# Radiative corrections on vortical spin polarization in hot QCD matter

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We investigate the radiative corrections on spin polarization of relativistic fermions induced by vortical fields in thermal-equilibrium QCD matter at weak coupling. Such corrections stem from the self-energy gradients in quantum kinetic theory, which are further obtained by a more systematic and general approach through the Keldysh equation. By applying the hard-thermal-loop approximation, we obtain new corrections upon the spin-polarization spectrum and also the axial-charge current in connection to the axial/chiral vortical effect for massive quarks up to the leading order of the QCD coupling. Further influence on spin alignment of vector mesons from similar effects is also analyzed.

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## I. INTRODUCTION

Relativistic heavy-ion collisions produce hot, dense matter governed by strong interactions, known as the quark-gluon plasma (QGP) [1,2]. This setting provides a platform for studying the properties of quantum chromodynamics (QCD) matter under extreme conditions, such as strong electromagnetic fields [3–5], strong vorticity [6–8], and anomalous transport [9]. One of the promising probes in this context is the spin observables. The spin polarization of hyperons reveals that the QGP is a fast-rotating system [6,10], while its momentum spectrum indicates an interplay between the fine structure of local vorticity and shear stress within the QGP [11–15]. Recent measurements of vector-meson spin alignment [16,17] further underscore the influence of strong force fields and their correlations [18–21].

Based on quantum field theory, quantum kinetic theory (QKT) [22–31] in terms of Wigner functions provides a unified framework to depict the microscopic dynamics of various degrees of freedom of particles, including particle number and spin. The perturbative solution of QKT can directly describe spin effects in the form of modified Cooper-Frye formulas [32,33] and the spin density matrix [34,35], provided the spin kinetic equations are solved. For example, in local equilibrium the spin polarization induced by vorticity and shear viscous tensor are explicitly derived [14,36].

Despite the success in probing macroscopic properties such as the vorticity of the QGP through global polarization, an intriguing and important question remains: How can we probe the microscopic properties associated with the strong interactions of QCD matter through spin transport? Related studies have investigated the spin alignment of vector mesons influenced by fluctuating glasma fields [21,37,38], turbulent color fields [20,39], and vector-meson fields [18,19,40]. Moreover, the nonequilibrium corrections dependent on interactions for local spin-polarization spectra have been extensively explored in recent years [36,41–51]. However, even under equilibrium conditions, interaction-dependent corrections on spin polarization may exist, as suggested by the radiative corrections to the axial-charge current under vorticity for massless fermions [52,53], known as the axial/chiral vortical effect (AVE/CVE) [54–56]. In QKT, such radiative corrections could potentially stem from self-energy corrections rather than collisions at weak coupling [46,57]. Nonetheless, obtaining a complete perturbative solution for general spin-$j$ Wigner functions is technically challenging [58], especially concerning their off-equilibrium and interaction corrections.

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In this paper, we investigate radiative corrections to the spin polarization spectrum induced by vorticity in thermal equilibrium, utilizing chiral kinetic theory for massless fermions and a novel approach based on the Keldysh equation for fermions of arbitrary mass. We further implement this theoretical framework to study quarks in a weakly coupled QCD environment using the hard-thermal-loop (HTL) approximation and also explore the axial-charge current. A similar study employing different methods and approximations for QED plasmas has been presented in Ref. [59].

The paper is organized as follows: We first review the chiral kinetic theory with self-energy corrections and propose the global equilibrium form of the interaction-corrected chiral Wigner functions in Sec. II. Then we propose the Keldysh equation framework as an off-equilibrium analog of the Schwinger-Dyson equation in the phase space in Sec. III. After that we present the follow-up applications of the Keldysh equation, calculating radiative corrections on spin polarization and AVE for massive fermions in Sec. IV and a similar contribution to vector-meson spin alignment in Sec. V. A summary and outlook are given in Sec. VI.

We utilize the natural unit by setting $\hbar = c = k_{\mathrm{B}} = 1$. We work in the Minkowski spacetime with mostly minus metric $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and use the notation $\epsilon^{0123} = \epsilon^{123} = +1$. We introduce the (anti)symmetric symbol

$$A_{(\mu\nu)} = \frac{A_{\mu\nu} + A_{\nu\mu}}{2}, \qquad A_{[\mu\nu]} = \frac{A_{\mu\nu} - A_{\nu\mu}}{2},$$

and the projection operator $\Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$ with $u^\mu$ a normalized timelike vector.

## II. CHIRAL KINETIC THEORY AND ITS GLOBAL EQUILIBRIUM FORM

The gauge-invariant Wigner functions for chiral fermions, with $\chi = \pm$ representing right- (R) and left-handed (L), respectively, are defined as [22,24–26,31,57]

$$\mathcal{W}_\chi^<(q,X) = \int \mathrm{d}^4 Y e^{-iq\cdot Y} U(x,y)\langle \psi_\chi^\dagger(y)\psi_\chi(x)\rangle, \quad (1)$$

where $x = X + \frac{Y}{2}$, $y = X - \frac{Y}{2}$, and $U(x,y)$ is a straight line gauge link [22]. The bracket denotes the ensemble average for a given initial-time density operator [60]. The Wigner functions and self-energies are decomposed into the basis of Pauli matrices [26], e.g., $\mathcal{W}_{\mathrm{R}}^< = \bar{\sigma}_\mu \mathcal{W}_{\mathrm{R}}^{<,\mu}$. In this work we mainly focus on the collisionless limit so that the lessor/greater self-energies are expected to be zero, $\Sigma^{\lessgtr} = 0$. By solving the equations of motion under quasiparticle approximations, we are able to obtain the perturbative solution of Eq. (1) up to $\mathcal{O}(\hbar^1, \bar{\Sigma}^1_{\chi,\mu})$ [57],

$$\mathcal{W}_\chi^{<,\mu}(q,X) = 2\pi\epsilon(q_0)\left[\delta(\tilde{q}^2)(\tilde{q}^\mu + \chi\hbar\tilde{S}^{\mu\nu}_{(n)}\tilde{\Delta}_\nu) + \frac{\chi\hbar}{2}\delta'(\tilde{q}^2)\epsilon^{\mu\nu\rho\sigma}\tilde{q}_\nu(F_{\rho\sigma} + 2\Delta_{[\rho}\bar{\Sigma}_{\chi,\sigma]})\right] f_\chi, \quad (2)$$

where $f_\chi$ is the chiral distribution function and $\epsilon(x) \equiv x/|x|$. Here we have denoted

$$\tilde{q}^\mu = q^\mu - \bar{\Sigma}^\mu_\chi, \qquad \bar{\Sigma}^\mu_\chi = \Sigma^{\delta,\mu}_\chi + \mathrm{Re}\Sigma^{\mathrm{r},\mu}_\chi, \quad (3)$$

where $\Sigma^{\delta,\mu}_\chi$ is a one-point potential and $\Sigma^{\mathrm{r},\mu}_\chi$ is the retarded self-energy. We define the frame-dependent modified spin tensor

$$\tilde{S}^{\mu\nu}_{(n)} = \epsilon^{\mu\nu\rho\sigma}\tilde{q}_\rho n_\sigma/(2\tilde{q}\cdot n), \quad (4)$$

where $n^\mu$ is a spacetime-dependent frame vector depending on the choice of a spin basis. For convenience, we also introduce the modified covariant derivative

$$\tilde{\Delta}_\mu = \Delta_\mu + (\Delta_\lambda\bar{\Sigma}_{\chi,\mu})\partial_q^\lambda - (\partial_{q,\lambda}\bar{\Sigma}_{\chi,\mu})\partial^\lambda, \qquad \Delta_\mu = \partial_\mu - F_{\mu\nu}\partial_q^\nu, \quad (5)$$

where the derivative $\partial_\mu = \partial_{X,\mu}$ operates only on the space coordinate $X$ and the $F_{\rho\sigma}$ is the field strength of background electromagnetic fields (EMF).

The accompanied chiral kinetic equation (CKE) [57] delineating the dynamical evolution of $f_\chi$ up to $\mathcal{O}(\hbar^1, \bar{\Sigma}^1_{\chi,\mu})$ reads

$$0 = \left\{\tilde{q}^\alpha + \chi\hbar\frac{S^{\mu\alpha}_{\tilde{q}}}{\tilde{q}\cdot n}[E_\mu + 2n^\beta(\Delta_{[\mu}\bar{\Sigma}_{\chi,\beta]}) - \tilde{q}^\beta(\tilde{\Delta}_\mu n_\beta)] + \chi\hbar\frac{\epsilon^{\mu\alpha\rho\sigma}\tilde{q}_\rho(\tilde{\Delta}_\mu n_\sigma)}{2\tilde{q}\cdot n} + \chi\hbar S^{\mu\alpha}_{\tilde{q}}\tilde{\Delta}_\mu\right\}\tilde{\Delta}_\alpha f_\chi, \quad (6)$$

where the on-shell condition $\tilde{q}^2 = -\chi\hbar\tilde{S}^{\mu\nu}_{(n)}\tilde{\Delta}_\mu\tilde{q}_\nu$ is imposed implicitly and $E_\mu = F_{\mu\nu}n^\nu$ as an electric field defined in terms of the frame vector $n^\mu$. Although the frame vectors are involved in both the Wigner function in Eq. (2) and the kinetic equation in Eq. (6), they are in fact frame independent. Note that $f_\chi$ with quantum corrections also follows a modified frame transformation. Consequently, the physical results obtained from the QKT are frame independent.

Our goal is to derive the lowest-order interaction corrections to the Wigner functions, beyond the EMF effects. These corrections, known to originate from the medium background, are calculated by $\bar{\Sigma}_\chi \sim \mathcal{O}(g^2)$, with collision contributions starting at $\mathcal{O}(g^4)$ [46]. However,

deriving these corrections is generally challenging, as the self-energies modify not only the momentum and on-shell conditions but also the chiral distribution functions. In this paper, we make an early-stage effort to derive such corrections at least in global equilibrium. These corrections are crucial for further understanding the local spin polarization of $\Lambda$ hyperons [11,61] and provide an alternative method to derive interaction corrections to anomalous transport effects, such as the chiral separation effects induced by vorticity [25,55,56].

In global equilibrium, at the leading order in $\hbar$ and the first order in $\bar{\Sigma}_\chi$, the chiral distribution function satisfies

$$\tilde{q}\cdot\tilde{\Delta}f_\chi^{(0)}=0, \tag{7}$$

where the on-shell condition is not imposed. Since $f_\chi^{(0)}$ should satisfy the detailed balance of classical collision terms and give conservation of the energy-momentum and particle number, it should only be a function of $\beta u\cdot q-\beta\mu_\chi$ with two Lagrangian multipliers $\{\beta^\mu=\beta u^\mu,\mu_\chi\}$. Here $\beta=T^{-1}$ and $\mu_\chi$ correspond to the inverse temperature and a chiral chemical potential of thermodynamics, respectively [62]. In this paper, we work in the fluid rest frame such that $u^\mu=(1,\boldsymbol{u})$ with $\boldsymbol{u}\approx 0$. In this case, we should have

$$f_\chi^{(0)}=\frac{1}{e^{\beta(u\cdot q-\mu_\chi)}+1}. \tag{8}$$

Then the Vlasov-type kinetic equation (7) reduces to

$$\begin{aligned}0=\{&q^\mu[\beta(u\cdot\partial\bar{\Sigma}_{\chi,\mu})-\bar{\Sigma}_{\chi,\nu}(\partial^\nu\beta u_\mu)-(\partial_{q,\nu}\bar{\Sigma}_{\chi,\mu})(\partial^\nu\beta u\cdot q)\\&+(\partial_{q,\nu}\bar{\Sigma}_{\chi,\mu})((\partial^\nu\beta\mu_\chi)+\beta E^\nu)-(\beta E_\mu+(\partial_\mu\beta\mu_\chi))]\\&+q^\mu q^\nu(\partial_\mu\beta u_\nu)+\bar{\Sigma}_{\chi,\nu}((\partial^\nu\beta\mu_\chi)+\beta E^\nu)\}\partial_{\beta u\cdot q}f_\chi^{(0)},\end{aligned} \tag{9}$$

from which one can read out the following equilibrium conditions by extracting the vanishing coefficients for independent bases, $\{1,q^\mu,q^\mu q^\nu\}$:

$$\begin{aligned}\partial^\nu(\beta\mu_\chi)+\beta E^\nu&=0,\\\partial_{(\mu}(\beta u_{\nu)})&=0,\end{aligned} \tag{10}$$

and

$$\beta u\cdot\partial\bar{\Sigma}_{\chi,\mu}-\bar{\Sigma}_{\chi,\nu}\partial^\nu(\beta u_\mu)-(\partial_{q,\nu}\bar{\Sigma}_{\chi,\mu})\partial^\nu(\beta u\cdot q)=0. \tag{11}$$

Equation (10) is the well-known global equilibrium conditions [25], while Eq. (11) gives a constraint for the self-energies in global equilibrium. Notably, the $\bar{\Sigma}_{\chi,\mu}$ obtained from the HTL approximation satisfies the constraint.

To obtain the $\mathcal{O}(\hbar^1)$ equilibrium distribution function, we demand the frame independence of Wigner functions [26,63]. The distribution function can further be decomposed to a frame-independent part $\delta f_\chi^{(u)}$ and a frame-dependent part $\delta f_\chi^{(n)}$, i.e., $f_\chi^{(1)}=\delta f_\chi^{(u)}+\delta f_\chi^{(n)}$, where the former can be an arbitrary correction, while the latter is proposed to be

$$\delta f_\chi^{(n)}=\frac{\hbar\chi}{\tilde{q}\cdot u}\tilde{S}^{\rho\nu}_{(n)}u_{[\nu}\tilde{\Delta}_{\rho]}f_\chi^{(0)}. \tag{12}$$

Using Schouten's identity and Eq. (7), we find

$$\delta(\tilde{q}^2)\tilde{q}^\mu\delta f_\chi^{(n)}=-\delta(\tilde{q}^2)\hbar\chi(\tilde{S}^{\mu\rho}_{(n)}-\tilde{S}^{\mu\rho}_{(u)})\tilde{\Delta}_\rho f_\chi^{(0)}. \tag{13}$$

Therefore, the $\mathcal{O}(\hbar^1)$ sector of chiral Wigner functions now becomes frame-independent,

$$\begin{aligned}&\delta(\tilde{q}^2)\,(\tilde{q}^\mu f_\chi^{(1)}+\chi\hbar\tilde{S}^{\mu\nu}_{(n)}\tilde{\Delta}_\nu f_\chi^{(0)})\\&\quad=\delta(\tilde{q}^2)\,(\tilde{q}^\mu\delta f_\chi^{(u)}+\chi\hbar\tilde{S}^{\mu\nu}_{(u)}\tilde{\Delta}_\nu f_\chi^{(0)}).\end{aligned} \tag{14}$$

Albeit no general guideline to determine $\delta f_\chi^{(u)}$, we may propose a generalization of the noninteracting case [36,63],

$$\delta f_\chi^{(u)}=\frac{\chi\hbar}{2}(\partial_{\beta q\cdot u}f_\chi^{(0)})\tilde{S}^{\rho\nu}_{(u)}\Omega_{\rho\nu}, \tag{15}$$

where we have defined the thermal vorticity

$$\Omega^{\alpha\beta}=\partial^{[\alpha}(\beta u^{\beta]}). \tag{16}$$

Then in the absence of external EMF, the equilibrium chiral Wigner function (2) now becomes

$$\begin{aligned}\mathcal{W}_\chi^{<,\mu}(q,X)=2\pi\epsilon(q_0)\Big\{&\delta(q^2)\Big[(q^\mu-\bar{\Sigma}_\chi^\mu)\\&-\frac{\chi\hbar}{4}\epsilon^{\mu\alpha\rho\nu}(q_\alpha-\bar{\Sigma}_{\chi,\alpha})\Omega_{\rho\nu}\partial_{\beta q\cdot u}\Big]\\&-\delta'(q^2)\Big[q\cdot\bar{\Sigma}_\chi\Big(2q^\mu-\frac{\chi\hbar}{2}\epsilon^{\mu\rho\nu\alpha}q_\alpha\Omega_{\rho\nu}\partial_{\beta q\cdot u}\Big)\\&-\chi\hbar\epsilon^{\mu\nu\rho\sigma}q_\nu(\partial_\rho\bar{\Sigma}_{\chi,\sigma})\Big]\Big\}f_\chi^{(0)},\end{aligned} \tag{17}$$

where we have kept the terms up to $\mathcal{O}(\bar{\Sigma}^1_{\chi,\mu})$ and used the equilibrium conditions (10) and (11). To verify whether Eq. (15) indeed represents the equilibrium distribution function, we insert the equilibrium chiral distribution function,

$$f_\chi=\frac{1}{e^{\beta g}+1},\quad g=u\cdot q-\mu_\chi+\frac{\chi\hbar}{2}\tilde{S}^{\rho\nu}_{(u)}\Omega_{\rho\nu}, \tag{18}$$

into the CKE in Eq. (6) with $n^\mu=u^\mu$, and we find that the equation is satisfied up to $\mathcal{O}(\hbar^1,\bar{\Sigma}^1_{\chi,\mu})$. Furthermore, we will derive such an equilibrium Wigner function (17) using the Keldysh equation formalism in the subsequent sections.

## III. KELDYSH EQUATIONS FOR WIGNER FUNCTIONS

Deriving the explicit form of the self-energy correction on the Wigner function from QKT is nontrivial, particularly when extending to massive fermions and vector bosons (including photons and vector mesons). More critically, generalizing these results to the near-equilibrium (local-equilibrium) scenario becomes even more complex, even if collision effects are disregarded. Furthermore, in strongly coupled systems, the quasiparticle paradigm is no longer applicable, rendering perturbative solutions of the Wigner functions unattainable. In this section, we propose an alternative approach parallel to QKT: the Keldysh equation [64] in phase space, which represents an integral-equation parallel of the kinetic equation manifested as a differential-integral equation. The Keldysh-equation formalism enables systematic incorporation of interaction and off-equilibrium corrections, and it can be extended beyond the quasiparticle limit.

We start from the time-ordered fermionic two-point Green's function along the Kadanoff-Baym (KB) contour

$$\begin{aligned} S_{ab}(x,y) &= \langle T_{\mathrm{c}}\psi_a(x)\bar{\psi}_b(y)\rangle \\ &= \theta_{\mathrm{c}}(x_0-y_0)S^{>}_{ab}(x,y)-\theta_{\mathrm{c}}(y_0-x_0)S^{<}_{ab}(x,y), \end{aligned} \tag{19}$$

where $T_{\mathrm{c}},\theta_{\mathrm{c}}$ are the time-ordering operator and Heaviside step function in the KB contour. We further denote $\langle\cdots\rangle_0$ as the ensemble average under a general reference density operator $\hat{\rho}_0$, which is taken as the noninteracting density operator in this work, and the corresponding correlation functions as $\mathcal{O}_0$. For example, in the global equilibrium case it takes the form of [64,65]

$$\hat{\rho}_0=\frac{\exp\ [-\beta_\mu\hat{P}^\mu_0+\alpha\hat{Q}_0+\varpi_{\mu\nu}\hat{\mathcal{J}}^{\mu\nu}_0]}{\mathrm{Tr}\ \exp\ [-\beta_\mu\hat{P}^\mu_0+\alpha\hat{Q}_0+\varpi_{\mu\nu}\hat{\mathcal{J}}^{\mu\nu}_0]}, \tag{20}$$

where $\hat{P}^\mu,\hat{Q},\hat{\mathcal{J}}^{\mu\nu}$ are the momentum operator, the U(1) charge operator, and the total angular momentum operator of the system, respectively. Accordingly, $\hat{H}_0=\hat{P}^0_0$ and $\{\beta_\mu,\alpha,\varpi_{\mu\nu}\}$ are the Lagrangian multipliers of the conserved charges of the system; the subscript "0" denotes the free operators. We also assume that the interaction information in the full density operator $\hat{\rho}$ only lies in the interaction sector of Hamiltonian $\hat{H}_{\mathrm{int}}=\hat{P}^0_{\mathrm{int}}$; e.g., this corresponds to set $\hat{\mathcal{J}}^{\mu\nu}_{\mathrm{int}}=0$ in the global equilibrium density operator (20), which only holds for $\varpi_{\mu\nu}=\varpi_{ij}\eta^i_\mu\eta^j_\nu$ [66]. We thus work in the real-time formalism and obtain the following Keldysh equation [67]:

$$\begin{aligned} S^{<}(x,y) = S^{<}_0(x,y) &- \int_{-\infty}^{+\infty}\mathrm{d}^4z_1\mathrm{d}^4z_2[S^{\mathrm{r}}(x,z_2)\Sigma^{\mathrm{r}}(z_2,z_1) \\ &\times S^{<}_0(z_1,y)+S^{\mathrm{r}}(x,z_2)\Sigma^{<}(z_2,z_1)S^{\mathrm{a}}_0(z_1,y) \\ &+S^{<}(x,z_2)\Sigma^{\mathrm{a}}(z_2,z_1)S^{\mathrm{a}}_0(z_1,y)] \\ &-i\int_{-\infty}^{+\infty}\mathrm{d}^4z_1(S^{\mathrm{r}}(x,z_1)\Sigma(z_1)S^{<}_0(z_1,y) \\ &+S^{<}(x,z_1)\Sigma(z_1)S^{\mathrm{a}}_0(z_1,y)), \end{aligned} \tag{21}$$

where the two-point self-energy and one-point potential are defined as

$$\Sigma_{cd}(z_2,z_1)=\left\langle T_{\mathrm{c}}i^2\int_{\mathrm{c}}\mathrm{d}^4z'\mathrm{d}^4z\frac{\delta\mathcal{L}_{\mathrm{int}}(z')}{\delta\bar{\psi}_c(z_2)}\frac{\delta\mathcal{L}_{\mathrm{int}}(z)}{\delta\psi_d(z_1)}\right\rangle, \tag{22}$$

$$\Sigma_{cd}(z_1)=\left\langle T_{\mathrm{c}}i\int_{\mathrm{c}}\mathrm{d}^4z\frac{\delta\mathcal{L}_{\mathrm{int}}(z)}{\delta\bar{\psi}_c(z_1)\delta\psi_d(z_1)}\right\rangle. \tag{23}$$

Here the subscripts of the fermion fields represent the spinor indices. Thus, $\Sigma_{cd}(z_1,z_2)$ does not acquire a singular part as in Ref. [60] and can also be written as Eq. (19),

$$\begin{aligned} \Sigma_{cd}(z_1,z_2) &= \theta_{\mathrm{c}}(t_1-t_2)\Sigma^{>}_{cd}(z_1,z_2) \\ &\quad -\theta_{\mathrm{c}}(t_2-t_1)\Sigma^{<}_{cd}(z_1,z_2). \end{aligned} \tag{24}$$

Here we have introduced (anti-)Feynman Green's functions and the retarded/advanced $(r/a)$ quantities for the Fermi-type correlators,

$$\begin{aligned} S^{\mathrm{f}}(x,y) &= [S^{\tilde{\mathcal{J}}}(x,y)]^\dagger \\ &= \theta(x_0-y_0)S^{>}(x,y)-\theta(y_0-x_0)S^{<}(x,y), \end{aligned} \tag{25}$$

and

$$\mathcal{O}_{\mathrm{r}}(x,y)=i\theta(x_0-y_0)(\mathcal{O}^{>}(x,y)+\mathcal{O}^{<}(x,y)), \tag{26}$$

$$\mathcal{O}_{\mathrm{a}}(x,y)=-i\theta(y_0-x_0)(\mathcal{O}^{>}(x,y)+\mathcal{O}^{<}(x,y)), \tag{27}$$

where $\mathcal{O}$ can be self-energies or Green's functions. For the Bose-type correlators, we can simply change the minus sign in front of $\mathcal{O}^{<}$ to the plus sign as opposed to the Grassmann nature of fermionic fields. It is easy to obtain the Keldysh equations in terms of Wigner functions,

$$\begin{aligned} S^{<}(q,X) = S^{<}_0(q,X) &- (S^{\mathrm{r}}(q,X)\star\Sigma^{\mathrm{r}}(q,X))\star S^{<}_0(q,X) \\ &-(S^{\mathrm{r}}(q,X)\star\Sigma^{<}(q,X))\star S^{\mathrm{a}}_0(q,X) \\ &-(S^{<}(q,X)\star\Sigma^{\mathrm{a}}(q,X))\star S^{\mathrm{a}}_0(q,X) \\ &-i(S^{\mathrm{r}}(q,X)\star\Sigma(X))\star S^{<}_0(q,X) \\ &-i(S^{<}(q,X)\star\Sigma(X))\star S^{\mathrm{a}}_0(q,X), \end{aligned} \tag{28}$$

where

$$H(q,X) = [f(q,X) \star g(q,X)] \star h(q,X) \tag{29}$$

stems from the Wigner transform of

$$H(x,y) = \int \mathrm{d}^4 z_1 \mathrm{d}^4 z_2 f(x,z_1) g(z_1,z_2) h(z_2,y) \tag{30}$$

and the Moyal product with associative properties is defined as

$$f \star g \equiv f \exp\left(-i\hbar \frac{\overleftarrow{\partial}\cdot\overrightarrow{\partial}_q - \overrightarrow{\partial}\cdot\overleftarrow{\partial}_q}{2}\right) g. \tag{31}$$

Equations (29) and (30) also hold for the situation involved with the one-point potential, where $g(z_1,z_2) = g(z_1)\delta^{(4)}(z_1 - z_2)$ and $g(q,X) = g(X)$.

Similarly, for the vector fields, we have

$$\begin{aligned} G^<_{\mu\nu}(q,X) &= G^{<,0}_{\mu\nu}(q,X) - (G^{\mathrm{r}}_{\mu\beta}(q,X) \star \Sigma^{\beta\alpha,\mathrm{r}}(q,X)) \star G^{0,<}_{\alpha\nu}(q,X) \\ &\quad - (G^{\mathrm{r}}_{\mu\beta}(q,X) \star \Sigma^{\beta\alpha,<}(q,X)) \star G^{0,\mathrm{a}}_{\alpha\nu}(q,X) \\ &\quad - (G^<_{\mu\beta}(q,X) \star \Sigma^{\beta\alpha,\mathrm{a}}(q,X)) \star G^{0,\mathrm{a}}_{\alpha\nu}(q,X), \end{aligned} \tag{32}$$

which originates from the Wigner transform of the lessor two-point function of a general complex vector field

$$G^<_{\mu\nu}(x,y) = \langle A^\dagger_\nu(y) A_\mu(x)\rangle, \tag{33}$$

and the greater function is $G^>_{\nu\mu}(y,x) = G^{<,\dagger}_{\nu\mu}(x,y)$. These Wigner functions are symmetric with respect to the Lorentz indices for the real parts and antisymmetric for imaginary parts. Namely, the antisymmetric part of $G^<_{\mu\nu}(p,X)$ is purely imaginary, which can be verified by evaluating the photonic Wigner functions [58,68].

We emphasize again that the Wigner functions with "0" subscripts in Eqs. (28) and (32) can be general out-of-equilibrium Wigner functions without interaction corrections. More concrete examples can be found in Refs. [69,70]. These Wigner functions serve as the initial input in the Keldysh-equation formalism and can be derived from statistical field theory [32,71–73], linear response theory [74,75], and the collisionless QKT [25,31]. Additionally, their validity can be further verified by quantum kinetic equations (QKEs) with vanishing vector and axial collision kernels [29,45,76]. Notably, some of us have developed a method to calculate the global equilibrium Wigner functions up to an arbitrarily high order in gradient or $\hbar$ expansions easily from the generalized Kubo-Martin-Schwinger (KMS) condition [68]. In this work, we use the global equilibrium Wigner functions with a general profile of fluid velocity $u^\mu$ for simplicity.

As an integral-equation parallel to QKT, the Wigner functions derived from Eqs. (28) and (32) should be equivalent to solving QKEs, provided $S^<_0, G^<_{\mu\nu,0}$ are appropriately specified. A key advantage of this formalism is the relative simplicity with which interaction corrections can be included. The use of derivatives in the Moyal product combined with the spacetime dependence of the Wigner functions and self-energies facilitates the execution of the gradient expansion and perturbation in the coupling constant order-by-order. At the leading order in gradients, we also expect that the results can be reproduced by linear response theory. Our goal in this work is to derive the lowest-order interaction corrections to fermionic spin-polarization pseudovector and spin density matrix elements within such a formalism.

## IV. FERMIONIC SPIN POLARIZATION AND AXIAL VORTICAL EFFECTS

In this section, we derive the interaction corrections to the fermionic spin-polarization pseudovector up to the leading order in gradients and lowest order in the coupling constant. We will first present the reproduction of the well-known polarization vector in a background U(1) field in Sec. IV A. Subsequently, we will explore the implications in a QCD background in Sec. IV B, ensuring consistency with the results from chiral kinetic theory presented in Sec. II and detailed in Ref. [46].

### A. Contribution from U(1) background field

In the presence of a background U(1) field, up to the leading order in the coupling constant, we may drop the two-point self-energies, and the Keldysh equation for fermions (28) reduces to

$$\begin{aligned} S^<(q,X) &= S^<_0(q) - gA_\mu(X)[S^{\mathrm{r}}_0(q)\gamma^\mu S^<_0(q) + S^<_0(q)\gamma^\mu S^{\mathrm{a}}_0(q)] \\ &\quad + i\hbar\partial_\nu g A_\mu(X)\left[S^{\mathrm{r}}_0(q)\frac{\overrightarrow{\partial}^\nu_q - \overleftarrow{\partial}^\nu_q}{2}\gamma^\mu S^<_0(q)\right. \\ &\quad \left. + S^<_0(q)\frac{\overrightarrow{\partial}^\nu_q - \overleftarrow{\partial}^\nu_q}{2}\gamma^\mu S^{\mathrm{a}}_0(q)\right] + \mathcal{O}(g^2,\partial^2), \end{aligned} \tag{34}$$

where we have taken $\Sigma(X) = -igA_\mu(X)\gamma^\mu$ with $A_\mu(X)$ being a spacetime-dependent Abelian background field and we have used the spacetime-independent fermionic $r/a$ Wigner functions in equilibrium (see also Appendix A for more details). If we omit the hydrodynamic gradient for simplicity, the global equilibrium fermionic Wigner function $S^<_0(q,X)$ acquires a simple form [77]

$$S^<_0(q) = 2\pi\epsilon(q_0)\delta(q^2 - m^2) f^<_{\mathrm{f}}(q_0)(\gamma^\alpha q_\alpha + m), \tag{35}$$

where the fermionic distribution function is Fermi-Dirac type $f^<_{\mathrm{f}}(q_0) = 1/[\exp(\beta(q_0 - \mu)) + 1]$ with $\mu$ being a U(1) vector-charge chemical potential. Inserting Eqs. (35) and (A6) into Eq. (34), we obtain

$$S^<(q,X) = S_0^<(q) - 2\pi\epsilon(q_0)\delta'(q^2-m^2)f_{\rm f}^<(q_0) \times \{gA_\mu(X)(\gamma^\alpha\gamma^\mu\gamma^\beta q_\beta q_\alpha + 2mq^\mu + m^2\gamma^\mu) + \hbar\frac{ig}{2}\partial_\nu A_\mu(X)[(\gamma^\nu\gamma^\mu\gamma^\alpha - \gamma^\alpha\gamma^\mu\gamma^\nu)q_\alpha + m(\gamma^\nu\gamma^\mu - \gamma^\mu\gamma^\nu)]\}, \tag{36}$$

where we have focused on the global equilibrium case with $\partial_i A_j \neq 0$ only. In the above calculations, we have used the following properties of the regularized Dirac Delta function [77,78]:

$$\frac{1}{x+i\epsilon}\delta(x) = -\frac{1}{2}\delta'(x) - i\pi(\delta(x))^2; \tag{37}$$

thus, the superficial divergence terms related to $[\delta(q^2-m^2)]^2$ are canceled automatically. Using the gamma matrices decomposition

$$\gamma^\mu\gamma^\nu\gamma^\rho = -i\epsilon^{\mu\nu\rho\sigma}\gamma^5\gamma_\sigma + \eta^{\mu\nu}\gamma^\rho + \eta^{\nu\rho}\gamma^\mu - \eta^{\mu\rho}\gamma^\nu,$$

we derive the $S^<(q,X)$ up to $\mathcal{O}(g^1,\partial^1)$ as

$$S^<(q,X) = 2\pi\epsilon(q_0)f_{\rm f}^<(q_0) \times \Big\{\delta[(q-gA)^2-m^2][\gamma^\alpha(q_\alpha - gA_\alpha)+m] + \delta'(q^2-m^2)\frac{\hbar}{2}gF_{\mu\nu}(\epsilon^{\mu\nu\rho\alpha}\gamma^5\gamma_\rho q_\alpha - m\gamma^{\mu\nu})\Big\}, \tag{38}$$

where the field strength tensor is defined as $F_{\alpha\beta} = 2\partial_{[\alpha}A_{\beta]}$ and $\gamma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$. Equation (38) is consistent with those derived from QKEs [33,79]. We notice that the canonical momentum $q_\mu$ is modified to the kinetic one $q_\mu - gA_\mu$ in Eq. (38), which is also observed in the previous studies [46,57,80]. This alteration ensures the gauge invariance of Wigner functions. More precisely, the $q_\mu - gA_\mu$ here should correspond to $q^\mu$ in Eq. (2).

The background-field-induced axial-vector and tensor components of the fermionic Wigner functions, proportional to $\gamma^5\gamma_\rho$ and $\gamma^{\mu\nu}$, respectively, manifest that the fermions are polarized by an external field. We also emphasize that the background field $A_\mu$ can be EMF or other effective meson fields. The nontrivial correlation from background effective meson fields can explain the global spin alignment of $\phi$ mesons measured by the STAR Collaboration [17,18,40]. From now on, we will set such a background EMF to zero because of its fast relaxation time and small magnitude in the QGP evolution [4]. Besides, in this work we are mainly interested in the polarization sources of $\mathcal{O}(\partial^1)$, so the EMF-vorticity coupling will be absent under this power counting.

### B. Contribution from QCD background and the corrections to spin polarization

We now consider the Wigner functions for massive quarks under a thermal QCD background. In our analysis, the one-point function for chromo-electromagnetic fields is set to zero, whereas the gluonic correlation functions remain nonvanishing. Consequently, the interaction-dependent corrections that we consider in this section are limited to terms of order $\mathcal{O}(g^2)$. We work in the Clifford components of Wigner functions by decomposing

$$S^\lessgtr = \mathcal{F}^\lessgtr + i\mathcal{P}^\lessgtr\gamma^5 + \gamma^\mu\mathcal{V}_\mu^\lessgtr + \gamma^5\gamma^\mu\mathcal{A}_\mu^\lessgtr + \frac{1}{2}\gamma^{\mu\nu}\mathcal{S}_{\mu\nu}^\lessgtr, \tag{39}$$

where $\mathcal{F}^\lessgtr$, $\mathcal{P}^\lessgtr$, $\mathcal{V}_\mu^\lessgtr$, $\mathcal{A}_\mu^\lessgtr$, and $\mathcal{S}_{\mu\nu}^\lessgtr$ correspond to the scalar, pseudoscalar, vector, axial-vector, and tensor components of Wigner functions, respectively. For simplicity, we focus on the color-averaged quantities and thus neglect the color decomposition. These components have well-established power counting by assuming spin effects are relatively small, i.e., $\mathcal{F},\mathcal{V}^\mu \sim \mathcal{O}(\partial^0)$; $\mathcal{A}^\mu,\mathcal{S}^{\mu\nu} \sim \mathcal{O}(\partial^1)$; and $\mathcal{P} \sim \mathcal{O}(\partial^2)$ (also see, e.g., Refs. [30,46,79]).

The self-energies can also be decomposed with the same manner. The imaginary part of $r/a$ self-energies contributes to the collision kernel, while the real parts play a similar role as background fields [26,46,57]. The collision effects are at least of $\mathcal{O}(g^4)$ [81], and $\Sigma^\lessgtr$ at the one-loop level corresponds to the splitting or collinear $2\to3$ processes in gauge theories [82,83]. Therefore, in the lowest order in a coupling constant, we only need to focus on the real parts of $r/a$ self-energy in one-loop, i.e.,

$$\mathrm{Re}\Sigma^{\rm r} = \mathrm{Re}\Sigma^{\rm a} = -\,\mathrm{Im}\,\Sigma_{\rm f}, \tag{40}$$

where $\mathrm{Im}\,\Sigma$ denotes imaginary parts of the Clifford components. Up to $\mathcal{O}(g^2)$,

$$\Sigma_{\rm f}(q,X) = -C_{\rm F}g^2\int\frac{{\rm d}^4p}{(2\pi)^4}G_{\rm f}^{\mu\nu}(q-p,X)\gamma_\mu S_{\rm f}(p,X)\gamma_\nu, \tag{41}$$

where we have factorized the color factor $t_a t_a = C_{\rm F}$ so that the gluonic propagator reduces to the photonic propagator. We insert the photonic and fermionic Wigner functions up to the first order in gradients, whose expression can be found in Appendix A and in Refs. [58,79], into the above equation for $\Sigma_{\rm f}(q,X)$. We find that in the HTL approximation and up to $\mathcal{O}(\partial^1)$, only the vector and axial-vector sectors of $\Sigma_{\rm f}(q,X)$ are nonzero,

$$\mathrm{Im}\,\Sigma_{\rm f}^{{\rm V},\beta} = -m_{\rm f}^2\left[\frac{u^\beta}{|\mathbf{q}|}Q_0\left(\frac{q_0}{|\mathbf{q}|}\right) + \frac{q_\perp^\beta}{|\mathbf{q}|^2}Q_1\left(\frac{q_0}{|\mathbf{q}|}\right)\right], \tag{42}$$

$$\mathrm{Im}\,\Sigma_{\rm f}^{{\rm A},\beta} = -\frac{C_{\rm F}g^2T\mu}{16\pi^2m_{\rm f}^2}\hbar\epsilon^{\beta\nu\rho\sigma}\Omega_{\rho\sigma}\,\mathrm{Im}\,\Sigma_{{\rm f},\nu}^{\rm V}. \tag{43}$$

Here we choose $u^\mu$ as the fluid velocity of the QCD background and introduce fermionic thermal mass $m_{\rm f}^2 = \frac{C_{\rm F} g^2}{8}(T^2 + \frac{\mu^2}{\pi^2})$. We decompose $q^\mu$ as

$$q^\mu = q_0 u^\mu + q_\perp^\mu, \quad q_0 = u\cdot q, \quad q_\perp^\mu = \Delta^{\mu\nu} q_\nu, \tag{44}$$

and define

$$|\mathbf{q}| = \sqrt{-q_\perp \cdot q_\perp}, \qquad \hat{q}_\perp^\mu = q_\perp^\mu |\mathbf{q}|^{-1}. \tag{45}$$

The auxiliary functions $Q(x)$ are defined as

$$Q_0(x) = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|, \quad Q_1(x) = xQ_0(x) - 1. \tag{46}$$

Taking the axial-vector Clifford component, $\mathcal{A}^{<,\mu}$, in Eq. (28) and expanding it up to the first order in gradient, we obtain

$$\begin{aligned}\mathcal{A}^{<,\mu}(q,X) = {}& \mathcal{A}_0^{<,\mu}(q,X) + 2\pi\epsilon(q_0)\delta(q^2-m^2) f_{\rm f}^< f_{\rm f}^> \\ & \times \frac{\hbar}{4}\epsilon^{\mu\rho\alpha\beta}\Omega_{\alpha\beta}\,\mathrm{Im}\,\Sigma^{\rm V}_{{\rm f},\rho} + 2\pi\epsilon(q_0)\delta'(q^2-m^2)\hbar \\ & \times \Big\{[\Omega_{\alpha\beta} q\cdot \mathrm{Im}\,\Sigma_{\rm f}^{\rm V} + \mathrm{Im}\,\Sigma^{\rm V}_{{\rm f},\beta}(q^\lambda \xi_{\alpha\lambda} - (\partial_\alpha \alpha))] \\ & \times \frac{1}{2}\epsilon^{\mu\rho\alpha\beta} q_\rho f_{\rm f}^< f_{\rm f}^> + [-\epsilon^{\mu\rho\alpha\beta} q_\rho (\partial_\alpha \mathrm{Im}\,\Sigma^{\rm V}_{{\rm f},\beta}) \\ & - 2q^\mu q\cdot \mathrm{Im}\,\Sigma_{\rm f}^{\rm A} + (m^2+q^2)\,\mathrm{Im}\,\Sigma_{\rm f}^{{\rm A},\mu}] f_{\rm f}^<\Big\},\end{aligned} \tag{47}$$

where $f_{\rm f}^> = 1 - f_{\rm f}^<$ and we have introduced the thermal chemical potential $\alpha = \beta\mu$ and thermal shear tensor $\xi_{\alpha\beta}$

$$\xi_{\alpha\beta} = \partial_{(\alpha}(\beta u_{\beta)}). \tag{48}$$

To derive the expression of $\mathcal{A}^{<,\mu}$, we have substituted equilibrium Wigner functions and carried out the following calculation with Eq. (37):

$$[\mathcal{F}_0^{\rm r} + \mathcal{F}_0^{\rm a}]\delta(q^2-m^2) = m\delta'(q^2-m^2), \tag{49}$$

where $\mathcal{F}_0^{\rm r}$ and $\mathcal{F}_0^{\rm a}$ are listed below Eq. (A6). Such a derivative applied to the Dirac delta function signifies corrections to the on-shell conditions induced by interactions, similar to those in Eq. (36). Notably, in the off-shell part, the term $\epsilon^{\mu\rho\alpha\beta} q_\rho \partial_\alpha \mathrm{Im}\,\Sigma^{\rm V}_{{\rm f},\beta}$ agrees with the findings in Ref. [46]. The off-global-equilibrium contribution $\partial_\mu\alpha$ and $\xi_{\alpha\beta}$ in Eq. (47) originates from the anisotropy of the fluid field.

Let us discuss the application to the spin polarization. We will focus on the global equilibrium case for fermionic spin polarization and AVE for simplicity. Integrating over $q_0$ of $\mathcal{A}^{<,\mu}$ in Eq. (47) and taking the normalization, we can obtain a full QCD interaction correction up to $\mathcal{O}(g^2)$ to the spin modified Cooper-Frye formulas [32,33]. These new corrections include the remaining terms from the so-called dynamical contributions neglected in our previous work [46]. Here, for simplicity, we list the new corrections to the spin polarization pseudovector,

$$\delta\mathcal{P}^\mu_{\rm therm}(t,\mathbf{q}) = -\frac{1}{4mN}\int_\Sigma \frac{m_{\rm f}^2}{E_{\mathbf{q}}^2} G_{\rm T}(\mathbf{q})\epsilon^{\mu\nu\alpha\beta} q_\nu \Omega_{\alpha\beta}, \tag{50}$$

$$\delta\mathcal{P}^\mu_{\rm acc}(t,\mathbf{q}) = \frac{1}{8mN}\int_\Sigma \frac{m_{\rm f}^2}{E_{\mathbf{q}}^3} G_{\rm a}(\mathbf{q})\epsilon^{\mu\nu\rho\sigma} q_\rho u_\sigma (u\cdot\partial) u_\nu, \tag{51}$$

$$\begin{aligned}\delta\mathcal{P}^\mu_{\rm vor}(t,\mathbf{q}) = {}& \frac{1}{4mN}\int_\Sigma \frac{m_{\rm f}^2}{E_{\mathbf{q}}^2}\Big[\omega^\mu G_{\rm vor1}(\mathbf{q}) \\ & - \frac{\omega\cdot q}{E_{\mathbf{q}}}\left(u^\mu G_{\rm vor2}(\mathbf{q}) + \frac{q_\perp^\mu}{E_q} G_{\rm vor3}(\mathbf{q})\right)\Big],\end{aligned} \tag{52}$$

where we define

$$\int_\Sigma = \int_\Sigma q\cdot d\sigma, \tag{53}$$

with $\Sigma$ being the chemical freeze-out hypersurface and $\mathrm{d}\sigma$ being the volume elements. The $N$ is given by

$$N = \int_\Sigma f_{\rm f}^<(E_{\mathbf{q}}, X). \tag{54}$$

The particles are set to be on-shell, and thus, $E_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + m^2}$. The lower index "therm," "acc," and "vor" means the corrections to the spin polarization induced by the thermal vorticity, fluid acceleration, and kinematic vorticity, respectively. The auxiliary functions, such as $G_{\rm T}(\boldsymbol{q})$, $G_{\rm a}(\boldsymbol{q})$, etc., for contributions from thermal vorticity, acceleration, and kinematic vorticity are listed in Appendix B 1.

Now, we comment on Eqs. (50)–(52). We emphasize that we have worked in the $\mu = 0$ and strictly global equilibrium case, characterized by a finite vorticity tensor

$$\omega_{\alpha\beta} = \Delta_{\mu\alpha}\Delta_{\nu\beta}\partial^{[\mu} u^{\nu]}, \tag{55}$$

and a finite fluid vorticity vector

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \omega_{\rho\sigma}. \tag{56}$$

Therefore, these contributions exist even for global polarization. Different from our previous work [14,15,46,47] in the massless case, the corrections to the spin polarization acquired above is for the massive quark. A smooth connection to the massless limit is straightforward to get. For phenomenological application, the so-called

strange-equilibrium scenario may be adopted with the assumption that the polarization spectrum of the strange quark is smoothly connected to the one for the corresponding $\Lambda$ hyperon [13,14,84]. We also remark that since the global polarization is already well-described by the current theoretical models [7] within the error bar, we do not expect a sizable modification to the global polarization can be made. On the other hand, the numerical simulation might be necessary to see whether the polarization spectrum can get influenced.

As a cross-check, we will recover the interaction modified chiral Wigner function (17) and equilibrium distribution (18) in the strict global equilibrium case, by imposing the Killing equation (10) which gives $\xi_{\mu\nu}=\partial_\mu\alpha=0$. Working in the massless limit and assuming zero chemical potential, Eq. (47) reduces to

$$\begin{aligned}\mathcal{A}^{<,\mu}(q,X)=&\mathcal{A}_0^{<,\mu}-2\pi\epsilon(q_0)\delta(q^2)f_{\mathrm{f}}^{<}f_{\mathrm{f}}^{>}\frac{\hbar}{4}\epsilon^{\mu\rho\alpha\beta}\Omega_{\alpha\beta}\bar{\Sigma}_{\mathrm{V},\rho}\\&+2\pi\epsilon(q_0)\delta'(q^2)\hbar\Big\{\epsilon^{\mu\rho\alpha\beta}q_\rho(\partial_\alpha\bar{\Sigma}_{\mathrm{V},\beta})f_{\mathrm{f}}^{<}\\&-\Omega_{\alpha\beta}q\cdot\bar{\Sigma}_{\mathrm{V}}\frac{1}{2}\epsilon^{\mu\rho\alpha\beta}q_\rho f_{\mathrm{f}}^{<}f_{\mathrm{f}}^{>}\Big\},\end{aligned}\tag{57}$$

where $\bar{\Sigma}_{\mathrm{V}}=\mathrm{Re}\Sigma_{\mathrm{V}}^{\mathrm{r}}$. Similarly, we work out the vector Wigner function up to $\mathcal{O}(\partial^1)$ from Eq. (28),

$$\begin{aligned}\mathcal{V}^{<,\mu}(q,X)=&\mathcal{V}_0^{<,\mu}+2\pi\epsilon(q_0)[\delta(q^2)(q^\mu-\bar{\Sigma}_{\mathrm{V}}^{\mu})\\&-\delta'(q^2)2q^\mu q\cdot\bar{\Sigma}_{\mathrm{V}}]f_{\mathrm{f}}^{<}.\end{aligned}\tag{58}$$

In the chiral basis, $\mathcal{W}_{\mathrm{R/L}}^{\lessgtr,\mu}=\mathcal{V}^{\lessgtr,\mu}\pm\mathcal{A}^{\lessgtr,\mu}$, $\bar{\Sigma}_{\mathrm{R/L}}^{\mu}=\bar{\Sigma}_{\mathrm{V}}^{\mu}$ taking the HTL approximation and from Eq. (A1),

$$\mathcal{A}_0^{<,\mu}=2\pi\epsilon(q_0)\delta(q^2-m^2)f_{\mathrm{f}}^{<}f_{\mathrm{f}}^{>}\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\Omega_{\nu\rho}q_\sigma,\tag{59}$$

we are able to exactly reproduce the equilibrium chiral Wigner function in Eq. (17) and the modified equilibrium distribution function in Eq. (18) without resorting to any knowledge of the modified equilibrium condition in Eq. (11) or the kinetic equations.

### C. Radiative corrections to massless and massive axial vortical effect

Since we obtain the axial-vector component of Wigner functions in thermal equilibrium with vorticity in the previous subsection, we further evaluate the coupling-dependent AVE for massive quarks in the thermal QCD background. We also refer to Refs. [52,53] for related studies on massless fermions, and to Ref. [59] for investigations of massive fermions under different conditions.

In global equilibrium with pure rotation, from Eq. (47), the axial-charge current is given by

$$\begin{aligned}J_{5,\mu}(X)&=4\int\frac{\mathrm{d}^4q}{(2\pi)^4}\mathrm{Tr}_{\mathrm{c}}\mathcal{A}_\mu^{<}(q,X)\\&=J_{5,\mu}^0(X)+\delta J_{5,\mu}(X).\end{aligned}\tag{60}$$

The zeroth-order result $J_{5,\mu}^0(X)$, independent of the interactions, is consistent with the previous finding [25,56]

$$J_\mu^{5,0}=\frac{N_{\mathrm{c}}T^2\omega_\mu}{\pi^2}I_0(\beta m),\tag{61}$$

$$I_0(a)=\int_0^{+\infty}\mathrm{d}x\frac{(2x^2+a^2)(x^2+a^2)^{-\frac{1}{2}}}{e^{\sqrt{x^2+a^2}}+1},\tag{62}$$

where $I_0(0)=\frac{\pi^2}{6}$. The $\mathcal{O}(g^2)$ correction, $\delta J_{5,\mu}(X)$, reads

$$\delta J_5^\mu(X)=\frac{2N_c m_{\mathrm{f}}^2\omega^\mu}{\pi^2}I_1(\beta m),\tag{63}$$

where

$$\begin{aligned}&I_1(a)\\&=\int_0^{+\infty}\frac{\mathrm{d}x}{e^{\sqrt{x^2+a^2}}+1}\Bigg\{-\frac{2a^2}{x^3}\ln\frac{\sqrt{x^2+a^2}+x}{a}\\&+\frac{2(x^2+3a^2)}{3x^2\sqrt{x^2+a^2}}-\frac{e^{\sqrt{x^2+a^2}}\left(e^{\sqrt{x^2+a^2}}-1\right)}{\left(e^{\sqrt{x^2+a^2}}+1\right)^2}\frac{x^2}{2\sqrt{x^2+a^2}}\\&+\frac{e^{\sqrt{x^2+a^2}}}{e^{\sqrt{x^2+a^2}}+1}\left(1-\frac{2a^2+x^2}{2x\sqrt{x^2+a^2}}\ln\frac{\sqrt{x^2+a^2}+x}{a}\right)\Bigg\}.\end{aligned}\tag{64}$$

In the calculation of $\delta J_5^\mu(X)$, we have inserted the normal ordering in the definition (19) to eliminate the ultraviolet (UV) divergence from vacuum.

Equation (63) is infrared (IR) safe and can be evaluated numerically for arbitrary nonzero mass since as $x\to 0$,

$$-\frac{a^2}{x^3}\ln\frac{\sqrt{x^2+a^2}+x}{a}+\frac{x^2+3a^2}{3x^2\sqrt{x^2+a^2}}\to\frac{2x^2}{15a^3},\tag{65}$$

$$1-\frac{2a^2+x^2}{2x\sqrt{x^2+a^2}}\ln\frac{\sqrt{x^2+a^2}+x}{a}\to 0.\tag{66}$$

This property shows that the integral kernel in Eq. (63) is nonanalytic for $m$ in the small $|\mathbf{q}|$ region, which means that if working in the massless limit, Eq. (63) is intrinsically IR divergent. More explicitly, we have

$$\delta J_{5,m=0}^\mu(X)=\frac{N_c C_{\mathrm{F}}g^2T^2\omega^\mu}{16\pi^2}+\frac{N_c m_{\mathrm{f}}^2\omega^\mu}{3\pi^2}\int_{\Lambda_{\mathrm{IR}}}^{+\infty}\frac{\mathrm{d}|\mathbf{q}|}{|\mathbf{q}|}f_{\mathrm{f}}^{<}(|\mathbf{q}|),\tag{67}$$

where an IR cutoff $\Lambda_{\rm IR}$ proportional to thermal mass has to be introduced to regularize the result. However, it is not surprising that Eq. (67) does not recover the exact result found in Refs. [52,53]. Because the HTL approximation excludes the energy scale of the phase-space momentum for $|\mathbf{q}| \ll T$, the integral range cannot cover the whole momentum space. Indeed, a careful analysis indicates that some $\mathcal{O}(T^2)$ contributions are missed in Eq. (67). A diagrammatic comparison between our phase-space formalism and the linear response theory [52,53] shows that the results without the HTL approximation are indeed equivalent [70].

## V. SPIN ALIGNMENT OF VECTOR MESONS FROM COALESCENCED QUARKS

Another important application of the Keldysh equation formalism is to derive the interaction correction to the spin density matrix of vector mesons. Instead of solving the spin Boltzmann equation for vector-meson fields [18,21,37,85] or using the linear response theory [86,87], the Keldysh equation provides another systematic way to derive the off-equilibrium corrections of spin density matrix elements. We focus on the boundary of the QGP and hadronic phase, where the self-energies of vector mesons receive corrections from quarks in thermal equilibrium through the effective quark-meson interactions. We also neglect the background strong (color) fields and concentrate on the contribution of leading hydrodynamic gradients for simplicity.

The phase-space spin density matrix elements is related to the vector-field Wigner function via the projection by polarization vectors $\epsilon^\mu_\lambda(q)$ with $\lambda = \pm 1, 0$ [34,35]:

$$\rho_{00}(\mathbf{q}, X) = \frac{\epsilon^\mu_0(q)\epsilon^{*,\nu}_0(q)G^<_{\mu\nu}(\mathbf{q}, X)}{\sum_\lambda \epsilon^\mu_\lambda(q)\epsilon^{*,\nu}_\lambda(q)G^<_{\mu\nu}(\mathbf{q}, X)} = \frac{1}{3} + \delta\rho_{00}(\mathbf{q}, X), \tag{68}$$

where

$$\delta\rho_{00}(\mathbf{q}, X) = \frac{\left[\epsilon^\mu_0(q)\epsilon^{*,\nu}_0(q) - \frac{1}{3}\sum_\lambda \epsilon^\mu_\lambda(q)\epsilon^{*,\nu}_\lambda(q)\right]\delta G^<_{\mu\nu}}{\sum_\lambda \epsilon^\mu_\lambda(q)\epsilon^{*,\nu}_\lambda(q)G^<_{\mu\nu,0}(\mathbf{q}, X)}. \tag{69}$$

Here, we decompose $G^<_{\mu\nu}$ as

$$G^<_{\mu\nu} \approx G^<_{\mu\nu,0} + \delta G^<_{\mu\nu}, \tag{70}$$

where $G^<_{\mu\nu,0}$ is independent of the interactions and $\delta G^<_{\mu\nu}$ is the leading-order interaction-dependent correction. In the second step of Eq. (68) we have used the expression for the interaction-free equilibrium vector-meson Wigner function up to $\mathcal{O}(\partial^1)$ which is derived from KMS relation [68]

$$G^<_{\mu\nu,0}(q, X) = -2\pi\epsilon(q_0)\delta(q^2 - m_V^2)\left[\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_V^2}\right)f^<_{\rm b}(q, X) + i\left(\frac{q_{[\nu}\Omega_{\mu]\alpha}q^\alpha}{m_V^2} - \Omega_{\mu\nu}\right)f^<_{\rm b}(q, X)f^>_{\rm b}(q, X)\right], \tag{71}$$

where $m_V$ represents the mass of vector mesons throughout this section. The polarization four-vector can be explicitly written as

$$\epsilon^\mu_\lambda(q) = \left(\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}_\lambda}{m_V}, \boldsymbol{\epsilon}_\lambda + \frac{(\boldsymbol{q}\cdot\boldsymbol{\epsilon}_\lambda)\mathbf{q}}{m_V(E^V_\mathbf{q} + m_V)}\right), \tag{72}$$

in terms of $\boldsymbol{\epsilon}_\lambda$ as the polarization in the rest frame of the vector meson. The $E^V_\mathbf{q}$ denotes the energy of vector mesons. The polarization four-vector satisfies the transverse condition, $\epsilon_\lambda \cdot q = 0$, polarization sum, $\sum_\lambda \epsilon^\mu_\lambda(q)\epsilon^{*,\nu}_\lambda(q) = -(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})$, and normalization condition, $\epsilon_\lambda \cdot \epsilon_{\lambda'} = -\delta_{\lambda\lambda'}$ [88]. In this work, we only discuss the out-of-plane spin alignment $\boldsymbol{\epsilon}_0 = (0, 1, 0)$ along the direction of global angular momentum [18,85]. In addition, the Wigner functions in Eq. (68) are actually on-shell, and we should integrate over $q_0$ for $G^<_{\mu\nu}$. In such a case, only the symmetric and real parts of $\delta G^<_{\mu\nu}$ are relevant since $\epsilon^\mu_0$ is real and the polarization contraction $\epsilon^\mu_0\epsilon^{*\nu}_0$ in Eq. (68) is symmetric with respect to the Lorentz indices. The terms proportional to $\{q_\mu, q_\nu, \eta_{\mu\nu}\}$ can be safely neglected since they are orthogonal to $\epsilon^\mu_0(q)\epsilon^{*,\nu}_0(q) - \frac{1}{3}\sum_\lambda \epsilon^\mu_\lambda(q)\epsilon^{*,\nu}_\lambda(q)$. In Eq. (68), we have also neglected the additional second-order gradient corrections from the gradient expansion of the polarization vectors in the Wigner transformation [89]. Also see Refs. [90,91] for related studies by Zubarev's approach.

Instead of considering the corrections from the background fields in Sec. IV, we focus on the in-medium coalescence and dissociation (leading-order scattering) contribution to the vector mesons, which means we may approximate the $r/a$ self-energies up to $\mathcal{O}(\partial^0)$ as

$$\Sigma^{\mu\nu}_{\rm r} \approx -\Sigma^{\mu\nu}_{\rm a} \approx \frac{i}{2}(\Sigma^{>,\mu\nu} - \Sigma^{<,\mu\nu}) = i\frac{e^{\beta(q\cdot u - \mu_{\rm b})} - 1}{2}\Sigma^{<,\mu\nu}, \tag{73}$$

where in the last step we have used the KMS relation for equilibrium self-energies [77]. Here $\mu_{\rm b}$ denotes the chemical potential of vector mesons. With these preparations, the nontrivial correction of a free vector-meson Wigner function up to $\mathcal{O}(\partial^1)$ can be calculated from Eq. (32),

$$\delta G^{<}_{\mu\nu} = -\mathrm{P.V.}\frac{1}{(q^2-m_V^2)^2}\Sigma^{<,0}_{\mu\nu} + 2\pi\epsilon(q_0)\delta''(q^2-m_V^2) \times\left[-\frac{1}{3}(q^\lambda q^\alpha \xi_{\lambda\alpha} - q^\lambda(\partial_\lambda\alpha))\Sigma^{<,0}_{\mu\nu} f^{>}_{\mathrm{b}} - \frac{2}{3}q^\lambda\partial_\lambda\Sigma^{<,0}_{\mu\nu}\right] + 2\pi\epsilon(q_0)\delta'(q^2-m_V^2)\frac{1}{4}f^{>}_{\mathrm{b}}\left[-2\Sigma^{<,0}_{\mu\alpha}\Omega^\alpha_\nu + \beta u\cdot\partial\Sigma^{<,0}_{\mu\nu} - (\partial^\lambda_q\Sigma^{<,0}_{\mu\nu})(q^\alpha\Omega_{\lambda\alpha} + q^\alpha\xi_{\lambda\alpha} - \partial_\lambda\alpha) + \frac{q^\alpha}{m_V^2}\Sigma^{<,0}_{\mu\alpha}(q^\lambda\xi_{\nu\lambda} - \partial_\nu\alpha)\right], \tag{74}$$

where we have used the interaction-free equilibrium form of $G^{<}_{\mu\nu}$ up to the leading gradient (A7) with $f^{<}_{\mathrm{b}} = f^{>}_{\mathrm{b}} - 1 = 1/[e^{\beta(u\cdot q-\mu_{\mathrm{b}})} - 1]$ the equilibrium mesonic distribution function. Here $\Sigma^{<,0}_{\mu\nu}$ is the leading-order self-energy with leading-order quark propagators. Similar properties of the Delta function such as Eq. (37) are used, which can be proven by using its Lorentzian-regularized definition. Similar to Eq. (47), the corrections (74) modify the on-shell conditions and possibly deviate the vector mesons from the quasiparticle picture.

The first term in Eq. (74) denotes the leading-order correction to $\rho_{00}$, which can be derived from the standard thermal field theory technique [77]. It also modifies the spectral functions. See Refs. [86,87] for relevant studies from linear response theory. In this work, we are interested in the remaining terms $\sim\mathcal{O}(\partial^1)$, which denote the hydrodynamic gradient contributions. From the Keldysh equation for vector-meson fields in Eq. (32), there are two possible contributions from the Moyal product and $\mathcal{O}(\partial^1)$ self-energy (or Wigner function). It can be shown that the mesonic self-energy $\Sigma_{\mu\nu}$ is symmetric with respect to the Lorentz indices in the leading gradient and its $O(\partial^1)$ part is antisymmetric. Inserting the leading-order vector-field Wigner function in Eq. (A7) and using the properties of the polarization vector, we find such a quantum self-energy makes no contribution. Therefore, the spin alignment at $\mathcal{O}(\partial^1)$ only comes from the quantum sector of vector-field Wigner functions and Moyal products.

The self-energies can be explicitly evaluated with the effective quark-meson interactions as an example. Here, we consider the bare vector vertex for $K^*$ [92],

$$\mathcal{L}^{\mathrm{vector},K*}_{qqV} = g_{\mathrm{V}}[\bar{u}\gamma^\mu K^{*+}_\mu s + \bar{d}\gamma^\mu K^{*0}_\mu s + \mathrm{c.c.}], \tag{75}$$

where $u$, $d$, $s$ are the constituent quark fields and c.c. is the charge conjugation of the first two terms. The self-energy for $K^{*,0}$ reads

$$\Sigma^{<,\mu\nu}_{K*0}(q,X) = -g_{\mathrm{V}}^2\int\frac{\mathrm{d}^4p}{(2\pi)^4}\theta(q_0)\mathrm{Tr}\,[\gamma^\mu S^{<}_{0,d}(q+p,X)\gamma^\nu S^{>}_{0,s}(p,X)]. \tag{76}$$

In RHIC energies, $e^{\beta m_s} \gg 1$ with $m_s \sim 0.42$ GeV being the constitute strange quark mass and $T \sim 0.15$ GeV as the freeze-out temperature, so the distribution of the $s$-quark can be approximated as Boltzmann-type, and the analytical expression of Eq. (76) can be obtained,

$$\Sigma^{<,\mu\nu}_{K*0}(q,X) \approx -\frac{g_{\mathrm{V}}^2 e^{\beta(-q_0+\mu_{K*0})}}{2\pi\beta|\mathbf{q}|}[-u^\mu u^\nu h_2(q,X) + (u^\nu q^\mu + u^\mu q^\nu)h_1(q,X) - q^\mu q^\nu h_3(q,X)], \tag{77}$$

where $\mu_{K^{*0}}$ is the chemical potential of the $K^{*0}$ meson and $h_{1,2,3}$ are defined in Appendix B 2. The gradients of the self-energy can be accordingly evaluated. Then performing the $q_0$ integration with $\delta^{(n)}(q^2-m^2) = (2q_0)^{-1}\partial_{q0}\delta^{(n-1)}\times(q^2-m^2)$ for Eq. (74), one can finally obtain $\delta\rho_{00}(\mathbf{q},X)$. However, such a calculation is rather involved, and we only carry out part of the calculation for illustration and an order-of-magnitude estimate. For example, for the first term after $\delta''(q^2-m^2)$ in Eq. (74), performing integration by parts only on $f^{>}_{\mathrm{b}}$, namely,

$$\int_0^{+\infty}\frac{\mathrm{d}q_0}{2\pi}\delta''(q^2-m_V^2)\times A\times f^{>}_{\mathrm{b}} \sim \int_0^{+\infty}\frac{\mathrm{d}q_0}{2\pi}\delta(q^2-m_V^2)\times A \times f^{>}_{\mathrm{b}}\left[\frac{3\beta}{4q_0^3}\left(f^{<}_{\mathrm{b}} + \frac{T}{q_0}\right) + \frac{\beta^2}{4q_0^2}(1+2f^{<}_{\mathrm{b}})f^{<}_{\mathrm{b}}\right] \tag{78}$$

with $A = -\frac{1}{3}(q^\lambda q^\alpha\xi_{\lambda\alpha} - q^\lambda(\partial_\lambda\alpha))\Sigma^{<,0}_{\mu\nu}$, its modification to the density operator reads

$$\delta\rho^{(2,1),y}_{00}(\mathbf{q},X) = \sum_{i=x,y,z}[I^{(2,1),i}_{\nabla\alpha}\beta\nabla^i\alpha + I^{(2,1),i}_{\nabla\beta}(\nabla^i\beta - (u\cdot\partial)\mathbf{u}^i)] + I^{(2,1)}_{D\alpha}\beta(u\cdot\partial)\alpha + I^{(2,1)}_{D\beta}(u\cdot\partial)\beta + I_\theta\beta(\partial\cdot u) + \sum_{i,j=x,y,z}I^{(2,1),ij}_\sigma\beta\sigma^{ij}, \tag{79}$$

where $I^{(2,1),i}$ are some complicated momentum-dependent coefficients with subscript 2,1 representing the contribution from the first part after the integration by parts for the term attached to the second-order derivative of the Delta function in Eq. (74) and $\sigma^{\mu\nu} = \Delta^{\mu(\alpha}\Delta^{\beta)\nu}\partial_\alpha u_\beta - \frac{1}{3}\Delta^{\mu\nu}(\partial\cdot u)$ is the shear-stress tensor.

For an order-of-magnitude estimate, in $\sqrt{s_{\mathrm{NN}}} = 19.6$ GeV, we choose the parameters, $\mu_d \simeq \mu_{K*0} = 0.07$ GeV [93], $T = 0.155$ GeV, and $m_s = 0.419$ GeV, $m_d = 0.220$ GeV [94], $m_{K*0} = 0.895$ GeV, and $g_{\mathrm{V}} = 1$. By considering the transverse momentum region $1.2 < q_{\mathrm{T}} < 5.4$ GeV, the

rapidity region $-1 < Y < +1$ and the full range of azimuthal angle $0 < \varphi < 2\pi$ [17], we find only the even moment of $\mathbf{q}^i$ survives,

$$\langle I_{D\alpha}^{(2,1)} \rangle = \frac{\int \mathrm{d}^3\mathbf{q} I_{D\alpha}(\mathbf{q}) f^<(q,X)}{\int \mathrm{d}^3\mathbf{q} f^<(q,X)} = -6.34 \times 10^{-4}, \quad (80)$$

and

$$\begin{aligned} &\langle I_{D\beta}^{(2,1)} \rangle = 6.78 \times 10^{-3}, \qquad \langle I_{\theta}^{(2,1)} \rangle = -4.79 \times 10^{-3}, \\ &\langle I_{\sigma}^{(2,1),xx} \rangle = -1.90 \times 10^{-3}, \qquad \langle I_{\sigma}^{(2,1),yy} \rangle = 6.48 \times 10^{-3}, \\ &\langle I_{\sigma}^{(2,1),zz} \rangle = 2.02 \times 10^{-4}. \end{aligned} \quad (81)$$

We expect the relevant coefficients as the counterpart of $I^{(2,1),i}$ for the remaining terms in Eq. (74) will be of the same order of magnitude. Similar calculations for $\phi$ mesons show the coefficients are in general larger than that of the $K^{*0}$ mesons in magnitude. On the other hand, the gradient of hydrodynamic variables are of $\mathcal{O}(10^{-2})$ in general, so these contributions are further suppressed by 2 orders of magnitude and do not significantly modify the global spin alignment of mesons, which is consistent with results from linear response theory [87] and the experimental data in Au-Au collsions at top RHIC energy [17]. We emphasize that the background gluon field correlations are not included temporarily and they could be an important source of vector meson spin alignment [18,19,21,37]. Systematic studies involving simulations combined with Eq. (79) and other possible effects on the spin alignment of vector mesons will be presented in future work.

## VI. SUMMARY AND OUTLOOK

We have investigated the radiative corrections to the spin-polarization spectrum and the AVE for massive fermions in a hot QCD background. To resolve the difficulty in determining the interaction corrections to Wigner functions up to the leading-order coupling, we formulate the Keldysh-equation framework, which is shown in Eq. (21) for fermions and Eq. (32) for bosons. These Keldysh equations are the integral-equation parallel of the Kadanoff-Baym differential-integral equations and the generalization of Dyson-Schwinger equation to the phase space in the off-equilibrium system and captures the nonperturbative features. In principle, it can be applicable to a strongly coupled system and large hydrodynamic gradient. However, solving the complete Keldysh equation in such cases is technically challenging in practical applications. Therefore, we focus on a weakly coupled, near-equilibrium system to illustrate how interaction corrections can be incorporated within this framework.

Up to the leading order in gradients, using the iteration method, we have recovered the results based on QKT and determined the interaction correction to a global equilibrium axial distribution function in the lowest order of the coupling constant. We present how to induce polarization from a background EMF by including the one-point potential. We further compute the complete lowest-order global equilibrium corrections to the spin polarization pseudovector from a QCD medium based on the HTL approximation in Eqs. (50)–(52), which also include the contributions from the previously omitted dynamical sector in QKT. Such contributions can also impact the global polarization of hyperons. Its local equilibrium generalization is also attainable from Eq. (21) if local equilibrium $\mathcal{O}(\partial^1)$ fermionic and gluonic Wigner functions are given.

We also compute the radiative corrections to both massless and massive AVE. Such corrections of the AVE for massive fermions in our study cannot reproduce the massless case in a smooth manner due to the illness of the infrared structure for HTL self-energies. Nonetheless, a diagrammatic comparison shows that such one-loop self-energy is indeed the origin of radiative corrections found in Refs. [52,53], which will be presented elsewhere [70].

At last, we evaluate $\delta\rho_{00}$ under the effective quark-meson interactions based on the Keldysh-equation formalism. Our result is indeed consistent with linear response theory [87]. As a complementary study, we will present the explicit calculation for the self-energy corrected fermionic Wigner function from the linear response theory in an upcoming paper [70].

Before ending this work, we would like to remark on the limitation of the framework based on the Keldysh equation. We emphasize that the Keldysh equation itself does not contain the complete information of the physical system. As shown in this work, it is necessary to incorporate the noninteracting Wigner functions derived from its equations of motion as inputs. A similar strategy is widely used in early works on quantum kinetic theory (see, e.g., Refs. [24,25]), in which the noninteracting Wigner functions at the leading order of $\hbar^0$ are assumed based on classical kinetic theory. Nevertheless, as long as given such input, we could incorporate various interaction corrections to the equilibrium and off-equilibrium Wigner functions.

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## DATA AVAILABILITY

No data were created or analyzed in this study.

## APPENDIX A: NONINTERACTING EQUILIBRIUM WIGNER FUNCTIONS

We present the Wigner functions for spin-half and spin-one particles in global equilibrium up to $\mathcal{O}(\partial^1)$, which can be diagrammatically derived by expanding the global equilibrium density operator up to the linear order [32,70,72,95] or from a generalized KMS relation [68]. For fermions,

$$\begin{aligned} &S_0^{\lessgtr}(q,X) \\ &= 2\pi\epsilon(q_0)\delta(q^2-m^2)\Big[(\gamma^\mu q_\mu + m) f_{\rm f}^{\lessgtr}(q,X) \\ &\quad \pm f_{\rm f}^{<}(q,X) f_{\rm f}^{>}(q,X)\frac{1}{4}\Omega_{\nu\rho}(\epsilon^{\mu\nu\rho\sigma}\gamma^5\gamma_\mu q_\sigma - m\sigma^{\nu\rho})\Big], \end{aligned} \tag{A1}$$

where $f_{\rm f}^{<}(q,X) = 1/[e^{\beta(u\cdot q-\mu)}+1]$. In this work we work in a near-equilibrium limit; i.e., the distribution functions do not eliminate the Boltzmann equations, so the configuration of $u^\mu$ can be general. It is noticed that for the fermionic field, the terms in lesser/greater Wigner functions of $\mathcal{O}(\partial^n)$, $n \geq 1$ are only different up to a minus sign. The $r/a$ quantities are related with each other via

$$S^{\rm r/a}(q,X) = \int \frac{{\rm d}q_0'}{2\pi}\frac{\rho(q_0',\mathbf{q};X)}{q_0'-q_0 \mp i\eta}, \tag{A2}$$

$$\Sigma^{\rm r/a}(q,X) = \int \frac{{\rm d}q_0'}{2\pi}\frac{\Gamma(q_0',\mathbf{q};X)}{q_0'-q_0 \mp i\eta}. \tag{A3}$$

The spectral function $\rho$ and width function $\Gamma$ are defined as

$$\rho = S^{>} + S^{<}, \quad \Gamma = \Sigma^{>} + \Sigma^{<}, \tag{A4}$$

where the sign before the lessor quantities are opposite to the bosonic case due to statistical properties. For the free equilibrium Wigner function [77],

$$\rho(q,X) = 2\pi\epsilon(q_0)\delta(q^2-m^2)(\gamma^\mu q_\mu + m), \tag{A5}$$

and the $r/a$ Wigner functions are easily calculated,

$$S_0^{\rm r/a}(q,X) = -\frac{\gamma^\mu q_\mu + m}{q^2 - m^2 \pm i q_0\eta}, \tag{A6}$$

which is spacetime-independent as expected. Especially, the scalar components read $\mathcal{F}_0^{\rm r/a}(q,X) = -\frac{m}{q^2-m^2\pm i q_0\eta}$. All other Wigner functions, such as (anti-)time order, can be expressed in terms of the $r/a$ and Wightman Wigner functions.

Similarly for the massive vector field,

$$\begin{aligned} G_{0,\mu\nu}^{<}(q,X) = &-2\pi\epsilon(q_0)\delta(q^2-m^2)\Big[\Big(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m^2}\Big) f_{\rm b}^{<}(q,X) \\ &+ i\Big(\frac{q_{[\nu}\Omega_{\mu]\alpha}q^\alpha}{m^2} - \Omega_{\mu\nu}\Big) f_{\rm b}^{<}(q,X) f_{\rm b}^{>}(q,X)\Big], \end{aligned} \tag{A7}$$

where the bosonic distribution function is defined as

$$f_{\rm b}^{<}(q,X) = \frac{1}{e^{\beta u\cdot q-\mu}-1}. \tag{A8}$$

For the bosonic field, opposite to the fermionic case, the terms in lesser/greater Wigner functions of $\mathcal{O}(\partial^n)$, $n \geq 1$ are the same. And the $r/a$ Wigner functions read

$$G_{0,\mu\nu}^{\rm r/a}(q,X) = \frac{\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m^2}}{q^2 - m^2 \pm i\eta q_0}, \tag{A9}$$

where we have dropped the normal-dependent term that does not contribute to the calculation of Feynman diagrams [88].

For the gauge field, the Wigner functions depend on the gauge choice [68]

$$\begin{aligned} G_{\mu\nu}^{<}(q,X) = &\,2\pi\epsilon(q_0)\delta(q^2)[P_{\mu\nu}^{(\xi)}(q) f_{\rm b}^{<}(q,X) \\ &- 2iP_{[\nu}^{(\xi),\alpha}\Omega_{\mu]\alpha} f_{\rm b}^{<}(q,X) f_{\rm b}^{>}(q,X)], \end{aligned} \tag{A10}$$

where $P_{\mu\nu}^{(\xi)}(q)$ is the polarization summation relying on the gauge choice [88]

$$\sum_{\lambda=1}^{2}\epsilon^\mu(\mathbf{q},\lambda)\epsilon^{*,\nu}(\mathbf{q},\lambda) = P_{(\xi)}^{\mu\nu}(q). \tag{A11}$$

for example, in the Feynman gauge $P_{(\xi)}^{\mu\nu}(q) = -\eta^{\mu\nu}$. The $r/a$ Wigner functions read

$$G_{\mu\nu,0}^{\rm r/a}(q,X) = \frac{\eta_{\mu\nu}}{q^2 \pm i\eta q_0}. \tag{A12}$$

## APPENDIX B: THE AUXILIARY FUNCTIONS

### 1. Auxiliary functions in spin Cooper-Frye formulas

We list some auxiliary functions appearing in Eqs. (50)–(52):

$$\begin{aligned} &G_{\rm T}(\mathbf{q}) \\ &= \frac{2T}{|\mathbf{q}|}\left(Q_0\left(\frac{E_\mathbf{q}}{|\mathbf{q}|}\right) + \frac{E_\mathbf{q}}{|\mathbf{q}|}Q_1\left(\frac{E_\mathbf{q}}{|\mathbf{q}|}\right)\right) f_{\rm f}^{<}(E_\mathbf{q}) \\ &\quad + \left[\frac{1}{2} + \frac{E_\mathbf{q}^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_\mathbf{q}}{|\mathbf{q}|}\right) + 2\frac{E_\mathbf{q}}{|\mathbf{q}|}R_1\left(\frac{E_\mathbf{q}}{|\mathbf{q}|}\right)\right] f_{\rm f}^{<}(E_\mathbf{q}) f_{\rm f}^{>}(E_\mathbf{q}) \\ &\quad + \left(\frac{\beta E_\mathbf{q}}{2} + \frac{\beta E_\mathbf{q}^2}{4|\mathbf{q}|}R_1\left(\frac{E_\mathbf{q}}{|\mathbf{q}|}\right)\right) f_{\rm f}^{<}(E_\mathbf{q}) f_{\rm f}^{>}(E_\mathbf{q})(1-2f_{\rm f}^{<}(E_\mathbf{q})), \end{aligned} \tag{B1}$$

$$
\begin{aligned}
&G_{\mathrm{a}}(\mathbf{q}) \\
&= \left(\frac{6E_{\mathbf{q}}}{|\mathbf{q}|}Q_0\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + \frac{6E_{\mathbf{q}}^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right)\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) \\
&\quad + \left(\frac{6\beta E_{\mathbf{q}}^2}{|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + \frac{2\beta E_{\mathbf{q}}^3}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right)\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}}) \\
&\quad + \frac{\beta^2 E_{\mathbf{q}}^3}{|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}})(1 - 2f_{\mathrm{f}}^{<}(E_{\mathbf{q}})), \qquad \text{(B2)}
\end{aligned}
$$

$$
\begin{aligned}
&G_{\mathrm{vor1}}(\mathbf{q}) \\
&= \left[\frac{4E_{\mathbf{q}}}{|\mathbf{q}|}Q_0\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + \frac{2E_{\mathbf{q}}^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + 2\right] f_{\mathrm{V}}^{<}(E_{\mathbf{q}}) \\
&\quad + \beta E_{\mathbf{q}}\left(1 + \frac{E_{\mathbf{q}}^2 + 2m^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + 4\frac{E_{\mathbf{q}}}{|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right)\right) \\
&\quad \times f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}}) \\
&\quad + \frac{\beta^2 E_{\mathbf{q}}^3}{2|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}})(1 - 2f_{\mathrm{f}}^{<}(E_{\mathbf{q}})), \qquad \text{(B3)}
\end{aligned}
$$

$$
\begin{aligned}
G_{\mathrm{vor2}}(\mathbf{q}) &= 6\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}Q_0\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) + \frac{E_{\mathbf{q}}^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right)\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) \\
&\quad + \frac{6\beta E_{\mathbf{q}}^2}{|\mathbf{q}|}Q_0\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}}) \\
&\quad + \frac{3\beta^2 E_{\mathbf{q}}^3}{|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}})(1 - 2f_{\mathrm{f}}^{<}(E_{\mathbf{q}})), \qquad \text{(B4)}
\end{aligned}
$$

and

$$
\begin{aligned}
&G_{\mathrm{vor3}}(\mathbf{q}) \\
&= \frac{2E_{\mathbf{q}}^2}{|\mathbf{q}|^2}\left(\frac{3E_{\mathbf{q}}^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) - 1\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) \\
&\quad + \frac{\beta E_{\mathbf{q}}^3}{|\mathbf{q}|^2}\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}R_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right) - \frac{2m^2}{|\mathbf{q}|^2}Q_1\left(\frac{E_{\mathbf{q}}}{|\mathbf{q}|}\right)\right) f_{\mathrm{f}}^{<}(E_{\mathbf{q}}) f_{\mathrm{f}}^{>}(E_{\mathbf{q}}), \qquad \text{(B5)}
\end{aligned}
$$

where we have introduced

$$
R_1(x) = Q_0(x) - xQ_1(x). \qquad \text{(B6)}
$$

### 2. Auxiliary functions in the vector-meson self-energies

We list the auxiliary functions appearing in vector-meson self-energy (77):

$$
\begin{aligned}
h_1 &= \left(\frac{q_0(q^2 + 3m_s^2 - m_d^2)}{2|\mathbf{q}|^2} + 3q_0\frac{(q^2 + m_s^2 - m_d^2)^2}{4|\mathbf{q}|^4}\right) \\
&\quad \times H_1(q_0) + \left(\frac{m_s^2 - m_d^2}{|\mathbf{q}|^2} - \frac{3q_0^2(q^2 + m_s^2 - m_d^2)}{|\mathbf{q}|^4}\right) \\
&\quad \times \frac{H_2(q_0)}{\beta} \qquad \text{(B7)} \\
&\quad + 3q_0\frac{q^2}{|\mathbf{q}|^4}\frac{H_3(q_0)}{\beta^2}, \\
h_2 &= \left((3q_0^2 - |\mathbf{q}|^2)\frac{(q^2 + m_s^2 - m_d^2)^2}{4|\mathbf{q}|^4} + m_s^2\frac{q^2}{|\mathbf{q}|^2}\right) H_1(q_0) \\
&\quad - \frac{3q^2}{|\mathbf{q}|^4}q_0(q^2 + m_s^2 - m_d^2)\frac{H_2(q_0)}{\beta} + \frac{3q^4}{|\mathbf{q}|^4}\frac{H_3(q_0)}{\beta^2}, \qquad \text{(B8)} \\
h_3 &= \left(\frac{3(q^2 + m_s^2 - m_d^2)^2}{4|\mathbf{q}|^4} + \frac{q^2 + 2m_s^2 - m_d^2}{|\mathbf{q}|^2}\right) H_1(q_0) \\
&\quad + \left(-\frac{3q_0(q^2 + m_s^2 - m_d^2)}{|\mathbf{q}|^4} - \frac{2q_0}{|\mathbf{q}|^2}\right)\frac{H_2(q_0)}{\beta} \\
&\quad + \frac{3q_0^2 - |\mathbf{q}|^2}{|\mathbf{q}|^4}\frac{H_3(q_0)}{\beta^2}, \qquad \text{(B9)}
\end{aligned}
$$

where the distribution function integral gives

$$
\begin{aligned}
H_1(q_0) &= -\beta m_s + \beta m_s \ln(e^{\beta(q_0 - \mu_d - m_s)} + 1), \\
H_2(q_0) &= \beta m_s \ln(e^{\beta(q_0 - \mu_d - m_s)} + 1) - \mathrm{Li}_2(-e^{\beta(q_0 - \mu_d - m_s)}), \\
H_3(q_0) &= (\beta m_s)^2 \ln(e^{\beta(q_0 - \mu_d - m_s)} + 1) \\
&\quad - 2\beta m_s \mathrm{Li}_2(-e^{\beta(q_0 - \mu_d - m_s)}) - 2\mathrm{Li}_3(-e^{\beta(q_0 - \mu_d - m_s)}),
\end{aligned}
$$

with $\mathrm{Li}_n(z)$ being the polylogarithm functions and $\mu_d$ the chemical potential of the $d$-quark.


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