

Self-interacting dark matter with observable ΔN_{eff}

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We propose a GeV-scale self-interacting dark matter (SIDM) candidate within a dark $U(1)_D$ gauged extension of the Standard Model (SM), addressing small-scale structure issues in Λ CDM while predicting an observable contribution to ΔN_{eff} in the form of dark radiation. The model introduces a fermionic DM candidate χ and a scalar ϕ , both charged under an unbroken $U(1)_D$ gauge symmetry. The self-interactions of χ are mediated by a light vector boson X^μ , whose mass is generated via the Stueckelberg mechanism. The relic abundance of χ is determined by thermal freeze-out through annihilations into X^μ , supplemented by a nonthermal component from the late decay of ϕ . Crucially, ϕ decays after the big bang nucleosynthesis (BBN) but before the cosmic microwave background (CMB) epoch, producing additional χ and a dark radiation species (ν_s). This late-time production compensates for thermal underabundance due to efficient annihilation into light mediators, while remaining consistent with structure formation constraints. The accompanying dark radiation yields a detectable ΔN_{eff} , compatible with Planck 2018 bounds and within reach of next-generation experiments such as SPT-3G, CMB-S4, and CMB-HD.

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I. INTRODUCTION

Self-interacting dark matter (SIDM) has emerged as a compelling alternative to collisionless cold dark matter (CDM), addressing small-scale structure anomalies such as the too-big-to-fail, missing satellite, and core-cusp problems that afflict CDM models [1–3]. The key distinction lies in SIDM's self-interaction cross section, parametrized as $\sigma/m \sim 1 \text{ cm}^2/\text{g} \approx 2 \times 10^{-24} \text{ cm}^2/\text{GeV}$ [4–7], for elastic scattering which can be achieved via light mediators. This interaction generates velocity-dependent effects that reconcile small-scale observations while preserving CDM's success at large scales [4–11].

Nevertheless, when dark matter (DM) couples to a light mediator with sizeable coupling, the annihilation rates become significantly enhanced, often leading to an underabundance of thermal relic for DM in the low-mass regime

(typically a few GeV). Although the light thermal DM scenario ($M_{\text{DM}} \lesssim \mathcal{O}(10 \text{ GeV})$) has garnered considerable attention, especially due to the relatively weak constraints from direct detection experiments [12], obtaining the correct relic density in this regime remains challenging [13–17]. For WIMP-like interactions, avoiding overclosure of the Universe generally imposes a lower bound on the DM mass around a few GeV [18,19]. However, by introducing light mediators or additional fields, it is possible to realize viable light thermal DM scenarios, as demonstrated in several recent works [20–26]. In the context of SIDM, the efficient annihilation into light mediators typically depletes the thermal relic density. Therefore, a hybrid production mechanism that combines both thermal and nonthermal contributions can be an attractive route to achieving the observed DM abundance [27–30].

In this work, we explore a gauged $U(1)_D$ framework that accommodates a GeV-scale SIDM candidate, along with observable deviations in the effective number of relativistic species, ΔN_{eff} . The dark sector is comprised of a fermionic DM candidate χ , a dark radiation state ν_s , a singlet scalar ϕ , and a vector mediator X^μ . The self-interactions of χ are mediated by the light vector boson X^μ , while the $U(1)_D$ symmetry remains unbroken, ensuring the stability of DM. The mass of X^μ considered to be generated via the Stueckelberg mechanism [31] for simplicity, although the

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Higgs mechanism is also viable. A central challenge in the GeV-scale SIDM scenario is that the efficient annihilation of χ into light mediators often results in an under-abundant relic density. In our model, this deficit is remedied by the late decay of the singlet scalar ϕ , which not only replenishes the DM population but also produces additional dark radiation (ν_S). The latter contributes to ΔN_{eff} , potentially yielding observable signatures in CMB experiments. As current CMB measurements constrain $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ (Planck 2018 [32]), with future experiments (SPT-3G [33], CMB-S4 [34], CMB-HD [35]) poised to probe deviations below $\Delta N_{\text{eff}} \sim 0.06$, this setup offers a complementary cosmological probe of the SIDM, while being consistent with constraints from structure formation and direct detection of DM. While we consider ν_S to be an additional light fermionic degree of freedom, ϕ can also decay into χ and active neutrinos ν_L at late epochs leading to enhanced ΔN_{eff} . Such an interaction $\phi\bar{\chi}\nu_L$ can be facilitated by active-sterile mixing below the electroweak symmetry breaking scale, similar to type-I seesaw model of neutrino mass. Such an UV completion does not require any new light degrees of freedom. In the present work, we remain agnostic about such UV completion and consider ν_S to be a new light degree of freedom.

This paper is organized as follows: in Sec. II we present the details the gauged $U(1)_D$ model under consideration. Section III investigates the parameter space where the correct DM relic density and observable ΔN_{eff} are simultaneously achieved while Sec. IV discusses the contributions to ΔN_{eff} and the relevant constraints. Direct detection prospects of DM are studied in Sec. V, while Sec. VI examines ϕ decay kinematics and DM free-streaming. We finally conclude in Sec. VII.

II. THE MODEL

The model extends the Standard Model (SM) gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with an additional $U(1)_D$ symmetry. The dark sector (DS) consists of: a Dirac fermion χ (DM candidate), a complex scalar ϕ , and a sterile neutrino ν_S . All new fields are singlets under the SM gauge group. The $U(1)_D$ charges are assigned such that χ and ϕ carry equal charge while ν_S is neutral. Crucially, ϕ does not acquire a vacuum expectation value (vev), preventing χ - ν_S mixing that would destabilize the DM. A schematic illustration of the dark sector is presented in Fig. 1. In our setup, the mass of X^μ arises due to Stueckelberg mechanism, as discussed in Appendix A.

The relevant Lagrangian is given by

$$\mathcal{L} \supset (D^\mu \phi)^\dagger (D_\mu \phi) + \mathcal{L}_{Z-\chi} - m_\chi \bar{\chi} \chi - (y \phi \bar{\chi} \nu_S + \text{H.c.}) - V(\phi, \Phi) \quad (1)$$

where the scalar potential is given by

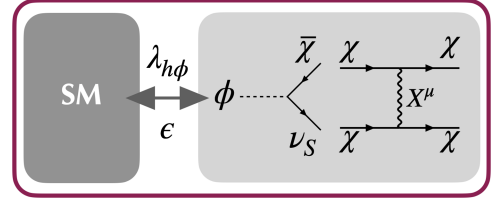


FIG. 1. Schematic of the proposed setup: dark matter (χ) with self-interactions mediated by a light dark gauge boson X^μ . Late time production of DM χ and DR (ν_S) from a scalar ϕ decay compensating for thermal underabundance of DM while also generating observable DR.

$$V(\phi, \Phi) = -\mu_h^2 \Phi^\dagger \Phi + \lambda_h (\Phi^\dagger \Phi)^2 + \mu_\phi^2 |\phi|^2 + \lambda_\phi |\phi|^4 + \lambda_{h\phi} |\phi|^2 \Phi^\dagger \Phi. \quad (2)$$

Here Φ denotes the SM Higgs doublet with h being the physical scalar after electroweak symmetry breaking. $\mathcal{L}_{Z-\chi}$, which describes the dynamics of the gauge fields in the interaction basis ($\tilde{W}_\mu^3, \tilde{B}_\mu, \tilde{X}_\mu$) reads

$$\begin{aligned} \mathcal{L}_{Z-\chi} \supset & -\frac{1}{4} \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} - \frac{1}{4} \tilde{W}_{3\mu\nu} \tilde{W}^{3\mu\nu} \\ & - \frac{1}{4} \tilde{X}_{\mu\nu} \tilde{X}^{\mu\nu} - \frac{\epsilon}{2} \tilde{X}_{\mu\nu} \tilde{B}^{\mu\nu} \\ & + j_{\text{SM}}^\mu \tilde{B}_\mu + j_\chi^\mu \tilde{X}_\mu. \end{aligned} \quad (3)$$

where $\tilde{V}_{\mu\nu}$ denotes the field strength tensors and ϵ parametrizes kinetic mixing between dark gauge boson field strength $\tilde{X}_{\mu\nu}$ and the hypercharge field strength $\tilde{B}_{\mu\nu}$. In Eq. (3), the SM electroweak neutral current is given by $j_{\text{SM}}^\mu = \{j_{\text{em}}^\mu, j_Z^\mu\}$, and the dark current is $j_\chi^\mu = g_d \bar{\chi} \gamma^\mu \chi$. To write the above Lagrangian in the canonical form one needs to diagonalize the kinetic terms of the gauge bosons.

After electroweak symmetry breaking, with $\Phi = (0, (v+h)/\sqrt{2})^T$, where h is the physical Higgs boson and v is the vacuum expectation value (vev), we rotate to the intermediate basis as

$$\begin{pmatrix} \tilde{W}_\mu^3 \\ \tilde{B}_\mu \\ \tilde{X}_\mu \end{pmatrix} = \begin{pmatrix} cw & sw & 0 \\ -sw & cw & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{Z}_\mu \\ \hat{A}_\mu \\ \hat{X}_\mu \end{pmatrix} \quad (4)$$

where $cw = \cos \theta_W$ and $sw = \sin \theta_W$ with θ_W being the Weinberg angle. Thus the Lagrangian in the new basis, including the Stueckelberg mass term for $\tilde{X}_\mu$, becomes [36]

$$\begin{aligned} \mathcal{L}_{Z-\chi} \supset & -\frac{1}{4} \hat{A}_{\mu\nu} \hat{A}^{\mu\nu} - \frac{1}{4} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} - \frac{1}{4} \hat{X}_{\mu\nu} \hat{X}^{\mu\nu} \\ & - \frac{\epsilon}{2} cw \hat{X}_{\mu\nu} \hat{A}^{\mu\nu} - \frac{\epsilon}{2} sw \hat{X}_{\mu\nu} \hat{Z}^{\mu\nu} + j_{\text{em}}^\mu \hat{A}_\mu \\ & + j_Z^\mu \hat{Z}_\mu + j_\chi^\mu \hat{X}_\mu + \frac{1}{2} m_X^2 \hat{X}_\mu \hat{X}^\mu + \frac{1}{2} m_Z^2 \hat{Z}_\mu \hat{Z}^\mu. \end{aligned} \quad (5)$$

Here, $j_Z^\mu = -\bar{f}\gamma^\mu(g_L^f P_L + g_R^f P_R)f$ and $j_{\text{em}}^\mu = -eQ_f\bar{f}\gamma^\mu f$. The mixing terms proportional to ϵ can be eliminated by a field redefinition as

$$\begin{pmatrix} \hat{A}_\mu \\ \hat{Z}_\mu \\ \hat{X}_\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{\epsilon c w}{\sqrt{1-\epsilon^2}} \\ 0 & 1 & -\frac{\epsilon s w}{\sqrt{1-\epsilon^2}} \\ 0 & 0 & -\frac{1}{\sqrt{1-\epsilon^2}} \end{pmatrix} \begin{pmatrix} A'_\mu \\ Z'_\mu \\ X'_\mu \end{pmatrix}. \quad (6)$$

In this new basis, the Lagrangian of Eq. (5) becomes

$$\begin{aligned} \mathcal{L}_{Z-\chi} \supset & -\frac{1}{4}A'_{\mu\nu}A'^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} - \frac{1}{4}X'_{\mu\nu}X'^{\mu\nu} \\ & + \frac{1}{2}m_Z^2 Z'_\mu Z'^\mu + \frac{1}{2}\frac{m_X^2 + m_Z^2\epsilon^2 s w^2}{1-\epsilon^2} X'_\mu X'^\mu \\ & + m_Z^2 \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} Z'_\mu X'^\mu + \frac{1}{\sqrt{1-\epsilon^2}} j_X^\mu X'_\mu \\ & + j_{\text{em}}^\mu \left(A'_\mu - \frac{\epsilon c w}{\sqrt{1-\epsilon^2}} X'_\mu \right) \\ & + j_Z^\mu \left(Z'_\mu + \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} X'_\mu \right) \end{aligned} \quad (7)$$

yielding a mass mixing between Z' and X' though A' field is achieved to be massless and diagonal. Thus performing a diagonalization via a rotation angle η

$$\begin{pmatrix} A'_\mu \\ Z'_\mu \\ X'_\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \eta & \sin \eta \\ 0 & -\sin \eta & \cos \eta \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ X_\mu \end{pmatrix} \quad (8)$$

gives the physical masses

$$\begin{aligned} M_X^2 &= \frac{m_X^2[m_X^2(1+\epsilon^2) - m_Z^2(1+cw^2\epsilon^2)]}{m_X^2 - m_Z^2}, \\ M_Z^2 &= \frac{m_Z^2[m_X^2 - m_Z^2(1+sw^2\epsilon^2)]}{m_X^2 - m_Z^2}, \end{aligned} \quad (9)$$

where the mixing angle η is given by

$$\begin{aligned} \tan(2\eta) &= 2 \frac{m_Z^2 s w \epsilon \sqrt{1-\epsilon^2}}{m_Z^2[1-\epsilon^2(1+sw^2)] - m_X^2} \\ &= 2 \frac{\epsilon s w}{1-\delta} + \mathcal{O}(\epsilon^3), \end{aligned} \quad (10)$$

with $\delta = m_X^2/m_Z^2$.

Thus, the interaction of the physical gauge boson X_μ with the SM fermions is described by Eq. (7) and is given by

$$X_\mu j_{\text{SM}}^\mu = -X_\mu \bar{f}\gamma^\mu \left\{ C_L^f P_L + C_R^f P_R \right\} f \quad (11)$$

where C_L^f and C_R^f are the left-handed and the right-handed couplings of SM fermions f to X_μ , and they are conveniently expressed in terms of the SM couplings as follows (see Appendix B)

$$\begin{aligned} C_L^f &= \left(\sin \eta + \frac{\cos \eta \epsilon s w}{\sqrt{1-\epsilon^2}} \right) g_L^f - \left(\frac{\cos \eta \epsilon c w}{\sqrt{1-\epsilon^2}} \right) e Q^f, \\ C_R^f &= \left(\sin \eta + \frac{\cos \eta \epsilon s w}{\sqrt{1-\epsilon^2}} \right) g_R^f - \left(\frac{\cos \eta \epsilon c w}{\sqrt{1-\epsilon^2}} \right) e Q^f, \end{aligned} \quad (12)$$

with

$$\begin{aligned} g_L^f &= \frac{g}{c w} (T_L^{3f} - s w^2 Q^f), \\ g_R^f &= -\frac{g}{c w} (s w^2 Q^f). \end{aligned} \quad (13)$$

Here, g is the SM $SU(2)_L$ gauge coupling and T_L^3 is the third component of the $SU(2)_L$ isospin.

Given the above-mentioned interactions, there are two ways the SM particles can interact with the DS particles: (1) the kinetic mixing parameter ϵ , and (2) the quartic coupling $\lambda_{h\phi}$ which quantifies the strength of $hh \rightarrow \phi^* \phi$ scattering. For the case of very small ϵ , the quartic coupling $\lambda_{h\phi}$ serves as the only portal (Fig. 1) through which the dark sector can thermalize with the SM bath. As shown in Eq. (2), the coupling $\lambda_{h\phi}$ plays a crucial role in determining the thermalization of ϕ . The processes through which ϕ can equilibrate with the SM bath are $h \leftrightarrow \phi\phi^*$, and $hh \leftrightarrow \phi\phi^*$. If $m_\phi > M_h/2$, the decay process is forbidden and the only thermalization is through $hh \leftrightarrow \phi\phi^*$ scattering. However, if $m_\phi \leq M_h/2$, both the processes can produce ϕ in the early Universe. In either of the cases, the production is possible and the thermalization is feasible for the scattering rate $\Gamma_{h \rightarrow \phi} \geq H$, where H is the Hubble expansion rate, which sets the coupling $\lambda_{h\phi}$ to a value $\geq 10^{-5}$.

A. Dark matter self-interactions

In our setup, the dark current interaction arises from the term $\frac{1}{\sqrt{1-\epsilon^2}} j_X^\mu X'_\mu$ in Eq. (7). Consequently, the relevant Lagrangian for self-interacting dark matter takes the form $(g_d \cos \eta / \sqrt{1-\epsilon^2}) X_\mu \bar{\chi} \gamma^\mu \chi$ or equivalently, $g_\chi X_\mu \bar{\chi} \gamma^\mu \chi$. This interaction generates a Yukawa potential between DM particles: $V(r) = \pm(\alpha_\chi/r) \exp[-M_\chi r]$, $\alpha_\chi \equiv g_\chi^2/4\pi$ where $+$ ($-$) corresponds to repulsive (attractive) interactions. Notably, while $\chi\bar{\chi}$ interactions are attractive, $\chi\chi$ and $\bar{\chi}\bar{\chi}$ interactions are repulsive. The DM self-scattering is conventionally characterized by the transfer cross section: $\sigma_T = \int d\Omega (1 - \cos \theta) \frac{d\sigma}{d\Omega}$, as discussed in Refs. [2,5,11]. A complete analysis requires computations beyond the perturbative limit. Depending on the DM mass ($m_\chi \equiv m_{\text{DM}}$), mediator mass (m_X), relative DM velocity (v_{rel}), and interaction strength (g_χ), three distinct regimes emerge:

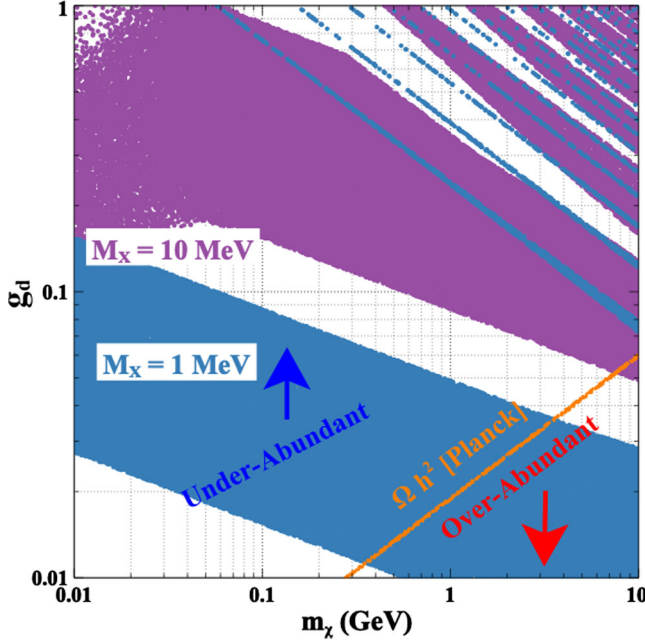


FIG. 2. The SIDM parameter space in the g_d vs. m_χ plane for DM self-interactions, with σ/m_χ ranging from 0.1 to 100 cm²/g. The orange line represents $\Omega h^2 = 0.120 \pm 0.001$.

Born regime: $g_\chi^2 m_\chi / 4\pi M_X \ll 1$, $m_\chi v_{\text{rel}} / M_X \geq 1$, Classical regime: $g_\chi^2 m_\chi / 4\pi M_X \geq 1$, Resonant regime: $g_\chi^2 m_\chi / 4\pi M_X \geq 1$, $m_\chi v_{\text{rel}} / M_X \leq 1$. Further details can be found in Appendix C.

If DM reaches thermal equilibrium in the early Universe, the same Lagrangian also governs DM annihilation into X mediators, contributing to its thermal relic abundance. The corresponding Feynman diagrams are given in Appendix D. The dominant thermally averaged annihilation cross section is $\langle \sigma v \rangle_{\chi\chi \rightarrow XX} \simeq \pi \alpha_\chi^2 / m_\chi^2$. For all practical purposes, we set $g_\chi = g_d$, since the kinetic mixing parameters ϵ and η are extremely small in the region of our interest. Since we focus on SIDM, the $U(1)_D$ coupling g_d lies within the range $[10^{-2}, 1]$. The thermally averaged annihilation cross section depends on g_d and m_χ , meaning different values of m_χ leading to varying annihilation cross sections, which impact the relic abundance of DM. Figure 2 illustrates the viable parameter space for two mediator masses, $M_X = 1$ MeV and 10 MeV, highlighting the regions where excessive DM annihilation results in a relic density below the observed value, indicated by the orange line.

III. DARK MATTER RELIC

A. Thermalization of dark sector particles

The dark sector thermalizes with the visible sector through two mechanisms: (1) the kinetic mixing parameter ϵ , and (2) the quartic coupling $\lambda_{h\phi}$. When ϵ is very small, the complex scalar ϕ serves as the primary portal (Fig. 1)

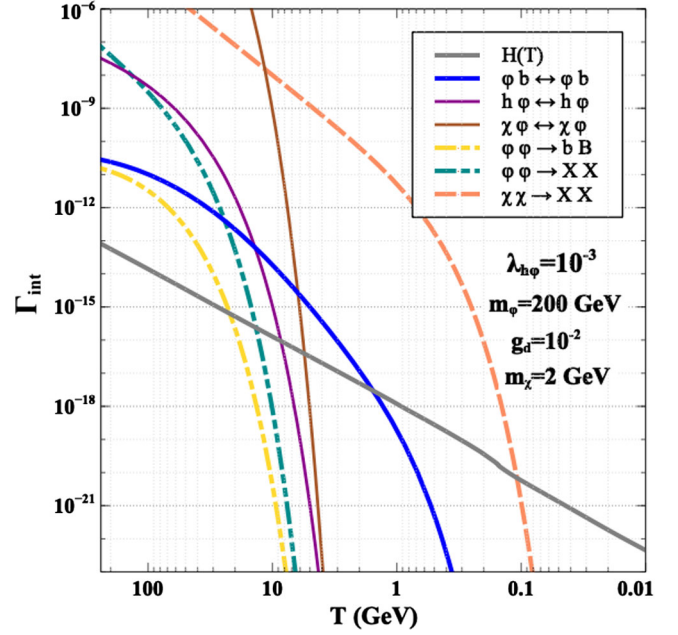


FIG. 3. Kinetic decoupling of ϕ particle from various processes for ϕ mass 200 GeV. For definiteness, we take $m_\chi = 2$ GeV and $g_d = 0.01$.

through which the dark sector thermalizes with the SM bath. Thermalization requires $\Gamma_{H \leftrightarrow \phi} \geq H$, translating to $\lambda_{h\phi} \geq 10^{-5}$. Once thermalized, ϕ maintains equilibrium via $\phi\text{SM} \leftrightarrow \phi\text{SM}$ scattering until kinetic decoupling.

For the process $\phi\text{SM} \leftrightarrow \phi\text{SM}$, the interaction rate can be computed as (see Appendix E)

$$\Gamma_{\text{int}} = \frac{g_{\text{SM}}}{4\pi^2 m_\phi^2 K_2(\frac{m_\phi}{T})} \int_{s_{\text{min}}}^{\infty} \sigma(s) p(s)^2 \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right) ds \quad (14)$$

where $s_{\text{min}} = (m_\phi + m_{\text{SM}})^2$ and the function $p(s)$ is given by

$$p(s) = \frac{[s - (m_\phi + m_{\text{SM}})^2]^{\frac{1}{2}} [s - (m_\phi - m_{\text{SM}})^2]^{\frac{1}{2}}}{2\sqrt{s}}. \quad (15)$$

The interaction rate Γ_{int} and the Hubble parameter H are plotted as functions of temperature for $\lambda_{h\phi} = 10^{-3}$, $m_\phi = 200$ GeV, $m_\chi = 2$ GeV in Fig. 3. From this plot, it is evident that the dominant process keeping ϕ in equilibrium with the SM at late times is $b\phi \leftrightarrow b\phi$ scattering, where b represents the bottom quark.

In addition to its quartic coupling to the SM Higgs, ϕ also directly couples to the $U(1)_D$ gauge boson X_μ through an interaction of the form $g_d^2 |\phi|^2 X_\mu X^\mu$. This strong coupling ensures sufficient interactions among ϕ , X_μ , and χ , leading to their thermalization within the dark sector.

B. Chemical decoupling of the hidden sector particles

For $\lambda_{h\phi} \geq 10^{-5}$, the scalar ϕ remains in thermal equilibrium with the SM bath, which in turn thermalizes dark matter χ . From Fig. 2, we observe that for sub-GeV DM masses, thermal relics are underabundant across most of the parameter space where self-interactions are significant ($g_d \geq 0.01$). This deficiency is compensated by the late decay of ϕ after its freeze-out. Here it is worth noticing that, the freeze-out abundance of ϕ is also controlled by the same coupling g_d along with its mass. The ϕ and χ number density changing processes are given in Appendix D, through which they decouple from the thermal bath and produce their thermal relic abundances. The thermally averaged annihilation cross section for $\phi\phi \leftrightarrow XX$ is $\langle\sigma v\rangle_{\phi\phi \rightarrow XX} \simeq 6\pi\alpha_d/m_\phi^2$ (see Appendix D for details).

To track the evolution of the comoving number densities of ϕ and χ , we solve the coupled Boltzmann equations considering all the number changing processes into account. The details are given in Appendix F. A representative solution for a benchmark set of parameters is shown in Fig. 4, where the solid red and green curves correspond to the comoving number densities of ϕ and χ , respectively. The dashed lines depict their respective equilibrium abundances, while the gray dotted horizontal line represents the observed relic abundance of χ for $m_\chi = 2.2$ GeV. The relevant parameter choices are indicated in the plot. As expected, for $m_\chi = 2.2$ GeV and $g_d = 0.038$, the thermal component of χ is underabundant. This deficit is compensated by the late-time decay of ϕ .

It is important to note that the decoupling and freeze-out abundances of both ϕ and χ are primarily governed by the

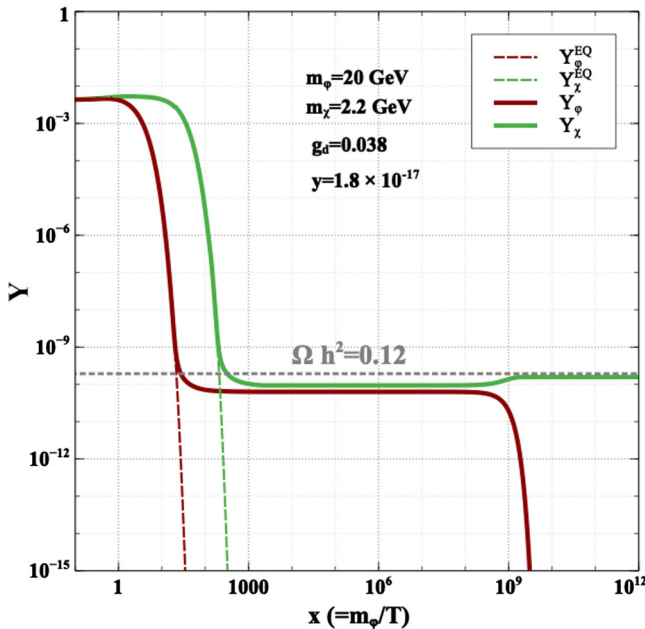


FIG. 4. The evolution of the comoving number densities of ϕ and χ as a function of $x(=m_\phi/T)$.

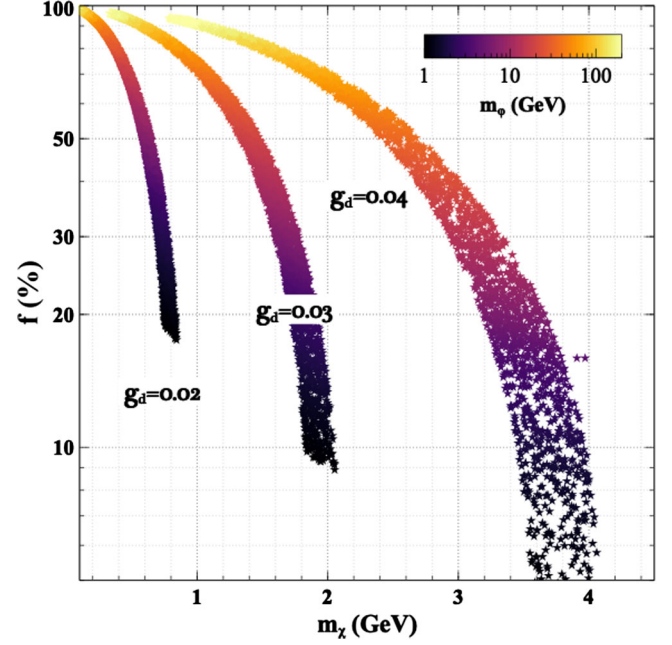
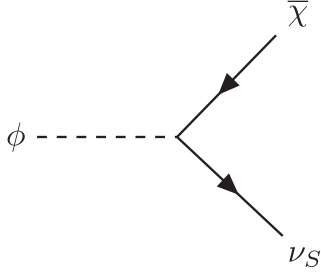


FIG. 5. The fraction of DM relic, $f = Y_\chi^{\text{nth}}/(Y_\chi^{\text{th}} + Y_\chi^{\text{nth}})$, shown as a function of m_χ . Here, th and nth in superscripts denote thermal and nonthermal components, respectively, under the constraint $(Y_\chi^{\text{th}} + Y_\chi^{\text{nth}}) = Y_\chi^{\text{Obs}}$.

parameters g_d , m_ϕ , and m_χ , with the ϕ abundance depending on g_d and m_ϕ , and the χ abundance controlled by g_d and m_χ . On the other hand, the late-time decay of ϕ , which converts its thermal relic into a nonthermal component of DM χ , is dictated by the parameters y and m_ϕ . Therefore, achieving the correct total DM relic abundance for a fixed g_d and m_χ requires a careful adjustment of m_ϕ to ensure that the nonthermal contribution from ϕ decay does not result in an overabundance of χ at late times.

To identify the viable parameter space that yields the correct DM relic density, we implement a scanning procedure with the following strategy. For fixed values of g_d and M_χ , we first determine the range of m_χ for which a purely thermal relic is insufficient to satisfy the observed relic density, while still respecting the self-interaction requirements for SIDM. For each such m_χ , we compute the thermal relic abundance of χ and quantify the resulting shortfall. With this information, we search for the corresponding m_ϕ that produces the appropriate thermal relic of ϕ such that its subsequent decay into χ and ν_S fills the deficit, thereby fulfilling the observed DM relic criteria. It is also worth mentioning that since M_χ is chosen to be of the order $\mathcal{O}(1)$ MeV, it does not significantly impact the freeze-out abundances of ϕ and χ , which lie in the $\mathcal{O}(10)$ GeV and $\mathcal{O}(1)$ GeV regimes, respectively.

The results of our parameter scan are presented in Fig. 5, where we show the nonthermal dark matter relic fraction $f = Y_\chi^{\text{nth}}/Y_\chi^{\text{Obs}}$ as a function of the dark matter mass m_χ for three different values of g_d . The color gradient in the plot

FIG. 6. Feynman diagram of the decay ϕ to χ and ν_S .

represents the corresponding values of m_ϕ required to obtain the correct relic density. We observe that, for a fixed g_d , increasing m_χ leads to a gradual decrease in the nonthermal fraction f , accompanied by a corresponding decrease in m_ϕ . This behavior can be understood as follows: as m_χ increases, the annihilation cross section $\langle\sigma v\rangle \propto 1/m_\chi^2$ decreases, resulting in a larger thermal contribution to the DM relic abundance. Consequently, a smaller nonthermal contribution is needed to match the observed relic density. To compensate, the thermal abundance of ϕ , which decays into χ must also be reduced. Since the annihilation cross section of ϕ scales as $\langle\sigma v\rangle \propto 1/m_\phi^2$, achieving a smaller ϕ relic requires a lower m_ϕ , explaining the observed trend.

IV. N_{eff} FROM ϕ DECAY

The late time decay of the scalar ϕ produces both the dark matter candidate χ and the dark radiation component ν_S , as illustrated in Fig. 6. This process contributes nonthermally to the effective number of relativistic species, N_{eff} . We are specifically interested in the scenario where the ϕ decay occurs after the epoch of BBN but before the epoch of CMB formation. The decay width of ϕ is given by

$$\Gamma_\phi = \frac{y^2 m_\phi}{16\pi} \left(1 - \frac{m_\chi^2}{m_\phi^2}\right)^2 \quad (16)$$

After ϕ freezes out and becomes nonrelativistic, its energy density ρ_ϕ scales as a^{-3} . Therefore, relative to the background radiation density ρ_{rad} (which scales as a^{-4}), the energy density of ϕ effectively increases over time. Although a long-lived ϕ could, in principle, come to dominate the energy density of the Universe, in our scenario it decays such that ρ_ϕ never overtakes radiation before its decay. Consequently, if ϕ decays when the ratio $\rho_\phi/\rho_{\text{rad}}$ is small, the injected dark radiation (and hence the change in N_{eff}) will be minimal. Conversely, a later decay occurring when $\rho_\phi/\rho_{\text{rad}}$ is relatively larger, yields a higher contribution to N_{eff} . Thus if ϕ decays shortly after BBN, the contribution of ν_S to N_{eff} remains small. However, if the decay occurs near the CMB epoch, the contribution can be

significantly enhanced and potentially detectable in future CMB experiments. However, relativistic χ particles produced nonthermally near the CMB epoch may disrupt structure formation [37]. Furthermore, such late-time decays can impact the primordial abundances of light elements due to the emission of charged leptons or quark pairs through higher-order three- or four-body decay channels [38–40]. Thus the Yukawa coupling y must be carefully chosen to ensure that decay of ϕ satisfies the χ relic density, provides an observable change in N_{eff} , remains consistent with constraints from light element abundances, and complies with structure formation constraints.

The additional relativistic degrees of freedom ΔN_{eff} at the time of CMB, can be expressed as

$$\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} = N_{\nu_S} \times \frac{\rho_{\nu_S}}{\rho_{\nu_L}} \Big|_{T_{\text{CMB}}} \quad (17)$$

where $\rho_{\nu_L} = 2 \frac{7}{8} \frac{\pi^2}{30} T_{\nu_L}^4$ denotes the energy density of one generation of light neutrinos (ν_L) is the energy density of one generation of SM neutrinos, and ρ_{ν_S} denotes the corresponding energy density of the sterile neutrino. N_{ν_S} is the number of ν_S generations. If neutrinos decouple instantaneously, then $N_{\text{eff}}^{\text{SM}} = 3$. Considering noninstantaneous decoupling of SM neutrinos along with flavor oscillations and plasma correction of quantum electrodynamics, the $N_{\text{eff}}^{\text{SM}}$ value shifts to $N_{\text{eff}}^{\text{SM}} = 3.045$ [41–43].¹ To track the evolution of the energy density of ν_S , we solve the Boltzmann equation

$$\frac{d\rho_{\nu_S}}{dx} = -\frac{4\beta\rho_{\nu_S}}{x} + \frac{1}{xH(x)} \langle E\Gamma_\phi \rangle (Y_\phi s(x)) \quad (18)$$

where $\beta = 1 + T \frac{dg_s/dT}{3g_s}$ and g_s is the effective relativistic degrees of freedom related to the entropy density s . The thermal-averaged energy transfer rate $\langle E\Gamma_\phi \rangle$ is given by [48,49]

$$\langle E\Gamma_\phi \rangle = g_\chi g_{\nu_S} \frac{|\mathcal{M}|_{\phi \rightarrow \chi \nu_S}^2}{32\pi m^2 \phi} (m_\phi^2 - m_\chi^2) \left(1 - \frac{m_\chi^2}{m_\phi^2}\right). \quad (19)$$

We numerically solve Eq. (18) together with the Boltzmann equations for DS particles X, χ, ϕ given in Appendix F simultaneously to track the evolution of ρ_{ν_S} along with the comoving abundances of ϕ and χ . Using the same benchmark parameter set employed in Fig. 4 for the ϕ and χ evolution, the resulting evolution of ρ_{ν_S} is shown in Fig. 7. The corresponding contribution to ΔN_{eff} is then computed using Eq. (17) and found to be 0.1047.

¹A few recent works [44–47] obtained a slightly different prediction $N_{\text{eff}}^{\text{SM}} = 3.044$.

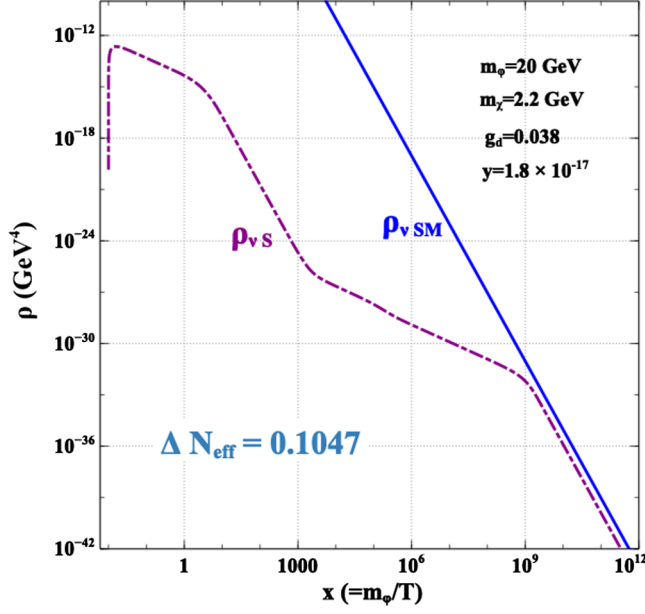


FIG. 7. The evolution of the energy density of ν_S and SM neutrinos illustrate their significant contribution to N_{eff} at the time of recombination epoch.

To explore the sensitivity of ΔN_{eff} to the model parameters g_d , m_ϕ , and y , we perform a random scan, ensuring in each case that the correct relic abundance for χ is reproduced. The variation of ΔN_{eff} for three representative combinations of m_ϕ , g_d is illustrated in Fig. 8, while fixing $m_\chi = 1$ GeV and $y = 10^{-16}$. The plot clearly demonstrates two key trends: (i) a smaller g_d leads to a larger thermal relic abundance of ϕ , which enhances the subsequent ΔN_{eff} , and (ii) increasing m_ϕ raises the energy density of ϕ (*i.e.* $\rho_\phi = m_\phi n_\phi$) (in addition to increasing freeze-out abundance of ϕ number density), which thus amplifies the energy density of DR ν_S when ϕ decays to ν_S thereby enhancing ΔN_{eff} . This can be seen by comparing the orange and green curves, both corresponding to $g_d = 0.05$, where increasing m_ϕ from 3 GeV (orange) to 100 GeV (green) significantly boosts ΔN_{eff} . Likewise, fixing $m_\phi = 100$ GeV and comparing the green ($g_d = 0.05$) and blue ($g_d = 0.1$) curves shows that reducing g_d enhances ΔN_{eff} , as expected.

In Fig. 8, the Yukawa coupling is fixed at $y = 10^{-16}$, while different values of m_ϕ and g_d are chosen to study their impact on ΔN_{eff} . To further illustrate this dependence as well as to see the effect of the Yukawa coupling, Fig. 9 presents the variation of ΔN_{eff} as a function of g_d for a fixed $m_\phi = 80$ GeV, while Fig. 10 shows ΔN_{eff} as a function of m_ϕ for a fixed $g_d = 0.03$ with the Yukawa coupling y varied as indicated by the color gradient in both the figures. As explained before, for a fixed y , ΔN_{eff} decreases with increasing g_d (Fig. 9) and increases with increasing m_ϕ (Fig. 10). Furthermore, for a fixed g_d or m_ϕ , a smaller Yukawa coupling y results in a larger ΔN_{eff} . This is because

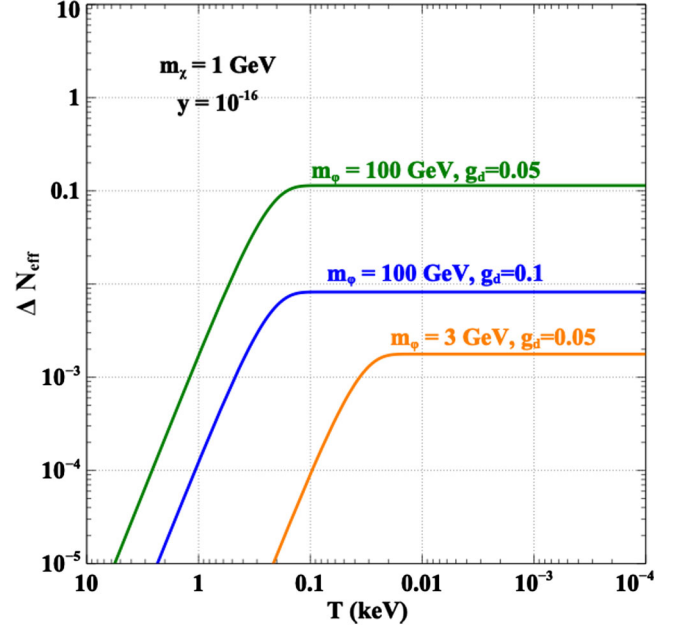


FIG. 8. Effect on ΔN_{eff} from the key parameters g_d and m_ϕ . Here, we fixed $m_\chi = 1$ GeV and $y = 10^{-16}$.

a smaller y implies a longer ϕ lifetime, allowing its energy density to grow relative to the background radiation before decay, thereby contributing more significantly to ΔN_{eff} . Throughout this parameter scan, the dark matter mass m_χ has been varied within the range of [1–10] GeV for both Figs. 9 and 10. For reference, both the figures also display the 1σ upper bound on ΔN_{eff} from Planck [32] and the

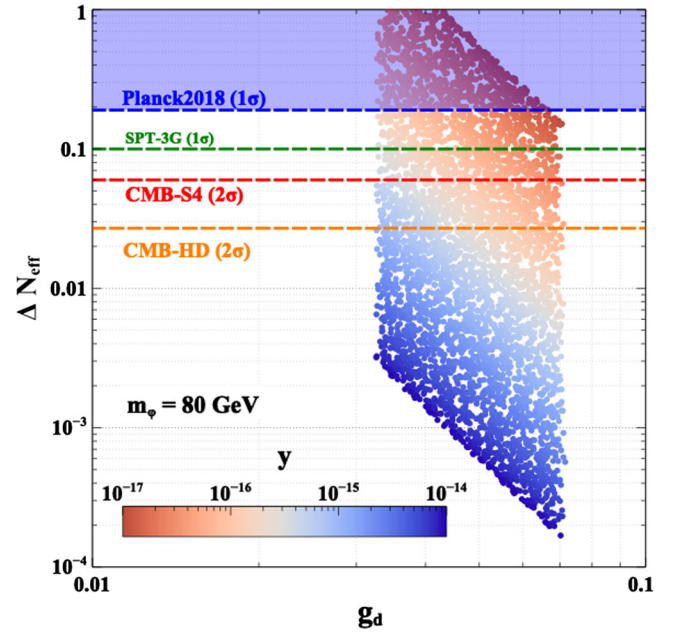


FIG. 9. Contribution to ΔN_{eff} as a function of g_d for $m_\phi = 80$ GeV. The Yukawa coupling y is varied as shown in the color bar.

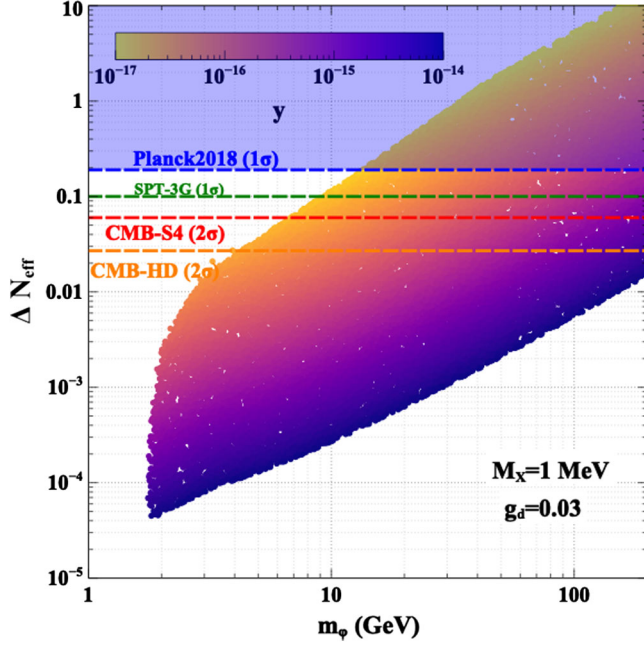


FIG. 10. The parameter space that will give the ΔN_{eff} from the process $\phi \rightarrow \chi \nu_S$ and the correct relic of the dark matter. All the points shown above satisfy the correct DM relic ballpark for m_χ , as shown in Fig. 5. Note that different values of g_d allows different ranges of Yukawa coupling y for the observable ΔN_{eff} .

projected sensitivities of upcoming CMB experiments such as SPT-3G [33] (green line), CMB-S4 [34] (red line), and CMB-HD [35] (orange line). Here, it is worth noting that we restrict the mass of the mediator M_χ to be 1 MeV or below, ensuring that it can decay only into SM neutrinos. The decay widths of X into SM fermions are given in Appendix G. This choice allows the scenario to safely evade stringent bounds from indirect detection experiments, as well as the constraints from CMB measurements on dark matter annihilation into charged final states mediated by X bosons [32,50–53] of the type $\chi\bar{\chi} \rightarrow XX \rightarrow 4\text{SM}$.

V. DARK MATTER DIRECT DETECTION

In this framework, dark matter can scatter off nucleons and electrons in terrestrial detectors either through gauge boson-mediated interactions induced by kinetic mixing, or via scalar portal interactions generated at the loop level with ν_S and ϕ circulating in the loop. However, the loop-induced scattering cross section remains extremely suppressed due to both the smallness of the Yukawa coupling y and the loop factor. Therefore, in this work, we focus solely on the gauge portal interaction. The corresponding Feynman diagram is shown in Fig. 11.

The effective interaction governing spin-independent DM-nucleon scattering via gauge boson mediation is described by the Lagrangian [54]

$$\mathcal{L}_{\text{eff}} = b_q (\bar{\chi} \gamma_\mu \chi) (\bar{q} \gamma^\mu q), \quad (20)$$

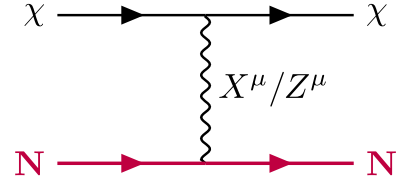


FIG. 11. Feynman diagram for the realization of dark matter detection in terrestrial laboratory. Here, N denotes the nucleons.

where the coupling b_q is defined as

$$b_q = \frac{g_d (C_L^{qX} + C_R^{qX})}{2M_\chi^2}. \quad (21)$$

At the nucleon level, the effective interactions with protons and neutrons can be written as

$$\mathcal{L}_p = b_p (\bar{\chi} \gamma_\mu \chi) (\bar{p} \gamma^\mu p), \quad \mathcal{L}_n = b_n (\bar{\chi} \gamma_\mu \chi) (\bar{n} \gamma^\mu n),$$

where the effective couplings b_p and b_n are related to the quark-level couplings by

$$b_p = 2b_u + b_d, \quad b_n = b_u + 2b_d.$$

With these expressions, the spin-independent DM-nucleon elastic scattering cross section is given by

$$\sigma_{\text{SI}} = \frac{1}{64\pi} \frac{\mu_r^2}{A^2} [Zb_p + (A-Z)b_n]^2, \quad (22)$$

where μ_r is the reduced mass of the DM-nucleus system, A is the atomic mass number, and Z is the atomic number of the target nucleus.

We present the spin-independent DM-nucleon scattering cross section as a function of dark matter mass in Fig. 12, for various choices of the gauge coupling g_d and kinetic mixing parameter ϵ . The plot also includes existing exclusion limits from XENON1T(M) [55] (green shaded), XENONnT [56] (red shaded), CRESST [57] (cyan region), and LUX-ZEPLIN [58,59] (yellow and light blue region). Additionally, future experimental sensitivities are illustrated using dashed lines: DSLM [60] (purple), CRESST [61] (dark green), and DARWIN [62,63] (orange). In this analysis, we consider $\epsilon = 10^{-9}$ or smaller, which remains consistent with constraints on the dark-photon search experiments as well as from SN1987A constraints [64,65] for the chosen mediator mass $M_\chi = 1$ MeV. Since the direct detection cross section scales with the gauge coupling g_d , lower values of g_d allow the model to evade current experimental bounds particularly for dark matter masses below 3 GeV. However, for $m_\chi \gtrsim 3$ GeV, the parameter set with $\epsilon = 10^{-9}$ is already excluded by current

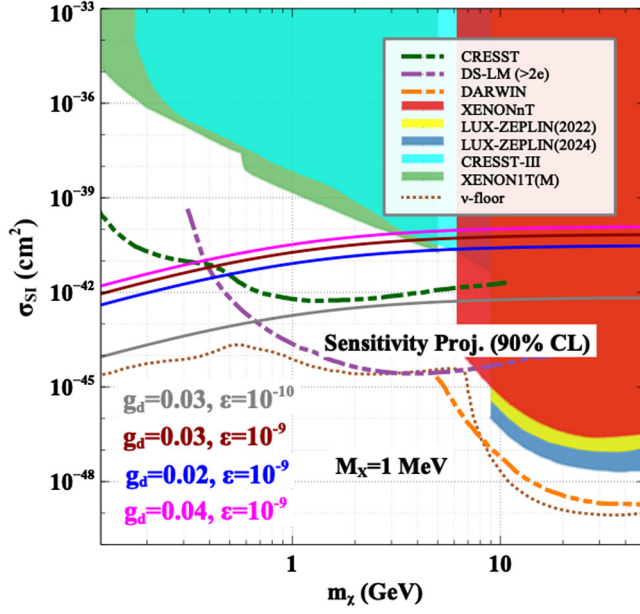


FIG. 12. Spin-independent DM-nucleon cross section for the direct detection is shown as a function of the DM mass, for a fixed $M_\chi = 1$ MeV.

direct detection experiments. A reduction of the kinetic mixing parameter to $\epsilon = 10^{-10}$ relaxes this constraint, allowing dark matter masses up to around 6 GeV to remain viable, which is represented by the gray curve in Fig. 12.

The recoil energy deposited by DM during scattering off a target particle depends on the DM mass. For light DM, scattering off electrons becomes more favorable than off heavier nuclei due to more efficient energy transfer. The cross section for DM–electron scattering ($\chi e \leftrightarrow \chi e$) in our model is given by [66]

$$\sigma_{\chi e \leftrightarrow \chi e} = \frac{\mu_{\chi e}^2}{\pi M_\chi^4} g_d^2 g_{Ve}^2 |F(q)|^2 \quad (23)$$

where the vector coupling is defined as $g_{Ve} = (C_L^e + C_R^e)/2$, as shown in Eq. (11), $\mu_{\chi e}$ is the DM–electron reduced mass, and $|F(q)| = 1$ since $M_\chi \gg am_e$.

Figure 13 displays existing experimental bounds from XENON1T [67] (blue shaded), PandaX-4T [68] (green shaded), and SENSEI@MINOS [69] (dark-magenta shaded). Projected sensitivity (90% CL) from DSLM [60] is also displayed for $4e^-$ threshold (orange dashed line) and $2e^-$ threshold (dark cyan dashed line). Our model predictions are shown as dark green, red, and dark blue lines corresponding to $\epsilon = 10^{-9}$ and $g_d = 0.04, 0.03$, and 0.02 , respectively. The light pink line represents the case with $g_d = 0.03$ and $\epsilon = 10^{-10}$. Since the DM mass is much larger than the electron mass, the reduced mass $\mu_{\chi e} \approx m_e$, leading to a direct detection cross section that remains constant across the DM mass range, as reflected in the plot.

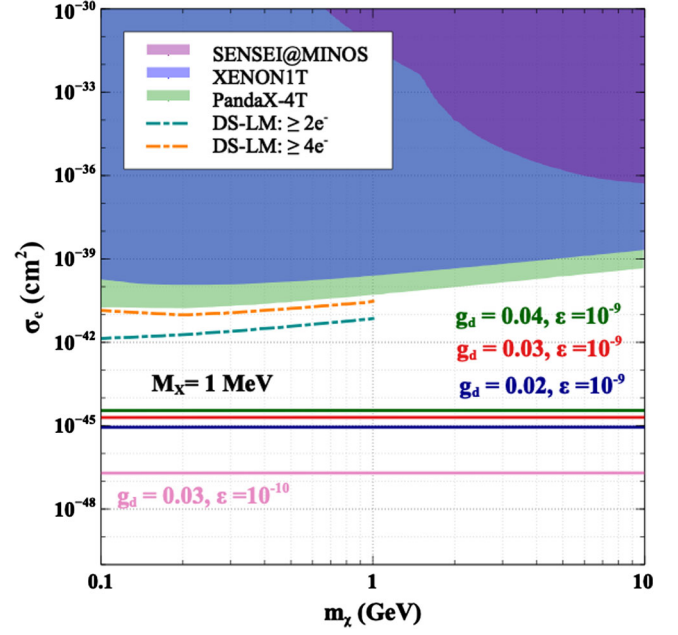


FIG. 13. DM-electron cross section for the direct detection is shown as a function of the DM mass, for a fixed $M_\chi = 1$ MeV.

VI. STRUCTURE FORMATION CONSTRAINT

In our framework, dark matter is produced nonthermally from the decay of a scalar field ϕ occurring after BBN but prior to the CMB epoch. Assuming the distribution of ϕ to be a delta function [70] justified at an epoch long after its thermal freeze-out, the maximum physical momentum imparted to χ is given by

$$p_{\text{phys}}^{\text{max}} = \frac{m_\phi^2 - m_\chi^2}{2m_\phi}. \quad (24)$$

This corresponds to a comoving momentum $p = a(t)p_{\text{phys}}^{\text{max}}$. The velocity of the daughter particle at any time t is then given by:

$$v_\chi(t) = \frac{p_{\text{phys}}(t)}{E(t)}, \quad E(t) = \sqrt{p_{\text{phys}}^2(t) + m_\chi^2}. \quad (25)$$

The comoving distance that the DM χ travels between its production time and conformal time τ is calculated by integrating the velocity as [71]

$$\begin{aligned} \lambda_{\text{fs}} &= \int_{\tau_{\text{dec}}}^{\tau} d\tau \mathbf{v}_\chi(t(\tau)) \\ &= \int_{a_{\text{dec}}}^a \frac{da'}{a'} \frac{1}{a'H(a')} \frac{p}{\sqrt{p^2 + a'^2 m_\chi^2}} \\ &= \int_{a_{\text{dec}}}^a \frac{da'}{a'} \frac{1}{a'H(a')} \frac{1}{\sqrt{1 + (a'/a_{\text{nr}})^2}} \end{aligned} \quad (26)$$

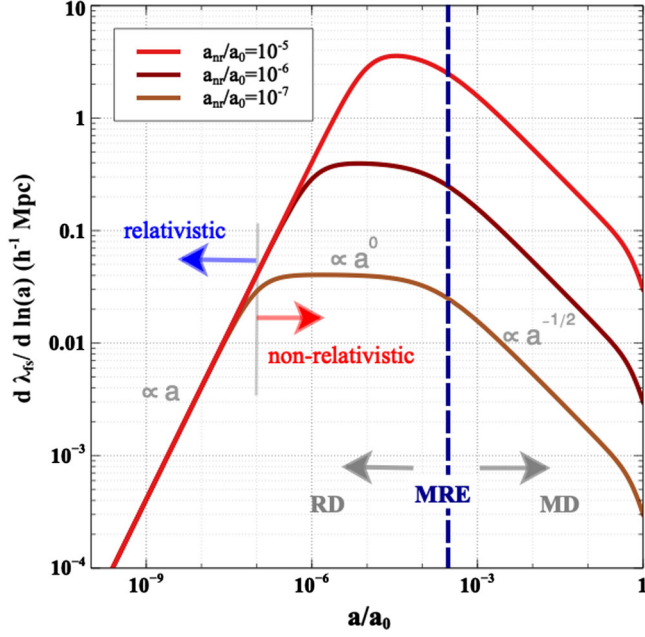


FIG. 14. Contributions to the free-streaming length integral.

Here, t_{dec} correspond to the time of ϕ decay, with a_{dec} as its associated scale factor. The quantity $a_{\text{nr}} = p/m_\chi$ denotes the scale factor at which the dark matter particle χ transitions from being relativistic to nonrelativistic, and $H(a)$ represents the Hubble expansion rate as a function of scale factor.

Figure 14 illustrates the evolution of $d(\lambda_{\text{fs}})/d \ln(a) = v_\chi/(aH(a))$ as a function of the scale factor a for different values of a_{nr} , which marks the transition of the dark matter particle χ from relativistic to nonrelativistic. While the particle remains relativistic ($a \leq a_{\text{nr}}$), its velocity $v_\chi \approx 1$ and the Hubble rate scales as $H \propto a^{-2}$ during the radiation-dominated era, making the integrand scale linearly with a . As the Universe expands and the particle becomes nonrelativistic ($a_{\text{nr}} < a < a_{\text{eq}}$), the velocity redshifts as $v_\chi \propto a^{-1}$, while H still follows the radiation-dominated scaling $H \propto a^{-2}$, causing the integrand to remain approximately constant (a^0). Beyond the epoch of matter-radiation equality ($a > a_{\text{eq}}$), the Universe enters the matter-dominated era, where $H \propto a^{-3/2}$ while the DM particle continues to redshift as $v_\chi \propto a^{-1}$. This results in the integrand scaling as $a^{-1/2}$. It is worth noting that the majority of the free-streaming length is accumulated during the

period when the particle is nonrelativistic but the Universe is still radiation dominated, i.e., in the range $a_{\text{nr}} < a < a_{\text{eq}}$.

In a generic two-body decay, the velocity imparted to the daughter particle at production known as the kick velocity is given by

$$v_{\text{kick}} = |v_\chi| = \frac{m_\phi^2 - m_\chi^2}{m_\phi^2 + m_\chi^2} \quad (27)$$

The kick velocity, dependent on m_ϕ and m_χ , can vary from relativistic to nonrelativistic values depending on the specific mass hierarchy. Whether this velocity leaves an imprint on cosmological structure formation is determined by the associated free-streaming length, which is constrained by Lyman- α observations and small-scale structure data. These constraints require: $\lambda_{\text{fs}} < 0.1$ Mpc to ensure consistency with the observation [72–74].

As $a_d \ll a_{\text{nr}} < a_{\text{eq}}$, where a_{eq} is the scale factor at matter-radiation equality, the free-streaming length is given by [75–77]

$$\begin{aligned} \lambda_{\text{fs}} &= \int_{a_d}^{a_{\text{eq}}} \frac{da'}{a'} \frac{1}{a' H(a')} \left(\frac{v_{\text{kick}} a_d}{a'} \right) \\ &= \int_{a_d}^{a_{\text{eq}}} \frac{da'}{a'^3} \frac{1}{H_0 \sqrt{\Omega_r a'^{-4}}} v_{\text{kick}} a_d \\ &= \frac{v_{\text{kick}} (\Gamma_\phi)^{-1}}{a_d} \ln \left(\frac{a_{\text{eq}}}{a_d} \right) \end{aligned} \quad (28)$$

where, in the last line, we noted that the decay happens when $\Gamma_\phi = H(a_d)$.

We summarize three representative benchmark scenarios in Table I, illustrating that, with an appropriate choice of parameters, it is indeed possible to satisfy the structure formation constraints. In Fig. 15, we present the parameter space points that simultaneously satisfy the observed dark matter relic abundance and yield ΔN_{eff} in the range [0.01, 0.2], while ensuring the free-streaming constraint $\lambda_{\text{fs}} < 0.1$ Mpc is respected. The plot displays λ_{fs} as a function of m_ϕ , with the color bar representing variations in the Yukawa coupling y . The DM mass is automatically adjusted so that the combined thermal and nonthermal contributions yield the correct relic density as shown in Fig. 5.

TABLE I. Benchmark points comparison.

	m_ϕ (GeV)	m_χ (GeV)	y	v_{kick}	a_d	a_{nr}	a_{eq}	λ_{fs} (Mpc)	ΔN_{eff}
BP1	22.4	2.82	1.8×10^{-17}	0.968	5.2×10^{-4}	1.76×10^{-3}	2.94×10^{-4}	0.256	0.13
BP2	46.56	1	8.1×10^{-16}	0.999	6.84×10^{-6}	1.56×10^{-4}	2.94×10^{-4}	0.006	0.023
BP3	127.32	0.1178	8.75×10^{-15}	0.999	3.83×10^{-7}	2.07×10^{-4}	2.94×10^{-4}	0.0005	0.04

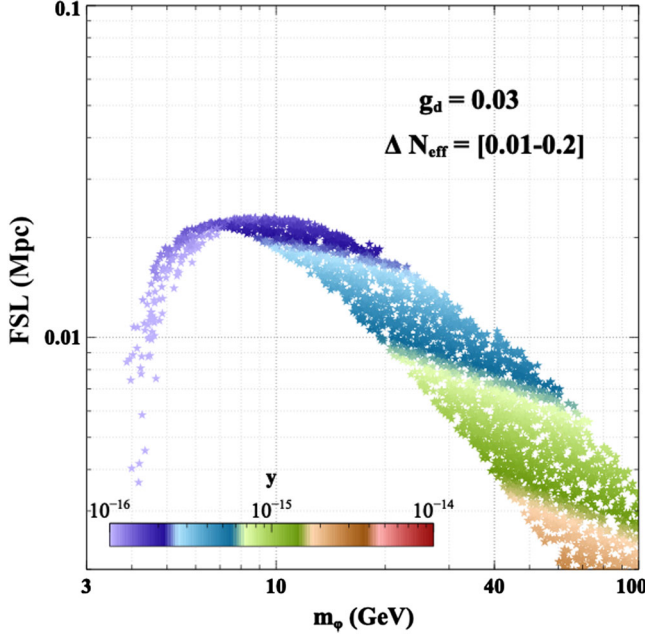


FIG. 15. Model parameters that gives $\lambda_{fs} < 0.1$ Mpc, while giving the correct dark matter relic density and the observable N_{eff} .

VII. SUMMARY AND CONCLUSION

In this work, we have proposed and analyzed a minimal yet phenomenologically rich framework for light self-interacting dark matter within a gauged $U(1)_D$ extension of the Standard Model. This setup not only addresses longstanding small-scale structure anomalies in the Λ CDM paradigm but also predicts observable signatures in the effective number of relativistic species, ΔN_{eff} . By combining thermal freeze-out with late time nonthermal production from scalar decays, the framework reconciles the relic abundance of light dark matter with large self interaction cross sections capable of resolving small scale issues. Simultaneously, the model generates a detectable ΔN_{eff} through dark radiation produced during scalar decays, offering complementary cosmological signatures.

The dark sector in our model consists of a fermionic dark matter candidate χ , a light mediator X_μ , a scalar ϕ , and a sterile neutrino ν_S . The relic abundance of χ is achieved through a hybrid mechanism involving both thermal freeze-out and nonthermal production from the late decay of ϕ , which occurs after BBN but before the CMB epoch. This decay not only replenishes the dark matter density but also produces dark radiation ν_S , contributing to ΔN_{eff} , thus offering an additional observational signature. Similar phenomenology can be achieved even if we allow ϕ decay to DM and active neutrinos facilitated via active-sterile mixing similar to seesaw models of neutrino mass. Our analysis demonstrates that this setup naturally accommodates the required strong self-interactions for χ , ensuring compatibility with astrophysical observations on small

scales, while simultaneously satisfying constraints from relic abundance, direct detection experiments, and structure formation constraint on the free-streaming length. We also showed that the model can give rise to a measurable ΔN_{eff} , lying within the projected sensitivity reach of future CMB experiments like SPT-3G, CMB-S4, and CMB-HD.

In summary, this framework presents a coherent and testable scenario that links the solution to small-scale structure anomalies with observable cosmological signatures. Future precision measurements in both cosmology and direct detection experiments will play a decisive role in probing and potentially validating this scenario.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: STUECKELBERG MECHANISM

The relevant part of the Lagrangian of the Stueckelberg extension of the SM is given by [36,78]

$$\mathcal{L} \supset -\frac{1}{4}C^{\mu\nu}C_{\mu\nu} - \frac{1}{2}(M_1C_\mu + M_2B_\mu + \partial_\mu\sigma)^2. \quad (\text{A1})$$

Here, C_μ is the gauge field of the $U(1)_D$, and B_μ is that for the $U(1)_Y$. The field σ is an axionic field which will be absorbed by the C_μ in the unitary gauge, thereby giving mass term for the C_μ field. The term M_2 which does not appear in the SM Lagrangian is explicitly written in the Stueckelberg extension to explain the Stueckelberg mass mixing [36]. However, we consider a simpler model without the Stueckelberg mass mixing, and hence we will take M_2 to be simply 0. Then the Lagrangian above can be made gauge invariant by splitting off the longitudinal degrees of freedom of σ through

$$\hat{X}_\mu = C_\mu + \frac{1}{M_1}\partial_\mu\sigma \quad (\text{A2})$$

along with the variations in the fields

$$\delta C_\mu = \partial_\mu \alpha, \quad \delta \sigma = -M_1 \alpha \quad (\text{A3})$$

where α is a gauge parameter. Then, the final Lagrangian for a massive gauge boson without violating the gauge structure of the theory takes the form

$$\mathcal{L} \supset -\frac{1}{4} \hat{X}^{\mu\nu} \hat{X}_{\mu\nu} - \frac{1}{2} M_X^2 \hat{X}^\mu \hat{X}_\mu. \quad (\text{A4})$$

APPENDIX B: X BOSON INTERACTION WITH SM FERMIONS

Various interactions induced due to the ϵ mixing.

Electromagnetic current: j_{em}^μ

$$\begin{aligned} \hat{A}_\mu j_{\text{em}}^\mu &= A'_\mu j_{\text{em}}^\mu + \frac{-\epsilon c w}{\sqrt{1-\epsilon^2}} X'_\mu j_{\text{em}}^\mu \\ &= A_\mu j_{\text{em}}^\mu - \frac{\epsilon c w}{\sqrt{1-\epsilon^2}} (\cos \eta X_\mu - \sin \eta Z_\mu) j_{\text{em}}^\mu. \end{aligned} \quad (\text{B1})$$

Neutral current: j_Z^μ

$$\begin{aligned} \hat{Z}_\mu j_Z^\mu &= Z'_\mu j_Z^\mu + \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} X'_\mu j_Z^\mu \\ &= (\cos \eta Z_\mu + \sin \eta X_\mu) j_Z^\mu \\ &\quad + \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} (\cos \eta X_\mu - \sin \eta Z_\mu) j_Z^\mu \\ &\simeq \left(\sin \eta + \cos \eta \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} \right) X_\mu j_Z^\mu \\ &\quad + \cos \eta Z_\mu j_Z^\mu. \end{aligned} \quad (\text{B2})$$

Now combining the result of the X_μ interaction with the SM j_Z^μ and j_{em}^μ , we have the final interaction terms given by

$$\begin{aligned} X_\mu j_{\text{SM}}^\mu &= X_\mu \left(\sin \eta + \cos \eta \frac{\epsilon s w}{\sqrt{1-\epsilon^2}} \right) j_Z^\mu \\ &\quad - X_\mu \left(\cos \eta \frac{\epsilon c w}{\sqrt{1-\epsilon^2}} \right) j_{\text{em}}^\mu. \end{aligned} \quad (\text{B3})$$

APPENDIX C: LOW ENERGY CROSS SECTIONS RELEVANT FOR THE SELF-INTERACTIONS OF DARK MATTER

Since DM self-interaction is realized due to the presence of the term like $g_\chi X_\mu \bar{\chi} \gamma^\mu \chi$, the nonrelativistic DM self-scattering can be well understood in terms of the attractive Yukawa potential

$$V(r) = \pm \frac{g_\chi^2}{4\pi r} e^{-M_X r} \quad (\text{C1})$$

where the $+$ ($-$) sign denotes repulsive (attractive) potential. To capture the relevant physics of forward scattering, the transfer cross section is defined as

$$\sigma_T = \int d\Omega (1 - \cos \theta) \frac{d\sigma}{d\Omega}$$

In the Born limit, $g_\chi^2 m_\chi / (4\pi M_X) \ll 1$,

$$\sigma_T^{\text{Born}} = \frac{g_\chi^4}{2\pi m_\chi^2 v_{\text{rel}}^4} \left[\log \left(1 + \frac{m_\chi^2 v_{\text{rel}}^2}{M_X^2} \right) - \frac{m_\chi^2 v_{\text{rel}}^2}{M_X^2 + m_\chi^2 v_{\text{rel}}^2} \right]$$

Outside the Born limit, where $g_\chi^2 m_\chi / (4\pi M_X) \geq 1$, there can be two different regions: the classical regime and resonance regime. In the classical regime ($m_\chi v_{\text{rel}} / M_X \geq 1$), solution for an attractive potential is given by [11,79,80]

$$\sigma_T^{\text{class}} = \begin{cases} \frac{4\pi}{M_X^2} \beta^2 \ln(1 + \beta^{-1}) & \beta > 1 \\ \frac{8\pi}{M_X^2} [\beta^2 / (1 + 1.5\beta^{1.65})] & 10^{-1} < \beta \leq 10^3 \\ \frac{\pi}{M_X^2} [\ln \beta + 1 - 1/2 \ln^{-1} \beta]^2 & \beta \geq 10^3 \end{cases}$$

and for the repulsive potential

$$\sigma_T^{\text{class}} = \begin{cases} \frac{2\pi}{M_X^2} \beta^2 \ln(1 + \beta^{-2}) & \beta \leq 1 \\ \frac{\pi}{M_X^2} [\ln(2\beta) - \ln(\ln 2\beta)]^2 & \beta \geq 1 \end{cases}$$

where $\beta = \frac{2g_\chi^2 M_X}{4\pi m_\chi v_{\text{rel}}^2}$.

Finally in the resonance region ($m_\chi v_{\text{rel}} / M_X \leq 1$), no analytical formula for σ_T is available. So approximating the Yukawa potential by Hulthen potential ($V(r) = \pm \frac{g_\chi^2}{4\pi} \frac{\delta e^{-\delta r}}{1 - e^{-\delta r}}$), the transfer cross section is obtained to be

$$\sigma_T^{\text{Hulthen}} = \frac{16\pi \sin^2 \delta_0}{m_\chi^2 v_{\text{rel}}^2}$$

where $l = 0$ phase shift δ_0 is given by

$$\delta_0 = \text{Arg} \left[\frac{i\Gamma(im_\chi v_{\text{rel}} / \kappa M_X)}{\Gamma(\lambda_+) \Gamma(\lambda_-)} \right]$$

with

$$\lambda_\pm = 1 + \frac{im_\chi v_{\text{rel}}}{2\kappa M_X} \pm i^r \sqrt{\frac{g_\chi^2 m_\chi}{4\pi \kappa M_X} + (-1)^{r+1} \frac{m_\chi^2 v_{\text{rel}}^2}{4\kappa^2 M_X^2}}$$

and $\kappa \approx 1.6$ is a dimensionless number and $r = 0(1)$ for attractive(repulsive) potential.

APPENDIX D: THERMALLY AVERAGED CROSS SECTION

The cross section for the process $\phi\phi \rightarrow XX$ (Fig. 16) and that of $\chi\chi \rightarrow XX$ (Fig. 17) are given by

$$\sigma_{\phi \rightarrow X} = \frac{g_d^4}{8\pi s(s - 4m_\phi^2)} \times \left(\frac{\sqrt{(s - 4m_\phi^2)(s - 4M_X^2)}(21M_X^4 + 16m_\phi^4 + m_\phi^2(20s - 88M_X^2))}{2(M_X^4 + m_\phi^2(s - 4M_X^2))} \right) \\ - \frac{(2s^2 + 4sM_X^2 - 24(s - M_X^2)m_\phi^2 + 16m_\phi^4 - 7M_X^4)}{(s - 2M_X^2)} \text{Log} \left(\frac{2M_X^2 - s + \sqrt{(s - 4m_\phi^2)(s - 4M_X^2)}}{2M_X^2 - s - \sqrt{(s - 4m_\phi^2)(s - 4M_X^2)}} \right),$$

$$\sigma_{\chi \rightarrow X} = \frac{g_d^4}{4\pi s(s - 4m_\phi^2)} \times \left(-\frac{\sqrt{(s - 4m_\chi^2)(s - 4M_X^2)}(sm_\chi^2 + 4m_\chi^4 - 2M_X^4)}{2(M_X^4 + m_\chi^2(s - 4M_X^2))} \right) \\ - \frac{(s^2 + 4(s - 2M_X^2)m_\chi^2 - 8m_\chi^4 + 4M_X^4)}{2(s - 2M_X^2)} \text{Log} \left(\frac{2M_X^2 - s + \sqrt{(s - 4m_\chi^2)(s - 4M_X^2)}}{2M_X^2 - s - \sqrt{(s - 4m_\chi^2)(s - 4M_X^2)}} \right).$$

To compute the analytic expression of the thermally averaged cross section suitable for the thermal freeze-out scenario, we write $s = 4m_\phi^2 + m_\phi^2 v^2$, and write σv in powers of v , as follows

$$\sigma_{\phi \rightarrow X} = \frac{g_d^4 \sqrt{-M_X^2 + m_\phi^2}}{16\pi v m_\phi^3 (2m_\phi^2 - M_X^2)^2} (24m_\phi^4 - 24m_\phi^2 M_X^2 + 7M_X^4) \\ + \mathcal{O}(v)$$

Ignoring the higher order terms in v , we get

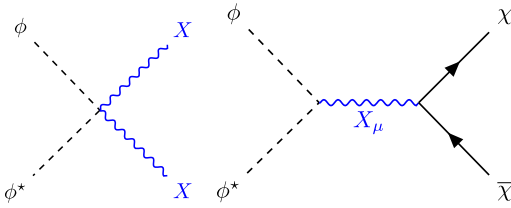


FIG. 16. Feynman diagrams for the number changing processes, relevant for the calculation of ϕ and χ abundance.

$$\sigma_{\phi \rightarrow X} \approx \frac{g_d^4}{16\pi m_\phi^3 v} \sqrt{-M_X^2 + m_\phi^2} \frac{24m_\phi^4 - 24m_\phi^2 M_X^2 + 7M_X^4}{(2m_\phi^2 - M_X^2)^2} \\ \approx \frac{g_d^4}{16\pi m_\phi^2 v} \sqrt{1 - \frac{M_X^2}{m_\phi^2}} \frac{24m_\phi^4}{(2m_\phi^2)^2} \\ = 6 \frac{g_d^4}{16\pi m_\phi^2 v} \sqrt{1 - \frac{M_X^2}{m_\phi^2}}$$

where, in the last two line, we have used the condition that $M_X < m_\phi$. Then, σv becomes

$$\sigma v|_{\phi \rightarrow X} = 6 \times \frac{g_d^4}{16\pi m_\phi^2} \sqrt{1 - \frac{M_X^2}{m_\phi^2}}. \quad (\text{D1})$$

Similarly, for $\chi\chi \leftrightarrow XX$, the cross section can be expanded in powers of v using $s = 4m_\chi^2 + m_\chi^2 v^2$ as

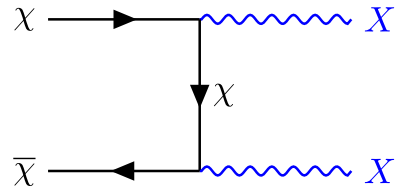


FIG. 17. Feynman diagram of χ annihilation to X .

$$\begin{aligned}
\sigma_{\chi \rightarrow X} &= \frac{g_d^4 (-M_X^2 + m_\chi^2)^{3/2}}{4\pi m_\chi v (2m_\chi^2 - M_X^2)^2} + \mathcal{O}(v) \\
&\approx \frac{g_d^4}{4\pi m_\chi v} \sqrt{-M_X^2 + m_\chi^2} \times \frac{(m_\chi^2 - M_X^2)}{(2m_\chi^2 - M_X^2)^2} \\
&\approx \frac{g_d^4}{4\pi v} \sqrt{1 - \frac{M_X^2}{m_\chi^2}} \times \frac{m_\chi^2}{(2m_\chi^2)^2} \\
&= \frac{g_d^4}{16\pi m_\chi^2 v} \sqrt{1 - \frac{M_X^2}{m_\chi^2}}
\end{aligned}$$

where, again, we ignored the higher power of v and realizing that $M_X < m_\chi$. Then, σv for the χ annihilation becomes

$$\sigma v|_{\chi \rightarrow X} = \frac{g_d^4}{16\pi m_\chi^2} \sqrt{1 - \frac{M_X^2}{m_\chi^2}}. \quad (\text{D2})$$

APPENDIX E: INTERACTION RATE OF SCATTERING PROCESSES

To find the thermally averaged cross section for a process $1 + 2 \leftrightarrow 3 + 4$, we use the general formula [81]

$$\langle \sigma v \rangle = \frac{\int \sigma v e^{-E_1/T} e^{-E_2/T} d^3 p_1 d^3 p_2}{\int e^{-E_1/T} e^{-E_2/T} d^3 p_1 d^3 p_2}. \quad (\text{E1})$$

Rewriting the integration measure as

$$\begin{aligned}
d^3 p_1 d^3 p_2 &= (4\pi |p_1| E_1 dE_1) (4\pi |p_2| E_2 dE_2) \frac{1}{2} d(\cos(\theta)) \\
&= (2\pi^2) E_1 E_2 dE_+ dE_- ds,
\end{aligned}$$

where the 2nd step is achieved via change of variables; $E_+ = E_1 + E_2$, $E_- = E_1 - E_2$ and $s = E_1^2 + E_2^2 + 2E_1 E_2 - 2|p_1||p_2|\cos(\theta)$. Noting the region of integration as

$$\begin{aligned}
s &\geq (m_1 + m_2)^2 \\
E_+ &\geq \sqrt{s} \\
\left| E_- E_+ + \frac{m_2^2 - m_1^2}{s} \right| &\leq 2p_{12} \sqrt{\frac{E_+^2 - s}{s}}
\end{aligned} \quad (\text{E2})$$

The numerator (N) of Eq. (E1) can be simplified by performing a first integration on dE_- as $\int dE_- = \frac{4p_{12}}{\sqrt{s}} \sqrt{E_+^2 - s}$ to get the following

$$\begin{aligned}
N &= 2\pi^2 \int ds dE_+ \sigma v E_1 E_2 \times \frac{4p_{12}}{\sqrt{s}} \sqrt{E_+^2 - s} e^{-E_+/T} \\
&= 2\pi^2 \int ds dE_+ \left(p_{12} \sqrt{s} \sigma \times \frac{4p_{12}}{\sqrt{s}} \right) \sqrt{E_+^2 - s} e^{-E_+/T} \\
&= 8\pi^2 \int ds \sigma(s) p_{12}^2(s) \times \int dE_+ e^{-E_+/T} \sqrt{E_+^2 - s} \quad (\text{E3})
\end{aligned}$$

where we have used $\sigma v E_1 E_2 = p_{12}(s) \sqrt{s} \sigma$ [82] to get the 2nd line for p_{12} defined by $p_{12}(s) = ([s - (m_1 + m_2)^2]^{1/2} [s - (m_1 - m_2)^2]^{1/2}) / 2\sqrt{s}$. Finally, integrating with respect to dE_+ , the numerator becomes

$$N = 8\pi^2 T \int ds \sigma(s) p_{12}^2(s) \sqrt{s} K_1(\sqrt{s}/T). \quad (\text{E4})$$

The denominator of Eq. (E1) can be simplified as

$$\begin{aligned}
D &= \int d^3 p_1 d^3 p_2 f_1 f_2 \\
&= \frac{(2\pi)^6}{g_1 g_2} \times \int g_1 \frac{d^3 p_1 f_1}{(2\pi)^3} \times \int g_2 \frac{d^3 p_2 f_2}{(2\pi)^3} \\
&= \frac{(2\pi)^6}{g_1 g_2} n_1 n_2. \quad (\text{E5})
\end{aligned}$$

Thus, the thermally averaged cross section for the scattering becomes

$$\begin{aligned}
\langle \sigma v \rangle &= \frac{g_1 g_2}{(2\pi)^6 n_1 n_2} 8\pi^2 T \\
&\times \int_{(m_1 + m_2)^2}^{\infty} ds \sigma(s) p_{12}^2(s) \sqrt{s} K_1(\sqrt{s}/T). \quad (\text{E6})
\end{aligned}$$

and the interaction rate is $n_{\text{bath}} \times \langle \sigma v \rangle$.

APPENDIX F: COMPLETE BOLTZMANN EQUATIONS

The complete Boltzmann equations concerning the evolution of X , ϕ , and χ are given by

$$\begin{aligned}
\frac{dY_X}{dx} &= \frac{s(m_\phi)}{H(m_\phi)} \frac{\langle \sigma v \rangle_{\phi \rightarrow X}}{x^2} \left[Y_\phi^2(x) - \frac{Y_X^2}{(Y_X^{eq}(xM_X/m_\phi))^2} (Y_\phi^{eq})^2(x) \right] \\
&\quad + \frac{s(m_\phi)}{H(m_\phi)} \frac{\langle \sigma v \rangle_{\chi \rightarrow X}}{x^2} \left[Y_\chi^2(x) - \frac{Y_X^2}{(Y_X^{eq}(xM_X/m_\phi))^2} (Y_\chi^{eq})^2(xm_\chi/m_\phi) \right] - \frac{x\Gamma_{X \rightarrow SM}}{H(m_\phi)} [Y_X(x) - Y_X^{eq}(xM_X/m_\phi)], \\
\frac{dY_\chi}{dx} &= -\frac{s(m_\phi)}{H(m_\phi)} \frac{\langle \sigma v \rangle_{\chi \rightarrow X}}{x^2} \left[Y_\chi^2(x) - \frac{Y_X^2}{(Y_X^{eq}(xM_X/m_\phi))^2} (Y_\chi^{eq})^2(xm_\chi/m_\phi) \right] + \frac{x\Gamma_\phi}{H(m_\phi)} Y_\phi(x), \\
\frac{dY_\phi}{dx} &= -\frac{s(m_\phi)}{H(m_\phi)} \frac{\langle \sigma v \rangle_{\phi \rightarrow X}}{x^2} \left[Y_\phi^2(x) - \frac{Y_X^2}{(Y_X^{eq}(xM_X/m_\phi))^2} (Y_\phi^{eq})^2(x) \right] - \frac{x\Gamma_\phi}{H(m_\phi)} Y_\phi(x),
\end{aligned} \tag{F1}$$

where $x = m_\phi/T$ is the dimensionless parameter, $s(m_\phi)$ and $H(m_\phi)$ are the corresponding entropy density and the Hubble expansion rate expressed in terms of m_ϕ (with x factored out). For most calculations, we adopt a simplified form of the Boltzmann equation that does not require tracking the comoving number density of X .

APPENDIX G: DECAY WIDTH OF X

The interaction given in Eq. (11) can be used to calculate the decay width of X to electron and neutrino. They are calculated as

$$\Gamma_{X \rightarrow \nu \bar{\nu}} = \frac{(C_L^e)^2 M_X}{24\pi}, \tag{G1}$$

$$\Gamma_{X \rightarrow e^+ e^-} = \frac{(C_{LR}^e)^2 M_X}{24\pi} \sqrt{1 - \frac{4m_e^2}{M_X^2}}, \tag{G2}$$

where the factor $(C_{LR}^e)^2$ is given by

$$((C_L^e)^2 + (C_R^e)^2) \left(1 - \frac{m_e^2}{M_X^2} \right) + 6 \frac{m_e^2}{M_X^2} C_L^e C_R^e. \tag{G3}$$

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