

Hydrogen 21-cm constraints on the photon's spin scale

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We explore the fundamental but untested possibility that the photon is a continuous spin particle (CSP) with a small but nonzero spin Casimir ρ . When $\rho \neq 0$, the familiar polarization modes of the photon transform nontrivially under Lorentz boosts, leading to deviations from familiar QED. Surprisingly, these deviations are strongest at low energy, but smoothly vanish in the $\rho \rightarrow 0$ limit. In this letter, we compute corrections to the hydrogen 21-cm transition rate, which is expected to be particularly sensitive given the small hyperfine energy splitting ω . We find deviations from $\text{QED} \propto \rho^2 \alpha^2 / \omega^2$ at leading order, suggesting experimental constraints $\rho \lesssim 1$ meV. Building on this work, we expect that a range of other atomic, molecular, or condensed matter systems could be used to provide even more stringent tests of ρ in electromagnetic interactions.

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I. INTRODUCTION

Beyond familiar boost-invariant helicity states, massless particles capable of mediating long-range forces also include a more general class: continuous spin particles (CSP) [1], for which their helicity quantum numbers transform nontrivially under Lorentz boosts. Both massive and massless particles within the Standard Model (SM) are characterized by a spin Casimir ρ , the “spin scale,” with units of momentum. In the massive case ρ has a discrete set of allowed values, proportional to the mass m . By contrast, in the massless case, ρ can take any non-negative value (the continuous range of allowed values is the origin of the term CSP). CSPs possess an infinite tower of helicity states ($h = 0, \pm 1, \pm 2, \dots$), which mix under Lorentz transformations in a way controlled by ρ . Individual helicity modes are only boost invariant when $\rho = 0$, as is generally assumed in deriving field theories of interacting scalar, vector, and tensor fields.

Historically, nonzero ρ in the SM was ignored because it was *assumed* that the infinite number of polarizations would be at odds with observation. However, the recent exploration of consistent interactions for CSPs revealed a very

different structure: for small ρ , the interactions of a CSP closely resemble those of a familiar gauge boson or graviton, with new—but still finite and physically well-behaved—effects appearing at long distances and low energies (see, e.g., Refs. [2–6]). This feature raises the intriguing possibility that known states in the SM, though well described thus far by familiar gauge theories, are in fact CSPs with small ρ .

The possibility that known forces are associated with mediators with nonzero ρ constitutes a fundamentally new kind of precision test of the Standard Model. All such directions are worth exploring experimentally in their own right. This one is particularly interesting given that the most prominent evidence for physics beyond the SM, in the form of dark matter and dark energy, is entirely derived from long distance observations—the regime where the effects of nonzero ρ become important. Furthermore, the framing of key theory puzzles about the SM—the hierarchy and strong CP problems—also assume $\rho = 0$, closing off exploration of possible approaches to these puzzles that might arise when this assumption is relaxed.

For these reasons, work towards understanding how to describe and make predictions for the new physics associated with CSPs has grown sharply, with numerous pure theory developments [7–30], and more recently progress towards computing novel physical effects associated with thermodynamics [6], gravitational scattering [29], atomic transitions [31], and gravitational waves [32] (see also a discussion of CSP neutrino interactions in Ref. [33]). The possibility under the best theoretical control is that the familiar photon in the SM has small but nonzero ρ , which could produce subtle deviations from Maxwell’s theory and QED that have escaped detection so far. Exploring this

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fundamental possibility is the purpose of this paper. We focus on atomic systems, which are a natural testing ground because of the low energy scales involved in atomic transitions and the high experimental precision that has been achieved in measuring them.

We first explored effects of $\rho \neq 0$, focusing only on electric dipole transitions in Ref. [31], where we derived a general framework for computing CSP photon mediated transitions. Using $O(1)$ constraints on the hydrogen 2s lifetime, the authors in Ref. [31] found that $\rho \gtrsim 0.1$ eV is excluded. Ultimately, the experimental sensitivity is limited by the fact that CSP effects in a wide range of systems are suppressed by $\rho v/\omega$, with v the velocity of the matter particle and ω the CSP energy.

The IR domination of $\rho \neq 0$ effects is what motivates the present work. The most sensitive experimental probes of ρ are those with low energy, and the lowest energy atomic transitions tend to be those that involve only a spin flip. The simplest and smallest of these is the transition between hyperfine split states of the hydrogen ground state, which gives the famous 21-cm line [34]. While we did not consider such transitions in Ref. [31], recent theoretical work provides a consistent method to couple a CSP photon to spin-1/2 fermions [30], enabling this study. We calculate explicit corrections to the 21-cm transition probability when $\rho \neq 0$, allowing us to place new, more stringent constraints on the photon's spin scale $\rho \lesssim 1$ meV, and to motivate a range of follow-up studies in atomic and related systems.

We note that deviations of stellar cooling rates, from the emission of the $h = 0, 2$ modes of the CSP photon, naively sets a stronger limit of $\rho \lesssim O(\text{neV})$. However, the region near $\rho \sim 1$ meV is close to the regime where these modes are trapped inside stars, while the $h = 3$ modes are too weakly coupled to generate significant cooling. Thus, laboratory probes sensitive to the range $\rho \sim 0.1\text{--}10$ meV are important to consider.

II. THEORETICAL FRAMEWORK

We start with a few remarks regarding the observation of the hyperfine transition in atomic systems. Directly observing the hyperfine transition rate in vacuum is impractical, as the expected lifetime is approximately 11 million years. However, stimulated emission can be induced in a cavity, as demonstrated in the hydrogen maser [35,36]. In this case, the matrix element for the transition remains effectively the same as in vacuum, but is multiplied by a Bose-enhancement factor proportional to the amplitude of the oscillating magnetic field in the cavity, $B_{\text{osc}} \propto \sqrt{N_\gamma}$. The cavity is then sensitive to transitions $|e\rangle|N_\gamma\rangle \leftrightarrow |g\rangle|N_\gamma + 1\rangle$, where $|e, g\rangle$ are the excited and ground states of the atom. This is the quantum mechanical mechanism of Rabi oscillations driven by an oscillating magnetic field, $\mathbf{B}_{\text{osc}} = B_{\text{osc}} \cos(\omega t) \mathbf{e}_B$, where $\mathbf{e}_B$ is a directional unit vector. For $\mathbf{e}_B$ aligned the

atoms' spin axis, and for $\omega \approx \omega_{eg}$, where $\omega_{eg} = E_e - E_g$ is energy splitting of the atomic states, a transition between the excited and ground state becomes near resonant. To leading order in the fine-structure constant α , the transition rate is controlled by the vacuum matrix element

$$M_{eg} = \langle g|H_I(t)|e\rangle = \cos(\omega t) \langle g|H'_I|e\rangle, \quad (1)$$

where $H_I(t)$ and H'_I are the time dependent and independent pieces of the interaction Hamiltonian. Under the rotating wave approximation, a matrix element that varies with time in this manner leads to oscillations from an initial excited state to a final ground state with time dependence given by

$$|P_g(t)|^2 = \frac{\Omega_R^2}{\Omega_D^2 + \Omega_R^2} \sin^2\left(\frac{t}{2} \sqrt{\Omega_D^2 + \Omega_R^2}\right), \quad (2)$$

where $\Omega_D = \omega_{eg} - \omega$ is the detuning of the cavity field from the transition frequency, and $\Omega_R = |\langle g|H'_I|e\rangle|$ is the Rabi frequency. In this experimental setup, CSP corrections to the photon coupling will change the Hamiltonian vacuum transition matrix element, and therefore alter the Rabi frequency relative to the QED expectation. It merits noting that the generation of the background B-field at frequency $\omega \simeq \omega_{eg}$ could lead to ρ dependent deviations at similar order to those of the atomic transition matrix element. However, we expect that such deviations would largely be absorbed into the definition of the field strength when tuning the cavity field prior to running the experiment. While future study of this subtlety is needed, we focus here on the leading-order ρ corrections to the vacuum matrix element, and set conservative bounds on this basis. Furthermore, we ignore any potential long range interactions or interference effects between hydrogen atoms. These are negligible for $\rho = 0$ QED. The precise modification to force-laws at nonzero ρ is unknown, but it is plausible that these modifications become $O(1)$ when $\rho v r \gtrsim 1$. Absent an (unexpected) parametric enhancement in interatomic forces, however, they should also be negligible for this experiment if $\rho \neq 0$.

We use the formalism in Refs. [2,3,30,37] to study CSP interactions with spin-1/2 matter particles. These works employ the quantum worldline formalism, wherein a matter particle's worldline Lagrangian is perturbatively coupled to a background photon field to compute amplitudes (see e.g. Refs. [38–41] for a review, and Ref. [37] for its application to CSP QED). As opposed to a quantum field theory (QFT), the worldline formalism restricts to an arbitrary but fixed number of input and output matter states, while the massless degrees of freedom are described using quantum fields. In many instances, this formalism renders calculations far simpler than their QFT counterparts.

The worldline formalism describes space-time spin-1/2 fermions using a Lagrangian with both Grassmann-even and Grassmann-odd degrees of freedom ($z^\mu(\tau)$ and $\psi^\mu(\tau)$, plus

ψ^5) that are each functions of a worldline time parameter τ . As described in Refs. [40,41], the appropriate free Lagrangian realizes a local worldline supersymmetry that transforms z^μ into ψ^μ and vice versa (a linear transformation for ψ^5 is recovered by introducing an auxiliary z^5). Upon quantizing the free theory, the wave function $|\psi\rangle$ for the matter state satisfies Hamiltonian constraints $(p^2 - m^2)|\psi\rangle = 0$ and $(p_\mu \psi^\mu + m\psi^5)|\psi\rangle = 0$, where p^μ is the momentum conjugate to z^μ . Moreover, quantization results in the ψ^μ operators satisfying the Dirac algebra $\{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$, permitting the identifications $\psi^\mu \rightarrow \gamma^5 \gamma^\mu$ and $\psi^5 \rightarrow \gamma^5$, so that the constraints above become the Klein-Gordon and Dirac equations.

Using the worldline formalism in familiar QED, the interaction Hamiltonian with a background photon plane wave $A_\mu(x) = \epsilon_\mu(k)e^{ik \cdot x} + \text{c.c.}$ of momentum k^μ takes the form

$$H_I^{\text{QED}}(t) = A_h \frac{q}{m} e^{ik \cdot z} (p \cdot \epsilon(\mathbf{k}) + (k \cdot \psi)(\epsilon(\mathbf{k}) \cdot \psi)), \quad (3)$$

where we have identified $t = z^0(\tau)$, q is the electric charge of the matter, m is its mass, and the addition by the complex conjugate (to make the Hamiltonian real) is left implicit. We have included a factor A_h to encode the occupation number associated with the background magnetic field. In particular, $A_h(\mathbf{k}) = B_{\text{osc}}/2\omega$.

Note that under a gauge transformation $\epsilon_\mu \rightarrow \epsilon_\mu + \beta k_\mu$, the first term transforms by a total τ derivative (from $p^\mu = m\partial_\tau z^\mu$), while the spin term is invariant because the ψ^μ are anticommuting. Thus, observables computed with this Hamiltonian are gauge invariant. When this Hamiltonian is inserted between initial and final spin eigenstates, the ψ^μ fields can be replaced by γ matrices via the correspondence $\psi^\mu \rightarrow \gamma^5 \gamma^\mu$. For example, for generic spin states $|S_i\rangle$ and $|S_f\rangle$, we have

$$\langle S_f | (k \cdot \psi)(\epsilon \cdot \psi) | S_i \rangle \Rightarrow -k_\mu \epsilon_\nu \langle S_f | \gamma^\mu \gamma^\nu | S_i \rangle, \quad (4)$$

where we have anticommuted the γ^5 matrices through and used the identity $\gamma^5 \gamma^5 = -1$. The position/momentum part of the Hamiltonian just becomes the usual derivative operators on the wave function.

While unfamiliar to most readers, the generalization to CSP interactions with vector correspondence in Refs. [3,30] is straightforward. For $\rho \neq 0$, the photon is identified with the $h = \pm 1$ modes of a field $\Psi(\eta, x)$ that depends on an auxiliary four-vector η^μ . Though not precise, the reader can think of the tower of helicity modes of the CSP as encoded in the tower of symmetric tensors that occur in the expansion of $\Psi(\eta, x) = \Psi_0(x) + \Psi_1^\mu(x)\eta_\mu + \dots$ in powers of η_μ . On-shell CSP wave functions can be decomposed into helicity eigenstates $\Psi_{h,k}(\eta, x) = \Psi_{h,k}(\eta)e^{ik \cdot x} + \text{c.c.}$, where $\Psi_{h,k}(\eta)$ is an appropriate eigenfunction, entirely analogous

to the polarization tensors $\epsilon_\mu(k)$ in the QED case. To first order in charge q , and now with $\rho \neq 0$, the interaction Hamiltonian is

$$H_I(t) = \frac{q}{m} A_h(\mathbf{k}) \int [\bar{d}^4 \eta] \Psi_{h,k}(\eta) e^{ik \cdot z} e^{-i\rho \frac{p \cdot p}{k \cdot p}} \times \sqrt{2}i \left(\frac{k \cdot p}{\rho} + (k \cdot \psi)(\eta \cdot \psi) \right), \quad (5)$$

where $[\bar{d}^4 \eta]$ is the appropriately regulated η space measure as defined in Ref. [3]. However, for the purpose of this work, integrations over η^μ can be accomplished using the following identity from Ref. [3]:

$$\int [\bar{d}^4 \eta] \Psi_{h,k}(\eta) F(\eta) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-ih\phi} F(\eta(\phi)), \quad (6)$$

where $\eta^\mu(\phi) = \epsilon_+^\mu(k)e^{i\phi} + \epsilon_-^\mu(k)e^{-i\phi}$, and $\epsilon_\pm^\mu$ are any pair of complex conjugate transverse polarization vectors. A convenient choice of η is to set $\eta^0 = 0$. The normalization is fixed by requiring $\eta^\mu(\phi)$ to have unit norm. We note that performing a full Legendre transformation on the CSP Lagrangian defined in Ref. [3] poses technical challenges, which motivated the path integral methods derived in Ref. [31]. Instead, in deriving Eq. (5), we have used the *free* Legendre transformation, i.e.,

$$p = m\dot{z} + \mathcal{O}(q), \quad \text{and} \quad p_\psi = \frac{i}{2}\dot{\psi} + \mathcal{O}(q), \quad (7)$$

(see Ref. [30] for more details) and the equation of motion $\dot{\psi} = 0 + \mathcal{O}(q)$. This is justified for the specific case of single photon emission and absorption, because the $\mathcal{O}(q)$ corrections would only contribute to the interaction Hamiltonian at $\mathcal{O}(q^2)$.

III. RESULTS

For the 21-cm transition between the hyperfine-split states of hydrogen, the matrix element that enters into Eq. (1) is

$$M_{eg} = \sum_h \frac{iq B_{\text{osc}}}{m} \frac{1}{2\omega} \int \frac{d\phi_\eta}{2\pi} e^{-ih\phi_\eta} \times \langle \xi_0, S_f | e^{ik \cdot z} e^{-i\rho \frac{p \cdot p}{k \cdot p}} \left(\frac{k \cdot p}{\rho} + k_\mu \eta_\nu \gamma^\mu \gamma^\nu \right) | \xi_0, S_i \rangle, \quad (8)$$

where ξ_0 is the ground state wave function of the hydrogen atom, $|S_{i,f}\rangle$ are the initial and final spin states, and we have set $A_h(\mathbf{k}) = B_{\text{osc}}/\sqrt{8}\omega$ [note that the $\sqrt{2}$ difference in $A_h(\mathbf{k})$ relative to that in Eq. (3) comes from normalization differences when defining a B-field in η space]. The two-spin system defined by the electron and proton of the hydrogen atom has a triplet excited state and singlet ground state. In the above, $|S_i\rangle$ can be any of the triplet states, while

$|S_f\rangle$ is the singlet. We note that since $\langle S_f | S_i = 0$, the first term of Eq. (8) (i.e., the term $\propto k \cdot p/\rho$) does not contribute to the matrix element. Furthermore, the γ matrices act only on the spin part of the wave function, while the position and momentum operators act only on the spatial part of the wave function ξ_0 . We can therefore factorize the relevant terms in the integrand of Eq. (8) as

$$\langle \xi_0 | e^{ik \cdot z} e^{-ip \frac{p}{k \cdot p}} | \xi_0 \rangle \times k_\mu \eta_\nu \langle S_f | \gamma^\mu \gamma^\nu | S_i \rangle. \quad (9)$$

It is clear from this form that a nonzero ρ only enters through the spatial wave function overlap factor in Eq. (9). Indeed, taking $\rho \rightarrow 0$ and the long-wavelength limit, the first factor approaches 1 while the second factor is simply the η -space formulation (see Ref. [3]) of the familiar QED result in the long-wavelength limit.

Since the 21-cm transition is not a relativistic process, for the rest of our treatment, we work at leading order in velocity while still considering all orders in the separate expansion parameter $\rho v/\omega$, which may be large even when v itself is small. In particular, we take $k \cdot z \approx \omega t$ and $k \cdot p \approx \omega m$, which allows us to expand the position space part of the Hamiltonian as

$$H_P = e^{ik \cdot z} e^{-ip \frac{p}{k \cdot p}} = e^{i\omega t} e^{ip \frac{\vec{p} \cdot \vec{\eta}}{\omega m}} + \mathcal{O}(v), \quad (10)$$

recalling that $\eta^0 = 0$. Using this approximation, we can rewrite this contribution to the matrix element as

$$\langle \xi_0 | H_P | \xi_0 \rangle = e^{i\omega t} e^{-\frac{|\rho| \alpha}{\omega}} \frac{|\rho|^2 \alpha^2 + 3|\rho| \alpha \omega + 3\omega^2}{3\omega^2}, \quad (11)$$

where we used $\vec{\eta}^2 = 1$ and worked in momentum space to simplify the calculation. As expected, in the limit that $\rho \rightarrow 0$, we recover the simple QED term $e^{i\omega t}$, and in the $\rho \rightarrow \infty$ limit the amplitude remains finite. With no dependence on the angular position of $\vec{\eta}$, this ρ dependent correction shows up as an overall constant change to the QED matrix element, but does not affect the allowed helicity of the emitted photon. Only $h = \pm 1$ modes are emitted. We verify this statement in detail by computing the spin dependent piece of Eq. (8) in the Appendix, and show that it matches the standard QED calculation. Putting the above together, and expanding in $\rho \alpha/\omega$ to leading order, the complete matrix element is

$$M_{eg} = \sum_{h=\pm 1} \frac{iq B_{\text{osc}}}{m} \frac{1}{4} \langle \xi_0 | H_P | \xi_0 \rangle = \frac{iq B_{\text{osc}}}{m} \frac{1}{2} \left(1 - \frac{|\rho|^2 \alpha^2}{6 \omega^2} \right) \cos(\omega t) + \mathcal{O}\left(\frac{|\rho|^4 \alpha^4}{\omega^4}\right), \quad (12)$$

where $\frac{q}{2m} B_{\text{osc}}$ is the QED ($\rho \rightarrow 0$) limit of the Rabi frequency Ω_R . The fact that the lowest-order correction to QED shows up at $\mathcal{O}(\rho^2)$ is not surprising, as an odd

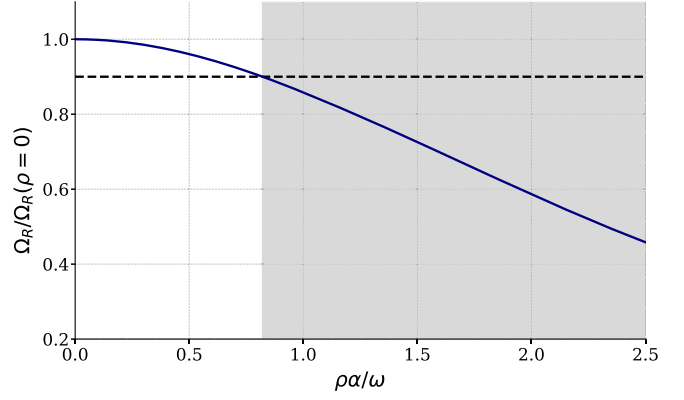


FIG. 1. Blue solid line shows the behavior of the Rabi frequency $\Omega_R = |\langle g | H_I | e \rangle|$ for $\rho \neq 0$, normalized to the standard case ($\rho = 0$), as a function of $\rho \alpha/\omega$. Based on the data in Ref. [42], we estimate that Ω_R can deviate by at most 10% (indicated by the dashed line) from the standard value. The shaded region represents the excluded parameter space for $\rho \alpha/\omega$.

power of ρ term would show up with an odd number of momentum operators, and thus vanish by parity.

If one were to use this transition to determine the electron's magnetic moment by defining $H_I = -\vec{\mu}_e \cdot \mathbf{B}_{\text{osc}}$, the ρ dependent term in Eq. (12) would manifest as a correction to the electron's Landé g-factor, g_e . However, this correction would not be the same for measurements of μ_e in other systems, and in particular is frequency dependent. As the Rabi frequency in Eq. (2) is directly proportional to the amplitude M_{eg} , any constraint on Ω_R in a cavity experiment will in turn bound the size of ρ/ω . For example, one can repurpose the experimental data used to take spectroscopic measurements of the hydrogen hyperfine energy splitting and immediately bound ρ . While hydrogen maser experiments offer one method, often additional spectroscopy is carried out via interferometry of the output photons rather than a direct measure of the power and hence transition probability. Therefore, we turn to more modern, “in-beam” experiments which measure the energy splitting by directly measuring the atomic transition rate as atoms pass through a microwave cavity parallel to a standing magnetic field [42]. If we take the energy splitting as a known constant, we can reinterpret the data in Ref. [42] to constrain the uncertainty on Ω_R . The cavity geometry of Ref. [42] means that the idealized transition probability given by Eq. (2) does not directly apply, and it is instead a more complicated function of Ω_D and Ω_R . Nonetheless, the ρ dependence of Ω_R itself is calculated reliably, and we bound deviations to Ω_R using the full analysis from Ref. [42]. In this way, we can conservatively say that Ω_R should not differ from the QED expectations by more than $\sim 10\%$, and hence $\rho \alpha/\omega \lesssim 0.8$ (see Fig. 1). Given the large uncertainties already present, we can safely neglect Zeeman effects in the energy splitting. Using the energy difference $\omega_{\text{hf}} = 6.9 \times 10^{-6}$ eV [43–45], we find that

$$\rho \lesssim 1 \text{ meV}. \quad (13)$$

Already, this is 2 orders of magnitude stronger than the constraint derived in Ref. [31].

IV. DISCUSSION AND CONCLUSION

In this paper, we presented a straightforward application of the formalism in Ref. [30] to compute the leading-order effect that a nonzero spin scale for the photon would have on the dynamics of the hydrogen 21-cm transition. We then used this result to derive a constraint on the photon's spin scale ρ . In particular, we found that for photons emitted in spin-flip transitions with energy ω , deviations from ordinary QED scale as $\propto \rho^2 \alpha^2 / \omega^2$ in the limit of small ρ . This is consistent with the expectation that the novel physics associated with the partner modes of the photon are only important in the IR, at low energy, if ρ is small. At higher energies, the interactions with the partner modes are suppressed—this is why the otherwise radical possibility that $\rho \neq 0$ for the familiar photon could have gone unnoticed.

As we noted in the Introduction, complementary constraints from stellar cooling are least sensitive in the range $\rho \sim 0.1\text{--}10$ meV, motivating laboratory tests of QED to decisively probe this range (see Ref. [6] for a discussion of CSP thermodynamics). One approach is to improve the precision of the measurements examined in this work, where we currently utilize only an $\mathcal{O}(10\%)$ constraint on the QED prediction. Another approach would be to utilize atomic or molecular transitions with even smaller energy splitting, or to use engineered systems with transitions tuned to be small. An example of such a system might include a one-electron quantum cyclotron, a setup traditionally used to measure the electron's magnetic moment [46,47] (note, however, that current experiments operate at cyclotron frequencies higher than that of the 21-cm transition). A calculation similar to that performed in this paper could be used to predict the leading-order deviations from the QED prediction for the rate of radiation emitted by the electron in this system. Examining changes to the cyclotron frequency, on the other hand, may yield better precision than a rate measurement, but requires solving the full equations of motion, a technically more difficult task. Yet another approach would be to use “light shining through a wall” style experiments to observe nearby helicity states of the photon [48]. Such experiments are limited by ρ/ω suppression for both the creation and detection of the photon partner modes, but might be able to overcome this limitation with the benefit of extremely low backgrounds and tunable emissions.

In addition to phenomenological studies, many open questions regarding the theory of CSP dynamics remain. Most pressing is that the couplings to matter derived in Refs. [3,37] are only strictly valid for on-shell CSP amplitudes. While the current formalism can include off-shell CSPs, it does not provide a unique prediction for their

effects. Beyond this limitation, many questions remain about generalizing the Abelian CSP theories explored thus far to non-Abelian theories applicable to the $SU(3)_c \times SU(2)_w$ part of the Standard Model, and also to their confined or Higgs phases. Initial progress has been made exploring deviations from general relativity for the case that gravity has $\rho \neq 0$ [32], but here too existing calculations are limited to the linear regime, and a full nonlinear theory has yet to be developed.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: SPIN CALCULATION AND INTEGRAL IN η

To calculate the spin-flipping part of the Hamiltonian, we first need to define our nonrelativistic spin states. In the limit where $v \rightarrow 0$ and in the Dirac basis, relativistic spinors take the following form:

$$|\uparrow\rangle_{\text{rel}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(v), \quad |\downarrow\rangle_{\text{rel}} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(v). \quad (\text{A1})$$

Using only the upper two indices of these spinors as our nonrelativistic bases vectors, we can rewrite the spin part of Eq. (9) as

$$k_\mu \eta_\nu \langle S_f | \gamma^\mu \gamma^\nu | S_i \rangle = i \langle S_f | \vec{\sigma} \cdot (\vec{k} \times \vec{\eta}) | S_i \rangle, \quad (\text{A2})$$

which looks very much like the equivalent QED term, with $\vec{\eta}$ replacing $\vec{A}$. Let us now consider the spin states of interest:

$$\begin{aligned} |g\rangle &= |F=0, m_F=0\rangle = \frac{1}{\sqrt{2}} (|\uparrow^e \downarrow^p\rangle - |\downarrow^e \uparrow^p\rangle), \\ |e\rangle &= |F=1, m_F=0\rangle = \frac{1}{\sqrt{2}} (|\uparrow^e \downarrow^p\rangle + |\downarrow^e \uparrow^p\rangle), \end{aligned} \quad (\text{A3})$$

where F is the total spin angular momentum of the atom, m_F is the $\hat{z}$ component of that angular momentum, and the

subscripts e and p refer to the spin states of the electron and proton, respectively. We have so far only considered coupling of the CSP photon to the electron, an approximation we will continue with as its coupling to the proton will be suppressed by a factor of m_e/m_p relative to the electron, rendering it negligible. We therefore consider only the identity acting on $|\uparrow^p\rangle, |\downarrow^p\rangle$ in the matrix element. This means that terms with opposite proton spins will vanish, and we are left with the computation of

$$\frac{i}{2} (\langle \uparrow^e | \vec{\sigma} \cdot (\vec{k} \times \vec{\eta}) | \uparrow^e \rangle - \langle \downarrow^e | \vec{\sigma} \cdot (\vec{k} \times \vec{\eta}) | \downarrow^e \rangle) = i(\vec{k} \times \vec{\eta})_z. \quad (\text{A4})$$

Because $\vec{k}$ and $\vec{\eta}$ are orthogonal, we can define the photon propagation direction to be $\hat{x}$ without loss of generality, so we are left with simply $i\omega\eta_y$. Finally, recalling that the norm of $\vec{\eta}$ is 1, we take our final integration over ϕ_η according to Eq. (6), as follows:

$$i\omega \int \frac{d\phi_\eta}{2\pi} e^{-ih\phi_\eta} \cos \phi_\eta = \pm \frac{\omega}{2} \delta_{h\pm 1}, \quad (\text{A5})$$

meaning that there only exists support for the helicity one mode. When we sum over photons emitted in the forward and backward direction with helicity $h = \pm 1$, we recover a simple factor of ω .

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