

Spin versus nonstabilizerness in gluon and graviton scattering

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The quantum property of *nonstabilizerness*, also known as *magic*, plays a key role in designing quantum computing systems. How to produce, manipulate, and enhance magic remains mysterious, such that concrete examples of physical systems that manifest magic behavior are sought after. In this paper, we study two-particle scattering of gluons and gravitons in Yang-Mills theory and general relativity, as well as their supersymmetric extensions. This provides an interesting case of two-qubit systems, differing only in the physical spin of the qubits. We show that magic is generically produced in both theories, and also show that magic typically decreases as the spin of the qubits increases. The maximal magic in each case is found to be substantially less than the known upper bound. Differences in the profile of magic generation can be traced to the known physics of each theory, as manifested in relations between their respective scattering amplitudes. Our case study may provide useful insights into understanding magic in other systems.

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I. INTRODUCTION

Ideas from quantum information theory continue to play an important role in the design and realization of quantum computers and their associated algorithms (see, e.g., Ref. [1] for a detailed review). There is also a growing community of people who are interested in applying similar ideas to high-energy quantum systems such as particle colliders. A key example of the latter is purported tests of entanglement, with a canonical example being the production of pairs of top quarks at the LHC [2–19] (see Ref. [20] for a more general review, and Refs. [21–24] for alternative perspectives). More recently, other quantum information ideas have been investigated in a collider setting [25–31] (see also Refs. [32–56] for related works),

and in the theory of particle scattering and decay processes [57–81], highlighting the various reasons for this increasing interest. Quantum information methods may inform ways to distinguish new physics from the Standard Model of particle physics, a point emphasized particularly by Refs. [4,28,31,37]. Going the other way, physical systems studied by high-energy physicists (however arcane or theoretical) may provide useful insights into open questions that concern the more traditional quantum theory community. It is this latter spirit that motivates the present study.

We will study a particular quantum information theory property called *magic*, also known as *nonstabilizerness*. To understand the latter term, it is sufficient to note that there is a well-defined family of special quantum states—called *stabilizer states*—that can be created using a particular type of quantum gate (i.e., *Clifford gates*). It is known (see, e.g., Ref. [1]) that quantum algorithms containing only stabilizer states lead to no computational advantage over equivalent classical computer algorithms. Thus, to build a genuinely powerful quantum computer, one must ensure that at least some intermediate states are nonstabilizers. “Magic” quantifies the degree to which this occurs, and is also known to be important for building fault-tolerant algorithms, such that the study of magic is related to the biggest open problems in real-world

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quantum computing. As such, there are now even entire conferences devoted to the study of magic in many-body systems.¹

Magic has been studied in a range of quantum systems [82–98], and in Ref. [25] was shown to be present in the same top quark pair system at the LHC that had been previously used to potentially probe entanglement and related new physics signatures. Since then, Ref. [26] considered theoretical upper bounds on magic production in two-qubit systems, and Ref. [27] looked at how efficiently one can generate magic in $2 \rightarrow 2$ scattering processes in quantum electrodynamics (QED). References [28,31,99] have considered how information measures including magic can be used as probes of new physics, and there has also been experimental verification that magic top quarks are routinely produced in nature [29]. Given the importance of magic for quantum computing, and the mystery surrounding how to produce and enhance this quantity in full generality for arbitrary quantum systems, there is a clear role for case studies that look at how magic is produced in given contexts, as has been done in Refs. [25] (for top quarks) and [27] (for QED processes). Furthermore, it is interesting to try to look for closely related systems, and to compare the profile of magic production when one or more physical parameters is varied. This may in turn lead to important insights that can be ported to other systems, including those in condensed matter and/or optics.

The aim of this paper is to perform such a case study, focusing on the particular context of $2 \rightarrow 2$ scattering processes involving gluons or gravitons. The relevant theories in each case are Yang-Mills theory,² and gravity in the form of general relativity. In both cases, the particles being scattered have two polarization states, and hence the initial and final states form a two-qubit system. The effect of the Yang-Mills or gravitational interactions is then to change the initial quantum state, which can possibly lead to the creation of magic in spin space. Furthermore, these theories provide an example of closely related situations: in moving from Yang-Mills to gravity, the (zero) mass and number of polarization states of the scattering particles remain the same, and only the spin changes: from spin 1 for gluons, to spin 2 for gravitons. Thus, these two theories provide a good example of two closely related types of theory, whose lessons may be fruitful for other systems which are related by a change of spin. We will see that both gluon and graviton scattering are capable of generating magic states, even when no magic is present in the initial state. The amount of magic depends upon the scattering

angle and, unsurprisingly, the profile of magic changes according to the nature of the scattering particles.

This leads us to another good reason to study Yang-Mills theory and gravity, which is that their theoretical structures are known to be mathematically very closely related. We will see this explicitly in what follows, in which we will need scattering amplitudes relating initial and final two-qubit states in the two theories. We will restrict ourselves to tree-level scattering, and it is then known that the relevant amplitudes in gravity can be obtained by those in Yang-Mills theory by the so-called *Kawai-Lewellen-Tye (KLT) relations*, originally derived in string theory [100], which state that gravity amplitudes can be written as certain sums of products of gauge theory ones. More recently, such relationships have been generalized to loop-level scattering amplitudes [101–104] and classical solutions [105–110], and the general program for relating quantities in gauge and gravity theories is known as the *double copy* (see, e.g., Refs. [111–113] for reviews). This tight relationship between gauge and gravitational physics means that we will be able to interpret *why* the profile of magic generation differs between our chosen types of theory, and why certain features are similar. This is as useful for interpreting the physics of the double copy as it is for drawing insights about magic. Also, the fact that double copy-like relationships have started to be extended to other theories of possible interest in condensed matter [113] suggests that our insights and methods from this paper may be highly portable. To illustrate this further, we will supplement our analysis of Yang-Mills and gravity with results from supersymmetric generalizations of these theories, whose amplitudes can also be obtained using KLT relations. The additional particles—gluinos and gravitinos—have half-integer spin, and will be seen to fit the same pattern of decreasing magic with increasing spin.

The structure of our paper is as follows. In Sec. II, we review key concepts needed for what follows. In Sec. III, we show how to calculate magic in both Yang-Mills theory and gravity, using known results from the study of scattering amplitudes. We present results for magic and other observables, comparing similarities and differences between the two theories. In Sec. IV, we broaden our results to particles of different spin by using supersymmetric generalizations of Yang-Mills theory and gravity. We discuss our results and conclude in Sec. V.

II. REVIEW OF NECESSARY CONCEPTS

In this section, we review salient details regarding magic of quantum states. For a fuller review of magic in a collider context, we refer the reader to Ref. [25]. Our starting point is to consider an n -qubit system, where each qubit has basis states $|0\rangle$ and $|1\rangle$, such that a basis for the full n -qubit Hilbert space is provided by the states

¹The many-body quantum magic series originated in 2024, in Abu Dhabi, and was held in Seattle in 2025.

²With gauge group $SU(3)$, Yang-Mills theory can be thought of as the theory of the strong force (quantum chromodynamics) without the quarks. However, we will not restrict to a particular gauge group unless otherwise stated.

$$|i\rangle \otimes |j\rangle \otimes \dots \otimes |l\rangle. \quad (2.1)$$

One may then consider the family of *Pauli string* operators

$$\mathcal{P}_n = P_1 \otimes P_2 \otimes \dots \otimes P_N, \quad P_a \in \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\}, \quad (2.2)$$

such that a Pauli or 2×2 identity matrix acts on each individual qubit. The Pauli strings then generate the *Pauli group*, which consists of the Pauli strings weighted by phases of $\pm 1, \pm i$. For n qubits, we define the *Clifford group* $\mathcal{C}_n$ as the normalizer of the Pauli group in $U(2^n)$; concretely,

$$\mathcal{C}_n \equiv \{U \in U(2^n) : U P U^\dagger = e^{i\theta} P'\}, \quad (2.3)$$

where P and P' are Pauli strings and $\theta \in \{0, \pi/2, \pi, 3\pi/2\}$. In quantum computing and quantum information theory, the members of this group are referred to as *Clifford gates*.

A general quantum state $|\psi\rangle$ can be classified by its *Pauli spectrum*

$$\text{spec}(|\psi\rangle) = \{\langle\psi|P|\psi\rangle, P \in \mathcal{P}_n\}, \quad (2.4)$$

namely by the set of 4^n expectation values of each Pauli string. For a given n , a special set of *stabilizer states* can be defined, whose Pauli spectrum has 2^n values equal to ± 1 , and the rest zero. Physically, these are the states obtained by acting on the state

$$|0\rangle \otimes |0\rangle \otimes \dots \otimes |0\rangle \quad (2.5)$$

with Clifford gates. An important result known as the Gottesmann-Knill theorem states that a quantum circuit comprised of only Clifford circuits (thus generating only stabilizer states) may be simulated in polynomial time on a classical computer. It is therefore useful to have a property that measures the “nonstabilizerlessness” of a quantum state $|\psi\rangle$, which can in turn be used to measure the degree of quantum computational advantage a given quantum circuit has over a classical computer. This property has become known as *magic* in the quantum information literature and can be defined in terms of the Pauli spectrum of $|\psi\rangle$, given that this is what decides whether a given state is a stabilizer or not. Various definitions of magic exist, and we will here adopt the family of *stabilizer Rényi entropies* (SREs) of Ref. [86]. For the general case of mixed quantum states, these can be written as

$$M_q(|\psi\rangle) = \frac{1}{1-q} \log_2 \left(\frac{\sum_{P \in \mathcal{P}_n} \langle\psi|P|\psi\rangle^{2q}}{\sum_{P \in \mathcal{P}_n} \langle\psi|P|\psi\rangle^2} \right). \quad (2.6)$$

There is a different SRE for each integer value $q \geq 2$, and the numerator and denominator in the logarithm can be seen to explicitly contain the Pauli spectrum, where there is a sum over all the different possible Pauli strings. One may

show that the quantities of Eq. (2.6) indeed vanish for stabilizer states, and are also additive when combining quantum systems. One may think of the set of values $\{M_q\}$ as characterizing the moments of the Pauli spectrum of $|\psi\rangle$, and it is often sufficient to use the second stabilizer Rényi entropy (SSRE) to quantify nonzero magic, as has been done in a collider context in Refs. [25,28,31].

In this work, we wish to analyze the $2 \rightarrow 2$ scattering of gluons. At leading order in perturbation theory, states of more than two gluons are not accessible, and the full multiparticle Fock space reduces to a two-particle Hilbert space. Generalizing the approach of Ref. [114], we partition this Hilbert space as $\mathcal{H} \simeq \mathcal{H}_{\text{hel}} \otimes \mathcal{H}_{\text{col}} \otimes \mathcal{H}_{\text{kin}}$, where $\mathcal{H}_{\text{hel}} \simeq \mathbb{C}^2 \otimes \mathbb{C}^2$ corresponds to the discrete helicity degrees of freedom, $\mathcal{H}_{\text{col}} \simeq \mathbb{C}^d \otimes \mathbb{C}^d$ to the discrete adjoint degrees of freedom of the d -dimensional gauge group, and $\mathcal{H}_{\text{kin}} \simeq L^2(\mathbb{R}^3 \otimes \mathbb{R}^3)$ to the continuous momentum degrees of freedom. States in these spaces are normalized in the canonical way,

$$\begin{aligned} \langle J|K\rangle &= \delta_{JK}, & \langle ab|cd\rangle &= \delta_{ac}\delta_{bd}, \\ \langle \mathbf{p}_1 \mathbf{p}_2 | \mathbf{p}_3 \mathbf{p}_4 \rangle &= 4E_1 E_2 (2\pi)^6 \delta^{(3)}(\mathbf{p}_3 - \mathbf{p}_1) \delta^{(3)}(\mathbf{p}_4 - \mathbf{p}_2), \end{aligned} \quad (2.7)$$

where $\{|J\rangle\} \equiv \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is a basis for the helicity space, and a, b , etc., denote adjoint (color) indices. The completeness relation is then fixed to

$$\mathbb{1} = \sum_J \sum_{ab} \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \frac{d^3 \mathbf{p}_2}{(2\pi)^3} \frac{1}{4E_1 E_2} |J; ab; \mathbf{p}_1 \mathbf{p}_2\rangle \langle J; ab; \mathbf{p}_1 \mathbf{p}_2|. \quad (2.8)$$

We assume an initial state of the form

$$|\text{in}\rangle = \frac{1}{\sqrt{V}} |J\rangle \otimes |a_1 a_2\rangle \otimes |\mathbf{p}_1 \mathbf{p}_2\rangle \equiv \frac{1}{\sqrt{V}} |J; a_1 a_2; \mathbf{p}_1 \mathbf{p}_2\rangle, \quad (2.9)$$

where $V = 4E_1 E_2 (2\pi)^6 \delta^{(3)}(0) \delta^{(3)}(0)$ is a normalization such that $\langle \text{in} | \text{in} \rangle = 1$.³ The final state is then given by $|\text{out}\rangle = S |\text{in}\rangle$, where $S = \mathbb{1} + iT$ is the S -matrix. We project out a particular choice of final-state color indices a_3, a_4 and momenta $\mathbf{p}_3, \mathbf{p}_4$ —this corresponds to a fictitious *measurement* of these quantum numbers.⁴ Assuming

³One could imagine using states that are not perfectly sharp in momentum, then, the normalization would not be formally infinite. In any event, this normalization will factor out shortly.

⁴Gluons (and quarks) are confined, and thus the asymptotic Hilbert space consists of color singlets. At high energies, though—before hadronization takes over—it is mathematically convenient to work with states that carry definite color indices. These states should be understood as formal artifacts: they are not physical asymptotic states, but they allow us to compute partonic cross sections and, in the present context, quantum-information-theoretic quantities.

$a_3 a_4 \neq a_1 a_2$, it can be shown using the completeness relation that the projected state is

$$\begin{aligned} |\psi\rangle &= |a_3 a_4; \mathbf{p}_3 \mathbf{p}_4\rangle \langle a_3 a_4; \mathbf{p}_3 \mathbf{p}_4 | \text{out}\rangle \\ &= \frac{(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)}{\sqrt{V}} \\ &\quad \times \sum_K iA(J \rightarrow K) |K; a_3 a_4; \mathbf{p}_3 \mathbf{p}_4\rangle, \end{aligned} \quad (2.10)$$

introducing the shorthand notation for the scattering amplitude

$$\begin{aligned} &(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) iA(J \rightarrow K) \\ &\equiv \langle K; a_3 a_4; \mathbf{p}_3 \mathbf{p}_4 | iT | J; a_1 a_2; \mathbf{p}_1 \mathbf{p}_2 \rangle \end{aligned} \quad (2.11)$$

(again, fixing a_1, a_2, a_3, a_4 and the external momenta). Since we have performed this projection, we are required to explicitly normalize the state. We thus write

$$\begin{aligned} |\psi\rangle &\rightarrow \frac{1}{\sqrt{\langle \psi | \psi \rangle}} |\psi\rangle \\ &= \frac{1}{\sqrt{\langle \psi | \psi \rangle}} \sum_K iA(J \rightarrow K) |K; a_3 a_4; \mathbf{p}_3 \mathbf{p}_4\rangle. \end{aligned} \quad (2.12)$$

Going forward, we will suppress the color and momentum labels when writing the states, but their presence should be implicitly assumed. We note that the combination of (i) starting in an initial state of the form of Eq. (2.9), (ii) the projection of Eq. (2.10) onto a color computational basis state, and (iii) the necessary normalization step of Eq. (2.12) are sufficient ingredients to ensure that the out state $|\psi\rangle$ has no color dependence. That is, nonvanishing out states subject to the aforementioned criteria are equal (up to a phase) for different initial color states and postscattering projections. This feature was called *universality* in Ref. [115]. We have verified explicitly that universality is *not* realized if the initial color state is a superposition of states with different color indices.

III. MAGIC IN YANG-MILLS THEORY AND GRAVITY

Having introduced the framework for calculating magic in $2 \rightarrow 2$ scattering processes of qubits, let us now turn to the particular case of gluon scattering. More precisely, we will consider the process shown in Fig. 1, such that we must keep track of the momentum p_i of each gluon and its color index a_i . For both the initial and final states, we must also choose a map from spin space to physical space, thus defining the meaning of the states $|0\rangle$ and $|1\rangle$ for each qubit. For both the initial and final states, we will take $|0\rangle$ and $|1\rangle$ to be the gluon states of positive and negative helicity, respectively, where the helicity measures whether the spin is (anti-)aligned with the momentum direction. In physical

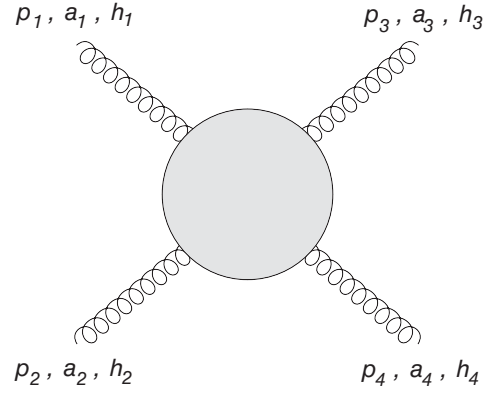


FIG. 1. The $2 \rightarrow 2$ scattering of gluons, where $\{p_i\}$, $\{a_i\}$, and $\{h_i\}$ label momenta, color indices, and helicities, respectively. Initial momenta are taken to be incoming, and final-state momenta outgoing.

terms, the positive (negative) helicity state corresponds to right (left) circular polarization of the gluon. From now on, we will make our choice of basis clear by relabeling as follows:

$$|J\rangle \in \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}. \quad (3.1)$$

We further denote the helicity of gluon i when needed by h_i , as shown in Fig. 1. We take the initial-state momenta to be *incoming*, and the final-state momenta to be *outgoing*, such that p_i corresponds to the 4-momentum one would observe in the physical scattering process. Our reason for stressing this is that relevant results in the quantum field theory literature are often written down for all outgoing momenta. To convert from an outgoing initial-state gluon to an incoming one, one must take $p_i \rightarrow -p_i$ and $h_i \rightarrow -h_i$ (i.e., the helicity also flips).

In our chosen basis, the scattering amplitudes appearing in Eq. (2.11) will be so-called *helicity amplitudes*, in which the helicity of all particles is fixed. That this choice is particularly convenient stems from the fact that helicity amplitudes are very well-studied quantities in both Yang-Mills theory and gravity (see, e.g., Refs. [116–118] for excellent contemporary reviews of helicity methods for scattering amplitudes). A first useful result is that the total tree-level result for an n -point amplitude (with all outgoing momenta) can be written as

$$\begin{aligned} \mathcal{A}_n^{\text{tot}}[1^{h_1} 2^{h_2} \dots n^{h_n}] &= g^{n-2} \sum_{\text{perms } \sigma} \mathcal{A}_n[1^{h_1} \sigma(2^{h_2} \dots n^{h_n})] \\ &\quad \times \text{Tr}[\mathbf{T}^{a_1} \sigma(\mathbf{T}^{a_2} \dots \mathbf{T}^{a_n})]. \end{aligned} \quad (3.2)$$

We have here used a common notation in which, e.g., $\mathcal{A}_4[1^+ 2^+ 3^+ 4^+]$ represents a four-point amplitude with all positive helicities, and color indices and momenta labeled according to Fig. 1 (but with all momenta outgoing). On the right-hand side of Eq. (3.2), g is the coupling constant, $\mathbf{T}^{a_1}$

is a color generator, and the sum is over all permutations σ of $(2\dots n)$. The quantity

$$\mathcal{A}_n[1^{h_1}\sigma(2^{h_2}\dots n^{h_n})] \quad (3.3)$$

is called a *color-ordered amplitude*, due to the fact that it is the coefficient of a particular ordering of color generators. Much is known about these quantities. For example, one may show that they vanish if either (i) all helicities are equal (for outgoing momenta) or (ii) all but one of the helicities are equal (for outgoing momenta). Thus, the first nonzero amplitudes occur if two helicities are different to all the rest, and such amplitudes are thus called *maximally helicity violating* (MHV). For our special case of $2 \rightarrow 2$ scattering, we have $n = 4$, and the only nonvanishing color-ordered amplitudes will have (for all outgoing momenta) two positive helicity states, and two negative ones. Translating back to incoming momenta in the initial state, this means that the only nonzero amplitudes are

$$\begin{aligned} &\mathcal{A}(++ \rightarrow ++), \mathcal{A}(-- \rightarrow --), \mathcal{A}(+- \rightarrow +-), \\ &\mathcal{A}(+- \rightarrow -+), \mathcal{A}(-+ \rightarrow -+), \mathcal{A}(-+ \rightarrow +-). \end{aligned}$$

A known result exists for tree-level MHV amplitudes, for any number of gluons. Returning to the case of all outgoing momenta, one has the so-called *Parke-Taylor formula* [119]

$$\mathcal{A}_n[1^+ \dots i^- \dots j^- \dots n^+] = \frac{\langle ij \rangle^4}{(12)\langle 23 \rangle \dots \langle n1 \rangle}, \quad (3.4)$$

written in terms of *spinor products*

$$\langle ij \rangle = e^{i\phi_{ij}} \sqrt{(p_i + p_j)^2}, \quad (3.5)$$

defined for outgoing momenta, and where ϕ_{ij} is a real phase. This provides all the ingredients needed for computing the amplitude matrix appearing in Eq. (2.11). First, one may evaluate all Parke-Taylor amplitudes using Eq. (3.5) which, upon switching back to incoming momenta in the initial state, can be written in terms of the Mandelstam invariants

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2. \quad (3.6)$$

It is also convenient to transform from the color basis appearing in Eq. (3.2) to one involving explicit products of structure constants, and a particularly convenient choice is the Del-Duca–Dixon–Maltoni (DDM) basis [120], given in terms of two independent color structures,

$$c_{1234} = f^{a_1 a_2 c} f^{a_3 a_4 c}, \quad c_{1324} = f^{a_1 a_3 c} f^{a_2 a_4 c}. \quad (3.7)$$

In this basis, the four-gluon amplitude is given by

$$\begin{aligned} \mathcal{A}[1^{h_1} 2^{h_2} 3^{h_3} 4^{h_4}] &= [c_{1234} \mathcal{A}_4[1^{h_1} 2^{h_2} 3^{h_3} 4^{h_4}] \\ &\quad + c_{1324} \mathcal{A}_4[1^{h_1} 3^{h_3} 2^{h_2} 4^{h_4}]], \end{aligned} \quad (3.8)$$

where color-ordered amplitudes $\mathcal{A}_4$ can be computed via the Parke-Taylor formula in Eq. (3.4). Using the relation between spinor products and Mandelstam variables, we can express the amplitudes as

$$\begin{aligned} \mathcal{A}(++ \rightarrow ++) &= \mathcal{A}(-- \rightarrow --) \\ &= g^2 \left[c_{1234} \left(\frac{s}{u} \right) + c_{1324} \left(\frac{s^2}{tu} \right) \right], \\ \mathcal{A}(+- \rightarrow ++) &= \mathcal{A}(-+ \rightarrow -+) \\ &= g^2 \left[c_{1234} \left(\frac{u}{s} \right) + c_{1324} \left(\frac{u}{t} \right) \right], \\ \mathcal{A}(+- \rightarrow -+) &= \mathcal{A}(-+ \rightarrow -+) \\ &= g^2 \left[c_{1234} \left(\frac{t^2}{su} \right) + c_{1324} \left(\frac{t}{u} \right) \right], \end{aligned} \quad (3.9)$$

where we have restricted ourselves to real, $2 \rightarrow 2$ kinematics, where the phase ϕ_{ij} in Eq. (3.5) is given by

$$\phi_{ij} = (1 - \Theta(\langle ij \rangle))\pi, \quad (3.10)$$

with $\Theta(x)$ being the Heaviside function.

Scattering amplitudes in quantum general relativity may be defined by expanding the full metric of spacetime in terms of the (vacuum) Minkowski part, plus a correction,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (3.11)$$

Here, $\kappa = \sqrt{32\pi G_N}$ in terms of the Newton constant G_N , and $h_{\mu\nu}$ is known as the graviton field, representing a spin two and massless particle. By expanding the action for general relativity in terms of the graviton, one obtains an infinite series of graviton interactions, and can apply the language of quantum field theory and Feynman rules in order to calculate scattering processes.⁵ It turns out, however, that there is a remarkably simple way to obtain $2 \rightarrow 2$ graviton amplitudes directly from the corresponding Yang-Mills ones. First, one may note that each graviton has two helicity states, so that one may define the basis of Eq. (3.1) also for gravitons. It then turns out that the same rules apply regarding individual helicity amplitudes, such that only MHV configurations are nonzero at four points. Finally, the tree-level four-point graviton amplitude for any

⁵General relativity is nonrenormalizable, but can be treated as a highly convergent effective theory involving increasingly complicated graviton interactions order-by-order in perturbation theory.

external helicity configuration is given by a product of color-ordered amplitudes,

$$\mathcal{A}^{(2)}[1^{h_1}2^{h_2}3^{h_3}4^{h_4}] = -\frac{\kappa}{2}t\mathcal{A}_4^{(1)}[1^{h_1}2^{h_2}3^{h_3}4^{h_4}] \times \mathcal{A}_4^{(1)}[1^{h_1}3^{h_3}2^{h_2}4^{h_4}]. \quad (3.12)$$

[We hereafter adopt the notation $\mathcal{A}^{(s)}$ for the amplitudes, where s denotes the spin of the appropriate particle species.] This is a special case of a general family of relations known as the *KLT relations* [100], which relate graviton amplitudes to sums of products of color-ordered gluon amplitudes. Note, however, that color degrees of freedom themselves are absent in Eq. (3.12), as they should be given that there is no color in gravity. Furthermore, the Yang-Mills coupling has been replaced with the gravitational coupling (up to a numerical factor). The KLT relations originated in string theory, in which closed string amplitudes can be written as products of open string amplitudes. Upon taking the low-energy limit, closed strings give rise to gravitons (roughly speaking), and open strings give rise to gluons, leading to results such as Eq. (3.12). As mentioned in the Introduction, more recent research has shown that relations between gauge theory and gravity go much further [101–104], such that studies of quantum information in the different theories can also be generalized significantly beyond the initial analysis of this paper. Applying Eq. (3.12) to the present case of tree-level $2 \rightarrow 2$ scattering, we find nonzero graviton amplitudes (for incoming initial-state momenta),

$$\begin{aligned} \mathcal{A}^{(2)}(++) \rightarrow ++ &= \mathcal{A}^{(2)}(-- \rightarrow --) = -\frac{\kappa^2}{2} \left(\frac{s^3}{ut} \right), \\ \mathcal{A}^{(2)}(+- \rightarrow +-) &= \mathcal{A}^{(2)}(-+ \rightarrow -+) = -\frac{\kappa^2}{2} \left(\frac{u^3}{st} \right), \\ \mathcal{A}^{(2)}(+- \rightarrow -+) &= \mathcal{A}^{(2)}(-+ \rightarrow +-) = -\frac{\kappa^2}{2} \left(\frac{t^3}{su} \right). \end{aligned} \quad (3.13)$$

Equations (3.9) and (3.13) may be used in Eq. (2.11) to convert an arbitrary initial gluon or graviton state into an unnormalized final state, from which one may calculate the magic according to Eq. (2.6) after normalization. In presenting results, we will begin by taking a particular initial state $|+-\rangle$. This has been considered recently in Ref. [115], in the context of studying how entanglement can be generated in gluon scattering. The $|++\rangle$ and $|--\rangle$ initial states create no entanglement in the final state, and results from $|+-\rangle$ are the same as those from $|+-\rangle$, due to the relations between amplitudes in Eqs. (3.9) and (3.13). Given that it is interesting to compare magic and entanglement in the final state, let us quantify the latter, as in Ref. [115], using the *concurrence*, which in our present notation reads

$$\Delta = |a_{++}a_{--} - a_{+-}a_{-+}|, \quad (3.14)$$

where it is understood that the final state $|\psi\rangle \equiv \sum_J a_J |J\rangle$ has already been normalized. We may then evaluate both the concurrence and magic (as measured by the SSRE M_2) in the center-of-mass frame, in which the momenta are given by

$$\begin{aligned} p_1 &= (E, 0, 0, E), \\ p_2 &= (E, 0, 0, -E), \\ p_3 &= (E, E \sin \theta, 0, E \cos \theta), \\ p_4 &= (E, -E \sin \theta, 0, -E \cos \theta), \end{aligned} \quad (3.15)$$

where E is the energy of each gluon or graviton, and θ is the scattering angle. The Mandelstam invariants are then given by

$$\begin{aligned} s &= 4E^2, \quad t = -4E^2 \sin^2 \left(\frac{\theta}{2} \right), \\ u &= -4E^2 \cos^2 \left(\frac{\theta}{2} \right). \end{aligned} \quad (3.16)$$

For our particular final state, we find that the concurrence and magic are given by

$$\begin{aligned} \Delta(|\psi\rangle) &= \frac{2t^2u^2}{t^4 + u^4}, \\ M_2(|\psi\rangle) &= -\log_2 \left(\frac{t^{16} + 14t^8u^8 + u^{16}}{(t^4 + u^4)^4} \right). \end{aligned} \quad (3.17)$$

The first equation agrees with the result found in Ref. [115], including the universality in color mentioned above. A plot of the concurrence and magic is shown in Fig. 2, as a function of the scattering angle. The entanglement rises from minimal in the forward direction $\theta = 0$, to maximal in the central direction $\theta = \pi/2$, as noted in

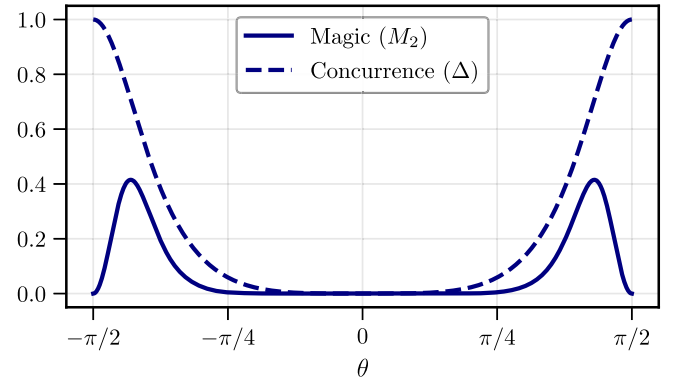


FIG. 2. The magic M_2 (solid) and concurrence Δ of the final state obtained from the initial gluon state $|+-\rangle$, as a function of the scattering angle θ .

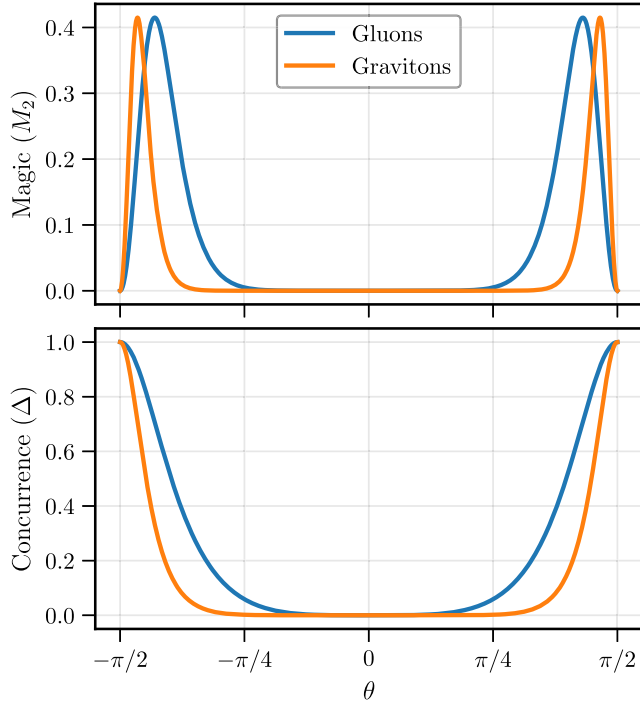


FIG. 3. Top: Magic M_2 of the final state obtained from the initial state $|+-\rangle$ for gluons (blue) and gravitons (orange). Bottom: Similar, but for the concurrence.

Ref. [115]. The magic profile, however, is very different. It rises slowly from zero at $\theta = 0$, reaching a maximum before decaying to zero at $\theta = \pi/2$. Indeed, this can be understood by examining the explicit form of the final state at extremal angles,

$$\begin{aligned} |\psi(\theta = 0)\rangle &= |+-\rangle, \\ |\psi(\theta = \pi/2)\rangle &= \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle). \end{aligned} \quad (3.18)$$

Both of these turn out to be stabilizer states, and hence the magic vanishes. Indeed, this follows a pattern previously seen in top quark pair production, whose final state consists of two massive spin-1/2 particles [25]. There, magic is large when entanglement is low, and vanishes in extremal regions of scattering angle. The detailed relationship between magic and entanglement is an ongoing research area, and our results confirm the view that magic provides very different information to entanglement in general.

In the top panel of Fig. 3, we compare the magic for gluons and gravitons, for the same final state as above (i.e., that obtained from the $|+-\rangle$ initial state). Interestingly, the qualitative shape of the magic profile is the same in both theories, but the region of nonzero magic is much more concentrated in gravity than in Yang-Mills theory. This can be traced directly to the fact that gravity amplitudes can be obtained by *multiplying* Yang-Mills amplitudes, via the

KLT relations. This introduces higher powers of Mandelstam invariants, as can be seen from the fact that the gravitational equivalent of Eq. (3.17) is found to be

$$M_2(|\psi\rangle_{\text{grav}}) = -\log_2 \left(\frac{t^{32} + 14t^{16}u^{16} + u^{32}}{(t^8 + u^8)^4} \right). \quad (3.19)$$

The higher powers in the numerator of the logarithm give rise to a slower rise of the magic from $\theta = 0$, as can be seen by comparing the Taylor expansions of the magic from Eqs. (3.17) and (3.19),

$$\begin{aligned} M_2(|\psi\rangle_{\text{YM}}) &= \frac{\theta^8}{64 \log(2)} + \mathcal{O}(\theta^{10}), \\ M_2(|\psi\rangle_{\text{grav}}) &= \frac{\theta^{16}}{16384 \log(2)} + \mathcal{O}(\theta^{18}). \end{aligned} \quad (3.20)$$

Given that double copy-like relationships are by no means limited to gauge theory and gravity, it may well be the case that a similar mechanism leads to concentration of magic for different types of qubit in other physical systems. For this particular final state, it is also interesting that the maximal value of the magic does not change in moving from Yang-Mills to gravity, as can clearly be seen in the top panel of Fig. 3. This will not turn out to be true for less special initial states, as we will see below. Before moving on, however, it is interesting to note that the concentration effect in moving to gravity is also seen for entanglement (for our particular initial state), as shown in the bottom panel of Fig. 3.

Inspired by Ref. [27], we can examine more general cases of magic by taking for the initial state each of the 60 two-qubit stabilizer states.⁶ Given that the magic of the initial state must then be zero by definition, any magic in the final state must be generated by the scattering process itself. It is not difficult to find cases in which the gravity magic profile is qualitatively different to the gluon one, and an example is shown in Fig. 4 based on the initial stabilizer state

$$|\psi\rangle = \frac{1}{2}(|++\rangle + |+-\rangle + |-+\rangle + |--\rangle). \quad (3.21)$$

Physically, this corresponds to the case of unpolarized gluon or graviton beams, and we see in this case that the magic is nonvanishing in the central region $\theta = \pi/2$ for both gluons and gravitons. However, the magic is maximized for gluons in the central region, but has a local minimum for gravity.

This leads us to examine the “amount” of magic in Yang-Mills theory and gravity. One way to examine this is to look at the maximum value of magic in the final state, upon cycling over all stabilizer states in the initial state.

⁶A full list of coefficients for the stabilizer states ($\{a_j\}$ in our notation) can be found in Appendix A of Ref. [27].

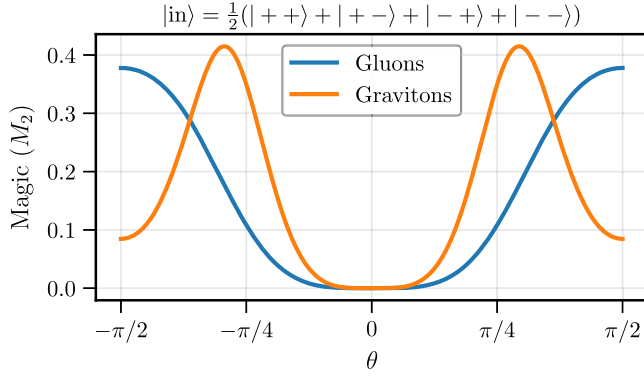


FIG. 4. Magic M_2 of the final state arising from the initial stabilizer state of Eq. (3.21), for gluons (blue) and gravitons (orange).

Numerically, we find values

$$M_2^{\max}|_{\text{YM}} = 0.530, \quad M_2^{\max}|_{\text{grav}} = 0.415. \quad (3.22)$$

This suggests that the typical amount of magic is less in gravity than in Yang-Mills theory, and indeed this can be made more precise by considering the *magic power*, defined as the mean final-state magic obtained from all

stabilizer initial states. Denoting the set of stabilizer states by $\mathcal{S}$ and the final state corresponding to a given initial state ξ as $|\psi(\xi)\rangle$, we may write the magic power as

$$\overline{M}_2 = \frac{1}{60} \sum_{\xi \in \mathcal{S}} M_2(|\psi(\xi)\rangle), \quad (3.23)$$

and find the following results in Yang-Mills theory and gravity:

$$\begin{aligned} \overline{M}_2|_{\text{YM}} = & -\frac{1}{15} \left[2\log_2 \left(\frac{s^{16} + 14s^8(t^2 - u^2)^4 + (t^2 - u^2)^8}{(s^4 + (t^2 - u^2)^2)^4} \right) \right. \\ & + 2\log_2 \left(\frac{s^{16} + 14s^8(t^2 + u^2)^4 + (t^2 + u^2)^8}{(s^4 + (t^2 + u^2)^2)^4} \right) \\ & + 8\log_2 \left(\frac{s^{16} + 14s^8(t^8 + u^8) + t^{16} + 14t^8u^8 + u^{16}}{(s^4 + t^4 + u^4)^4} \right) \\ & \left. + \log_2 \left(\frac{t^{16} + 14t^8u^8 + u^{16}}{(t^4 + u^4)^4} \right) \right], \end{aligned} \quad (3.24)$$

$$\begin{aligned} \overline{M}_2|_{\text{grav}} = & -\frac{1}{15} \left[2\log_2 \left(\frac{s^{32} + 14s^{16}(t^4 - u^4)^4 + (t^4 - u^4)^8}{(s^8 + (t^4 - u^4)^2)^4} \right) + \log_2 \left(\frac{s^{32} + 14s^{16}(t^4 + u^4)^4 + (t^4 + u^4)^8}{(s^8 + (t^4 + u^4)^2)^4} \right) \right. \\ & \left. + 8\log_2 \left(\frac{s^{32} + 14s^{16}(t^{16} + u^{16}) + t^{32} + 14t^{16}u^{16} + u^{32}}{(s^8 + t^8 + u^8)^4} \right) + \log_2 \left(\frac{t^{32} + 14t^{16}u^{16} + u^{32}}{(t^8 + u^8)^4} \right) \right]. \end{aligned} \quad (3.25)$$

These expressions are not unique, due to the ability to recombine terms using $s + t + u = 0$. However, we note the close relationship in form between the results, where powers of Mandelstam invariants in the Yang-Mills theory result are doubled in the gravity result. Again, this is directly traceable to the KLT relations (alternatively, the double copy), which multiply together kinematic contri-

butions from Yang-Mills amplitudes. In Fig. 5, we plot the magic power as a function of the scattering angle. Due to the different combinations of Mandelstam invariants occurring, and the interplay between different logarithmic terms, we see that the magic power peaks closer to $\theta = 0$ for gravity, and extends to higher values of θ on average for gluons. Consistent with the above remarks, the peak of the magic power for gravity is lower than that for Yang-Mills theory. The integrated magic power—which we can evaluate numerically—is also lower,

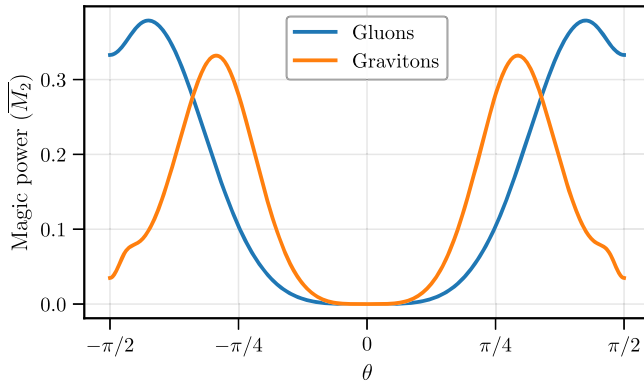


FIG. 5. The magic power of Eq. (3.23) for gluons (blue) and gravitons (orange), as a function of scattering angle θ .

$$\begin{aligned} \int_0^{\pi/2} \overline{M}_2(\theta)|_{\text{YM}} d\theta &= 0.245, \\ \int_0^{\pi/2} \overline{M}_2(\theta)|_{\text{grav}} d\theta &= 0.208. \end{aligned} \quad (3.26)$$

Thus, the amount of magic generated in gravity is indeed typically lower than that in a lower-spin theory. Qualitatively similar results are obtained for higher SREs. For example, in Fig. 6 we plot the behavior of $\overline{M}_4$ and $\overline{M}_{10}$, defined similarly to Eq. (3.23).

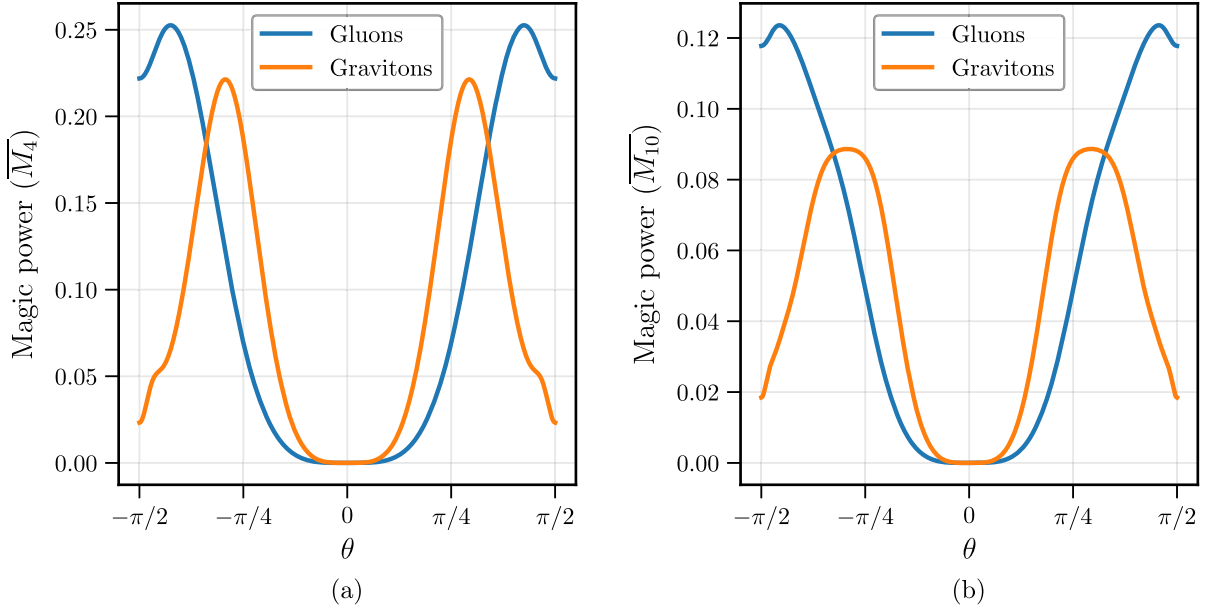


FIG. 6. (a) The magic power $\overline{M}_4$ as a function of scattering angle, for gluons (blue) and gravitons (orange). (b) Similar, but for $\overline{M}_{10}$.

Reference [26] conjectured (with strong numerical evidence) a theoretical upper bound for the maximum SSRE of a two-qubit state,

$$M_2^{\max} = \log_2\left(\frac{16}{7}\right) \simeq 1.193. \quad (3.27)$$

It is interesting to compare the maximum magic we have found in Eq. (3.22) with this bound, and we find that it is significantly lower in both Yang-Mills theory and gravity. This also matches the observation of Ref. [27], which found that QED is unable to generate the maximum value of magic for most $2 \rightarrow 2$ scattering processes. Nevertheless, that reference found higher values than observed here, in processes involving spin-1/2 qubits. Combined with our results above, this suggests that typical values of magic may decrease as the spin of qubits increases. This is a very different behavior to, e.g., entanglement, given that maximum entanglement is always possible, regardless of the spin.

IV. RESULTS FOR OTHER SPINS

To investigate further the issue of how spin affects the typical amount of magic, we can replace the theories considered thus far with their supersymmetric extensions. Supersymmetry adds extra particle content to field theories, where the spins of the additional particles differ from their partners by half integers. Thus, by considering supersymmetric extensions of Yang-Mills theory and

gravity, we can repeat our calculations for the scattering of spin-1/2 *gluinos*, and spin-3/2 *gravitinos*. Whilst the applicability of supersymmetric gauge theory to our own Universe may be an open question, the novelty of such theories in our present context is that they provide additional data points for what happens when one changes the spin of a qubit.

In order to derive the gluino amplitudes, we can use the supersymmetric Ward identities that relate color-ordered gluon amplitudes with color-ordered gluino amplitudes [121–124]. Again restricting ourselves to real, $2 \rightarrow 2$ kinematics, these identities are given by

$$\begin{aligned} \mathcal{A}_4^{(1/2)}[1_A^- 2_B^- 3_C^+ 4_D^+] &= \frac{t\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{s} \mathcal{A}_4^{(1)}[1^- 2^- 3^+ 4^+], \\ \mathcal{A}_4^{(1/2)}[1_A^- 2_B^+ 3_C^- 4_D^+] &= \frac{s\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{t} \mathcal{A}_4^{(1)}[1^- 2^+ 3^- 4^+], \\ \mathcal{A}_4^{(1/2)}[1_A^- 2_B^+ 3_C^+ 4_D^-] &= \frac{s\delta_{AC}\delta_{BD} + t\delta_{AD}\delta_{BC}}{u} \mathcal{A}_4^{(1)}[1^- 2^+ 3^+ 4^-], \end{aligned} \quad (4.1)$$

where A, B, C, D are $SU(\mathcal{N})_R$ flavor indices (see Appendix). The first thing to note is that when there is only a single flavor, as is the case in $\mathcal{N} = 1$ supersymmetry (SUSY), the gluino and gluon amplitudes are identified up to a phase, as can be seen applying $s + t + u = 0$ above along with $A = B = C = D$. Note that the color ordering does not affect the prefactors, so we can write out the full gluino amplitude in the DDM basis, flipping to incoming

momentum in the initial state as before to find

$$\begin{aligned}\mathcal{A}^{(1/2)}[1_A^+ 2_B^+ \rightarrow 3_C^+ 4_D^+] &= g^2 \left(\frac{t\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{s} \right) \left[c_{1234} \left(\frac{s}{u} \right) + c_{1324} \left(\frac{s^2}{tu} \right) \right], \\ \mathcal{A}^{(1/2)}[1_A^+ 2_B^- \rightarrow 3_C^- 4_D^+] &= g^2 \left(\frac{s\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{t} \right) \left[c_{1234} \left(\frac{t^2}{su} \right) + c_{1324} \left(\frac{t}{u} \right) \right], \\ \mathcal{A}^{(1/2)}[1_A^+ 2_B^- \rightarrow 3_C^+ 4_D^-] &= g^2 \left(\frac{s\delta_{AC}\delta_{BD} + t\delta_{AD}\delta_{BC}}{u} \right) \left[c_{1234} \left(\frac{u}{s} \right) + c_{1324} \left(\frac{u}{t} \right) \right].\end{aligned}\quad (4.2)$$

We now take particles (1,3) to share the same flavor, and (2,4) likewise, with the two flavors being distinct. In this setup, the amplitudes reduce to

$$\begin{aligned}\mathcal{A}^{(1/2)}[1_A^+ 2_B^+ \rightarrow 3_A^+ 4_B^+] &= g^2 \left[c_{1234} \left(\frac{t}{u} \right) + c_{1324} \left(\frac{s}{u} \right) \right], \\ \mathcal{A}^{(1/2)}[1_A^+ 2_B^- \rightarrow 3_A^- 4_B^+] &= g^2 \left[c_{1234} \left(\frac{t}{u} \right) + c_{1324} \left(\frac{s}{u} \right) \right], \\ \mathcal{A}^{(1/2)}[1_A^+ 2_B^- \rightarrow 3_A^+ 4_B^-] &= g^2 \left[c_{1234} + c_{1324} \left(\frac{s}{t} \right) \right].\end{aligned}\quad (4.3)$$

The KLT relations in Eq. (3.12) are valid even when the two color-ordered amplitudes belong to different particle species. Pairing up the four-gluon amplitude with the four-gluino amplitude with the same helicity configuration results in the four-gravitino amplitude. This is an example of the double copy for supersymmetric theories, where such particles are naturally a part of the larger SUSY multiplet.

Picking the first amplitude in the KLT relations to be a color-ordered gluino amplitude, written in terms of a color-ordered gluon amplitude, tells us that the gravitino amplitudes satisfy the same Ward identities, and are therefore given by

$$\begin{aligned}\mathcal{A}^{(3/2)}[1_A^+ 2_B^+ \rightarrow 3_C^+ 4_D^+] &= \left(\frac{t\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{s} \right) \mathcal{A}^{(2)}(++ \rightarrow ++), \\ \mathcal{A}^{(3/2)}[1_A^+ 2_B^- \rightarrow 3_C^- 4_D^+] &= \left(\frac{s\delta_{AC}\delta_{BD} + u\delta_{AD}\delta_{BC}}{t} \right) \mathcal{A}^{(2)}(+- \rightarrow -+), \\ \mathcal{A}^{(3/2)}[1_A^+ 2_B^- \rightarrow 3_C^+ 4_D^-] &= \left(\frac{s\delta_{AC}\delta_{BD} + t\delta_{AD}\delta_{BC}}{u} \right) \mathcal{A}^{(2)}(+- \rightarrow +-).\end{aligned}\quad (4.4)$$

Again restricting to $A = C$ and $B = D$ this gives

$$\begin{aligned}\mathcal{A}^{(3/2)}[1_A^+ 2_B^+ \rightarrow 3_A^+ 4_B^+] &= -\frac{\kappa^2}{2} \left(\frac{s^2}{u} \right), \\ \mathcal{A}^{(3/2)}[1_A^+ 2_B^- \rightarrow 3_A^- 4_B^+] &= -\frac{\kappa^2}{2} \left(\frac{t^2}{u} \right), \\ \mathcal{A}^{(3/2)}[1_A^+ 2_B^- \rightarrow 3_A^+ 4_B^-] &= -\frac{\kappa^2}{2} \left(\frac{u^2}{t} \right).\end{aligned}\quad (4.5)$$

We will consider the same initial state as in the last section, from which we may compute the magic M_2 and concurrence. For gluinos, we find that this is

$$\Delta(|\psi\rangle_{\text{gluino}}) = \frac{2tu}{t^2 + u^2}, \quad M_2(|\psi\rangle_{\text{gluino}}) = -\log_2 \left(\frac{t^8 + 14t^4u^4 + u^8}{(t^2 + u^2)^4} \right), \quad (4.6)$$

whereas for gravitinos we get

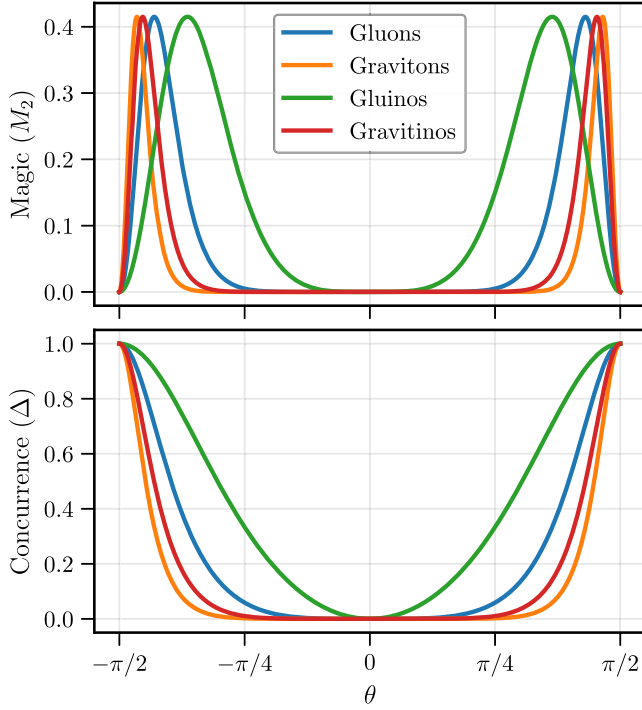


FIG. 7. Top: Magic M_2 of the final state obtained from the initial state $|+\rangle$ for gluons (blue), gravitons (orange), gluinos (green), and gravitinos (red). Bottom: Similar, but for the concurrence.

$$\Delta(|\psi\rangle_{\text{gravitino}}) = \frac{2t^3u^3}{t^6 + u^6},$$

$$M_2(|\psi\rangle_{\text{gravitino}}) = -\log_2\left(\frac{t^{24} + 14t^{12}u^{12} + u^{24}}{(t^6 + u^6)^4}\right), \quad (4.7)$$

and we find that the magic and concurrence for massless particles of any spin s are given by

$$\Delta(|\psi\rangle_s) = \frac{2t^{2s}u^{2s}}{t^{4s} + u^{4s}},$$

$$M_2(|\psi\rangle_s) = -\log_2\left(\frac{t^{16s} + 14t^{8s}u^{8s} + u^{16s}}{(t^{4s} + u^{4s})^4}\right). \quad (4.8)$$

We plot these in Fig. 7, together with our previous results for gluons and gravitons. As the spin increases from 1/2 to 2, the concentration of the magic and entanglement toward higher scattering angles becomes progressively stronger, thus corroborating the effect seen earlier. Similarly as for gluons and gravitons, we can compute the magic power for gluinos and gravitinos, finding

$$\begin{aligned} \overline{M}_2|_{\text{gluino}} &= -\frac{1}{15} \left[\log_2\left(\frac{t^8 + 14t^4u^4 + u^8}{(t^2 + u^2)^4}\right) + 8\log_2\left(\frac{16t^8 + 28t^4u^4 + u^8}{(2t^2 + u^2)^4}\right) \right. \\ &\quad \left. + 2\log_2\left(\frac{256t^{16} - 512t^{14}u^2 + 896t^{12}u^4 - 208t^8u^8 + 56t^4u^{12} - 8t^2u^{14} + u^{16}}{(4t^4 + u^4)^4}\right) \right], \\ \overline{M}_2|_{\text{gravitino}} &= -\frac{1}{15} \left[2\log_2\left(\frac{s^{16}t^8 + 14s^8t^4(t^3 + u^4)^4 + (t^3 + u^3)^8}{(s^4t^2 + (t^3 + u^3)^2)^4}\right) \right. \\ &\quad + 2\log_2\left(\frac{s^{16}t^8 + 14s^8(t^4 - tu^3)^4 + (t^3 - u^3)^8}{(s^4t^2 + (t^3 - u^3)^2)^4}\right) \\ &\quad + 8\log_2\left(\frac{s^{16}t^8 + 14s^8t^4(t^{12} + u^{12}) + t^{24} + 14t^{12}u^{12} + u^{24}}{(s^4t^2 + t^6 + u^6)^4}\right) \\ &\quad \left. + \log_2\left(\frac{t^{24} + 14t^{12}u^{12} + u^{24}}{(t^6 + u^6)^4}\right) \right]. \end{aligned} \quad (4.9)$$

The variation of magic power with scattering angle is shown in Fig. 8, and it shows interesting qualitative differences as the spin increases. We note in particular that the interesting structure of peaks and dips in the gravitino results is already present (albeit less pronounced) in the gluino result. Thus, the half-integer spin profiles are more closely related to each other than to the integer spin results. The integrated magic power for gluinos and gravitinos is found to be

$$\int_0^{\pi/2} \overline{M}_2(\theta)|_{\text{gluinos}} = 0.407,$$

$$\int_0^{\pi/2} \overline{M}_2(\theta)|_{\text{gravitinos}} = 0.220, \quad (4.10)$$

and we summarize the comparison with gluons and gravitons in Fig. 9. In particular, we confirm the pattern of monotonically decreasing magic, as the spin of the qubits increases.

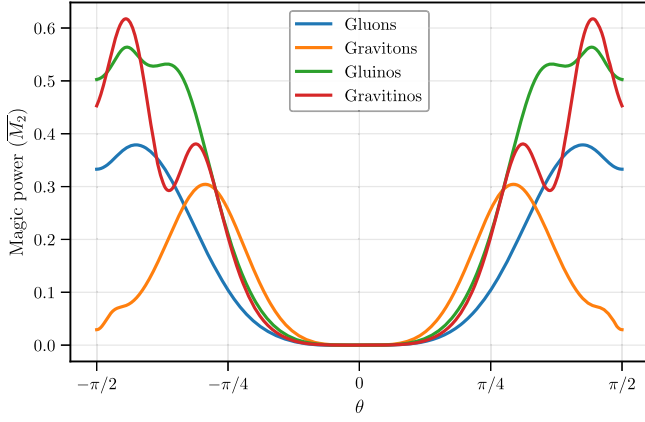


FIG. 8. The magic power for gluons (blue), gravitons (orange), gluinos (green), and gravitinos (red) as a function of scattering angle θ .

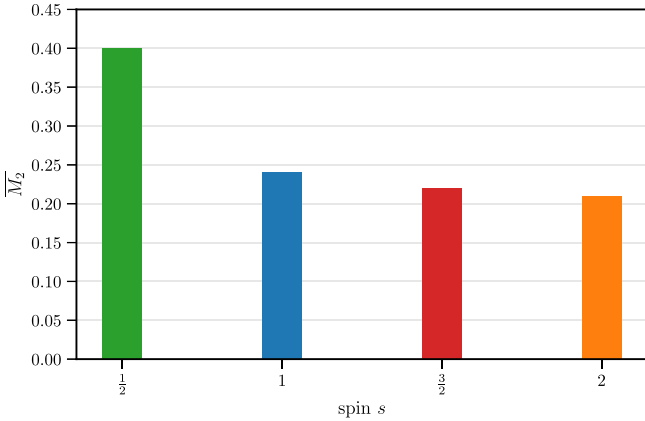


FIG. 9. The integrated magic power for gluons (blue), gravitons (orange), gluinos (green), and gravitinos (red), for nonminimal supersymmetry ($\mathcal{N} > 1$).

V. CONCLUSION

Recent years have seen increasing focus on the quantum property of *magic* (or *nonstabilizerness*), due to its crucial role in developing quantum computers with genuine computational advantage, and that are fault-tolerant. How to produce, enhance, and manipulate magic are open problems, such that case studies that explore these issues in given physical systems can be useful for generating insights that may in turn be more general. In this paper, we have considered two-qubit systems of gluons or gravitons in Yang-Mills theory and general relativity, respectively. We have shown that scattering of such particles indeed generates magic, where the amount of magic depends—as in other recent studies [25,27–29,31]—on the kinematic properties of the final state (i.e., the scattering angle).

Our results provide an interesting case of two closely related physical systems that differ by the value of a parameter (i.e., whether, in the nonsupersymmetric case, the qubits are spin 1 or spin 2). There is also a tight

theoretical relationship between them, given that amplitudes in gravity are related to products of gauge theory amplitudes by the KLT relations [100] (more generally, the double copy [101–104]). Thus, we are able to understand differences in magic between our two theories in terms of known results in the scattering amplitude literature, and in particular the generation of higher powers of Mandelstam invariants in expressions for the magic, in transitioning from gluons to gravitons. Given that double copy relationships between different field theories go much wider than traditional gauge theories and gravity (see, e.g., Ref. [113] for a recent review), this suggests that our conclusions may well be portable to other theories. In particular, we see that the typical amount of magic decreases as the spin of qubits increases, which is also consistent with previous results for spin-1/2 qubits in QED [27]. To corroborate this conclusion, we have also calculated results for supersymmetric extensions of Yang-Mills theory and gravity. Such theories contain half-integer spin partners of the gluon and graviton, and show a clear pattern of monotonically decreasing magic as the qubit spin increases. It would be interesting to know if this conclusion also holds in low-energy quantum systems, such as those being explored in condensed matter physics. We hope that our results provide a useful contribution to the ongoing investigation of magic and its uses, and look forward to further work in this area.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: KINEMATICS AND WARD IDENTITIES

In the center-of-mass frame, we consider the momentum of the incoming particles along the z -axis, given by

$$\begin{aligned} p_1^\mu &= (E, 0, 0, E), \\ p_2^\mu &= (E, 0, 0, -E), \end{aligned} \quad (\text{A1})$$

for the incoming gluons and

$$\begin{aligned} p_3^\mu &= (E, E \sin \theta, 0, E \cos \theta), \\ p_4^\mu &= (E, -E \sin \theta, 0, -E \cos \theta), \end{aligned} \quad (\text{A2})$$

for the outgoing ones, where θ is the scattering angle. The Mandelstam variables are expressed in terms of E and θ , or $s_{ij} = 2p_i \cdot p_j$, as

$$\begin{aligned} s &= 4E^2 = s_{12}, \quad t = -4E^2 \sin^2\left(\frac{\theta}{2}\right) = -s_{13}, \\ u &= -4E^2 \cos^2\left(\frac{\theta}{2}\right) = -s_{14}. \end{aligned} \quad (\text{A3})$$

In terms of spinor-helicity variables, the momentum can be written as

$$p_i^\mu = \langle i | \sigma^\mu | i \rangle = \lambda_i^a \sigma_{a\dot{a}}^\mu \tilde{\lambda}_{\dot{i}}^{\dot{a}}. \quad (\text{A4})$$

For the kinematics above, the spinors are then real, with $|i\rangle = |i\rangle$, and can be chosen to be

$$\begin{aligned} |1\rangle &= \sqrt{2E} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |2\rangle = \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ |3\rangle &= \sqrt{2E} \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} \end{pmatrix}, \quad |4\rangle = \sqrt{2E} \begin{pmatrix} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} \end{pmatrix}. \end{aligned} \quad (\text{A5})$$

The Mandelstam invariants are then given by $s_{ij} = \langle ij \rangle [ij] = \langle ij \rangle^2$, and we can therefore express the spinor brackets in terms of Mandelstam variables via $\langle ij \rangle = \sqrt{s_{ij}} e^{i\phi_{ij}}$, where $\phi_{ij} = 0$ or π . For the specific choice of spinors above, we find

$$\begin{aligned} \langle 12 \rangle &= \sqrt{s} = -\langle 34 \rangle, \quad \langle 13 \rangle = \sqrt{-t} = -\langle 24 \rangle, \\ \langle 14 \rangle &= \sqrt{-u} = \langle 23 \rangle \end{aligned} \quad (\text{A6})$$

and, therefore, that

$$\phi_{12} = \phi_{13} = \phi_{14} = \phi_{23} = 0, \quad \phi_{34} = \phi_{24} = \pi \quad (\text{A7})$$

for this choice of kinematics. We can use this to derive the Ward identities used in the main text, where we start from the spinor representation given by [121–124]

$$\begin{aligned} \mathcal{A}_4^{(1/2)}[1_A^- 2_B^- 3_C^+ 4_D^+] &= \frac{\langle 13 \rangle \langle 24 \rangle \delta_{AC} \delta_{BD} - \langle 14 \rangle \langle 23 \rangle \delta_{AD} \delta_{BC}}{\langle 12 \rangle^2} \\ &\times \mathcal{A}_4^{(1)}[1^- 2^- 3^+ 4^+]. \end{aligned} \quad (\text{A8})$$

Using the expressions above, this becomes

$$\begin{aligned} \mathcal{A}_4^{(1/2)}[1_A^- 2_B^- 3_C^+ 4_D^+] &= \frac{-\sqrt{-t} \sqrt{-t} \delta_{AC} \delta_{BD} - \sqrt{-u} \sqrt{-u} \delta_{AD} \delta_{BC}}{s} \\ &\times \mathcal{A}_4^{(1)}[1^- 2^- 3^+ 4^+] \\ &= \frac{t \delta_{AC} \delta_{BD} + u \delta_{AD} \delta_{BC}}{s} \mathcal{A}_4^{(1)}[1^- 2^- 3^+ 4^+], \end{aligned} \quad (\text{A9})$$

and similarly for the other channels.

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Correction: A project number in the fourth sentence of the Acknowledgments contained an error and has been fixed.