

Naturally resonant dark matter from extra dimensions

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We explore the mass-resonance structure that naturally arises in extradimensional models. The resonance can enhance both dark matter annihilation and self-interactions. We demonstrate such a resonance structure by considering the fermionic dark matter and dark photon models on an $S^1/(Z_2 \times Z'_2)$ orbifold. We also note that this model embeds a dark matter axial-vector coupling to the dark photon, thereby opening up the viable dark matter parameter space. We then present the unique predictions for direct-detection experiments and accelerator searches.

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Introduction. Despite overwhelming observational evidence for dark matter (DM) and its profound impact on particle physics, astrophysics, and cosmology, the detailed properties of DM remain unknown. More specifically, its interactions beyond gravity are yet to be discovered. One of the most compelling DM candidates, which features a predictive thermal history and interaction strength with Standard Model (SM) particles, is thermal DM. Currently, direct-detection and accelerator experiments have explored a large portion of the parameter space of thermal DM models mediated through dark photons [1,2], except in scenarios where an enhancement of annihilation occurs due to the presence of a Breit-Wigner resonance [3–5]. These phenomenological mass-resonance models need more underlying UV theories and theoretical motivations.

Resonant structures are ubiquitous in the Standard Model, and it has been proposed that DM could exhibit a similar resonant behavior [6]. Such resonances can have intriguing implications, including opening up a valid parameter space by introducing resonance between mediators and DM [4,5]. Furthermore, they can address small-scale structure problems by introducing self-interactions and resonances among DM particles [7], which can also affect the self-freeze-out process [8,9]. While such structures can arise from strongly interacting sectors or supersymmetry [10], the possibility

that resonant DM candidates originate from extradimensional theories remains underexplored.

Extradimensional theories were first introduced to unify gravity and electromagnetism [11], and the development of string theory later provided a consistent framework for quantum gravity, wherein extra dimensions are compactified at scales near the Planck length. Furthermore, to address the hierarchy problem and the observed weakness of gravity, models with flat extra dimensions were proposed [12], followed by warped extradimensional scenarios [13]. Unlike some of these frameworks, where only gravity propagates in the extra dimensions, the universal extra dimension (UED) model allows all SM particles to traverse the extradimensional space [14,15]. In this broad landscape, the implications of extra dimensions for resonant structures, particularly in connection with DM, remain an area of ongoing investigation.

This work represents one of the first attempts to investigate resonant structures within extradimensional theories and their application to DM. Specifically, we explore the resonance structure of DM in the framework of simple Kaluza-Klein (KK) models that naturally accommodate resonant behavior. Our model is inspired by the five-dimensional (5D) UED framework, where the DM and dark photon KK modes are embedded in an extra dimension compactified on an S^1/Z_2 or $S^1/(Z_2 \times Z'_2)$ orbifold. Unlike UED [16–19], we will focus on the scenarios in which only the dark sector propagates in the extra dimension while the SM is localized on the 4D spacetime. We demonstrate that this model can naturally achieve the desired resonance level between DM and dark photons with minimal fine-tuning, thereby opening up a new region of parameter space for the kinetic mixing parameter of invisible dark photons. Additionally, we briefly discuss the parameter space required to achieve resonance levels capable of mitigating small-scale structure problems.

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TABLE I. Summary of the boundary conditions imposed on KK particles.

States	$y = 0$	$y = \pi R/2$	$y = \pi R$
RH singlet fermion	Neumann	Dirichlet	Neumann
LH singlet fermion	Dirichlet	Neumann	Dirichlet
Dark photon (B_μ)	Neumann	Neumann	Neumann
Dark photon (B_5)	Dirichlet	Dirichlet	Dirichlet

Fermionic dark matter model. Throughout this paper, we adopt the convention that the coordinates in $(3+1)$ dimensions are denoted by x^μ (where $\mu = 0, 1, 2, 3$), while y represents the fifth extra dimension. The metric signature used is mostly negative, $g_{MN} = (+ - - - -)$.

We consider a model that is characterized by the size of the orbifold, R , with a compactification scale $M_c = 1/R$. This model consists of a fermion field in addition to the dark photon gauge field, where these KK modes are compactified on an $S^1/(Z_2 \times Z'_2)$ orbifold. The matter content of the model consists of an $SU(2)_L$ singlet fermion, $E(x, y)$, and a dark photon (γ_D), $B_N(x, y)$, where $N = \mu, 5$. Each field obeys specific boundary condition on each boundary point. A summary of the boundary conditions is presented in Table I. The Supplemental Material [20] provides further details on $S^1/(Z_2 \times Z'_2)$ and compares the convention used in this paper with those in the literature, including Ref. [21]. The implications of the conditions are noteworthy. To satisfy these boundary conditions, dark photons must consist of only n -even modes, while right-handed (RH) singlet fermions and left-handed (LH) singlet fermions must consist of only n -odd modes. Furthermore, we imply the zero-flux condition [22], $\oint B_\mu dy = 0$, to eliminate the zeroth mode of the gauge field $B_\mu^{(0)}$, as follows:

$$E(x, y) = \sqrt{\frac{4}{\pi R}} \left[\sum_{n=1}^{\infty} E_R^{(2n-1)}(x) \cos\left(\frac{(2n-1)y}{R}\right) + E_L^{(2n-1)}(x) \sin\left(\frac{(2n-1)y}{R}\right) \right], \quad (1)$$

$$B_\mu(x, y) = \sqrt{\frac{4}{\pi R}} \left[\sum_{n=1}^{\infty} B_\mu^{(2n)}(x) \cos\left(\frac{2ny}{R}\right) \right],$$

$$B_5(x, y) = \sqrt{\frac{4}{\pi R}} \sum_{n=1}^{\infty} B_5^{(2n)}(x) \sin\left(\frac{2ny}{R}\right). \quad (2)$$

The superscript n represents the KK number.

The Lagrangian in $(3+1)$ dimensions is constructed by integrating out the extradimensional coordinate y . The Lagrangians involving fermions and dark photons are

$$\mathcal{L}_E = \int_0^{\pi R/2} dy [i \bar{E} \Gamma^M D_M E], \quad (3)$$

$$\mathcal{L}_{\gamma_D} = \int_0^{\pi R/2} dy \left(-\frac{1}{4} B_{MN} B^{MN} - \frac{1}{2} m_B^2 B_N B^N \right), \quad (4)$$

where B_{MN} is the 5D field strength tensor of the dark photon. The covariant derivative acting on the field in the Lagrangian is given by

$$D_M E = (\partial_M + i Q_E g_1^{(5)} B_M) E, \quad (5)$$

where B_M is the KK dark photon, and Q_E and $g_1^{(5)}$ are the associated charge and 5D gauge coupling, respectively. Throughout this work, Q_E is set to unity. Here, $g_1^{(5)}$ is a dimensionful quantity in 5D and is related to the dimensionless 4D gauge coupling, g_1 , by

$$g_1 = \frac{g_1^{(5)}}{\sqrt{L}}, \quad (6)$$

where $L = \pi R/2$ for the $S^1/(Z_2 \times Z'_2)$ orbifold. The γ matrices and partial derivative operators in 5D are given by

$$\Gamma^M = (\gamma^\mu, i\gamma^5), \quad (7)$$

$$\partial_M = (\partial_\mu, \partial_y). \quad (8)$$

The fifth component of the partial derivative, ∂_y , introduces an additional contribution to the mass for each KK mode, in addition to the bare mass. Therefore, the total mass of the n th mode for each particle is

$$m_E^{(n)} = \frac{n}{R}, \quad (9)$$

$$m_{\gamma_D}^{(n)} = \sqrt{m_B^2 + \frac{n^2}{R^2}}, \quad (10)$$

at tree level. Here, we assume that a bare mass term for the fermion is forbidden due to the lack of a Higgs mechanism, while the dark photon acquires a bare mass through a different mechanism. The three-point vertex essential for the resonant structure, $EE \rightarrow \gamma_D$, is given in Eq. (3). Due to the structure of the model, chirality-conserving vertices occur only with B_μ , while chirality-flipping vertices occur only with B_5 .

It can be shown that the only stable particle in the model is $E^{(1)}$. Given that $B^{(2)}$ decays into two electrons, the decay channels for the remaining particles are as follows: $E^{(2n+1)} \rightarrow E^{(2n-1)} B^{(2)*} \rightarrow E^{(2n-1)} e^+ e^-$ and $B^{(2n+2)} \rightarrow E^{(2n-1)} E^{(3)*} \rightarrow E^{(2n-1)} E^{(1)} B^{(2)*} \rightarrow E^{(2n-1)} E^{(1)} e^+ e^-$, where the superscript “*” denotes an off-shell particle.

Resonant structures and interactions with SM. Here, we briefly discuss the sub-GeV thermal DM, in our model, that freezes out to produce the correct DM relic abundance.

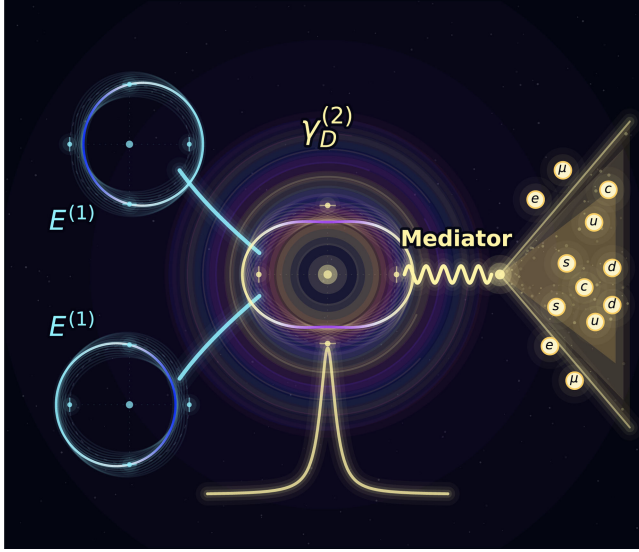


FIG. 1. This figure schematically illustrates resonant annihilation of dark matter into Standard Model particles through a dark photon mediator. The small dots surrounding each particle, represented by the larger dot, indicate the boundaries of the $(S^1/(Z_2 \times Z_2))$ orbifold. The blue-cyan circles and yellow-purple oval show the oscillatory behavior of the corresponding modes near the boundary, taking into account the imposed boundary conditions. The opaque circles around the dark photon visually represent the resonance. For clarity, we also include a simplified graph highlighting the resonant structure.

The interaction with the SM can be achieved through kinetic mixing between dark photons and SM photons. This kinetic mixing arises through particles charged under both $U(1)_{\text{em}}$ and $U(1)_D$. For instance, if ψ is a fermion charged under both $U(1)_{\text{em}}$ and $U(1)_D$, and behaves as a KK particle similar to E , then a diagram (shown in Fig. 2) enables the interaction of DM with the SM. A key feature of this diagram is the resonance effect during DM annihilation into SM fermions. To examine the resonance structure in detail, we must analyze the radiative corrections to the masses, as these can lead to substantial changes in the mass spectrum.

Potential resonant self-interactions. Studies have shown that small-scale structure problems could be mitigated by

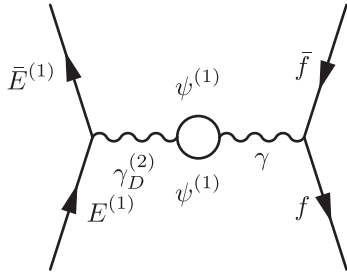


FIG. 2. Dark matter interaction with SM.

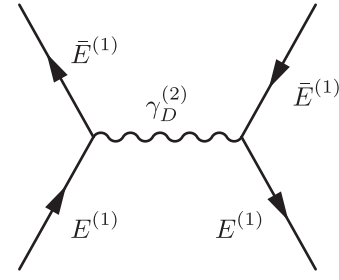


FIG. 3. A diagram that can lead to resonant structure in the self-interaction.

introducing self-interactions among DM particles [23]. Additionally, it has been proposed that resonant self-interacting DM could help reconcile simulation results with experimental data, thereby reducing existing tensions [6,7]. Achieving this, however, requires a strong resonance effect, which may necessitate some fine-tuning of parameters. The desired resonance level, ϵ_R , specified in Eq. (13), should range between 10^{-8} and 10^{-4} . This level of resonance can be achieved, with minimal fine-tuning, when the 4D gauge coupling g_1 is less than 0.01. A brief discussion on small 4D gauge coupling is provided at the end of the next section. The relevant resonant structure is illustrated in the diagram in Fig. 3.

Radiative corrections to masses and the level of resonance. In this section, we discuss the one-loop radiative mass corrections to the states of interest, $B^{(2)}$ and $E^{(1)}$, and examine how these corrections impact the resonance structure. General prescriptions for calculating the radiative corrections in extradimensional models are provided in Refs. [24–27], and we focus on our special consideration below. The mass corrections to the states of interest at one loop level are given by

$$m'_{B^{(2)}} = \sqrt{m_B^2 + \left(\frac{2}{R}\right)^2 - \frac{g_1^2 \zeta(3)}{16\pi^4 R^2}}, \quad (11)$$

$$m'_{E^{(1)}} = \frac{1}{R} \left(1 + \frac{9}{64\pi^2} g_1^2 \ln \frac{\Lambda^2}{\mu^2} \right). \quad (12)$$

We define the “level of resonance”, ϵ_R [5], as follows:

$$\epsilon_R \equiv \frac{(m'_{B^{(2)}})^2 - (2m'_{E^{(1)}})^2}{(2m'_{E^{(1)}})^2}. \quad (13)$$

A positive value of ϵ_R ($\epsilon_R > 0$) corresponds to the case of an invisible dark photon, while a negative value ($\epsilon_R < 0$) corresponds to a visible dark photon. In the Supplemental Material [20], we discuss and plot the level of resonance as a function of m_B for fixed Λ/μ , with other parameters set to $g_1 = 0.1, 1$ and $1/R = 1$ GeV. It is particularly noteworthy

that, for $g_1 = 0.1$, when $m_B \sim 1/R$, the resonance level remains $\epsilon_R \lesssim \mathcal{O}(0.1)$ across a wide range of cutoff scales, Λ/μ . This feature enables resonance over a large parameter space while consistently preserving the invisibility of the dark photon. Compared with the SM $U(1)$ gauge coupling, $g' = 0.357$, both $g_1 = 0.1$ and $g_1 = 1$ represent viable benchmark choices.

Moreover, very small gauge couplings may be considered unnatural if the 4D gauge coupling is viewed as a fundamental parameter. However, in the context of 5D, $g_1^{(5)}$ is not a fundamental parameter since it is dimensionful. This implies that there is no reason to consider $g_1^{(5)} \sim \sqrt{L}$ as more natural (for a related discussion, see Ref. [28]). Therefore, one can always select $g_1^{(5)}$ and L such that the 4D gauge coupling, $g_1 = g_1^{(5)}/\sqrt{L}$, is very small. This approach further enhances the level of resonance, potentially enabling the realization of resonant self-interacting DM to help alleviate small-scale structure problems [7].

Resonance freeze-out of the relic abundance. This section explores the DM parameter space where the mass resonance can be realized as a concrete model. The dark photon decays invisibly to the dark sector, meaning $m'_{B(2)} > 2m'_{E(1)}$. Therefore, it is sufficient to identify the parameter space where $0 < \epsilon_R < \delta$, where $\delta \lesssim 0.5$, to ensure a sufficient resonance effect.

Here, we project our model to the experimental constraints for the case of invisible dark photons, following related thermal-target studies of vector-portal dark matter [29,30]. The 4D Lagrangian that is responsible for the interaction given in Fig. 2 is given as

$$\begin{aligned} \mathcal{L} \supset & -g'_1 \bar{E}^{(1)} \gamma^\mu \gamma^5 E^{(1)} B_\mu^{(2)} - \Sigma_f \kappa e q_f \bar{f} \gamma^\mu f B_\mu^{(2)} \\ & - g'_1 \bar{E}^{(1)} \gamma^5 E^{(1)} B_5^{(2)}, \end{aligned} \quad (14)$$

where $g'_1 = g_1/\sqrt{2}$, κ , e , and q_f are 4D gauge coupling, kinetic mixing parameter, electric charge, and $U(1)_{\text{em}}$ quantum number, respectively. The dark fine structure constant, α_D , is constructed from g'_1 . Interestingly, the coupling between DM and dark photon is given by axial vector coupling. This kind of coupling is general in the UED model where the first mode of fermion couples to the second mode of vector boson via axial vector coupling; see Ref. [15]. This leads to interesting phenomenology that we highlight at the end of this section. Note that the third term does not contribute to the annihilation cross section since the fifth component of the second mode of dark photon cannot mix with the SM photon. This is because there is no two-point vertex available between the scalar field (B_5) and the vector field (SM photon).

We show the detailed calculation of the relic abundance in the Supplemental Material [20].

Targets for direct-detection experiments. The DM-dark photon coupling will induce the DM-electron scattering cross section that can be tested by future direct detection measurements. We present our model predictions in Fig. 4 using the reference cross section, $\bar{\sigma}_e$, that has been defined in Refs. [31,32],

$$\bar{\sigma}_e = \frac{16\pi\mu_{e,E^{(1)}}^2 \alpha\alpha_D \kappa^2 v^2}{((m'_{B(2)})^2 + \alpha^2 m_e^2)^2}, \quad (15)$$

where $\mu_{e,E^{(1)}}$ is the reduced mass of electron and dark matter and $v = 10^{-3}$ is the typical velocity of DM halo. Interestingly, we have additional velocity suppression in the cross section. This is because the interaction between the dark photon and the DM, in this model, is given by axial vector coupling, while the effective interaction between the dark photon and the SM fermions remains as a vector coupling.

In Fig. 4, we present the naturally resonant dark matter predictions with different levels of resonances, along with limits and projections from direct-detection experiments. The solid black contours shown in Fig. 4 correspond to parameter points that reproduce the observed total dark matter abundance. For a dark photon mass of $m'_{B(2)} = 100$ MeV, which is related to m_χ through Eq. (13), we find $2 \times 10^{-13} < \kappa^2 < 3 \times 10^{-7}$ over the range of ϵ_R values considered. Smaller values of κ correspond to smaller values of ϵ_R . The existing limits are from experiments including DAMIC-M [41], DAMIC-SNOLAB (DAMIC at SNOLAB, i.e., Dark Matter in CCDs at SNOLAB) [42], SENSEI [43], Expérience pour DETecter Les Wimps En Site Souterrain [44], SuperCDMS [45], CDEX-10 (China dark matter experiment) [46], DarkSide-50 [47], XENON1T [48,49], PandaX-II [50], and from the energy loss of supernova SN1987A [39]. The supernova SN1987A bound shown in Fig. 4 should be interpreted only as a reference. This is because our projections correspond to $\alpha_D = 0.04, 0.0004$, and a proper, consistent derivation of the corresponding supernova constraints for this setup would require a dedicated analysis beyond the scope of this work. Additionally, we show projected future sensitivities from SuperCDMS-SNOLAB [35], SENSEI and DAMIC-M [31,33], Oscura [34], DM scattering off of molecules [36], and HeRALD [37], along with the neutrino fog specific for silicon detectors [38]. More specifically, our model, with a minimal level of resonance, may be discovered by the Oscura experiment or the second generation of the HeRALD experiment. Furthermore, higher resonance level scenarios could also be potentially probed by third- and fourth-generation HeRALD experiments. Accelerator searches such as Belle II [51], the NA64 experiment at CERN (NA64) [52], forward liquid argon experiment [53], mini Beam Dump eXperiment [54], light dark matter eXperiment [55,56], and proposed millicharged/fractionally charged particle searches at intensity-frontier facilities [57], and

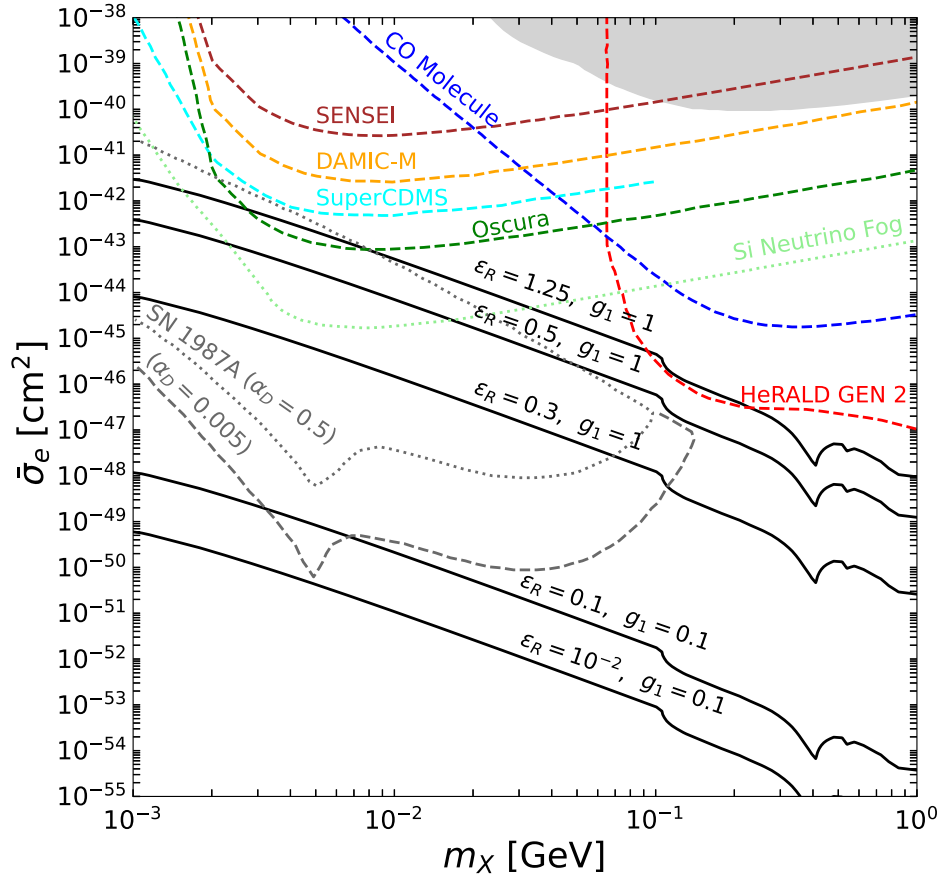


FIG. 4. Solid black contours correspond to the projections of the model for different values of ϵ_R , where $\epsilon_R = 1.25$ corresponds to the nonresonant case when $m'_{B^{(2)}} = 3m'_{E^{(1)}}$. For $\epsilon_R = 1.25, 0.5, 0.3$, we set $g_1 = 1$ ($\alpha_D \sim 0.04$) and for $\epsilon_R = 0.1, 0.01$, we set $g_1 = 0.1$ ($\alpha_D \sim 0.0004$). The gray-shaded region is the excluded region. We also include projected sensitivities, dashed lines, from future experiments, including Sub-Electron-Noise Skipper CCD Experimental Instrument (SENSEI) [31], Dark Matter in CCDs at Modane (DAMIC-M) [33], Oscura [34], super cryogenic dark matter search (SuperCDMS)-SNOLAB (SuperCDMS) [35], DM scattering off of molecules (CO molecule) [36], and a superfluid helium experiment (HeRALD (helium roton apparatus for light dark matter)) [37]. The dotted green curve corresponds to the neutrino fog for silicon detectors (Si neutrino fog) [38]. The dotted/dashed gray curve is the constraint from supernova SN1987A for $\alpha_D = 0.5/0.005$ [39,40]. For SuperCDMS, we present only the projection curve from the cited paper, but the sensitivity can be extended to higher masses.

LongQuest-inspired inelastic-dark-matter searches [58] can potentially discover or constrain the DM model we presented, which is currently not constrained by limits from *BABAR* [60], Compact Muon Solenoid [61], NA64_e [62], E137 [63], liquid scintillator neutrino detector [64], and mini booster neutrino experiment [65].

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Data availability. The data that support the findings of this article are available from the corresponding authors upon reasonable request.

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