

Proton and neutron electromagnetic form factors in the continuum limit using lattice QCD ensembles with physical pion masses

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We compute the electromagnetic form factors of the proton and neutron using lattice QCD. We employ $N_f = 2 + 1 + 1$ twisted mass clover-improved fermions with quark masses tuned to their physical values. Three ensembles with lattice spacings of $a = 0.080$ fm, 0.068 fm, and 0.057 fm and approximately the same physical volume allow us to obtain the continuum limit directly at the physical pion mass. For each ensemble, we use several values of the sink-source time separation, ranging from 0.5 to 1.5 fm, to allow for a thorough analysis of excited-state effects via multistate fits. The disconnected contributions are also analyzed using high statistics combined with techniques to mitigate stochastic noise in the estimation of the fermion loop. These techniques include low-mode deflation, dilution in the color and spin components, and hierarchical probing. We study the momentum-transfer dependence of the form factors using the z -expansion and dipole Ansätze, thereby enabling the extraction of the electric and magnetic radii and the magnetic moments, as well as the Zemach and Friar radii in the continuum limit. Results for the proton and neutron electric and magnetic mean square radii are $\sqrt{\langle r_E^2 \rangle^p} = 0.860(38)(23)$ fm, $\langle r_E^2 \rangle^n = -0.147(48)$ fm², $\sqrt{\langle r_M^2 \rangle^p} = 0.870(53)(15)$ fm, and $\sqrt{\langle r_M^2 \rangle^n} = 0.913(67)(19)$ fm, and for the proton and neutron magnetic moments, $\mu^p = 2.849(92)(52)$ and $\mu^n = -1.819(76)(29)$, respectively. In all cases, the first error is statistical, and the second is systematic, where the latter includes an estimate of the error from the fits to the momentum dependence of the form factors and from the continuum extrapolation.

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I. INTRODUCTION

The study of proton and neutron electromagnetic form factors offers insights into the rich internal electromagnetic structure of the hadronic world. The proton, being a stable particle, has been used as the prototype system for experimental investigations of hadronic bound states. Thus, the proton electromagnetic form factors are known to high precision from electron-proton scattering [1]. The neutron form factors, on the other hand, are accessed indirectly experimentally through electron-deuteron or electron-helium scattering, and therefore quantities, such as the neutron electric charge radius, remain less well known compared to the proton. Despite the many years of experimental and theoretical studies [2], the electromagnetic form factors of the proton and the neutron still remain an active area of research. In particular, new experiments are reaching

high precision in the extraction of charges, moments, and radii of the proton [1,3,4] but also improving the accuracy of the corresponding neutron quantities [5]. The computation of these fundamental quantities from lattice QCD, especially for the proton, thus provides a benchmark for the lattice formulation validating nonperturbative calculations. This in turn and most importantly enables us to use lattice QCD to evaluate hadron structure quantities that are less well known experimentally.

In the past five years, lattice QCD simulations have been carried out with quark mass parameters approximately equal to their physical values, thus enabling us to extract hadron structure quantities without the need of a chiral extrapolation. However, to date, computations of the electromagnetic form factors either lack a continuum extrapolation or rely on the inclusion of ensembles with heavier-than-physical quark masses to carry out the continuum and chiral extrapolation [6–10]. In this work, we provide a calculation of the electromagnetic form factors of the proton and neutron using lattice QCD on three ensembles of clover-improved twisted mass fermions with two degenerate light, strange, and charm quarks ($N_f = 2 + 1 + 1$) with masses tuned to their physical values (physical point). The lattice spacings span $a = 0.080$ fm, 0.068 fm, and 0.057 fm, allowing us to take

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the continuum limit directly at the physical pion mass, thus eliminating the need for chiral extrapolations.

The remainder of this paper is organized as follows. In Sec. II, we define the nucleon electromagnetic form factors, radii, and magnetic moments. In Sec. III, we provide details on the setup of our lattice QCD calculation, including details of the ensembles used and statistics. In Sec. IV, we explain the techniques used to extract the form factors from the lattice QCD correlators, including the methods used for disconnected contributions, our approaches for identifying ground-state dominance, and the fits of the momentum-transfer dependence of the form factors. The results on the proton and neutron form factors for each gauge ensemble together with the fits to their momentum-transfer dependence are carried out in Sec. V, where we also include their continuum extrapolation. Final results and comparisons with the literature are carried out in Sec. VI, and in Sec. VII, we summarize and provide our conclusions. For completeness, we include in Appendix B the decomposition of the nucleon matrix elements in terms of the form factors, and in Appendix D, we provide tables with the numerical results for the electric and magnetic form factors as well as the fit parameters that reproduce our final result.

II. NUCLEON ELECTROMAGNETIC FORM FACTORS

We consider the electromagnetic form factors of the nucleon in the SU(2) flavor isospin symmetric limit, i.e., neglecting isospin-breaking effects due to QED interactions and the u–d quark mass difference. The proton and neutron are thus degenerate, and the form factors are given in terms of the matrix element of the electromagnetic current with nucleon states in Minkowski space,

$$\begin{aligned} \langle N(p', s') | j_\mu | N(p, s) \rangle &= \sqrt{\frac{m_N^2}{E_N(\vec{p}') E_N(\vec{p})}} \bar{u}_N(p', s') \\ &\times \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2(q^2) \right] \\ &\times u_N(p, s), \end{aligned} \quad (1)$$

where $N(p, s)$ is the nucleon with initial (final) momentum p (p') and spin s (s'), with energy $E_N(\vec{p})$ [$E_N(\vec{p}')$] and mass m_N ; u_N is the nucleon spinor; j_μ is the vector current; and $q^2 \equiv q_\mu q^\mu$ is the momentum transfer squared with $q_\mu = (p'_\mu - p_\mu)$. In Eq. (1), the form factors of the nucleon are given in terms of the Dirac (F_1) and Pauli (F_2) form factors. Alternatively, we can compute the electric and magnetic Sachs form factors $G_E(q^2)$ and $G_M(q^2)$, which are related to the Dirac and Pauli form factors via

$$\begin{aligned} G_E(q^2) &= F_1(q^2) + \frac{q^2}{4m_N^2} F_2(q^2), \\ G_M(q^2) &= F_1(q^2) + F_2(q^2). \end{aligned} \quad (2)$$

The local vector current is given by

$$j_\mu = \sum_{f=u,d,s,c} e_f j_\mu^f = \sum_{f=u,d,s,c} e_f \bar{\psi}_f \gamma_\mu \psi_f, \quad (3)$$

where the sum over f runs over the up-, down-, strange- and charm-quark flavors (u, d, s, and c, respectively) and e_f is the electric charge of the quark with flavor f . In the twisted mass formulation, the lattice conserved vector current after symmetrization is given by

$$\begin{aligned} j_\mu^f(x) &= \frac{1}{4} [\bar{\psi}_f(x + \hat{\mu}) U_\mu^\dagger(x) (1 + \gamma_\mu) \psi_f(x) \\ &+ \bar{\psi}_f(x) U_\mu^\dagger(x - \hat{\mu}) (1 + \gamma_\mu) \psi_f(x - \hat{\mu}) \\ &- \bar{\psi}_f(x) U_\mu(x) (1 - \gamma_\mu) \psi_f(x + \hat{\mu}) \\ &- \bar{\psi}_f(x - \hat{\mu}) U_\mu(x - \hat{\mu}) (1 - \gamma_\mu) \psi_f(x)], \end{aligned} \quad (4)$$

where $U_\mu(x)$ is the gauge link of lattice site x in direction $\hat{\mu}$. Within SU(2) considered in this work, we can extend the notation of the flavor composition of the current and use $j_\mu^{u-d} = j_\mu^u - j_\mu^d$ to denote the isovector current and $j_\mu^{u+d} = j_\mu^u + j_\mu^d$ to denote the isoscalar current. The standard electromagnetic current is then given as

$$j_\mu = \frac{2}{3} j_\mu^u - \frac{1}{3} j_\mu^d = \frac{1}{2} j_\mu^{u-d} + \frac{1}{6} j_\mu^{u+d}, \quad (5)$$

and the proton (G_E^p and G_M^p) and neutron (G_E^n and G_M^n) electromagnetic form factors are given in terms of the isovector (G_E^{u-d} and G_M^{u-d}) and isoscalar (G_E^{u+d} and G_M^{u+d}) form factors, via

$$\begin{aligned} G_X^p(q^2) &= \frac{1}{2} G_X^{u-d}(q^2) + \frac{1}{6} G_X^{u+d}(q^2) \\ G_X^n(q^2) &= -\frac{1}{2} G_X^{u-d}(q^2) + \frac{1}{6} G_X^{u+d}(q^2), \end{aligned} \quad (6)$$

where $X = E, M$.

At zero momentum transfer ($q^2 = 0$), the electric form factor yields the charge of the hadron and the magnetic form factor the magnetic moment, i.e.,

$$\begin{aligned} G_E^p(0) &= 1, & G_E^n(0) &= 0, \\ G_M^p(0) &= \mu_p, & G_M^n(0) &= \mu_n. \end{aligned} \quad (7)$$

When using the lattice conserved current, these relations hold by symmetry, with no renormalization of the currents required.

The electric and magnetic mean-squared radii are defined by the slope of the corresponding Sachs form factor as $q^2 \rightarrow 0$, namely,

$$\langle r_X^2 \rangle^f = \frac{-6}{G_X^f(0)} \left. \frac{\partial G_X^f(q^2)}{\partial q^2} \right|_{q^2=0}. \quad (8)$$

III. LATTICE SETUP

A. Lattice correlation functions

Nucleon matrix elements are computed within lattice QCD via appropriate combinations of two- and three-point correlation functions. These are defined in a Euclidean space-time, and we will thus use notation for Euclidean quantities from here on, including expressing the form

factors in terms of $Q^2 = -q^2$. We use the standard nucleon interpolating field,

$$\chi_N(\vec{x}, t) = \epsilon^{abc} u^a(x) [u^{b\dagger}(x) C \gamma_5 d^c(x)], \quad (9)$$

where $C = \gamma_0 \gamma_2$ is the charge conjugation matrix. The two-point function in momentum space is given by

$$C(\Gamma_0, \vec{p}; t_s, t_0) = \sum_{\vec{x}_s} e^{-i(\vec{x}_s - \vec{x}_0) \cdot \vec{p}} \text{Tr}[\Gamma_0 \langle \chi_N(t_s, \vec{x}_s) \bar{\chi}_N(t_0, \vec{x}_0) \rangle], \quad (10)$$

and the three-point function is given by

$$C_\mu(\Gamma_\nu, \vec{q}, \vec{p}'; t_s, t_{\text{ins}}, t_0) = \sum_{\vec{x}_{\text{ins}}, \vec{x}_s} e^{i(\vec{x}_{\text{ins}} - \vec{x}_0) \cdot \vec{q}} e^{-i(\vec{x}_s - \vec{x}_0) \cdot \vec{p}'} \text{Tr}[\Gamma_\nu \langle \chi_N(t_s, \vec{x}_s) j_\mu(t_{\text{ins}}, \vec{x}_{\text{ins}}) \bar{\chi}_N(t_0, \vec{x}_0) \rangle]. \quad (11)$$

The initial lattice site where the nucleon is created is denoted by x_0 and referred to as the *source*, the lattice site where the current couples to a quark x_{ins} is referred to as the *insertion*, and the site where the nucleon is annihilated x_s is referred to as the *sink*. Γ_ν is a projector acting on Dirac indices, with $\Gamma_0 = \frac{1}{2}(1 + \gamma_0)$ yielding the unpolarized and $\Gamma_k = \Gamma_0 i \gamma_5 \gamma_k$ the polarized matrix elements. Without loss of generality, we will take t_s and t_{ins} relative to the source time t_0 in what follows.

Inserting a complete set of states in Eq. (10) yields

$$C(\vec{p}, t_s) = \sum_n c_n(\vec{p}) e^{-E_n(\vec{p}) t_s}, \quad \text{where} \\ c_n(\vec{p}) = \sum_s \text{Tr}[\Gamma_0 \langle \chi_N | n, \vec{p}, s \rangle \langle n, \vec{p}, s | \bar{\chi}_N \rangle], \quad (12)$$

with $|n, \vec{p}, s\rangle$ a QCD eigenstate with the quantum numbers of the nucleon, spin s , momentum $\vec{p}$, and energy $E_n(\vec{p})$. Similarly, inserting two complete sets of states in Eq. (11) yields

$$C_\mu(\Gamma_\nu, \vec{q}, \vec{p}'; t_s, t_{\text{ins}}) = \sum_{n,m} \mathcal{A}_\mu^{n,m}(\Gamma_\nu, \vec{q}, \vec{p}') e^{-E_n(\vec{p}') (t_s - t_{\text{ins}}) - E_m(\vec{q}) t_{\text{ins}}}, \quad \text{where} \\ \mathcal{A}_\mu^{n,m}(\Gamma_\nu, \vec{q}, \vec{p}') = \sum_{s,s'} \text{Tr}[\Gamma_\nu \langle \chi_N | n, \vec{p}', s' \rangle \langle n, \vec{p}', s' | j_\mu | m, \vec{p}, s \rangle \langle m, \vec{p}, s | \bar{\chi}_N \rangle]. \quad (13)$$

The desired nucleon matrix element is obtained for $m = n = 0$ in Eq. (13) and after canceling the overlaps $\langle \chi_N | 0, \vec{p}', s' \rangle$ and $\langle 0, \vec{p}, s | \bar{\chi}_N \rangle$. One way to achieve this is by using an optimized ratio composed of two- and three-point functions, given by

$$R_\mu(\Gamma_\nu, \vec{p}, \vec{p}'; t_s, t_{\text{ins}}) = \frac{C_\mu(\Gamma_\nu, \vec{p}, \vec{p}'; t_s, t_{\text{ins}})}{C(\Gamma_0, \vec{p}'; t_s)} \sqrt{\frac{C(\Gamma_0, \vec{p}; t_s - t_{\text{ins}}) C(\Gamma_0, \vec{p}'; t_{\text{ins}}) C(\Gamma_0, \vec{p}'; t_s)}{C(\Gamma_0, \vec{p}'; t_s - t_{\text{ins}}) C(\Gamma_0, \vec{p}; t_{\text{ins}}) C(\Gamma_0, \vec{p}; t_s)}}. \quad (14)$$

In the limit of large time separations $(t_s - t_{\text{ins}}) \gg$ and $t_{\text{ins}} \gg$, the ratio in Eq. (14) converges to the nucleon ground-state matrix element, which we can express in terms of the amplitudes in Eqs. (12) and (13) as

$$\Pi^\mu(\Gamma_\nu; \vec{p}', \vec{q}) = \frac{\mathcal{A}_\mu^{0,0}(\Gamma_\nu, \vec{q}, \vec{p}')}{\sqrt{c_0(\vec{p}) c_0(\vec{p}')}}. \quad (15)$$

In Sec. IV, we detail the analysis we follow to extract the ground-state ($n = m = 0$) matrix element via a combination

of multistate fits to the two- and three-point functions. The ratio in Eq. (14) will be used only for visualization purposes of the fits to the two- and three-point functions.

B. Gauge ensembles and statistics

We use the twisted-mass fermion discretization scheme [11,12], which provides automatic $\mathcal{O}(a)$ improvement [13]. The bare light quark parameter μ_l is tuned to reproduce the isosymmetric pion mass $m_\pi = 135$ MeV [14,15], while the heavy quark parameters, μ_s and μ_c , are tuned using the kaon

TABLE I. Parameters of the three $N_f = 2 + 1 + 1$ ensembles analyzed in this work. From the leftmost to rightmost columns, we provide the name of the ensemble and its short acronym in parenthesis, the lattice volume, $\beta = 6/g^2$ with g the bare coupling constant, the lattice spacing, the pion mass, and the value of $m_\pi L$. Lattice spacings and pion masses are taken from Ref. [16].

Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	β	a (fm)	m_π (MeV)	$m_\pi L$
cB211.072.64 (B)	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)	3.62
cC211.060.80 (C)	$80^3 \times 160$	1.836	0.06821(13)	136.7(2)	3.78
cD211.054.96 (D)	$96^3 \times 192$	1.900	0.05692(12)	140.8(2)	3.90

mass and an appropriately defined ratio between the D-meson mass and decay constant as well as the ratio between charm and strange quark mass, following the procedure of Refs. [14,15]. A clover term is included in the action that reduces isospin-breaking effects. The parameters of the three ensembles analyzed in this work can be found in Table I. The lattice spacings and pion masses are taken from Ref. [16]. We quote the values of the lattice spacing obtained from the meson sector, which are compatible with the values determined from the nucleon mass in Ref. [17].

To increase the overlap of the interpolating field with the nucleon ground state, so that excited states are suppressed at earlier time separations, we use Gaussian smeared point sources [18,19],

$$\psi^{\text{sm}}(\vec{x}, t) = \sum_{\vec{y}} [1 + a_G H(\vec{x}, \vec{y}; U(t))]^{N_G} \psi(\vec{y}, t), \quad (16)$$

where the hopping matrix is given by

$$H(\vec{x}, \vec{y}; U(t)) = \sum_{i=1}^3 [U_i(x) \delta_{x, y-\hat{i}} + U_i^\dagger(x - \hat{i}) \delta_{x, y+\hat{i}}]. \quad (17)$$

The parameters a_G and N_G are tuned [8,20] in order to approximately give a root-mean-square (rms) radius for the nucleon of 0.5 fm. For the links entering the hopping matrix, we apply APE smearing [21] to reduce statistical errors due to ultraviolet fluctuations. The smearing parameters are tuned separately for each value of the lattice spacing [22] and are given in Table II.

TABLE II. Number of Gaussian smearing iterations (N_G) and Gaussian smearing parameter a_G used for each ensemble. We also provide the number of APE-smearing iterations n_{APE} and parameter α_{APE} applied to the links that enter the Gaussian smearing hopping matrix. The resulting quark rms radius is given in the last column as $\langle r^2 \rangle_{\psi^{\text{sm}}} = \sum_{\vec{r}} r^2 |\psi^{\text{sm}}(\vec{r})|^2 / \sum_{\vec{r}} |\psi^{\text{sm}}(\vec{r})|^2$, where the error is due to the uncertainty in the lattice spacing.

Ensemble	N_G	a_G	n_{APE}	α_{APE}	$\sqrt{\langle r^2 \rangle_{\psi^{\text{sm}}}}$ (fm)
cB211.072.64	125	0.2	50	0.5	0.461(2)
cC211.060.80	140	1.0	60	0.5	0.516(2)
cD211.054.96	200	1.0	60	0.5	0.502(3)

Carrying out the quark field contractions for the three-point function in Eq. (11) for a general current insertion gives rise to both connected and disconnected contributions, as shown in Fig. 1. For the case of the isovector combination, only the connected diagram contributes since the disconnected contribution cancels due to the degeneracy between the up- and down-quark fermion loops at the continuum limit. For the connected contribution, we restrict to $\vec{p}' = 0$, meaning the source momentum $\vec{p}$ is determined via momentum conservation by the momentum transfer as $\vec{p} = -\vec{q}$. The all-to-all propagator required due to the sums over $\vec{x}_{\text{ins}}$ and $\vec{x}_s$ is computed using a sequential inversion through the sink, i.e., using the so-called *fixed-sink* method. Thus, the same sequential propagator yields all insertion momenta $\vec{q}$; however, new inversions are required for each sink time t_s and projector Γ_ν . To reliably extract the nucleon matrix element of interest, we compute the connected three-point function for multiple values of t_s and seek convergence to the asymptotic ground-state matrix element via multistate fits. We increase statistics with increasing t_s to counter the expected exponential increase of statistical uncertainty. The statistics used for each ensemble and each value of t_s are shown in Table III, which also conveys our strategy of keeping the number of configurations fixed

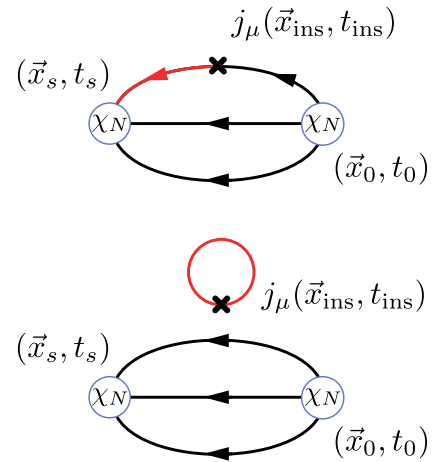


FIG. 1. Connected (top) and disconnected (bottom) contributions to the nucleon three-point function. The red lines denote all-to-all propagators.

TABLE III. Statistics used for the computation of the connected contributions for cB211.072.64 (left), cC211.060.80 (center), and cD211.054.96 (right). For each ensemble, we give the number of configurations (n_{conf}), the sink-source separations in lattice units (t_s/a , first column) and physical units (t_s , second column), and the number of source positions per configuration (n_{src} , third column).

cB211.072.64		
$n_{\text{conf}} = 750$		
t_s/a	t_s (fm)	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128

cC211.060.80		
$n_{\text{conf}} = 400$		
t_s/a	t_s (fm)	n_{src}
6	0.41	1
8	0.55	2
10	0.69	4
12	0.82	10
14	0.96	22
16	1.10	48
18	1.24	45
20	1.37	116
22	1.51	246

cD211.054.96		
$n_{\text{conf}} = 500$		
t_s/a	t_s (fm)	n_{src}
8	0.46	1
10	0.57	2
12	0.68	4
14	0.80	8
16	0.91	16
18	1.03	32
20	1.14	64
22	1.25	16
24	1.37	32
26	1.48	64

while increasing the number of randomly chosen source-positions per configuration with increasing t_s .

The disconnected three-point functions involve correlating the two-point function of Eq. (10) with the quark loop with the electromagnetic current insertion, which results in

$$L_f(t_{\text{ins}}, \vec{q}) = \sum_{\vec{x}_{\text{ins}}} \text{Tr}[D_f^{-1}(x_{\text{ins}}; x_{\text{ins}})\gamma_\mu] \cdot e^{i\vec{q} \cdot \vec{x}_{\text{ins}}}, \quad (18)$$

TABLE IV. Parameters for the computation of the disconnected three-point function. For the three ensembles (indicated in the first column), we give the number of configurations (n_{conf} , second column) and the number of vectors that are inverted per configuration (n_{vec}) resulting from color and spin dilution (n_{dil} , third column), hierarchical probing (n_{had} , fourth column), and the number of stochastic vectors used (n_{stoch} , fifth column). We indicate the number of low-lying eigenvectors treated exactly (n_{ev} , sixth column) for the two ensembles where deflation is used and the number of source positions for the two-point functions (n_{src}).

Ensemble	n_{conf}	$n_{\text{vec}} = n_{\text{dil}} \times n_{\text{had}} \times n_{\text{stoch}}$	n_{ev}	n_{src}
cB211.072.64	750	$12 \times 512 \times 1$	200	477
cC211.060.80	400	$12 \times 512 \times 1$	450	650
cD211.054.96	500	$12 \times 512 \times 8$	...	480

for each quark flavor f . Therefore, the selection of $\vec{q}$ and t_s is independent of the computation of the insertion operator, and we can thus obtain all insertion sites and sink times. In practice, however, this is limited by statistical uncertainties, which grow with t_s . Note that the disconnected three-point function can also be obtained for nonzero sink momentum $\vec{p}'$. We compute the isoscalar combination $L_{u+d}(t_{\text{ins}}, \vec{q})$ stochastically, via the *generalized one-end trick* [23], where the trace of the sum of the up- and down-quark propagators are written in terms of the trace of a product, namely,

$$\begin{aligned} L_{u+d}(t_{\text{ins}}, \vec{q}) &= \sum_{\vec{x}_{\text{ins}}} e^{i\vec{q} \cdot \vec{x}_{\text{ins}}} [\text{Tr}[D_u^{-1}(x_{\text{ins}}; x_{\text{ins}})\gamma_\mu] + \text{Tr}[D_d^{-1}(x_{\text{ins}}; x_{\text{ins}})\gamma_\mu]] \\ &= \sum_{\vec{x}_{\text{ins}}} e^{i\vec{q} \cdot \vec{x}_{\text{ins}}} \text{Tr}[D_u^{-1}(x_{\text{ins}}; x) D_W(x; y) D_d^{-1}(y; x_{\text{ins}})\gamma_\mu], \quad (19) \end{aligned}$$

where D_u (D_d) is the twisted mass operator for $+\mu_l$ ($-\mu_l$) and D_W is the standard Wilson operator. The trace in the last line of Eq. (19) can be obtained using the one-end trick with volume stochastic sources, as also used in our previous studies [8,24–27].

The parameters used for the computation of the disconnected three-point function are summarized in Table IV. For all three ensembles, we use the same number of configurations as in the connected three-point functions and fully dilute in color and spin ($n_{\text{dil}} = 12$).

Furthermore, we use hierarchical probing [28] to distance eight in the four-dimensional volume, which results in 512 Hadamard vectors ($n_{\text{had}} = 512$). Hierarchical probing suppresses stochastic noise from the off-diagonal spatial indices in the trace of Eq. (18), and therefore, as indicated in Eq. (18), for the disconnected contribution we use the nonconserved local vector current $\bar{\psi}_f \gamma_\mu \psi_f$, which only couples the diagonal components of the spatial indices of the current. For the disconnected contributions, we therefore need to renormalize the current, as will be explained in Sec. III C.

For the ensembles, cB211.072.64 and cC211.060.80, we use 1 stochastic source ($n_{\text{stoch}} = 1$) and deflate [29] the lowest 200 and 450 eigenvectors (n_{ev}), respectively. For the cD211.054.96 ensemble with the largest lattice size, deflation was impractical due to memory requirements at the time of its analysis, and we therefore use instead $n_{\text{stoch}} = 8$. On each configuration, we compute two-point functions on $\mathcal{O}(10^2)$ source positions (n_{src}), as indicated in the last column of Table IV. Beyond increasing the statistics of the disconnected three-point function, these two-point functions improve the analysis of multistate fits to the connected three-point functions.

C. Renormalization

The connected parts of the isovector and isoscalar matrix elements are calculated using the conserved vector current, for which no renormalization is required. However, the disconnected part of the isoscalar combination is computed using the local operator, which needs renormalization. To address this, we determine the renormalization factors of both flavor nonsinglet and flavor singlet vector operator, denoted as Z_V^{ns} and Z_V^s , respectively, using the RI'/MOM scheme [30]. In the massless limit, where $\text{SU}(N_f = 4)$ flavor symmetry holds, the isoscalar combination j_μ^{u+d} can potentially mix with j_μ^{s+c} (or equivalently with the singlet combination $j_\mu^{u+d+s+c}$), where u,d,s,c are the $N_f = 4$ quark flavors. In particular, the renormalized isoscalar vector current $(j_\mu^{u+d})^R$ is obtained through

$$\begin{aligned} (j_\mu^{u+d})^R &= \frac{1}{2} (j_\mu^{u+d+s+c})^R + \frac{1}{6} (j_\mu^{u+d+s-3c})^R + \frac{1}{3} (j_\mu^{u+d-2s})^R \\ &= \frac{1}{2} Z_V^s j_\mu^{u+d+s+c} + \frac{1}{6} Z_V^{ns} j_\mu^{u+d+s-3c} + \frac{1}{3} Z_V^{ns} j_\mu^{u+d-2s} \\ &= Z_V^{ns} j_\mu^{u+d} + \frac{1}{2} (Z_V^s - Z_V^{ns}) j_\mu^{u+d+s+c}, \end{aligned} \quad (20)$$

where the nonsinglet combinations $j_\mu^{u+d+s-3c}$ and j_μ^{u+d-2s} as well as the singlet combination $j_\mu^{u+d+s+c}$ renormalize multiplicatively with Z_V^{ns} and Z_V^s , respectively. Thus, the calculation of the matrix element of $j_\mu^{u+d+s+c}$ is required in addition to the matrix element of j_μ^{u+d} and the renormalization factors Z_V^{ns} and Z_V^s . However, as shown in Ref. [16], using symmetry arguments and confirmed by our analysis, Z_V^s and Z_V^{ns} coincide, and thus the second term of Eq. (20) is eliminated, and the computation of the matrix element of $j_\mu^{u+d+s+c}$ is no longer necessary.

The RI'/MOM condition used for extracting Z_V^{ns} and Z_V^s is [31]

$$(Z_f^{\text{RI}'})^{-1} Z_V^{\text{ns(s)}} \frac{1}{36} \sum_\mu \text{Tr} \left[\Lambda_{V_\mu}^{\text{ns(s)}}(p) \left(\gamma_\mu - \frac{\not{p}}{4\tilde{p}_\mu} \right) \right] \Big|_{p^2=\mu_0^2} = 1, \quad (21)$$

where $\tilde{p}_\mu \equiv \sin(ap_\mu) \neq 0$, μ_0 represents the RI'/MOM scale, and $\Lambda_{V_\mu}^{\text{ns(s)}}(p)$ is the amputated vertex function of the nonsinglet (singlet) vector operator with external quark fields. Note that a disconnected diagram enters the calculation of $\Lambda_{V_\mu}^s(p)$. $Z_f^{\text{RI}'}$ is the renormalization factor of the quark field, calculated through the quark propagator.

The vertex function $\Lambda_{V_\mu}^{\text{ns(s)}}(p)$ is calculated using the momentum source approach [32], which gives high statistical accuracy using only $\mathcal{O}(10)$ configurations. For the disconnected part of $\Lambda_{V_\mu}^s(p)$, we employ the generalized one-end trick [23] along with the hierarchical probing algorithm [28] to mitigate stochastic noise.

Our strategy can be summarized as follows: we first perform chiral extrapolations using a set of ensembles specifically simulated for our renormalization program (see Ref. [33]), which features four mass-degenerate quarks ($N_f = 4$) at the same β values as the three physical-point ensembles used in our analysis of the matrix elements. A linear fit with respect to the twisted-mass parameter μ_{sea} adequately removes the mild dependence on quark mass, consistent with findings from previous studies, e.g., Ref. [34].

To minimize lattice artifacts and rotational $O(4)$ breaking effects, we employ spatially isotropic momenta of the form

$$(ap) \equiv 2\pi \left\{ \frac{n_t + 0.5}{T/a}, \frac{n_x}{L/a}, \frac{n_x}{L/a}, \frac{n_x}{L/a} \right\}, \quad (22)$$

where $n_i \in \mathbb{Z}$ and $L(T)$ is the spatial (temporal) extent of the lattice. We impose momentum cuts defined by $\sum_\mu p_\mu^4 / (\sum_\mu p_\mu^2)^2 < 0.3$. Additionally, we improve our nonperturbative results by subtracting one-loop discretization errors calculated through lattice perturbation theory, leading to a reduced dependence of the renormalization factors on $(a^2 p^2)$. Further details can be found in similar studies by our group [34].

The vector current is scheme and scale independent for both nonsinglet and singlet operators. Consequently, unlike other bilinear operators, we do not encounter systematic errors from perturbative conversion and evolution functions.

Next, we apply a linear fit in μ_0^2 to eliminate any residual dependence on the renormalization scale resulting from discretization effects. Figure 2 displays the momentum fits of Z_V^{ns} and Z_V^s for the three different β values. The fit range is set to 12–50 GeV^2 , where the linear fit can sufficiently describe the remaining momentum dependence.

The extrapolated values of Z_V^{ns} and Z_V^s at $\mu_0^2 = 0$ are given in Table V. The two values are in strong agreement within the small statistical uncertainties, as expected. Our analysis provides robust support for the theoretical proof established in Ref. [16]. A more comprehensive analysis of

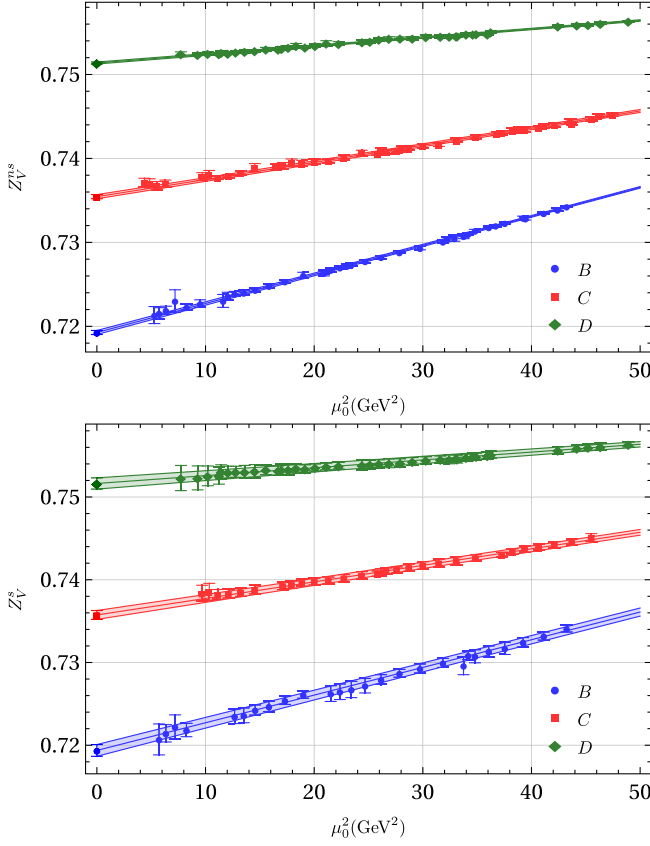


FIG. 2. Renormalization factors of the flavor nonsinglet (top) and flavor singlet (bottom) vector operator for the three physical-point ensembles: cB211.072.64 (“B”), cC211.060.80 (“C”), and cD211.054.96 (“D”).

the nonsinglet quark bilinear operators using a total of five lattice spacings will be provided in a forthcoming publication by ETMC [31].

Obtaining the disconnected contributions using the local current suppresses stochastic errors in the quark loop that would arise from the product of off-diagonal space-time points had the conserved current been used. The use of the conserved current for the connected matrix elements and the renormalized local current for the disconnected may affect the approach to the continuum. Practically though, any systematic arising from this is much smaller than our statistical errors. Indeed, the difference between the isoscalar matrix elements obtained using the renormalized local current and the conserved current is at most 30% of

TABLE V. Renormalization factors of the flavor nonsinglet and flavor singlet vector operator for the three physical-point ensembles.

	cB211.072.64	cC211.060.80	cD211.054.96
Z_V^{ns}	0.7193(2)	0.7354(2)	0.7514(1)
Z_V^s	0.7194(6)	0.7357(5)	0.7516(7)

the statistical error for all Q^2 and values of a considered in this work.

IV. EXTRACTION OF NUCLEON MATRIX ELEMENTS

The bare form factors at each value of the momentum transfer squared, Q^2 , are obtained by appropriate combinations of Γ_ν and μ depending on the momenta $\vec{p}'$ and $\vec{q}$ in $\Pi^\mu(\Gamma_\nu; \vec{p}', \vec{q})$ of Eq. (15) in order to isolate $G_E(Q^2)$ and $G_M(Q^2)$. For the case of the connected contributions, the fixed sink approach [8] requires us to fix the sink momentum, which we set to $\vec{p}' = 0$, as explained in the previous section. For this case, the expressions yielding $G_E(Q^2)$ and $G_M(Q^2)$ can be disentangled via

$$\Pi^0(\Gamma_0, \vec{q}) = C \frac{E_N + m_N}{2m_N} G_E(Q^2), \quad (23)$$

$$\Pi^i(\Gamma_0, \vec{q}) = C \frac{q_i}{2m_N} G_E(Q^2), \quad (24)$$

$$\Pi^i(\Gamma_k, \vec{q}) = C \frac{\epsilon_{ijk} q_j}{2m_N} G_M(Q^2) \quad (25)$$

where

$$C = \sqrt{\frac{2m_N^2}{E_N(E_N + m_N)}}, \quad (26)$$

and $E_N = E_N(\vec{q})$. In practice, combining Eqs. (23) and (24) does not improve the statistical precision of our results for $G_E(Q^2)$ compared to using Eq. (23) alone, and therefore in what follows, we extract $G_E(Q^2)$ from Eq. (23) and $G_M(Q^2)$ from Eq. (25). In the disconnected contributions, the sink momentum can be chosen in the two-point function independently of the disconnected loop and at no additional computational cost. We thus take sink momenta $\vec{p}' = \frac{2\pi}{L} \vec{k}$ with $\vec{k}^2 = 1$ and 2 in addition to the case $\vec{p}' = 0$. With these additional momenta, the expressions yielding G_E and G_M cannot be disentangled (see, for example, Appendix B); i.e., each combination of Γ_ν, μ , and sink and insertion momenta in $\Pi^\mu(\Gamma_\nu, \vec{p}', \vec{q})$ relates to a linear combination of $G_E(Q^2)$ and $G_M(Q^2)$. The set of equations is overdetermined, and we thus resort to using a singular value decomposition (SVD) to solve for the electromagnetic form factors. Our procedure, including how statistical errors are propagated, is the same as the one used in Ref. [20] where we extracted the generalized form factors of the nucleon.

To benefit from the availability of kinematics with $\vec{p}' \neq 0$ in the disconnected case, as well as the additional separations, we proceed by carrying out the excited-state analysis by fitting the connected and disconnected contributions separately and then adding the obtained ground-state contributions. As will be shown in Sec. IV B, the

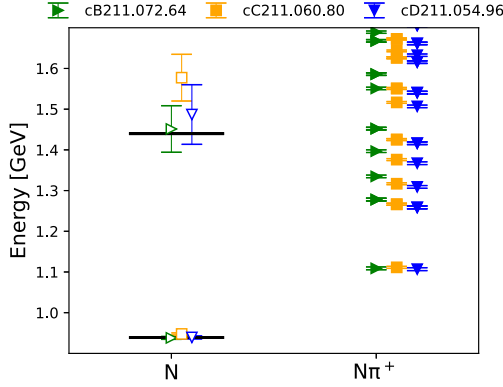


FIG. 3. The ground-state and first excited-state energies obtained from a three-state fit to the nucleon two-point function (open symbols) and the energies of the noninteracting π^+N states (filled symbols, green for the B ensemble, orange for the C ensemble, and blue for the D ensemble) for all three ensembles and for total momentum $\vec{p}^2 = (\frac{2\pi}{L})^2$. The black lines correspond to the energies of the nucleon and Roper resonance, respectively.

additional kinematics are crucial for obtaining a good signal in the disconnected case. The alternative approach, whereby the connected and disconnected three-point functions are added first and the excited-state analysis follows, would require us to restrict our analysis to using the disconnected contributions with sink momentum zero ($\vec{p}' = 0$) and only for the sink-source separations that are also available for the connected case. For completeness, in Appendix A, we compare the two approaches, showing that they lead to ground-state matrix elements which are consistent within errors.

A. Connected

With multiple sink-source time separations and high statistics for both connected three- and two-point functions, we can carry out a thorough analysis of excited-state effects. In particular, our data allow for fits to the three-point functions including the first excited state, namely, with $(m, n) = (0, 0), (0, 1), (1, 0),$ and $(1, 1)$ in Eq. (13), and fits to the two-point functions including up to the second excited state, namely, with $n = 0, 1,$ and 2 in Eq. (12). An important component of our multistate fit analysis is to allow for different excited-state energies in the two- and three-point function. Indeed, a strong coupling of the electromagnetic current to a multiparticle excited state may amplify those terms in the spectrum of the three-point function while otherwise being suppressed in the two-point function. This has also been pointed out in Ref. [35], where a study in chiral perturbation theory suggests that this may be the case for pion-nucleon states coupling to the isovector electromagnetic current. As an example, in Fig. 3, we show the ground-state and first excited-state energies obtained via three-state fits to the nucleon two-point function and compare with the noninteracting π^+N energies using the

mass of π^+ taken from Table I for each ensemble. The first excited-state energies of the two-point function for all ensembles considered coincide with the Roper resonance and are thus higher than several possible pion-nucleon states.

Our multistate fitting procedure for the connected contributions ($\vec{p}' = 0$) proceeds as follows; for each value of the momentum transfer squared $Q^2 = 2m_N(E_N(\vec{q}) - m_N)$, we fit together the two-point functions $C(\vec{0}, t_s)$ and $C(\vec{q}, t_s)$ and the isovector and isoscalar three-point functions $C_\mu^{u-d}(\Gamma_\nu, \vec{q}; t_s, t_{\text{ins}})$ and $C_\mu^{u+d}(\Gamma_\nu, \vec{q}; t_s, t_{\text{ins}})$, respectively, for both the electric and magnetic form factors. We allow different fit parameters between two- and three-point functions. The fit parameters for a single value of $\vec{q}$ are described below:

- (i) The nucleon ground-state energy at rest $E_N(0) = m_N$ is a common fit parameter, and the dispersion relation is used to set the nucleon ground-state energy at momentum $\vec{q}$, i.e., $E_N(\vec{q}) = \sqrt{m_N^2 + \vec{q}^2}$.
- (ii) For each two-point function, we do a three-state fit that has three coefficient parameters and two parameters corresponding to the first and second excited-state energies. This leads to 11 parameters so far.
- (iii) The two-state fits to the isovector and isoscalar three-point functions are carried out in the same fit as the two-point functions above, thus sharing common ground-state energies $E_N(0)$ and $E_N(\vec{q})$. However, we allow for the isovector and isoscalar three-point functions to have different excited-state energies, i.e., two excited-state energies for the isovector and two for the isoscalar, which are different from those of the two-point functions. We thus have four parameters for the energies of the excited states of the three-point functions. Furthermore, we have 16 amplitudes as fit parameters, i.e., four for each $G_E^{u-d}(Q^2)$, $G_M^{u-d}(Q^2)$, $G_E^{u+d}(Q^2)$, and $G_M^{u+d}(Q^2)$.

Thus, each fit has 31 fit parameters. While we do not use any priors in the final fit with the 31 fit parameters, a series of fits precedes the final fit to determine the initial parameters. Namely, we first fit to the individual two-point functions and then fit the three-point functions omitting the $\mathcal{A}^{1,1}$ term of Eq. (13), using the obtained parameters at each stage as starting values for subsequent fits. An exception is a few cases with $Q^2 > 0.6 \text{ GeV}^2$ where the two-state fit to the three-point function is unstable, likely due to weak excited-state contamination. In these cases, we use a prior for $\mathcal{A}^{0,0}$ centered around the result from a plateau fit and with width 10 times the error of the plateau fit. In all fits, we use the covariance matrix taking into account correlations between three-point functions of different separations (t_s) and correlations with the two two-point functions.

We vary the following time separations in our fitting procedure:

TABLE VI. The values of $t_s^{\text{low,3pt}}$, $t_s^{\text{low,2pt}}$, and $t_{\text{ins}}^{\text{sink}}$ varied in our multistate fits of the two- and connected three-point functions. For each value of $t_{\text{ins}}^{\text{sink}}$, we use three values for $t_{\text{ins}}^{\text{source}}$, namely, $t_{\text{ins}}^{\text{source}} = t_{\text{ins}}^{\text{sink}}$, $t_{\text{ins}}^{\text{sink}} + 1$, and $t_{\text{ins}}^{\text{sink}} + 2$.

Ensemble	$t_s^{\text{low,3pt}}/a$	$t_s^{\text{low,2pt}}/a$	$t_{\text{ins}}^{\text{sink}}/a$
cB211.072.64	8, 10, 12, 14	1, 2, 3	2, 3, 4
cC211.060.80	8, 10, 12, 14	1, 2, 3, 4	2, 3, 4
cD211.054.96	8, 10, 12, 14	1, 2, 3, 4, 5	2, 3, 4

- (i) $t_s^{\text{low,3pt}}$: the smallest sink-source separation of the connected three-point function included, the largest being the maximum separation listed in Table III.
- (ii) $t_s^{\text{low,2pt}}$: the smallest sink-source separation of the two-point function included.
- (iii) $t_{\text{ins}}^{\text{source}}$: the smallest insertion-source separation of the connected three-point function included.
- (iv) $t_{\text{ins}}^{\text{sink}}$: the smallest sink-insertion separation of the connected three-point function included.

The current insertion times included in a given fit are thus $t_{\text{ins}} \in [t_{\text{ins}}^{\text{source}}, t_s - t_{\text{ins}}^{\text{sink}}]$. Since the sink is at rest and the source has momentum, the magnitude of the excited-state contamination at a given time distance from the source and sink of the three-point function may be different, which is why we allow $t_{\text{ins}}^{\text{source}} \neq t_{\text{ins}}^{\text{sink}}$. The values used for each ensemble are provided in Table VI.

An example of this fitting procedure is shown in Fig. 4, for the case of the connected isoscalar and isovector form factors for the cD211.054.96. In particular, in the left columns, we show the ratio of Eq. (14) after multiplying with the relevant kinematic factor for isolating $G_E(Q^2)$ and $G_M(Q^2)$ according to Eqs. (23) and (25). The data in the second column show the value of the ratio at the midpoint $t_{\text{ins}} = t_s/2$, in order to visualize the convergence of the data as t_s is increased. We note that the data fitted to in our multistate fits are the two- and three-point functions and not the ratios shown in Fig. 4; i.e., the ratio is formed for visualization purposes only. In the third column, we show results for the most probable model in the subset of models obtained after fixing $t_s^{\text{low,3pt}}$ to demonstrate the variation in the results while varying the $t_s^{\text{low,3pt}}$. In the last column, we show our result for the form factor as a function of the fit probability, which we determine via the Akaike Information Criterion (AIC) explained in what follows. The fit with the highest probability is shown with the open symbol and the horizontal band spanning the x axis. Using the fit parameters of the most probable fit, we construct the ratio as a function of t_{ins} and t_s , yielding the bands in the first and second columns. As can be seen, the fit range combinations having probabilities greater than 1% are compatible with each other and with the most probable fit.

1. Model averaging

For each combination of fit ranges, as given in Table VI, we obtain the χ^2 value of the fit and evaluate its probability via the AIC [36,37] to obtain combined statistical and systematic errors of the ground-state matrix elements. We follow the model-average procedure that was used for the analysis of the axial form factors using the same three ensembles [38]. In brief, for each combination of fit ranges (j) and jackknife sample (i), we obtain $\mathcal{O}_j^i$, the desired form factor at the given value of Q^2 . We associate a weight, $w_{i,j}$, defined as

$$\log(w_{i,j}) = -\frac{\chi_{i,j}^2}{2} + N_{i,j}^{\text{dof}}, \quad (27)$$

where $N_{i,j}^{\text{dof}} = N_{i,j}^{\text{data}} - N_{i,j}^{\text{params}}$ is the number of degrees of freedom, given as the difference between the number of data, $N_{i,j}^{\text{data}}$, and the number of parameters, $N_{i,j}^{\text{params}}$, used in the corresponding fit. $\chi_{i,j}^2$ is defined as

$$\chi_{i,j}^2 = (\vec{r}_{i,j})^\top (C_j)^{-1} \vec{r}_{i,j} \quad \text{with} \quad \vec{r}_{i,j} = \vec{y}_{i,j} - f(\vec{x}_{i,j}), \quad (28)$$

where, for each fit j , C_j is the covariance matrix of the data $\vec{y}_{i,j}$ determined via jackknife resampling, $\vec{y}_{i,j}$ is the jackknife sample i of the data, and $\vec{r}_{i,j}$ is the residual, computed using the selected fit function and ranges $f(\vec{x}_{i,j})$. The probability associated with each combination of fit ranges is

$$p_{i,j} = \frac{w_{i,j}}{Z_i} \quad \text{with} \quad Z_i = \sum_j w_{i,j}, \quad (29)$$

and the model-averaged value of our observable, $\mathcal{O}^i$, in jackknife bin i is

$$\mathcal{O}^i = \sum_j p_{i,j} \mathcal{O}_j^i. \quad (30)$$

The central value and error of the observable is then obtained via the standard jackknife procedure.

It should be noted that with this procedure all three-point functions for the isovector and isoscalar currents with the different projectors in Eqs. (23) and (25) are fitted together, meaning that a single probability is associated for a given fit range. The advantage is that when combining the isoscalar and isovector form factors to obtain the proton and neutron form factors the combination can be done per fit range and the same value of the probability can be carried through and assigned to the proton and neutron combinations.

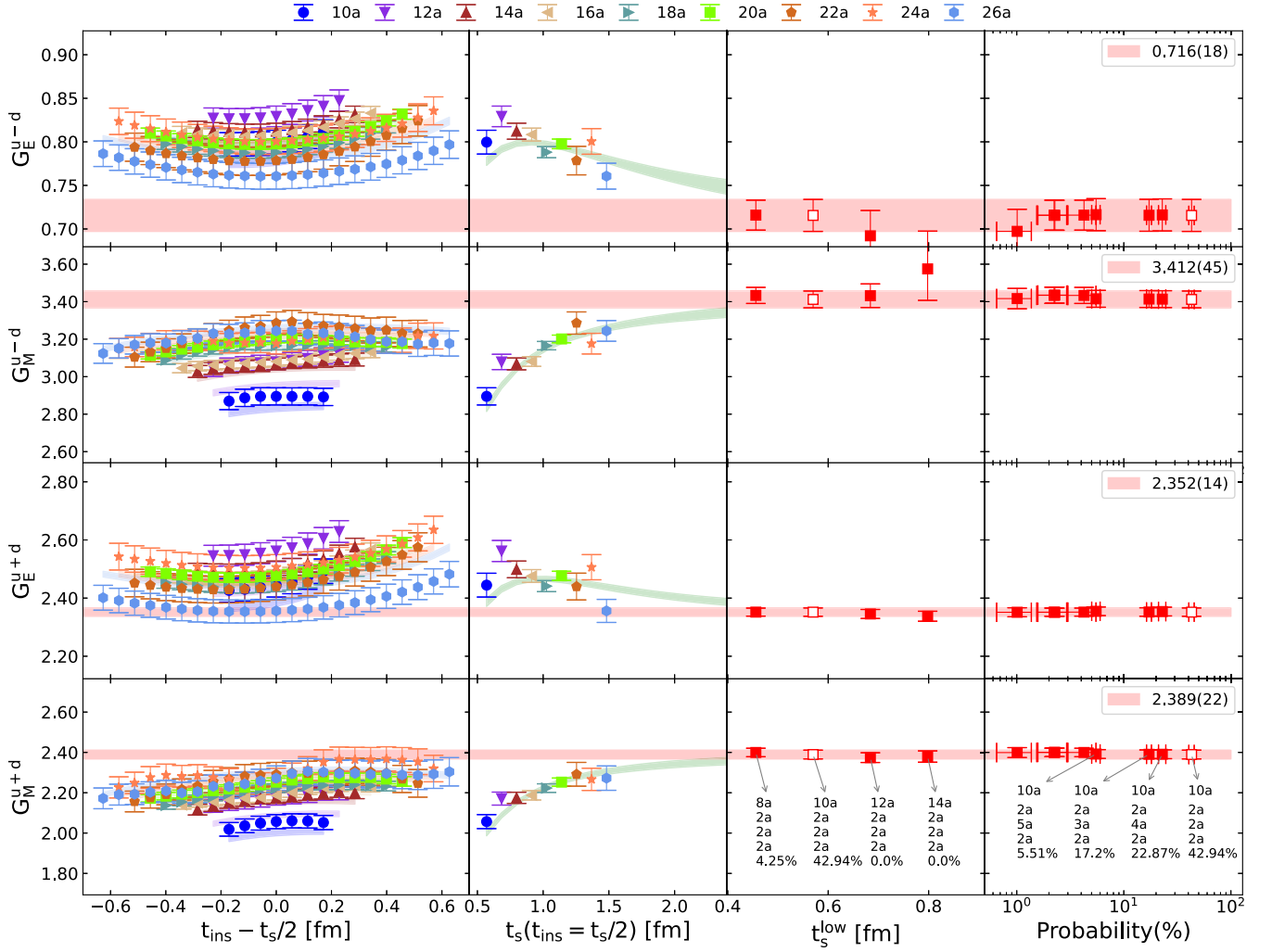


FIG. 4. The left column shows the ratio of three- to two-point functions as defined in Eq. (14) vs $t_{\text{ins}} - t_s/2$ from which the isovector (isoscalar) electric and magnetic form factors are extracted in the first (third) and second (fourth) rows, respectively. Different symbols denote different t_s as shown in the header. Results are for $Q^2 = 0.0996 \text{ GeV}^2$ and the cD211.054.96 ensemble. The second column shows the value of the ratio for $t_{\text{ins}} = t_s/2$. The third column shows the result of the most probable fit for a given $t_s^{\text{low,3pt}}$, and the right column gives the result of each fit versus its fit probability, evaluated as explained in the text. The bands in the left column and the light green band in the second column are obtained by constructing the ratio using the fit parameters of the most probable fit. The open symbol in the third and fourth columns and the horizontal red band is the value for the form factor obtained using the fit parameters of the most probable fit. The annotation in the third and fourth columns in the last row corresponds to the vector $[t_s^{\text{low,3pt}}, t_{\text{ins}}^{\text{source}}, t_{\text{ins}}^{\text{sink}}, t_s^{\text{low,2pt}}, \text{Probability}(\%)]$. The horizontal errors in last column are obtained via jackknife.

2. Energy spectrum and dispersion relation

Since in our fits we allow for different excited-state energies in the two-point functions and the isoscalar and isovector three-point functions, it is instructive to check and compare the spectrum with the Roper and π^+N free energies. In Fig. 5, we collect the excited-state energies obtained from the fits to the isovector and isoscalar connected three-point functions. For each value of the momentum $\vec{q}^2 = \vec{p}^2$, the fit yields two excited-state energies: one from the sink which is at rest ($E_{1,\vec{0}}$) and one from the source which carries the opposite momentum of the

insertion ($E_{1,\vec{p}}$, $\vec{p} = -\vec{q}$). From this plot, we observe the following:

- (i) The first excited-state energy at sink in the isoscalar case, $E_{1,\vec{0}}^{u+d}$, is compatible with the Roper for values of $\vec{p}^2 \lesssim 0.4 \text{ GeV}^2$. Then, it increases as $\vec{p}^2$ increases. This behavior may indicate residual admixture of two particle states. We note that the excited energy from the two-point function, on the other hand, follows the dispersion relation.
- (ii) The first excited-state energy at the source for the isoscalar case $E_{1,\vec{p}}^{u+d}$ lies between the Roper moving

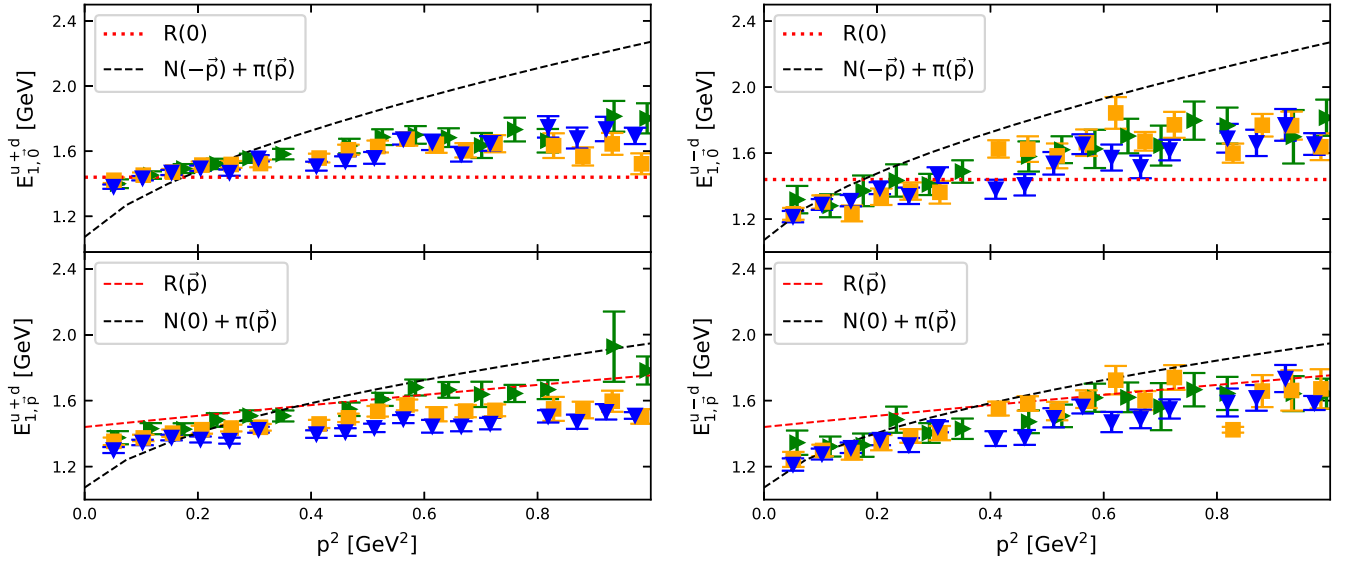


FIG. 5. Excited-state energies in the three-point function, as obtained from the combined fits to the two- and three-point functions explained in the text, for the B (green right-pointing triangles), C (yellow squares), and D (blue down-pointing triangles) ensemble. In the top (bottom), we show the first excited state at sink (source) for the isoscalar (left) $E_{1,0}^{u+d}$ ($E_{1,0}^{u+d}$) and isovector (right) $E_{1,0}^{u-d}$ ($E_{1,0}^{u-d}$) three-point functions. The red line shows the Roper mass (top) and the Roper energy with momentum $\vec{p}$ (bottom). Top: the dashed black lines show the energy of the noninteracting nucleon-pion state with total momentum zero and back-to-back relative momentum $\vec{p}$. Bottom: the dashed (dotted) black lines show the noninteracting nucleon-pion energy with total momentum $\vec{p}$ when $\vec{p}$ is carried by the pion (nucleon).

with finite momentum and the nucleon-pion state with total momentum $\vec{p}$, when the momentum is carried by the nucleon.

- (iii) The first excited-state energy at the sink in the isovector case $E_{1,0}^{u-d}$ is compatible with the nucleon-pion state with relative back-to-back momentum $\vec{p}$ for values of $\vec{p}^2$ where this energy is smaller than the Roper mass. When $\vec{p}^2$ is such that the nucleon-pion energy crosses the Roper mass, $E_{1,0}^{u-d}$ lies between the Roper mass and the nucleon-pion energy indicating a possible admixture between the Roper and the π^+N state.
- (iv) The first excited-state energy at the source for the isovector case $E_{1,p}^{u-d}$, unlike the isoscalar case, $E_{1,p}^{u-d}$ follows the nucleon-pion state at lower $\vec{p}^2$.

The deviation from the Roper energy, which is the excited-state energy obtained in the two-point function, shows that the choice of allowing for different excited-state energies between the two- and three-point functions is justified. The difference between the excited-state energies of isoscalar and isovector cases, especially at low Q^2 values, also explains the need for different excited-state energy parametrization in multistate fits for the two channels.

B. Disconnected contributions

For the disconnected contributions, we take sink momenta $\vec{p}' = \frac{2\pi}{L}\vec{k}$ with $\vec{k}^2 = 1$ and 2 in addition to the

case $\vec{p}' = 0$. The set of equations yielding G_E and G_M cannot be disentangled and are overdetermined (see, for example, Appendix B), and we thus use the SVD of the system of equations to solve for the electromagnetic form factors. To summarize, for a given Q^2 , with N equations, we have

$$\Pi^\mu(\Gamma_\nu; \vec{p}', \vec{p}) = \mathcal{G}^{\mu\nu}(\vec{p}', \vec{p})F(Q^2), \quad (31)$$

where the matrix $\mathcal{G}$ is an $N \times 2$ array of kinematic factors and $F(Q^2)^T = [G_E(Q^2), G_M(Q^2)]$. Taking a SVD of the coefficient matrix $\mathcal{G}$,

$$\mathcal{G} = U\Sigma V, \quad (32)$$

we can then extract the form factors by taking

$$F(Q^2) = V^\dagger \Sigma^{-1} U^\dagger \Pi(\vec{p}, \vec{p}'): \quad (33)$$

- (i) For each Q^2 , we segregate the equations corresponding to unique $(\vec{p}^2, \vec{p}'^2)$.
- (ii) The nucleon mass and energy that appears in the coefficient matrix, $\mathcal{G}$ is set from the fits carried out for the connected contributions.
- (iii) After we obtain the SVD of $\mathcal{G}$ for a given $(\vec{p}^2, \vec{p}'^2)$, compute the rotated three-point function $\tilde{C}_{\mu\nu}(\vec{p}, \vec{p}'; t_s, t_{\text{ins}}) \equiv U^\dagger C_{\mu\nu}^{u+d, \text{disc}}(\Gamma_\nu, \vec{p}, \vec{p}'; t_s, t_{\text{ins}})$.

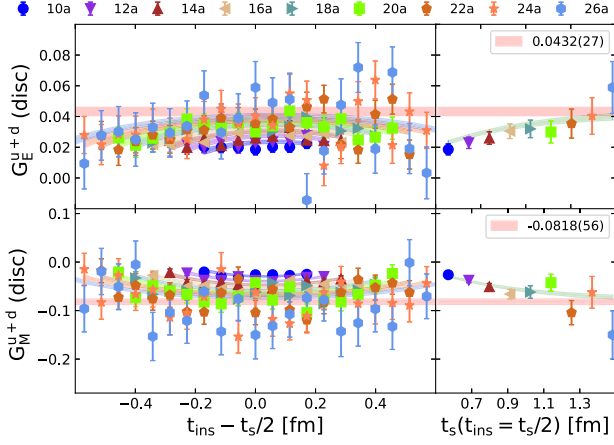


FIG. 6. The renormalized isoscalar disconnected $G_E(Q^2)$ and $G_M(Q^2)$ for $Q^2 = 0.307 \text{ GeV}^2$ for the cD211.054.96 ensemble. The left column shows G_E (top) and G_M (bottom) for source-sink separation as indicated in the header, for all insertion times t_{ins} . The colored bands in the left and the light green band in the right column correspond to the fit results from the two-state fit. The right column shows the value of the ratio for $t_{\text{ins}} = t_s/2$.

- (iv) Only the first two rows of $\tilde{C}_{\mu\nu}(\vec{p}, \vec{p}'; t_s, t_{\text{ins}})$ contribute to the form factors; therefore, we fit these rows with the two-point functions $C(\vec{p}, t_s)$ and $C(\vec{p}', t_s)$.
- (v) This results in a 14-parameter fit, with three-state fits to each of the two-point functions (three coefficient parameters and two excited-state energy parameters) and a two-state fit to the rotated three-point function (four coefficient parameters for each row). The first excited-state energies between the two- and three-point functions are shared in this case, since the larger statistical errors for the disconnected do not allow following the procedure of the connected

analysis. The nucleon ground-state energy at rest is fixed, and the dispersion relation is used to set the nucleon ground-state energy at source or sink momentum. We obtain the ground-state matrix element using Eq. (15).

- (vi) Following the fit, we multiply the resulting two-element vector with $V^\dagger \Sigma^{-1}$.
- (vii) We average over the results obtained for multiple $(\vec{p}, \vec{p}')$ pairs contributing to the same Q^2 .

To demonstrate this fitting procedure, in Fig. 6, we show the ratio formed for the isoscalar disconnected form factor for the cD211.054.96 at $Q^2 = 0.307 \text{ GeV}^2$ for $G_E(Q^2)$ and $G_M(Q^2)$. To show convergence in t_s , in the second column, the value of the ratio at the midpoint $t_{\text{ins}} = t_s/2$ is shown.

The inclusion of momenta at sink leads to a total number of Q^2 values in the range of $[0,1)$ to 224 for ensemble cB211.072.64 and 262 each for ensembles cC211.060.80 and cD211.054.96. In Fig. 7, we present a comparison of the disconnected data obtained for ensemble cD211.054.96 when $\vec{p}'^2 = 0$ (maroon points) and when $p'^2 \in (0, 1, 2)$ (blue points) for $Q^2 \in [0, 0.5] \text{ GeV}^2$. Furthermore, we find that the range of fit ranges which yield stable fits are limited and that systematic effects from varying within these fit ranges are suppressed compared to the relatively larger statistical errors. We therefore take our result for the disconnected form factors from the single fit-range combinations listed in Table VII. For all fits, the maximum source-sink separation used is kept the same as in the connected case.

The isovector and connected and disconnected contributions to the isoscalar form factors are shown in Fig. 8 as a function of Q^2 for the three ensembles analyzed in this work. The errors are as described above; namely, for the connected, they are a result of the model-average procedure, while for the disconnected, they are statistical from

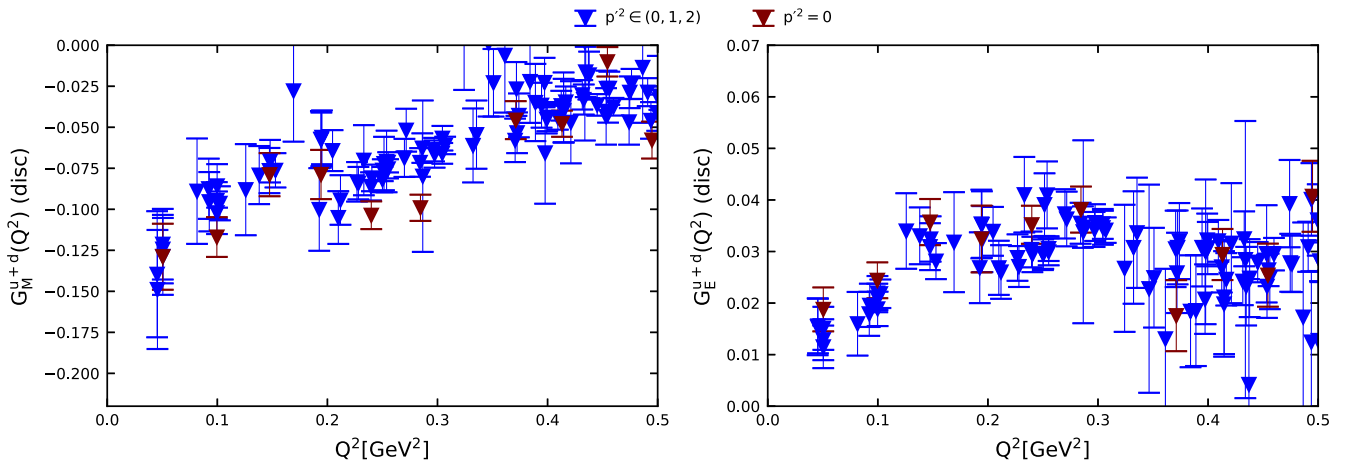


FIG. 7. $G_M(Q^2)$ (left) and $G_E(Q^2)$ (right), disconnected isoscalar form factors as a function of Q^2 for the ensemble cD211.54.96 obtained while allowing finite sink momenta $p'^2 \in (0, 1, 2)$ (blue downward-pointing triangles) and while setting sink momenta to zero (maroon downward-pointing triangles).

TABLE VII. Values of lowest source-sink separation in three point function $t_s^{\text{low},3\text{pt}}$, minimum insertion time used $t_{\text{ins}}^{\text{source}}$ ($t_{\text{ins}}^{\text{sink}} = t_{\text{ins}}^{\text{source}}$), and lowest source-sink separation in two-point function $t_s^{\text{low},2\text{pt}}$ used in the fits to the disconnected data.

Ensemble	$t_s^{\text{low},3\text{pt}}/a$	$t_{\text{ins}}^{\text{source}}/a$	$t_s^{\text{low},2\text{pt}}/a$
cB211.72.64	8	2	2
cC211.60.80	10	2	3
cD211.54.96	10	2	4

the fit ranges indicated in Table VII. Furthermore, for the case of the disconnected contributions, the large number of combinations of $\vec{q}$ and $\vec{p}'$ yields values of Q^2 that would be too close to each other to discern in Fig. 8. Therefore, for

the purpose of a clear visualization, the Q^2 axis has been subdivided into 50 bins, and the form factor values are averaged in their respective bins, weighted by their errors (see Appendix D).

V. ANALYSIS OF EXTRACTED FORM FACTORS

In this section, we describe our approach for fitting the Q^2 dependence of the form factors and taking the continuum limit. This is carried out for the proton and neutron form factors rather than the isovector and isoscalar. These are obtained via the combinations in Eq. (6), which we construct per jackknife bin. Within our errors, fitting the isoscalar and isovector form factors and combining them to

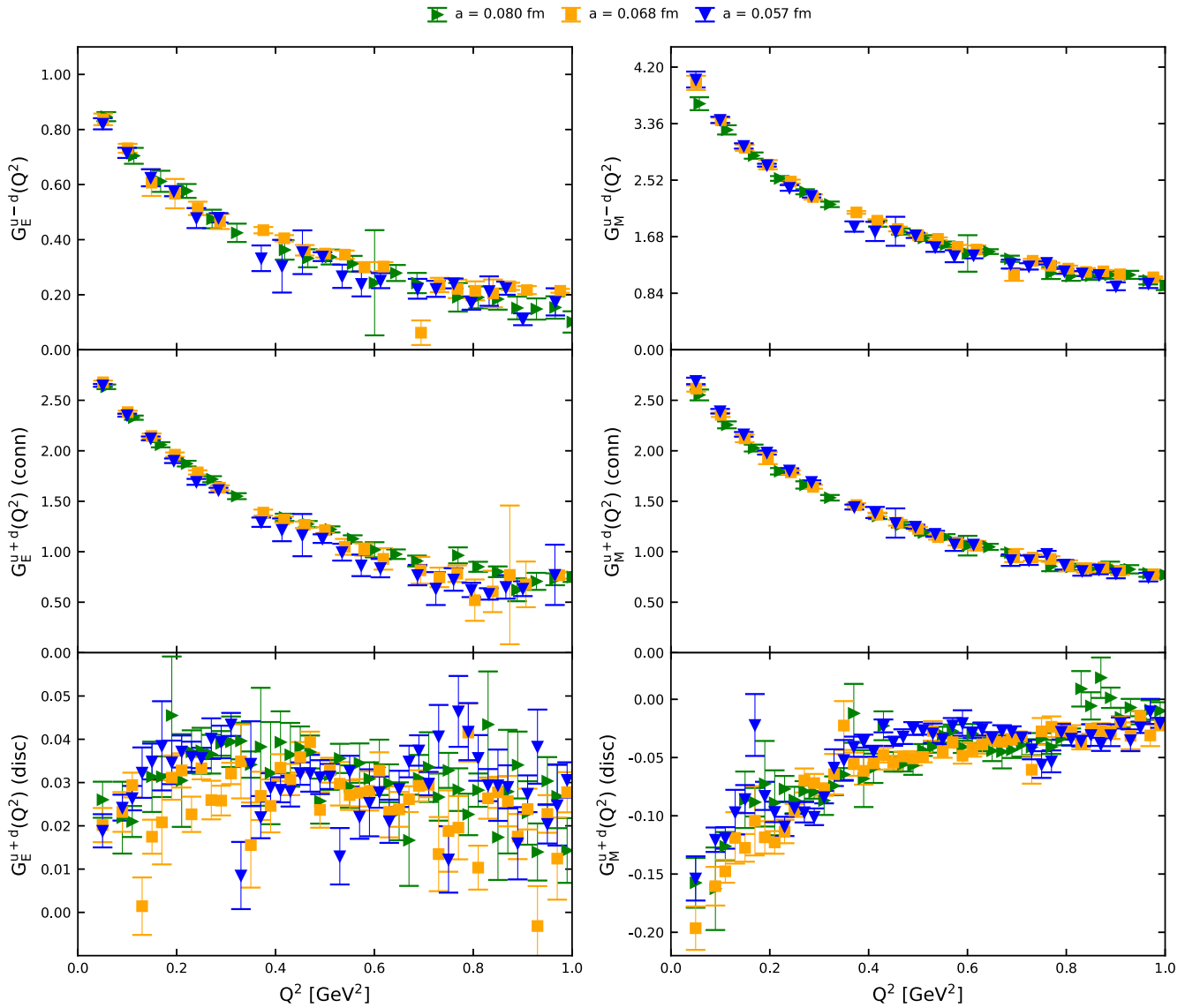


FIG. 8. $G_E(Q^2)$ (left) and $G_M(Q^2)$ (right), connected isovector (top), connected isoscalar (center), and disconnected isoscalar (bottom) form factors as a function of Q^2 for the three ensembles analyzed here, namely, cB211.72.64 (green right-pointing triangles), cC211.60.80 (orange squares), and cD211.54.96 (blue downward-pointing triangles).

the proton and neutron form factor at the continuum limit yields consistent results.

For adding the isoscalar connected and disconnected contributions, which are obtained at different Q^2 values due to the inclusion of additional sink momenta in the disconnected, we first fit the Q^2 dependence of the disconnected data and take the values that match the connected from the fit. We use the Galster-like form for the electric and the dipole form for the magnetic form factor, which are explained in the remainder of this section.

In general, we carry out the analysis of the form factors in two ways. In what we will refer to as the *two-step approach*, we fit the Q^2 dependence of each ensemble individually, and then in a second step, we take the continuum limit as an extrapolation in a^2 of either the form factor at a given set of Q^2 values or the observables of interest, i.e., the radii and magnetic moments. In what we will refer to as a *one-step approach*, we fit the form factors of the three ensembles together in a single fit, either including a^2 -dependent terms in the fit of the Q^2 dependence or with no a^2 dependence, i.e., assuming no cutoff effects. In either case, the jackknife mean and covariance matrix are used to take into account correlations between data within the same ensemble. For fitting the Q^2 dependence, we use both z expansion and dipole forms. For the case of the neutron electric form factor which is zero for $Q^2 = 0$, we use the Galster-like parametrization.

A. Dipole form

The dipole form is given by

$$G(Q^2) = \frac{g}{(1 + \frac{Q^2}{M^2})^2}, \quad (34)$$

with g and M^2 the fit parameters. Employing Eq. (8) for the radius, we have $\langle r^2 \rangle = \frac{12}{M^2}$.

For the one-step approach, we incorporate cutoff effects by including the a^2 dependence of the observables, namely, the charge and the radius,

$$g(a^2) = g_0 + a^2 g_2 \quad \text{and} \quad \langle r^2(a^2) \rangle = \langle r \rangle_0^2 + a^2 \langle r^2 \rangle_2,$$

which, when substituting in Eq. (34), gives

$$G(Q^2, a^2) = \frac{g(a^2)}{[1 + \frac{Q^2}{12} \langle r^2(a^2) \rangle]^2}. \quad (35)$$

In the one-step approach, we fit all three ensembles to Eq. (35). The radius and charge at the continuum limit are given by the fit parameters g_0 and $\langle r^2 \rangle_0$.

B. Galster-like parametrization

The neutron electric form factor and disconnected contribution to the isoscalar electric form factor vanishes at $Q^2 = 0$. We use the Galster-like parametrization [39], which is zero at $Q^2 = 0$,

$$G(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^2}, \quad (36)$$

where A and B are the fit parameters. In this case, the radius is given by

$$\langle r^2 \rangle = -\frac{3A}{2m_N^2}. \quad (37)$$

While the a^2 dependence can be incorporated in the same way as for the dipole, as we will see, errors for the neutron electric form factor are too large to identify any dependence on the lattice spacing, and we thus fit all three ensembles simultaneously to Eq. (37).

C. z expansion

The z expansion is given by

$$G(Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k(Q^2), \quad (38)$$

where c_k are the fit parameters and

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} + t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} + t_0}} \quad (39)$$

with t_{cut} being the particle production threshold and t_0 being an arbitrary choice in $(-t_{\text{cut}}, \infty)$. We set $t_0 = 0$, which simplifies the relation of the charge and radii to the fit parameters, namely,

$$g = c_0 \quad \text{and} \quad \langle r^2 \rangle = -\frac{3c_1}{2c_0 t_{\text{cut}}} \quad \text{with} \quad t_0 = 0. \quad (40)$$

The dependence on the lattice spacing is introduced in the coefficients of the z expansion, similar to the procedure adopted in Ref. [38],

$$c_k(a^2) = c_{k,0} + a^2 c_{k,2}, \quad (41)$$

which modifies the z expansion as

$$G(Q^2, a^2) = \sum_{k=0}^{k_{\max}} c_k(a^2) z^k(Q^2). \quad (42)$$

We furthermore enforce smooth convergence of the form factor to zero at $Q^2 \rightarrow \infty$ [40], which requires

TABLE VIII. Results on the proton electric charge radius $\langle r_E^2 \rangle^p$ and $\tilde{\chi}^2$ obtained when using dipole fits. The first three rows show the values obtained from fits to the individual ensembles. The continuum result (marked $a = 0$) is obtained either by single fit to the combined Q^2 and a^2 dependence (one-step approach) or by linear extrapolation of the values of the radii obtained at the individual ensembles (two-step approach).

Ensemble	$\langle r_E^2 \rangle^p$ (fm ²)	$\tilde{\chi}^2$
cB211.72.64	0.619(31)	0.518
cC211.60.80	0.609(17)	0.635
cD211.54.96	0.635(20)	1.969
$a = 0$, 1-step	0.650(52)	1.042
$a = 0$, 2-step	0.650(52)	0.552

$$\sum_{k=0}^{k_{\max}} c_k \frac{d^n z^k}{dz^n} \Big|_{z=1} = 0 \quad \text{with} \quad n = 0, 1, 2, 3. \quad (43)$$

In our case, it is sufficient to enforce this condition for $n = 0$, which implies $\sum_{k=0}^{k_{\max}} c_k = 0$, meaning that for a given $k_{\max}$, instead of $k_{\max} + 1$ fit parameters, we have $k_{\max}$, with one being fixed by this condition. We follow our approach used in Ref. [38] for the axial form factors, which was adopted from Refs. [41,42]. Namely, we use Gaussian priors for the parameters not directly related to the charge and radius; i.e., for $k \geq 2$,

$$c_{k,0} \sim 0(w/k), \quad c_{k,2} \sim 0(20w/k) \quad \text{for } k \geq 2. \quad (44)$$

$w \leq 5$ is a hyperparameter controlling the width of the priors and which we vary together with the $k_{\max}$.

D. Proton electric form factor

We first present our analysis of the proton electric form factor $G_E^p(Q^2)$.

1. Dipole analysis

Since we use the lattice conserved current, the form factor is equal to 1 at $Q^2 = 0$ for any lattice spacing by symmetry, and we can therefore set $g = 1$ in Eq. (34) and $g_0 = 1$, $g_2 = 0$ in Eq. (35).

In Table VIII, we summarize the values of $\langle r_E^2 \rangle^p$ obtained when fitting the individual ensembles and at the continuum limit when using either the one- or two-step approaches. In Fig. 9, we plot the data for each ensemble and the bands corresponding to the dipole fits. We vary the largest momentum transfer (Q_{cut}^2) that we include in the fit and monitor the $\tilde{\chi}^2 \equiv \chi^2/N_{\text{dof}}$, where N_{dof} are the number of degrees of freedom. We find that a value of $Q_{\text{cut}}^2 = 0.4 \text{ GeV}^2$ describes all three ensembles well, as can be seen in Fig. 9, as well as via the values of $\tilde{\chi}^2$ quoted in Table VIII.

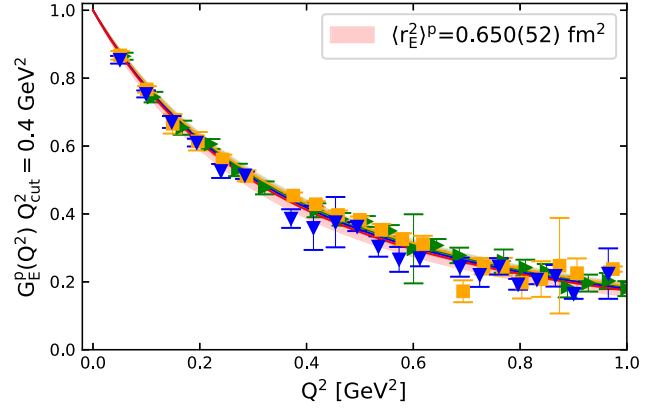


FIG. 9. Results on the proton electric form factor $G_E^p(Q^2)$ for each ensemble denoted by the same symbols as in Fig. 8. The bands show the results obtained by fitting the dipole form to each ensemble (green band for cB211.72.64, orange band for cC211.60.80, and blue band for cD211.54.96) and at the continuum limit (red band) using the “one-step” approach to fit the combined (Q^2, a^2) dependence using $Q_{\text{cut}}^2 = 0.4 \text{ GeV}^2$. We find $\langle r_E^2 \rangle^p = 0.650(52) \text{ fm}^2$ at the continuum obtained from the one-step approach. The fit $\tilde{\chi}^2 = 1.042$.

A comparison of the one- and two-step approaches is shown in Fig. 10, where the linear extrapolation of the radius to the continuum limit is shown for the case of the two-step approach and compared against the value obtained from the combined fit in the one-step approach. As can be seen, the two approaches yield the same result.

The analysis using the dipole fits has shown that the two- and one-step approaches yield the same results at the continuum limit. We note that an advantage of the one-step approach is that it provides for a single $\tilde{\chi}^2$ value to the combined continuum extrapolation and Q^2 -dependence fit. The one-step approach is thus more easily incorporated within a model-averaging procedure rather than the two-step approach.

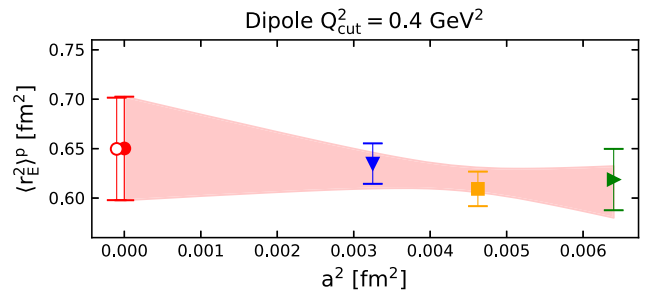


FIG. 10. $\langle r_E^2 \rangle^p$ obtained using dipole fits to the cB211.72.64 (green right-pointing triangle), cC211.60.80 (orange square), and cD211.54.96 (blue downward-pointing triangle) ensembles. A linear fit to the three values of the radii is also shown (red band) yielding the result at $a^2 = 0$ (filled red circle), which corresponds to the two-step approach. The one-step approach result is also shown at $a^2 = 0$, slightly shifted for clarity (open red circle).

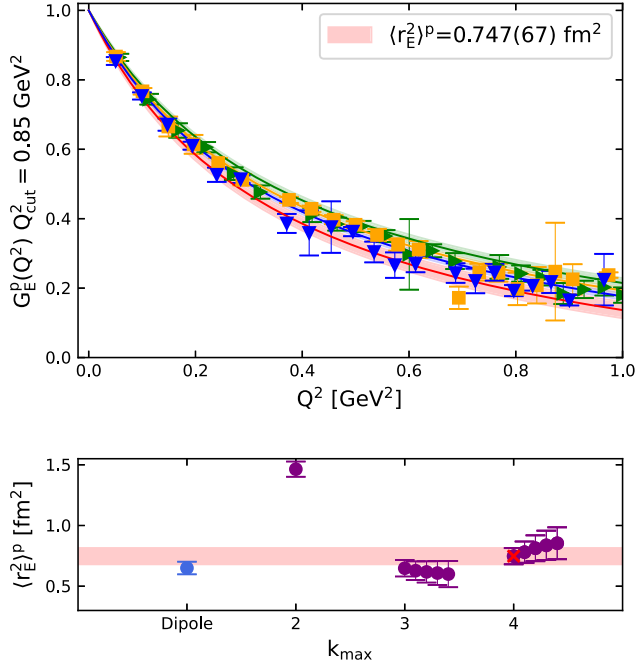


FIG. 11. The z -expansion fit to $G_E^p(Q^2)$ with $Q_{\text{cut}}^2 = 0.85 \text{ GeV}^2$ (top panel) and the dependence of the extracted radius on the z -expansion order (k_{max}) and prior widths (w) (bottom panel). In the top panel, the green, orange, and blue bands show the result of the one-step z -expansion fit for $a = 0.080, 0.068$, and 0.057 fm , respectively, and the red band for $a = 0$. In the bottom panel, for $k_{\text{max}} = 3$ and 4 , $w = 1$ for the leftmost point and increases by 1 up to $w = 5$. For $k_{\text{max}} = 2$, no priors are employed, as explained in the text. In the top panel, we use $k_{\text{max}} = 4$ and $w = 1$, indicated by the red cross and the red band spanning horizontally the bottom panel. The blue point is the result from the one-step dipole fit, shown for comparison.

2. z -expansion results

For the z -expansion analysis, we restrict to presenting results using the one-step approach only.

For the case of the proton electric form factor, we set $c_{0,0} = 1$ and $c_{0,2} = 0$ so that $G(0, a^2) = 1$ for any value of a^2 . For the z -expansion fits, we vary Q_{cut}^2 , the order of the expansion k_{max} , and the width of the priors w . In Fig. 11, we show an example fit with $Q_{\text{cut}}^2 = 0.85 \text{ GeV}^2$, as well as the

TABLE IX. The proton electric charge radius $\langle r_E^2 \rangle^p$ obtained using the one-step approach and the z -expansion with $k_{\text{max}} = 4$ and $w = 1$ as we increase Q_{cut}^2 . The values of $\tilde{\chi}^2$ are also provided.

$Q_{\text{cut}}^2 \text{ (GeV}^2\text{)}$	$\langle r_E^2 \rangle^p \text{ (fm}^2\text{)}$	$\tilde{\chi}^2$
0.40	0.700(76)	0.770
0.50	0.713(72)	0.638
0.70	0.716(69)	1.016
0.85	0.747(67)	1.032
1.00	0.816(64)	1.261

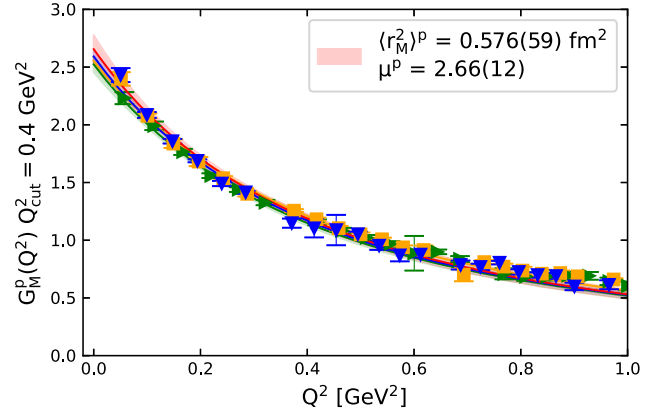


FIG. 12. The same as in Fig. 9 but for $G_M^p(Q^2)$.

value obtained for the radius when varying k_{max} and w . For comparison, we also show the result of the dipole fit for the radius. Note that for $k_{\text{max}} = 2$ no prior is used since c_0 is fixed, c_1 is free with no prior, and $c_2 = -(c_0 + c_1)$, making this effectively a single parameter fit. For $k_{\text{max}} = 3$ and 4 , we employ priors as in Eq. (44) starting with $w = 1$ and increasing by 1 from left to right until $w = 5$. We see agreement between the z -expansion fits for $k_{\text{max}} = 3$ and 4 and relatively weak dependence on w . The bands showing the form factor Q^2 dependence in Fig. 11 are from the fits with $k_{\text{max}} = 4$ and $w = 1$.

Once the choice for the order of z expansion and the width of the prior are set, we vary the maximum momentum transfer used for the fits (Q_{cut}^2), obtaining the results shown in Table IX.

E. Proton magnetic form factor

1. Dipole results

The same analysis is carried out for the proton magnetic form factor, starting with the dipole fits. In this case, the proton magnetic moment (μ^p) is a fit parameter, in addition to the magnetic radius $\langle r_M^2 \rangle^p$. In Fig. 12, we show the fit

TABLE X. The proton magnetic moment (μ^p), magnetic radius ($\langle r_M^2 \rangle^p$), and $\tilde{\chi}^2$ extracted from dipole fits to the proton magnetic form factor for each ensemble, along with continuum extrapolation results for both one- and two-step approaches, as in Table VIII. For the two-step approach, the $\tilde{\chi}^2$ is given as $(\tilde{\chi}_{(r^2)}^2, \tilde{\chi}_\mu^2)$ where the former corresponds to the fit for $\langle r_M^2 \rangle^p$ and the latter corresponds to the fit for μ^p .

Ensemble	μ^p	$\langle r_M^2 \rangle^p \text{ (fm}^2\text{)}$	$\tilde{\chi}^2$
cB211.72.64	2.524(67)	0.562(34)	1.016
cC211.60.80	2.553(37)	0.527(17)	2.230
cD211.54.96	2.592(49)	0.569(24)	2.732
$a = 0$, 1-step	2.66(12)	0.576(59)	2.326
$a = 0$, 2-step	2.66(12)	0.569(60)	(2.182, 0.031)

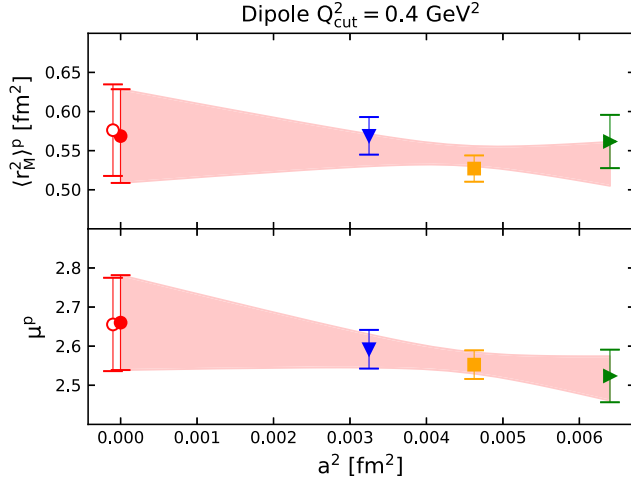


FIG. 13. Results on $\langle r_M^2 \rangle^p$ (top panel) and μ^p (bottom panel) obtained using dipole fits to the three ensembles. The notation is the same as that of Fig. 10.

results using the same convention as used for the proton electric form factor. For completeness, we carry out both one- and two-step approaches as in the case of the dipole fits to the electric form factor, with results summarized in Table X. A continuum extrapolation in a^2 results in consistent values for the radius and the magnetic moment when using either one- or two-step approaches, as shown in Fig. 13.

2. *z*-expansion results

The analysis of the proton magnetic form factors using the z expansion follows that of the proton electric form factor, where now the $k = 0$ coefficients, $c_{0,0}$ and $c_{0,2}$, are fit parameters. The investigation of k_{max} and prior widths w is shown in Fig. 14. A stronger dependence on w is observed compared to the electric case, with convergence achieved for $w = 3$. At this value of w , we observe agreement between $k_{\text{max}} = 3$ and 4. We take $k_{\text{max}} = 3$, with results shown in Table XI. We note that $k_{\text{max}} = 3$ results in the same number of fit parameters as the electric case for $k_{\text{max}} = 4$.

F. Neutron electric form factor

The neutron electric form factor at $Q^2 = 0$ is zero by current conservation. At finite Q^2 , due to the small absolute magnitude of the signal arising from the subtraction in Eq. (6), our data have large relative statistical errors, as shown in Fig. 15. Within these errors, we cannot discern a dependence on the lattice spacing, and we thus fit all data using the Galster-like parametrization without including any a^2 dependence. Our fit is thus constrained by the data with the smaller statistical errors, and we find that taking $Q_{\text{cut}}^2 = 0.3 \text{ GeV}^2$ yields a fit with $\tilde{\chi}^2 = 0.656$. The value of the neutron electric radius extracted from this fit is $\langle r_E^2 \rangle^n = -0.147(48) \text{ fm}^2$.

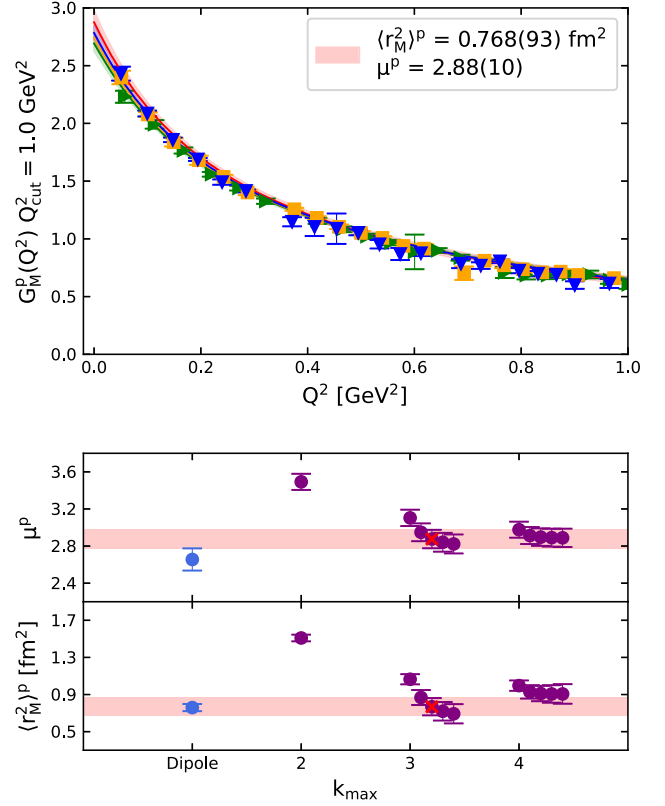


FIG. 14. Results on $G_M^p(Q^2)$ using the z -expansion with $Q_{\text{cut}}^2 = 1.0 \text{ GeV}^2$ and $k_{\text{max}} = 4$ and $w = 1$ (top panel) and the dependence of the extracted magnetic moment and radius on the z -expansion order (k_{max}) and prior widths (w) (bottom panel). In the top panel, the green, orange, and blue bands show the result of the one-step z -expansion fit for the B, C, and D ensembles, respectively, and the red band corresponds to the continuum limit. In the bottom panel, we show results on the magnetic moment μ^p and square radius when using the z expansion with $k_{\text{max}} = 2, 3$, and 4, and with $w = 1$ for the leftmost points and increasing by 1 up to $w = 5$. The blue point is the result from the one-step dipole fit, shown for comparison.

G. Neutron magnetic form factor

1. Dipole results

Our analysis of the neutron magnetic form factor follows that of the proton. The dipole fits to the form factor are

TABLE XI. The proton magnetic moment (μ^p), magnetic radius ($\langle r_M^2 \rangle^p$), and $\tilde{\chi}^2$ obtained using the one-step approach and the z expansion with $k_{\text{max}} = 3$ and $w = 3$ as we increase Q_{cut}^2 .

$Q_{\text{cut}}^2 \text{ (GeV}^2\text{)}$	μ^p	$\langle r_M^2 \rangle^p \text{ (fm}^2\text{)}$	$\tilde{\chi}^2$
0.40	2.97(12)	0.98(12)	1.172
0.50	2.92(12)	0.91(11)	1.007
0.70	2.89(10)	0.80(10)	1.311
0.85	2.86(10)	0.79(10)	1.338
1.00	2.88(10)	0.768(93)	1.282

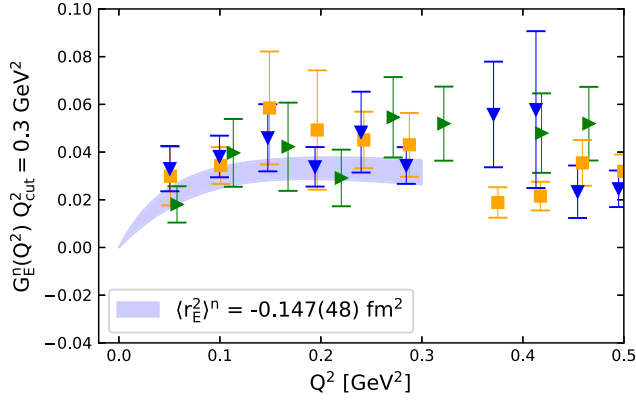


FIG. 15. Galster-like fit to $G_E^n(Q^2)$ with $Q_{\text{cut}}^2 = 0.3 \text{ GeV}^2$. The blue band shows the resulting fit to all ensembles neglecting any a^2 dependence. We find $\langle r_E^2 \rangle^n = -0.147(48) \text{ fm}^2$ and $\tilde{\chi}^2 = 0.656$.

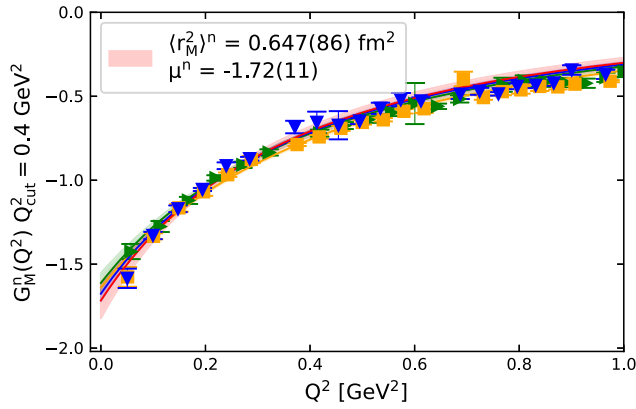


FIG. 16. The same as in Fig. 12 but for $G_M^n(Q^2)$.

shown in Fig. 16, and in Fig. 17, we compare the continuum extrapolation between the one- and two-step approaches, showing that also in this case the two approaches yield consistent results. In Table XII, we

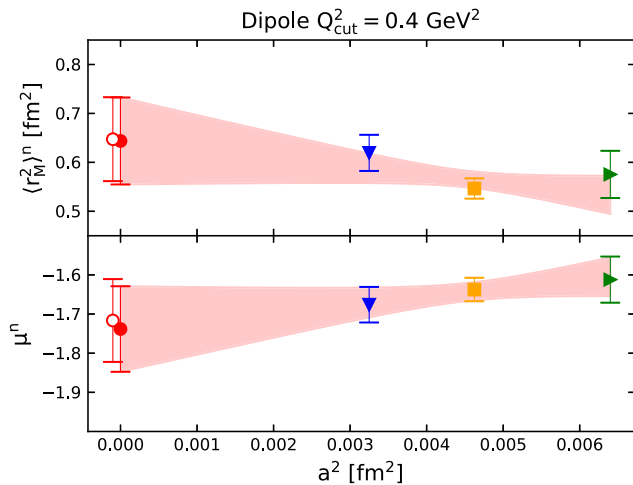


FIG. 17. The same as in Fig. 13 but for the neutron magnetic radius $\langle r_M^2 \rangle^n$ (top) and the neutron magnetic moment μ^n (bottom).

TABLE XII. The neutron magnetic moment (μ^n), magnetic radius ($\langle r_M^2 \rangle^n$), and $\tilde{\chi}^2$ extracted from dipole fits to the neutron magnetic form factor for each ensemble, along with continuum extrapolation results for both one- and two-step approaches, as in Table X.

Ensemble	μ^n	$\langle r_M^2 \rangle^n \text{ (fm}^2\text{)}$	$\tilde{\chi}^2$
cB211.72.64	-1.612(59)	0.575(48)	0.770
cC211.60.80	-1.637(30)	0.547(21)	1.883
cD211.54.96	-1.676(45)	0.619(37)	2.182
$a = 0$, 1-step	-1.72(11)	0.647(86)	2.072
$a = 0$, 2-step	-1.74(11)	0.644(89)	(2.207, 0.052)

summarize the values of the neutron magnetic radius and the neutron magnetic moment obtained using the one- and two-step approaches.

2. z-expansion results

We proceed to carry out the analysis using the z expansion following the analysis presented for the proton case. In Fig. 18, we show the z -expansion fits, with the analysis of k_{max} and w in the bottom panel. We see that as in the proton case, the fits with $k_{\text{max}} = 3$ and $w = 3$ achieve convergence for both the radius and the moment. Fixing these parameters, we vary Q_{cut}^2 and give the results in Table XIII.

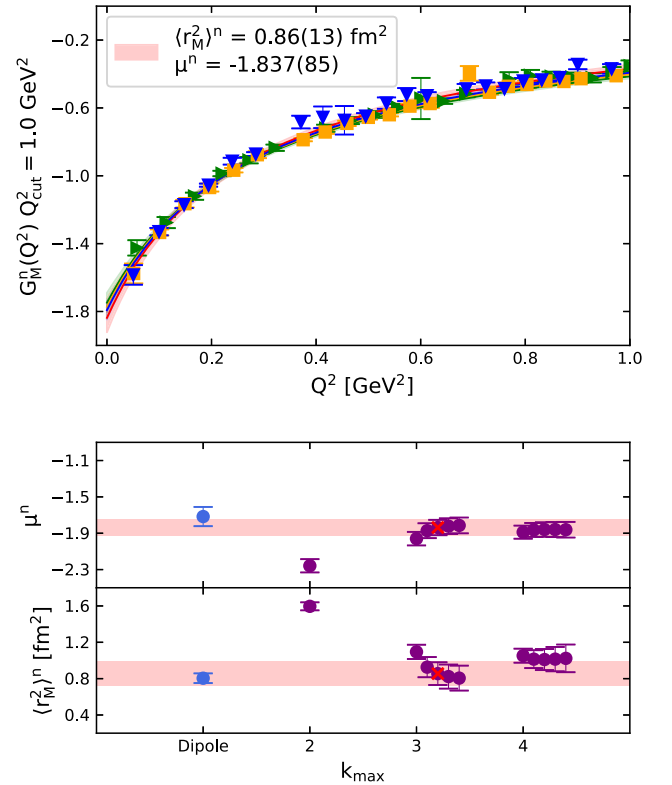


FIG. 18. The notation is the same as the one in Fig. 14 but for $G_M^n(Q^2)$.

TABLE XIII. The neutron magnetic moment (μ^n), magnetic radius ($\langle r_M^2 \rangle^n$), and $\tilde{\chi}^2$ obtained using the one-step approach and the z -expansion with $k_{\max} = 3$ and $w = 3$ as we increase Q_{cut}^2 .

Q_{cut}^2 (GeV ²)	μ^n	$\langle r_M^2 \rangle^n$ (fm ²)	$\tilde{\chi}^2$
0.40	-1.95(12)	1.11(17)	0.987
0.50	-1.88(11)	1.02(16)	0.890
0.70	-1.856(92)	0.90(14)	1.113
0.85	-1.838(91)	0.88(14)	1.024
1.00	-1.837(85)	0.86(13)	1.012

VI. RESULTS AND COMPARISON TO OTHER STUDIES

A. Final results on the electric and magnetic radii and moments

For our final results, we use the one-step approach with either a linear or no a^2 dependence since, as already explained, compared to the two-step approach, it allows for a single value of $\tilde{\chi}^2$ for the combined Q^2 dependence and continuum extrapolation fit. The models used are given in Table XIV and include one-step dipole and z -expansion fits with varying values of Q_{cut}^2 . We note the exception of the neutron electric form factor, for which we use a single, Galster-like fit for the description of the Q^2 dependence. The probability associated with each fit is provided using the AIC.

A comparison of these fits based on the fit probability shows that for the electric and magnetic proton form factors and for the magnetic neutron form factor, the one-step z -expansion parametrization with linear a^2 dependence has

the highest weight, with the dipole fit effectively not contributing to the model average. Furthermore, all three cases favor a higher Q_{cut}^2 , with the proton electric favoring $Q_{\text{cut}}^2 = 0.85$ GeV² and the magnetic proton and neutron form factors favoring $Q_{\text{cut}}^2 = 1$ GeV².

For the final row of Table XIV, we quote our statistical and systematic errors separately, the latter derived from the Q_{cut}^2 and model dependence. Namely, if p_j is the probability of the combination of Q_{cut}^2 and model j , with observable given by $\bar{\mathcal{O}}_j(\sigma_j)$, we define

$$\begin{aligned} \langle \mathcal{O} \rangle &= \bar{\mathcal{O}}(\sigma^{\text{stat}})(\sigma^{\text{sys}}) \quad \text{where,} \quad \bar{\mathcal{O}} = \sum_j \bar{\mathcal{O}}_j p_j, \\ (\sigma^{\text{stat}})^2 &= \sum_j \sigma_j^2 p_j \quad \text{and,} \quad (\sigma^{\text{sys}})^2 = \sum_j \bar{\mathcal{O}}_j^2 p_j - \bar{\mathcal{O}}^2. \end{aligned} \quad (45)$$

Our results for the radii and magnetic moments are shown in Fig. 19 for varying Q_{cut}^2 and for the one-step approach with and without an a^2 dependence. We overall observe consistent results among the variations. The model-averaged result, also shown, is consistent with the Particle Data Group (PDG) values [43] for these quantities.

Our final results for the proton and neutron electromagnetic form factors are shown in Fig. 20, where we also compare with experimental fits and data. For our continuum limit band, we show the highest probability fit for each case, namely, the z expansion with $Q_{\text{cut}}^2 = 0.85$ GeV² for G_E^p and $Q_{\text{cut}}^2 = 1$ GeV² for G_M^p and G_M^n and using the Galster-like parametrization for G_E^n . For comparison with experiment, for the proton form factors, we use a

TABLE XIV. Results for the proton and neutron electric and magnetic radii squared and for their magnetic moments. We use the one-step approach and a z expansion or dipole with a linear a^2 dependence [“ z -exp(a^2)” and “Dipole(a^2)” in the first column, respectively] or with no a^2 dependence [“ z -exp” and “Dipole” in the first column, respectively]. For each combination of model and Q_{cut}^2 , we indicate the probability from the model-averaging procedure in the columns labeled “prob.” For the neutron electric case, we use the Galster-like parametrization with a single Q_{cut}^2 and no a^2 dependence as explained in the text.

Model	Q_{cut}^2 (GeV ²)	$\langle r_E^2 \rangle^p$ (fm ²)	Prob (%)	μ^p	$\langle r_M^2 \rangle^p$ (fm ²)	Prob (%)	μ^n	$\langle r_M^2 \rangle^n$ (fm ²)	Prob (%)	$\langle r_E^2 \rangle^n$ (fm ²)
z -exp(a^2)	0.40	0.700(76)	0.000	2.97(12)	0.98(12)	0.000	-1.95(12)	1.11(17)	0.000	...
z -exp(a^2)	0.50	0.713(72)	0.186	2.92(12)	0.91(11)	0.027	-1.88(11)	1.02(16)	0.000	...
z -exp(a^2)	0.70	0.716(69)	0.565	2.89(10)	0.80(10)	0.033	-1.856(92)	0.90(14)	0.000	...
z -exp(a^2)	0.85	0.747(67)	82.151	2.86(10)	0.79(10)	0.735	-1.838(91)	0.88(14)	0.613	...
z -exp(a^2)	1.00	0.816(64)	5.983	2.88(10)	0.768(93)	79.124	-1.837(85)	0.86(13)	70.807	...
z -exp	0.40	0.652(49)	0.000	2.849(68)	0.93(11)	0.000	-1.855(65)	1.03(16)	0.000	...
z -exp	0.50	0.658(48)	0.301	2.828(66)	0.88(10)	0.010	-1.840(63)	0.98(15)	0.000	...
z -exp	0.70	0.651(47)	0.644	2.763(63)	0.75(10)	0.010	-1.795(59)	0.85(14)	0.000	...
z -exp	0.85	0.636(46)	10.158	2.757(61)	0.739(94)	0.277	-1.775(55)	0.79(13)	0.302	...
z -exp	1.00	0.667(46)	0.009	2.744(58)	0.705(87)	19.784	-1.772(52)	0.78(12)	28.278	...
Dipole(a^2)	0.4	0.650(52)	0.000	2.66(12)	0.576(59)	0.000	-1.72(11)	0.647(86)	0.000	...
Dipole	0.4	0.620(12)	0.000	2.551(27)	0.545(13)	0.000	-1.626(22)	0.563(17)	0.000	...
Galster-like	0.3	...	...	...	...	...	...	...	...	-0.147(48)
Final results		0.739(64)(39)		2.849(92)(52)	0.756(92)(25)		-1.819(76)(29)	0.83(12)(03)		-0.147(48)

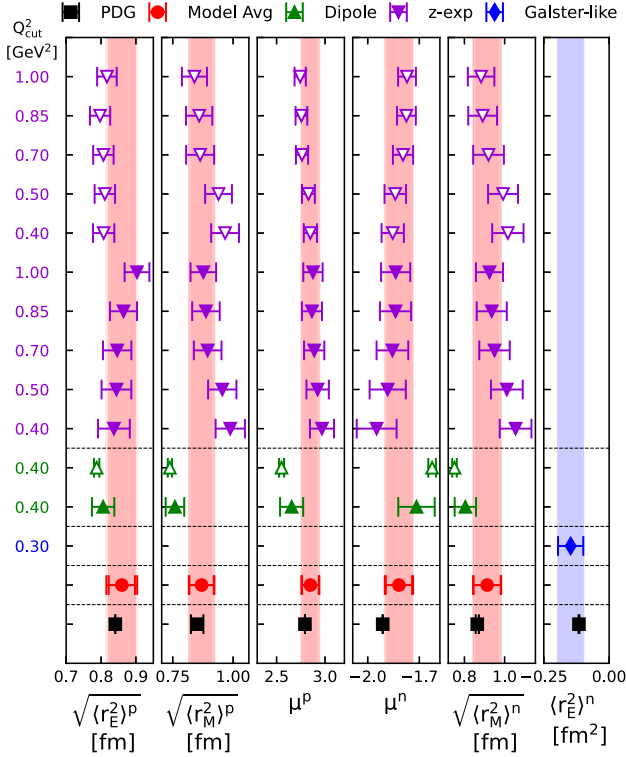


FIG. 19. Electric and magnetic radii and magnetic moments of the proton and neutron for all combinations of Q_{cut}^2 and models considered in Table XIV. Namely, we show the z expansion (purple downward-pointing triangles) and dipole fits (green triangles) using the one-step approach with (filled symbols) or without (open symbols) the a^2 dependence. The blue diamond and blue band corresponds to the Galster-like fit to $G_E^n(Q^2)$. The red point and band denoted “Model average” is obtained by weighting according to the AIC as explained in Sec. IV.

z -expansion fit to experimental data from Ref. [44], and for the neutron form factors, we show experimental data from Refs. [4,45–51,51–63]. The values for the radii and magnetic moments that correspond to the final results of the form factors are included in Fig. 19.

For completeness, we summarize our final result for electric and magnetic radii and magnetic moment of the proton,

$$\begin{aligned} \langle r_E^2 \rangle^p &= 0.739(64)(39) \text{ fm}^2, \\ \langle r_M^2 \rangle^p &= 0.756(92)(25) \text{ fm}^2, \\ \mu^p &= 2.849(92)(25); \end{aligned} \quad (46)$$

the neutron,

$$\begin{aligned} \langle r_E^2 \rangle^n &= -0.147(48) \text{ fm}^2, \\ \langle r_M^2 \rangle^n &= 0.83(12)(03) \text{ fm}^2, \\ \mu^n &= -1.819(76)(29); \end{aligned} \quad (47)$$

and the isovector combinations,

$$\begin{aligned} \langle r_E^2 \rangle^{u-d} &= 0.886(80)(39) \text{ fm}^2, \\ \langle r_M^2 \rangle^{u-d} &= 0.786(73)(20) \text{ fm}^2, \\ \mu^{u-d} &= 4.67(12)(6). \end{aligned} \quad (48)$$

B. Comparison of electric and magnetic radii and moments with other lattice QCD studies

The electromagnetic form factors of the nucleon were among the first nucleon structure quantities studied within lattice QCD, and therefore results, especially for the isovector case in which disconnected contributions cancel, go back several decades. For the purposes of comparing with similar results in the literature, in Fig. 21, we compare our results to a recent calculation by the Mainz Collaboration, which includes a continuum extrapolation using one physical pion mass ensemble and 10 additional ensembles at heavier than physical pion mass [6]. This calculation employs $N_f = 2 + 1$ Coordinated Lattice Simulations (CLS) ensembles and includes disconnected contributions. The summation method is used for the treatment of their excited states, and they perform a simultaneous chiral, continuum, and infinite volume fit using an expression from chiral perturbation theory. Their final result is obtained after a cut in the pion mass of $M_\pi \leq 0.3$ GeV that leaves six of the 11 ensembles contributing. Beyond their quoted final result, we include their result obtained using the z -expansion (denoted as Mainz’23 z -exp. in Fig. 21) for a direct comparison to our results. We observe good agreement with the results by the Mainz group within 1.5σ .

Other recent results in the literature which quote proton and neutron form factors and that include physical point ensembles but lack a continuum limit include two previous studies by the ETMC [8,64] and PACS [9,65]:

- (i) ETMC results include one study using a single ensemble of $N_f = 2$ twisted mass fermions with lattice spacing $a = 0.094$ fm [64] and physical pion mass, as well as a more recent result obtained using only the B ensemble used in this work [8]. Our current work, which includes a continuum extrapolation, yields larger values for the radii compared to our previous results. Beyond the continuum limit extrapolation, this work includes a more thorough excited-state analysis with systematic errors obtained from model averaging.
- (ii) The PACS Collaboration has two results using $N_f = 2 + 1$ stout-smeared, $\mathcal{O}(a)$ -improved Wilson-clover fermions, with $a = 0.084$ fm and lattice volume $128^3 \times 128$ [65] and with $a = 0.063$ fm and lattice volume $160^3 \times 160$ [9]. They performed plateau fits to obtain their final results using source sink separations ranging between 0.82 to 1.35 fm. Their results on proton and neutron electromagnetic form

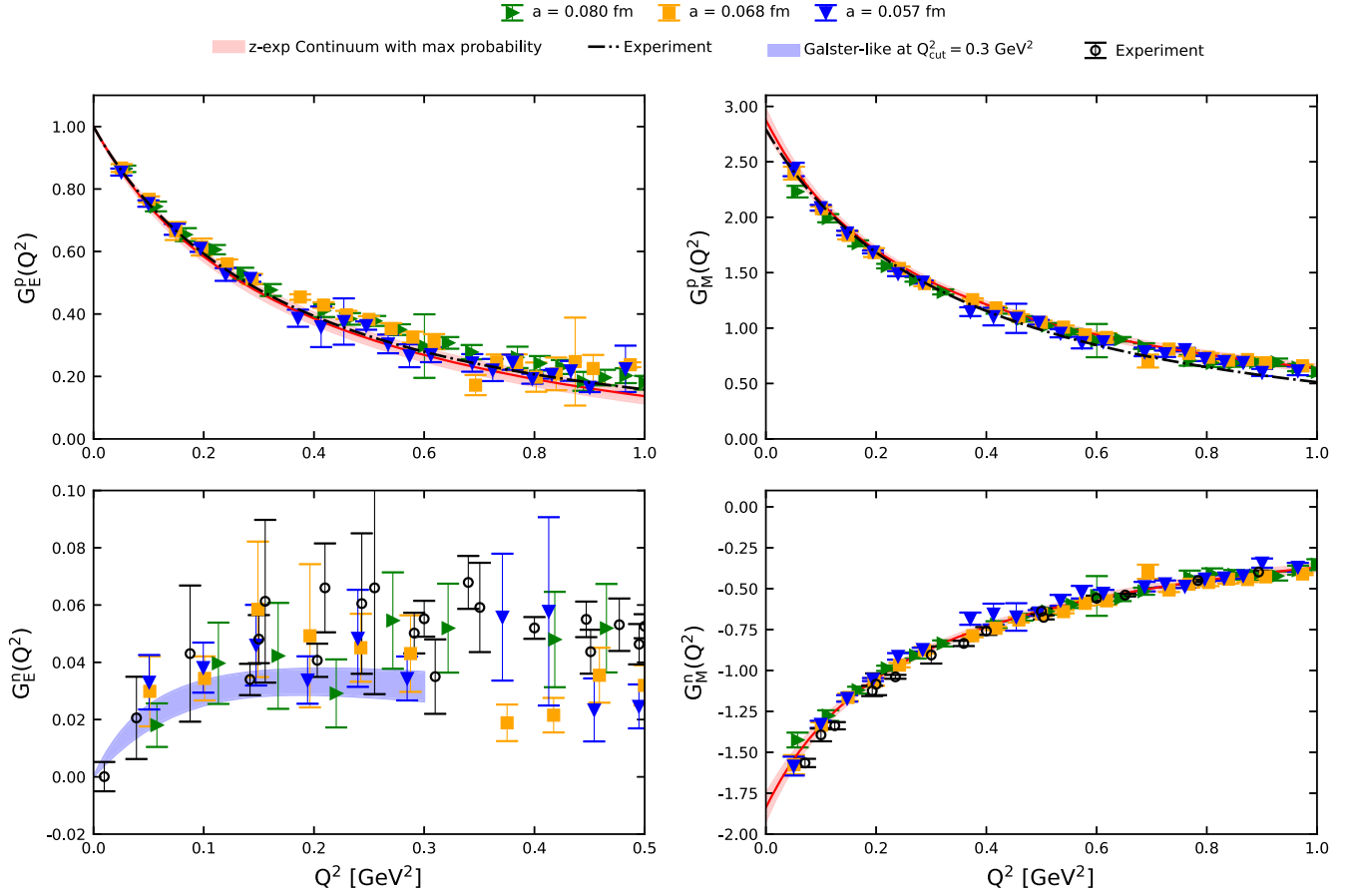


FIG. 20. $G_E(Q^2)$ (left) and $G_M(Q^2)$ (right), proton (top) and neutron (bottom) form factors as a function of Q^2 for the three ensembles analyzed. The red bands indicate the continuum limit using the z expansion with $Q_{\text{cut}}^2 = 0.85 \text{ GeV}^2$ for the case of proton electric and $Q_{\text{cut}}^2 = 1 \text{ GeV}^2$ for the proton and neutron magnetic form factors. The light blue band corresponds to the Galster-like fit to the neutron electric form factor with $Q_{\text{cut}}^2 = 0.3 \text{ GeV}^2$. The black dashed lines for the proton case are z expansion fits to experimental results [44]. The black circles for the neutron case are from a collection of experimental results [4,45–51,51–63].

factors are obtained by combining with only the connected contribution to the isoscalar form factor since they do not perform the calculation of disconnected contributions. There is qualitative

agreement between our results and those of the PACS Collaboration which suggests that finite volume effects are likely within our combined statistical and systematic errors.

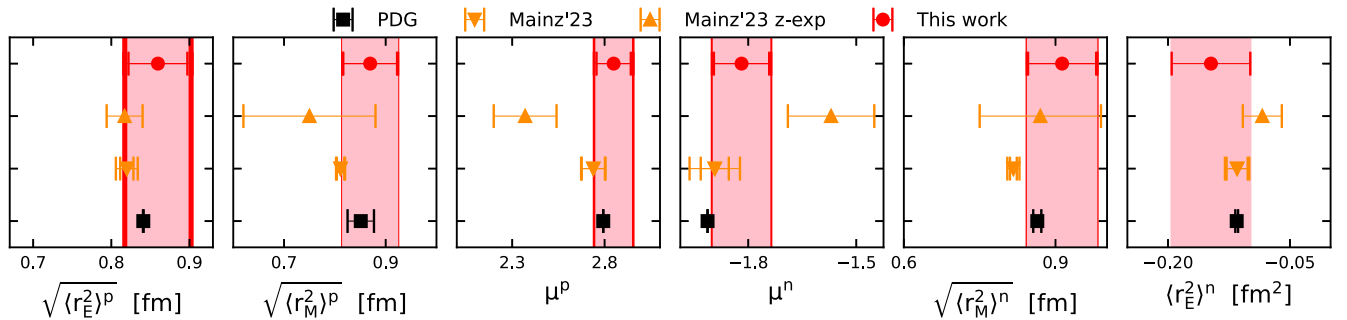


FIG. 21. Results for the electric and magnetic radii and the magnetic moments of the proton and neutron obtained within this work (red circles). The light vertical bands correspond to statistical error, and the dark red bands corresponds to the total error, obtained by adding statistical and systematic errors in quadrature. We compare to results by the Mainz Collaboration (Mainz'23 and Mainz'23 z-exp. [6]) as well as results from the PDG [43].

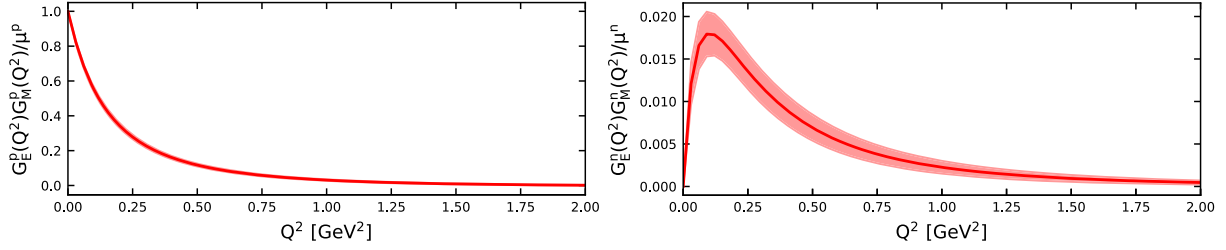


FIG. 22. Product of proton (left) and neutron (right) electric and magnetic form factors as a function of Q^2 , normalized by the respective magnetic moment.

C. Zemach and Friar radii

With our results for the form factors at the continuum limit, we can further analyze the Q^2 dependence to study moments of the charge and magnetization distribution for both proton and neutron, known as the Zemach moments. Experimentally, the Zemach moments enter the determination of the proton radius from Lamb shifts, such as for example the high-precision determination using muonic hydrogen [66] which led to the proton radius puzzle. The Zemach moments can be related to the electromagnetic form factors [67–69], for example, the Zemach radius is given by

$$r_Z = -\frac{2}{\pi} \int_0^\infty \frac{dQ^2}{(Q^2)^{3/2}} \left[\frac{G_E(Q^2)G_M(Q^2)}{\mu_M} - 1 \right], \quad (49)$$

while the third Zemach moment is given by

$$\langle r_F^3 \rangle = \frac{24}{\pi} \int_0^\infty \frac{dQ^2}{(Q^2)^{5/2}} [(G_E(Q^2))^2 + \eta(Q^2)], \quad (50)$$

where $\eta(Q^2) = (-1 + \frac{1}{3}\langle r_E^2 \rangle^p Q^2)$ for proton and 0 for neutron. $\langle r_F^3 \rangle^{1/3}$ corresponds to the Friar radius.

We use our final result for the form factors at the continuum limit as presented in Fig. 20 to compute the Zemach and Friar radii. Namely, using the parameters given in Table XX, which correspond to the most probable z -expansion fits to G_E^p , G_M^p , and G_M^n , and the Galster-like fit to G_E^n , we carry out numerically the two integrals in Eqs. (49) and (50). In Fig. 22, we plot the product of the electric and magnetic form factors for the proton and neutron, normalized by the respective magnetic moment, which forms the main part of the integrand of Eq. (49), which yields the Zemach radius.

Our results for the Zemach and Friar radii are given in Table XV and plotted in Fig. 23, where we also compare

TABLE XV. Results for the Zemach and Friar radii for proton and neutron.

r_Z^p (fm)	$\langle r_F^3 \rangle^p$ (fm ³)	r_Z^n (fm)	$\langle r_F^3 \rangle^n$ (fm ³)
1.091(34)	2.54(43)	−0.0405(81)	0.0097(41)

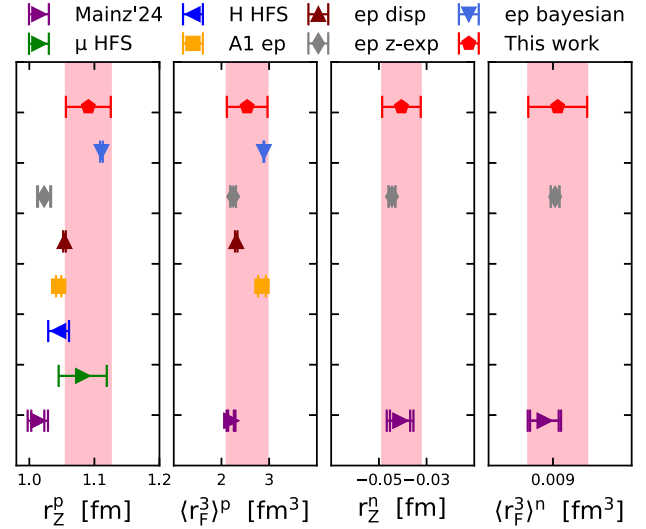


FIG. 23. Results for the Zemach and Friar proton (left two columns) and neutron (right two columns) radii as obtained within this work (red circles and vertical bar) and compared to lattice results by the Mainz Collaboration (Mainz'24, [70]), and experimental results ([66–69,71,72]).

with other works in the literature. Errors are obtained by carrying out the numerical integration over bootstrap samples drawn according to the errors and correlations in the z expansion and Galster-like fit parameters. The only other available lattice study for these quantities is by the Mainz Collaboration [70], where we observe agreement within 1σ for all quantities except proton Zemach radius, where we agree within 2σ . Our results are also in agreement with experimental results based on ep -scattering and electronic and muonic hydrogen hyperfine splitting experiments, which we also compare to Refs. [66–69,71,71,72].

VII. CONCLUSIONS

An analysis of the electromagnetic form factors of the nucleon is presented. We use three ensembles of $N_f = 2 + 1 + 1$ twisted mass fermions at three lattice spacings simulated at approximately physical pion mass. We carry out an extended excited-state analysis which employs multistate fits and data with high statistics. This allows

us to use different excited states when fitting the two- and three-point functions. We carry out the calculation of the disconnected contributions to the isoscalar form factors, which allows us to obtain the proton and neutron electromagnetic form factors.

The Q^2 dependence of the proton and neutron magnetic form factors are fitted with both dipole and z -expansion forms to extract the related radii and magnetic moments. The neutron electric form factor carries larger errors, and we use a Galster-like parametrization. Our final results include statistical and systematic errors, where the latter are obtained from a model average when considering the different fits to extract the ground-state matrix elements and using the dipole and z -expansion Ansätze and varying Q_{cut}^2 . Our results on the form factors agree with experimental results. For the neutron electric form factor, our results are more precise than the experimental ones. For the radii and magnetic moments, we extract the following values: $\langle \sqrt{r_E^2} \rangle^p = 0.860(38)(23)$ fm and $\langle r_E^2 \rangle^n = -0.147(48)$ fm² for the proton and neutron electric radius, respectively; $\mu^p = 2.849(92)(52)$ and $\mu^n = -1.819(76)(29)$ for the proton and neutron magnetic moments, respectively; and $\sqrt{\langle r_M^2 \rangle}^p = 0.870(53)(15)$ fm and $\sqrt{\langle r_M^2 \rangle}^n = 0.913(67)(19)$ fm for the proton and neutron magnetic radii, respectively. We also proceed to compute the Zemach and Friar radii, obtaining $r_Z^p = 1.091(34)$ fm and $\langle r_F^3 \rangle^p = 2.54(43)$ fm³ for the proton and $r_Z^n = -0.0405(81)$ fm and $\langle r_F^3 \rangle^n = 0.0097(41)$ fm³ for the neutron. Our results for the electric and magnetic radii and magnetic moments are consistent with the PDG values for these quantities within our combined statistical and systematic errors. Our result for the proton electric radius carries an error that cannot distinguish between the larger value extracted from historical electron-proton scattering results [73] and the smaller radius measured in muonic hydrogen spectroscopy experiments [66]. Increasing the accuracy of the results for each ensemble but, even more importantly, including more gauge ensembles with smaller lattice spacing will help reduce this error. Furthermore, an analysis including gauge ensembles with larger physical volumes simulated at physical pion mass recently by ETMC [74] will allow for more lattice momenta close to $Q^2 = 0$, which can potentially lead to more robust fits of the Q^2 dependence in turn reducing systematic errors in the determination of the radii and magnetic moments.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: EXCITED-STATE ANALYSIS OF CONNECTED AND DISCONNECTED CONTRIBUTIONS

We compare the ground-state element obtained for the isoscalar case when (i) the connected and disconnected contributions are fitted separately, as done in the main text, and (ii) the connected and disconnected contributions are added first and then fitted. For the purposes of this comparison, we use the same data for both cases, i.e., in both i and ii, we use the disconnected contributions with $\vec{p}' = 0$ and only use the sink-source separations available for the connected case.

The comparison is shown in Fig. 24. The data plotted are the sum of connected and disconnected contributions for

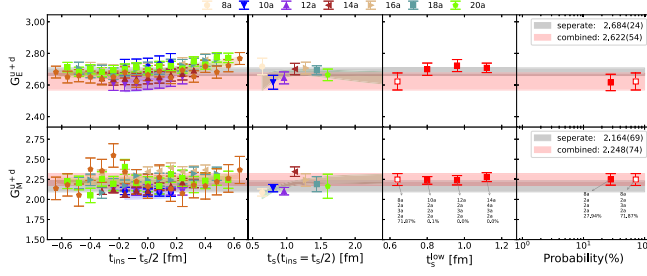


FIG. 24. Excited-state analysis of the isoscalar matrix elements for the B ensemble for the first nonzero momentum transfer. The procedure and notation is the same as for the connected case, shown in Fig. 4, but after adding the connected and disconnected three-point function data. The gray band is obtained by adding the connected and disconnected contributions to the ground-state matrix element after fitting them separately.

each case for the B ensemble. Otherwise, the procedure followed is the same as that followed for the connected contribution in the main text (see Fig. 4 and related discussion). As can be seen, the ground-state matrix

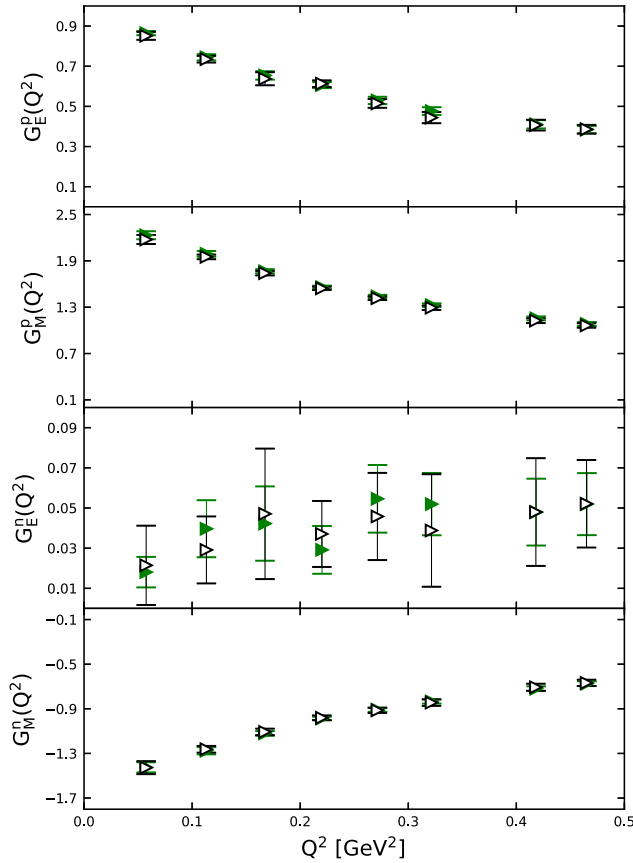


FIG. 25. Comparison of the electric and magnetic proton and neutron form factors when the ground-state analysis is carried out for the connected and disconnected contributions separately (solid triangles) and when the connected and disconnected contributions are added first and then fitted for the ground state (open triangles). We show the B ensemble up to $Q^2 = 0.5 \text{ GeV}^2$.

element of the total contribution is consistent between the two approaches within statistical errors.

The comparison is carried out for additional values of Q^2 , as shown in Fig. 25, where we show that the two approaches yield consistent results for the B ensemble for all form factors up to $Q^2 = 0.5 \text{ GeV}^2$.

APPENDIX B: DECOMPOSITION OF LATTICE MATRIX ELEMENTS

We provide the general equations, in Euclidean space, corresponding to the ratio in Eq. (14) when considered for the case of $p' \neq 0$. For compact representation, we take $G_E \equiv G_E(Q^2)$, $G_M \equiv G_M(Q^2)$, $\Pi_{\mu,0} \equiv \Pi^\mu(\Gamma_0; \vec{p}', \vec{p})$, $\Pi_{\mu,k} \equiv \Pi^\mu(\Gamma_k; \vec{p}', \vec{p})$, where $\vec{p}$ ($\vec{p}'$) corresponds to the initial (final) momentum. The four momenta are then given by $\{E, \vec{p}\}$ ($\{E', \vec{p}'\}$), where E (E') is the initial (final) energy,

$$\begin{aligned} \Pi_{\mu,0} = & -iCG_E[(p'_\mu + p_\mu)(m(E' + E + m) - p'_\rho p_\rho)] \\ & + \frac{CG_M}{2m}[\delta_{\mu 0}(4m^2 + Q^2)(m^2 + p'_\rho p_\rho) - iEQ^2 p'_\mu \\ & + 2im^2(E' - E)(p'_\mu - p_\mu) - iE'Q^2 p_\mu \\ & - imQ^2(p'_\mu + p_\mu)(2m^2 + Q^2 + 2p'_\rho p_\rho)], \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \Pi_{\mu,k} = & CG_E[\epsilon_{\mu k 0 \rho}(p'_\rho - p_\rho)(m^2 - p'_\sigma p_\sigma) \\ & - i\epsilon_{\mu k \rho \sigma}p'_\rho p_\sigma(E' + E) + \epsilon_{\mu 0 \rho \sigma}p'_\rho p_\sigma(p'_k + p_k)] \\ & + \frac{CG_M}{2m}\left[m\epsilon_{\mu k 0 \rho}(p'_\rho - p_\rho)(2m^2 + Q^2 + 2p'_\sigma p_\sigma) \right. \\ & + 2im\epsilon_{\mu k \rho \sigma}p'_\rho p_\sigma\left(2m + E' + E + \frac{Q^2}{2m}\right) \\ & \left. - 2m\epsilon_{\mu 0 \rho \sigma}p'_\rho p_\sigma(p'_k + p_k)\right], \end{aligned} \quad (\text{B2})$$

where C is a kinematic factor given by

$$C = \frac{m(4m^2 + Q^2)^{-1}}{E(E' + m)} \sqrt{\frac{E(E' + m)}{E'(E + m)}}. \quad (\text{B3})$$

APPENDIX C: COMPARISON WITH PREVIOUS WORK

Here, we compare the results obtained using the current method with our previous work presented in Ref. [8]. A major difference between the analysis procedure of the previous work and the current work is the treatment of excited states, where previous results were obtained using plateau fits. Our current work includes all systematics due to presence of excited states. As can be seen in Fig. 26, while most of the results are compatible within 2σ , the minor deviations at lower Q^2

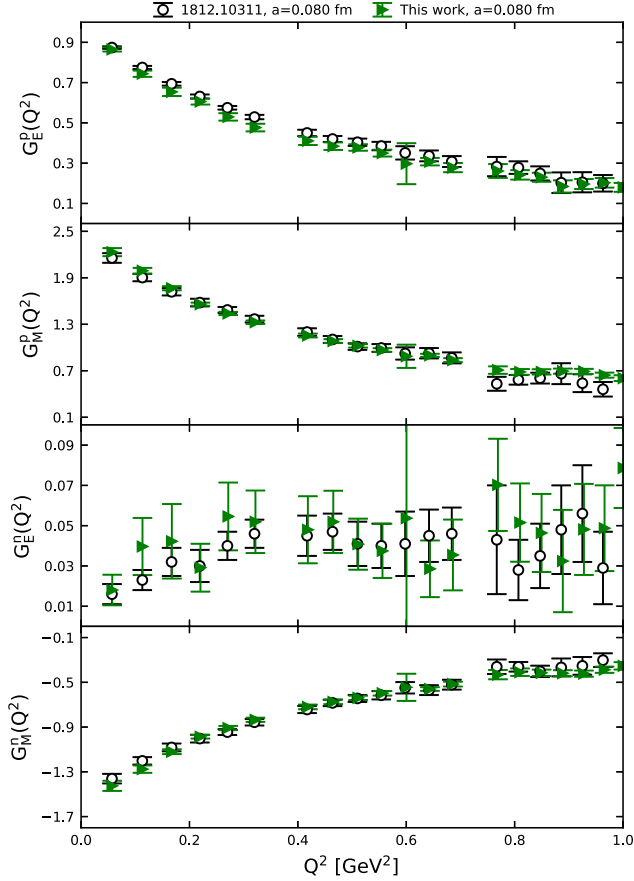


FIG. 26. Comparison of proton and neutron electromagnetic form factors for the ensemble cB211.072.64, in Ref. [8] (open black circles) and current work (green right-pointing triangles).

region, especially for the magnetic form factors, explain the shift in the magnetic moments obtained for the given ensemble.

APPENDIX D: TABULATED RESULTS

We provide the results of the form factors obtained for each ensemble. In Table XVI, we provide our values for the electric and magnetic form factors of the proton, neutron, isoscalar connected, and isovector combinations for the cB211.072.64 ensemble, where the results for the proton and neutron have the disconnected contributions included. Similarly, Tables XVII and XVIII tabulate the results obtained for ensembles cC211.060.80 and cD211.054.96, respectively. The disconnected contributions are also provided in Table XIX. We note that the inclusion of momenta at the sink leads to 224 Q^2 values for the cB211.072.64 ensemble and 262 for the cC211.060.80 and cD211.054.96 ensembles when restricting to Q^2 values smaller than ~ 1 GeV². These results shown in the table have been averaged into 50 equal bins within the interval $Q^2 = [0, 1)$ GeV². More precisely, for bin i starting from Q_i^2 and ending at $Q_{i+1}^2 = Q_i^2 + \delta Q^2$, we average all form factor values with $Q_i^2 \leq Q^2 < Q_{i+1}^2$ weighted by their errors. We use $\delta Q^2 = 0.02$ GeV², $Q_0^2 = 0.0$ GeV², and $Q_{50}^2 = 1.0$ GeV².

In addition, in Table XX, we provide the fit parameters and their associated covariance matrix for the most probable fit to the Q^2 dependence of the form factors, i.e., those used to plot the bands in Fig. 20.

TABLE XVI. Results for the electromagnetic form factors using the cB211.072.64 ensemble for the proton, neutron, isoscalar connected, and isovector connected combinations.

Q^2	G_E^p	G_E^n	$G_E^{u+d}(\text{conn})$	G_E^{u-d}	G_M^p	G_M^n	$G_M^{u+d}(\text{conn})$	G_M^{u-d}
0.0575	0.865(10)	0.0181(76)	2.634(23)	0.847(17)	2.231(52)	-1.425(45)	2.553(53)	3.656(96)
0.1133	0.744(15)	0.040(14)	2.328(21)	0.704(29)	1.991(37)	-1.276(33)	2.257(34)	3.267(69)
0.1675	0.654(21)	0.042(18)	2.060(26)	0.612(38)	1.765(26)	-1.120(21)	2.027(34)	2.886(46)
0.2202	0.606(15)	0.029(12)	1.873(28)	0.577(25)	1.559(20)	-0.987(16)	1.797(32)	2.546(35)
0.2716	0.530(18)	0.055(17)	1.718(30)	0.475(34)	1.439(21)	-0.908(19)	1.661(36)	2.348(38)
0.3217	0.477(19)	0.052(16)	1.551(30)	0.425(33)	1.326(23)	-0.835(19)	1.535(28)	2.161(41)
0.4185	0.411(21)	0.048(17)	1.341(36)	0.363(35)	1.155(25)	-0.720(21)	1.353(30)	1.875(45)
0.4654	0.384(19)	0.052(15)	1.273(31)	0.332(33)	1.083(25)	-0.675(21)	1.266(28)	1.757(45)
0.5113	0.378(16)	0.041(13)	1.221(30)	0.337(27)	1.022(23)	-0.639(19)	1.189(28)	1.661(41)
0.5561	0.350(17)	0.037(13)	1.128(34)	0.312(29)	0.968(23)	-0.598(21)	1.147(28)	1.566(43)
0.6004	0.30(10)	0.054(91)	1.020(75)	0.24(19)	0.89(15)	-0.54(12)	1.060(97)	1.43(27)
0.6437	0.307(19)	0.029(14)	0.976(47)	0.279(29)	0.899(20)	-0.560(16)	1.048(30)	1.459(35)
0.6862	0.278(23)	0.035(18)	0.908(53)	0.242(37)	0.837(25)	-0.516(22)	0.991(33)	1.353(46)
0.7692	0.261(34)	0.070(23)	0.964(79)	0.191(52)	0.709(48)	-0.431(42)	0.856(49)	1.140(88)
0.8099	0.242(23)	0.052(19)	0.851(50)	0.190(40)	0.685(37)	-0.409(33)	0.850(32)	1.095(69)
0.8497	0.230(22)	0.046(19)	0.802(53)	0.184(38)	0.685(34)	-0.415(29)	0.831(39)	1.100(62)
0.8889	0.184(30)	0.032(25)	0.62(11)	0.152(41)	0.695(33)	-0.419(27)	0.848(54)	1.114(58)

(Table continued)

TABLE XVI. (*Continued*)

Q^2	G_E^p	G_E^n	$G_E^{u+d}(\text{conn})$	G_E^{u-d}	G_M^p	G_M^n	$G_M^{u+d}(\text{conn})$	G_M^{u-d}
0.9278	0.196(25)	0.048(23)	0.707(84)	0.148(38)	0.691(33)	-0.422(28)	0.826(41)	1.114(60)
0.9659	0.202(24)	0.049(21)	0.727(59)	0.154(41)	0.643(32)	-0.387(28)	0.787(36)	1.031(59)
1.0007	0.180(22)	0.079(20)	0.750(49)	0.101(39)	0.604(37)	-0.353(33)	0.771(32)	0.957(69)
1.0748	0.186(38)	0.073(19)	0.75(13)	0.114(44)	0.573(81)	-0.336(56)	0.728(94)	0.91(14)

TABLE XVII. Results for the electromagnetic form factors using the cC211.060.80 ensemble for the proton, neutron, isoscalar connected, and isovector connected combinations.

Q^2	G_E^p	G_E^n	$G_E^{u+d}(\text{conn})$	G_E^{u-d}	G_M^p	G_M^n	$G_M^{u+d}(\text{conn})$	G_M^{u-d}
0.0511	0.867(13)	0.030(12)	2.678(17)	0.837(24)	2.398(58)	-1.576(59)	2.617(32)	3.97(12)
0.1008	0.7669(94)	0.0344(77)	2.383(14)	0.733(17)	2.078(21)	-1.334(18)	2.357(21)	3.412(38)
0.1492	0.665(29)	0.059(24)	2.146(25)	0.607(52)	1.837(37)	-1.165(22)	2.124(51)	3.001(59)
0.1964	0.614(27)	0.049(25)	1.961(22)	0.565(52)	1.681(40)	-1.069(24)	1.929(53)	2.750(63)
0.2425	0.560(14)	0.045(12)	1.786(21)	0.515(25)	1.537(17)	-0.967(13)	1.792(21)	2.505(30)
0.2876	0.512(16)	0.043(13)	1.634(24)	0.469(28)	1.402(23)	-0.876(20)	1.648(20)	2.278(43)
0.3752	0.4541(89)	0.0189(64)	1.388(30)	0.435(12)	1.256(13)	-0.7876(92)	1.464(20)	2.044(21)
0.4174	0.4285(90)	0.0215(60)	1.319(29)	0.407(12)	1.182(13)	-0.742(11)	1.372(18)	1.924(23)
0.4590	0.397(13)	0.0355(96)	1.267(32)	0.362(20)	1.105(20)	-0.691(15)	1.288(24)	1.796(34)
0.4999	0.383(10)	0.0319(70)	1.214(28)	0.351(15)	1.051(16)	-0.656(13)	1.231(19)	1.707(28)
0.5401	0.353(18)	0.006(12)	1.049(79)	0.347(15)	1.008(14)	-0.6403(98)	1.145(27)	1.649(22)
0.5794	0.326(18)	0.0246(92)	1.022(60)	0.301(19)	0.938(17)	-0.590(13)	1.084(26)	1.528(29)
0.6181	0.311(24)	0.008(13)	0.93(11)	0.304(17)	0.915(39)	-0.574(25)	1.058(48)	1.490(63)
0.6933	0.172(32)	0.109(33)	0.82(13)	0.063(48)	0.702(58)	-0.397(43)	0.948(86)	1.099(99)
0.7315	0.246(24)	0.011(17)	0.746(97)	0.235(27)	0.811(17)	-0.508(13)	0.938(32)	1.318(29)
0.7676	0.245(26)	0.025(17)	0.783(82)	0.220(34)	0.771(25)	-0.472(22)	0.924(29)	1.244(46)
0.8032	0.198(47)	-0.016(30)	0.52(20)	0.214(38)	0.737(33)	-0.462(22)	0.851(60)	1.199(53)
0.8394	0.208(52)	0.002(29)	0.61(20)	0.206(50)	0.710(34)	-0.442(27)	0.830(47)	1.152(59)
0.8739	0.25(14)	0.017(89)	0.77(69)	0.231(53)	0.716(22)	-0.444(15)	0.838(30)	1.160(37)
0.9066	0.225(44)	0.008(33)	0.68(23)	0.218(16)	0.687(15)	-0.428(11)	0.800(33)	1.115(24)
0.9739	0.2376(78)	0.0250(50)	0.767(34)	0.2126(69)	0.6602(96)	-0.4103(66)	0.769(17)	1.070(15)

TABLE XVIII. Results for the electromagnetic form factors using the cD211.054.96 ensemble for the proton, neutron, isoscalar connected, and isovector connected combinations.

Q^2	G_E^p	G_E^n	$G_E^{u+d}(\text{conn})$	G_E^{u-d}	G_M^p	G_M^n	$G_M^{u+d}(\text{conn})$	G_M^{u-d}
0.0505	0.854(11)	0.0330(95)	2.648(14)	0.821(20)	2.431(61)	-1.584(58)	2.692(32)	4.02(12)
0.0996	0.754(10)	0.0382(87)	2.352(14)	0.715(18)	2.086(25)	-1.329(22)	2.391(23)	3.415(47)
0.1475	0.671(17)	0.046(14)	2.123(19)	0.625(31)	1.858(20)	-1.170(20)	2.163(23)	3.028(39)
0.1942	0.610(11)	0.0338(83)	1.901(22)	0.576(19)	1.688(13)	-1.0552(91)	1.982(20)	2.743(21)
0.2398	0.527(21)	0.048(17)	1.692(29)	0.478(36)	1.491(23)	-0.914(20)	1.805(21)	2.405(42)
0.2846	0.514(11)	0.0344(77)	1.611(23)	0.479(17)	1.415(13)	-0.872(11)	1.691(15)	2.287(24)
0.3710	0.386(27)	0.056(22)	1.293(46)	0.331(47)	1.148(40)	-0.683(37)	1.443(25)	1.831(77)
0.4130	0.359(65)	0.058(33)	1.22(11)	0.301(96)	1.104(79)	-0.655(64)	1.389(55)	1.76(14)
0.4542	0.376(74)	0.023(11)	1.16(21)	0.352(80)	1.09(13)	-0.673(84)	1.28(14)	1.76(22)
0.4948	0.363(13)	0.0246(77)	1.130(43)	0.338(16)	1.052(22)	-0.648(17)	1.248(20)	1.701(39)
0.5346	0.304(30)	0.038(21)	0.996(83)	0.266(43)	0.953(37)	-0.572(33)	1.177(30)	1.525(69)
0.5734	0.266(37)	0.032(20)	0.87(11)	0.234(47)	0.869(52)	-0.521(38)	1.073(65)	1.390(89)
0.6122	0.270(24)	0.019(16)	0.838(91)	0.250(28)	0.876(25)	-0.528(21)	1.071(29)	1.404(45)
0.6872	0.243(28)	0.022(21)	0.77(10)	0.221(36)	0.785(39)	-0.488(29)	0.917(57)	1.273(66)
0.7243	0.221(35)	0.0002(256)	0.64(16)	0.220(30)	0.768(28)	-0.470(21)	0.916(47)	1.239(47)
0.7603	0.246(24)	0.005(17)	0.73(11)	0.241(18)	0.805(15)	-0.486(10)	0.978(30)	1.291(24)

(Table continued)

TABLE XVIII. (Continued)

Q^2	G_E^p	G_E^n	$G_E^{u+d}(\text{conn})$	G_E^{u-d}	G_M^p	G_M^n	$G_M^{u+d}(\text{conn})$	G_M^{u-d}
0.7962	0.193(16)	0.023(18)	0.621(71)	0.170(24)	0.725(23)	-0.441(17)	0.871(47)	1.166(37)
0.8315	0.207(14)	-0.004(13)	0.582(56)	0.211(19)	0.698(23)	-0.435(17)	0.809(46)	1.133(37)
0.8661	0.218(32)	0.008(13)	0.65(12)	0.210(29)	0.690(24)	-0.421(17)	0.824(46)	1.111(39)
0.9004	0.165(15)	0.054(18)	0.634(74)	0.111(22)	0.600(32)	-0.344(28)	0.785(48)	0.943(59)
0.9652	0.224(74)	0.040(26)	0.77(30)	0.184(50)	0.614(39)	-0.369(27)	0.748(42)	0.983(66)

TABLE XIX. Results for isoscalar disconnected electromagnetic form factors. Results have been binned as explained in the text. Bins that do not contain any values are omitted.

Q^2	cB211.072.64		cC211.060.80		cD211.054.96	
	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$
0.05	0.0260(42)	-0.158(21)	0.0202(39)	-0.196(19)	0.0188(38)	-0.154(19)
0.09	0.0219(83)	-0.163(35)	0.0232(46)	-0.160(17)	0.0242(30)	-0.121(10)
0.11	0.0210(36)	-0.126(12)	0.0294(29)	-0.1476(98)	0.0264(28)	-0.1190(92)
0.13	...	...	0.0014(66)	-0.119(22)	0.0323(58)	-0.097(19)
0.15	0.0313(81)	-0.086(24)	0.0175(39)	-0.128(12)	0.0349(41)	-0.086(12)
0.17	0.0313(47)	-0.088(12)	0.0208(97)	-0.104(30)	0.038(10)	-0.022(27)
0.19	0.045(14)	-0.072(37)	0.0311(51)	-0.118(14)	0.0347(50)	-0.083(12)
0.21	0.030(11)	-0.089(28)	0.0328(40)	-0.1228(97)	0.0372(37)	-0.0967(88)
0.23	0.0371(50)	-0.077(11)	0.0227(41)	-0.113(10)	0.0360(35)	-0.1091(83)
0.25	0.0374(53)	-0.086(11)	0.0333(25)	-0.0954(56)	0.0357(24)	-0.0931(53)
0.27	0.0365(45)	-0.0788(86)	0.0260(45)	-0.070(10)	0.0402(47)	-0.0969(87)
0.29	0.0390(28)	-0.0796(56)	0.0258(34)	-0.0719(78)	0.0394(38)	-0.1008(71)
0.31	0.0394(58)	-0.087(12)	0.0321(25)	-0.0774(53)	0.0434(27)	-0.0840(53)
0.33	0.0396(42)	-0.0742(81)	0.0349(86)	-0.064(17)	0.0085(78)	-0.059(16)
0.35	0.0341(30)	-0.0645(55)	0.0156(98)	-0.022(21)	0.0343(98)	-0.052(17)
0.37	0.038(14)	-0.012(25)	0.0269(56)	-0.056(10)	0.0220(49)	-0.0407(87)
0.39	0.032(12)	-0.068(24)	0.0246(61)	-0.061(12)	0.0289(33)	-0.0359(56)
0.41	0.0393(71)	-0.059(13)	0.0334(31)	-0.0550(53)	0.0283(29)	-0.0446(48)
0.43	0.0364(74)	-0.023(12)	0.0307(36)	-0.0478(63)	0.0280(36)	-0.0228(55)
0.45	0.0382(48)	-0.0536(84)	0.0358(35)	-0.0543(58)	0.0321(26)	-0.0362(37)
0.47	0.0371(38)	-0.0557(63)	0.0393(24)	-0.0490(40)	0.0321(42)	-0.0320(63)
0.49	0.0257(52)	-0.0570(78)	0.0237(41)	-0.0507(60)	0.0312(39)	-0.0251(55)
0.51	0.0318(35)	-0.0435(51)	0.0329(29)	-0.0495(45)	0.0314(29)	-0.0265(39)
0.53	0.0356(34)	-0.0408(49)	0.0297(50)	-0.0271(84)	0.0130(65)	-0.0292(85)
0.55	0.0291(49)	-0.0303(68)	0.0271(34)	-0.0447(54)	0.0326(36)	-0.0340(45)
0.57	0.0345(45)	-0.0274(57)	0.0276(32)	-0.0368(46)	0.0221(51)	-0.0225(64)
0.59	0.0309(38)	-0.0408(52)	0.0281(45)	-0.0484(68)	0.0253(77)	-0.020(11)
0.61	0.0335(64)	-0.0294(82)	0.0328(42)	-0.0434(59)	0.0278(36)	-0.0288(46)
0.63	0.0298(44)	-0.0387(59)	0.0232(59)	-0.0387(81)	0.0210(50)	-0.0255(56)
0.65	0.0234(67)	-0.0441(89)	0.0238(40)	-0.0339(52)	0.0285(42)	-0.0329(48)
0.67	0.017(11)	-0.045(14)	0.0262(30)	-0.0364(39)	0.0349(37)	-0.0269(47)
0.69	0.0310(54)	-0.0435(69)	0.0292(36)	-0.0343(45)	0.0374(35)	-0.0278(46)
0.71	0.0336(81)	-0.039(10)	0.0295(29)	-0.0317(35)	0.0298(27)	-0.0325(36)
0.73	0.0266(54)	-0.0411(68)	0.0135(86)	-0.060(12)	0.0407(72)	-0.0432(83)
0.75	0.0328(40)	-0.0338(48)	0.0188(94)	-0.027(11)	0.0122(77)	-0.056(11)
0.77	0.0284(49)	-0.0409(60)	0.0196(73)	-0.0236(89)	0.0464(82)	-0.0532(94)
0.79	0.0226(48)	-0.0336(58)	0.0415(67)	-0.0274(73)	0.0418(66)	-0.0279(66)
0.81	0.0282(38)	-0.0297(46)	0.0103(51)	-0.0283(61)	0.0357(53)	-0.0338(58)
0.83	0.043(12)	0.009(15)	0.0264(47)	-0.0373(57)	0.0294(37)	-0.0358(46)
0.85	0.0174(99)	-0.006(12)	0.0277(36)	-0.0254(42)	0.0294(82)	-0.0307(92)

(Table continued)

TABLE XIX. (Continued)

Q^2	cB211.072.64		cC211.060.80		cD211.054.96	
	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$	$G_E^{u+d}(\text{disc})$	$G_M^{u+d}(\text{disc})$
0.87	0.028(14)	0.019(17)	0.0257(58)	-0.0268(62)	0.0290(50)	-0.0386(57)
0.89	0.0341(77)	0.0013(87)	0.0175(52)	-0.0279(62)	0.0160(84)	-0.0308(94)
0.91	0.0230(73)	-0.0155(83)	0.0239(47)	-0.0194(51)	0.0274(42)	-0.0203(47)
0.93	0.0140(66)	-0.0071(75)	-0.0032(92)	-0.031(12)	0.0383(86)	-0.0352(93)
0.95	0.0303(55)	-0.0158(56)	0.0227(43)	-0.0148(47)	0.0204(45)	-0.0240(49)
0.97	0.0268(74)	-0.0092(83)	0.0124(95)	-0.0310(92)	0.0248(93)	-0.010(11)
0.99	0.0143(75)	-0.0099(74)	0.0278(46)	-0.0216(50)	0.0306(41)	-0.0202(42)

TABLE XX. Fit parameters for the most probable fits shown in Fig. 20 at $a = 0$. For each form factor indicated in the first column, the Q_{cut}^2 and fit form of the most probable fit is provided in the second column, and the symbols used in the text for the fit parameters are provided in the third column. The values and errors of the fit parameters are given in the fourth column, and their corresponding covariance matrices are given in the fifth.

	Fit	Fit parameters, $\vec{p}$	Results for $\vec{p}$	Covariance of fit parameters, $\text{cov}(\vec{p})$
G_E^p	z-exp, $Q_{\text{cut}}^2 = 0.85 \text{ GeV}^2$	$\begin{pmatrix} c_{1,0} \\ c_{2,0} \\ c_{3,0} \end{pmatrix}$	$\begin{pmatrix} -0.936(84) \\ -1.16(26) \\ -0.54(45) \end{pmatrix}$	$\begin{pmatrix} 0.007 & -0.015 & 0.013 \\ -0.015 & 0.07 & -0.098 \\ 0.013 & -0.098 & 0.203 \end{pmatrix}$
G_M^p	z-exp, $Q_{\text{cut}}^2 = 1 \text{ GeV}^2$	$\begin{pmatrix} c_{0,0} \\ c_{1,0} \\ c_{2,0} \end{pmatrix}$	$\begin{pmatrix} 2.88(10) \\ -2.77(42) \\ -4.17(66) \end{pmatrix}$	$\begin{pmatrix} 0.01 & -0.038 & 0.039 \\ -0.038 & 0.176 & -0.238 \\ 0.039 & -0.238 & 0.441 \end{pmatrix}$
G_M^n	z-exp, $Q_{\text{cut}}^2 = 1 \text{ GeV}^2$	$\begin{pmatrix} c_{0,0} \\ c_{1,0} \\ c_{2,0} \end{pmatrix}$	$\begin{pmatrix} -1.837(85) \\ 1.97(37) \\ 2.39(58) \end{pmatrix}$	$\begin{pmatrix} 0.007 & -0.028 & 0.032 \\ -0.028 & 0.136 & -0.188 \\ 0.032 & -0.188 & 0.337 \end{pmatrix}$
G_E^n	Galster-like, $Q_{\text{cut}}^2 = 0.3 \text{ GeV}^2$	$\begin{pmatrix} A \\ B \end{pmatrix}$	$\begin{pmatrix} 2.23(73) \\ 23(14) \end{pmatrix}$	$\begin{pmatrix} 0.537 & 9.144 \\ 9.144 & 187.346 \end{pmatrix}$

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