent the respective $N_\tau = 12_{[b]}$ results. The unfilled colored markers for $T > 176$ MeV represent $N_\tau = 8_{[old]}$ results taken from Ref. [10]. The corresponding QM-HRG predictions are shown with the dashed lines of the same colors, whereas the horizontal solid lines on the left-hand side represent the respective ideal charm quark gas limits. The yellow bands represent T_{pc} with its uncertainty.

of χ_{22}^{SC}/χ_4^C from HRG can be due to the imprecise or incomplete knowledge of $|S| = 2$ baryon spectrum. By using Eq. (5), it can be easily deduced that unlike $|S| = 1$ charmed hadrons, $|S| = 2$ hadrons contribute with a higher absolute weight to χ_{22}^{SC} in comparison to χ_{13}^{SC} :

$$\begin{aligned}\chi_{22}^{SC} &= P_M^{C,S=1} + P_B^{C,S=1} + 4P_B^{C,S=2}, \\ \chi_{13}^{SC} &= P_M^{C,S=1} - P_B^{C,S=1} - 2P_B^{C,S=2},\end{aligned}\quad (13)$$

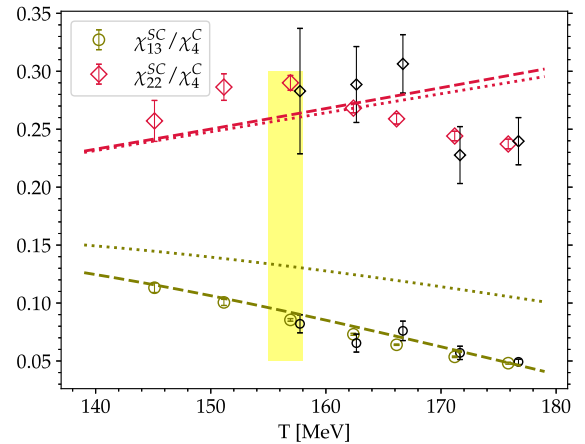


FIG. 11. Shown are the two SC correlations normalized by χ_4^C . The dashed (dotted) lines with the same color as the markers represent the respective QM-HRG (PDG-HRG) predictions. The black points with the same symbols as the $N_\tau = 8_{[b]}$ results, represent the respective $N_\tau = 12_{[b]}$ data.

where $P_M^{C,S=1}$ is the partial pressure for strange charmed mesons, $P_B^{C,S=1}$ is the partial pressure for strange charmed baryons with strangeness one, and $P_M^{C,S=2}$ is the partial pressure of strange charmed baryons with strangeness two. The deviations observed between QM-HRG calculations and our data for the ratio χ_{22}^{SC}/χ_4^C may be due to missing $|S|=2$ charmed baryons not even calculated in quark models so far. Moreover, in our QM-HRG list we use masses obtained in the relativistic quark model calculations [37]. Nonrelativistic quark models [39,40], however, often give predictions for the masses of strange charmed baryons that are up to 100 MeV lower than the prediction of the relativistic quark model. Therefore, we also estimated the ratio χ_{22}^{SC}/χ_4^C using a QM-HRG model, where all the missing $|S|=2$ charmed baryon masses are shifted down by 100 MeV. We find that this version of QM-HRG shows much smaller deviations from the lattice QCD results for χ_{22}^{SC}/χ_4^C but still agrees with the lattice QCD results for χ_{13}^{SC}/χ_4^C . Lastly, we also see that the $N_\tau = 8_{[b]}$ and $N_\tau = 12_{[b]}$ data for these ratios agree within errors.

To summarize this section: we see that except for χ_{22}^{SC}/χ_4^C , various ratios of generalized susceptibilities below T_{pc} agree well with the corresponding values obtained in QM-HRG and are significantly above the ones obtained in PDG-HRG. Thus our lattice QCD results provide clear evidence for missing charmed hadrons, especially charmed baryons. The QM-HRG underpredicts χ_{22}^{SC}/χ_4^C , possibly indicating uncertainties in the spectrum of the doubly strange charmed hadrons in the current QM-HRG list. While this is not always evident, the QM-HRG description breaks down just above T_{pc} signaling the appearance of new degrees of freedom. Finally, the cutoff effects largely cancel out in the ratios of the generalized susceptibilities.

VI. PARTIAL CHARM PRESSURES AND THE NATURE OF CHARM DEGREES OF FREEDOM

A. Quasiparticle description

The breakdown of QM-HRG for several quantities above T_{pc} signals the possible appearance of new degrees of freedom. In Ref. [11] a quasiparticle model has been proposed in which the charm pressure is written in terms of the partial pressures of noninteracting charm quarks, charmed mesons and charmed baryons:

$$P_C(T, \vec{\mu}) = P_M^C(T, \vec{\mu}) + P_B^C(T, \vec{\mu}) + P_q^C(T, \vec{\mu}). \quad (14)$$

The above model simply extends the HRG relation introduced in Eq. (3) to incorporate charm quark pressure given by Eq. (10). In this model as well, χ_4^C is a proxy for the total charm pressure. The proxies of charm quark, charmed meson and charmed baryon partial pressures at $\vec{\mu} = 0$ can be expressed in terms of the generalized susceptibilities as [11]

$$P_q^C = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2, \quad (15)$$

$$P_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2, \quad (16)$$

$$P_M^C = \chi_4^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC}. \quad (17)$$

Equation (15) is not the only way to define the pressure from charm quark quasiparticles. As already pointed out, in our previous work [22], independent constructions of operators that project onto observables with quantum numbers of charm quarks were discussed. We showed that three such operators projecting onto states with quantum numbers (i) $|B|=1/3$, $|C|=1$, (ii) $|Q|=2/3$, $|C|=1$, (iii) $|B|=1/3$, $|Q|=2/3$, $|C|=1$, vanish below T_{pc} , and give consistent results at temperatures above T_{pc} . This consistency can only be achieved if the proposed quasiparticle describes the charm thermodynamics in the explored temperature range. Such an approach has been used very successfully to define operators for various ordinary hadron quantum number channels. Here we use the first operator with $|B|=1/3$, $|C|=1$, given in Eq. (15), to extend the existing HRG model (3) to the quasiparticle model in Eq. (14).

The high statistics $N_\tau = 8_{[b]}$ results of the charm quark, charmed meson and charmed baryon partial pressures normalized by χ_4^C has been accurately determined in [22]. It also has been shown that the charm quark partial pressure below T_{pc} is zero within errors, whereas the charmed baryon and meson partial pressures remain to be significant above T_{pc} [22]. The continuum limit of χ_4^C obtained in Sec. IV, enables us to convert fractional contributions of coexisting charm quarks and charmed hadrons to the total charm pressure, calculated in [22], to absolute contributions.

In Fig. 12 we show the continuum estimates of the partial charmed meson and baryon pressures obtained in the quasiparticle model and compare to various HRG model calculations. As expected, QM-HRG describes the partial pressures of charmed mesons and baryons up to the crossover temperature, and PDG-HRG clearly underestimates both pressures. In particular, at T_{pc} , the lattice-QCD result of P_B^C is 1.95(23) times the PDG-HRG prediction of P_B^C , which implies that the partial charmed baryon pressure receives 51(6)% contribution from experimentally known charmed baryons, while the rest comes from not-yet-discovered charmed baryons. On the other hand, such an enhancement factor for P_M^C is 1.13(9), which implies that at the chiral crossover, the partial charmed meson pressure receives 88(7)% contribution from experimentally known charmed mesons. Therefore, as expected, the charmed meson sector is more complete in comparison to the charmed baryon sector. These conclusions align with the prediction shown in Fig. 1.

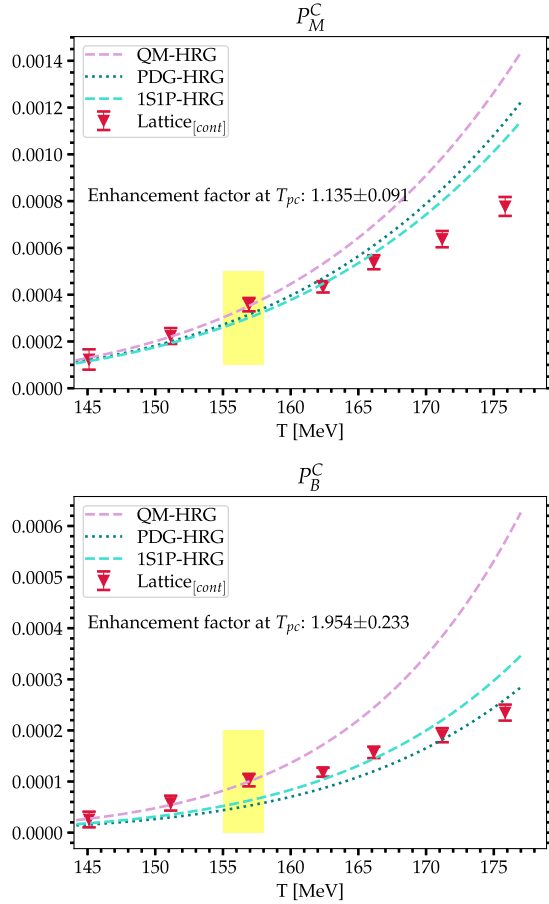


FIG. 12. Shown are the continuum estimates of the charmed meson (top) and charmed baryon (bottom) partial pressures as a function of the temperature across the chiral crossover. We compare the lattice QCD results with various versions of hadron resonance gas models. Enhancement factors are ratios of shown lattice quantities to their respective PDG-HRG counterparts. Note that after adding a few missing isospin partners to the PDG-HRG list, these factors slightly changed in comparison to our previous publication [29].

Above the crossover temperature, QM-HRG overpredicts the lattice results on charmed mesons and baryon pressures. This may imply that many of the charmed hadron states melt above the crossover temperature. To explore this possibility a bit further we constructed a hadron resonance gas model which only contains the ground state charmed mesons and baryons and their parity partners, i.e., the first orbital excitation of these hadrons with vacuum masses. In the language of quark model, this means that we only consider the 1S and 1P mesons and baryons given in Tables I, II and VII of [36]. Therefore, we call this model the 1S1P-HRG model. The 1S1P-HRG model is closer to the lattice results on the charmed meson and charmed baryon pressures above the crossover temperature compared to the QM-HRG model. We note, however, that lattice QCD calculations of charmed meson and charmed baryon correlation functions indicate that around

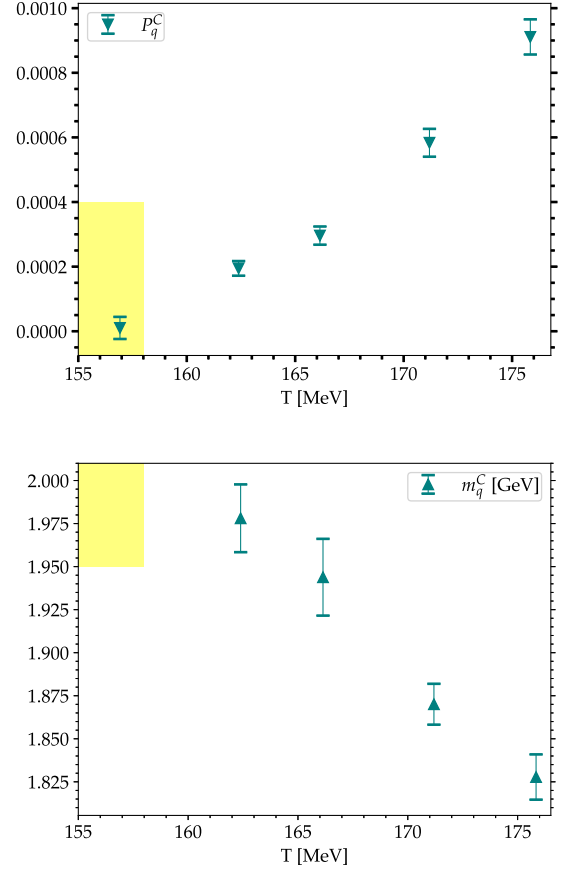


FIG. 13. Shown are the continuum estimates of the partial pressure of charm quarklike excitation above T_{pc} (top) and the temperature dependent in-medium mass of a charm quarklike excitation above T_{pc} (bottom). The yellow bands represent T_{pc} with its uncertainty. Note that m_q^C differs from [28], where we only considered a gas of charm quarks and not quarks-antiquarks.

and above the crossover temperature in-medium modification of charmed hadrons masses could be significant [23,24]. In particular it was shown that the single charmed baryon masses are sensitive to the chiral crossover [24]. Therefore, the 1S1P-HRG model can only serve as a rough guidance on charmed meson and charmed baryon pressure above T_{pc} .

In the coexistence phase of charmed hadron and quarklike excitations, m_q^C in Eq. (10) becomes temperature dependent and can be interpreted as the mass of a quasiparticle with quantum numbers of charm quark. Given $P_q^C(T)$ as a function of temperature, one can solve Eq. (10) at each temperature and obtain temperature dependence of m_q^C . In Fig. 13 (bottom), at each temperature, the error on m_q^C is the standard deviation of m_q^C values obtained after solving Eq. (10) for 50 fake Gaussian samples. Figure 13 (bottom) shows that at $T = 162.4$ MeV, m_q^C is around the mass of the D-meson, and starts decreasing with temperature. The quasiparticle model in Eq. (14) considers a noninteracting gas of charmed hadron and quarklike excitations, but the

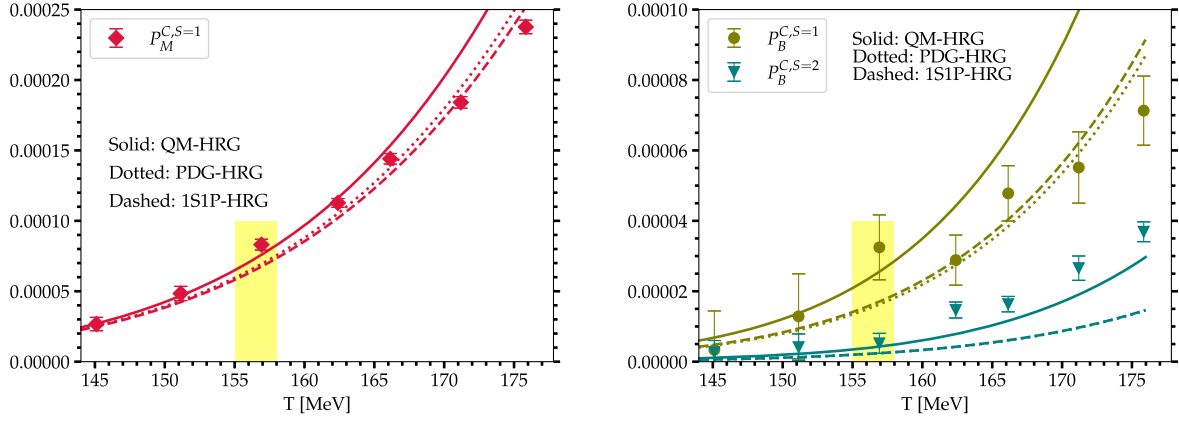


FIG. 14. Shown are the continuum estimates of the strange charmed meson (left) and strange charmed baryon (right) pressures as functions of the temperature across the chiral transition. We compare the lattice QCD results with various versions of the hadron resonance gas model. Dashed, QM-HRG; dotted, PDG-HRG; solid, 1S1P-HRG.

temperature dependence of m_q^C encodes the in-medium interactions. In the noninteracting quark gas limit, m_q^C will become the pole mass of charm quark.

B. Strange charmed degrees of freedom

The charmed meson and charmed baryon pressures can be further decomposed into the partial pressures of mesons and baryons corresponding to different strangeness content. Using lattice QCD results on the generalized susceptibilities involving strangeness, χ_{kmn}^{BSC} one can construct $P_M^{C,S=1}$, $P_B^{C,S=1}$ and $P_M^{C,S=2}$. The following explicit formulas for $P_M^{C,S=1}$, $P_B^{C,S=1}$ and $P_M^{C,S=2}$ in terms of χ_{kmn}^{BSC} are taken from Ref. [11]:

$$P_M^{C,S=1} = \chi_{13}^{SC} - \chi_{112}^{BSC} \quad (18)$$

$$P_B^{C,S=1} = \chi_{13}^{SC} - \chi_{22}^{SC} - 3\chi_{112}^{BSC} \quad (19)$$

$$P_B^{C,S=2} = (2\chi_{112}^{BSC} + \chi_{22}^{SC} - \chi_{13}^{SC})/2. \quad (20)$$

Our continuum estimates for $P_M^{C,S=1}$ ($P_M^{C,S}$), $P_B^{C,S=1}$ and $P_M^{C,S=2}$ are shown in Fig. 14. The corresponding partial pressures normalized by χ_4^C for $N_\tau = 8_{[b]}$ were shown in [28]. We compare continuum estimates for these partial pressures with QM-HRG, PDG-HRG and 1S1P-HRG. These nonvanishing strangeness-charm correlations above T_{pc} support the existence of charmed hadrons in QGP. On the other hand the existence of strange charmed diquarks is not supported by the lattice results on generalized charm susceptibilities [28]. As shown in Fig. 14 (left), the QM-HRG description works well for $P_M^{C,S}$ at low temperatures and breaks down above the crossover temperature. On the other hand, given the current errors, it is difficult to establish disagreement with PDG-HRG and 1S1P-HRG in the strange charmed baryon sector; see Fig. 14 (right).

Above T_{pc} , QM-HRG fails to describe the lattice results for strange charmed hadrons. Interestingly, for $|S| = 2$ baryons, QM-HRG slightly underpredicts the lattice results above T_{pc} . This again may signal an inadequate knowledge of the $|S| = 2$ charmed baryon sector that is used in our QM-HRG list. To explicitly see this, we analyze the continuum estimate of a lesser noisy observable which receives contributions only from the strange charmed baryon sector:

$$\chi_{22}^{SC} - \chi_{13}^{SC} = 2P_B^{C,S=1} + 6P_B^{C,S=2}. \quad (21)$$

Figure 15 shows the continuum estimate of (21) normalized by its QM-HRG counterpart. At T_{pc} , the central value of continuum result differs by 25% from the QM-HRG prediction. If we then lower the masses of the missing $|S| = 2$ charmed baryons by 100 MeV we find that the continuum estimate for this quantity, normalized by the corresponding QM-HRG result, is much closer to unity.

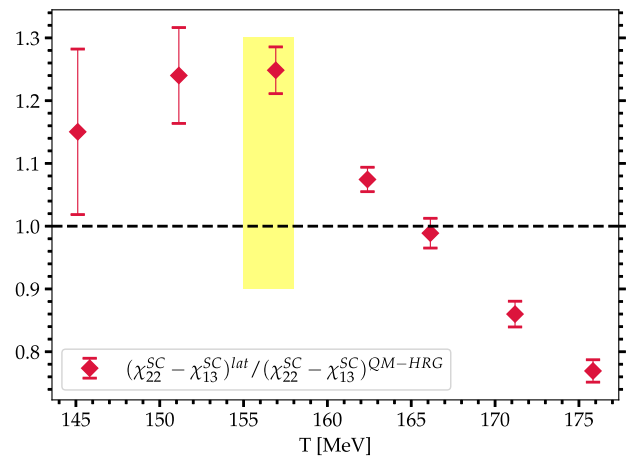


FIG. 15. Shown is the continuum estimate of the proxy for $2P_B^{C,S=1} + 6P_B^{C,S=2}$ normalized by its QM-HRG expectation.

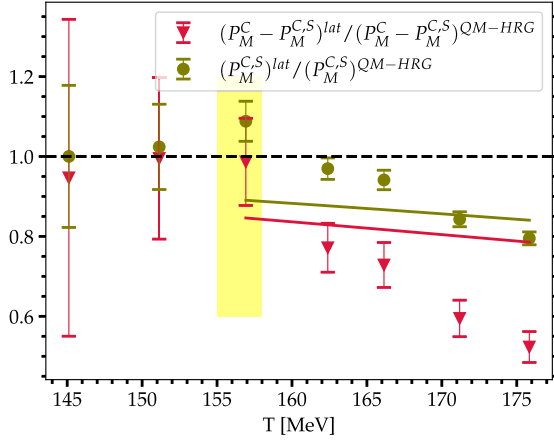


FIG. 16. Shown are the continuum estimates of the lattice results for nonstrange and strange charmed meson pressures normalized by their corresponding QM-HRG counterparts. The solid lines show 1S1P-HRG predictions normalized by the corresponding QM-HRG predictions.

Having estimated the pressure of strange charmed hadrons we can also estimate the partial pressures of nonstrange charmed hadrons, e.g., $P_M^C - P_M^{C,S}$ is the pressure of nonstrange charmed mesons. In Fig. 16, we show the ratios of the partial pressures of strange and nonstrange charmed mesons to their respective counterparts calculated in the QM-HRG model. These ratios equal unity within errors below T_{pc} as expected from Figs. 12 (top) and 14 (left). Above T_{pc} , these ratios are below 1. However, we see that the deviations from unity are significantly larger for nonstrange charmed mesons than for strange charmed mesons. As can be seen, 1S1P-HRG can describe the continuum estimate of nonstrange charmed meson partial pressure for temperatures slightly above T_{pc} , whereas for strange charmed meson partial pressure, it works in a small temperature range above 170 MeV. In Fig. 17 we show similar ratios for strange, $P_B^{C,S} = P_B^{C,S=1} + P_B^{C,S=2}$, and nonstrange, $P_B^C - P_B^{C,S}$, charmed baryons, and again we see larger deviations from unity for nonstrange charmed baryons. This may imply that more strange charmed hadrons can survive above the QCD crossover than nonstrange charmed hadrons. In other words nonstrange charmed hadrons are more susceptible to the chiral crossover. This is a reflection of QM-HRG underpredicting the partial pressures of strange charmed baryons due to additional missing states. In Figs. 16 and 17 we also show the partial pressures of strange and nonstrange charmed hadrons in 1S1P-HRG as solid lines. We see that at the highest two temperatures the 1S1P-HRG lies closer to the lattice data for strange hadron pressures than for nonstrange hadron pressures, i.e., 1S1P-HRG captures the lattice results for the strange hadron pressure better at the highest two temperatures. However, if one could account for missing strange charmed hadrons in 1S1P-HRG, solid

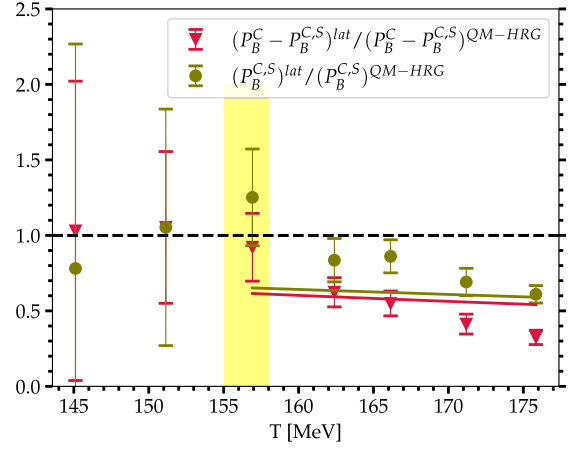


FIG. 17. Shown are the continuum estimates of the lattice results for nonstrange and strange charmed baryon pressures normalized by their corresponding QM-HRG counterparts. The solid lines show 1S1P-HRG predictions normalized by the corresponding QM-HRG predictions.

lines might be able to describe both strange and nonstrange charmed hadron partial pressures in the small temperature range above T_{pc} .

VII. CONCLUSIONS

In this paper we presented a comprehensive study of generalized charm susceptibilities at three values of the lattice spacing with the aim to gain insights into the change of charm degrees of freedom across the chiral crossover. It turned out that a large part of the cutoff effects is associated with the choice of the charm quark mass in the region of the lattice spacings used in our study. We showed that by setting the charm quark mass through the D -meson mass one minimizes the cutoff effects in generalized charm susceptibilities. Furthermore, we found that the ratios of generalized susceptibilities are largely independent of the lattice spacing and the precise value of the charm quark mass. Based on these observations we could provide continuum estimates of the generalized charm susceptibilities.

Above T_{pc} lattice QCD results on generalized charm susceptibilities can be well described by a hadron resonance gas model, which includes the so-called missing hadron states predicted by quark model calculations, the QM-HRG. In a comprehensive study of various generalized charm susceptibilities we also showed that the QM-HRG model breaks down just above T_{pc} , even though this breakdown is not apparent in some of the considered quantities, e.g., χ_4^C .

To explain the temperature dependence of generalized susceptibilities above T_{pc} we used the quasiparticle model first proposed in Ref. [11], in which the charm pressure is written as a sum of partial pressures for charmed mesons, charmed baryons and charm quarks. From the charm quark pressure, which assumes a nonzero value above T_{pc} , we extracted the quasiparticle mass of charm quarks.

The partial charmed meson and baryon pressures drop below the corresponding QM-HRG values for $T > T_{pc}$, indicating that many higher lying charmed hadron states melt above the chiral crossover. Within the quasiparticle model of Ref. [111] we further decomposed the partial pressure of charmed baryons and mesons into the partial pressures of strange and nonstrange charmed baryons, and strange and nonstrange charmed mesons. We found that the partial pressure of strange charmed hadrons shows smaller deviations from QM-HRG above T_{pc} compared to the partial pressures of nonstrange charmed hadrons. This may indicate a different melting pattern of strange and nonstrange charmed hadrons. We also argued that χ_{22}^{SC} is very sensitive to both the number and the masses of missing strangeness two charmed baryons, and that the masses of these baryons could be up to 100 MeV smaller than predicted by the quark model of Ref. [37].

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DATA AVAILABILITY

All data from our calculations, presented in the figures of this paper, can be found in Ref. [33].

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