

Renormalization of $U(1)$ gauge boson kinetic mixing

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Quantum field theories containing fields with the same quantum numbers allow for mixed kinetic terms in the Lagrangian, leading to off-diagonal elements in the tree-level two-point function. After removing the mixing by a field rotation, the off-diagonal UV divergences cannot be subtracted by a counterterm, still, one can show that the theory is renormalizable. We study kinetic mixing of $U(1)$ gauge bosons in an extension of quantum electrodynamics with a massive “dark” photon at one-loop order. In the general covariant R_ξ -gauge, the gauge fixing function naively obstructs the removal of tree-level mixing, but we show that these off-diagonal gauge-dependent contributions cancel. We compare two renormalization schemes: one with and one without kinetic mixing, and relate them via a scale-dependent field transformation, showing that the schemes are equivalent.

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I. INTRODUCTION

Renormalizability is key for a quantum field theory to be consistent. In many standard cases, the UV divergences can be subtracted through counterterms at the level of each individual amputated Green’s function. However, this is not always possible if the fields mix at loop level. Known examples with this feature are theories with several fields Φ_i with the same quantum numbers. In these cases, one may allow these fields to mix through nondiagonal kinetic terms in the Lagrangian,

$$\mathcal{L} \supset \sum_{i < j} \frac{\kappa_{ij}}{2} \partial_\mu \Phi_i \partial^\mu \Phi_j.$$

These terms may be rotated away at tree level but mixing will reappear at loop level. Divergences from the off-diagonal two-point Green’s functions G_{ij} must be absorbed by a sort of nondiagonal renormalization. Thus, even if one rotates away κ_{ij} initially, one might think that it must be reintroduced at tree level to provide a counterterm that can cancel this divergence. For scalar fields, it was decisively shown to one-loop order in [1] that κ_{ij} can be rotated away

at tree level and that the theory can be consistently renormalized without reintroducing κ_{ij} . The resolution lies in the basic fact that it is only physical quantities that are in need of renormalization, and the two-point functions themselves are not physical, only their poles and S -matrix elements.

As is well known, in theories without kinetic mixing, the reduction formula of Lehmann, Symanzik, and Zimmermann (LSZ) [2] relates S -matrix elements directly to amputated, renormalized correlation functions. It is important to note that these two objects differ in a theory with kinetic mixing, since one has to modify the LSZ theorem to account for mixing on the external legs [3] when computing S -matrix elements. Then, the off-diagonal pieces of G_{ij} contribute to the S -matrix elements, and thus, they do enter the computation of physical quantities. However, it is possible to define a consistent scheme, without reintroducing κ_{ij} , where these divergences are absorbed in counterterms of the couplings.

In this work, we address this matter in the context of kinetic mixing of two $U(1)$ gauge bosons. We work in the general covariant R_ξ -gauge and demonstrate to one-loop order that the theory can be renormalized consistently, both in a basis with and without kinetic mixing, and that the results in the two bases match. Note that renormalization of kinetic mixing has also been discussed in the literature for scalar [4] and Abelian [5] standard model extensions, and in the context of additional Abelian sectors in grand unified theories [6].

On the level of the Lagrangian, removing the off-diagonal kinetic terms produces nondiagonal gauge fixing

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functions, which still lead to a nondiagonal tree-level propagator. We show, however, that these gauge-dependent off-diagonal contributions cancel and that the field redefinition is consistent. We work with standard quantum electrodynamics (QED) with an additional massive, *dark photon*, which does not couple directly to the electron but interacts only through a mixed kinetic term with the photon. This model setup is inspired by popular standard model extensions, e.g., [7–15], where many of the phenomenological consequences of a kinetically mixed massive dark photon are explored, in particular the possibility of dark photon dark matter. To give a nonvanishing gauge-invariant mass to the dark photon, we employ the Stückelberg mechanism [16,17].

This paper is organized as follows. First, in Sec. II we introduce the QED extension with kinetic mixing, renormalize it at one-loop order, and derive the renormalization group equations (RGEs). In Sec. III we do the same in a basis without kinetic mixing, where we only renormalize the physical parameters. We relate the two schemes in Sec. IV and conclude in Sec. V, where we also comment on the role of the specific choice of mass generating mechanism in our analysis.

II. QED EXTENSION WITH $U(1)$ GAUGE BOSON KINETIC MIXING

We consider QED with kinetic mixing between the photon and an additional, massive $U(1)$ gauge boson, which we will refer to as a dark photon. The theory is defined by the (bare) Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(F_{1b}^{\mu\nu})^2 + i\bar{\psi}_b\gamma_\mu(\partial^\mu + ie_b A_{1b}^\mu)\psi_b - m_b\bar{\psi}_b\psi_b \\ & -\frac{1}{2\xi_{1b}}(\partial_\mu A_{1b}^\mu)^2 - \frac{\kappa_b}{2}F_{1b}^{\mu\nu}F_{2b,\mu\nu} \\ & -\frac{1}{4}(F_{2b}^{\mu\nu})^2 + \frac{M_b^2}{2}\left(A_{2b}^\mu - \frac{1}{M_b}\partial^\mu\phi_b\right)^2 \\ & -\frac{1}{2\xi_{2b}}(\partial_\mu A_{2b}^\mu + \xi_{2b}M_b\phi_b)^2, \end{aligned} \quad (1)$$

where A_{1b} describes the photon, A_{2b} the dark photon with mass M_b , ψ_b the electron with mass m_b , and we define $F_{ib}^{\mu\nu} = \partial^\mu A_{ib}^\nu - \partial^\nu A_{ib}^\mu$, $i = 1, 2$, as usual. We denote the symmetry group by $U(1)_{\text{QED}} \times U(1)_{\text{dark}}$. The second line in the above Lagrangian is the Stückelberg Lagrangian. The

Stückelberg mechanism [16,17] allows us to give a mass to the dark photon in a gauge-invariant way, by introducing the Stückelberg field ϕ_b . For the Lagrangian (1) to be invariant under the gauge transformations

$$\begin{aligned} A_{1b}^\mu(x) & \xrightarrow{U(1)_{\text{QED}}} A_{1b}^\mu(x) - \frac{1}{e_b}\partial^\mu\alpha_1(x) \quad \text{and} \\ A_{2b}^\mu(x) & \xrightarrow{U(1)_{\text{dark}}} A_{2b}^\mu(x) - \partial^\mu\alpha_2(x), \end{aligned} \quad (2)$$

we require the Stückelberg field to transform trivially under $U(1)_{\text{QED}}$ and as

$$\phi_b(x) \xrightarrow{U(1)_{\text{dark}}} \phi_b(x) - M_b\alpha_2(x) \quad (3)$$

under $U(1)_{\text{dark}}$, such that the mass term of the dark photon is invariant. For the gauge fixing term of A_{2b} to be invariant as well, we impose for $\alpha_2(x)$ the Klein-Gordon equation

$$(\partial^2 - \xi_{2b}M_b^2)\alpha_2(x) = 0. \quad (4)$$

To see more clearly that the dark photon field A_{2b} acquires a mass in this model, note that, up to a total derivative, the Stückelberg Lagrangian may be rewritten as

$$\begin{aligned} \mathcal{L}_{\text{Stückelberg}} = & -\frac{1}{4}(F_{2b}^{\mu\nu})^2 - \frac{1}{2\xi_{2b}}(\partial_\mu A_{2b}^\mu)^2 + \frac{M_b^2}{2}(A_{2b}^\mu)^2 \\ & + \frac{1}{2}(\partial_\mu\phi_b)^2 - \xi_{2b}\frac{M_b^2}{2}\phi_b^2. \end{aligned} \quad (5)$$

For a more detailed review of the Stückelberg mechanism, see, for example, [18].

In our Lagrangian in Eq. (1), there is no direct fermion–dark photon coupling. The only coupling of the dark photon is through the kinetic mixing to the photon, parametrized by κ_b . We will now renormalize this theory at one-loop and derive the RGEs. First, we compute the gauge boson propagator and full two-point function, which will be matrix-valued due to the mixing.

A. Two-point function and masses

The tree-level propagator P is found from a fundamental solution to the equations of motion. The equations of motion from the Lagrangian (1) are

$$\begin{pmatrix} \partial^2 g_{\mu\nu} - (1 - \frac{1}{\xi_{1b}})\partial_\mu\partial_\nu & \kappa_b(\partial^2 g_{\mu\nu} - \partial_\mu\partial_\nu) \\ \kappa_b(\partial^2 g_{\mu\nu} - \partial_\mu\partial_\nu) & (\partial^2 + M_b^2)g_{\mu\nu} - (1 - \frac{1}{\xi_{2b}})\partial_\mu\partial_\nu \end{pmatrix} \begin{pmatrix} A_{1b}^\mu \\ A_{2b}^\mu \end{pmatrix} = \begin{pmatrix} J_{b,\nu}^{\text{QED}} \\ 0 \end{pmatrix}, \quad (6)$$

where $J_{b,\nu}^{\text{QED}} = \bar{\psi}_b(x)\gamma_\nu\psi_b(x)$ is the standard QED current. By Fourier transforming this matrix and inverting it as a block matrix, we find the matrix-valued propagator

$$iP = \begin{pmatrix} \frac{-i}{p^2} \left(A(p^2) \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \xi_{1b} \frac{p_\mu p_\nu}{p^2} \right) & \frac{i\kappa_b}{(1-\kappa_b^2)p^2 - M_b^2} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \\ \frac{i\kappa_b}{(1-\kappa_b^2)p^2 - M_b^2} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) & \frac{-i}{(1-\kappa_b^2)p^2 - M_b^2} \left(g^{\mu\nu} - \frac{p^\mu p^\nu (1 - \xi_{2b}(1-\kappa_b^2))}{(p^2 - \xi_{2b}M_b^2)} \right) \end{pmatrix}, \quad (7)$$

where we have defined

$$A(p^2) = \frac{1}{(1 - \kappa_b^2)} \frac{p^2 - M_b^2}{p^2 - M_b^2 / (1 - \kappa_b^2)}. \quad (8)$$

For two fields A_i^μ and A_j^ν , the full two-point function $G_{ij}^{\mu\nu}$ is defined as

$$\langle \Omega | T \{ A_i^\mu(x) A_j^\nu(y) \} | \Omega \rangle = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} iG_{ij}^{\mu\nu}(p^2), \quad (9)$$

and we denote its tree-level contribution by $iP_{ij}^{\mu\nu}$. Furthermore, let $i\Pi_{ij}^{\mu\nu}(p^2)$ be the all-order one-particle irreducible (1PI) contribution. Then, suppressing the Lorentz indices $\mu\nu$, the two-point function G_{ij} can be written as

$$\begin{aligned} iG_{ij} &= i \text{ (tree) } + i \text{ (1PI) } + i \text{ (2PI) } + \dots \\ &= iP + iP(i\Pi iP) + iP(i\Pi iP)^2 + \dots \\ &= iP(1 + \Pi P)^{-1}. \end{aligned} \quad (10)$$

Due to gauge invariance, $\Pi_{ij}^{\mu\nu}(p^2)$ is transversal, i.e., $p_\mu \Pi_{ij}^{\mu\nu}(p^2) = 0$. Thus, the Lorentz structure can be factored out, and we may write

$$\Pi_{ij}^{\mu\nu}(p^2) = -(p^2 g^{\mu\nu} - p^\mu p^\nu) \Pi_{ij}(p^2). \quad (11)$$

In the next section, we compute the one-loop UV divergences and renormalize our theory.

B. Renormalization

To renormalize the theory, we introduce the rescalings

$$\begin{aligned} \psi_b &= \sqrt{Z_2} \psi, & A_{1b}^\mu &= \sqrt{Z_3} A_1^\mu, & A_{2b}^\mu &= \sqrt{Z_4} A_2^\mu, & m_b &= Z_m m, & e_b &= \mu^\epsilon Z_e e, \\ M_b &= \sqrt{Z_M} M, & \kappa_b &= Z_\kappa \kappa, & \frac{1}{\xi_{1b}} &= Z_{\xi_1} \frac{1}{\xi_1}, & \frac{1}{\xi_{2b}} &= Z_{\xi_2} \frac{1}{\xi_2}, & \phi_b &= \sqrt{Z_\phi} \phi. \end{aligned}$$

The renormalization factors are decomposed into counterterms as $Z_i = 1 + \delta_i$, and we define the two combined renormalization factors

$$\begin{aligned} Z_1 &:= Z_e Z_2 \sqrt{Z_3} \Rightarrow \delta_1 := \delta_e + \delta_2 + \frac{1}{2} \delta_3, \\ Z_{\text{mix}} &:= Z_\kappa \sqrt{Z_3} \sqrt{Z_2} \Rightarrow \delta_{\text{mix}} := \delta_\kappa + \frac{1}{2} (\delta_3 + \delta_4). \end{aligned}$$

The original Lagrangian (1) can now be written as a “renormalized” Lagrangian plus the counterterm Lagrangian,

$$\begin{aligned} \mathcal{L}_\delta &= -\frac{\delta_3}{4} (F_1^{\mu\nu})^2 + i\delta_2 \gamma^\mu \bar{\psi} \partial^\mu \psi - e\delta_1 \gamma_\mu \bar{\psi} A_1^\mu \psi - m(\delta_m + \delta_2) \bar{\psi} \psi \\ &\quad - \frac{\delta_3 + \delta_{\xi_1}}{2\xi_1} (\partial_\mu A_1^\mu)^2 - \kappa \frac{\delta_{\text{mix}}}{2} F_1^{\mu\nu} F_{2\mu\nu} - \frac{\delta_4}{4} (F_2^{\mu\nu})^2 - \frac{\delta_4 + \delta_{\xi_2}}{2\xi_2} (\partial_\mu A_2^\mu)^2 \\ &\quad + (\delta_M + \delta_4) \frac{M^2}{2} (A_2^\mu)^2 + \frac{\delta_\phi}{2} (\partial_\mu \phi)^2 - \xi_2 \frac{M^2}{2} (\delta_{\xi_2} + \delta_M + \delta_\phi) \phi^2. \end{aligned} \quad (12)$$

The Feynman rules for the counterterm Lagrangian are

$$\begin{aligned}
 \mu \text{---}\text{wavy}\text{---}\nu &= -i\delta_3(p^2 g^{\mu\nu} - p^\mu p^\nu) - i\frac{\delta_3 + \delta_{\xi_1}}{\xi_1} p^\mu p^\nu, \\
 \mu \text{---}\text{wavy}\text{---}\nu &= -i\delta_4(p^2 g^{\mu\nu} - p^\mu p^\nu) + i(\delta_M + \delta_4)M^2 g^{\mu\nu} - i\frac{\delta_4 + \delta_{\xi_2}}{\xi_1} p^\mu p^\nu, \\
 \mu \text{---}\text{wavy}\text{---}\nu &= -i\delta_{\text{mix}}\kappa(p^2 g^{\mu\nu} - p^\mu p^\nu), \\
 \text{---}\otimes\text{---} &= i(\not{p}\delta_2 - m(\delta_m + \delta_2)), \\
 \text{---}\otimes\text{---} &= i(p^2\delta_\phi - \xi_2 M^2(\delta_{\xi_2} + \delta_M + \delta_\phi)), \\
 \text{---}\otimes\text{---} &= -ie\delta_1\gamma^\mu.
 \end{aligned} \tag{13}$$

Throughout this paper, for any quantity f , we write $f^{[l]}$ for the l -loop contribution to f . There are no amputated loop diagrams with external dark photons or Stückelberg fields, so there are only three one-loop diagrams we need to compute: the photon and electron self-energy, and the vertex correction, respectively,

$$\begin{aligned}
 i\Pi_{11}^{[1]\mu\nu}(p^2) &= \text{wavy loop} + \text{wavy}\text{---}\otimes\text{---}, & i\Sigma^{[1]}(\not{p}) &= \text{wavy}\text{---}\text{---}\text{---} + \text{---}\otimes\text{---}, \\
 i\Gamma_1^{[1]\mu}(p^2) &= \text{wavy}\text{---}\text{---}\text{---} + \text{---}\otimes\text{---}.
 \end{aligned}$$

The latter two of these objects differ from the ones in standard QED by the fact that we have to use the modified propagator iP_{11} for the photon, defined through Eq. (7). However, since we only extract the UV poles, the extra factor A simplifies to

$$A(p^2) \xrightarrow{p \rightarrow \infty} \frac{1}{1 - \kappa^2}. \tag{14}$$

The tensor structures might be simplified, for example, using FeynCalc [19,20]. After a straightforward computation, we find the UV poles

$$\begin{aligned}
 i\Pi_{11}^{[1]\mu\nu}(p^2) &= \frac{-e^2}{12\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu)\frac{1}{\epsilon} - i\delta_3(p^2 g^{\mu\nu} - p^\mu p^\nu) - i\frac{\delta_3 + \delta_{\xi_1}}{\xi_1} p^\mu p^\nu, \\
 i\Sigma^{[1]}(\not{p}) &= \frac{ie^2}{16\pi^2\epsilon} \left(\xi_1(\not{p} - m) - 3m\frac{1}{1 - \kappa^2} \right) + i(\not{p}\delta_2 - m(\delta_m + \delta_2)), \\
 i\Gamma_1^{[1]\mu}(p^2) &= -\frac{ie^3}{16\pi^2\epsilon}\xi_1\gamma^\mu - ie\delta_1\gamma^\mu.
 \end{aligned} \tag{15}$$

From here, we find the minimal subtraction counterterms at one-loop order,

$$\begin{aligned}
 \delta_3^{[1]} = -\delta_{\xi_1}^{[1]} &= -\frac{e^2}{12\pi^2}\frac{1}{\epsilon}, & \delta_2^{[1]} &= -\frac{e^2}{16\pi^2\epsilon}\xi_1, \\
 \delta_m^{[1]} &= -\frac{e^2}{16\pi^2\epsilon}\frac{3}{1 - \kappa^2}, & \delta_1^{[1]} &= -\frac{e^2}{16\pi^2\epsilon}\xi_1.
 \end{aligned} \tag{16}$$

Since there are no one-loop diagrams for the dark-photon self-energy or the mixed photon–dark photon two-point function, we simply have that the corresponding counter-terms in Eq. (13) vanish,

$$\delta_4^{[1]} = \delta_M^{[1]} = \delta_{\xi_2}^{[1]} = 0, \quad \delta_{\text{mix}}^{[1]} = 0 \Rightarrow \delta_k^{[1]} = -\frac{1}{2}\delta_3^{[1]}. \quad (17)$$

C. Renormalization group

Here we derive the RGEs to one-loop order. As usual, we define the beta function for a generic coupling g as $\beta(g) := \mu \frac{d}{d\mu} g|_{\epsilon=0}$ and the anomalous dimension of a generic parameter Λ as $\gamma_\Lambda := \frac{\mu}{\Lambda} \frac{d}{d\mu} \Lambda|_{\epsilon=0}$. Using that the bare parameters are independent of μ , we get

$$0 = \mu \frac{d}{d\mu} e_b = \mu \mu^\epsilon e Z_e \left(\epsilon + \frac{\mu}{e} \frac{de}{d\mu} + \frac{\mu}{Z_e} \frac{dZ_e}{d\mu} \right), \quad (18)$$

and for the β function

$$\beta^{[1]}(e) = \frac{e^3}{12\pi^2}, \quad (19)$$

as in standard QED. We also have that

$$0 = \mu \frac{d}{d\mu} m_b = Z_m m \left(\frac{\mu}{m} \frac{dm}{d\mu} + \frac{\mu}{Z_m} \frac{dZ_m}{d\mu} \right), \quad (20)$$

so the anomalous dimension for m is

$$\gamma_m^{[1]} = -\frac{3e^2}{8\pi^2} \frac{1}{1-\kappa^2}, \quad (21)$$

which reproduces the standard QED result upon taking the limit $\kappa \rightarrow 0$. Furthermore, we capture the running of κ in the quantity

$$\gamma_\kappa^{[1]} = \frac{e^2}{12\pi^2}. \quad (22)$$

Finally, for the gauge fixing parameter, we find

$$\gamma_{\xi_1^{-1}}^{[1]} = \frac{e^2}{6\pi^2}. \quad (23)$$

The remaining parameters in our Lagrangian do not run at one-loop in this basis. The picture changes once we perform a change of basis in field space to remove the mixing term $-\frac{\kappa_b}{2} F_{1b}^{\mu\nu} F_{2b,\mu\nu}$ in the Lagrangian, which will be the subject of the next section.

III. BASIS WITHOUT KINETIC MIXING

We can rotate in field space to trade the mixing term in the Lagrangian (1) for an explicit dark photon–fermion coupling and nondiagonal gauge fixing functions. To do so, we start from our original Lagrangian and substitute

$$\begin{pmatrix} A_{1b}^\mu \\ A_{2b}^\mu \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\kappa_b}{\sqrt{1-\kappa_b^2}} \\ 0 & \frac{1}{\sqrt{1-\kappa_b^2}} \end{pmatrix} \cdot \begin{pmatrix} \tilde{A}_{1b}^\mu \\ \tilde{A}_{2b}^\mu \end{pmatrix} \quad (24)$$

to remove the kinetic mixing. Note that the choice of rotation matrix is not unique. One can always apply an additional orthogonal rotation keeping the kinetic terms canonically normalized, but we do not need this extra degree of freedom here. Dropping the tilde on the field variables, the original Lagrangian becomes

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(F_{1b}^{\mu\nu})^2 - \frac{1}{4}(F_{2b}^{\mu\nu})^2 + i\bar{\psi}_b \gamma_\mu \partial^\mu \psi_b - \tilde{m}_b \bar{\psi}_b \psi_b \\ & - e_{1b} \bar{\psi}_b \gamma_\mu A_{1b}^\mu \psi_b - e_{2b} \bar{\psi}_b \gamma_\mu A_{2b}^\mu \psi_b \\ & - \frac{1}{2\tilde{\xi}_{1b}} \left(\partial_\mu A_{1b}^\mu + \frac{e_{2b}}{e_{1b}} \partial_\mu A_{2b}^\mu \right)^2 - \frac{1}{2\tilde{\xi}_{2b}} (\partial_\mu A_{2b}^\mu)^2 \\ & + \frac{\tilde{M}_b^2}{2} (A_{2b}^\mu)^2 + \frac{1}{2} (\partial_\mu \phi_b)^2 - \tilde{\xi}_{2b} \frac{\tilde{M}_b^2}{2} \phi_b^2, \end{aligned} \quad (25)$$

with the new parameters satisfying

$$\begin{aligned} \tilde{m}_b &= m_b, \quad \tilde{M}_b = \frac{M_b}{\sqrt{1-\kappa_b^2}}, \quad e_{1b} = e_b, \\ e_{2b} &= -\frac{e_b \kappa_b}{\sqrt{1-\kappa_b^2}}, \quad \tilde{\xi}_{1b} = \xi_{1b}, \quad \tilde{\xi}_{2b} = (1-\kappa_b^2) \xi_{2b}. \end{aligned} \quad (26)$$

Now we have an explicit electron–dark photon coupling, but the kinetic mixing is removed. The rotation also produced nondiagonal gauge fixing functions, but we will show explicitly how the off-diagonal contributions cancel when computing any correlation function in the next section.

A. Tree-level propagator

Due to the nondiagonal gauge fixing function, the matrix-valued gauge boson propagator still has nonzero off-diagonal elements. The Fourier transformed quadratic terms are

$$P^{-1} := \begin{pmatrix} -p^2 g_{\mu\nu} + \left(1 - \frac{1}{\xi_{1b}}\right) p_\mu p_\nu & -\frac{e_{2b}}{e_{1b} \xi_{1b}} p_\mu p_\nu \\ -\frac{e_{2b}}{e_{1b} \xi_{1b}} p_\mu p_\nu & (-p^2 + M_b^2) g_{\mu\nu} + \left(1 + \frac{e_{2b}}{e_{1b} \xi_{1b}} - \frac{1}{\xi_{2b}}\right) p_\mu p_\nu \end{pmatrix}, \quad (27)$$

and thus we find for the propagator

$$iP^{\mu\nu} = \begin{pmatrix} -\frac{i}{p^2} \left(g^{\mu\nu} - \left(\frac{1-\xi_{1b}}{p^2} - \frac{e_{2b}}{e_{1b}} C(p^2) \right) p^\mu p^\nu \right) & iC(p^2) \frac{p^\mu p^\nu}{p^2} \\ iC(p^2) \frac{p^\mu p^\nu}{p^2} & -\frac{i}{(p^2 - M_b^2)} \left(g^{\mu\nu} - \frac{(1-\xi_{2b})p^\mu p^\nu}{(p^2 - M_b^2 \xi_{2b})} \right) \end{pmatrix}, \quad (28)$$

where

$$C(p^2) = \xi_{2b} \frac{e_{2b}}{e_{1b}} \frac{1}{p^2 - M_b^2 \xi_{2b}}. \quad (29)$$

We see that, even having removed the κ term in the Lagrangian, the off-diagonal elements are still nonvanishing. However, they are unphysical, as indicated by the overall factor of ξ_{2b} . To see this explicitly in a general gauge, let us define

$$iP_{11}^{\mu\nu} := i\tilde{P}_{11}^{\mu\nu} - i \frac{e_{2b}}{e_{1b}} C(p^2) \frac{p^\mu p^\nu}{p^2}, \quad (30)$$

i.e., split $P_{11}^{\mu\nu}$ into the usual photon propagator $\tilde{P}_{11}^{\mu\nu}$ plus an additional piece.

Now, let us consider an arbitrary Feynman diagram in this theory. For every gauge boson propagator that appears in the diagram, one also has to consider the diagrams obtained by replacing that propagator with either a mixed propagator or the one of the other gauge boson. Summing over these three contributions, taking into account the different couplings of attaching each of these propagators to the rest of the diagram, gives

$$e_{1b}^2 P_{11}^{\mu\nu} + e_{1b} e_{2b} P_{12}^{\mu\nu} + e_{2b}^2 P_{22}^{\mu\nu} = e_{1b}^2 \tilde{P}_{11}^{\mu\nu} + e_{2b}^2 P_{22}^{\mu\nu}. \quad (31)$$

This shows that the off-diagonal part of the gauge fixing term has no effect and the theory behaves as if we had a massless photon and a massive vector boson without kinetic mixing, i.e., with diagonal tree-level propagators.

B. UV poles

In this basis without the mixed kinetic term, loop corrections still induce a nonvanishing off-diagonal gauge boson two-point function, even though there is no tree-level term in the Lagrangian. Without the parameter κ , we cannot make the individual off-diagonal two-point functions finite. Instead, as renormalization conditions we impose all observables, i.e., the masses and S -matrix elements, to be finite (cf. [1]). The two-point functions are not physical observables, and thus, they are not required to be finite. Accordingly, we introduce renormalization constants only for the two masses and two couplings of the theory (in the same fashion as in [1] for the scalar case),

$$\begin{aligned} \tilde{m}_b &= Z_{\tilde{m}} \tilde{m}, & e_{1b} &= Z_{e_1} e_1, \\ \tilde{M}_b &= \sqrt{Z_{\tilde{M}}} \tilde{M}, & e_{2b} &= Z_{e_2} e_2. \end{aligned} \quad (32)$$

Following [1], we do not introduce field rescalings here but, equivalently, we will instead include the residues of the two-point functions explicitly in the LSZ theorem in Sec. III C. As before, we decompose the renormalization factors as $Z_i = 1 + \delta_i$, and the counterterm Lagrangian then reads

$$\begin{aligned} \mathcal{L}_\delta &= -e_1 \delta_{e_1} \gamma_\mu \bar{\psi} A_1^\mu \psi - e_2 \delta_{e_2} \gamma_\mu \bar{\psi} A_2^\mu \psi - \tilde{m} \delta_{\tilde{m}} \bar{\psi} \psi \\ &\quad + \delta_{\tilde{M}} \frac{\tilde{M}^2}{2} (A_2^\mu)^2 - \xi_2 \frac{\tilde{M}^2}{2} \delta_{\tilde{M}} \phi^2. \end{aligned} \quad (33)$$

The Feynman rules for the counterterms are

$$\begin{aligned} \mu \text{---}\otimes\text{---}\nu &= +i\delta_{\tilde{M}} \tilde{M}^2 g^{\mu\nu}, & \text{---}\otimes\text{---} &= -i\tilde{m}\delta_{\tilde{m}}, & \text{---}\otimes\text{---} &= -i\xi_2 \tilde{M}^2 \delta_{\tilde{M}} \\ \text{---}\otimes\text{---}\mu &= -ie_1 \delta_{e_1} \gamma^\mu, & \text{---}\otimes\text{---}\mu &= -ie_2 \delta_{e_2} \gamma^\mu. \end{aligned} \quad (34)$$

Next, we will compute the one-loop UV divergences. As we have shown in Sec. III A, we may work with the standard diagonal propagators

$$\begin{aligned} \mu \text{---}\nu &= \frac{-i}{p^2} \left(g_{\mu\nu} - (1 - \tilde{\xi}_1) \frac{p_\mu p_\nu}{p^2} \right), \\ \mu \text{---}\nu &= \frac{-i}{p^2 - \tilde{M}^2} \left(g_{\mu\nu} - (1 - \tilde{\xi}_2) \frac{p_\mu p_\nu}{p^2 - \tilde{\xi}_2 \tilde{M}^2} \right). \end{aligned} \quad (35)$$

The relevant one-loop quantities are 4 two-point functions, which we denote by

$$\begin{aligned}
 i\Pi_{11}^{[1]\mu\nu} &= \text{diagram 1}, & i\Pi_{12}^{[1]\mu\nu} &= \text{diagram 2}, \\
 i\Pi_{22}^{[1]\mu\nu} &= \text{diagram 3} + \text{diagram 4}, \\
 i\Sigma^{[1]} &= \text{diagram 5} + \text{diagram 6} + \text{diagram 7},
 \end{aligned}$$

and the 2 three-point functions

$$\begin{aligned}
 i\Gamma_1^{[1]\mu} &= \text{diagram 8} + \text{diagram 9} + \text{diagram 10}, \\
 i\Gamma_2^{[1]\mu} &= \text{diagram 11} + \text{diagram 12} + \text{diagram 13}.
 \end{aligned}$$

Many of these objects only differ by constants originating from the vertex factors. We find the UV poles

$$\begin{aligned}
 i\Pi_{11}^{[1]\mu\nu} &= \frac{-ie_1^2}{12\pi^2} (p^2 g^{\mu\nu} - p^\mu p^\nu) \frac{1}{\epsilon}, & i\Pi_{12}^{[1]\mu\nu} &= -\frac{ie_1 e_2}{12\pi^2} (p^2 g^{\mu\nu} - p^\mu p^\nu) \frac{1}{\epsilon}, \\
 i\Pi_{22}^{[1]\mu\nu} &= \frac{-ie_2^2}{12\pi^2} (p^2 g^{\mu\nu} - p^\mu p^\nu) \frac{1}{\epsilon} + i\delta_{\tilde{M}} \tilde{M}^2 g^{\mu\nu}, \\
 i\Sigma^{[1]} &= \frac{i}{16\pi^2 \epsilon} ((e_1^2 \tilde{\xi}_1 + e_2^2 \tilde{\xi}_2) \not{p} - (e_1^2 (\tilde{\xi}_1 + 3) + e_2^2 (\tilde{\xi}_2 + 3)) \tilde{m}) - i\tilde{m} \delta_{\tilde{m}}, \\
 i\Gamma_1^{[1]\mu} &= -\frac{ie_1}{16\pi^2 \epsilon} \gamma^\mu (\tilde{\xi}_1 e_1^2 + \tilde{\xi}_2 e_2^2) - ie_1 \delta_{e_1} \gamma^\mu, \\
 i\Gamma_2^{[1]\mu} &= -\frac{ie_2}{16\pi^2 \epsilon} \gamma^\mu (\tilde{\xi}_1 e_1^2 + \tilde{\xi}_2 e_2^2) - ie_2 \delta_{e_2} \gamma^\mu.
 \end{aligned} \tag{36}$$

C. Resummed Green's functions, LSZ theorem, and renormalization

The crucial observation is that in this basis, even if we were to renormalize all parameters, there is no counterterm able to absorb the divergence of $\Pi_{12}^{[1]\mu\nu}$, so the off-diagonal element of the full Green's function, G_{12} , cannot be made finite. This is not a problem, as it is only the poles of G_{ij} (i.e., the masses of the particles) that are observables and in need of renormalization. The other observables that have to be finite are the S -matrix elements calculated from the LSZ theorem, which differ from amputated, renormalized Green's functions due to the mixing on the external legs. After accounting for this [3], the S -matrix elements may receive a contribution from G_{12} . However, this divergence can be absorbed by the coupling counterterms.

The physical masses of the photon and dark photon are the poles of the matrix-valued two-point function $G^{\mu\nu}$. They may be found as the zeros of the determinant of the inverse of $G^{\mu\nu}$ [3],

$$\det(G^{-1}) = \det((1 + \Pi P)P^{-1}) = \det(P^{-1} + \Pi), \tag{37}$$

where we again suppressed the Lorentz indices. This allows us to determine the poles without explicitly resumming all 1PI contributions. The determinant evaluates to

$$\begin{aligned}
 \det(G^{-1}) &= \frac{(p^2)^4}{\xi_1 \xi_2} (p^2 \Delta - \tilde{M}^2 (1 + \Pi_{11}))^3 (p^2 - \tilde{\xi}_2 \tilde{M}^2), \\
 \Delta &= \det \begin{pmatrix} \Pi_{11} + 1 & \Pi_{12} \\ \Pi_{12} & \Pi_{22} + 1 \end{pmatrix}.
 \end{aligned} \tag{38}$$

The two gauge-independent pole masses $\tilde{m}_{\gamma,p}^2$ and $\tilde{M}_p^2$ are, respectively, the solutions of $p^2 = 0$ (i.e., the photon mass) and $\tilde{M}^2 (1 + \Pi_{11}) - p^2 \Delta = 0$, which determines the mass of the dark photon.

As our propagators have a nonstandard structure, we also resum the 1PI diagrams to provide the explicit results for the resummed Green's functions in the general R_ξ -gauge.

Note that also in this basis, $\Pi_{ij}^{\mu\nu}$ is transversal due to gauge invariance. In standard QED, the resummed photon two-point function is

$$iG^{\mu\nu} = \frac{-i}{p^2(1+\Pi)} \left(g^{\mu\nu} - \frac{(1-(1+\Pi)\xi)p^\mu p^\nu}{p^2} \right). \quad (39)$$

Once we add the dark photon, the expressions become slightly more involved and matrix-valued. In this basis, the components of the resummed Green's function are

$$\begin{aligned} G_{11}^{\mu\nu} &= \frac{-1}{p^2} \left(\frac{p^2(1+\Pi_{22}) - \tilde{M}^2}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \right. \\ &\quad \left. + \left(\tilde{\xi}_1 + \frac{e_2^2}{e_1^2} \frac{p^2 \tilde{\xi}_2}{p^2 - \tilde{\xi}_2 \tilde{M}^2} \right) \frac{p^\mu p^\nu}{p^2} \right), \\ G_{12}^{\mu\nu} &= \frac{\Pi_{12}}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \\ &\quad + \frac{e_2}{e_1} \frac{\tilde{\xi}_2}{p^2 - \tilde{\xi}_2 \tilde{M}^2} \frac{p^\mu p^\nu}{p^2}, \\ G_{22}^{\mu\nu} &= \frac{-1}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \left((1+\Pi_{11}) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2 - \tilde{\xi}_2 \tilde{M}^2} \right) \right. \\ &\quad \left. + \tilde{\xi}_2 \Delta \frac{p^\mu p^\nu}{p^2 - \tilde{\xi}_2 \tilde{M}^2} \right). \end{aligned} \quad (40)$$

Note that the poles coincide with the zeros of $\det(G^{-1}) = 0$. Furthermore, note that, even though the kinetic mixing is removed in this basis, these results are modified due to the presence of the off-diagonal gauge fixing terms in the tree-level propagator. However, just as at tree level, there is a cancellation of the same type as in Eq. (31) also here, on the level of the resummed Green's function.

For the pole mass $\tilde{M}_P^2$ up to one-loop order, we find

$$(\tilde{M}_P^2)^{[0]} + (\tilde{M}_P^2)^{[1]} = \tilde{M}^2 \left(1 - \frac{e_2^2}{12\pi^2} \frac{1}{\epsilon} + \delta_M^{[1]} \right), \quad (41)$$

and thus,

$$\delta_M^{[1]} = \frac{e_2^2}{12\pi^2} \frac{1}{\epsilon}. \quad (42)$$

For the electron, the resummed two-point function is

$$\begin{aligned} G_e(\not{p}) &= \frac{1}{\not{p} - \tilde{m} + \Sigma(\not{p})} = \frac{1}{\not{p}(1+\Sigma_V) - \tilde{m}(1-\Sigma_S)}, \\ \Sigma(\not{p}) &= \not{p}\Sigma_V + \tilde{m}\Sigma_S. \end{aligned} \quad (43)$$

We require the physical pole mass to one-loop order,

$$\tilde{m}_P^{[0]} + \tilde{m}_P^{[1]} = \tilde{m}(1 + \delta_m^{[1]} - \Sigma_S^{[1]} - \Sigma_V^{[1]}), \quad (44)$$

to be finite, which fixes the counterterm

$$\delta_m^{[1]} = \Sigma_V^{[1]} + \Sigma_S^{[1]} = -\frac{3}{16\pi^2\epsilon}(e_1^2 + e_2^2). \quad (45)$$

At this pole we define the residue

$$Z_e = \left(\lim_{\not{p} \rightarrow \tilde{m}_P} (\not{p} - \tilde{m}_P) G_e(\not{p}) \right) = \frac{1}{1 + \Sigma_V} \Big|_{\not{p} = \tilde{m}_P}. \quad (46)$$

The Källén-Lehmann representation for vector bosons in the covariant gauge may be written as [21]

$$\begin{aligned} -G_{ij}(p^2) &= Z_{ij}^{(k)} \left(\frac{g^{\mu\nu} - p^\mu p^\nu / M_k^2}{p^2 - M_k^2 + i\epsilon} + z_{ij}^{(k)} \frac{p^\mu p^\nu / M_k^2}{p^2 - \xi M_k^2 + i\epsilon} \right) \\ &\quad + \int_{M_0^2}^{\infty} \rho_{ij}(s) \frac{g^{\mu\nu} - p^\mu p^\nu / s}{p^2 - s + i\epsilon} ds, \end{aligned} \quad (47)$$

where $Z_{ij}^{(k)}$ and $z_{ij}^{(k)}$ are scalar numbers and $\rho_{ij}(s)$ is a positive measure. The factor $Z_{ij}^{(k)}$ is simply the residue at the pole of the ij two-point function at the state with mass M_k^2 . For practical purposes it is easiest to find the residue by fixing the gauge to the Feynman gauge, $\tilde{\xi}_1 \rightarrow 0$, $\tilde{\xi}_2 \rightarrow 0$. In particular, we find

$$\begin{aligned} Z_{11}^{(1)} &= \lim_{p^2 \rightarrow 0} p^2 \left(\frac{1}{p^2} \frac{p^2(1+\Pi_{22}) - \tilde{M}^2}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \right) = \frac{1}{1+\Pi_{11}} \Big|_{p^2=0}, \\ Z_{22}^{(2)} &= \lim_{p^2 \rightarrow \tilde{M}_P^2} (p^2 - \tilde{M}_P^2) \frac{(1+\Pi_{11})}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \\ &= \frac{1+\Pi_{11}}{\frac{d}{dp^2} (\Delta p^2 - \tilde{M}^2(1+\Pi_{11}))} \Big|_{p^2=\tilde{M}_P^2}, \\ Z_{12}^{(1)} &= \lim_{p^2 \rightarrow 0} p^2 \left(\frac{-\Pi_{12}}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \right) = 0, \\ Z_{12}^{(2)} &= \lim_{p^2 \rightarrow \tilde{M}_P^2} (p^2 - \tilde{M}_P^2) \left(\frac{-\Pi_{12}}{\Delta p^2 - \tilde{M}^2(1+\Pi_{11})} \right) \\ &= \frac{-\Pi_{12}}{\frac{d}{dp^2} (\Delta p^2 - \tilde{M}^2(1+\Pi_{11}))} \Big|_{p^2=\tilde{M}_P^2}. \end{aligned} \quad (48)$$

We denote the two physical three-point amplitudes for $e\bar{e}\gamma$ and $e\bar{e}\gamma_{\text{dark}}$ by $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively. They may be computed from the LSZ theorem after accounting for mixing on the external legs, as was demonstrated for a scalar theory in [3]. In our case, the S -matrix elements are then given by

$$\begin{aligned}
\mathcal{A}_i^\mu &= \left(\lim_{\not{p} \rightarrow \tilde{m}_P} (\not{p} - \tilde{m}_P) G_e(\not{p}) \right)^2 \lim_{p^2 \rightarrow \tilde{m}_i^2} (p^2 - \tilde{m}_i^2) G_{ij}^{\mu\nu}(p^2) \frac{1}{\sqrt{Z_e^2 Z_{ii}^{(i)}}} \Gamma_{j,\nu} \\
&= \left(\lim_{\not{p} \rightarrow \tilde{m}_P} (\not{p} - \tilde{m}_P) G_e(\not{p}) \right) \lim_{p^2 \rightarrow \tilde{m}_i^2} (p^2 - \tilde{m}_i^2) G_{ii}^{\mu\nu}(p^2) \frac{1}{\sqrt{Z_{ii}^{(i)}}} \Gamma_{i,\nu} \\
&\quad + \left(\lim_{\not{p} \rightarrow \tilde{m}_P} (\not{p} - \tilde{m}_P) G_e(\not{p}) \right) \left(\lim_{p^2 \rightarrow \tilde{m}_i^2} (p^2 - \tilde{m}_i^2) G_{ij}^{\mu\nu}(p^2) \frac{1}{\sqrt{Z_{ii}^{(i)}}} \Gamma_{j,\nu} \right) \Big|_{i \neq j},
\end{aligned} \tag{49}$$

with an implicit sum over the repeated index j . Here, we separated the contribution from attaching diagonal or off-diagonal gauge boson legs. The divergent piece of the amplitude with an external photon at one-loop is given by

$$\mathcal{A}_1^{[1]\mu} \sim -e_1 \gamma^\mu \left(\delta_{e_1}^{[1]} - \frac{e_1^2}{24\pi^2 \epsilon} \right) \Rightarrow \delta_{e_1}^{[1]} = \frac{e_1^2}{24\pi^2 \epsilon}. \tag{50}$$

Similarly, for the dark photon amplitude, we find

$$\begin{aligned}
\mathcal{A}_2^{[1]\mu} &\sim -e_2 \gamma^\mu \left(\delta_{e_2}^{[1]} - \frac{e_2^2}{24\pi^2 \epsilon} - \frac{e_1^2}{12\pi^2 \epsilon} \right) \\
\Rightarrow \delta_{e_2}^{[1]} &= \frac{e_1^2}{12\pi^2 \epsilon} + \frac{e_2^2}{24\pi^2 \epsilon}.
\end{aligned} \tag{51}$$

Note that the extra contribution in $\delta_{e_2}^{[1]}$ proportional to e_1^2 comes from the fact that we also need to absorb the divergence originating from G_{12} . Next, we study the RGEs for all physical parameters in this basis and then compare them to the results in the basis with kinetic mixing.

D. Renormalization group

Again, we give the RGEs here to one-loop order. For e_1 , we find the same as in standard QED,

$$\beta^{[1]}(e_1) = \frac{e_1^3}{12\pi^2}. \tag{52}$$

However, for e_2 we have

$$\begin{aligned}
\beta^{[1]}(e_2) &= -e_2 \left(\epsilon + \frac{\mu}{Z_{e_2}} \left(\frac{dZ_{e_2}}{de_1} \frac{de_1}{d\mu} + \frac{dZ_{e_2}}{de_2} \frac{de_2}{d\mu} \right) \right) \\
&= \frac{e_1^2 e_2}{6\pi^2} + \frac{e_2^3}{12\pi^2}.
\end{aligned} \tag{53}$$

The anomalous dimension for $\tilde{m}$ is

$$\begin{aligned}
\gamma_{\tilde{m}}^{[1]} &= -\frac{\mu}{Z_{\tilde{m}}} \frac{dZ_{\tilde{m}}}{d\mu} = -\frac{\mu}{Z_{\tilde{m}}} \left(\frac{dZ_{\tilde{m}}}{de_1} \frac{de_1}{d\mu} + \frac{dZ_{\tilde{m}}}{de_2} \frac{de_2}{d\mu} \right) \\
&= -\frac{3}{8\pi^2} (e_1^2 + e_2^2).
\end{aligned} \tag{54}$$

In this basis, we also have an anomalous dimension for the dark photon mass $\tilde{M}$,

$$\gamma_{\tilde{M}}^{[1]} = -\frac{\mu}{\sqrt{Z_{\tilde{M}}}} \frac{d\sqrt{Z_{\tilde{M}}}}{d\mu} = -\frac{1}{2} \frac{d\tilde{M}}{de_2} \mu \frac{de_2}{d\mu} = \frac{e_2^2}{12\pi^2}. \tag{55}$$

IV. COMPARISON OF THE TWO BASES

For the convenience of the reader, we give again the full set of all nonvanishing RGEs in the two bases. In the basis with kinetic mixing, we found

$$\begin{aligned}
\beta^{[1]}(e) &= \frac{e^3}{12\pi^2}, & \gamma_m^{[1]} &= -\frac{3e^2}{8\pi^2} \frac{1}{1-\kappa^2}, \\
\gamma_\kappa^{[1]} &= \frac{e^2}{12\pi^2}, & \gamma_{\xi^{-1}}^{[1]} &= \frac{e^2}{6\pi^2}.
\end{aligned} \tag{56}$$

In the basis without kinetic mixing, we have

$$\begin{aligned}
\beta^{[1]}(e_1) &= \frac{e_1^3}{12\pi^2}, & \beta^{[1]}(e_2) &= \frac{e_1^2 e_2}{6\pi^2} + \frac{e_2^3}{12\pi^2}, \\
\gamma_{\tilde{m}}^{[1]} &= -\frac{3}{8\pi^2} (e_1^2 + e_2^2), & \gamma_{\tilde{M}}^{[1]} &= \frac{e_2^2}{12\pi^2}.
\end{aligned} \tag{57}$$

The relations between the parameters in the two bases have the same form for the renormalized parameters as for the bare ones. Thus, according to Eq. (26), we have

$$\tilde{m} = m, \quad \tilde{M} = \frac{M}{\sqrt{1-\kappa^2}}, \quad e_1 = e, \quad e_2 = -\frac{e\kappa}{\sqrt{1-\kappa^2}}. \tag{58}$$

Here, we give only the relations for the physical parameters, since we will be interested in matching their RGEs only.

The matching of e with e_1 and of m with $\tilde{m}$ is trivial: expressing e_1 and e_2 on the right-hand side of the RGEs in terms of e and κ , we immediately find that

$$\beta^{[1]}(e) = \beta^{[1]}(e_1), \quad \gamma_m^{[1]} = \gamma_m^{[1]}. \quad (59)$$

To match the remaining physical parameters, we first differentiate the relations of the parameters in the two bases,

$$\begin{aligned} \gamma_M^{[1]} &= \frac{\mu}{\tilde{M}} \frac{d}{d\mu} \tilde{M} = \sqrt{1-\kappa^2} \frac{\mu}{M} \frac{d}{d\mu} \left(\frac{M}{\sqrt{1-\kappa^2}} \right) \\ &= \gamma_M^{[1]} + \sqrt{1-\kappa^2} \mu \frac{d}{d\mu} \frac{1}{\sqrt{1-\kappa^2}} \end{aligned} \quad (60)$$

and

$$\begin{aligned} \beta^{[1]}(e_2) &= \mu \frac{d}{d\mu} e_2 = \mu \frac{d}{d\mu} \left(-\frac{e\kappa}{\sqrt{1-\kappa^2}} \right) \\ &= -\frac{\kappa}{\sqrt{1-\kappa^2}} \beta^{[1]}(e) - e\mu \frac{d}{d\mu} \frac{\kappa}{\sqrt{1-\kappa^2}}. \end{aligned} \quad (61)$$

From Eq. (60) we have

$$\sqrt{1-\kappa^2} \mu \frac{d}{d\mu} \frac{1}{\sqrt{1-\kappa^2}} = \gamma_M^{[1]} - \gamma_M^{[1]}, \quad (62)$$

and with the chain rule, we find

$$\frac{\kappa}{1-\kappa^2} \mu \frac{d\kappa}{d\mu} = \gamma_M^{[1]} - \gamma_M^{[1]} = \frac{e_2^2}{12\pi^2} \Rightarrow \gamma_\kappa = \frac{e^2}{12\pi^2}. \quad (63)$$

In the same fashion, from Eq. (61) we get

$$\mu \frac{d}{d\mu} \frac{\kappa}{\sqrt{1-\kappa^2}} = -\frac{1}{e} \left(\frac{\kappa}{\sqrt{1-\kappa^2}} \beta(e) + \beta(e_2) \right), \quad (64)$$

and thus

$$\begin{aligned} \frac{1}{(1-\kappa^2)^{3/2}} \mu \frac{d\kappa}{d\mu} &= -\frac{1}{e} \left(\frac{\kappa}{\sqrt{1-\kappa^2}} \frac{e^3}{12\pi^2} + \frac{e_1^2 e_2}{6\pi^2} + \frac{e_2^3}{12\pi^2} \right) \\ \Rightarrow \gamma_\kappa &= \frac{e^2}{12\pi^2}. \end{aligned} \quad (65)$$

This shows that the RGEs of all physical parameters in the two bases can be matched consistently with a rotation matrix that runs itself.

V. DISCUSSION AND OUTLOOK

We have shown at one-loop order that in a theory with two $U(1)$ gauge bosons, one can choose a renormalization scheme without kinetic mixing. This means that after the kinetic mixing has been rotated away, it is not necessary to reintroduce this parameter to absorb divergences of off-diagonal two-point functions. We showed this by explicitly matching two renormalization schemes for two different bases in field space: one where an additional $U(1)$ gauge

boson (dark photon) was added to QED with kinetic mixing and one where this mixing was rotated away. The calculations are done in the general covariant R_ξ -gauge and, naively, the gauge fixing function obstructs the diagonalizability of the tree-level propagator. However, this obstruction is explicitly gauge dependent, and we have shown that it drops out at each loop order once all contributing diagrams are summed.

Importantly, in the second scheme, we only renormalize the physical observables, i.e., the masses and scattering amplitudes, and we show that the two schemes can be matched using scale-dependent field redefinitions. Note that the renormalizability in the second basis, where the mixing appears not directly in the Lagrangian but is induced only via loop corrections, never relied on its direct relation to the first basis with kinetic mixing in the Lagrangian. Thus, if one is interested only in the setup where the mixing is loop-induced, one can perform the renormalization as described in this work, without ever having to refer to another field basis.

Another natural question is to ask how sensitive the results presented here are to the specific choice of the mass generating mechanism for the dark photon. Suppose, for example, to replace the Stückelberg by the Higgs mechanism and generate the dark photon mass through spontaneous symmetry breaking. Then, one could repeat the exact same analysis. In the basis with kinetic mixing, there are additional divergent one-loop diagrams, but again all divergences can be absorbed through counterterms straightforwardly. Now note that the dark photon mass terms in the Stückelberg and Higgs Lagrangian are independent of the fermion and photon. If we perform the transformation to the basis without mixing, as given in Eq. (24), the dark photon field is simply rescaled, but no structurally new terms are generated. Therefore, the renormalization in the second scheme and the matching between the schemes can be carried out in the same way as above, also for the Higgs mechanism. In general, these renormalization schemes are expected to be insensitive to the precise nature of the mass generating mechanism.

In the future, we plan to extend the analysis in this work to two loops and beyond. At this level, the physical principles described here imply nontrivial cancellations between sub-diagrams to ensure renormalizability. Elucidating this cancellation and proving it to all loop orders would be very illuminating.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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