

Infinite heat order in 3+1 dimensions

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We investigate whether spontaneous symmetry breaking can persist up to arbitrarily high temperature in ultraviolet-complete quantum field theories in four spacetime dimensions. We focus on completely asymptotically free models with gauge group $SU(N_{c1}) \times SU(N_{c2})$ and two complex scalar fields, each transforming in the fundamental representation of one gauge factor and singlet under the other. The scalar potential contains quartic self-interactions together with a negative portal coupling between the two sectors. In the Veneziano limit, this class of theories was previously shown to admit fixed-flow trajectories for which one scalar acquires a negative thermal mass at asymptotically large temperature, leading to symmetry nonrestoration. Here we extend that analysis to finite numbers of colors and flavors. We derive the finite- N fixed-flow equations, compute the leading $1/N$ corrections to the large- N solutions, and solve the full finite- N system numerically. We find explicit finite- N benchmark theories for which the scalar potential remains bounded from below, the gauge sector is asymptotically free, and one scalar thermal mass stays negative at arbitrarily high temperature. This provides an explicit perturbative example of infinite heat order in a four-dimensional ultraviolet-complete quantum field theory with a finite field content.

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I. INTRODUCTION

Does spontaneous symmetry breaking survive at arbitrarily high temperatures? Standard thermodynamic intuition suggests a negative answer: in the free energy $F = E - TS$, the entropic contribution typically dominates at large T , favoring disordered (symmetric) phases. Quantum field theory (QFT) can evade this expectation. Weinberg showed long ago that multiscalar theories can display high-temperature symmetry breaking through negative thermal masses induced by interactions [1]. This phenomenon is known as *symmetry nonrestoration*. A modern perspective was provided by Han, Huang, Komargodski, Lucas, and Popov, who identified a general mechanism termed *entropic order*: ordering one sector can increase the available phase space of another sector, so

that the ordered phase can carry more entropy than the disordered one [2].

The possibility of symmetry nonrestoration was developed in particle-physics settings by Mohapatra and Senjanović [3,4]. Cosmological applications include potential resolutions of the monopole [5–7], domain-wall [8,9] and false-vacuum problems, as well as implications for baryogenesis [10–15] and inflation [16]. Symmetry nonrestoration can also be driven by large conserved charges, through Bose-Einstein condensation or superconducting phases [17–24], with phenomenological motivations ranging from neutrino physics to beyond-the-Standard-Model dynamics [25–30]. Supersymmetric theories generically restore their symmetries at high temperature [31–33], except in the presence of flat directions [34,35] or fixed charges. For nonsupersymmetric models, the phenomenon has been investigated with a range of perturbative and nonperturbative techniques [36–42]; some claims remain controversial [43–47]. Extensions to lower and noninteger dimensions were also explored [48–52]; see [53,54] for reviews. The topic was also explored via the AdS/CFT correspondence [55–58] and further discussed within nonlocal theories in [59,60]. Examples with an infinite number of fields can be found in [61–64]. Finally, spontaneous breaking of time-reversal symmetry was investigated in [65].

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Despite this extensive literature, except for large- N examples such as those in [61,62], essentially all four-dimensional examples have been *effective field theories*. Scalar self-interactions (and often Yukawa couplings) grow in the ultraviolet and typically hit a Landau pole, so the theory ceases to exist above some cutoff. In such a situation, statements about the strict $T \rightarrow \infty$ limit are not meaningful: the thermal scale inevitably exceeds the regime of validity of the effective description.

A sharp formulation of the problem, therefore, requires UV-complete theories. In Wilson's renormalization-group picture [66,67], a local QFT is well defined at arbitrarily short distances only if its RG flow approaches a UV fixed point. In four dimensions, the best-understood possibilities are a Gaussian fixed point (asymptotic freedom) [68,69] or an interacting fixed point (asymptotic safety) [70]. Only in such theories can one consistently send the temperature to arbitrarily large values without leaving the domain of the microscopic theory.

In our previous work [62] we initiated a systematic study of infinite heat order in UV-complete four-dimensional gauge-Yukawa theories. Working mainly in the Veneziano limit, we found that complete asymptotic freedom can coexist with symmetry nonrestoration in models with two gauged scalars transforming under a product gauge group $SU(N_{c1}) \times SU(N_{c2})$, while the asymptotically safe examples studied there displayed high-temperature symmetry restoration.

Related progress was obtained recently in lower dimensions. Komargodski and Popov [71] constructed a UV-complete, local, and unitary QFT in $2+1$ dimensions with a finite field content that breaks a $\mathbb{Z}_2$ symmetry at any temperature. Shortly thereafter, Han *et al.* [2] demonstrated that the underlying mechanism, entropic order, arises broadly across lattice systems and continuum QFT and provides a unifying explanation for high-temperature ordering phenomena.

In this paper, we provide an explicit UV-complete example in $3+1$ dimensions with a finite number of degrees of freedom that exhibits spontaneous symmetry breaking at arbitrarily high temperature. The theory is completely asymptotically free: all couplings flow to a Gaussian UV fixed point, so the microscopic description remains valid at all scales. The analysis is perturbative and controlled by one-loop RG evolution together with the high-temperature effective potential.

Our minimal model contains two complex scalars ϕ_1 and ϕ_2 , each transforming in the fundamental representation of an independent non-Abelian gauge group $SU(N_{c1})$ and $SU(N_{c2})$, respectively. The renormalizable scalar potential includes a quartic cross-coupling $\lambda(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2)$ analogous to Weinberg's original setup [1]. Imposing the fixed-flow conditions required for complete asymptotic freedom restricts the couplings to a surface parametrized by the ratio $x = N_{c1}/N_{c2}$. For an appropriate range of x , one of the

scalar thermal masses becomes negative asymptotically, leading to symmetry nonrestoration. We demonstrate this mechanism both in the Veneziano limit and at finite N_{ci} , combining analytic $1/N_{ci}$ corrections with a direct numerical solution of the finite- N RG equations. We also exhibit an explicit anomaly-free benchmark point at finite N , confirming that the construction involves a genuinely finite field content.

The product gauge-group structure is essential. Models with a single $SU(N_c)$ gauge factor and fundamental scalars, as well as the singlet-scalar class studied in [62], restore their symmetries at high temperature once complete asymptotic freedom is imposed. Two independent gauge sectors provide sufficient parametric freedom for the fixed-flow constraints and the negative-thermal-mass condition to be simultaneously satisfied, thereby realizing the entropic-order mechanism of [2] in four dimensions.

The paper is organized as follows. Section II A reviews the large- N analysis of [62], including Weinberg's two-scalar model and the $SU(N_{c1}) \times SU(N_{c2})$ model exhibiting infinite heat order in the Veneziano limit. Section II B extends the analysis to finite N_{ci} , providing analytic next-to-leading corrections and numerical solutions. Section III contains our conclusions. The appendices collect one-loop RG equations and thermal-mass formulas for generic gauge-Yukawa theories, together with details for the specific models analyzed in the main text.

II. INFINITE HEAT ORDER WITH FINITE DEGREES OF FREEDOM

A. The infinite degrees of freedom limit recap

We start by briefly reviewing the results in [62], where symmetry nonrestoration at infinite temperature (infinite heat order) was first discovered in four dimensions within asymptotically free theories. It generalizes the results of Weinberg [1], which is valid for effective theories, meaning theories with an ultraviolet cutoff. Weinberg's original example [1] features two scalars interacting via a quartic potential with a cross-coupling term that preserves a discrete $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry,

$$V = \frac{\lambda_1}{4} \phi_1^4 + \frac{\lambda_2}{4} \phi_2^4 - \frac{\lambda}{2} \phi_1^2 \phi_2^2. \quad (2.1)$$

For the model to be viable, the potential has to be bounded from below, imposing constraints on the couplings

$$\lambda_{1,2} > 0, \quad \lambda^2 < \lambda_1 \lambda_2. \quad (2.2)$$

At high temperatures, the potential acquires a leading-order thermal correction

$$\Delta V_T = \frac{T^2}{24} ((3\lambda_1 - \lambda)\phi_1^2 + (3\lambda_2 - \lambda)\phi_2^2), \quad (2.3)$$

which for $\lambda > 3\lambda_2$ will ensure that ϕ_2 acquires a negative thermal mass, resulting in a nonzero vacuum expectation value at high temperatures. However, as mentioned above, this model is not UV complete, since the required large cross-coupling increases with the energy scale, eventually reaching a Landau pole.

In this work, we close the gap and show that we can extend asymptotically free models supporting infinite heat order [62] to a finite number of degrees of freedom. The model introduced in [62] consists of the gauge-group product $SU(N_{c1}) \times SU(N_{c2})$, featuring two complex scalar fields φ_i each transforming according to the fundamental representation of their respective gauge group $SU(N_{ci})$ while staying a singlet under the other; the complete field content is summarized in Table I. The most general renormalizable potential is

$$V = \frac{\lambda_1}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)^2 + \frac{\lambda_2}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)^2 - \lambda(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)(\vec{\varphi}_2^* \cdot \vec{\varphi}_2), \quad (2.4)$$

which naturally mimics that of Weinberg's model (2.1) by including the cross-coupling term between the two sectors, which is essential for generating a negative mass squared. The global symmetry over the scalars is still the Weinberg one.

To verify the presence of symmetry nonrestoration, we first consider the effective potential ΔV_T at large N_{ci} . The logic is simple: we impose fixed-flow trajectories, solve the resulting algebraic constraints for the rescaled couplings, and then inspect the sign of the thermal masses. If one of them is negative while the zero-temperature potential remains bounded from below, then the corresponding scalar condenses even at asymptotically high temperature.

In this process, we parametrize the couplings in terms of RG time $t = \log(\mu/\mu_0)$ to enforce the fixed flow solution. Further, the couplings are scaled by N_{ci} , to ensure stable renormalization group equations (RGEs) in the Veneziano limit, and are given by

$$g_i^2 = \frac{16\pi^2 \tilde{\alpha}_i}{N_{ci} t}, \quad \lambda_i = \frac{16\pi^2 \tilde{\lambda}_i}{N_{ci} t}, \quad \lambda = \frac{16\pi^2 \tilde{\lambda}}{\sqrt{N_{c1} N_{c2}} t}, \quad (2.5)$$

for $i = 1, 2$. We further introduce the variables $\tilde{\lambda}_{\pm} = \frac{1}{2}(\tilde{\lambda}_1 \pm \tilde{\lambda}_2)$, and $\tilde{\alpha}_{\pm} = \frac{1}{2}(\tilde{\alpha}_1 \pm \tilde{\alpha}_2)$. Using these variables

TABLE I. Field content of the model. $\square$ denotes the fundamental representation, **adj** the adjoint. The fermions Ψ_i are Dirac and carry N_{fi} flavours each; the scalars φ_i are a single copy.

Field	Spin	$SU(N_{c1})$	$SU(N_{c2})$	Multiplicity
$A_{\mu}^{(1)}$	1	adj	1	1
$A_{\mu}^{(2)}$	1	1	adj	1
φ_1	0	$\square$	1	1
φ_2	0	1	$\square$	1
Ψ_1	$\frac{1}{2}$	$\square$	1	N_{f1}
Ψ_2	$\frac{1}{2}$	1	$\square$	N_{f2}

and enforcing fixed flow trajectories, the RGEs yield the following solutions

$$\tilde{\alpha}_- = 0, \quad \tilde{\lambda}_+ = \frac{6\tilde{\alpha}_+ - 1}{4}, \quad \tilde{\lambda}_-^2 + \tilde{\lambda}^2 = \frac{1}{16}(24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1), \quad (2.6)$$

which are valid for

$$\tilde{\alpha}_+ \geq (3 + \sqrt{3})/12. \quad (2.7)$$

Using positive $\tilde{\lambda} = |\tilde{\lambda}|$ and considering the negative root for $\tilde{\lambda}_-$

$$\tilde{\lambda}_- = -\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16}} - |\tilde{\lambda}|, \quad (2.8)$$

from the effective potential ΔV_T the thermal mass parameters μ_1^2 and μ_2^2 are extracted

$$\mu_1^2 = \frac{12\tilde{\alpha}_+ - 1}{2} - 2 \left(\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16}} - |\tilde{\lambda}|^2 + \sqrt{\frac{N_{c2}}{N_{c1}}} |\tilde{\lambda}| \right), \quad (2.9)$$

$$\mu_2^2 = \frac{12\tilde{\alpha}_+ - 1}{2} + 2 \left(\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16}} - |\tilde{\lambda}|^2 - \sqrt{\frac{N_{c1}}{N_{c2}}} |\tilde{\lambda}| \right). \quad (2.10)$$

Here, the $|\tilde{\lambda}|$ term in μ_1^2 is the crucial one: the negative contribution proportional to $\sqrt{N_{c2}/N_{c1}} |\tilde{\lambda}|$ can dominate for sufficiently small color ratio $x = N_{c1}/N_{c2}$. The large- N analysis, therefore, already isolates the basic mechanism that will survive at finite N : the portal interaction can overcome the positive gauge and quartic contributions in one scalar channel while keeping the potential bounded from below.

Checking the second condition (2.2) for a potential bounded from below $\lambda_1 \lambda_2 - \lambda^2 > 0$ and ensuring real fixed flow solutions, we find a constraint on the ratio of the number of fermion flavours to colors

$$\frac{1}{2}(2 + 3\sqrt{3}) \leq \frac{N_{fi}}{N_{ci}} < \frac{11}{2}. \quad (2.11)$$

Ultimately, this framework successfully yields a UV-complete model where one thermal mass turns negative for a given ratio x . Stability of the vacuum and the requirement for real algebraic solutions to the RGEs necessitate a specific bound on the ratio of fermion flavors to colors, alongside a symmetric gauge structure $\tilde{\alpha}_1 = \tilde{\alpha}_2$. This preserves the broken symmetry at high energies.¹

¹Note that this model also has an IR Banks-Zaks fixed circle. Previous analysis [62] found that symmetry nonrestoration could also be realized near this IR fixed point.

The analytical success of the model with two scalars, respectively, gauged under $SU(N_{c1}) \times SU(N_{c2})$, was achieved by simplifications of the Veneziano limit. This limit decoupled and simplified the RGEs. A relevant question to ask is whether the characteristic of negative thermal mass persists beyond this limit. Thus, in the next Section, we will identify the parameter space for a negative thermal mass in the case of finite N .

B. Finite number of degrees of freedom

Having established the general conditions under which infinite heat order can arise, we now turn to the more restrictive and physically relevant case in which the number of degrees of freedom is finite. This setting is important for two reasons. First, it shows that the phenomenon is not tied to large- N idealizations or to parametrically large field multiplicities. Second, it allows one to identify explicitly the minimal structures required for the persistence of ordered phases at arbitrarily high temperatures. The analysis in this Section, therefore, serves to clarify how much of the mechanism survives once the theory is reduced to a genuinely finite field content.

Following the derivation in Appendix A 1 and introducing the parameter x through $N_{c1} = xN_{c2}$, the finite- N RG equations take the form

$$\begin{aligned} -\tilde{\lambda}_1 &= \left(2 + \frac{8}{xN_{c2}}\right)\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 - 6\left(1 - \frac{1}{x^2N_{c2}^2}\right)\tilde{\alpha}_1\tilde{\lambda}_1 \\ &\quad + \frac{3}{2}\tilde{\alpha}_1^2\left(1 + \frac{1}{xN_{c2}} - \frac{4}{x^2N_{c2}^2} + \frac{2}{x^3N_{c2}^3}\right), \\ -\tilde{\lambda}_2 &= \left(2 + \frac{8}{N_{c2}}\right)\tilde{\lambda}_2^2 + 2\tilde{\lambda}^2 - 6\left(1 - \frac{1}{N_{c2}^2}\right)\tilde{\alpha}_2\tilde{\lambda}_2 \\ &\quad + \frac{3}{2}\tilde{\alpha}_2^2\left(1 + \frac{1}{N_{c2}} - \frac{4}{N_{c2}^2} + \frac{2}{N_{c2}^3}\right), \\ -\tilde{\lambda} &= \tilde{\lambda}\left[-\frac{4}{N_{c2}\sqrt{x}}\tilde{\lambda} + 2\left(1 + \frac{1}{xN_{c2}}\right)\tilde{\lambda}_1 + 2\left(1 + \frac{1}{N_{c2}}\right)\tilde{\lambda}_2\right. \\ &\quad \left.- 3\left(\tilde{\alpha}_1\left(1 - \frac{1}{x^2N_{c2}^2}\right) + \tilde{\alpha}_2\left(1 - \frac{1}{N_{c2}^2}\right)\right)\right]. \end{aligned} \quad (2.12)$$

Moreover, the finite- N thermal potential is

$$\begin{aligned} \Delta V_T &= (4\pi)^2 \frac{T^2}{24 \log T} \left\{ \left[2\left(\tilde{\lambda}_1\left(1 + \frac{1}{N_{c1}}\right) - \sqrt{\frac{N_{c2}\tilde{\lambda}}{N_{c1}}}\right) + 3\tilde{\alpha}_1\left(1 - \frac{1}{N_{c1}^2}\right) \right] (\vec{\varphi}_1^* \cdot \vec{\varphi}_1) \right. \\ &\quad \left. + \left[2\left(\tilde{\lambda}_2\left(1 + \frac{1}{N_{c2}}\right) - \sqrt{\frac{N_{c1}\tilde{\lambda}}{N_{c2}}}\right) + 3\tilde{\alpha}_2\left(1 - \frac{1}{N_{c2}^2}\right) \right] (\vec{\varphi}_2^* \cdot \vec{\varphi}_2) \right\}, \end{aligned} \quad (2.13)$$

yielding quadratic thermal masses

$$\mu_i^2 = 3\tilde{\alpha}_i\left(1 - \frac{1}{N_{ci}^2}\right) + 2\tilde{\lambda}_i\left(\frac{1}{N_{ci}} + 1\right) - 2\sqrt{\frac{N_{cj}\tilde{\lambda}}{N_{ci}}}, \quad (2.14)$$

for $(i, j) = (1, 2), (2, 1)$.

These expressions make the finite- N problem concrete. Once the algebraic RG constraints are solved, the sign of μ_i^2 can be read off directly from (2.14). The strategy in the remainder of this Section is therefore to proceed in two complementary ways: first by constructing the leading $1/N_{ci}$ deformation of the Veneziano-limit solution, and then by solving the full finite- N equations numerically.

1. Symmetry nonrestoration at NLO

The goal of this Subsection is to exhibit analytically how the relevant thermal contributions reorganize themselves in a finite system, and to determine under which conditions the ordered phase can be maintained as the temperature increases. This provides the simplest concrete realization of infinite heat order at finite N , and it will serve as a useful benchmark for the more general constructions discussed

below. Our roadmap is the following: we expand the couplings around the Veneziano-limit solution, solve the $1/N_{ci}$ algebraic constraints, and finally insert the result into the thermal mass in order to determine whether the ordered phase persists. In the finite N_{ci} system of Eqs. (2.12), we redefine the couplings defined in Sec. II A by separating their leading order (LO) and next leading order (NLO) terms, i.e.,

$$\begin{aligned} \tilde{\lambda}_\pm &= \tilde{\lambda}_\pm^{(0)} + \frac{1}{N_{c2}}\tilde{\lambda}_\pm^{(1)}, \\ \tilde{\lambda} &= \tilde{\lambda}^{(0)} + \frac{1}{N_{c2}}\tilde{\lambda}^{(1)}, \\ \tilde{\alpha}_\pm &= \tilde{\alpha}_\pm^{(0)} + \frac{1}{N_{c2}}\tilde{\alpha}_\pm^{(1)}. \end{aligned} \quad (2.15)$$

Here the color ratio $x = N_{c1}/N_{c2}$ is kept fixed, while $1/N_{c2}$ is the actual expansion parameter. By plugging these expressions into the finite- N RG equations and truncating at order $1/N_{c2}$, the coefficients $\tilde{\lambda}^{(0)}, \tilde{\lambda}_\pm^{(0)}, \tilde{\alpha}_\pm^{(0)}$ are identified with the large- N solutions (2.6)–(2.8) of the LO equations (A3)–(A5). The $1/N_{c2}$ terms then determine the

NLO corrections $\tilde{\lambda}^{(1)}, \tilde{\lambda}_{\pm}^{(1)}, \tilde{\alpha}_{\pm}^{(1)}$, shown in Eqs. (A6)–(A8). From these, we find

$$\tilde{\alpha}_{-}^{(1)} = \frac{1}{12x(4\tilde{\alpha}_{+}^{(0)} - 1)} \left[3 - 16\tilde{\lambda}^{(0)2} + 3K - 8\sqrt{x}\tilde{\lambda}^{(0)}K + 6\tilde{\alpha}_{+}^{(0)}(17\tilde{\alpha}_{+}^{(0)} - 3(2K)) + x(-3 + 16\tilde{\lambda}^{(0)2} + 3K - 6\tilde{\alpha}_{+}^{(0)}(-6 + 17\tilde{\alpha}_{+}^{(1)} + 3K)) \right], \quad (2.16)$$

$$\tilde{\lambda}_{+}^{(1)} = \frac{1}{8x} \left[1 + x - 6(1 + x)\tilde{\alpha}_{+}^{(0)} + 12x\tilde{\alpha}_{+}^{(1)} + 8\sqrt{x}\tilde{\lambda}_{+}^{(0)} + (1 - x)K \right], \quad (2.17)$$

$$\begin{aligned} \tilde{\lambda}_{-}^{(1)} = & \frac{1}{4xK} \left[63\tilde{\alpha}_{+}^{(0)2} - 12\tilde{\alpha}_{+}^{(0)}(2 + K) + 2(1 - 8\tilde{\lambda}^{(0)2} + K) \right. \\ & + x(2 + 6\tilde{\alpha}_{+}^{(1)} - 2K + 6\tilde{\alpha}_{-}^{(1)}K \\ & + 3\tilde{\alpha}_{+}^{(0)}(-8 + 21\tilde{\alpha}_{+}^{(0)} - 8\tilde{\alpha}_{+}^{(1)} + 4K) \\ & \left. + 16\tilde{\lambda}^{(0)}(\tilde{\lambda}^{(1)} - \tilde{\lambda}^{(0)}) \right], \end{aligned} \quad (2.18)$$

where $K = \sqrt{1 + 12\tilde{\alpha}_{+}^{(0)}(2\tilde{\alpha}_{+}^{(0)} - 1) - 16\tilde{\lambda}^{(0)2}}$.

The final step to see that symmetry nonrestoration persists with a finite number of fields is to plug the LO and NLO solutions into the thermal mass (2.14) and truncate at order $1/N_{c2}$. As an example, we consider the thermal mass μ_1^2 . Without loss of generality, we set $\tilde{\alpha}_{+}^{(1)} = \tilde{\lambda}^{(1)} = 0$, as allowed by the RG equations, and therefore drop the superscripts on $\tilde{\alpha}_{+}^{(0)}$ and $\tilde{\lambda}^{(0)}$. This gives

$$\begin{aligned} \mu_1^2 = & 3\tilde{\alpha}_{+} - \frac{2\tilde{\lambda}}{\sqrt{x}} + \frac{1}{2x^2N_{c2}^2(4\tilde{\alpha}_{+} - 1)K} \{ (1 + xN_{c2})[-1 - 12(x - 7)\tilde{\alpha}_{+}^3 - K \\ & - xN_{c2}(4\tilde{\alpha}_{+} - 1)(1 + 24\tilde{\alpha}_{+}^2 - 16\tilde{\lambda}^2 + K - 6\tilde{\alpha}_{+}(2 + K)) + 4\tilde{\lambda}(-2\tilde{\lambda}K \\ & + 2x\tilde{\lambda}(K - 2) - \sqrt{x}(1 - 16\tilde{\lambda}^2 + K)) + \tilde{\alpha}_{+}(13 + 48\tilde{\lambda}^2 + 7K + 16\sqrt{x}\tilde{\lambda}(K + 3) \\ & + x(-3 + 112\tilde{\lambda}^2 + 3K)) - 3\tilde{\alpha}_{+}^2(17 + 32\sqrt{x}\tilde{\lambda} + 3K + x(-7 + 5K))] \}. \end{aligned} \quad (2.19)$$

In Fig. 1, we show that there exists a region in the $\tilde{\alpha}_{+} - \tilde{\lambda}$ parameter space where this thermal mass becomes negative.

2. On the lower bound on the color degrees of freedom

Having established that symmetry nonrestoration survives at subleading order in the inverse number of colors, we now determine, within perturbation theory, the lowest

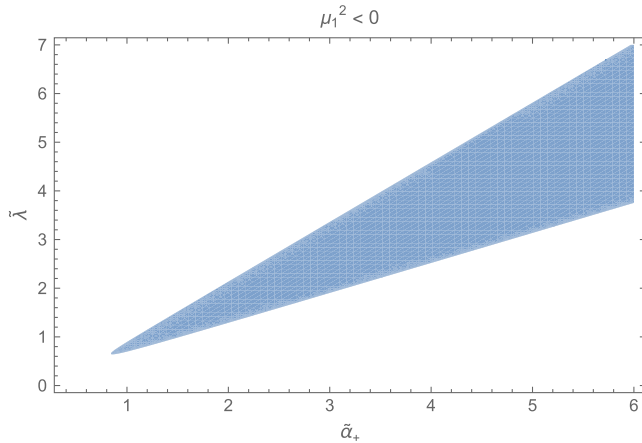


FIG. 1. Shaded region shows where in the $\tilde{\alpha}_{+} - \tilde{\lambda}$ parameter space the squared mass of φ_1 is negative. Here we set $\tilde{\alpha}_{+}^{(1)} = \tilde{\lambda}^{(1)} = 0$ (as allowed by the RG equations), $x = 0.1$ and $N_{c2} = 1000$.

possible bound on N_{c1} and N_{c2} below which symmetry must restore at high temperature. To analyze this situation, we recall the equations guaranteeing UV freedom in Eq. (2.12) at a finite number of colors as well as that the leading high temperature one-loop correction to the potential is (2.13), with the dimensionless thermal masses given in (2.14). The symmetry nonrestoration condition implies

$$\mu_1^2 \equiv -m^2 \leq 0 \quad (2.20)$$

where, to ease notation, we introduced the positive m^2 parameter and start computing $\tilde{\lambda}$ from μ_1^2 in Eq. (2.14). This yields,

$$\tilde{\lambda} = \sqrt{\frac{N_{c1}}{N_{c2}}} \left(\frac{m^2}{2} + \tilde{\lambda}_1 \left(1 + \frac{1}{N_{c1}} \right) + \frac{3}{2}\tilde{\alpha}_1 \left(1 - \frac{1}{N_{c1}^2} \right) \right). \quad (2.21)$$

We then substitute it into the first RG equation in the system (2.12). It is useful to transform the variables $\tilde{\lambda}_1, \tilde{\alpha}_1$ into ℓ_1, a_1 via

$$\begin{aligned} \tilde{\lambda}_1 &= \ell_1 - a(N_{c1}, N_{c2})a_1 - b(N_{c1}, N_{c2}, m^2), \\ \tilde{\alpha}_1 &= a_1 + c(N_{c1}, N_{c2}, m^2), \end{aligned} \quad (2.22)$$

with a, b, c functions constructed in the following way, such that we can eliminate $\ell_1 a_1, \ell_1$ and a_1 terms in the first RG equation of (2.12),

$$a(N_{c1}, N_{c2}) = \frac{3(N_{c1}^2 - 1)(N_{c1} - N_{c2} + 1)}{2N_{c1}((N_{c1} + 1)^2 + (N_{c1} + 4)N_{c2})}, \quad (2.23)$$

$$b(N_{c1}, N_{c2}, m^2) = \frac{N_{c1}(N_{c1} + 1)(2m^2(4N_{c1}^2 + 2N_{c1} - 5) + 3N_{c1}^2 - 3) + N_{c1}(N_{c1}(N_{c1} + 2) - 2)N_{c2}}{4N_{c1}(9N_{c2} + N_{c1}(N_{c1}(10N_{c1} - 2N_{c2} + 31) + 3(N_{c2} + 4)) - 29) - 20(N_{c2} + 4)}, \quad (2.24)$$

$$c(N_{c1}, N_{c2}, m^2) = -\frac{N_{c1}^2(N_{c1} + 1)(2m^2(2N_{c1} + 5) - N_{c1} + N_{c2} - 1)}{2N_{c1}(9N_{c2} + N_{c1}(N_{c1}(10N_{c1} - 2N_{c2} + 31) + 3(N_{c2} + 4)) - 29) - 10(N_{c2} + 4)}. \quad (2.25)$$

Therefore the first RG equation in the system (2.12) now reads

$$c_{\ell_1}(N_{c1}, N_{c2})\ell_1^2 + c_{a_1}(N_{c1}, N_{c2})a_1^2 = c_1(N_{c1}, N_{c2}, m^2), \quad (2.26)$$

with

$$c_{\ell_1}(N_{c1}, N_{c2}) = \frac{(N_{c1} + 1)^2 + (N_{c1} + 4)N_{c2}}{N_{c1}N_{c2}}, \quad (2.27)$$

$$c_{a_1}(N_{c1}, N_{c2}) = \frac{3(N_{c1} - 1)((N_{c1}(10N_{c1} + 11) - 20)(N_{c1} + 1)^2 + (N_{c1}((3 - 2N_{c1})N_{c1} + 9) - 5)N_{c2})}{4N_{c1}^3((N_{c1} + 1)^2 + (N_{c1} + 4)N_{c2})}, \quad (2.28)$$

$$c_1(N_{c1}, N_{c2}, m^2) = \frac{c_1^u(N_{c1}, N_{c2}, m^2)}{c_1^d(N_{c1}, N_{c2})}, \quad (2.29)$$

$$c_1^u(N_{c1}, N_{c2}, m^2) = N_{c1}(4m^4(2N_{c1} - 1)((N_{c1} - 1)N_{c1} - 5) + 4m^2(N_{c1} + 1)(4N_{c1}^2 + 2N_{c1} - 5)) + 3(N_{c1} - 1)(N_{c1} + 1)^2 + (N_{c1}(N_{c1} + 2) - 2)N_{c2}, \quad (2.30)$$

$$c_1^d(N_{c1}, N_{c2}) = 16N_{c1}(9N_{c2} + N_{c1}(N_{c1}(10N_{c1} - 2N_{c2} + 31) + 3(N_{c2} + 4)) - 29) - 80(N_{c2} + 4). \quad (2.31)$$

The solution of the quadratic equation in (2.26) can be written as

$$\ell_1 = \sqrt{\frac{c_1}{c_{\ell_1}}} \cos \phi, \quad (2.32)$$

$$a_1 = \sqrt{\frac{c_1}{c_{a_1}}} \sin \phi. \quad (2.33)$$

Additionally, the solution should further satisfy the constraints

$$\tilde{\lambda}_1 > 0 \rightarrow \sqrt{\frac{c_1}{c_{\ell_1}}} \cos \phi - a \sqrt{\frac{c_1}{c_{a_1}}} \sin \phi - b > 0, \quad (2.34)$$

$$\tilde{\alpha}_1 > 0 \rightarrow \sqrt{\frac{c_1}{c_{a_1}}} \sin \phi + c > 0. \quad (2.35)$$

We have that (2.34) can be satisfied if

$$c_1 \left(\frac{1}{c_{\ell_1}} + \frac{a^2}{c_{a_1}} \right) - b^2 = -\frac{m^4 N_{c1}^2 (N_{c1}^2 + 2N_{c1} - 2)}{4(10N_{c1}^4 + (31 - 2N_{c2})N_{c1}^3 + 3(N_{c2} + 4)N_{c1}^2 + (9N_{c2} - 29)N_{c1} - 5(N_{c2} + 4))} \geq 0. \quad (2.36)$$

This can be satisfied when

$$N_{c2} \geq \frac{10N_{c1}^4 + 31N_{c1}^3 + 12N_{c1}^2 - 29N_{c1} - 20}{2N_{c1}^3 - 3N_{c1}^2 - 9N_{c1} + 5}. \quad (2.37)$$

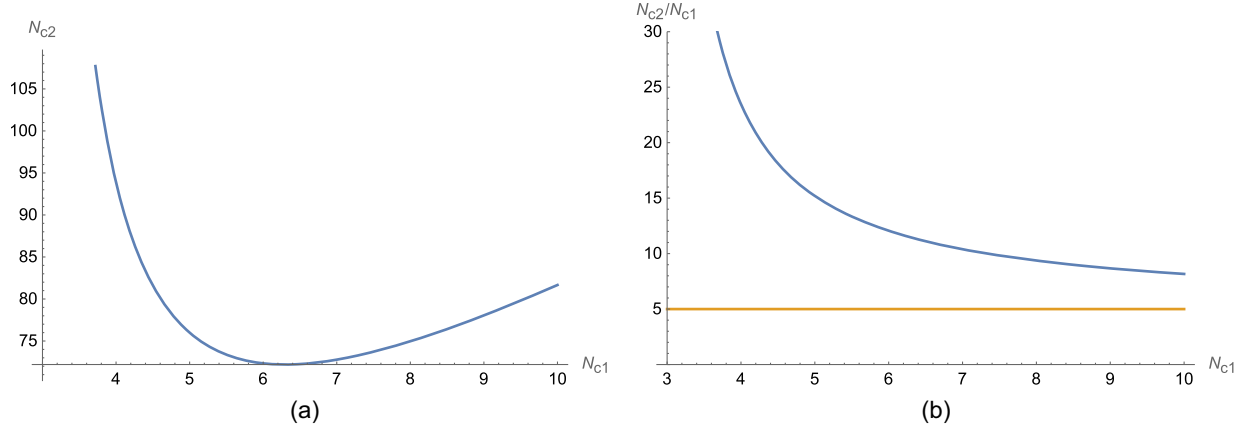


FIG. 2. (a) The necessary condition for symmetry non-restoration to happen in the theory is that the ratio N_{c2}/N_{c1} lies above the blue curve. (b) The necessary condition for the solution to exist is that N_{c2} is above the blue curve. The yellow line denotes the lowest possible bound for N_{c2}/N_{c1} occurring for $N_{c1} \rightarrow \infty$.

In this range we have $a < 0$ and $c > 0$, so that there is a simultaneous solution for (2.34) and (2.35) when ϕ lies in the first quadrant. We observe that (2.37) implies that $N_{c2} \geq 5N_{c1}$ with the equality recovered in the limiting case of $N_{c1} \rightarrow \infty$ as shown in Fig. 2(a).

In Fig. 2(b) we show the lower bound stemming from (2.37) from which it is clear that the lowest possible integer value for N_{c2} is $(N_{c2})_{\min} = 73$ for either $N_{c1} = 6$ or $N_{c1} = 7$.

We can therefore conclude that any color ratio with $x < 5$ cannot support symmetry nonrestoration. Among the possible pairs of N_{c1} and N_{c2} satisfying (2.37) only those that further abide

will be physical solutions. We will now move to the numerical evaluation of the constraints to identify which physical theory supports symmetry nonrestoration.

3. Numerical results

We next extend the analysis numerically to achieve a wider class of finite degrees of freedom theories, within the same class of models, in order to assess the robustness of the mechanism. In particular, this Subsection is meant to show which features survive at a smaller number of colors and flavors. This comparison is essential for bridging the gap between analytic results and possible phenomenological future applications.

We start our analysis here by defining

$$\tilde{\lambda}_2 > 0, \quad \tilde{\alpha}_2 > 0, \quad \tilde{\lambda}_1 \tilde{\lambda}_2 > \tilde{\lambda}^2, \quad (2.38)$$

$$\tilde{\alpha}_{1,2} = \frac{1}{2b_{1,2}}, \quad b_{1,2} = \frac{11}{3} - \frac{1}{6N_{c1,2}} - \frac{2N_{f1,2}}{3N_{c1,2}}. \quad (2.39)$$

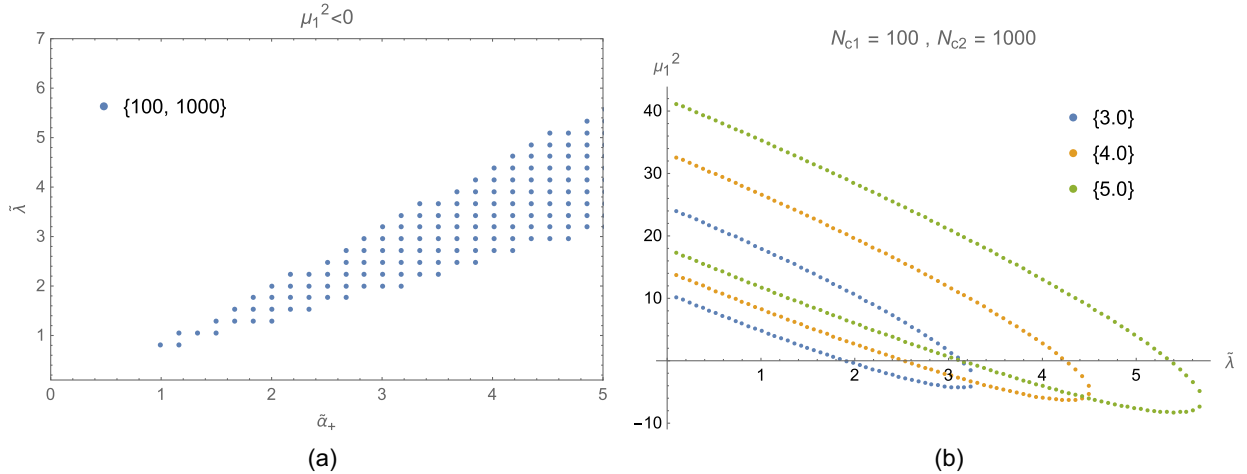


FIG. 3. (a) Region in the $\tilde{\alpha}_+ - \tilde{\lambda}$ plane with negative thermal mass square for $(N_{c1}, N_{c2}) = (100, 1000)$. This is in agreement with Fig. 1. (b) Mass square as a function of $\tilde{\lambda}$. Different colors represent different values of $\tilde{\alpha}_+$. For each $\tilde{\alpha}_+$ and $\tilde{\lambda}$ there are two solutions for the other parameters $\tilde{\alpha}_-, \tilde{\lambda}_{1,2}$.

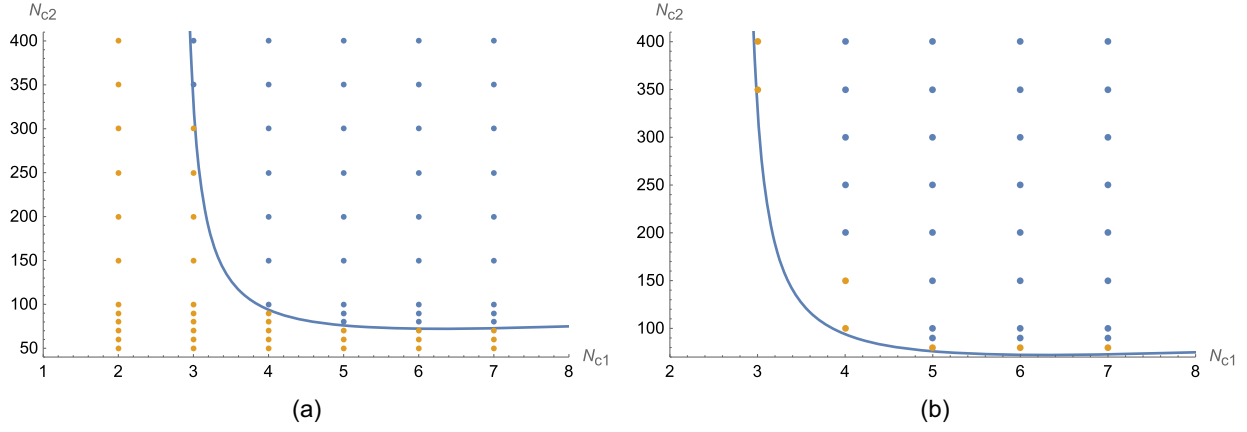


FIG. 4. (a) Regions in the $N_{c1} - N_{c2}$ plane with (blue dots) and without (yellow dots) a solution with symmetry nonrestoration. The boundary between them is well described by the constraint (2.37). Here $\tilde{\alpha}_{1,2}$ are arbitrary real numbers, not $1/(2b_{1,2})$. (b) Taking into account (2.39) and varying integers $N_{f1,2}$, we plot the possible solutions above the blue continuous curve in panel (a). We see that some blue points, which represented allowed solutions before, become yellow, i.e., they are not solutions. This happens near the lower boundary.

We look for solutions of the finite N RGEs providing

$$\tilde{\lambda}_{1,2} > 0, \quad |\tilde{\alpha}_-| \leq \tilde{\alpha}_+, \quad \tilde{\lambda}_1 \tilde{\lambda}_2 > \tilde{\lambda}^2. \quad (2.40)$$

The solutions are shown in Fig. 3.

We still have to find explicitly $N_{f1,2}$ for which there is such a solution. For infinite N_c , this is always possible since N_f/N_c is continuous, but in our case $N_{f1,2}$ are integers. As a benchmark point, we find,

$$(N_{c1}, N_{c2}, N_{f1}, N_{f2}, \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}) \\ = (100, 1000, 534, 5342, 2.74077, 11.0171, 3.57732), \quad (2.41)$$

for which $\tilde{\alpha}_+ = 63050/13251 = 4.75813$, $\tilde{\alpha}_- = 50/13251 = 0.0037733$ and

$$(\mu_1^2, \mu_2^2) = (-2.80429, 34.0568). \quad (2.42)$$

The resulting regions in the $N_{c1} - N_{c2}$ plane admitting solutions with symmetry nonrestoration are displayed in Fig. 4(a), where the boundary is well described by the analytical constraint (2.37). Once the additional restriction to integer flavor numbers is imposed, some of the allowed points are lost, as shown in Fig. 4(b).

III. CONCLUSIONS

We studied whether symmetry nonrestoration at arbitrarily high temperature can occur in an ultraviolet-complete quantum field theory in four spacetime dimensions with a finite field content. The key point is that the strict $T \rightarrow \infty$ limit is meaningful only if the microscopic theory remains valid at arbitrarily short distances. This naturally

leads to the study of completely asymptotically free theories.

Our starting point was the large- N analysis of the $SU(N_{c1}) \times SU(N_{c2})$ model with two scalar fundamentals and a quartic cross-coupling. In the Veneziano limit, fixed-flow solutions were previously shown to exist for which one scalar acquires a negative thermal mass at high temperature, thereby realizing symmetry nonrestoration. In this work, we asked whether this mechanism survives when the number of degrees of freedom is kept finite.

We derived the finite- N fixed-flow equations and the corresponding thermal masses, and then analyzed the system in two complementary ways. First, we expanded around the large- N solution and obtained the leading $1/N$ corrections analytically. This shows that the region of parameter space with negative thermal mass is not an artifact of the Veneziano limit. Second, we solved the finite- N equations numerically and identified explicit benchmark points with integer numbers of colors and flavors. These benchmarks satisfy asymptotic freedom and boundedness of the scalar potential while still exhibiting a negative thermal mass for one scalar field.

Our results, therefore, provide an explicit perturbative realization of infinite heat order in a four-dimensional ultraviolet-complete quantum field theory with finite field content. The essential ingredient is the product gauge-group structure, which gives enough parametric freedom for complete asymptotic freedom and symmetry nonrestoration to coexist. This sharply distinguishes the model from simpler single-gauge-factor constructions, where the corresponding constraints are typically too restrictive.

A natural question concerns the role of higher-loop corrections. In an asymptotically free theory, all couplings vanish as $1/\log T$ at high temperature. The one-loop fixed-flow analysis therefore captures the leading behavior, and

n -loop corrections to both the beta functions and the thermal effective potential enter at relative order $\tilde{\alpha}^{n-1} \sim 1/(\log T)^{n-1}$, which is parametrically suppressed in the $T \rightarrow \infty$ limit. As is standard in quantum field theory, the perturbative series is expected to be asymptotic rather than convergent. However, this does not affect our conclusions: what matters is that, for sufficiently high temperature, the coupling is arbitrarily small and any finite truncation of the series provides a controlled approximation. In particular, the sign of the thermal mass squared is determined at one loop and is a continuous function of the couplings; higher-loop contributions cannot reverse it once the effective expansion parameter becomes small enough.

What about gravity? Gravitational corrections are generally expected to become important near the Planck scale and to modify the running of the couplings, typically strengthening the asymptotic freedom of gauge and Yukawa interactions, while the fate of scalar self-couplings remains dependent on the specific ultraviolet completion of gravity, as summarized in [72]. If, however, the underlying particle theory is already completely asymptotically free below the Planck scale, it is natural to expect it to enjoy an enhanced degree of robustness against such Planckian effects. From this viewpoint, the theories studied here, when interpreted as dark sectors coupled only through gravity, are expected to preserve their infinite heat order up to, and plausibly slightly beyond, the Planck scale, until a complete theory of quantum gravity takes over.

The framework presented here also suggests several directions for future work. It would be valuable to classify more systematically the finite- N theories that exhibit this behavior, to assess whether analogous mechanisms can be realized in phenomenologically motivated settings, and to explore the possible cosmological implications of dark and visible sectors that become increasingly ordered at high temperature.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: THE ONE-LOOP RG EQUATIONS

1. $SU(N_{c1}) \times SU(N_{c2})$ with two scalar fundamentals

The model under consideration has the gauge symmetry $SU(N_{c1}) \times SU(N_{c2})$, where each scalar field φ_i transforms

in the fundamental representation of its respective $SU(N_{ci})$ and is a singlet under the other. The most general potential is

$$V = \frac{\lambda_1}{2} (\vec{\varphi}_1^* \cdot \vec{\varphi}_1)^2 + \frac{\lambda_2}{2} (\vec{\varphi}_2^* \cdot \vec{\varphi}_2)^2 - \lambda (\vec{\varphi}_1^* \cdot \vec{\varphi}_1) (\vec{\varphi}_2^* \cdot \vec{\varphi}_2). \quad (\text{A1})$$

To derive the one-loop RGEs, we follow the approach established in [62], to which we refer for the details.

Again, to ensure that all the couplings are asymptotically free, i.e., that the model exhibits complete asymptotic freedom (CAF), we parametrize the couplings with a factor $1/t$

$$i=1,2: g_i^2 = \frac{16\pi^2 \tilde{\alpha}_i}{N_{ci}t}, \quad \lambda_i = \frac{16\pi^2 \tilde{\lambda}_i}{N_{ci}t}, \quad \lambda = \frac{16\pi^2 \tilde{\lambda}}{\sqrt{N_{c1}N_{c2}}t}. \quad (\text{A2})$$

This ensures that the couplings all vanish at the same rate in the UV, a behavior known as fixed flow [73]. With this parametrization, the RGEs reduce to a set of coupled polynomial equations, which for the model here discussed are the ones in Eq. (2.12). Real solutions of these algebraic equations correspond to models exhibiting CAF, as long as the gauge coupling is asymptotically free, i.e., $b_0 > 0$. For the model to be physically viable, it must also satisfy (2.2) such that its potential is bounded from below.

Plugging in Eqs. (2.12), the coupling redefinitions (2.15), and solving perturbatively order by order, we have that the LO couplings must satisfy the system

$$-\tilde{\lambda}_1^{(0)} = 2\tilde{\lambda}_1^{(0)2} + 2\tilde{\lambda}^{(0)2} - 6\tilde{\alpha}_1^{(0)}\tilde{\lambda}_1^{(0)} + \frac{3}{2}\tilde{\alpha}_1^{(0)2}, \quad (\text{A3})$$

$$-\tilde{\lambda}_2^{(0)} = 2\tilde{\lambda}_2^{(0)2} + 2\tilde{\lambda}^{(0)2} - 6\tilde{\alpha}_2^{(0)}\tilde{\lambda}_2^{(0)} + \frac{3}{2}\tilde{\alpha}_2^{(0)2}, \quad (\text{A4})$$

$$-\tilde{\lambda}^{(0)} = 2(\tilde{\lambda}_1^{(0)} + \tilde{\lambda}_2^{(0)})\tilde{\lambda}^{(0)} - 3(\tilde{\alpha}_1^{(0)} + \tilde{\alpha}_2^{(0)})\tilde{\lambda}^{(0)}, \quad (\text{A5})$$

which is the system solved for the infinite- N case in [62]. On the other hand, the NLO couplings will need to satisfy the system of equations

$$\begin{aligned} &39\tilde{\alpha}_+^{(0)2} + (1 - 4\tilde{\lambda}_-^{(0)})^2 - 12\tilde{\alpha}_+^{(0)}(1 - 4\tilde{\lambda}_-^{(0)}) \\ &+ x[1 + 39\tilde{\alpha}_+^{(0)2} + 6\tilde{\alpha}_+^{(1)2} + 16\tilde{\lambda}^{(0)}\tilde{\lambda}^{(1)} \\ &- 12\tilde{\alpha}_+^{(0)}(1 + 2\tilde{\alpha}_+^{(1)} + 4\tilde{\lambda}_-^{(0)}) \\ &+ 8\tilde{\lambda}_-^{(0)}(1 - 3\tilde{\alpha}_-^{(1)} + 2\tilde{\lambda}_-^{(0)} + 2\tilde{\lambda}_-^{(1)})] = 0, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} &39\tilde{\alpha}_+^{(0)2} + (1 - 4\tilde{\lambda}_-^{(0)})^2 - 12\tilde{\alpha}_+^{(0)}(1 - 4\tilde{\lambda}_-^{(0)}) \\ &- x[1 + 39\tilde{\alpha}_+^{(0)2} + 6\tilde{\alpha}_+^{(1)2}(4\tilde{\alpha}_+^{(0)} - 1) - 12\tilde{\alpha}_+^{(0)}(1 + 4\tilde{\lambda}_-^{(0)}) \\ &+ 8\tilde{\lambda}_-^{(0)}(1 + 3\tilde{\alpha}_+^{(1)} + 2\tilde{\lambda}_-^{(0)} - 2\tilde{\lambda}_+^{(1)})] = 0, \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} &-1 + 6\tilde{\alpha}_+^{(0)} - 8\sqrt{x}\tilde{\lambda}^{(0)} + 4\tilde{\lambda}_-^{(0)} \\ &+ x(-1 + 6\tilde{\alpha}_+^{(0)} - 12\tilde{\alpha}_+^{(1)} - 4\tilde{\lambda}_-^{(0)} + 8\tilde{\lambda}_+^{(1)}) = 0. \end{aligned} \quad (\text{A8})$$

The thermal effective potential is given by

$$\Delta V_T = \frac{T^2}{48} \sum_{i=1}^2 \sum_{a=1}^{N_{ci}} \left(4 \frac{\partial^2 V}{\partial \varphi_i^a \partial \varphi_{ia}^*} + 6 g_i^2 \frac{N_{ci}^2 - 1}{N_{ci}} \varphi_i^a \varphi_{ia}^* \right), \quad (\text{A9})$$

which for the potential (A1) yields

$$\begin{aligned} \Delta V_T = (4\pi)^2 \frac{T^2}{24 \log T} & \left(\left(2 \left(\tilde{\lambda}_1 \left(1 + \frac{1}{N_{c1}} \right) - \sqrt{\frac{N_{c2} \tilde{\lambda}}{N_{c1}}} \right) + 3 \tilde{\alpha}_1 \left(1 - \frac{1}{N_{c1}^2} \right) \right) (\vec{\varphi}_1^* \cdot \vec{\varphi}_1) \right. \\ & \left. + \left(2 \left(\tilde{\lambda}_2 \left(1 + \frac{1}{N_{c2}} \right) - \sqrt{\frac{N_{c1} \tilde{\lambda}}{N_{c2}}} \right) + 3 \tilde{\alpha}_2 \left(1 - \frac{1}{N_{c2}^2} \right) \right) (\vec{\varphi}_2^* \cdot \vec{\varphi}_2) \right). \end{aligned} \quad (\text{A10})$$

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