

Nonlocality induces isometry and factorisation in holography

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In AdS/CFT holography, two manifestations of the black hole information paradox are given by the nonisometric nature of the bulk-boundary map and by the factorisation puzzle. By considering time-shifted microstates of the eternal black hole, we demonstrate that both puzzles may be simultaneously resolved by taking into account nonlocal quantum corrections that correspond to wormholes arising from state averaging. This is achieved by showing, using a resolvent technique, that the resulting Hilbert space for an eternal black hole is finite-dimensional with a discrete energy spectrum. The latter gives rise to a transition to a type I von Neumann algebra.

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The holographic principle [1,2] states that large N holographic conformal field theories (CFTs) [3–5] in d spacetime dimensions possess dual descriptions in terms of semiclassical theories of gravity in asymptotically anti-de Sitter (AdS) spacetimes in $d + 1$ dimensions [6–8]. This duality provides our most powerful insight into the quantum nature of gravity. It is quantified by equating the partition functions of both theories. However, in the semiclassical limit, this identification appears to be inconsistent, leading to several puzzles that amount to manifestations of the information paradox.

In the present work we deal with two apparently distinct manifestations of these puzzles, the first being the *nonisometric nature of the bulk-boundary map*. This is the statement that the effective bulk Hilbert space, corresponding to the Fock space of quantum fields propagating in a curved background, is much larger than its holographic counterpart [9–12].

The second aspect of the information paradox that we address is the *factorisation puzzle* [13–15], which arises from conflicting statements about the bulk and boundary Hilbert space of a two-sided eternal black hole in AdS. Following the ER = EPR proposal [16,17], the black hole is dual to two entangled CFTs living on the asymptotic boundaries [18] (see Fig. 1), and their Hilbert space is factorized. However, the bulk Hilbert space is not factorized due to the presence of a geometric wormhole. From an

operator algebraic perspective, the nonfactorisation of the Hilbert space is a consequence of the local observables forming a type III₁ von Neumann (vN) algebra in the semiclassical bulk description [19], which does not have irreducible representations. Nevertheless, it is expected that including nonperturbative corrections, the algebra of observables will become type I [20]. A type I algebra admits irreducible representations, and the definition of pure black hole microstates which belong to a factorized Hilbert space, solving the factorisation puzzle.

In this work, we show that these two faces of the information paradox can be resolved together by invoking the fundamentally nonlocal nature of the quantum theory of gravity. We demonstrate this by considering an infinite family of states, namely time-shifted thermofield double (TFD) states [21–27],

$$|\text{TFD}_\alpha\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} e^{i\alpha_n} |E_n\rangle_R |E_n\rangle_L, \quad (1)$$

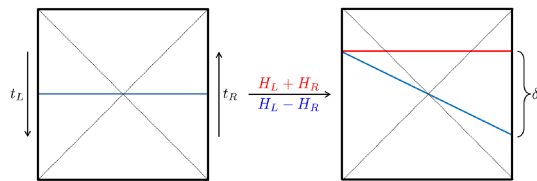


FIG. 1. The left figure shows the Kruskal diagram of an eternal black hole, in which the horizon is depicted by dashed lines. The TFD state (2) corresponds to this geometry, together with an identification of the left and right boundary times along the blue Cauchy slice. Since time runs in different directions on the asymptotic boundaries, this identification is preserved under the evolution with the difference of boundary Hamiltonians, shown on the right-hand side. Evolving with the sum of the boundary Hamiltonians, as indicated by the red line, induces a time shift δ , changing the TFD state to a generalized TFD state (1).

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where the α_n are additional phases. These states are entanglement-equivalent to the TFD state

$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2} E_n} |E_n\rangle_R |E_n\rangle_L, \quad (2)$$

where $Z(\beta)$ is the canonical thermal partition function of each boundary theory, and the $|E_n\rangle_{R/L}$ are the energy eigenstates of the right and left CFT with corresponding energy eigenvalue E_n . The generalized TFD states (1) are holographically dual to an eternal black hole in AdS_{d+1} , with different gluing of the left and the right boundaries to the bulk spacetime, as depicted in Fig. 1 [21,22]. The details of this gluing cannot be detected by a bulk observer infalling from the boundaries and therefore quantify the inherent nonlocal nature of gravitational spacetimes.

In the strict large N , or equivalently in the semiclassical $G_N \rightarrow 0$ limit, the generalized TFD states are orthogonal to each other, and thus span an infinite dimensional Hilbert space. This leads to a nonisometric map from the effective to the fundamental bulk Hilbert space [28], which according to the central dogma of black hole physics has a finite dimension, given by the exponential of the black hole entropy.

In the fundamental Hilbert space, the microstates (1) have exponentially small (in G_N) overlaps, arising from nonperturbative wormhole corrections to the gravitational path integral (PI). These overlaps serve as input for the resolvent method [29,32–34], from which we calculate the dimension of the Hilbert space, spanned by the linearly independent microstates. We indeed find that the dimension of the black hole Hilbert space is reduced to $e^{S_{\text{BH}}}$, where S_{BH} is the Bekenstein-Hawking entropy. This agrees with the central dogma, and demonstrates how the isometric nature of the bulk-boundary map is retrieved. Crucially, the reduction in the dimension of the Hilbert space is achieved via Euclidean replica wormhole contributions to the gravitational PI. We show that these replica wormholes arise from state-averaging over the Hilbert space of phase-shifted TFD states. Since these states are dual to a Lorentzian two-sided black holes, this demonstrates a novel connection between two seemingly different notions of Euclidean and Lorentzian nonlocality. Therefore, our argument does not require the inclusion of additional external degrees of freedom such as branes [34–36] or matter fields [37–39], but only relies on the fundamental nonlocality of gravity. Moreover, our setup takes the state dependence in the holographic bulk reconstruction [21,22] into account, confirming its proposed relation to nonisometry [12].

Furthermore, we show how the reduction in the dimension of the Hilbert space serves as a simple demonstration of a transition from a type III_1 vN algebra to a type I_D vN algebra corresponding to all operators acting on the $D = e^{S_{\text{BH}}}$ -dimensional black hole Hilbert space. We show that this transition is due to the energy spectrum becoming discrete once the nonperturbative corrections are included. The countable energy eigenstates then span a factorized

Hilbert space, which resolves the factorisation puzzle. In particular, we show the factorisation directly at the level of the Hilbert space, without relying on the factorisation of the two-point function [40–42].

Brief review of generalized TFD states. From the perspective of the boundary theory, which is an ordinary quantum theory without gravity, the TFD state (2) can be thought of as a split state [43–45] satisfying

$$\text{Tr}(\rho_{\text{TFD}} A_L B_R) = \text{Tr}_L(\rho_L A_L) \text{Tr}_R(\rho_R B_R), \quad (3)$$

where $\rho_{\text{TFD}} = |\text{TFD}\rangle\langle\text{TFD}|$ is the TFD density matrix. $\rho_{L/R}$ are density matrices, and A_L, B_R are observables defined for the left and right system respectively. In this way the TFD state (2) can be thought of as the purification of a thermal state represented by the thermal density matrix ρ_R . From this perspective, the von Neumann entropy associated with ρ_R may also be interpreted as the entanglement entropy between the left and the right systems. The interpretation of the TFD state as a split state is a more precise restatement of the factorisation of the boundary Hilbert space.

The ER = EPR proposal [16,17] provides a holographic bulk interpretation of the TFD state as the Hartle-Hawking wave function of a two-sided black hole, i.e., a wormhole in AdS [18]. Consequently, the entanglement entropy associated with the reduced density matrix ρ_R is identified with the thermal entropy of the two-sided eternal black hole [23],

$$S(\rho_R) = -\text{Tr}_R \rho_R \log(\rho_R) = S_{\text{BH}} = \frac{A}{4G_N}, \quad (4)$$

where A is the area of the black hole horizon. This implies that the thermal partition function $Z(\beta)$ appearing in (2) corresponds to the Gibbons-Hawking partition function [46]. We note that this interpretation crucially relies on the fact that for any local quantum mechanical system, such as the boundary CFT, the split property is satisfied, and consequently reduced density matrices exist.

However, the gravity theory in the bulk is manifestly nonlocal. Therefore, the split property, which is a statement about locality, ceases to exist in the bulk [47]. In particular, this implies that density matrices associated with a local subregion do not exist in gravity. Consequently, the TFD state dual to such a wormhole geometry can no longer be identified as the purification of a thermal state, and the identification (4) does not hold.

The nonlocality of gravity manifests itself through the absence of a time-reversal symmetry, or equivalently an origin of time. For the two-sided eternal black hole in AdS, the bulk nonlocality leads to a large class of states, namely the generalized TFD states (1). These states arise from one-sided time evolutions of the TFD state,

$$|\text{TFD}_\alpha\rangle = e^{iH_L t} |\text{TFD}\rangle, \quad (5)$$

identifying $\alpha_n = E_n t$, H_L being the left boundary Hamiltonian. These states share the same dual black hole geometry with different identifications of the left and the right boundary points. The original TFD state (2) corresponds to an identification of boundary times along the Cauchy slice shown in blue on the left-hand side of Fig. 1, with support at $t_L = t_R = 0$.

For observers infalling from one of the boundaries, all the states (5) yield the same correlation functions [21,22]. This fact reflects, quantitatively, the freedom to independently choose the origins of time for each CFT. Since time runs in different directions in the left and right exterior, this identification is respected by time evolution generated by the difference of the boundary Hamiltonians,

$$U_-(t) = e^{i\frac{t}{2}(H_L - H_R)}, \quad (6)$$

and therefore, the TFD state remains invariant under this evolution. A direct consequence of this fact is that we can also obtain the generalized TFD states (5) from global time evolution, which is generated by

$$|\text{TFD}_\alpha\rangle = U_+(t)U_-(t)|\text{TFD}\rangle = U_+(t)|\text{TFD}\rangle, \quad (7)$$

where the unitary transformation U_+ is given by the sum of the boundary Hamiltonians,

$$U_+(t) = e^{i\frac{t}{2}(H_L + H_R)}. \quad (8)$$

The global time evolution is illustrated on the right-hand side of Fig. 1. Since the evolved Cauchy slice contains a larger portion of the black hole interior, we may also interpret the evolved state as being dual to a Lorentzian wormhole of different length.

Every state in the infinite family (1) is a microstate of the black hole [48], which shows that the bulk Hilbert space appears to have way too many states when probed by a local observer. However, it is worth emphasizing that although the relative time-shift between the left and the right boundaries remains undetected by a single infalling observer, the phases can be detected through nonlocal measurements [23,49]. In what follows, we will demonstrate how nonlocal effects can actually lift this infinite microstate degeneracy.

Microstates and overlaps. We now show that the infinitely many black hole microstates (1) have exponentially small overlaps, which arise from wormholes, i.e., nonlocal contributions in the gravitational PI.

In our case, the nonlocal nature of gravity is encoded in the phases of the generalized TFD states (1). At large N , and large time of the order of the Poincaré recurrence time [23], these phases are random, since they are related to the random energy spectrum of the holographic CFT (5). Therefore, the states satisfy an orthogonality condition of the form,

$$\langle \text{TFD}_\alpha | \text{TFD}_\gamma \rangle = \sum_n p_n^2 e^{i(\gamma_n - \alpha_n)} = \delta_{\alpha\gamma} + \mathcal{O}(e^{-S_{\text{BH}}/2}), \quad (9)$$

where $p_n = e^{-\beta E_n/2}/\sqrt{Z}$ is the Boltzmann probability [23]. This equation implies that in the semiclassical limit, these phase-shifted microstates form an infinite basis of the black hole Hilbert space at late enough times.

In Euclidean gravity the phase-shifted TFD state corresponds to the Hartle-Hawking state representing an Euclidean eternal black hole with a given identification of the asymptotic boundaries [18]. The overlap (9) between these states appears as the leading order contribution in the topological expansion of the Euclidean gravitational PI (see supplemental material [50] for more details). The different identifications between boundary times manifest through an ambiguity in imposing boundary field data on this PI. This ambiguity is associated with the lack of a manifest time reversal symmetry, or equivalently, the bulk nonlocality giving rise to the additional phases (5). Due to appearance of connected geometries in the topological expansion of the gravitational path the orthogonality relation (9) receives exponentially small corrections. To show this, we compute the second moment,

$$\frac{1}{\mathcal{N}} \sum_\gamma |\langle \text{TFD}_\alpha | \text{TFD}_\gamma \rangle|^2 = \frac{\delta_{\alpha\alpha}}{\mathcal{N}} + \frac{Z(2\beta)}{Z^2(\beta)}, \quad (10)$$

with $Z(\beta)$ the thermal partition function, and $\mathcal{N}$ is the number of phases contributing to the sum. In the derivation of (10) we have explicitly used the randomness of the phases at large timescale. The details of this calculation are included in the supplemental material [50].

The first term in (10) arises from the square of (9), which from the point of the gravitational PI corresponds to two disconnected copies of an Euclidean black hole. The second term in (10), given by the ratio of partition functions, arises from a Euclidean wormhole connecting two boundaries, known as replica wormhole [23,34]. In particular, the second term in (10) represents the properly normalized partition function of this wormhole [54]. Schematically, the topological expansion corresponding to (10) is represented as shown in Fig. 2. These wormhole corrections to the overlap are nonperturbative in nature, since they are exponential in the black hole entropy, that is of $\mathcal{O}(e^{-S_{\text{BH}}})$. Equivalently, they are nonperturbative in an expansion in Newton's constant.

The appearance of connected contributions in (10) may also be explained as arising from an averaging procedure.

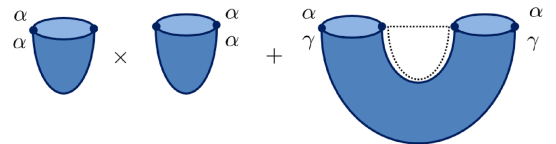


FIG. 2. Geometric representation of the right hand side of (10). The first term corresponds to two disconnected topologies, while the second represents a replica wormhole arising from state averaging. The averaging over the states labeled by γ is depicted through the dashed line in this diagram.

From that perspective, the quantum amplitude $\langle \text{TFD}_\alpha | \text{TFD}_\gamma \rangle$ is interpreted as the expectation value of a random matrix,

$$\mathcal{M}_{\alpha\gamma} = \delta_{\alpha\gamma} + e^{-S_{\text{BH}}/2} \mathcal{R}_{\alpha\gamma}, \quad (11)$$

where $\mathcal{R}_{\alpha\gamma}$ is a random matrix with zero mean [34]. Thus, (9) is reproduced by taking the average of this matrix $\mathcal{M}$, and the higher moment (10) represents the variance of $\mathcal{R}$. Thus, the gravitational PI computes an average over the fundamental degrees of freedom [28], which are encoded in the random matrix [34]. In particular, in our case the averaging over $\mathcal{M}$ is a state averaging over the Hilbert space of phase-shifted states (1).

Note that these phases are due to bulk nonlocality and are not detectable by any local measurements. Moreover, the states (1) are holographically dual to Lorentzian Einstein-Rosen wormholes of different lengths. However in order to derive (10), we only need the definition (1) without any reference to the Lorentzian origin of these phases. This brings us to a salient feature of our construction which prompts that, even though the phases appearing in (1) manifest Lorentzian nonlocality, at the level of the Euclidean PI, the same phases lead to replica wormhole contributions given by (10). These two notions of Lorentzian and Euclidean wormholes are *a priori* different and their relation had so far been elusive. Our choice of microstates, and their interpretation given above, thus demonstrates how replica wormholes arise from an average over states corresponding to different Lorentzian wormholes, thereby connecting two different manifestations of nonlocality in a quantum theory of gravity.

Counting the microstates. Due to (10), not all of the infinitely many microstates (1) contribute to the dimension of the black hole Hilbert space $\mathcal{H}_{\text{BH}}$. We now show that the dimension of this Hilbert space is finite and given by $D = e^{S_{\text{BH}}}$.

For this purpose we count the number of linearly independent microstates in a fixed microcanonical energy window $[E_0, E_0 + \Delta E]$, where the microstates take the form [55],

$$|\psi_i\rangle = \sum_n e^{-\beta(E_n - E_0)/2} e^{i\alpha_{i,n}} f(E_n - E_0) |E_n\rangle_L |E_n\rangle_R. \quad (12)$$

Here f is an arbitrary smooth function, supported in the respective energy window, and f is chosen such that the states (12) are normalized. In the large N limit the microcanonical TFD states are indistinguishable from their canonical counterpart, since correlation functions for simple operators in these states match [55]. In order to count the microstates we consider the Hilbert space,

$$\mathcal{H}_\Omega = \text{span}\{|\psi_i\rangle | i = 1, \dots, \Omega\}, \quad (13)$$

spanned by Ω of the black hole microstates, and analyze its dimension d_Ω as a function of Ω . Since $\mathcal{H}_\Omega$ is a subspace of $\mathcal{H}_{\text{BH}}$ we have

$$d_\Omega := \dim(\mathcal{H}_\Omega) \leq \min(D, \Omega), \quad (14)$$

and the dimension D of the black hole Hilbert space is obtained from the limit value of d_Ω as $\Omega \rightarrow \infty$. We introduce the Gram matrix G with entries,

$$G_{ij} = \langle \psi_i | \psi_j \rangle, \quad i, j = 1, \dots, \Omega, \quad (15)$$

which is positive semidefinite by construction, and d_Ω is then equal to the rank, i.e., the number of positive eigenvalues of G [37,38,56]. Since the Gram matrix may be interpreted as a random matrix (11) a suitable method to determine d_Ω is the resolvent method [29,32–34]. We use this method in order to find the eigenvalue density of G , which is defined as

$$D(\lambda) := \text{Tr}(\delta(\lambda \mathbb{1} - G)) = \sum_{i=1}^{\Omega} \delta(\lambda - \lambda_i), \quad (16)$$

where the λ_i are the eigenvalues of G . The rank of the Gram matrix is then obtained from the eigenvalue density as

$$d_\Omega = \lim_{\epsilon \rightarrow 0^+} \int_{\epsilon}^{\infty} d\lambda D(\lambda) = \Omega - \text{Ker}(G), \quad (17)$$

where $\text{Ker}(G)$ is the dimension of the kernel of the Gram matrix, and the second equality is the rank-nullity theorem. The dimension of the kernel corresponds to the number of zero eigenvalues of G , which can be obtained from (16). Using the resolvent method [32–34], we calculate the explicit form of the eigenvalue density and find

$$D(\lambda) = \delta(\lambda)(\Omega - e^{S_{\text{BH}}})\Theta(\Omega - e^{S_{\text{BH}}}) + \frac{e^{S_{\text{BH}}}}{2\pi\lambda} \times \sqrt{\left[\lambda - \left(1 - \sqrt{\frac{\Omega}{e^{S_{\text{BH}}}}}\right)^2\right] \left[\left(1 + \sqrt{\frac{\Omega}{e^{S_{\text{BH}}}}}\right)^2 - \lambda\right]}. \quad (18)$$

The details of this calculation are given in the supplemental material [50]. The singular term of the eigenvalue density (18), i.e., the term proportional to $\delta(\lambda)$, corresponds to the zero eigenvalues of G , while the continuous term gives the distribution of the positive eigenvalues. We note that the appearance of the Bekenstein-Hawking entropy (4) in (18) arises from the overlaps (10), since S_{BH} is contained in the ratio of the Gibbons-Hawking partition functions. These overlaps are encoded in the higher moments of the Gram matrix that appear in the definition of the resolvent, as explained in the supplemental material [50].

Combining (18), with (17), we finally determine the rank of G , i.e., the dimension of the Hilbert space $\mathcal{H}_\Omega$ as

$$d_\Omega = \min(\Omega, e^{S_{\text{BH}}}). \quad (19)$$

The result of this calculation shows that if the number of microstates Ω in the Hilbert space $\mathcal{H}_\Omega$ is smaller than $e^{S_{\text{BH}}}$, the microstates span a proper subspace of the black hole Hilbert space of dimension Ω . Moreover, following (17),

for this case the Gram matrix does not have zero eigenvalues, i.e., it has a trivial kernel. Moreover, we find from (19) for $\Omega > e^{S_{\text{BH}}}$ that the rank of the Gram matrix reaches a saturation value, given by the Bekenstein-Hawking entropy $e^{S_{\text{BH}}}$. This indeed shows that the black hole Hilbert space has dimension $D = e^{S_{\text{BH}}}$.

Nonisometric map. The result (19) shows in an explicit example how the Hilbert space of the effective theory $\mathcal{H}_{\text{eff}}$ can be reduced to the Hilbert space of the fundamental theory $\mathcal{H}_{\text{BH}}$ [28] upon inclusion of nonlocal features of quantum gravity. In the effective description, (9) does not receive corrections, and thus the Hilbert space is spanned by all of the infinitely many black hole microstates, violating the holographic entropy bound of [57]. However, the fundamental Hilbert space is spanned by finitely many microstates (19), and the bound is satisfied. Therefore, the embedding map is nonisometric [9,10],

$$\mathcal{F}_{\text{embedding}}: \mathcal{H}_{\text{eff}} \rightarrow \mathcal{H}_{\text{BH}}. \quad (20)$$

The nonisometry implies the existence of a large class of null states that the map annihilates. These null states are identified with the kernel of the Gram matrix (15), which is nontrivial, as soon as $\Omega = e^{S_{\text{BH}}}$ states are included in $\mathcal{H}_{\Omega}$ (18). In order to have a well-defined low-energy effective description the existence of the null states should not be detectable by a local observer, performing simple measurement. Since the overlaps (10) are $\mathcal{O}(e^{-S_{\text{BH}}})$ this is indeed the case, as their detection requires an operation of exponential complexity.

It is worth emphasizing that in our approach, the nonisometric map does not require the introduction of additional external nongravitational degrees of freedom. Instead the nonisometry is a direct consequence of the emergent bulk locality in the low-energy description. The isometry of the bulk-boundary map is recovered by exploiting inherent nonlocality of quantum gravity.

Algebra and factorisation. We now provide an algebraic definition of the effective Hilbert space and the fundamental (black hole) Hilbert space [28] appearing in the nonisometric map (20). A brief summary of the type classification of vN algebras is given in the supplemental material [50]. We show how the algebraic considerations automatically resolve the factorisation puzzle.

The vN algebra of observables associated to semiclassical gravity is of type III_1 , which is a consequence of the fact that in the $G_N \rightarrow 0$ limit, or equivalently large N limit, the energy spectrum becomes continuous since energy gaps scale as $1/N$ [58]. Due to this continuity, or equivalently due to the randomness of the energy spectrum, the microstates (1) are orthogonal (9), and span an infinite-dimensional Hilbert space. In this limit the TFD state is no longer a split state (3), and accordingly does not belong to a factorized Hilbert space $\mathcal{H}_L \otimes \mathcal{H}_R$. Instead the TFD state is contained in a GNS Hilbert space [59] given by

$$\mathcal{H}_{\text{GNS}} = \{|a\rangle, a \in \mathcal{A}_L | \langle a|b\rangle := \langle \text{TFD} | a^* b | \text{TFD} \rangle\}, \quad (21)$$

where $\mathcal{A}_L$ is the type III_1 von Neumann algebra generated by the single trace operators of the left CFT [60,61]. The TFD state is associated to the identity operator of $\mathcal{A}_L$, and the GNS Hilbert space is not factorized. Since a type III algebra prohibits defining a trace, it does not contain density matrices, which is consistent with the fact that in the semiclassical limit, the TFD state cannot be identified with the purification of a thermal density matrix.

Furthermore, each black hole microstates (1) belongs to a different infinite dimensional GNS Hilbert space, associated with a fixed value of α , defined as

$$\mathcal{H}_{\text{GNS}}^\alpha = \{|a\rangle, a \in \mathcal{A}_L | \langle a|b\rangle := \langle \text{TFD}_\alpha | a^* b | \text{TFD}_\alpha \rangle\}. \quad (22)$$

Each of these Hilbert spaces can be thought of as a low-energy effective Hilbert space, corresponding to the Fock space of quantum fields propagating in curved spacetime. In this Hilbert space (22) any local measurement is insensitive to α . The Hamiltonian H_L , which is needed to transition between TFD states with different phases (5) is not included in the type III algebra $\mathcal{A}_L$. Therefore, at large N the black hole Hilbert space, which contains all TFD_α states, is given by a disjoint union of these different GNS Hilbert spaces (see Fig. 3) [62]. Since we work in the microcanonical ensemble (12), we can include the left boundary Hamiltonian H_L into the algebra, which leads to a transition to a type II_∞ algebra [55]. This algebra acts on an extended Hilbert space,

$$\hat{\mathcal{H}} = \mathcal{H}_{\text{GNS}} \otimes L^2(\mathbb{R}), \quad (23)$$

where L^2 are the square integrable functions. The resulting perturbative Hilbert space $\hat{\mathcal{H}}$ includes all black hole microstates (1), and corresponds to the infinite dimensional effective Hilbert space $\mathcal{H}_{\text{eff}}$ (20).

Due to nonperturbative corrections the orthogonality relation (10) is broken, and the black hole Hilbert space is finite dimensional (19). The algebra of observables is thus of type I_D , where $D = e^{S_{\text{BH}}}$. There is a simple yet precise way to justify this transition to a type I algebra through the

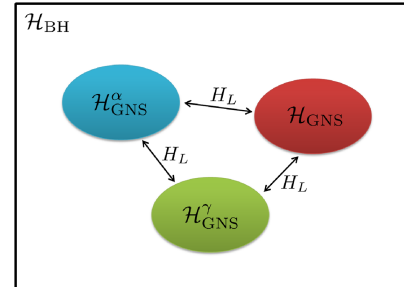


FIG. 3. At large N , the black-hole Hilbert space $\mathcal{H}_{\text{BH}}$ splits into GNS Hilbert spaces around the TFD state, labeled by $\mathcal{H}_{\text{GNS}}$, and the microstates (1), labeled $\mathcal{H}_{\text{GNS}}^{\alpha/\gamma}$. Due to (5), the left boundary Hamiltonian allows us to transition between different GNS Hilbert spaces.

discreteness of the energy spectrum [58], arising as follows. If we assume that the spectrum is continuous, the phases $\alpha_n = E_n t$ are random, and consequently microstates are orthogonal (9). By including nonperturbative wormhole corrections we find that the microstates (1) are not orthogonal, and consequently the assumption that the spectrum is continuous has to break down. Thus, the energy spectrum is discrete, and the TFD state can be written in terms of direct products over energy eigenstates $|E_n\rangle_{L/R}$, which span a factorized Hilbert space

$$\mathcal{H}_L \otimes \mathcal{H}_R = \text{span}(|E_n\rangle_L \otimes |E_m\rangle_R). \quad (24)$$

This confirms in an explicit example the conjecture of [20] that a nonperturbative description of black holes leads to a transition to a type I algebra, and thereby resolves the factorisation puzzle.

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