

Leptogenesis in a Majoron plus triplet model

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We discuss leptogenesis in a Majoron model extended by a right-handed $SU(2)_L$ triplet fermion. We study several different parameter assignments and find that the interactions of neutrinos with the new particles in the Majoron plus triplet model can significantly alter the way leptogenesis proceeds. We show that for large parts of the considered parameter space, it is essential to solve the set of coupled Boltzmann equations for the evolution of the neutrinos and the additional particles rather than solving the Boltzmann equations for the neutrino evolution only.

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I. INTRODUCTION

The Standard Model (SM) of particle physics has been remarkably successful on many levels. Despite its predictive power however, it fails to explain the origin of the baryon asymmetry of the Universe (BAU) [1] and how the light neutrino masses arise [2–4].

The type I seesaw mechanism [5] is an elegant way of generating small neutrino masses. Extending the SM by heavy right-handed singlet neutrinos, the seesaw mechanism naturally explains the smallness of the light neutrino masses as the consequence of a suppression by the mass scale of the heavy neutrinos. An inherent feature of the seesaw mechanism is that it inhibits all ingredients necessary to explain the BAU via leptogenesis [6]. In the most simple version of the leptogenesis mechanism, often called vanilla leptogenesis (VL), CP violating out-of-thermal-equilibrium decays of a heavy neutrino generate a lepton asymmetry which is subsequently converted to a baryon asymmetry via sphaleron transitions [7]. A compelling extension of the seesaw mechanism is the singlet Majoron model [8–10] where rather than including the masses of the Majorana neutrinos *ad hoc*, their origin is based on the spontaneous symmetry breaking (SSB) of a global $U(1)_{L'}$ symmetry where the index L' denotes lepton number. This SSB gives rise to a Goldstone boson, called Majoron, that has been shown to be a viable dark matter (DM) candidate if it obtains a small mass [11–15]. As in the type I seesaw mechanism, a lepton asymmetry can be generated via neutrino decays [16–18].

In this paper, we focus on the extension of the Majoron model by a right-handed triplet fermion. This extension of the Majoron model is originally motivated by the aim to prevent the appearance of cosmological domain walls as discussed in [19]. However, we note that in the absence of additional explicit baryon number violation on the Lagrangian level, $SU(2)_L$ instantons do not generate a Majoron potential [20–24] and hence no domain walls appear. Nevertheless, the presence of the right-handed triplet fermion has interesting consequences on the dynamics of leptogenesis and additionally, it changes the $[SU(2)_L]^2 \times U(1)_{L'}$ anomaly factor associated with the instanton transitions and therefore affects the conversion rate with which a lepton asymmetry is converted to a baryon asymmetry via sphaleron transitions [25]. For simplicity, we restrict ourselves to a model where an additional Z_2 symmetry forbids a Yukawa coupling of the triplet fermion with the lepton doublet and the Higgs. We will refer to this setup as the “Majoron plus triplet model.” In contrast to VL, neutrino interactions in the Majoron plus triplet model with the new particles introduce several additional parameters which can severely affect the dynamics that create the lepton asymmetry. Moreover, we find that the creation of the lepton asymmetry is independent from the presence of the triplet if the corresponding Yukawa coupling is sufficiently small.

We analyze how the parameters relevant in the Majoron plus triplet model affect the evolution of the lepton asymmetry compared to VL. We find that the lepton asymmetry that can be generated depends significantly on the Majorana neutrino Yukawa coupling and the effective neutrino mass while the initial abundances of the new particles are only relevant for a specific subcategory of parameters. Depending on the parameters, a sizable lepton asymmetry can be generated while in other cases, the asymmetry is significantly diminished.

This paper is structured as follows. In Sec. II, we give an introduction to the basic ingredients of the Majoron plus

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triplet model. In Sec. III, we give an overview on the leptogenesis mechanism both in VL and in the Majoron plus triplet model, present the Boltzmann equations for the evolution of the particle abundances and the lepton asymmetry and introduce the parameters that we used to solve these Boltzmann equations. In Sec. IV, we present our results for the efficiency factor that is relevant for leptogenesis and discuss the solutions of various scenarios in greater detail. In Sec. V, we discuss dark matter constraints on the parameters of the model. In Sec. VI, we focus on a more specific realization of the model and discuss the lepton and baryon asymmetries that can be realized in the Majoron plus triplet model. In Sec. VII, we conclude with a summary. In Appendix, we present useful formulas.

II. MODEL

In the Majoron plus triplet model, the SM is extended by a singlet complex scalar $\hat{\sigma}$, three right-handed neutrinos N_R^i , and a fermion triplet T_R ,

$$T_R = \begin{pmatrix} \frac{T_R^0}{\sqrt{2}} & T_R^+ \\ T_R^- & -\frac{T_R^0}{\sqrt{2}} \end{pmatrix}, \quad (2.1)$$

transforming under $(SU(3)_C \times SU(2)_L \times U(1)_Y)_{L'}$ as

$$\hat{\sigma} \sim (1, 1, 0)_{-2}, \quad N_R^{1,2,3} \sim (1, 1, 0)_1, \quad T_R \sim (1, 3, 0)_1, \quad (2.2)$$

where the index L' denotes the lepton number. The $U(1)_{L'}$ invariant scalar potential is given by

$$V = -\mu_\sigma^2 |\hat{\sigma}|^2 - \lambda_\sigma |\hat{\sigma}|^4 - \mu_H^2 |H|^2 - \lambda_H |H|^4 + 2\lambda_{\text{mix}} |\hat{\sigma}|^2 |H|^2, \quad (2.3)$$

where $H = (\phi^+)$ is the Higgs doublet. Moreover, the relevant Yukawa and kinetic terms are given by

$$\mathcal{L}_{\text{Yuk}} = -y_\nu \bar{L} \tilde{H} N_R - \frac{1}{2} g_{N_i} \overline{(N_R^i)^c} N_R^i \hat{\sigma} - \frac{1}{2} g_t \text{Tr} \left[\overline{(T_R)^c} T_R \hat{\sigma} \right] + \text{H.c.}, \quad (2.4)$$

$$\mathcal{L}_{\text{kin}} = i \bar{N}_R \not{D} N_R + i \text{Tr} [\bar{T}_R \not{D} T_R], \quad (2.5)$$

where L are the lepton doublets and $\tilde{H} = i\sigma_2 H^*$. In the Lagrangian above, we invoked an additional Z_2 symmetry under which T_R is odd while all other particles are even. This assumption simplifies the discussion for two main reasons. First, it prevents the triplet from mixing with the SM leptons via the Yukawa term $\bar{L} \tilde{H} T_R$. Second, as we discuss later, the absence of this term ensures that the only lepton number violating interactions in the Lagrangian are

neutrino decays, thereby simplifying the Boltzmann equations.

At the seesaw scale f , the $U(1)_{L'}$ symmetry is broken as $\hat{\sigma}$ obtains its vacuum expectation value (VEV) with an expansion around its ground state given by

$$\hat{\sigma} = \frac{1}{\sqrt{2}} (f + \sigma + iJ), \quad (2.6)$$

where J denotes the CP -odd Majoron and σ is CP -even. As J is a Goldstone boson, it is massless. However, the Majoron may obtain a small mass due to e.g., radiative corrections or gravitational instanton processes and thereby become a dark matter candidate [11–14]. For simplicity, we neglect λ_{mix} so that the potentials for $\hat{\sigma}$ and H decouple and we obtain,

$$V_\sigma = \frac{m_\sigma^2}{2} \sigma^2 + k_\sigma J^2 \sigma + k_\sigma \sigma^3 + \lambda_\sigma J^2 \sigma^2 + \frac{\lambda_\sigma}{4} J^4 + \frac{\lambda_\sigma}{4} \sigma^4 + \text{const}, \quad (2.7)$$

for the singlet potential with

$$f = \frac{\mu_\sigma}{\sqrt{\lambda_\sigma}}, \quad k_\sigma = \lambda_\sigma f, \quad m_\sigma = \sqrt{\lambda_\sigma} f. \quad (2.8)$$

After SSB at the seesaw scale, we define the heavy Majorana neutrinos as

$$N_i \equiv N_R^i + (N_R^i)^c = (N_i)^c \quad i = 1, 2, 3, \quad (2.9)$$

with the corresponding Majorana masses given by $M_{N_i} \equiv \frac{g_{N_i}}{\sqrt{2}} f$. Similarly, we write the triplet as

$$T \equiv \begin{pmatrix} \frac{T^0}{\sqrt{2}} & T^+ \\ T^- & -\frac{T^0}{\sqrt{2}} \end{pmatrix} = T_R + (T_R)^c = T^c, \quad (2.10)$$

with $M_T = \frac{g_T}{\sqrt{2}} f$ so that $T^0 = T_R^0 + (T_R^0)^c$ is a Majorana fermion while the charged components are combined into a Dirac spinor as

$$T^\pm = T_R^\pm + (T_R^\pm)^c = (T^\pm)^c. \quad (2.11)$$

Consequently, we can write the Yukawa terms (2.4) as

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & \left(-y_\nu \bar{L} \tilde{H} N + \text{H.c.} \right) - \frac{1}{2} M_N \bar{N} N - \frac{1}{2\sqrt{2}} g_N \bar{N} N \sigma \\ & - \frac{1}{2\sqrt{2}} g_N \bar{N} \gamma_5 N J - \frac{1}{2} M_T \text{Tr} [\bar{T} T] - \frac{1}{2\sqrt{2}} g_T \text{Tr} [\bar{T} T \sigma] \\ & - \frac{1}{2\sqrt{2}} g_T \text{Tr} [\bar{T} \gamma_5 T J], \end{aligned} \quad (2.12)$$

while the kinetic terms become

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= \frac{i}{2} \bar{N} \not{\partial} N + \frac{i}{2} \text{Tr}[\bar{T} \not{\partial} T] \\
&= \frac{i}{2} \bar{N} \not{\partial} N + \frac{g}{2} \left(\bar{T}^0 W^+ T^- + \bar{T}^0 W^- T^+ \right) \\
&\quad + e \bar{T}^+ A T^+ + \frac{g}{c_W} (1 - s_W^2) \bar{T}^+ Z T^+, \quad (2.13)
\end{aligned}$$

where we omitted generation indices for brevity. After electroweak symmetry breaking (EWSB), we have

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0 \end{pmatrix}, \quad (2.14)$$

where $v = 246$ GeV is the SM VEV and h^0 is the Higgs boson. Moreover, the neutrinos obtain a Dirac mass $m_D = \frac{y_\nu}{\sqrt{2}} v$ and we can write the neutrino mass terms as

$$\begin{aligned}
\mathcal{L}_{\text{mass}}^\nu &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \overline{(N_R)^c} \end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} (\nu_L)^c \\ N_R \end{pmatrix} \\
&\quad + \text{H.c.} \quad (2.15)
\end{aligned}$$

The Majorana mass matrix $\mathcal{M}$ can be diagonalized with a unitary matrix U as

$$U^T \mathcal{M} U = \text{diag}(m_1, \dots, m_6), \quad (2.16)$$

where $m_1, \dots, m_6$ are the physical neutrino masses. In the seesaw limit $M_N \gg m_D$, block diagonalization of $\mathcal{M}$ yields three light neutrinos with masses of order $-\frac{m_D m_D^T}{M_N}$ and three heavy neutrinos with masses of order M_N . Without loss of generality, we can assume that M_N is diagonal and define the heavy neutrino masses as $m_{4,5,6} = M_{N_{1,2,3}}$ while $m_{1,2,3}$ are the light neutrino masses.

In the following, we will assume that $M_{N_1} \ll M_{N_{2,3}}$ so that N_1 is the lightest of the heavy neutrino mass eigenstates. For brevity, we will generally omit the index 1 and stress that unless otherwise specified, g_N , M_N refer to the Yukawa coupling and the mass of $N \equiv N_1$.

III. LEPTOGENESIS

In this section, we give a brief overview of leptogenesis. For an overview of the basic formulas and conventions, we refer the reader to Appendix A 1. The relevant Feynman diagrams are shown in Appendix A 2, the relevant cross sections are given in Appendix A 3 and a more detailed derivation of the Boltzmann equations in the Majoron plus triplet model is presented in Appendix A 4.

A. Overview

In the VL scenario, the first term given in (2.12) is already sufficient to produce a lepton asymmetry

via CP -violating out-of-equilibrium neutrino decays $N \rightarrow LH, \bar{L} \bar{H}$ with a decay rate given by

$$\Gamma_D = \frac{(y_\nu^\dagger y_\nu)_{11} M_N}{8\pi} = \frac{M_N^2}{8\pi v^2} \tilde{m}, \quad (3.1)$$

where $\tilde{m} = (y_\nu^\dagger y_\nu)_{11} v^2 M_N^{-1}$ is the effective neutrino mass [26]. In our computations, we also include scattering processes involving third-generation quarks, induced via the top Yukawa coupling y_t , $Q_3 U_3 \leftrightarrow NL$ and $L \bar{Q}_3 \leftrightarrow N \bar{U}_3$, while neglecting gauge boson contributions. The corresponding Feynman diagrams can be found in Fig. 30. In the one flavor approximation, the Boltzmann equation for the neutrino abundance can then be written as [27]

$$s H z_N \frac{dY_N^{\text{VL}}}{dz_N} = -(\delta_N - 1) \gamma_D - 2(\delta_N - 1) \gamma_Q, \quad (3.2)$$

where the abundance Y_i is defined as the number density n_i normalized to the entropy density s (A13) so that $Y_i = \frac{n_i}{s}$ and $\delta_i \equiv \frac{Y_i}{Y_i^{\text{eq}}}$ where Y_i^{eq} is the equilibrium abundance. Moreover, we defined $z_N \equiv \frac{M_N}{T}$ while γ_D and $\gamma_Q \equiv \gamma_{Q_3 U_3 NL} + \gamma_{L \bar{Q}_3 N \bar{U}_3} + \gamma_{L \bar{U}_3 N \bar{Q}_3}$ are the thermal rates for neutrino decays and quark scatterings, respectively.

The CP violation in VL is a result of the interference of the tree- and one-loop-level diagrams of $N \rightarrow LH, \bar{L} \bar{H}$ presented in Fig. 1 and in the case of strongly hierarchical heavy neutrinos with $M_{N_{2,3}} \gg M_{N_1}$, it is given by [28]

$$\begin{aligned}
\varepsilon &= \frac{\Gamma(N \rightarrow LH) - \Gamma(N \rightarrow \bar{L} \bar{H})}{\Gamma(N \rightarrow LH) + \Gamma(N \rightarrow \bar{L} \bar{H})} \\
&\approx \frac{1}{8\pi} \frac{1}{(y_\nu^\dagger y_\nu)_{11}} \sum_{i=2,3} \text{Im} \left[(y_\nu^\dagger y_\nu)_{1i}^2 \right] f \left(\frac{M_{N_i}^2}{M_{N_1}^2} \right), \quad (3.3)
\end{aligned}$$

where

$$f(x) = \sqrt{x} \left[\frac{x-2}{x-1} - (1+x) \log \left(\frac{1+x}{x} \right) \right]. \quad (3.4)$$

Using the Davidson-Ibarra (DI) bound [29] and assuming $\frac{M_{N_{2,3}}}{M_{N_1}} \rightarrow \infty$, an upper limit on the CP violation can be given as

$$|\varepsilon^{\text{DI}}| \leq \frac{3}{16\pi} \frac{M_N}{v^2} (m_3 - m_1). \quad (3.5)$$

If the light neutrinos follow a normal hierarchy with $m_1 \ll m_2 \ll m_3$, (3.5) can be written as

¹Here, T is the temperature.

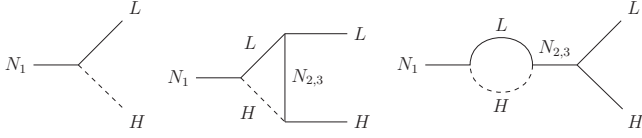


FIG. 1. Feynman diagrams contributing to the CP -violating N decay.

$$|\varepsilon^{DI}| \leq \frac{3}{16\pi} \frac{M_N}{v^2} \sqrt{\Delta m_{atm}^2}, \quad (3.6)$$

where Δm_{atm}^2 is the atmospheric neutrino mass splitting with [30]

$$\Delta m_{atm}^2 = m_3^2 - m_2^2 \approx 2.55 \times 10^{-3} \text{ eV}^2. \quad (3.7)$$

It is convenient to express the lepton asymmetry $Y_{\mathcal{L}} \equiv Y_{L'} - Y_{\bar{L}'}$ in terms of an efficiency factor η [31],

$$Y_{\mathcal{L}}(z_N) = \varepsilon Y_N^0 \eta(z_N), \quad (3.8)$$

where $Y_N^0 \equiv Y_N(z_N \rightarrow \infty)$ is a normalization factor, yielding a Boltzmann equation for the efficiency independent from ε ,

$$sH z_N \frac{d\eta}{dz_N} = \frac{1}{Y_N^0} (\delta_N - 1) \gamma_D - \frac{\eta}{Y_L^{eq}} \left(\frac{\gamma_D}{2} + \delta_N \gamma_{Q_3 U_3 N L} + 2 \gamma_{L \bar{Q}_3 N \bar{U}_3} \right). \quad (3.9)$$

The final lepton asymmetry is given by $Y_{\mathcal{L}}(z_N \rightarrow \infty) = \varepsilon Y_N^0 \eta$ where we defined the final efficiency as $\eta \equiv \eta(z_N \rightarrow \infty)$. In (3.9), the first term is the source term that produces the lepton asymmetry while the term proportional to η is called the washout (WO) term which reduces the efficiency via inverse decays and quark scatterings.

Compared to VL, the interactions given in (2.4), (2.5), and (2.7) induce a vast number of new interactions between the additional particles as shown in Figs. 31 and 32. While these interactions affect the neutrino evolution, the only lepton number violating processes are still the (inverse) neutrino decays and quark scatterings and consequently, the corresponding Boltzmann equation for the efficiency is the same as in VL. We also assume that no additional CP violation compared to VL is present in the Majoron plus triplet model.

Clearly, the lepton asymmetry is directly linked to the neutrino evolution and consequently, the most relevant processes are these that change the neutrino abundance (see Fig. 31). It is therefore convenient to combine these processes to a summed thermal scattering rate γ_S as

$$\gamma_S \equiv \gamma_{NNTT} + \gamma_{NNJJ} + \gamma_{NN\sigma\sigma} + \gamma_{NN\sigma J} + \text{Br}_{\sigma, NN} \gamma_{\sigma, NN}, \quad (3.10)$$

where we introduced the shorthand notations,

$$\gamma(12 \leftrightarrow 34) \equiv \gamma_{1234}, \quad \gamma(1 \leftrightarrow 23) \equiv \gamma_{1,23}, \quad (3.11)$$

for the thermal rates γ and $\text{Br}_{\sigma, NN} \equiv \frac{\Gamma_{\sigma \rightarrow NN}}{\sum_{i=N,T,J} \Gamma_{\sigma \rightarrow ii}}$ is the branching ratio of $\sigma \rightarrow NN$. In contrast to the VL scenario, we expect that a large scattering rate γ_S effectively thermalizes the neutrinos, thus allowing the generation of a lepton asymmetry only once the scattering processes decouple. This effect is similar to type III leptogenesis [32–37] where the lepton asymmetry is produced via the decays of a $SU(2)_L$ triplet fermion. An appealing feature of type III leptogenesis is that gauge interactions thermalize the triplet, rendering leptogenesis independent from the triplets initial abundance while in type I leptogenesis, the asymmetry is independent from the initial neutrino abundance only in the so called strong washout regime where $\tilde{m} \gtrsim \times 10^{-3} \text{ eV}$. In contrast to type III leptogenesis though, the triplet in the model discussed here does not directly contribute to the lepton asymmetry due to the additional Z_2 symmetry and further, we can treat the couplings g_N , g_T , λ_σ as free parameters and explore the effects of changes in these couplings on the generation of the asymmetry. For example, if γ_S is small, neutrinos might not be thermalized as efficiently as in type III leptogenesis. One complicating factor when calculating the lepton asymmetry however are the abundances of T , σ , and J . Their evolution depends on the same couplings as the evolution of the neutrinos which suggests that a coupled set of Boltzmann equations for T , σ , J , and N needs to be solved. However, in order to explore qualitative implications, it is convenient to begin with the discussion of a simplified scenario where we assume that T , σ , and J are in thermal equilibrium, i.e., $\delta_{T,\sigma,J} = 1$, and consequently, the Boltzmann equation for N is independent from their abundance.

In the next section, we present the full set of Boltzmann equations relevant for the evolution of N , T , σ , and J and the Boltzmann equation for Y_N in the simplified scenario where $\delta_{T,\sigma,J} = 1$. Afterwards, we specify the parameters we used to solve the Boltzmann equations.

B. Boltzmann equations

The full set of coupled Boltzmann equations for the evolution of Y_N , Y_T , Y_σ , and Y_J is given by

$$sH z_N \frac{dY_N}{dz_N} = \frac{dY_N^{VL}}{dz_N} - 2(\delta_N^2 - \delta_T^2)\gamma_{NNTT} - 2(\delta_N^2 - \delta_J^2)\gamma_{NNJJ} - 2(\delta_N^2 - \delta_\sigma^2)\gamma_{NN\sigma\sigma} - 2(\delta_N^2 - \delta_J\delta_\sigma)\gamma_{NN\sigma J} + 2\delta_{\text{sub}}\gamma_{\sigma,NN} \\ \equiv -(\delta_N - 1)\gamma_D - 2(\delta_N - 1)\gamma_Q - 2(\delta_N^2 - 1)\gamma_S - 2\rho, \quad (3.12)$$

$$sH z_N \frac{dY_T}{dz_N} = -2(\delta_T^2 - \delta_N^2)\gamma_{TTNN} - 2(\delta_T^2 - \delta_J^2)\gamma_{TTJJ} - 2(\delta_T^2 - \delta_\sigma^2)\gamma_{TT\sigma\sigma} - 2(\delta_T^2 - \delta_\sigma\delta_J)\gamma_{TT\sigma J} - 2(\delta_T^2 - 1)\gamma_{T,\text{gauge}} \\ + 2\delta_{\text{sub}}\gamma_{\sigma,TT}, \quad (3.13)$$

$$sH z_N \frac{dY_\sigma}{dz_N} = -2(\delta_\sigma^2 - \delta_N^2)\gamma_{\sigma\sigma NN} - 2(\delta_\sigma^2 - \delta_T^2)\gamma_{\sigma\sigma TT} - 2(\delta_\sigma^2 - \delta_J^2)\gamma_{\sigma\sigma JJ} - (\delta_\sigma\delta_J - \delta_T^2)\gamma_{\sigma JTT} - (\delta_\sigma\delta_J - \delta_N^2)\gamma_{\sigma JNN} \\ - (\delta_N\delta_\sigma - \delta_N\delta_J)\gamma_{N\sigma NJ} - (\delta_T\delta_\sigma - \delta_T\delta_J)\gamma_{T\sigma TJ} - (\delta_\sigma - \delta_N^2)\gamma_{\sigma,NN} - (\delta_\sigma - \delta_T^2)\gamma_{\sigma,TT} - (\delta_\sigma - \delta_J^2)\gamma_{\sigma,JJ}, \quad (3.14)$$

$$sH z_N \frac{dY_J}{dz_N} = -2(\delta_J^2 - \delta_N^2)\gamma_{JJNN} - 2(\delta_J^2 - \delta_\sigma^2)\gamma_{JJ\sigma\sigma} - 2(\delta_J^2 - \delta_T^2)\gamma_{JJTT} - (\delta_J\delta_\sigma - \delta_N^2)\gamma_{J\sigma NN} - (\delta_J\delta_\sigma - \delta_T^2)\gamma_{J\sigma TT} \\ - (\delta_J\delta_N - \delta_\sigma\delta_N)\gamma_{JN\sigma N} - (\delta_J\delta_T - \delta_\sigma\delta_T)\gamma_{JT\sigma T} + 2\delta_{\text{sub}}\gamma_{\sigma,JJ}, \quad (3.15)$$

where we wrote the Boltzmann equation for T as the sum of the three triplet components $n_T \equiv n_{T^0} + n_{T^+} + n_{T^-}$, the summed scattering rate γ_S is given by (3.10) and we defined,

$$\rho \equiv (1 - \delta_J^2)(\gamma_{NNJJ} - \text{Br}_{\sigma,JJ}\gamma_{\sigma,NN}) + (1 - \delta_T^2)(\gamma_{NNTT} - \text{Br}_{\sigma,TT}\gamma_{\sigma,NN}) + (1 - \delta_\sigma^2)\gamma_{NN\sigma\sigma} + (1 - \delta_\sigma\delta_J)\gamma_{NN\sigma J} + (1 - \delta_\sigma)\gamma_{\sigma,NN} \\ = (1 - \delta_J^2)\gamma_{NNJJ} + (1 - \delta_T^2)\gamma_{NNTT} + (1 - \delta_\sigma^2)\gamma_{NN\sigma\sigma} + (1 - \delta_\sigma\delta_J)\gamma_{NN\sigma J} \\ + [(1 - \delta_\sigma) - (1 - \delta_J^2)\text{Br}_{\sigma,JJ} - (1 - \delta_T^2)\text{Br}_{\sigma,TT}]\gamma_{\sigma,NN}, \quad (3.16)$$

in order to isolate the terms in the Boltzmann equation for the neutrino evolution that explicitly contain $\delta_{T,\sigma,J}$.² Moreover,

$$\delta_{\text{sub}} = \delta_\sigma - \delta_J^2\text{Br}_{\sigma,JJ} - \delta_N^2\text{Br}_{\sigma,NN} - \delta_T^2\text{Br}_{\sigma,TT} \quad (3.17)$$

appears due to the subtraction of on shell scattering as explained in Appendix A 4 and $\gamma_{T,\text{gauge}}$ is the reaction density of the triplet from interactions involving gauge bosons.

In the simplified scenario where $\delta_{T,\sigma,J} = 1$, only the Boltzmann equation for the neutrinos remains which simplifies to

$$sH z_N \frac{dY_N}{dz_N} = -(\delta_N - 1)\gamma_D - 2(\delta_N - 1)\gamma_Q - 2(\delta_N^2 - 1)\gamma_S. \quad (3.18)$$

Note that we recover (3.2) for $\rho, \gamma_S \rightarrow 0$ in (3.12) while we have (3.12)→(3.18) for $\rho \rightarrow 0$.

C. Parameters

Before discussing the solutions of the Boltzmann equations in detail, a short examination of the parameters appearing in the Boltzmann equations is in order. In VL,

²We stress however that $\delta_{T,\sigma,J}$ and δ_N are clearly not independent.

the thermal rates γ_D, γ_Q govern the neutrino evolution and are mainly determined by the effective neutrino mass $\tilde{m}$ while the thermal rates in the Majoron plus triplet model depend on four additional parameters; the VEV f and the couplings g_N, g_T, λ_σ . Solving the Boltzmann equations introduces additional parameters in form of the initial conditions. We will therefore distinguish between various different cases, depending on the couplings, the initial conditions and whether the simplified scenario or the full set of Boltzmann equations is considered. As it is reasonable to assume that changes in g_T and λ_σ have comparably small effects on the asymmetry compared to changes in g_N , we vary g_N in the range $g_N \in [0.1, 1]$ in each case. We then distinguish between cases $A, \hat{A}$ where $g_T = \lambda_\sigma = 1$ and $B, \hat{B}$, where $g_T = 10^{-7}, \lambda_\sigma = 1$ where A, B correspond to the case where the full set of Boltzmann equations is considered while $\hat{A}, \hat{B}$ correspond to the simplified scenario. In each case, we vary $\tilde{m}$ in the range $\tilde{m} \in [0.5 \times 10^{-4}, 0.1]$ eV.³ and fix the VEV at $f = 10^{10}$ GeV, neglecting any temperature dependence.⁴

In contrast to VL, the lepton asymmetry in the Majoron plus triplet model is produced only after the $U(1)_{L'}$

³In case of normal ordering, neutrino data suggest $m_1 \lesssim m_3 \sim 0.05$ eV, although a larger $\tilde{m}$ is in principle possible if cancellations occur [38].

⁴We discuss the effect of the VEV on the efficiency briefly in Sec. VI.

TABLE I. Parameters and initial conditions used to solve the Boltzmann equations (3.2), (3.9), (3.12)–(3.15), and (3.18). Cases $\hat{A}$, $\hat{B}$ correspond to the simplified scenario where $\delta_{T,\sigma,J} = 1$ while in cases A , B , the full set of Boltzmann equations for N , T , σ , J is considered. The index $t(z)$ denotes that the corresponding Boltzmann equations are solved with thermal (vanishing) initial particle abundances. Note that M_N does not have an effect on η_{VL} .

Case	g_N	g_T	λ_σ	f/GeV	$\tilde{m}/\text{eV}$	$Y_N(z_I)$	$Y_{\sigma,J,T}(z_I)$	$\delta_{T,\sigma,J}(z_N) = 1$
$\hat{A}_z$	[0.1, 1]	1	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	0		Yes
$\hat{A}_t$	[0.1, 1]	1	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	$Y_N^{eq}(z_I)$		Yes
$\hat{B}_z$	[0.1, 1]	10^{-7}	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	0		Yes
$\hat{B}_t$	[0.1, 1]	10^{-7}	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	$Y_N^{eq}(z_I)$		Yes
A_z	[0.1, 1]	1	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	0	0	No
A_t	[0.1, 1]	1	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	$Y_N^{eq}(z_I)$	$Y_{\sigma,J,T}^{eq}(z_I)$	No
B_z	[0.1, 1]	10^{-7}	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	0	0	No
B_t	[0.1, 1]	10^{-7}	1	10^{10}	$[5 \times 10^{-5}, 10^{-1}]$	$Y_N^{eq}(z_I)$	$Y_{\sigma,J,T}^{eq}(z_I)$	No
VL _z					$[5 \times 10^{-5}, 10^{-1}]$	0		
VL _t					$[5 \times 10^{-5}, 10^{-1}]$	$Y_N^{eq}(z_I)$		

symmetry is broken at f and thus the relevant range for the asymmetry is $z_N \geq z_I \equiv \frac{g_N}{\sqrt{2}}$. For simplicity, we solve the Boltzmann equations for the particle abundances in that range as well and hence specify the initial conditions at $z_N = z_I$. We then broadly distinguish between the two limiting scenarios where either all relevant particles have a thermal initial abundance or a vanishing initial abundance at z_I . Thus, for cases $\hat{A}$, $\hat{B}$, the neutrinos have either a thermal initial abundance,

$$\hat{A}_t, \hat{B}_t \text{ initial abundance: } Y_N(z_I) = Y_N^{eq}(z_I), \quad (3.19)$$

denoted by a subscript t , or a vanishing initial abundance,

$$\hat{A}_z, \hat{B}_z \text{ initial abundance: } Y_N(z_I) = 0, \quad (3.20)$$

denoted by a subscript z . In cases A , B , we assume for simplicity that the initial abundances of N , σ , J , T are the same and similarly solve the Boltzmann equations for their evolution for a scenario where the new fields have either a thermal initial abundance,

$$A_t, B_t \text{ initial abundance: } Y_{N,\sigma,J,T}(z_I) = Y_{N,\sigma,J,T}^{eq}(z_I), \quad (3.21)$$

denoted with a subscript t , or a vanishing initial abundance,

$$A_z, B_z \text{ initial abundance: } Y_{N,\sigma,J,T}(z_I) = 0, \quad (3.22)$$

denoted with a subscript z . Assuming that no other mechanisms generated an lepton asymmetry for $z_N < z_I$, we solve the Boltzmann equation for the efficiency with the initial condition $\eta(z_I) = 0$.

For comparison, we also solve the Boltzmann equations in the VL scenario. We choose the same initial conditions as cases $\hat{A}$, $\hat{B}$ with the distinction that the initial conditions are given at $z_I^{VL} \equiv z_N = 0$. The parameter assignments are summarized in Table I. Even though it does not affect the

numerical results, we explicitly included a small but non-zero Majoron mass $m_J \sim \times 10^{-3}$ GeV inspired by previous works on Majoron DM [11].

The sizes of the couplings g_N , g_T , λ_σ are particularly relevant when considering the decays $\sigma \rightarrow NN, TT, JJ$ with decay rates given by

$$\begin{aligned} \Gamma_{\sigma \rightarrow NN} &= \frac{g_N^2}{8\pi} \sqrt{\frac{m_\sigma^2}{4} - M_N^2}, & \Gamma_{\sigma \rightarrow TT} &= 3 \frac{g_T^2}{8\pi} \sqrt{\frac{m_\sigma^2}{4} - M_T^2}, \\ \Gamma_{\sigma \rightarrow JJ} &= \frac{\pi k_\sigma^2}{16\pi^2 m_\sigma^2} \sqrt{\frac{m_\sigma^2}{4} - m_J^2}. \end{aligned} \quad (3.23)$$

Clearly, $\sigma \rightarrow NN$ and $\sigma \rightarrow TT$ are kinematically allowed only when $g_N \leq 0.7$ and $g_T \leq 0.7$, respectively. This implies

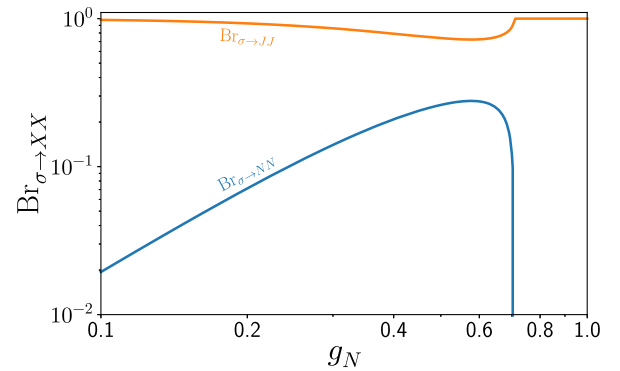


FIG. 2. Branching ratios of $\sigma \rightarrow NN$ and $\sigma \rightarrow JJ$ as functions of g_N . We stress that in cases $A, \hat{A}$ the decay $\sigma \rightarrow TT$ is kinematically forbidden while in cases $B, \hat{B}$, the branching ratio $\text{Br}_{\sigma \rightarrow TT}$ is of order 10^{-13} and can therefore be neglected. As a result, the branching ratios $\text{Br}_{\sigma \rightarrow JJ}$ and $\text{Br}_{\sigma \rightarrow NN}$ are practically the same in all considered scenarios. Moreover, we note that σ -decays to a pair of Majorons are clearly the dominant decay channel over the full g_N range. Above $g_N = 0.7$, σ decays to a pair of neutrinos are kinematically forbidden, resulting in $\text{Br}_{\sigma \rightarrow JJ} = 1$.

that $\sigma \rightarrow TT$ is forbidden in cases $A, \hat{A}$ and allowed in cases $B, \hat{B}$. However, we find $\text{Br}_{\sigma \rightarrow TT} \sim 10^{-13}$ for $g_T = 10^{-7}$ so that we can safely neglect $\sigma \rightarrow TT$ even in cases $B, \hat{B}$. As $m_J \ll m_\sigma$ independent of the couplings, $\sigma \rightarrow JJ$ is always allowed. In Fig. 2, we show $\text{Br}_{\sigma \rightarrow JJ, NN}$ as a function of g_N . We clearly see that $\sigma \rightarrow JJ$ is the preferred decay channel while $\text{Br}_{\sigma \rightarrow NN} \ll \text{Br}_{\sigma \rightarrow NJJ}$ peaks at $g_N = \frac{1}{\sqrt{3}}$.

We further note that we distinguish the thermal rates where T appears in the initial or final state for cases $A, \hat{A}$ and cases $B, \hat{B}$ while the other thermal rates do not depend on g_T and are therefore identical for all cases.

IV. RESULTS

We numerically solve the Boltzmann equations given in (3.2), (3.9), (3.12)–(3.15), and (3.18) with the parameters given in Table I. As our main interest is the effect of the different parameter assignments on the final efficiency, we will begin with a brief overview of the corresponding results before discussing the evolutions of the particle abundances and the efficiency in the respective cases in greater detail.

A. Overview

In Figs. 3 and 4, we show density plots of the efficiency η in the g_N - $\tilde{m}$ plane (left panels) and $\eta(\tilde{m})$ (middle panels),

and $\eta(g_N)$ (right panels) for exemplary values of g_N and $\tilde{m}$, respectively.

Note that we find that η is independent from the initial conditions in cases $A, \hat{A}, \hat{B}$ while the efficiency in case B depends on the initial conditions. Therefore, we will not distinguish between the initial conditions in the discussion of the final efficiency for cases $A, \hat{A}, \hat{B}$, i.e.,

$$\eta_{\hat{A}} \equiv \eta_{\hat{A}}^{t,z}, \quad \eta_{\hat{B}} \equiv \eta_{\hat{B}}^{t,z}, \quad \eta_A \equiv \eta_A^{t,z}. \quad (4.1)$$

First, we discuss cases $A, \hat{A}, \hat{B}, B_i$ as they display some similar features. Concerning the dependence on $\tilde{m}$, the efficiencies $\eta_{\hat{A}}, \eta_A, \eta_{\hat{B}}, \eta_B^t$ initially increase with $\tilde{m}$ until they reach a maximum at some $\tilde{m}_{\text{max}}$ and subsequently decrease. For $\tilde{m} > \tilde{m}_{\text{max}}$, we find that similarly to what happens in VL, all efficiencies eventually align and obtain a common efficiency we define as η^{WO} . This holds except for two notable exceptions. In cases $\hat{A}, \hat{B}$ with $g_N \approx 0.7$, the efficiency stays below η^{WO} while in case A with $g_N \approx 1$, the efficiency slightly exceeds η^{WO} .

For $\tilde{m} \lesssim \tilde{m}_{\text{max}}$, we find that the efficiencies for cases A, B exceed the efficiencies reached in the cases $\hat{A}, \hat{B}$. This is highlighted in Fig. 5 (upper panels) where we show density plots of $\frac{\eta_{\hat{A}}}{\eta_A}(g_N, \tilde{m})$ (left) and $\frac{\eta_{\hat{B}}}{\eta_B}(g_N, \tilde{m})$ (right). However, the efficiencies in the considered scenarios of the Majoron plus triplet model for $\tilde{m} \lesssim \tilde{m}_{\text{max}}$ are generally smaller than η_{VL}^t

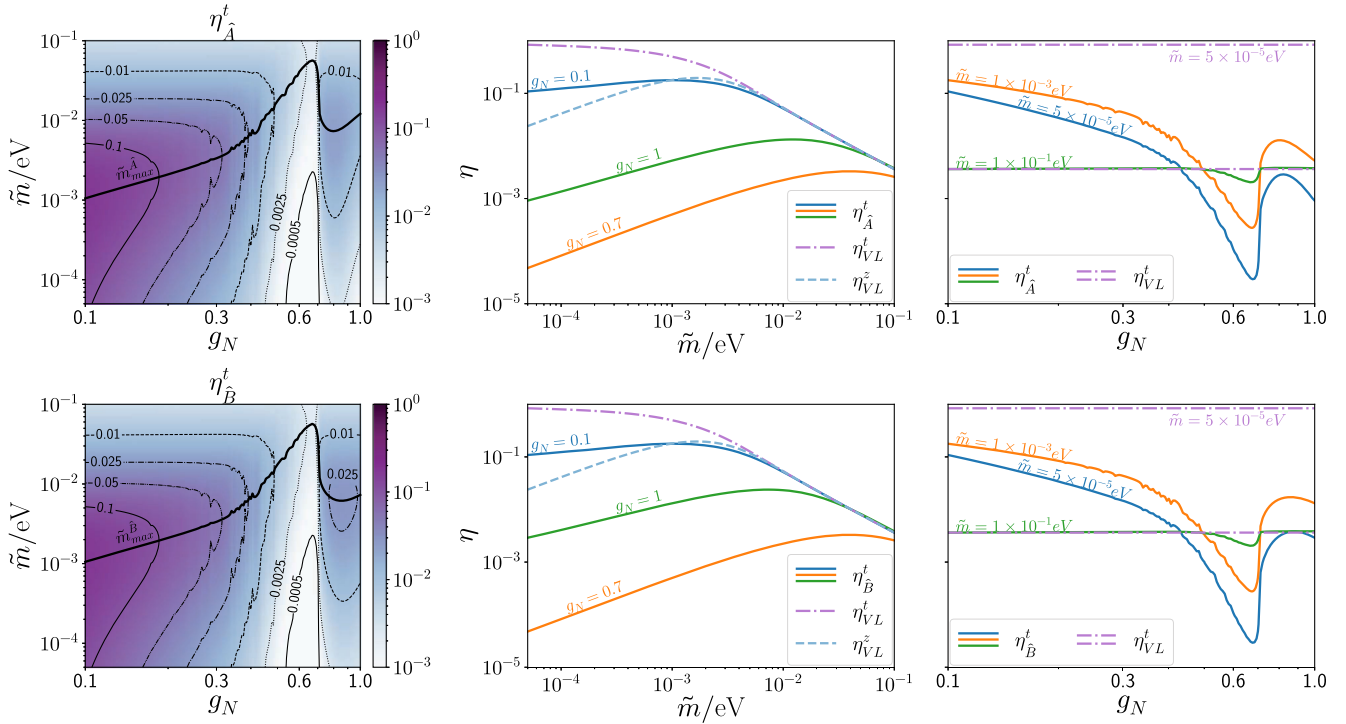


FIG. 3. *Left:* Density plots of $\eta_{\hat{A}}$ (upper plot) and $\eta_{\hat{B}}$ (lower plot) in the g_N - $\tilde{m}$ plane. The black lines indicate at which $\tilde{m}$ the efficiency reaches its maximum. *Middle:* $\eta_{\hat{A}}$ (upper plot) and $\eta_{\hat{B}}$ (lower plot) as functions of g_N . *Right:* $\eta_{\hat{A}}$ (upper plot) and $\eta_{\hat{B}}$ (lower plot) as functions of $\tilde{m}$. See text for discussion.

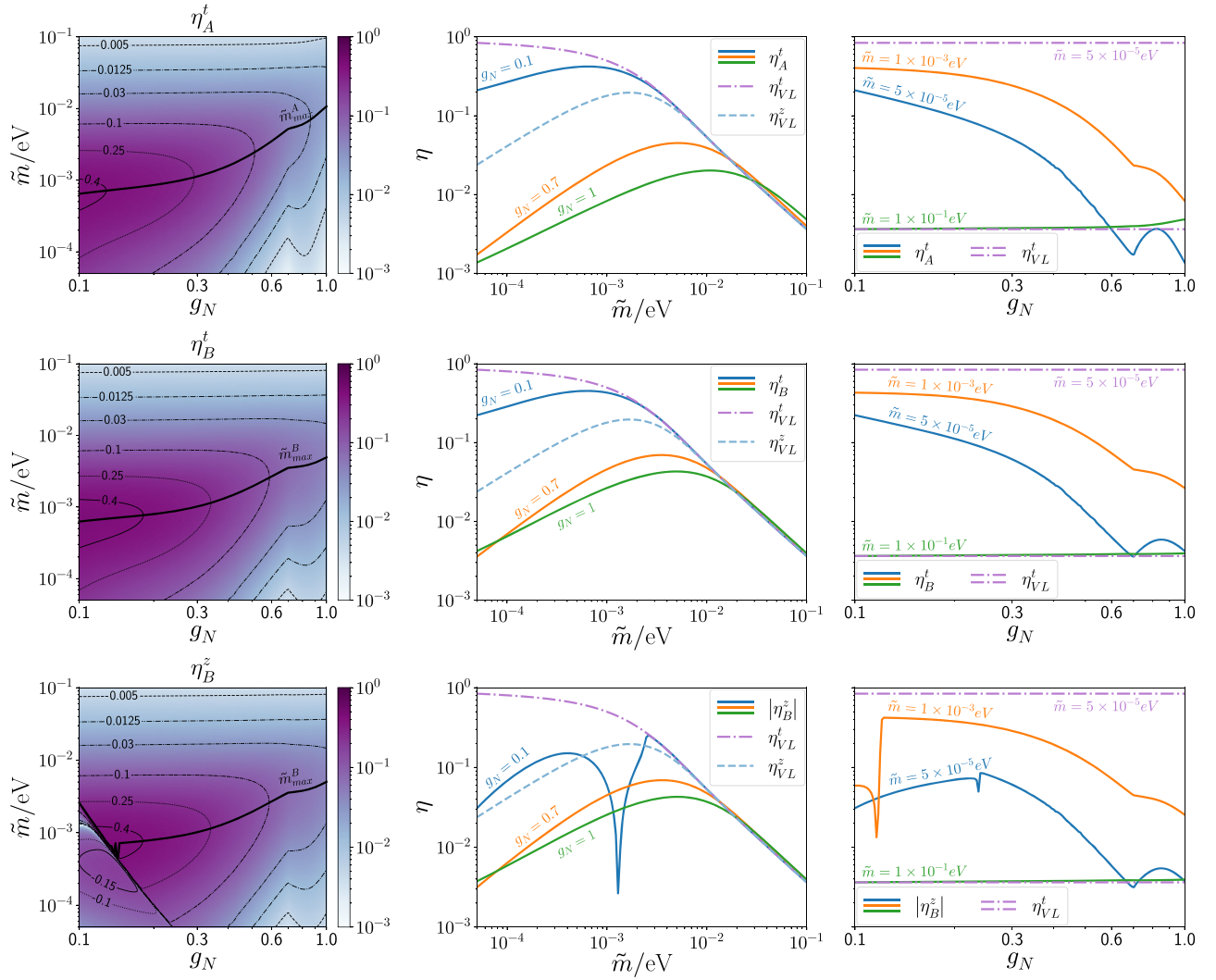


FIG. 4. *Left:* Density plots of η_A (top column), η_B^t (middle column), and η_B^z (bottom column) in the $g_N - \tilde{m}$ plane. The black lines indicate at which $\tilde{m}$ the efficiency reaches its maximum. *Middle:* η_A (top column), η_B^t (middle column), and η_B^z (bottom column) as functions of g_N . *Right:* η_A (top column), η_B^t (middle column), and η_B^z (bottom column) as functions of $\tilde{m}$. See text for discussion.

and unless g_N is small, they are also smaller than η_{VL}^z (see Figs. 3 and 4).

Considering the dependence on g_N , we find that the efficiencies in the simplified scenario $\eta_{\hat{A},\hat{B}}(g_N)$ depend strongly on g_N and irrespective of $\tilde{m}$, they reach a striking minimum around $g_N \sim 0.7$. Overall, we note that for $g_N \ll 0.7$ and $g_N \gg 0.7$, $\eta_{\hat{A},\hat{B}}(g_N)$ decrease with g_N . Similarly, the efficiencies obtained from the full set of Boltzmann equations $\eta_{A,B}^t(g_N)$ decrease with g_N for $g_N \ll 0.7$ and $g_N \gg 0.7$. On the other hand, the minimum of $\eta_{A,B}^t$ around $g_N \sim 0.7$ is significantly less striking compared to the simplified scenario and even disappears for $\tilde{m} \rightarrow 10^{-1} \text{ eV}$. Thus, as can be seen in Fig. 5 (upper panels), the relative deviations $\frac{\eta_{\hat{A}}}{\eta_A}(g_N, \tilde{m})$ and $\frac{\eta_{\hat{B}}}{\eta_B}(g_N, \tilde{m})$ are maximal around $g_N \sim 0.7$. Finally, as can be seen in Fig. 5 (lower panels) where we show density plots of $\frac{\eta_{\hat{A}}}{\eta_A}(g_N, \tilde{m})$ (left) and $\frac{\eta_{\hat{B}}}{\eta_B}(g_N, \tilde{m})$ (right), the efficiencies

$\eta_{\hat{A}}, \eta_{\hat{B}}$ are close to identical for $g_N < 0.7$ while $\eta_{\hat{B}}$ slightly exceeds $\eta_{\hat{A}}$ for $g_N > 0.7$. Similarly, the relative deviation $\frac{\eta_{\hat{A}}}{\eta_B}(g_N, \tilde{m})$ is maximal for $g_N > 0.7$ but in contrast to the simplified scenario, η_B slightly exceeds η_A already for $g_N < 0.7$.

Next, let us discuss η_B^z . In contrast to the previous cases, we find that η_B^z is negative in a triangular region spanning from $0.1 \leq g_N \lesssim 0.24$ and $5 \times 10^{-5} \text{ eV} \leq \tilde{m} \lesssim 2.5 \times 10^{-3} \text{ eV}$ while outside of this region, η_B^z is positive and we have $\eta_B^t \approx \eta_B^z$ as highlighted in Fig. 6 (left) where we show $\frac{\eta_B^z}{\eta_B^t}(g_N, \tilde{m})$. Thus, we separate $|\eta_B^z(\tilde{m}, g_N)|$ into an initial abundance (IA) regime where the initial abundances of N, σ, J, T are relevant and into an $\bar{\text{IA}}$ regime where the initial abundances have no significant effect so that $\eta_B^t \approx \eta_B^z$. This is displayed in Fig. 6 (right) where we also

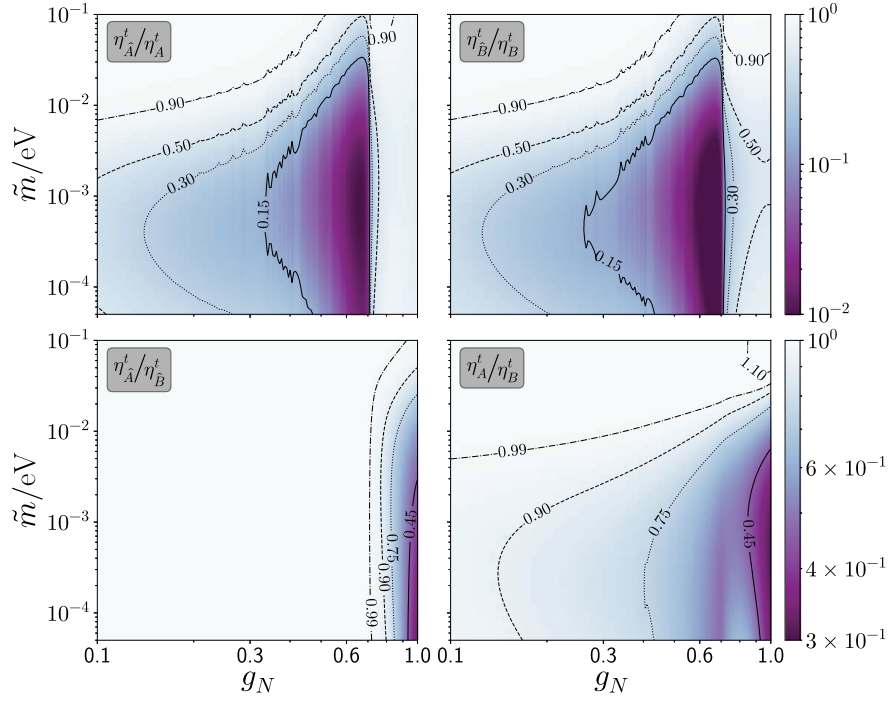


FIG. 5. *Upper plots:* Density plots of $\frac{\eta_A^t}{\eta_A^t}$ (left) and $\frac{\eta_B^t}{\eta_B^t}$ (right) in the g_N - $\tilde{m}$ plane. *Lower plots:* Density plots of $\frac{\eta_A^t}{\eta_B^t}$ (left) and $\frac{\eta_A^t}{\eta_B^t}$ (right) in the g_N - $\tilde{m}$ plane. See text for discussion.

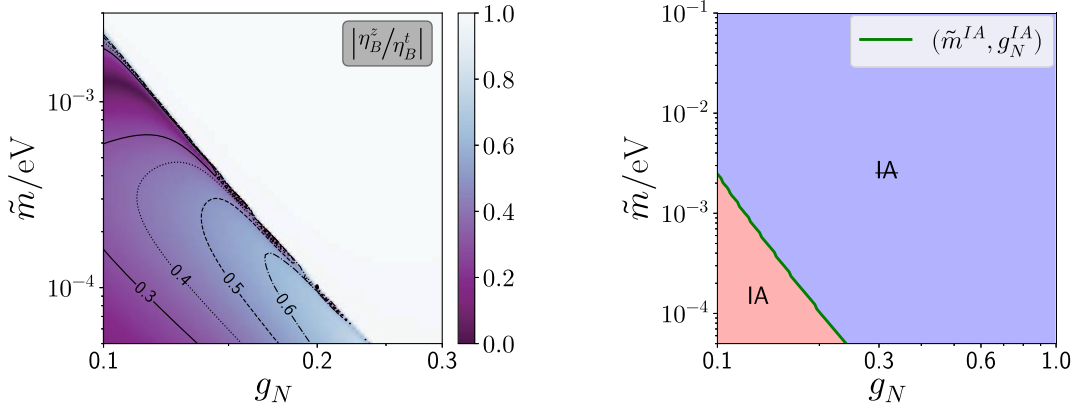


FIG. 6. *Left:* Density plot of $|\frac{\eta_B^z}{\eta_B^t}|$ in the g_N - $\tilde{m}$ plane. *Right:* Schematic density plot of IA, IA and $(g_N^{IA}, \tilde{m}^{IA})$. See text for discussion.

introduced $(g_N^{IA}, \tilde{m}^{IA})$ in order to quantify when the transition from IA to the IA regime occurs.⁵

To summarize, we find that while the additional scattering processes generally diminish the efficiency that can be reached compared to VL, certain sets of parameters still allow for a sizable efficiency. Moreover, solving the full Boltzmann equations including T , σ , and J can have significant effects on the final efficiency. Further, we note that solving the full Boltzmann equations can also result in a dependence on the initial abundances if g_N , g_T , and $\tilde{m}$ are

small while for most of the parameters sets we considered, the initial abundances are irrelevant. In the following, we will discuss the dynamics of the respective cases in greater detail.

B. Cases $\hat{A}, \hat{B}$: $\delta_{T,\sigma J} = 1$

In this section, we discuss the cases $\hat{A}, \hat{B}$ in more detail. Recall that we found in the previous section that the initial abundance of neutrinos does not affect the final efficiency in this scenario. Additionally, we found that the final efficiencies in both cases differ only slightly. We will begin with a discussion of the relevant thermal rates and

⁵We will properly define $(g_N^{IA}, \tilde{m}^{IA})$ in Sec. IV D.

how they depend on $\tilde{m}$ and g_N . We then proceed with a discussion of the neutrino evolution and how this affects the efficiency. Note that for brevity, we will only distinguish between case $\hat{A}$ and case $\hat{B}$ when necessary and introduce the convention to refer to generic quantities of the simplified scenario with a hat so that e.g., $\hat{\eta} = \eta_{\hat{A},\hat{B}}$.

1. Discussion of γ_S

First, let us discuss the overall behavior of the summed scattering rate γ_S . In Fig. 8, we show γ_S and the relevant scattering rates that contribute to γ_S for exemplary values of g_N . It is apparent that for small z_N , we have

$$\frac{\gamma_S}{n_N^{eq} H}(z_N) > 1, \quad (4.2)$$

and thus the scattering interactions thermalize the neutrinos. In fact, the scattering processes are so fast around $z_N = z_I$ that neutrinos immediately reach thermal equilibrium even if $\hat{Y}_N(z_I) = 0$. For convenience, we will therefore not distinguish between $\hat{Y}_N^I(z_N)$ and $\hat{Y}_N^z(z_N)$ for the remainder of this section. We note however that $\frac{\gamma_S}{n_N^{eq} H}(z_N)$ is Boltzmann suppressed and therefore the scatterings decouple when

$$\frac{\gamma_S}{n_N^{eq} H}(z_N) \leq 1, \quad (4.3)$$

i.e., scattering processes keep neutrinos close to thermal equilibrium until $z_N = z_S^{eq}$ defined via

$$\frac{\gamma_S}{n_N^{eq} H}(g_N, z_N = z_S^{eq}) = 1. \quad (4.4)$$

In Fig. 7, we show $z_S^{eq}(g_N)$ as a function of g_N . We can easily see that for $g_N < 0.7$, $z_S^{eq,\hat{A}} = z_S^{eq,\hat{B}}$ and additionally, both reach a peak at $g_N \lesssim 0.7$ before rapidly decreasing. After reaching a local minimum, both $z_S^{eq,\hat{A}}$ and $z_S^{eq,\hat{B}}$ start to increase again with $z_S^{eq,\hat{A}}$ growing faster than $z_S^{eq,\hat{B}}$. This behavior can be explained by considering the individual terms that appear in γ_S .

Both for case $\hat{A}$ and case $\hat{B}$, we can see in Fig. 8 that γ_S is dominated by $\gamma_{\sigma,NN}$ and γ_{NNJJ} when $g_N \leq 0.7$, thus explaining why z_S^{eq} is identical for case $\hat{A}$ and case $\hat{B}$ in that range. On the other hand, for $g_N > 0.7$, we note that in case $\hat{A}$, γ_S is dominated by $\gamma_{NNTT}^{\hat{A}}$ while in case $\hat{B}$, the dominant processes in γ_S are γ_{NNJJ} and $\gamma_{NN\sigma J}$. The change of behavior around $g_N \sim 0.7$ can be traced back to $\sigma \rightarrow NN$ being forbidden for $g_N > 0.7$ and a changing Boltzmann suppression due to the change in g_N .

In particular, let us consider γ_{NNJJ} and $\gamma_{\sigma,NN}$ in the range $g_N \leq 0.7$. As g_N increases, the overall magnitudes of γ_{NNJJ} and $\gamma_{\sigma,NN}$ increase and as a consequence, $NN \leftrightarrow JJ$ scatterings and $\sigma \leftrightarrow NN$ (inverse) decays are thermalized longer, translating to an increasing z_S^{eq} . Moreover, the

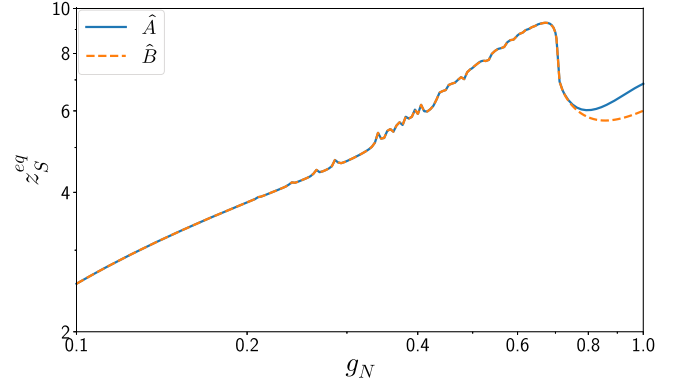


FIG. 7. $z_S^{eq}(g_N)$ as defined in (4.3) with the blue solid line corresponding to case $\hat{A}$ and the orange dashed line corresponding to case $\hat{B}$. For $g_N \ll 0.7$ and $g_N > 0.7$, both $z_S^{eq,\hat{A}}$ and $z_S^{eq,\hat{B}}$ increase with g_N due to the Boltzmann suppression of the relevant thermal rates appearing in γ_S being damped as g_N increases. Moreover, for $g_N < 0.7$, $z_S^{eq}(g_N)$ is identical for cases $\hat{A}$ and $\hat{B}$ as the processes dominating γ_S are independent from g_T . At $g_N \sim 0.7$, both $z_S^{eq,\hat{A}}$ and $z_S^{eq,\hat{B}}$ rapidly drop with can be attributed to $\gamma_{\sigma,NN}$ becoming forbidden, thus decreasing the overall magnitudes of $\gamma_S^{\hat{A}}$ and $\gamma_S^{\hat{B}}$ (see Fig. 8). For $g_N > 0.7$, we have $z_S^{eq,\hat{A}} > z_S^{eq,\hat{B}}$ as the summed scattering rate γ_S in case $\hat{B}$ decouples faster than in case $\hat{A}$.

Boltzmann suppression of $\frac{\gamma_{\sigma,NN}}{n_N}$ does change notably with g_N . In general, we have

$$\frac{\gamma_{\sigma,NN}}{n_N^{eq}} \propto \frac{n_{\sigma}^{eq}}{n_N^{eq}}, \quad (4.5)$$

where n_{σ}^{eq} and n_N^{eq} are given by (A8) and depend on $z_{\sigma} = \frac{m_{\sigma}}{T}$ and z_N , respectively. With

$$z_{\sigma} = \frac{m_{\sigma}}{M_N} z_N = \frac{\sqrt{2}}{g_N} z_N, \quad (4.6)$$

we can easily see that the minimal value z_{σ} can take is given by $z_{\sigma}^{\min} = \frac{\sqrt{2}}{g_N} z_I^{\min} = 1$, i.e., n_{σ}^{eq} is Boltzmann suppressed over the full range of interest, irrespective of g_N . On the other hand, we have $z_N^{\min} = z_I(g_N) < 1$, meaning that n_N^{eq} is not initially Boltzmann suppressed and we need to distinguish between $z_N < 1$ and $z_N > 1$, i.e.,

$$\frac{\gamma_{\sigma,NN}}{n_N^{eq}} \propto \begin{cases} \left(\frac{\sqrt{2}}{g_N}\right)^{\frac{3}{2}} e^{-\frac{\sqrt{2}}{g_N} z_N}, & z_N < 1, \\ \frac{m_{\sigma}}{M_N} e^{-\left(\frac{\sqrt{2}}{g_N}-1\right) z_N}, & z_N > 1. \end{cases} \quad (4.7)$$

In both regimes however, we find that the overall Boltzmann suppression of $\frac{\gamma_{\sigma,NN}}{n_N^{eq}}$ decreases with increasing g_N . Additionally, it is evident from Fig. 7 that $\Gamma_{\sigma,NN}$

⁶Here, T is the temperature.

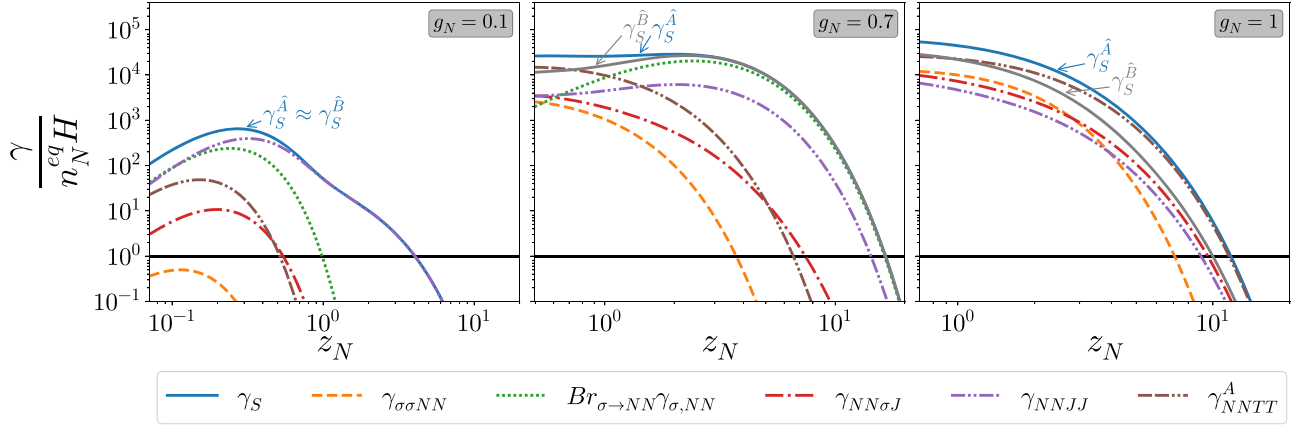


FIG. 8. Thermal rates of scattering processes that appear in the Majoron plus triplet model for $g_N = 0.1$ (left), $g_N = 0.7$ (middle), and $g_N = 1$. Note that we show $\gamma_{NNTT}^A = \gamma_{NNTT}^{\hat{A}}$ while $\gamma_{\sigma\sigma NN}$, $\gamma_{NN\sigma J}$, γ_{NNJJ} , and $\gamma_{\sigma, NN}$ are identical for cases $\hat{A}$ and $\hat{B}$. For $g_N = 0.1$, $\gamma_{NNTT}^{\hat{A}}$ is tiny, resulting in $\gamma_S^{\hat{A}} \approx \gamma_S^{\hat{B}}$. As g_N increases, the contribution of $\gamma_{NNTT}^{\hat{A}}$ to $\gamma_S^{\hat{A}}$ grows so that eventually $\gamma_S^{\hat{A}} > \gamma_S^{\hat{B}}$. This is especially striking for $g_N = 1$, where $\gamma_S^{\hat{B}} \approx \gamma_{\sigma\sigma NN} + \gamma_{NN\sigma J} + \gamma_{NNJJ}$ whereas $\gamma_S^{\hat{A}}$ is dominated by $\gamma_{NNTT}^{\hat{A}}$ while the contributions from $\gamma_{\sigma\sigma NN}$, $\gamma_{NN\sigma J}$, and γ_{NNJJ} are subdominant. Moreover, as g_N increases, the Boltzmann suppression of the thermal rates is shifted to larger z_N and consequently, the corresponding interactions are coupled to the plasma longer. We further stress that we used z_I as the lower limit for the z_N range presented and consequently, the respective plot range changes with g_N .

increases with g_N until it peaks at $g_N = \frac{1}{\sqrt{3}}$. Putting all these observations together, it is clear that z_S^{eq} increases with g_N while $g_N \leq 0.7$.

On the other hand, when $g_N > 0.7$, the previously dominant process $\gamma_{\sigma, NN}$ is forbidden and z_S^{eq} is determined by the processes $\gamma_{NNTT}^{\hat{A}}$ (case $\hat{A}$) and $\gamma_{NN\sigma J}$, γ_{NNJJ} , $\gamma_{\sigma\sigma NN}$ (case $\hat{B}$) which have an overall smaller magnitude, thereby explaining the rapid drop in z_S^{eq} .

2. Neutrino abundance

Next, we will compare the behavior of γ_S with respect to γ_D in more detail. In Fig. 9, we show $\frac{\gamma_{S,D,Q}}{n_N^{eq} H}$ which are the relevant quantities for the neutrino evolution (3.18).

It is apparent that $\frac{\gamma_S}{n_N^{eq} H}$ initially significantly exceeds $\frac{\gamma_D}{n_N^{eq} H}$ until $\frac{\gamma_S}{n_N^{eq} H}$ drops quickly due to Boltzmann suppression setting in. As $\frac{\gamma_D}{n_N^{eq} H}$ is not Boltzmann suppressed, it eventually exceeds $\frac{\gamma_S}{n_N^{eq} H}$. On the other hand, quark scatterings never dominate the neutrino evolution. Thus, neutrinos initially scatter with T , σ , and J before they decay until these scattering processes become inefficient and neutrinos can freely decay via γ_D . Consequently, we can distinguish between two regions; in the *scatter* regime, scattering processes dominate the neutrino evolution while in the *decay* regime, the neutrino evolution is driven by (inverse) neutrino decays. From (3.18), one can deduce that scattering processes dominate the neutrino evolution as long as the scatter term exceeds the decay term, i.e.,

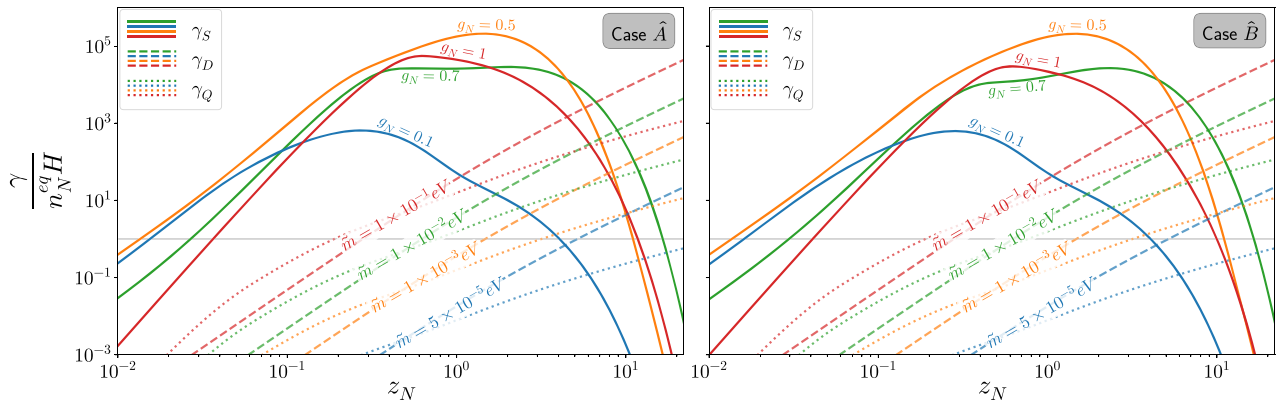


FIG. 9. Thermal rates relevant for the evolution of Y_N (3.18) for $g_N = [0.1, 0.5, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-3}, 10^{-2}, 10^{-1}]$ eV for case $\hat{A}$ (left) and case $\hat{B}$ (right). Note that $\frac{\gamma_S}{n_N^{eq} H}$ exceeds $\frac{\gamma_D}{n_N^{eq} H}$ until it drops due to being Boltzmann suppressed and subsequently, $\frac{\gamma_D}{n_N^{eq} H}$ becomes dominant while γ_Q is always subdominant to either γ_S or γ_D .

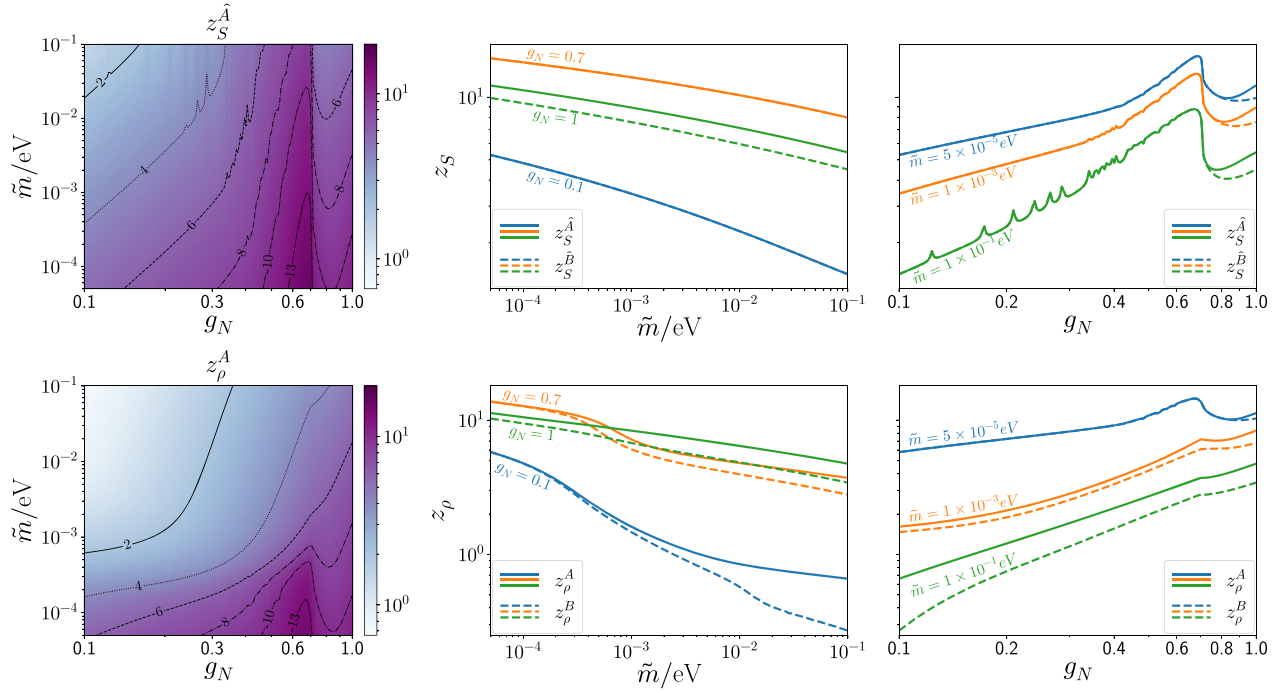


FIG. 10. *Upper plots:* Left: Density plot of z_S^A in the g_N - $\tilde{m}$ plane. Middle: z_S^A (solid) and z_S^B (dashed) as functions of $\tilde{m}$. Right: z_S^A (solid) and z_S^B (dashed) as functions of g_N . Note that for $g_N \leq 0.7$, we have $z_S^A \sim z_S^B$. We stress the striking similarity in the behavior of z_S compared to z_S^{eq} (see Fig. 7). *Lower plots:* Left: Density plot of z_ρ^A in the g_N - $\tilde{m}$ plane. Middle: z_ρ^A (solid) and z_ρ^B (dashed) as functions of $\tilde{m}$. Right: z_ρ^A (solid) and z_ρ^B (dashed) as functions of g_N .

$$4\gamma_S > \gamma_D. \quad (4.8)$$

Thus, we can make the approximation,

$$sH z_N \frac{d\hat{Y}_N}{dz_N} \approx \begin{cases} sH z_N \frac{d\hat{Y}_N^{scatter}}{dz_N} = -(\hat{\delta}_N - 1)4\gamma_S, & z_N \lesssim z_S, \\ sH z_N \frac{d\hat{Y}_N^{decay}}{dz_N} = -(\hat{\delta}_N - 1)\gamma_D, & z_N \gtrsim z_S, \end{cases} \quad (4.9)$$

where $z_S(\tilde{m}, g_N)$ is defined via

$$4\gamma_S(z_N = z_S, \tilde{m}) = \gamma_D(z_N = z_S, g_N). \quad (4.10)$$

Note that while γ_S is independent from $\tilde{m}$, we have $\gamma_D \sim \tilde{m}$ and conclusively, we expect that z_S increases with $\tilde{m}$. On the other hand, γ_D is independent of g_N and the dependence of z_S on g_N is dictated by γ_S . If $\frac{\gamma_S}{n_N^{eq} H}$ is already Boltzmann suppressed and consequently drops quickly around z_S , we expect that z_S depends similarly on g_N than z_S^{eq} . In Fig. 10 (upper panels), we show a density plot of $z_S^A(g_N, \tilde{m})$ (left) and $z_S(\tilde{m})$ (middle) and $z_S(g_N)$ (right) for exemplary values of g_N and $\tilde{m}$, respectively, which clearly display these dependencies. However, recalling the previous discussion, note that scattering processes keep neutrinos close to thermal equilibrium only for $z_N < z_S^{eq}$. If $z_S \gg z_S^{eq}$, this implies that for $z_S^{eq} < z_N < z_S$, scattering processes

dominate the neutrino evolution over decays but are not fast enough to ensure that neutrinos stay in thermal equilibrium. As a result, the neutrino abundance at z_S exceeds the thermal abundance at z_S . On the other hand, $z_S \lesssim z_S^{eq}$ results in a thermal neutrino abundance at z_S . This behavior is also highlighted in Fig. 11 where we show $Y_{N_A}(z_N)$ and $Y_{N_{VL}}(z_N)$ for $g_N = 0.1$ (left) and $g_N = 1$ (right). It is apparent that $Y_{N_A}^t(z_N)$ follows $Y_N^{eq}(z_N)$ closely until scattering processes fall out of thermal equilibrium around z_S^A while $Y_{N_{VL}}^t(z_N)$ deviates from thermal equilibrium significantly stronger and already for smaller z_N . Note that for $g_N = 0.1$, we have $Y_{N_A} \approx Y_{N_{VL}}^z$ around z_S so that Y_{N_A} approaches $Y_{N_{VL}}^z$ in the decay regime, indicating that the neutrino evolution in both cases is determined by γ_D . In the right panel with $g_N = 1$, the neutrino abundance in case $\hat{A}$ at the transition from the scatter to the decay regime is significantly smaller than $Y_{N_{VL}}^z$ and thus Y_{N_A} does not approach $Y_{N_{VL}}^z$ in the z_N range relevant for leptogenesis.

3. Efficiency

Let us now discuss the evolution of the efficiency. To that end, we show $\eta_A^{t,z}(z_N)$ and $\eta_{VL}^{t,z}(z_N)$ for $g_N = [0.1, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-3}, 10^{-1}]$ eV in Fig. 12.

Let us first discuss the case of a vanishing initial neutrino abundance. In VL, it is well-known that at first, a negative efficiency is produced via lepton number violating inverse

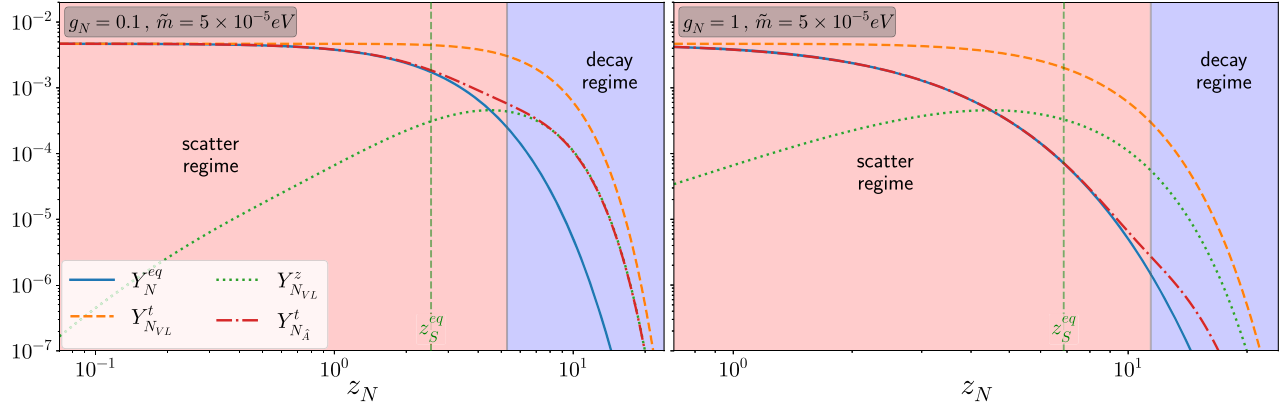


FIG. 11. Evolution of $Y_{N_{\hat{A}}}^t(z_N)$ and $Y_{N_{VL}}^{t,z}(z_N)$, compared to Y_N^{eq} . The red region denotes the scatter regime where the neutrino evolution in the Majoron plus triplet model is dominated by scattering processes while the blue region denotes the decay regime where (inverse) neutrino decays are dominant. The vertical line denotes z_S^{eq} where the scattering processes decouple. Note that the lower limits of the z_N range are given by z_I and hence depend on g_N .

decays and quark scatterings until neutrinos are thermalized at z_{eq}^{VL} so that

$$Y_{N_{VL}}^z(z_N = z_{eq}^{VL}) = Y_N^{eq}(z_N = z_{eq}^{VL}). \quad (4.11)$$

For $z_N > z_{eq}^{VL}$, a positive efficiency is produced and the final efficiency can be written as [39]

$$\eta_{VL}^z = \eta_{VL}^-|_0^{z_{eq}^{VL}} + \eta_{VL}^+|_{z_{eq}^{VL}}^\infty. \quad (4.12)$$

As $\eta_{VL}^+ > |\eta_{VL}^-|$, $\eta_{VL}^z(z_N)$ eventually changes sign at some $z_{VL}^\pm$ so that the final efficiency η_{VL}^z is positive. In case $\hat{A}$ in the Majoron plus triplet model on the other hand, neutrinos are thermalized mainly via the scattering processes in γ_S and only a fraction of neutrinos are produced via lepton number violating inverse decays and quark scatterings. Moreover, neutrinos are thermalized so quickly that inverse decays can only create a negative efficiency around $z_N \gtrsim z_I$ while for $z_N > z_N^\pm \gtrsim z_I$, the efficiency $\eta_{\hat{A}}^z(z_N)$ is positive. Thus, the negative efficiency created around z_I is significantly less sizable compared to the negative efficiency created in VL and $\eta_{\hat{A}}^z(z_N)$ approaches $\eta_{\hat{A}}^t(z_N)$ already for small z_N (see Fig. 12).⁷ Hence, the initial conditions are never relevant and for the remainder of this section, we will focus on $\eta_{\hat{A}}^t(z_N)$. The same considerations apply for case $\hat{B}$.

⁷From the panels (h) and (i) in Fig. 12, we can deduce that for these parameters sets, η_{VL}^z changes sign for smaller z_N compared to case $\eta_{\hat{A}}^z$. We stress however that this is merely due to the different z_I used in the Boltzmann equations; recall that in VL, we used $z_I^{VL} = 0$ while in the Majoron plus triplet model, leptogenesis can only take place for $z_{eq}^N \geq z_I$. In other words, η_{VL}^z changes sign for smaller z_N compared to $\eta_{\hat{A}}^z$ as the leptogenesis era begins earlier.

As can be seen in Fig. 13, the inverse decays $\frac{\gamma_D}{n_L^{eq} H}$ that are relevant for washout of the efficiency are Boltzmann suppressed and additionally, they are never thermalized when $\tilde{m}$ is small. In this case, washout processes are not effective and we can write the Boltzmann equation for the efficiency (3.9) as

$$sH z_N \frac{d\hat{\eta}}{dz_N} = \frac{1}{Y_N^0} (\delta_N - 1) \gamma_D. \quad (4.13)$$

In order to examine the evolution of the efficiency in the scatter and decay regimes, we insert (4.9) in (4.13), yielding

$$\frac{d\hat{\eta}}{dz_N} \approx \begin{cases} -\frac{1}{Y_N^0} \frac{\gamma_D}{4\gamma_S} \frac{d\hat{Y}_N}{dz_N}, & z_N \lesssim z_S, \\ -\frac{1}{Y_N^0} \frac{d\hat{Y}_N}{dz_N}, & z_N \gtrsim z_S, \end{cases} \quad (4.14)$$

and the final efficiency can be written as

$$\hat{\eta}(z_N \rightarrow \infty) = \frac{1}{Y_N^0} \left(\hat{Y}_N(z_S) - \int_{z_I}^{z_S} dz' \frac{\gamma_D}{4\gamma_S} \frac{d\hat{Y}_N}{dz_N} \right). \quad (4.15)$$

It is apparent that the efficiency consists of two parts; for $z_N < z_S$, the efficiency is suppressed by $4\gamma_S$ as neutrinos dominantly scatter while decays are ineffective. On the other hand, for $z_N > z_S$, the suppression is lifted as neutrinos dominantly decay via γ_D and thus the main portion of the efficiency is dictated by the neutrino abundance around z_S . However, the efficiency created around z_S is still indirectly suppressed as the scattering processes diminish the neutrino abundance that is left to decay at z_S . We can therefore deduce that the final efficiency evolves similarly to $\frac{1}{z_S}$; for increasing z_S , $\hat{\eta}(z_N)$ is suppressed over a larger range and as the neutrino abundance decreases with z_N , it reduces the neutrino

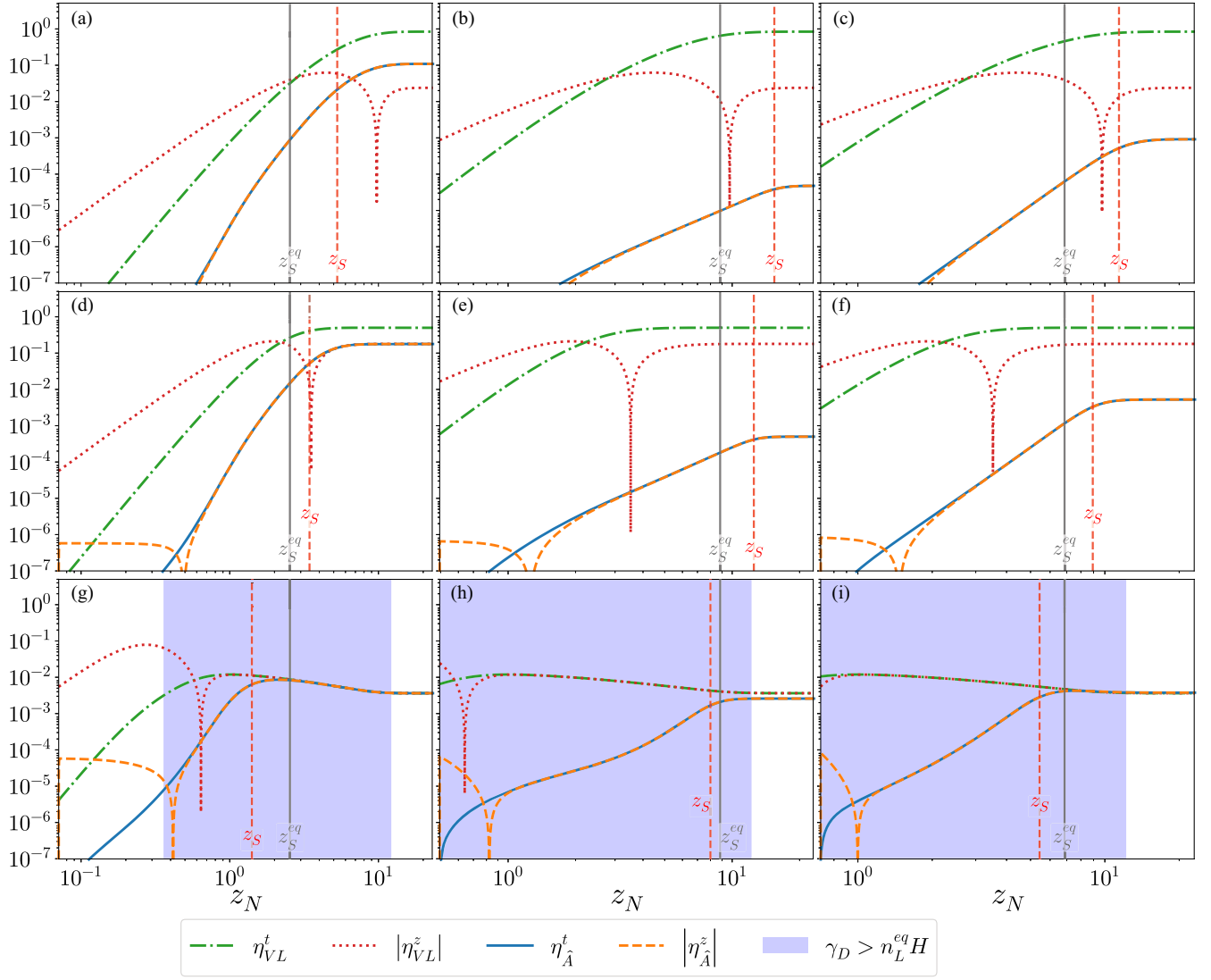


FIG. 12. Evolution of the efficiencies $\eta_{VL}^{t,z}$ and $\eta_A^{t,z}$ for $g_N = [0.1, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-3}, 10^{-1}]$ eV. The blue region in panels (g)–(i) indicates where inverse neutrino decays are in thermal equilibrium, the solid vertical line denotes z_S^{eq} and the dashed vertical line denotes z_S . Note that the lower limits of the z_N range are given by z_l and hence depend on g_N . As the efficiency in VL does not depend on g_N , $\eta_{VL}^{t,z}$ are the same in each column. The efficiencies $\eta_A^z(z_N)$ and $\eta_{VL}^z(z_N)$ are initially negative and change sign at $z_N^\pm$ and $z_{VL}^\pm$ (located at the striking singularities), respectively. In VL, neutrino production proceeds only via lepton number violating inverse neutrino decays and quark scatterings while in case $\hat{A}$, the scattering processes in γ_S are significantly more effective at populating the plasma with neutrinos, resulting in $\text{Max}(|\eta_{VL}^z(z_N < z_{VL}^\pm)|) \gg \text{Max}(|\eta_A^z(z_N < z_N^\pm)|)$. As $\tilde{m}$ increases, neutrino production via inverse decays and quark scatterings becomes more effective, hence $\text{Max}(|\eta_A^z(z_N < z_N^\pm)|)$ increases with $\tilde{m}$ [see e.g., panels (a),(d),(e)] while scattering processes with σ, J , and T are more effective as g_N increases, i.e., $\text{Max}(|\eta_A^z(z_N < z_N^\pm)|)$ decreases with g_N [see e.g., panels (d)–(f)]. Comparing η_A^t and η_{VL}^t , the suppression of η_A^t while $z_N < z_S$ is striking. In particular, even if inverse decays are in thermal equilibrium [panels (g)–(i)], they are mostly ineffective at diminishing η_{VL}^t until the suppression is lifted at z_S . (a) $g_N = 0.1$, $\tilde{m} = 5 \times 10^{-5}$ eV. (b) $g_N = 0.7$, $\tilde{m} = 5 \times 10^{-5}$ eV. (c) $g_N = 1$, $\tilde{m} = 5 \times 10^{-5}$ eV. (d) $g_N = 0.1$, $\tilde{m} = 1 \times 10^{-3}$ eV. (e) $g_N = 0.7$, $\tilde{m} = 1 \times 10^{-3}$ eV. (f) $g_N = 1$, $\tilde{m} = 1 \times 10^{-3}$ eV. (g) $g_N = 0.1$, $\tilde{m} = 1 \times 10^{-1}$ eV. (h) $g_N = 0.7$, $\tilde{m} = 1 \times 10^{-1}$ eV. (i) $g_N = 1$, $\tilde{m} = 1 \times 10^{-1}$ eV.

abundance at z_S . Recalling our previous discussion, this implies that $\hat{\eta}(\tilde{m})$ increases with $\frac{1}{z_S(\tilde{m})} \sim \tilde{m}$ while $\hat{\eta}(g_N)$ displays a similar behavior as $\frac{1}{z_S(g_N)} \sim \frac{1}{z_S^{eq}(g_N)}$, resulting in a dip in the efficiency around $g_N \sim 0.7$. This behavior is

striking when comparing Fig. 3 with Fig. 10 (top panels). In particular, we stress how $z_S^{\hat{B}} < z_S^{\hat{A}}$ for $g_N > 0.7$ translates to $\eta_{\hat{B}} > \eta_{\hat{A}}$.

This picture changes drastically when washout effects become relevant for larger $\tilde{m}$. In VL, $\eta_{VL}(z_N)$ is diminished

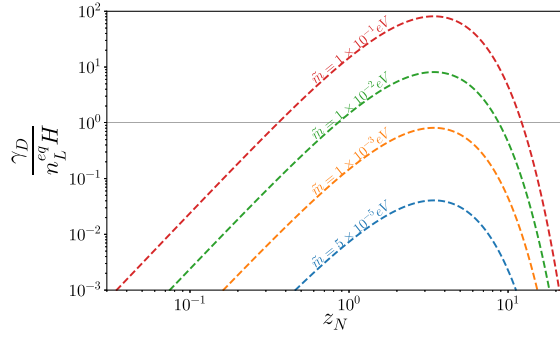


FIG. 13. Thermal rate $\frac{\gamma_D}{n_L^{\text{eq}} H}$ relevant for (3.9). We can easily see that inverse decays are thermalized only for $\tilde{m} > \times 10^{-3}$ eV.

by inverse decays and quark scatterings soon after they reach thermal equilibrium, resulting in $\eta_{VL} \sim \frac{1}{\tilde{m}}$ for $\tilde{m} \gtrsim \times 10^{-3}$ eV. Due to the additional scattering processes however, inverse decays being thermalized does not necessarily imply that washout processes reduce the efficiency $\hat{\eta}$.

Instead, recall that $\hat{\eta}$ is suppressed as long as $z_N < z_S$. As the washout terms are proportional to the efficiency, this implies that washout is mostly ineffective even in thermal equilibrium if $z_N < z_S$ and only relevant once a sizable efficiency is created for $z_N > z_S$. More precisely, if inverse decays are in thermal equilibrium, i.e., $\frac{\gamma_D}{n_L^{\text{eq}} H} > 1$ in the range $z_{D_1} < z_N < z_{D_2}$, the region where inverse decays can have a sizable effect on the efficiency is reduced to $\text{Max}[z_{D_1}, z_S] < z_N < z_{D_2}$ and may even completely vanish if $z_S > z_{D_2}$. This can easily be seen in Fig. 12 [panels (g)–(i)] where the blue region indicates where inverse decays are in thermal equilibrium. It is apparent that $\hat{\eta}$, unlike η_{VL} , is affected by WO effects only for $z_N \gtrsim z_S$ even if inverse decays are in thermal equilibrium. Consequently, for a given g_N , we can find the smallest $\tilde{m} =: \tilde{m}_S$ for which the interval $[\text{Max}[z_{D_1}, z_S], z_{D_2}]$ is not empty as

$$\frac{\gamma_D}{n_L^{\text{eq}} H}(\tilde{m}_S, z_S) = 1. \quad (4.16)$$

This implies that WO processes are relevant only for $\tilde{m} > \tilde{m}_S$, resulting in $\eta_{\hat{A}} \sim \frac{1}{\tilde{m}}$ (“WO regime”), while for $\tilde{m} < \tilde{m}_S$, WO processes can be neglected and therefore $\hat{\eta} \sim \tilde{m}$ (“VO regime”). Thus, $\tilde{m}_S$ separates the VO regime where the efficiency depends strongly on the additional scattering processes from the WO regime where leptogenesis in the Majoron plus triplet model proceeds very similar to the VL scenario with initially thermalized neutrinos. We stress however that this is only a rough lower bound since the WO term is generally Boltzmann suppressed for $z_N > z_S$ and thus for $\tilde{m} \gtrsim \tilde{m}_S$, it is thermalized too briefly to have a notable effect, i.e., we expect that WO becomes

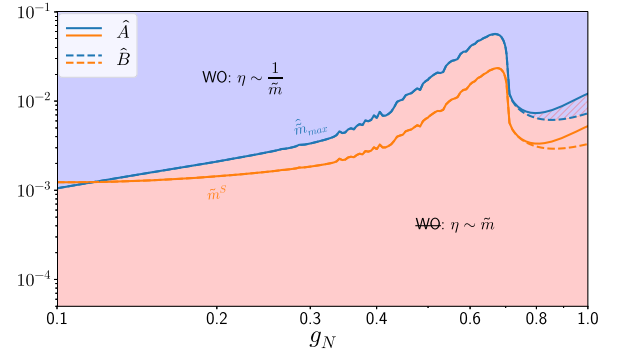


FIG. 14. Comparison of $\tilde{m}_S$ and $\hat{m}_{\text{max}}$ for case $\hat{A}$ (solid) and case $\hat{B}$ (dashed). In the red region, the final efficiency is determined by scattering processes and neutrino decays and thus proportional to $\tilde{m}$. On the other hand, the final efficiency in the blue regions is determined by washout processes, resulting in $\eta_{\hat{A}, \hat{B}} \sim \frac{1}{\tilde{m}}$. The hatched region corresponds to the WO regime in case $\hat{A}$ and to the VO regime in case $\hat{B}$. We can easily see how the strong suppression via scattering processes around $g_N \sim 0.7$ translates to peaks in $\tilde{m}_S$ and $\hat{m}_{\text{max}}$. With the suppression in case $\hat{B}$ being slightly weaker for $g_N > 0.7$, we also find that this results in $\tilde{m}_S^{\hat{A}} \gtrsim \tilde{m}_S^{\hat{B}}$ and $\hat{m}_{\text{max}}^{\hat{A}} \gtrsim \hat{m}_{\text{max}}^{\hat{B}}$. We also note that although $\tilde{m}_S$ slightly underestimates $\hat{m}_{\text{max}}$, it does display the same qualitative features and thus is in reasonably good agreement with $\hat{m}_{\text{max}}$.

effective for $\tilde{m}$ slightly larger than $\tilde{m}_S$. Moreover, we neglected the effect of quark scatterings in this definition of $\tilde{m}_S$.⁸

Nevertheless, these considerations imply that $\hat{\eta}(\tilde{m})$ reaches a maximum around $\tilde{m} \gtrsim \tilde{m}_S$. From the definition of $\tilde{m}_S$ it is also clear that $\tilde{m}_S$ depends similarly on g_N as z_S . Using our results for $\eta_{\hat{A}, \hat{B}}(g_N, \tilde{m})$, we further determine $\tilde{m} =: \tilde{m}_{\text{max}}^{\hat{A}, \hat{B}}$ where $\eta_{\hat{A}, \hat{B}}(g_N, \tilde{m})$ actually become maximal. In Fig. 14, we show $\tilde{m}_S$ and $\hat{m}_{\text{max}}$ and we find indeed $\hat{m}_{\text{max}} \gtrsim \tilde{m}_S \sim z_S$. As expected from the previous discussion, we also find $\tilde{m}_S^{\hat{A}} = \tilde{m}_S^{\hat{B}}$ for $g_N < 0.7$ and $\tilde{m}_S^{\hat{A}} > \tilde{m}_S^{\hat{B}}$ for $g_N > 0.7$. Thus, the suppression due to the scattering processes significantly diminishes the efficiency which in turn severely restricts the $\tilde{m}$ range in which WO processes have an effect on the efficiency compared to VL. This is especially striking for $g_N \sim 0.7$ where the suppression is so strong that $\eta_{\hat{A}, \hat{B}} < \eta^{\text{WO}}$ and WO barely has an effect.

To summarize, we find that similarly to type III leptogenesis, the initial neutrino abundance is irrelevant as neutrinos are rapidly thermalized due to the presence of the new scattering processes. However, we find that over a

⁸This treatment is similar to the discussions in [32,33] regarding type III leptogenesis. We note however that the results differ fundamentally due to the different particle content of the models.

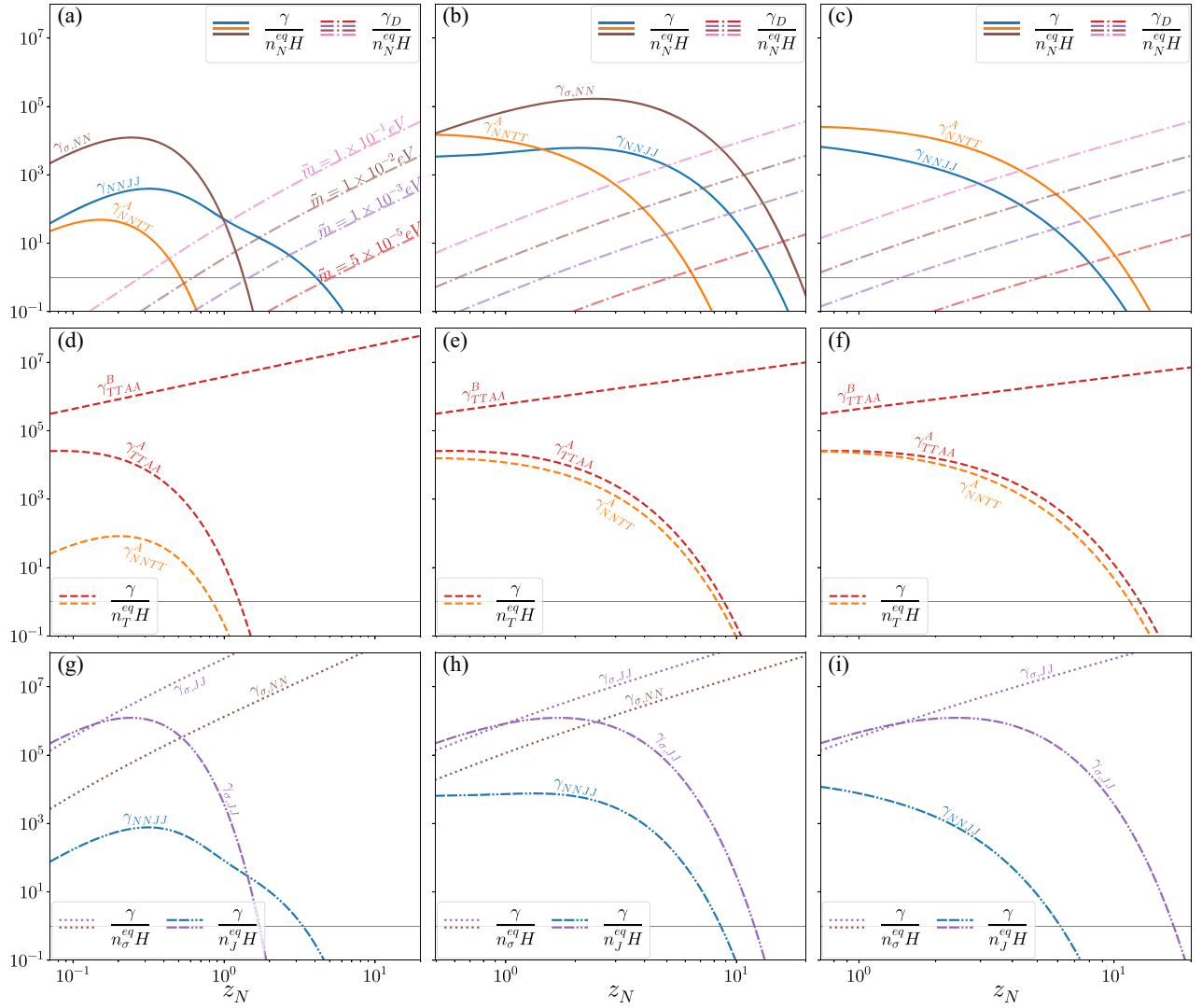


FIG. 15. Thermal rates relevant for the evolution of N (3.12) [panels (a)–(c)], T (3.13) [panels (d)–(f)], σ (3.14) [panels (g)–(i)] and J (3.15) [panels (g)–(i)] for $g_N = 0.1$ (left), $g_N = 0.7$ (middle) and $g_N = 1$ (right). The thermal rates without T in the initial or final state are identical for cases A and B. Note that the lower limits of the z_N range are given by z_I and hence depend on g_N . See text for details. (a) $g_N = 0.1$. (b) $g_N = 0.7$. (c) $g_N = 1$. (d) $g_N = 0.1$. (e) $g_N = 0.7$. (f) $g_N = 1$. (g) $g_N = 0.1$. (h) $g_N = 0.7$. (i) $g_N = 1$.

significant parameter space, scattering processes not only effectively thermalize the neutrinos but also stop them from decaying up to late z_N , thereby preventing the creation of a sizable efficiency. In the next section, we will explore how the efficiency is affected when we drop the assumption that σ , J , T are in thermal equilibrium.

C. Case A: $\delta_{T,\sigma,J} \neq 1, g_T = \lambda_\sigma = 1$

In this section, we discuss the efficiency for case A in more detail. Compared to the simplified scenarios $\hat{A}$, $\hat{B}$, the Boltzmann equations are significantly more complex and depend delicately on the effectiveness with which N , σ , J , and T are thermalized. For example, $\delta_T > \delta_N$ implies that γ_{NNTT} enhances the neutrino abundance while $\delta_T < \delta_N$

implies that the neutrino abundance is diminished. In the following, we therefore only highlight some fundamental features that have notable effects on the neutrino evolution and stress that the details are highly nontrivial.

1. Thermal rates

In Fig. 15, we show a subset of the thermal rates appearing in the Boltzmann equations (3.12)–(3.15), normalized to the respective number density, for $g_N \in [0.1, 0.7, 1]$. For clarity, we do not present all thermal rates but focus in these that are the most relevant for the overall perspective. For example, T interacts dominantly via gauge scatterings T , gauge^A and even if $Y_{T_A}(z_I) = 0$, these gauge scatterings instantly thermalize T . They are, however, Boltzmann suppressed with

$$\frac{T, \text{gauge}^A}{n_T^{eq}} \sim \left(\frac{g_N}{z_N}\right)^{\frac{3}{2}} \exp^{-\frac{z_N}{g_N}}, \quad (4.17)$$

and consequently, T freezes out at $z_T = \frac{z_N}{g_N} \sim 10$ (see Fig. 17). Clearly, T freezes out at larger z_N as g_N increases.

At $z_N \sim z_I$, interactions between N , σ , J , and T are so strong that N , σ , and J are instantly thermalized as well. Thus, as in the simplified case, we find that the initial abundances of N , σ , J , and T are irrelevant, i.e., $Y_{(N,\sigma,T,J)_A}^z(z_N > z_I) \sim Y_{(N,\sigma,T,J)_A}^t(z_N > z_I)$. Moreover, J interacts dominantly via $\frac{\gamma_{\sigma JJ}}{n_J^{eq} H}$ and $\frac{\gamma_{NNJJ}}{n_J^{eq} H}$ which are Boltzmann suppressed as

$$\frac{\gamma_{\sigma JJ}}{n_J^{eq}} \sim \frac{n_\sigma^{eq}}{n_J^{eq}} \sim \left(\frac{m_\sigma z_N}{M_N}\right)^{\frac{3}{2}} \exp^{-\frac{\sqrt{2} z_N}{g_N}}, \quad (4.18)$$

$$\frac{\gamma_{NNJJ}}{n_J^{eq}} \sim \frac{n_N^{eq2}}{n_J^{eq}} \sim \begin{cases} \left(\frac{M_N}{z_N}\right)^3, & z_N < 1, \\ M_N^3 \exp^{-2z_N}, & z_N > 1, \end{cases} \quad (4.19)$$

i.e., the respective Boltzmann suppressions are damped as g_N grows and J freezes out at larger z_N . As already discussed in Sec. IV B, the suppression of the scattering processes dominating the neutrino evolution is similarly less pronounced for larger g_N . On the other hand, the evolution of σ is determined by the decays $\frac{\gamma_{\sigma JJ}}{n_\sigma^{eq}}, \frac{\gamma_{\sigma NN}}{n_\sigma^{eq}}$ which are not Boltzmann suppressed. This has drastic consequences for the evolution of N as we demonstrate in the following.

2. Abundances

Similar to the discussion in Sec. IV B, let us first distinguish between a scattering regime where the neutrino evolution is determined by scattering processes and a decay regime where neutrino decays are the dominant processes, i.e.,

$$sH z_N \frac{dY_{N_A}}{dz_N} \approx sH z_N \begin{cases} \frac{dY_{N_A}^{\text{scatter}}}{dz_N} = -2(\delta_{N_A}^2 - 1)\gamma_S - 2\rho_A, & z_N < z_\rho^A, \\ \frac{dY_{N_A}^{\text{decay}}}{dz_N} = -(\delta_{N_A} - 1)\gamma_D, & z_N > z_\rho^A, \end{cases} \quad (4.20)$$

where at z_ρ^A the transition from scatter to decay regime occurs and we neglected quark scatterings. Note the similarities to (4.9). In the scatter regime, the first term proportional to γ_S in (4.20) generally reduces the neutrino abundance as $\delta_{N_A}, \gamma_S > 1$. On the other hand, the sign of the ρ_A -term is not immediately clear due to the subtractions of on shell scatterings [see (3.16)]. We do however find that

irrespective of g_N and $\tilde{m}$, $\rho_A(z_N)$ is always negative. Concerning the neutrino evolution, this implies that $-2\rho_A(z_N)$ gives a positive contribution to $\frac{dY_{N_A}}{dz_N}(z_N)$, i.e., the deviations from thermal equilibrium of σ , J , and T can enhance the neutrino abundance if $\rho_A(z_N)$ is sufficiently sizable. We stress however that for the most part, we have $[(\delta_{N_A}^2 - 1)\gamma_S](z_N) > |\rho_A(z_N)|$ so that $\frac{dY_{N_A}}{dz_N}(z_N) < 0$, i.e., the neutrino abundance overall decreases. In the decay regime, it is apparent that the neutrino evolution is mainly determined by the same Boltzmann equation as in the simplified scenario and thus we expect that the difference between the neutrino evolution in both scenarios is mainly due to the differing neutrino abundances at z_S and z_ρ^A , respectively.

In order to examine the neutrino evolution in the scatter and decay regimes in more detail, it is convenient to parametrize the deviation from thermal equilibrium of a particle i as

$$\Delta_i \equiv \delta_i - 1, \quad (4.21)$$

and discuss the evolutions of $\Delta_{N_A}(z_N)$ rather than $\frac{dY_{N_A}}{dz_N}(z_N)$ as shown in Fig. 16 for $g_N = [0.1, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-1}]$ eV. Moreover, in Fig. 10 (lower panels), we show a density plot of $z_\rho^A(\tilde{m}, g_N)$ (left) and $z_\rho^A(\tilde{m})$ (middle), and $z_\rho^A(g_N)$ (right) for exemplary values of g_N and $\tilde{m}$, respectively.

First, let us discuss the effect of $\tilde{m}$ on the neutrino evolution in the scatter regime. As the thermal rates that dominate in the scatter regime do not depend on $\tilde{m}$, it is clear that $\Delta_{N_A}(z_N)$ is independent from $\tilde{m}$ as long as $z_N < z_\rho^A$, see Fig. 16. Additionally, as $\gamma_D \sim \tilde{m}$, we conclude that z_ρ^A decreases with $\tilde{m}$, similarly to z_S^A . Comparing z_S^A and z_ρ^A , we find however that z_ρ^A decreases faster with $\tilde{m}$ than z_S^A (see left panels in Fig. 10) which can be traced back to effects of ρ_A on the transition from the scatter to the decay regime. More precisely, the transition occurs when

$$\left| \frac{dY_{N_A}^{\text{scatter}}}{dz_N} \right| = \left| \frac{dY_{N_A}^{\text{decay}}}{dz_N} \right| \text{ which yields the condition, } \gamma_D \approx 4\gamma_S + \frac{2\rho_A}{\epsilon} \lesssim 4\gamma_S, \quad z_N = z_\rho^A, \quad (4.22)$$

if neutrinos are close to equilibrium, $\delta_{N_A} = 1 + \epsilon$ with $0 < \epsilon \ll 1$ and $\rho_A < 0$. This implies that z_ρ^A can easily be smaller than z_S^A and in particular, it results in z_ρ^A so small that the Boltzmann suppression of γ_S is not yet very effective, resulting in a significantly stronger dependence on $\tilde{m}$ compared to z_S^A .

The effects of g_N on the neutrino evolution in the scatter regime are more evolved. For $g_N \lesssim 0.7$, we find that $\Delta_{N_A}(z_N) \gg \Delta_{N_A}(z_N)$, i.e., neutrinos deviate significantly more from thermal equilibrium compared to the simplified

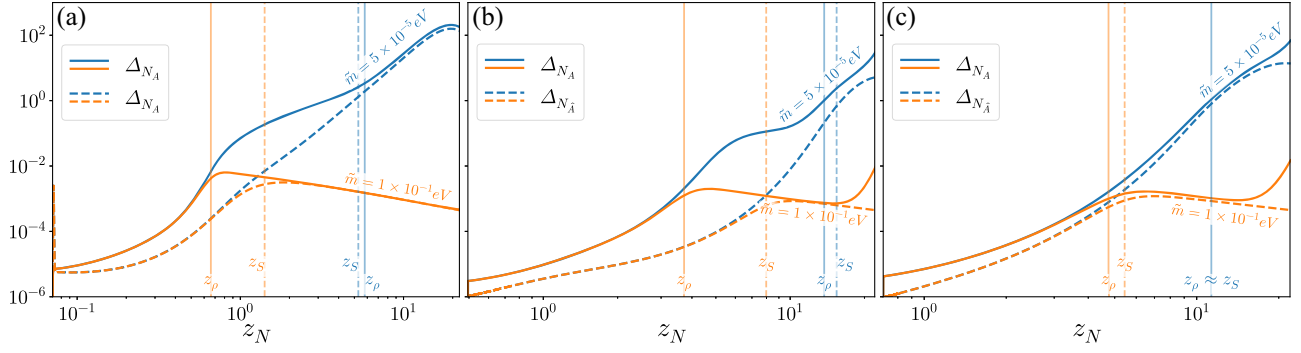


FIG. 16. Evolution of $\Delta_{N_A} = \delta_{N_A} - 1$ (solid lines) and $\Delta_{N_{\hat{A}}} = \delta_{N_{\hat{A}}} - 1$ (dashed lines) for $\tilde{m} = [5 \times 10^{-5}, 10^{-1}]$ eV and $g_N = 0.1$ (a), $g_N = 0.7$ (b), $g_N = 1$ (c). The vertical solid lines indicate z_ρ^A while the vertical dashed lines denote $z_S^{\hat{A}}$. Note that the lower limits of the z_N range are given by z_I and hence depend on g_N . For $z_N < z_\rho^A$ and $z_N < z_S^{\hat{A}}$, respectively, the neutrino evolution is determined by scattering processes with T , σ , and J and thus $\Delta_{N_A}(z_N < z_\rho^A)$ and $\Delta_{N_{\hat{A}}}(z_N < z_S^{\hat{A}})$ are independent from $\tilde{m}$. In case A for $z_N < z_\rho^A, z_S^{\hat{A}}$, neutrinos deviate significantly more from thermal equilibrium compared to the simplified scenario $\hat{A}$, i.e., $\Delta_{N_A}(z_N < z_\rho^A, z_S^{\hat{A}}) \gg \Delta_{N_{\hat{A}}}(z_N < z_\rho^A, z_S^{\hat{A}})$. In panel (a), $\Delta_{N_A}(z_N)$ approaches $\Delta_{N_{\hat{A}}}(z_N)$ for $z_N > z_\rho^A$ as the neutrino evolution in this regime is entirely driven by inverse neutrino decays. In panels (b) and (c), $\Delta_{N_A}(z_N)$ initially approaches $\Delta_{N_{\hat{A}}}(z_N)$ for $z_N > z_\rho^A$ but eventually starts to increase again due to freeze out of T .

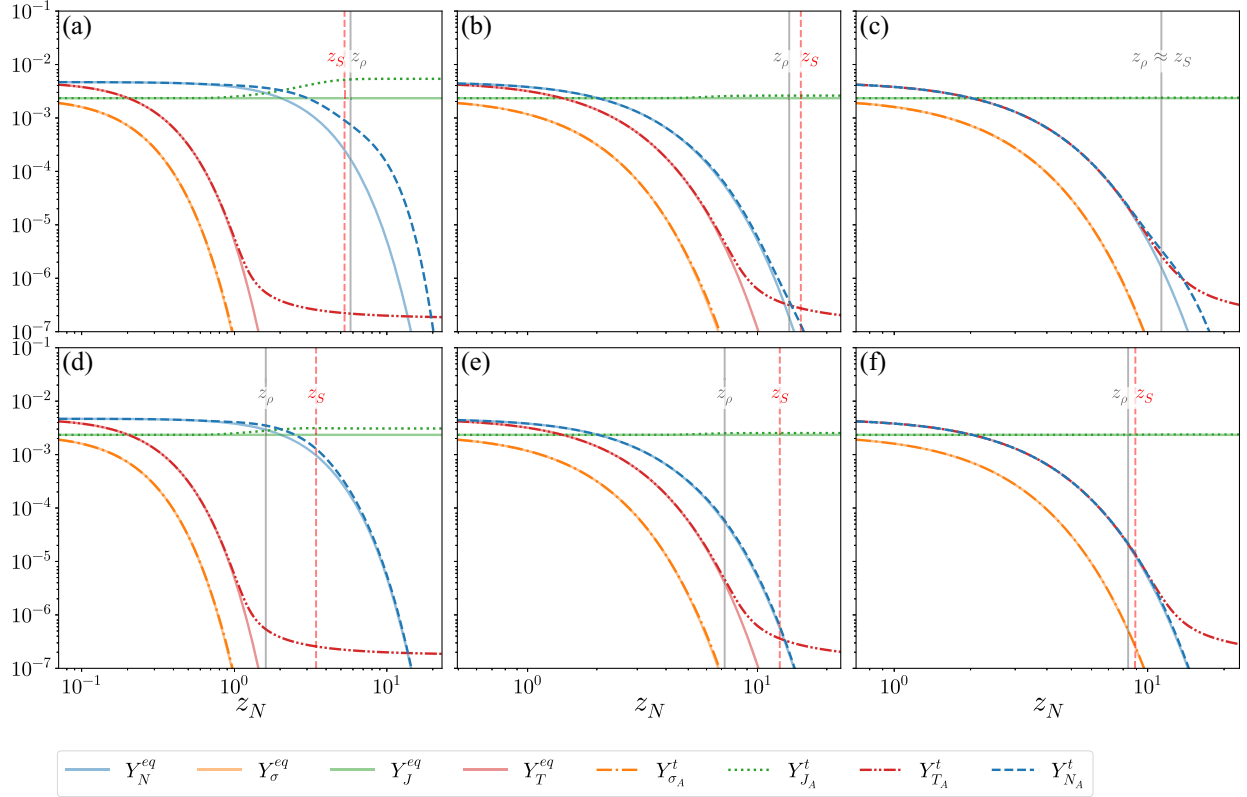


FIG. 17. Evolution of the abundances of N , σ , T , and J for $g_N = [0.1, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-2}]$ eV. In the right plot, we have $g_N = g_T$ and therefore $Y_{N_A}^{eq} = Y_T^{eq}$. Note that the lower limits of the z_N range are given by z_I and hence depend on g_N . Scattering processes and inverse decays initially keep N , T , σ , and J close to thermal equilibrium. While σ never deviates significantly from thermal equilibrium, T freezes out and eventually obtains a constant abundance once gauge interactions decouple at $z_T = \frac{z_N}{g_N} \sim 10$, independently from $\tilde{m}$. The evolutions of both neutrinos and Majorons on the other hand are significantly affected by g_N and $\tilde{m}$ and if g_N and $\tilde{m}$ are small, their abundances deviate significantly from thermal equilibrium. In particular, J freezes out once interactions with σ and N decouple and for e.g., $g_N = 0.1$ and $\tilde{m} = 5 \times 10^{-5}$ eV (a), the Majoron abundance after freeze out notably exceeds the corresponding thermal abundance. (a) $g_N = 0.1$, $\tilde{m} = 5 \times 10^{-5}$ eV. (b) $g_N = 0.7$, $\tilde{m} = 5 \times 10^{-5}$ eV. (c) $g_N = 1$, $\tilde{m} = 5 \times 10^{-5}$ eV. (d) $g_N = 0.1$, $\tilde{m} = 1 \times 10^{-3}$ eV. (e) $g_N = 0.7$, $\tilde{m} = 1 \times 10^{-3}$ eV. (f) $g_N = 1$, $\tilde{m} = 1 \times 10^{-3}$ eV.

case [panels (a) and (b) in Fig. 16]. While the details are highly nontrivial, two essential ingredients are that interactions of N , σ , and J with T are too weak to thermalize them effectively (recall that T is efficiently thermalized via gauge interactions) while fast σ decays create nonthermal N and J abundances as demonstrated in Fig. 17. Moreover, note that with increasing g_N , $\Delta_{N_A}(z_N)$ deviates from thermal equilibrium up to larger z_N , as one would expect due to the shift of the Boltzmann suppression to larger z_N . For $g_N > 0.7$ on the other hand, interactions with T are more efficient at thermalizing N , σ , and J . Additionally, $\frac{\gamma_{\sigma JJ}}{n_J H}$ is fast up to larger z_N compared to $g_N < 0.7$ while $\sigma \leftrightarrow NN$ is kinematically forbidden. As a consequence, the neutrino abundance is not significantly enhanced in the scatter regime and we have $\Delta_{N_A}(z_N) \gtrsim \Delta_{N_{\hat{A}}}(z_N)$ for $z_N < z_\rho^A$ [panel (c) in Fig. 16].

Next, let us discuss the neutrino evolution in the decay regime. For $g_N < 0.7$, we find that irrespective of the value of $\tilde{m}$, $\Delta_{N_A}(z_N)$ eventually approaches $\Delta_{N_{\hat{A}}}(z_N)$ for $z_N \gg z_\rho^A, z_S^A$, i.e., the neutrino evolution is determined only by γ_D while scattering processes with the new particles are irrelevant, similarly to the simplified scenario [panel (a) in Fig. 16]. As discussed above, this is to be expected given that the evolution of the neutrinos in the decay regimes in cases $\hat{A}$ and A is governed by the same Boltzmann equation. This simple picture changes however for $g_N \geq 0.7$. We find that initially, $\Delta_{N_A}(z_N)$ still tracks $\Delta_{N_{\hat{A}}}(z_N)$, although $\Delta_{N_A}(z_N)$ slightly exceeds $\Delta_{N_{\hat{A}}}(z_N)$ even for $\tilde{m} \rightarrow 10^{-1}$ eV. Eventually, $\Delta_{N_A}(z_N)$ begins to increase while $\Delta_{N_{\hat{A}}}(z_N)$ keeps decreasing as for $g_N < 0.7$ [panels (b) and (c) in Fig. 16]. This is especially striking for $\tilde{m} \rightarrow 10^{-1}$ eV. The reason for this behavior is that although decay processes dominate the neutrino evolution for $g_N \geq 0.7$, the neutrino abundance is slightly enhanced due to the freeze out of T . In the decay regime, neutrinos are thermalized mainly via γ_D and as $\frac{T_{\text{gauge}}^A}{n_T^q}$ drops, T couples weaker to the plasma than the neutrinos, resulting in $\delta_{T_A}(z_N) > \delta_{N_A}(z_N)$ and consequently, the respective term in the Boltzmann equation for the neutrino evolution gives a positive contribution to the neutrino abundance. As a result, $\left| \frac{dY_{N_A}}{dz_N} \right|$ drops slower than in a scenario where only decays are relevant, thereby enhancing $\Delta_{N_A}(z_N)$. Just before T completely freezes out and decouples, $\delta_{T_A}(z_N)$ begins to increase significantly as $Y_T^{eq}(z_N)$ becomes Boltzmann suppressed while $Y_{T_A}(z_N)$ becomes constant, resulting in a short but striking enhancement of $\Delta_{N_A}(z_N)$. Thus, despite decay processes being dominant for $z_N > z_\rho^A$, scattering processes are still relevant and affect the neutrino evolution. To summarize, we find that neutrinos can deviate significantly more from thermal equilibrium compared to the simplified case as the (inverse) decays and neutrino scatterings involving σ , J , and T introduce a source of

nonequilibrium which is redistributed amongst N , σ , J , and T . Although the distinction between a scatter and a decay regime is not entirely accurate for $g_N > 0.7$, it does grasp the overall qualitative features of the neutrino evolution. In the following, we will discuss how this affects the evolution of the efficiency.

3. Efficiency

In Fig. 19, we show $\eta_A(z_N)$ for exemplary values of $\tilde{m}$ and g_N . Similarly to the simplified scenario, we find that in the case of vanishing initial abundances of N , σ , T , and J , the initially negative efficiency η_A^c quickly changes sign as neutrinos are mainly produced via scattering processes involving σ , T , and J . However, neutrino production via lepton number violating inverse decays and quark scatterings plays a larger role in case A compared to the simplified scenario as in the former case, the initial abundances of T , σ , and J vanish as well. Nevertheless, η_A^c approaches η_A^l very and for simplicity, we will focus the following discussion on η_A^l .

Let us at first focus on a regime where $\tilde{m}$ is small so that WO can be neglected. The Boltzmann equations in the scatter and decay regimes, respectively, can be then written as (neglecting quarks)

$$\begin{aligned} \frac{d\eta_A}{dz_N} &= \frac{1}{sH z_N} (\delta_{N_A} - 1) \gamma_D \frac{1}{Y_N^0} \\ &\approx \begin{cases} -\frac{1}{Y_N^0} \frac{\gamma_D}{4\gamma_S} \left(\frac{dY_{N_A}}{dz_N} + \frac{2\rho_A}{sH z_N} \right), & z_N < z_\rho^A, \\ -\frac{1}{Y_N^0} \frac{dY_{N_A}}{dz_N}, & z_N > z_\rho^A, \end{cases} \end{aligned} \quad (4.23)$$

and we can write the final efficiency as

$$\eta_A(\infty) = \eta_A(z_\rho^A) + \frac{1}{Y_N^0} Y_{N_A}(z_\rho^A). \quad (4.24)$$

We can easily see that as in the simplified scenario, the efficiency in the scatter regime is suppressed by $\frac{\gamma_D}{4\gamma_S}$. However, neutrinos deviate significantly more from thermal equilibrium in the scatter regime compared to the simplified scenario which counteracts the suppression due to scattering processes. Thus, while the main portion of the efficiency in the simplified scenario is created at the transition from the scatter to the decay regime at z_S^A , this picture changes drastically in case A as the enhanced neutrino abundance allows the creation of an substantial part of the final efficiency already in the scatter regime. This is especially striking for $\tilde{m} = 5 \times 10^{-5}$ eV in Fig. 19 [panels (a)–(c)] where $z_\rho^A \sim z_S^A$ and $\eta_A(z_\rho^A) \gg \eta_A^c(z_\rho^A)$. Moreover, as the transition from the scatter to the decay regime in case A generally occurs for smaller z_N than the transition in the simplified scenario, it is reasonable to assume that $Y_{N_A}(z_\rho^A)$ exceeds $Y_{N_{\hat{A}}}(z_S^A)$ which additionally

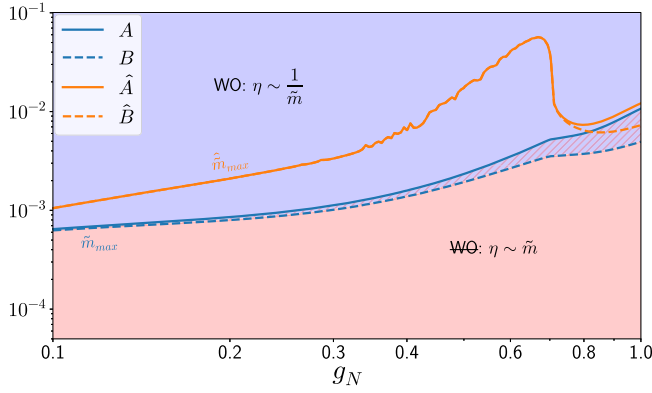


FIG. 18. Comparison of $\tilde{m}_{\max}$ and $\hat{m}_{\max}$ for case $\hat{A}$, A (solid) and case $\hat{B}$, B (dashed). In the red region, the final efficiency in cases A , B is determined by scattering processes and neutrino decays and thus proportional to $\tilde{m}$. On the other hand, the final efficiency in the blue regions is determined by washout processes, resulting in $\eta_{A,B} \sim \frac{1}{\tilde{m}}$. The hatched region corresponds to the WO regime in case A and to the $\mathbb{W}\Theta$ regime in case B . We can immediately see that the strong suppression of the efficiency in the simplified scenario that results in a peak around $g_N \sim 0.7$ in $\tilde{m}_{\max}$ is absent in cases A , B . Additionally, neutrinos are thermalized weaker in cases A , B compared to the simplified scenario, translating to $\tilde{m}_{\max} < \hat{m}_{\max}$. Even more precisely, in case B , the absence of interactions with T renders the scattering processes less efficient in thermalizing the neutrinos compared to case A , resulting in $\tilde{m}_{\max}^B < \tilde{m}_{\max}^A$.

enhances the efficiency that can be generated in the decay regime of case A compared to the simplified scenario. Thus, as we already saw in Fig. 4 (right), the efficiency in case A for small $\tilde{m}$ is suppressed similarly to the simplified scenario but the suppression is overall much less pronounced.

As in the simplified scenario, increasing $\tilde{m}$ damps the suppression of the efficiency and while WO effects can be neglected, we have $\eta_A \sim \tilde{m}$ until a maximum is reached at $\tilde{m}_{\max}$ and WO processes subsequently dominate, resulting in $\eta_A \sim \frac{1}{\tilde{m}}$, see Fig. 4 (middle). However, the regime where WO processes are effective in case A differs significantly from the simplified scenario. As can be seen in Fig. 19 [panels (g)–(i)], the suppression of $\eta_A(z_N)$ is significantly weaker compared to $\eta_{\hat{A}}(z_N)$ but still sufficiently strong in order to render WO processes ineffective up to $z_N \sim z_{\rho}^A$, similar to the discussion in Sec. IV B. As is demonstrated in Fig. 18, this results in $\tilde{m}_{\max}^A \sim z_{\rho}^A \ll \hat{m}_{\max}^A \sim z_S$ and in particular, the striking peak around $g_N \sim 0.7$ in $\tilde{m}_{\max}^A$ is missing in $\hat{m}_{\max}^A$.

Finally, let us shortly comment on the efficiency for $g_N \rightarrow 1$ and $\tilde{m} \rightarrow 10^{-1}$ eV. From Fig. 4, recall that the final efficiency in this case slightly exceeds the “common” efficiency in the strong WO regime η^{WO} . In light of our previous discussion, this unexpected behavior can be explained as a combination of two effects. For $g_N \sim 1$, the suppression of $\eta_A(z_N)$ in the scatter regime is maximal

and thus WO processes effectively diminish the efficiency only for $\tilde{m} \gtrsim 10^{-2}$ eV (see Fig. 18). As can be seen in Fig. 19 [panel (i)] though, even for $\tilde{m} = 10^{-1}$ eV, washout is effective only for $z_N \gtrsim 6$. Just after that, T begins to freeze out which enhances the neutrino abundance and therefore counteracts the washout of the efficiency. Thus, even though WO processes dictate the overall evolution of $\eta_A(\tilde{m})$ in the WO regime, leptogenesis is not entirely driven by VL interactions. Note that the significant enhancement Δ_{N_A} around $z_N \sim 20$ we observed in Fig. 16 does not have a significant effect as due to the strong Boltzmann suppression of $Y_N(z_N)$ around $z_N \sim 20$, even a sizable deviation from thermal equilibrium in this regime only corresponds to a tiny enhancement of the neutrino abundance.

D. Case B : $\delta_{T,\sigma J} \neq 1, g_T = 10^{-7}, \lambda_{\sigma} = 1$

In this section, we discuss the efficiency in case B where in contrast to case A , we have $g_T = 10^{-7}$ which results in a dependence of the efficiency on the initial conditions. We begin with a short discussion of the thermal rates before separately analyzing the evolution of the efficiency for the distinct initial conditions. In the discussion of case B_z , we focus in particular on the regime where the initial conditions are relevant for the final efficiency.

1. Thermal rates

In Fig. 15, we show thermal rates relevant for the Boltzmann equations (3.12)–(3.15) and as in case A , we focus only on the most relevant ones. In contrast to the thermal rates in case A , the interactions of T with N , σ , and J in case B are practically irrelevant while gauge interactions thermalize T over the full range of z_N that is of interest for leptogenesis. In particular, T does not freeze out during the leptogenesis era. On the other hand, the interactions between N , σ , and J that do not involve T are not affected and identical to A . As a result, the evolutions of N , σ , and J are decoupled from the evolution of T .

2. Discussion of case B_t

After discussing the efficiency for cases $\hat{A}$, $\hat{B}$, and A , applying the findings to case B_t is straightforward. First, note that gauge interactions keep T close to thermal equilibrium, independently of g_N and $\tilde{m}$. On the other hand, due to the absence of notable interactions with T , we find that in the scatter regime, N , σ , and J deviate from thermal equilibrium slightly more compared to case A . This can be seen in Fig. 20 where we show $\Delta_{N_A}(z_N)$ and $\Delta_{N_B}(z_N)$ for different values of g_N and $\tilde{m}$. Clearly, this translates to a larger final efficiency compared to case A . Additionally, as is highlighted in Fig. 10, the transition from the scatter to the decay regime occurs for a slightly smaller z_N compared to case A , i.e., $z_{\rho_B} \lesssim z_{\rho_A}$, as the absence of interactions with T slightly reduces the overall magnitude of scattering processes involving neutrinos.

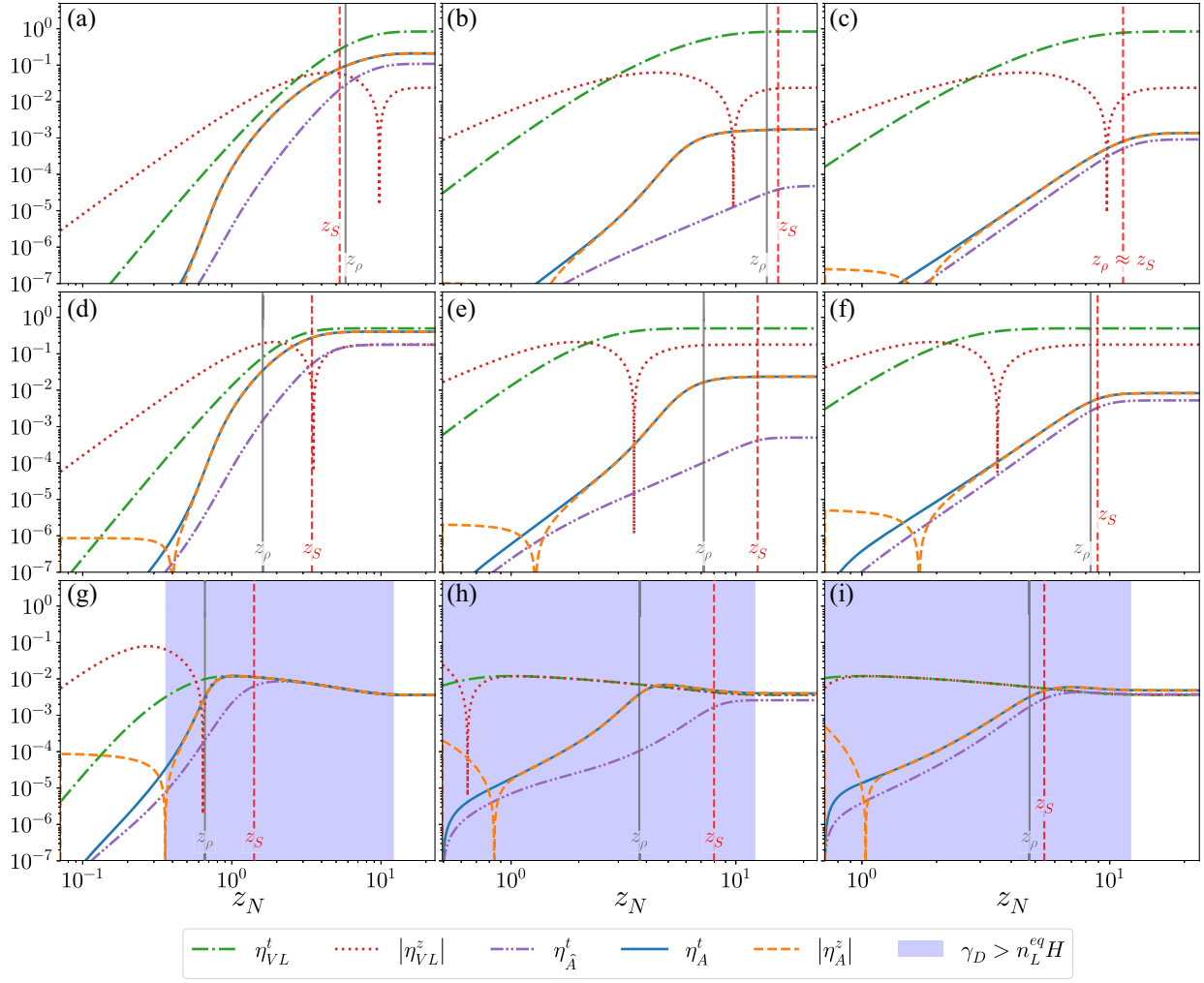


FIG. 19. Efficiencies $\eta_A^{t,z}$, η_A^t , and $\eta_{VL}^{t,z}$ as functions of z_N . The blue region indicates where inverse decays are in thermal equilibrium, the solid vertical line denotes z_ρ and the dashed vertical line denotes z_S . Note that the lower limits of the z_N range are given by z_I and hence depend on g_N . As the efficiency in VL does not depend on g_N , $\eta_{VL}^{t,z}$ are the same in each column. In case A, neutrinos are coupled weaker to the plasma compared to case $\hat{A}$, resulting in a weaker suppression of η_A^t compared to $\eta_{\hat{A}}^t$. In panels [(g)–(i)], it is apparent that WO processes diminish η_A for smaller z_N compared to $\eta_{\hat{A}}$. As the initial abundances of T , σ , and J vanish in case A_z (in contrast to case $\hat{A}_z$ where they have thermal abundances), a larger portion of the neutrino production is via lepton number violating inverse decays and quark scatterings compared to case $\hat{A}$ (see Fig. 12). Hence, $\text{Max}(|\eta_A^z(z_N < z_N^\pm)|) \gg \text{Max}(|\eta_{\hat{A}}^z(z_N < z_N^\pm)|)$. Nevertheless, η_A^z eventually approaches $\eta_{\hat{A}}^z$, irrespective of $\tilde{m}$ and g_N . (a) $g_N = 0.1$, $\tilde{m} = 5 \times 10^{-5} \text{ eV}$. (b) $g_N = 0.7$, $\tilde{m} = 5 \times 10^{-5} \text{ eV}$. (c) $g_N = 1$, $\tilde{m} = 5 \times 10^{-5} \text{ eV}$. (d) $g_N = 0.1$, $\tilde{m} = 1 \times 10^{-3} \text{ eV}$. (e) $g_N = 0.7$, $\tilde{m} = 1 \times 10^{-3} \text{ eV}$. (f) $g_N = 1$, $\tilde{m} = 1 \times 10^{-3} \text{ eV}$. (g) $g_N = 0.1$, $\tilde{m} = 1 \times 10^{-1} \text{ eV}$. (h) $g_N = 0.7$, $\tilde{m} = 1 \times 10^{-1} \text{ eV}$. (i) $g_N = 1$, $\tilde{m} = 1 \times 10^{-1} \text{ eV}$.

This is particularly striking for $g_N > 0.7$ where in case A, γ_{NNTT} is the dominating process which is absent in case B, similarly to our discussion regarding $\hat{A}$, $\hat{B}$. Further, as can be seen in Fig. 18, the increased efficiency implies that the transition from the $\mathbb{W}\Theta$ to the WO regime takes place at smaller $\tilde{m}$ compared to case A and therefore $\tilde{m}_{\text{max}}^{B,t} < \tilde{m}_{\text{max}}^A$.

3. Discussion of case B_z

Abundances. Due to the strong gauge interactions, T is thermalized instantly at z_I irrespective of g_N and $\tilde{m}$ and as

in case B_t , its evolution is independent from N , σ and J . On the other hand, the thermalization of N , σ , and J depends strongly on g_N and $\tilde{m}$. This is highlighted in Fig. 21 where we show density plots of $z_{eq}^{(N,\sigma,J)}(\tilde{m}, g_N)$ where $z_{eq}^{(N,\sigma,J)}$ are defined via

$$Y_{i_B}^z(z_{eq}^i) = Y_i^{eq}(z_{eq}^i) \quad (4.25)$$

with

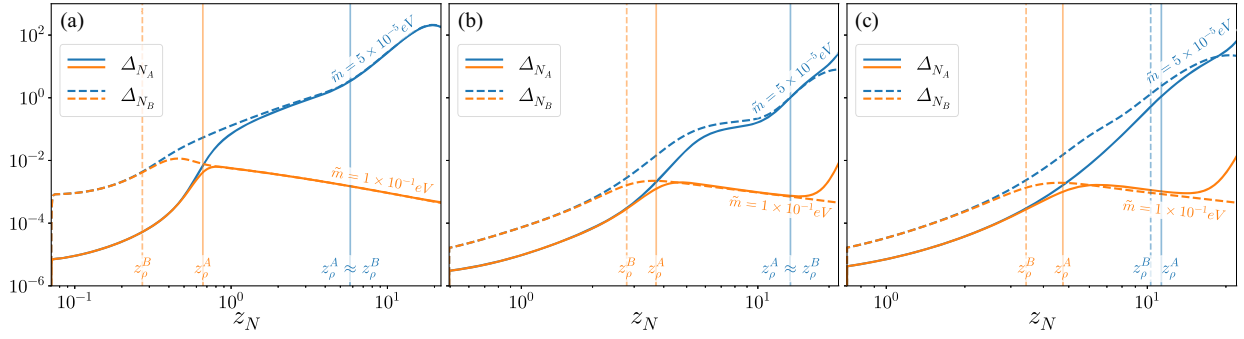


FIG. 20. Evolution of $\Delta_{N_A} = \delta_{N_A} - 1$ (solid lines) and $\Delta_{N_B} = \delta_{N_B} - 1$ (dashed lines) for different values of $\tilde{m} = [5 \times 10^{-5}, 10^{-1}] \text{ eV}$ and $g_N = 0.1$ (a), $g_N = 0.7$ (b), $g_N = 1$ (c). The vertical solid lines indicate z_ρ^A while the vertical dashed lines belong to z_ρ^B . Note that the lower limits of the z_N range are given by z_I and hence depend on g_N . For $z_N \lesssim z_\rho^B$, the neutrino abundance in case *B* deviates more from thermal equilibrium than in case *B* as interactions with T are absent. For $g_N \geq 0.7$, the neutrino evolution in case *B* is entirely driven by inverse neutrino decays, hence $\Delta_{N_B}(z_N \gg z_\rho) < \Delta_{N_A}(z_N \gg z_\rho)$.

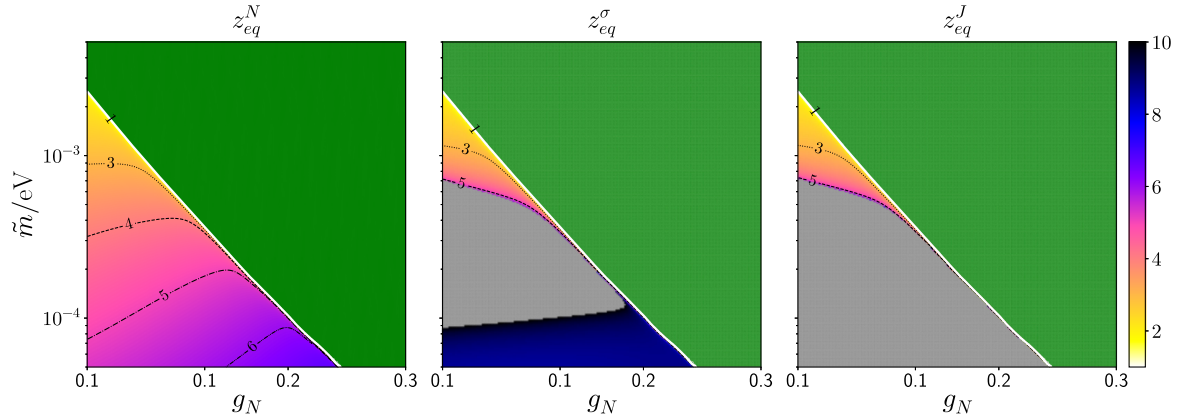


FIG. 21. Density plots of $z_{eq}^{(N,σ,J)}$ in the $g_N - \tilde{m}$ plane. In the green region, thermalization proceeds rapidly so that $z_{eq}^i \ll 1$ while in the gray region, the respective particle is not thermalized until $z_N = 10$. The white line indicates $z_{eq}^i = 1$. Note that we do not show the full $g_N - \tilde{m}$ plane. See text for details.

$$Y_{i_B}^z(z_N) \begin{cases} < Y_i^{eq}(z_N), & z_N < z_{eq}^i, \\ > Y_i^{eq}(z_N), & z_N > z_{eq}^i. \end{cases} \quad (4.26)$$

Further, we define $(g_N^{IA}, \tilde{m}^{IA})$ which we already introduced in Sec. IV A so that

$$z_{eq}^N(\tilde{m}^{IA}, g_N^{IA}) = 1, \quad (4.27)$$

with

$$z_{eq}^N(\tilde{m} < \tilde{m}^{IA}, g_N < g_N^{IA}) > 1, \quad (4.28)$$

$$z_{eq}^N(\tilde{m} > \tilde{m}^{IA}, g_N > g_N^{IA}) < 1. \quad (4.29)$$

In Fig. 21, we show $(\tilde{m}^{IA}, g_N^{IA})$ as a white line while in the green regions, we have $z_{eq}^i < 1$. We stress that there is no fundamental reason to explicitly choose $z_{eq}^N(\tilde{m}^{IA}, g_N^{IA}) = 1$ as the implicit definition of $\tilde{m}^{IA}, g_N^{IA}$. As Boltzmann

suppression of neutrinos sets in at $z_N \approx 1$, it is reasonable to assume that once $z_{eq}^N \lesssim 1$, the effect of the initial abundance on the neutrino evolution and the efficiency is small which motivates our choice of (4.27). Moreover, comparing Figs. 6 and 21, we can easily see that this definition of $(g_N^{IA}, \tilde{m}^{IA})$ reasonably well separates the IA and the $\bar{\text{IA}}$ regime. As we will see later, the explicit value used in (4.27) is irrelevant as long as it is close to 1. Additionally, we used a cutoff at $z_N = 10$ to simplify the visualization of z_{eq}^σ and z_{eq}^J .⁹

From Fig. 21, we easily see that σ and J do not reach thermal equilibrium until $z_N = 10$ over a significant parameter space where $(g_N, \tilde{m}) < (g_N^{IA}, \tilde{m}^{IA})$. On the other

⁹Due to Boltzmann suppression, we expect that around $z_N \approx 10$, the effects of σ and J on the neutrino evolution are no longer significant which motivates the choice of $z_N = 10$. The exact value is however of no relevance for the following discussion.

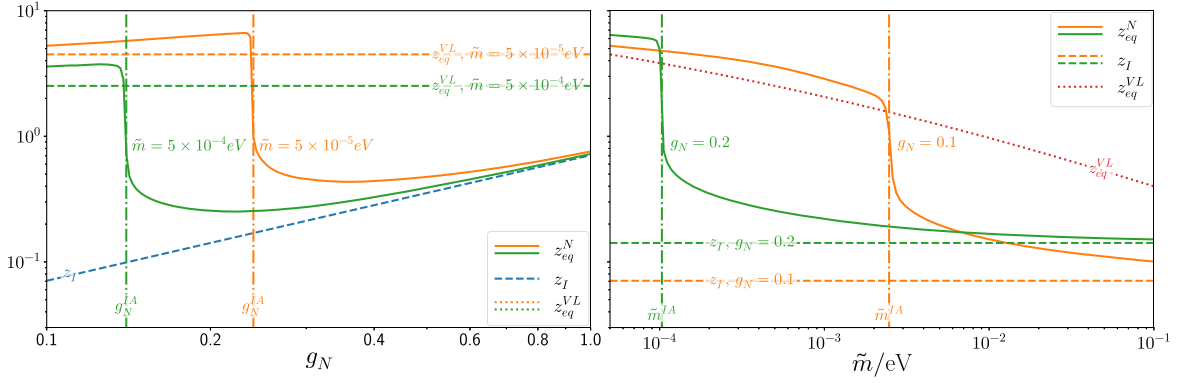


FIG. 22. *Left:* z_{eq}^N as a function of g_N for $\tilde{m} = [5 \times 10^{-5}, 510^{-4}]$ eV, compared to z_{eq}^{VL} and z_I . *Right:* z_{eq}^N as a function of $\tilde{m}$ for $g_N = [0.1, 0.2]$, compared to z_{eq}^{VL} and z_I . We note that $z_{eq}^{VL}(z_I)$ is constant in the left (right) panel as it does not depend on g_N ($\tilde{m}$). The dash-dotted line denotes g_N^{IA} (left) and $\tilde{m}^{IA}$ (right), respectively. See text for details.

hand, similarly to the neutrinos, we find that σ and J are quickly thermalized once $(g_N, \tilde{m}) > (g_N^{IA}, \tilde{m}^{IA})$, indicating that there is a close correlation between their thermalizations. In the following, we will discuss this in greater detail.

As the initial abundances of N , σ , and J vanish, the Boltzmann equation for the neutrino evolution at $z_N \approx z_I$ is the same as in VL,

$$sH z_N \frac{dY_{N,B,VL}^z}{dz_N} \approx \gamma_D + 2\gamma_Q, \quad (4.30)$$

and the plasma is populated with neutrinos via inverse decays and quark scatterings. As the interactions of neutrinos with σ and J are significantly stronger than $\gamma_{D,Q}$, neutrinos soon begin to populate the plasma with σ and J and the Boltzmann equation for case B_z becomes

$$sH z_N \frac{dY_{N,B}^z}{dz_N} \approx (\gamma_D + 2\gamma_Q) + \kappa, \quad (4.31)$$

where we defined

$$\begin{aligned} \kappa \equiv & -2(\overline{\delta_N^2} - \overline{\delta_J^2})\gamma_{NNJJ} - 2(\overline{\delta_N^2} - \overline{\delta_\sigma^2})\gamma_{NN\sigma\sigma} \\ & - 2(\overline{\delta_N^2} - \overline{\delta_J \delta_\sigma})\gamma_{NN\sigma J} + 2\overline{\delta_{\text{sub}}}\gamma_{\sigma,NN}, \end{aligned} \quad (4.32)$$

with

$$\overline{\delta_i} \equiv \delta_{i_B}^z, \quad (4.33)$$

and (4.31) holds as long as $\overline{\delta_N} \ll 1$. Here, κ plays an essential role in the neutrino evolution as depending on the sign, it can either enhance the neutrino abundance via scatterings and σ decays to neutrinos or diminish it via the inverse processes. In the first case, neutrinos are thermalized faster than in VL while in the second case, neutrino thermalization is slower compared to VL.

In order to make the effect of the scattering processes appearing in κ more clear, let us discuss the effects of $\tilde{m}$ and g_N on z_{eq}^N individually. To that end, we show $z_{eq}^N(g_N)$ and $z_{eq}^N(\tilde{m})$ in Fig. 22 for $g_N = [0.1, 0.2, 0.24]$ and $\tilde{m} = [5 \times 10^{-5}, 510^{-4}, 2.5 \times 10^{-3}]$ eV, respectively, while in Fig. 23, we show $Y_{(N,\sigma,J)_B}^z(z_N)$ and compare them to the respective thermal abundances. First, let us discuss the effect of $\tilde{m}$ on z_{eq}^N and for definiteness, we set $g_N = 0.1$. For $\tilde{m} = 5 \times 10^{-5}$ eV [panel (a) in Fig. 23], the plasma is populated with neutrinos very slowly as γ_D and γ_Q are initially small. As a result, the scatterings in κ become Boltzmann suppressed before thermal σ , J abundances can be created. As the scatterings to σ and J reduce the neutrino abundance, we have $Y_{N_B}^z(z_N) < Y_{N_{VL}}^z(z_N)$ and consequently, neutrinos are thermalized later than in VL, i.e., $z_{eq}^N > z_{eq}^{VL}$.

As $\tilde{m}$ increases to e.g., $\tilde{m} = 510^{-4}$ eV [panel (d) plot in Fig. 23], inverse decays and quark scatterings populate the plasma with neutrinos faster and a larger abundance of σ , J can be produced before the corresponding scattering processes become Boltzmann suppressed. As in VL, this implies that z_{eq}^N and z_{eq}^{VL} decrease with $\tilde{m}$ (right plot in Fig. 22) while $z_{eq}^N > z_{eq}^{VL}$ still holds.

Eventually, as $\tilde{m} \approx 2.5 \times 10^{-3}$ eV [panel (g) in Fig. 23], a sufficiently large neutrino abundance is created for small z_N , allowing efficient interactions between N , σ , J before they become Boltzmann suppressed. Thus, N , σ , J are thermalized significantly faster than before and $z_{eq}^N(\tilde{m})$ drops rapidly and approaches z_I (right plot in Fig. 22). On the other hand, the neutrino thermalization in VL depends only on $\gamma_{D,Q} \sim \tilde{m}$ and consequently $z_{eq}^{VL}(\tilde{m})$ smoothly decreases, eventually resulting in $z_{eq}^N < z_{eq}^{VL}$.

Next, let us discuss the effect of g_N on z_{eq}^N . As discussed previously, the Boltzmann suppression of the scattering rates involving N , σ , J is shifted to larger z_N and the

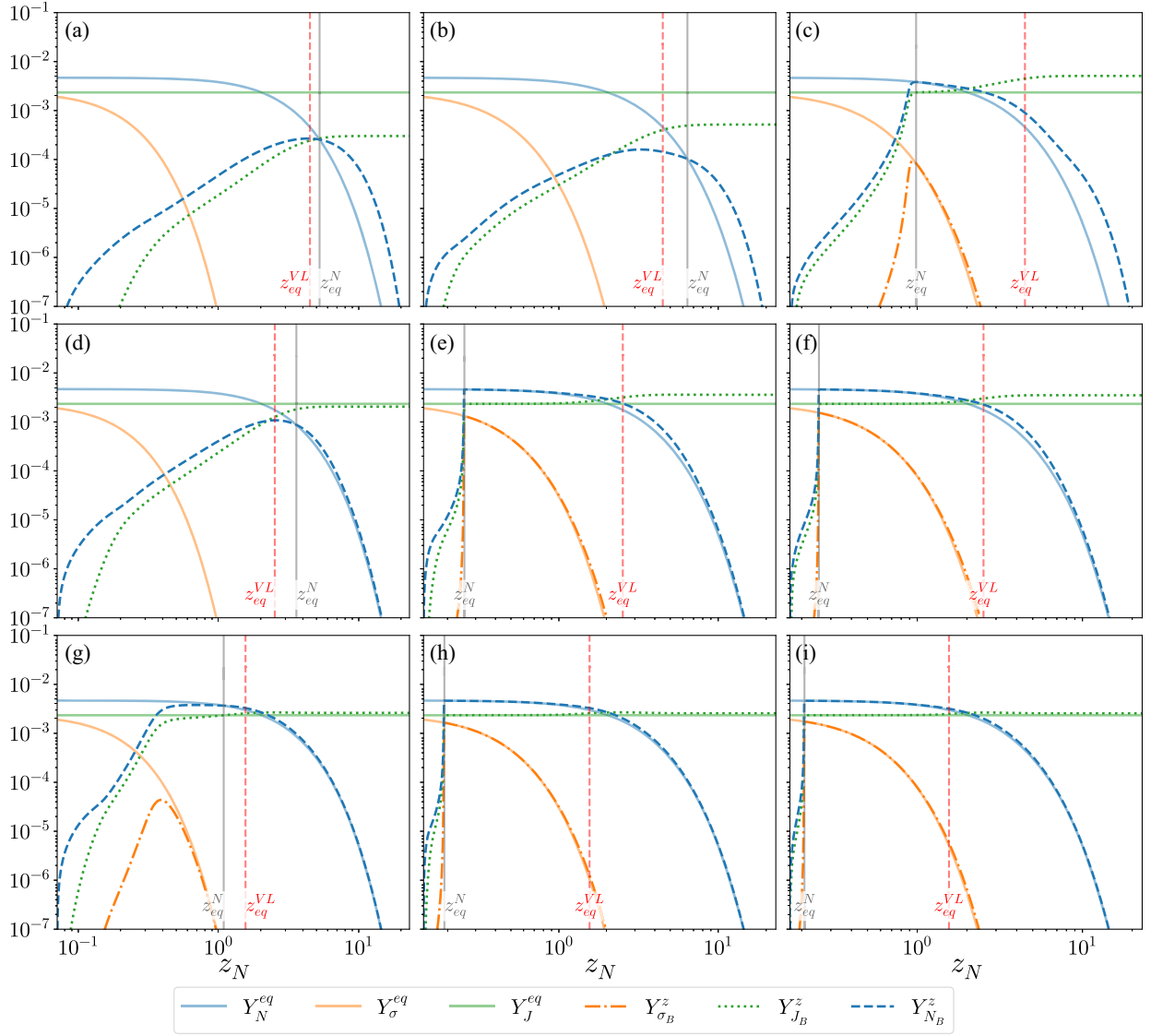


FIG. 23. Evolution of $Y_{N,\sigma,J}(z_N)$, compared to the respective equilibrium abundances, for $g_N = [0.1, 0.2, 0.24]$ and $\tilde{m} = [5 \times 10^{-5}, 5 \times 10^{-4}, 2.5 \times 10^{-3}]$ eV. The vertical solid line indicates z_{eq}^N while the vertical dashed line indicates z_{eq}^{VL} . We do not show Y_T as it is always close to thermal equilibrium. For the parameter choices in panels (a),(b),(d), no sizable Y_σ abundance is produced and therefore it does not appear in the respective figures. In the same panels, the Majoron is never thermalized and hence freezes in once the interactions with N and σ decouple, resulting in a relic abundance smaller than the thermal one. Comparing e.g., panels (a) and (b), it is apparent how slightly increasing g_N slows the neutrino thermalization down due to an enhanced scattering rate of neutrinos to Majorons until g_N reaches a value that results in sufficiently strong interactions between N , σ , and J that briefly thermalized them before these interactions decouple (c). Increasing $\tilde{m}$ enhances the neutrino production and therefore indirectly the production of σ and J , thermalizing them faster [see e.g., panels (b),(d),(h)]. (a) $g_N = 0.1$, $\tilde{m} = 5 \times 10^{-5}$ eV. (b) $g_N = 0.2$, $\tilde{m} = 5 \times 10^{-5}$ eV. (c) $g_N = 0.24$, $\tilde{m} = 5 \times 10^{-5}$ eV. (d) $g_N = 0.1$, $\tilde{m} = 5 \times 10^{-4}$ eV. (e) $g_N = 0.2$, $\tilde{m} = 5 \times 10^{-4}$ eV. (f) $g_N = 0.24$, $\tilde{m} = 5 \times 10^{-4}$ eV. (g) $g_N = 0.1$, $\tilde{m} = 2.5 \times 10^{-3}$ eV. (h) $g_N = 0.2$, $\tilde{m} = 2.5 \times 10^{-3}$ eV. (i) $g_N = 0.24$, $\tilde{m} = 2.5 \times 10^{-3}$ eV.

magnitude of the overall thermal rates increases as we move to larger g_N . Considering for example $g_N = 0.2$ and $\tilde{m} = 5 \times 10^{-5}$ eV [panel (b) in Fig. 23], we find that this initially slows the neutrino thermalization down compared to $g_N = 0.1$. This can be traced back to a larger number of neutrinos scattering to σ and J while the interaction rates are not yet fast enough to thermalize them,

consequently reducing the neutrino abundance. Thus, $z_{eq}^N(g_N)$ initially increases with g_N , while z_{eq}^{VL} is constant with $z_{eq}^N(g_N) > z_{eq}^{VL}$. Eventually though, at $g_N \approx 0.24$ [panel (c) in Fig. 23], the scattering processes between N , σ , J are fast enough to thermalize them efficiently and $z_{eq}^N(g_N)$ drops rapidly and approaches z_I , resulting in $z_{eq}^N(g_N) < z_{eq}^{VL}$ (left plot in Fig. 22).

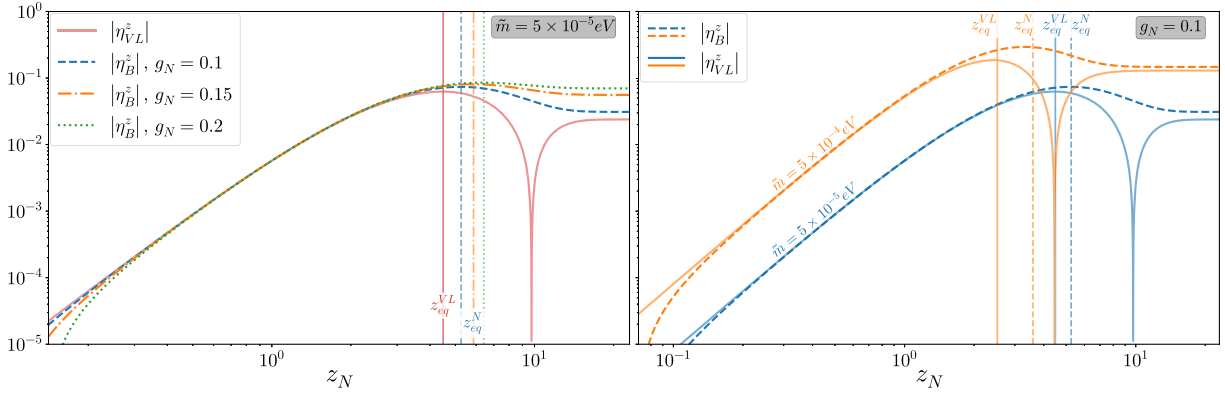


FIG. 24. *Left*: Evolution of $|\eta_B^z|(z_N)$ for $g_N = [0.1, 0.15, 0.2]$ (dashed, dash-dotted, dotted) and $\tilde{m} = 5 \times 10^{-5}$ eV, compared to $|\eta_{VL}^z|$ (solid). For $z_N \ll z_{eq}^N, z_{eq}^{VL}$, the evolution of the efficiency is dominated by (inverse) neutrino decays and therefore independent from g_N . The efficiencies $|\eta_B^z|(z_N)$ and $|\eta_{VL}^z|(z_N)$ increase until they peak at z_{eq}^N and z_{eq}^{VL} , respectively. In VL, $\eta_{VL}^z(z_N)$ eventually changes sign while $\eta_B^z(z_N)$ stays negative. As z_{eq}^N increases with g_N , we have $\eta_B^z(z_{eq}^N)|_{g_N=0.1} < \eta_B^z(z_{eq}^N)|_{g_N=0.15} < \eta_B^z(z_{eq}^N)|_{g_N=0.2}$. *Right*: Evolution of $|\eta_B^z|(z_N)$ (solid) for $g_N = 0.1$ and $\tilde{m} = [5 \times 10^{-5}, 5 \times 10^{-4}]$ eV compared to $|\eta_{VL}^z|$ (dashed). For $z_N \ll z_{eq}^N, z_{eq}^{VL}$, the evolution of the efficiency is determined by $\gamma_D \sim \tilde{m}$, i.e., a larger $\tilde{m}$ enhances the negative part of η_B^z and η_{VL}^z produced in $z_N \ll z_{eq}^N$ and $z_N \ll z_{eq}^{VL}$, respectively. As neutrinos are thermalized faster, z_{eq}^N and z_{eq}^{VL} decrease with $\tilde{m}$.

To summarize, we find that while $(\tilde{m}, g_N) < (\tilde{m}^{IA}, g_N^{IA})$, the scattering processes in κ suppress the neutrino abundance as neutrinos scatter to σ and J , resulting in a slower thermalization compared to VL. On the other hand, for $(\tilde{m}, g_N) > (\tilde{m}^{IA}, g_N^{IA})$, we find that the scattering processes enhance the thermalization of N, σ, J . We further note that since $z_{eq}^N(\tilde{m}, g_N)$ drops rapidly around $(\tilde{m}^{IA}, g_N^{IA})$, they are very insensitive to the exact value of z_{eq}^N used in the definition (4.27).

Efficiency. In the IA regime, $\tilde{m}$ is small so that we can safely neglect WO effects and the Boltzmann equation for η_B^z becomes

$$sH z_N \frac{d\eta_B^z}{dz_N} \approx \frac{1}{Y_N^0} (\bar{\delta}_N - 1) \gamma_D. \quad (4.34)$$

Clearly, before neutrinos are thermalized at z_{eq}^N , we have $\bar{\delta}_N - 1 < 0$ and a negative efficiency is created. We therefore define

$$\eta_B^- \equiv \eta(z_{eq}^N) < 0. \quad (4.35)$$

For $z_N \ll z_{eq}^N$, we can simplify (4.34) to

$$sH z_N \frac{d\eta_B^z}{dz_N} \approx -\frac{1}{Y_N^0} \gamma_D, \quad (4.36)$$

and as can be seen in Fig. 24, this implies that $\eta_B^z(z_N \ll z_{eq}^N)$ closely follows $\eta_{VL}^z(z_N \ll z_{eq}^N)$ and is mostly independent from g_N . For $z_N > z_{eq}^N$ on the other hand, a positive efficiency is created with

$$\eta_B^+ \equiv \int_{z_{eq}^N}^{\infty} dz_N \frac{1}{Y_N^0} (\bar{\delta}_N - 1) \gamma_D \frac{1}{sH z_N} > 0. \quad (4.37)$$

Assuming that the scattering processes involving N, σ, J are already Boltzmann suppressed and (inverse) decays $N \leftrightarrow LH$ are the only relevant processes for $z_N > z_{eq}^N$, we can simplify this to

$$\eta_B^+ \approx \frac{1}{Y_N^0} Y_{N_B}^z(z_{eq}^N). \quad (4.38)$$

Thus, the final efficiency is given by

$$\eta_B^z(\tilde{m}, g_N) = \eta_B^+(\tilde{m}, g_N) + \eta_B^-(\tilde{m}, g_N), \quad (4.39)$$

where $\eta_B^+(\tilde{m}, g_N)$ and $\eta_B^-(\tilde{m}, g_N)$ clearly depend significantly on $z_{eq}^N(\tilde{m}, g_N)$ and in particular, $\eta_B^+(\tilde{m}, g_N) < |\eta_B^-(\tilde{m}, g_N)|$ results in a negative final efficiency. From Fig. 4, recall that η_B^z is indeed negative if $\tilde{m}$ and g_N are small and for convenience, we define that $\eta_B^z(\tilde{m}, g_N)$ changes sign at $(\tilde{m}^\pm, g_N^\pm)$, i.e.,

$$\eta_B^+(\tilde{m}^\pm, g_N^\pm) = |\eta_B^-(\tilde{m}^\pm, g_N^\pm)|, \quad (4.40)$$

so that

$$\eta_B^z(\tilde{m}, g_N) < 0, \quad (\tilde{m}, g_N) < (\tilde{m}^\pm, g_N^\pm), \quad (4.41)$$

$$\eta_B^z(\tilde{m}, g_N) > 0, \quad (\tilde{m}, g_N) > (\tilde{m}^\pm, g_N^\pm). \quad (4.42)$$

In Fig. 25, we show $|\eta_B^-|, \eta_B^+$, and η_B^z as functions of g_N (left) and $\tilde{m}$ (right) with $\tilde{m} = 5 \times 10^{-5}$ eV and $g_N = 0.1$, respectively. In the blue regions, the initial conditions are relevant

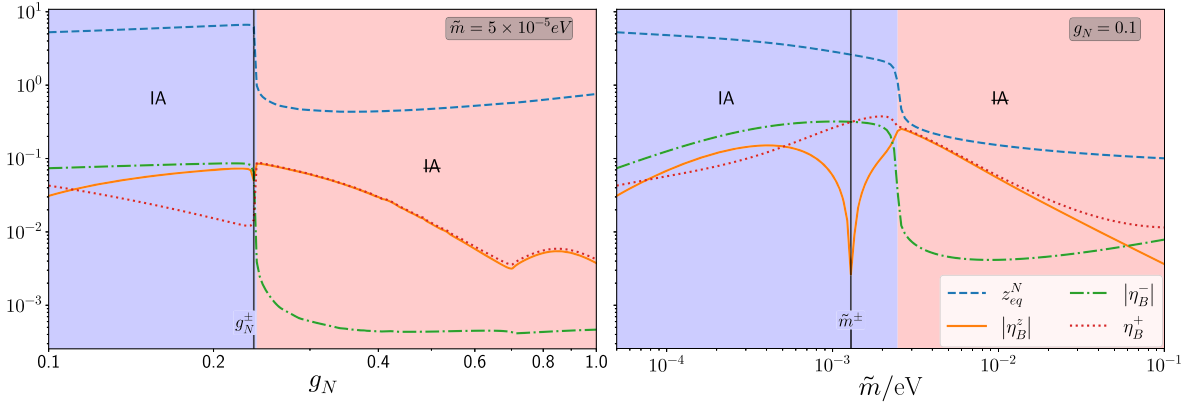


FIG. 25. *Left:* $|\eta_B^z|$, $|\eta_B^-|$, η_B^+ , and z_{eq}^N as functions of g_N with $\tilde{m} = 5 \times 10^{-5}$ eV. In the blue region, the final efficiency depends on the initial abundances while in the red region, the final efficiency is independent from the initial abundance. The vertical line denotes $g_N^\pm$. *Right:* $|\eta_B^z|$, $|\eta_B^-|$, η_B^+ , and z_{eq}^N as functions of $\tilde{m}$ with $g_N = 0.1$. In the blue region, the final efficiency depends on the initial abundances while in the red region, the final efficiency is independent from the initial abundance. The vertical line denotes $\tilde{m}^\pm$. Note that as $\tilde{m} \gtrsim 10^{-2}$ eV, $|\eta_B^-|$ begins to increase again. This is due to washout processes reducing the efficiency and independent from the scattering processes in the Majoron plus triplet model.

while in the red regions, the initial conditions are irrelevant for the final efficiency.

Let us first focus on the blue region in the left plot. We immediately see that η_B^z changes sign at $g_N^\pm \approx g_N^{IA}$ and for simplicity, we will assume equality in the following discussion so that the blue region also corresponds to $\eta_B < 0$. From (4.36) it is clear that the dependence of η_B^- on g_N stems only from $z_{eq}^N(g_N)$. More precisely, as z_{eq}^N increases with g_N , the z_N range in which inverse decays produce a sizable negative efficiency increases, resulting in $|\eta_B^-(g_N)| \sim z_{eq}^N(g_N)$. This is further highlighted in Fig. 24 (left). On the other hand, the neutrino abundance is Boltzmann suppressed with

$$Y_{NB}^z(z_{eq}^N) \sim e^{-z_{eq}^N}, \quad (4.43)$$

and consequently, an increasing $z_{eq}^N(g_N)$ diminishes $\eta_B^+(g_N)$. Thus, in the blue region, $|\eta_B^-(g_N)|$ increases while $\eta_B^+(g_N)$ decreased and as $|\eta_B^-(g_N)| > \eta_B^+(g_N)$, this clearly results in η_B^z decreasing (or $|\eta_B^z|$ increasing since $\eta_B^z < 0$) with g_N . Once neutrinos are quickly thermalized for $g_N \gtrsim 0.24$ (red region in Fig. 25), the initial conditions are irrelevant and the contribution of η_B^- can be neglected, resulting in $\eta_B^z(g_N) \sim \eta_B^+(g_N) > 0$.

Next, let us consider the blue region in the right plot in Fig. 25. First, we note that $\tilde{m}^\pm < \tilde{m}^{IA}$, i.e., η_B^z changes sign in the blue region before the initial conditions are irrelevant. From (4.36), it is clear that a larger $\tilde{m}$ enhances the neutrino production via inverse decays already at $z_N \ll z_{eq}^N$ [see also Fig. 24(right)] and consequently, $|\eta_B^-|$ increases with $\tilde{m}$. On the other hand, η_B^+ depends strongly on $z_{eq}^N(\tilde{m})$ due to the Boltzmann suppression of neutrinos. More precisely, recalling that $z_{eq}^N(\tilde{m})$ decreases, this implies that the Boltzmann

suppression at z_{eq}^N decreases as well and consequently, $\eta_B^+(\tilde{m})$ increases. In contrast to the scenario shown in the left-hand plot, the sign change of the efficiency is not due to the rapid drop of $|\eta_B^-|$ at $\tilde{m}^{IA}$ but due to η_B^+ approaching and eventually overtaking $|\eta_B^-|$ at $\tilde{m}^\pm < \tilde{m}^{IA}$. Clearly, this results in a severely diminished final efficiency around $\tilde{m}^\pm \approx 1.3 \times 10^{-3}$ eV. Only once neutrino thermalization becomes fast and z_{eq}^N drops at $\tilde{m}^{IA} \sim 2.5 \times 10^{-3}$ eV, we find that $|\eta_B^-|$ drops as well, resulting in $\eta(\tilde{m}) \sim \eta_B^+(\tilde{m})$ (red region) and the initial conditions are rendered irrelevant. Clearly, the $\tilde{m}$ -range around $\tilde{m}^\pm$ where $|\eta_B^z|$ almost vanishes depends crucially on the difference between $\tilde{m}^\pm$ and $\tilde{m}^{IA}$. As g_N increases, $\tilde{m}^\pm$ approaches $\tilde{m}^{IA}$ and the $\tilde{m}$ range where the efficiency is diminished shrinks until at $g_N \approx 0.15$, we have $\tilde{m}^\pm \approx \tilde{m}^{IA}$.

To summarize, we find that η_B^z significantly depends on how fast neutrinos are thermalized. While the evolution of the efficiency for small z_N is not significantly changed compared to VL, the suppression of the neutrino abundance for $(\tilde{m}, g_N) < (\tilde{m}^{IA}, g_N^{IA})$ results in a later thermalization compared to VL and thus reduces the number of neutrinos that can contribute to η_B^+ . Moreover, for $(\tilde{m}, g_N) \approx (\tilde{m}^\pm, g_N^\pm)$, the negative and the positive contribution almost cancel each other, resulting in an almost vanishing final efficiency η_B^z .

E. General remarks

Before discussing other implications of the model, let us address some additional points. So far, we have kept λ_σ constant and discussed the effect of $g_T = 10^{-7}$ compared to $g_T = 1$ on the efficiency. Before we proceed, let us comment in greater detail on the effect these couplings have on the efficiency. To that end, we show $\eta(g_T, \tilde{m})$ (left) and $\eta(\lambda_\sigma, \tilde{m})$ (right) for $g_N = 1$ in the scenario where all particles are in thermal equilibrium at z_I in Fig. 26.

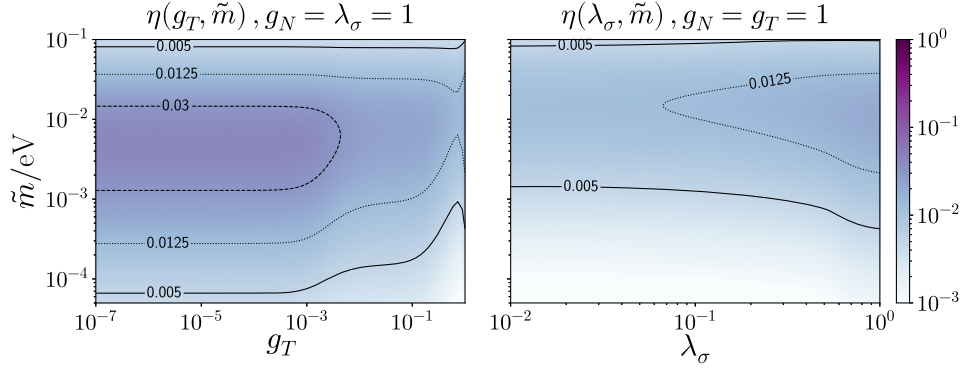


FIG. 26. Efficiency $\eta(g_T, \tilde{m})$ (left) and $\eta(\lambda_\sigma, \tilde{m})$ (right) for $g_N = 1$. Note that we used the same color scaling as in Fig. 4. We can easily see that the efficiencies do not change significantly with g_T and λ_σ , respectively.

As expected, the sizes of g_T and λ_σ have significantly smaller effects on η compared to g_N . What is more, below $g_T \sim 10^{-3}$, $\eta(g_T)$ becomes constant, i.e., the Yukawa coupling of the triplet is irrelevant for the value of the final efficiency. In fact, we checked explicitly that the final efficiency in case *B* agrees with the final efficiency in scenario where $g_T = 0$. Thus, the results in case *B* are applicable to a broader class of models, in particular to the conventional singlet Majoron model, and hold even for larger values of g_T than considered in case *B*. Similarly, $\eta(\lambda_\sigma)$ is barely affected by the changes in λ and becomes almost constant for $\lambda_\sigma \lesssim 0.1$.

Next, recall that we neglected the doublet-singlet mixing term $\sim \lambda_{\text{mix}}$ in (2.3). We expect that depending on the size of the λ_{mix} , including this term could drive the efficiencies in cases *A*, *B* closer to $\hat{A}$, $\hat{B}$ as the additional interaction with H could result in a more efficient thermalization of σ and J .

After these general considerations, we will focus on a more realistic version of the model and examine dark matter constraints on the new particles.

V. DARK MATTER

The most precise measurements of the DM energy density today come the cosmic microwave background (CMB) anisotropies with an observed value of [1],

$$\Omega_{\text{DM}} h_0^2 = 0.120 \pm 0.0001, \quad (5.1)$$

where h_0 is the scaling factor for the Hubble parameter today, H_0 ,

$$h_0 = 0.674 \pm 0.0005, \quad (5.2)$$

with

$$H_0 \equiv h_0 100 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (5.3)$$

A DM candidate needs to be nonrelativistic at the onset of galaxy formation, nonbaryonic, have at most weak

interactions with the SM and it has to be stable on cosmological timescales. As we will discuss in the following, both the Majoron J and the triplet fermion T in the Majoron plus triplet model can in principle fulfill these requirements.

A. Majoron

The Majoron as a DM candidate has already been extensively discussed in various scenarios [11–15]. In this work, we do not specify a mechanism that generates the necessary Majoron mass m_J and merely focus on the implications in light of the Majoron plus triplet model. Moreover, we note that the Majoron can decay to a pair of light neutrinos if m_J exceeds the light neutrino masses. This implies that in order to be a viable DM candidate, the Majoron decay width needs to be sufficiently small in order to render the Majoron stable on cosmological timescales. For simplicity, we assume for the remainder of this section that the Majoron is indeed long lived enough to constitute DM and explore the limits the Majoron plus triplet model places on the majoron mass.

During the leptogenesis era, the Majoron interacts via scatterings and inverse decays with N , σ , and T . As discussed in Sec. IV, these interactions are very effective at populating the plasma with Majorons in cases *A* and *B*, for some parameter sets even reaching an abundance larger than the thermal one. Those interactions decouple at $z_N^d \sim \mathcal{O}(10)$ and the Majoron abundance reaches a constant value $Y_J(z_N^d) \gtrsim Y_J^{eq}(z_N^d)$. In case *B*, with $(\tilde{m}, g_N) < (\tilde{m}^{IA}, g_N^{IA})$ on the other hand, the Majoron abundance tends to be smaller than the corresponding thermal abundance when the interactions with N , σ , and T decouple, resulting in $Y_J(z_N^d) \lesssim Y_J^{eq}(z_N^d)$.

In both cases, a Majoron relic density,

$$\Omega_J = \frac{Y_J(z_N \rightarrow \infty) m_J s_0}{\rho_c}, \quad (5.4)$$

arises. In the equation above, s_0 is the entropy density today, defined via the photon density today $n_{\gamma,0}$ as

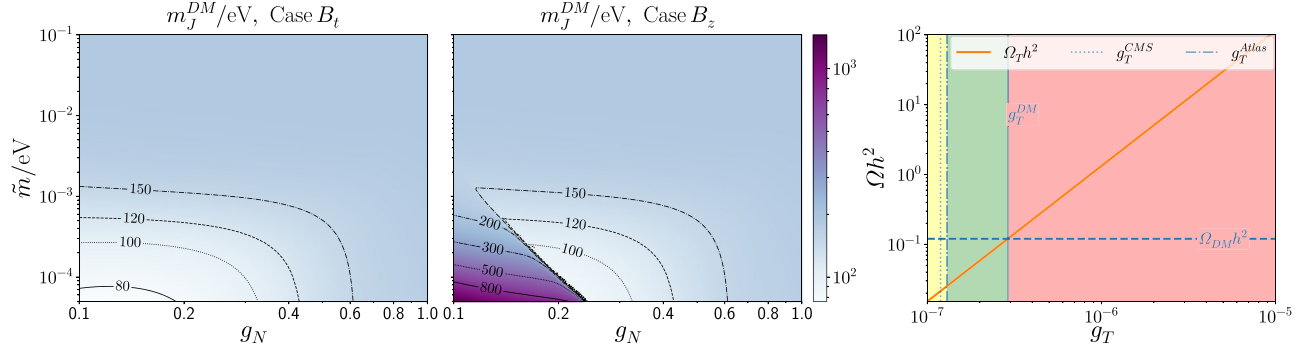


FIG. 27. *Left, middle:* Upper limits on the Majoron mass that does not overproduce DM for case A, B_t (left), and B_z (middle) in the g_N - $\tilde{m}$ plane. Note that these constraints are only valid if Majoron decays to light neutrinos are sufficiently slow. *Right:* Triplet relic density $\Omega_T h^2$ as a function of g_T , compared to the DM relic density $\Omega_{\text{DM}} h^2$. In the green region, the triplet relic density does not exceed the DM relic density and is in agreement with bounds from Atlas and CMS placed on the triplet mass. In the red region, DM is overproduced while in the yellow region, bounds from Atlas are violated.

$$s_0 \approx 7.04 n_{\gamma,0}, \quad (5.5)$$

while ρ_c is the critical density,

$$\rho_c = (1.87834 \pm 0.00004) \times 10^{-29} h^2 \text{ g cm}^{-3}. \quad (5.6)$$

From the condition $\Omega_J(m_J^{\text{DM}}) \stackrel{!}{=} \Omega_{\text{DM}}$, we determine the maximal acceptable Majoron mass m_J^{DM} that does not result in a Majoron relic density that exceeds the DM relic density. Thus, if $m_J = m_J^{\text{DM}}$, the Majoron constitutes all DM while for $m_J < m_J^{\text{DM}}$, the Majoron accounts only for a fraction of the DM relic density, requiring contributions from additional DM particles. In Fig. 27, we show density plots of m_J^{DM} in the g_N - $\tilde{m}$ plane for case B_t (left) and case B_z (middle).¹⁰ We can immediately see that in the case of thermal initial abundances, the large Majoron abundance constrains the Majoron mass to be of order $m_J \sim 10^2$ eV. In case B_z with $\tilde{m} \sim 5 \times 10^{-5}$ eV, $g_N \sim 0.1$, i.e., in the parameter range where the Majoron is not thermalized, values up to $m_J \sim 10^3$ eV are allowed. In the context of DM, this mass range is however problematic as the Majoron with such a low mass would be relativistic at the onset of galaxy formation and constitute hot DM. This implies that in this setup, the Majoron mass needs to be significantly smaller than m_J^{DM} in order to result in a negligible contribution to the DM relic density. We stress however that we implicitly assumed that the Majoron is in kinetic equilibrium when the relic density freezes out and that Majoron decays to neutrinos are slow. If the former assumption does not hold, even a small Majoron mass could result in experimentally viable cold Majoron DM (similarly to the axion) while in the case that the latter assumption does not hold, the Majoron would be unstable and not a DM candidate. Moreover, depending on the mechanism that is employed

in order to generate the Majoron mass, the limits derived in this section can drastically change. Thus, whether the Majoron in the Majoron plus triplet model is a viable DM candidate or not depends significantly on the origin of the Majoron mass.

B. Triplet

As discussed previously, the triplet T is effectively thermalized via gauge interactions while the scattering processes with N, σ, J tend to be subdominant. Thus, once gauge interactions decouple at $z_T \sim 10$, the triplet freezes out and Y_T reaches a constant value. This implies that the relic density is determined mostly by g_T and rather independent from g_N and λ_σ . For simplicity, we therefore neglect all scattering processes except these involving gauge bosons and accordingly solve a simple Boltzmann equation for the triplet evolution,

$$sH z_N \frac{dY_T}{dz_N} = -2(\delta_T^2 - 1)\gamma_{T,\text{gauge}}, \quad (5.7)$$

for $10^{-7} \leq g_T \leq 10^{-5}$. The corresponding relic density given by

$$\Omega_T = \frac{Y_T(z_N \rightarrow \infty) M_T s_0}{\rho_c} \quad (5.8)$$

is shown in Fig. 27 (left). We can immediately see that the triplet relic density exceeds the observed DM relic density already for $g_T \gtrsim 2.9 \times 10^{-7}$ or equivalently $M_T \gtrsim 2$ TeV. Searches for triplet fermions in the framework of the type III seesaw by Atlas [40] and CMS [41] place lower bounds on the triplet mass of

$$M_T^{\text{Atlas}} \approx 910 \text{ GeV}, \quad M_T^{\text{CMS}} \approx 840 \text{ GeV}, \quad (5.9)$$

corresponding to

¹⁰Note that the constraints from case A and B_t result in almost identical constraints which is why we present only case B_t .

$$g_T^{\text{Atlas}} \approx 1.19 \times 10^{-7}, \quad g_T^{\text{CMS}} \approx 1.29 \times 10^{-7}. \quad (5.10)$$

In terms of the discussed scenarios of Majoron plus triplet leptogenesis, this implies that case *A* is excluded while case *B* is tightly constrained but remains viable.¹¹ We stress nonetheless that the discussion of the effects on the efficiency for different values of g_T gives valuable insights on the dynamics of the leptogenesis scenario despite being largely experimentally forbidden. We also stress that these constraints are only valid in the absence of the Z_2 forbidden mixing term with SM leptons and the Higgs doublet. Reintroducing this term allows lepton number violating decays of T and thus renders the triplet unstable. We note however that this the inclusion of the mixing term is nontrivial and beyond the scope of this work.

VI. BARYON ASYMMETRY

In this section, we finally discuss the baryon asymmetry,

$$Y_B \equiv Y_{B'} - Y_{\overline{B'}}, \quad (6.1)$$

that can be created in the Majoron plus triplet model. As we found in the previous section that DM constraints exclude case *A*, we will focus on case *B*.

A. Sphaleron rate

As discussed in [25], the presence of the triplet changes the $[SU(2)_L]^2 \times U(1)_{L'}$ anomaly factor $\mathcal{A}_{L'}$ from $\mathcal{A}_{L'} = 3$ in the SM to $\mathcal{A}_{L'} = -1$ in the Majoron plus triplet model. As a consequence, sphaleron transitions no longer conserve $Y_B - Y_L$ as in the SM but instead $Y_B + 3Y_L$, thereby affecting the rate with which sphaleron processes convert the lepton asymmetry to a baryon asymmetry.

For simplicity, we calculate the conversion rate at a temperature $T \sim \text{TeV}$ where all gauge and Yukawa interactions are fast. This induces relations between the chemical potentials of the SM particles,

$$\mu_{W^-} = \mu_{\phi^0} + \mu_{\phi^-} = \mu_{d_L} - \mu_{u_L} = \mu_{e_L} - \mu_{\nu_L}, \quad (6.2)$$

$$\mu_{\phi^0} = \mu_{u_R} - \mu_{u_L} = \mu_{d_L} - \mu_{u_R} = \mu_{e_L} - \mu_{e_R}. \quad (6.3)$$

With g_T of order 10^{-7} , gauge interactions of the triplet are still fast as well, resulting in the additional relation,

$$\mu_{W^\pm} = \mu_{T^\pm}, \quad (6.4)$$

while the chemical potentials of N, T^0, σ, J vanish. Moreover, fast sphaleron transitions in the Majoron plus triplet model imply,

¹¹Recall from Sec. IV E that the exact value of g_T is not relevant in case *B* as the triplet does not affect the efficiency.

$$\mu_{d_R} - 6\mu_{e_L} + 11\mu_{e_R} - 6\mu_{\nu_L} + 8\mu_{u_R} = 0, \quad (6.5)$$

while in the SM, the corresponding relation reads,

$$2\mu_{d_L} + \mu_{u_L} + \mu_{\nu_L} = 0. \quad (6.6)$$

In the relativistic limit, the asymmetry in a charge q can be expressed as

$$Y_q \equiv \frac{n_q - n_{\bar{q}}}{s} = \frac{T^2}{6s} \left[\sum_{i \in f} q_i g_i \mu_i + 2 \sum_{j \in f} q_j g_j \mu_j \right], \quad (6.7)$$

where the sum over $i(j)$ is for fermions (bosons), $q_{i,j}$ is the charge and $g_{i,j}$ are the numbers of degrees of freedom of the corresponding particle. Before EWSB, the plasma is charge and hypercharge neutral which imposes additional conditions between the chemical potentials by means of (6.7), finally resulting in [25],

$$Y_B = \frac{76}{679} Y_{(B+3L)}, \quad (6.8)$$

while the respective relation in the SM reads,

$$Y_B = \frac{28}{79} Y_{(B-L)}. \quad (6.9)$$

Assuming that no other sources of baryon number violation are present, we find that the baryon asymmetry is given by

$$|Y_B| = \begin{cases} 3 \times \frac{76}{679} Y_L^B & \approx 0.35443 |Y_L^B|, \\ \frac{28}{79} Y_L^{VL} & \approx 0.335788 |Y_L^{VL}|, \end{cases} \quad (6.10)$$

i.e., apart from the different signs, we find that the baryon asymmetries that can be generated from a given lepton asymmetries in each model do not differ significantly.

B. Comparison with experimental data

From observations, the baryon asymmetry of the Universe is given by

$$Y_B^{\text{exp}} \equiv \frac{n_{B'} - n_{\overline{B'}}}{s} \Big|_0 \approx \frac{274 \times 10^{-10}}{7.04} \Omega_{B'} h^2 \approx 8.718 \times 10^{-11}, \quad (6.11)$$

where

$$\Omega_{B'} h^2 = 0.0224 \pm 0.0001 \quad (6.12)$$

is the baryon density obtained from CMB data [1].

Combining (6.10) and (6.11) we can determine the CP violation that is necessary in the Majoron plus triplet model

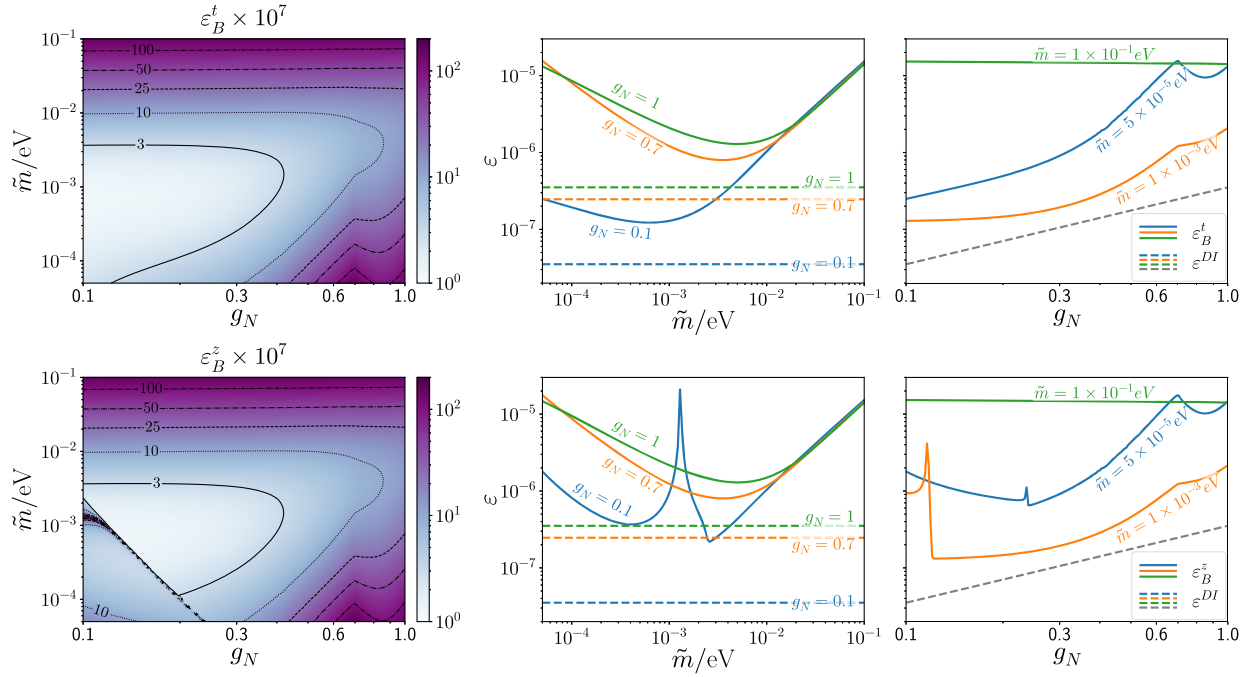


FIG. 28. *Left*: Density plots of the CP violation $|\epsilon_B^t|$ (top) and $|\epsilon_B^z|$ (bottom) required to explain the experimentally observed baryon asymmetry in case B in the Majoron plus triplet model. *Middle*: $|\epsilon_B^t|$ (top) and $|\epsilon_B^z|$ (bottom) as functions of $\tilde{m}$ (solid lines) for $g_N = [0.1, 0.7, 1]$. The dashed lines corresponds to ϵ^{DI} . *Right*: $|\epsilon_B^t|$ (top) and $|\epsilon_B^z|$ (bottom) as functions of g_N (solid lines) for $\tilde{m} = [5 \times 10^{-5}, 10^{-3}, 10^{-1}]$ eV. The dashed line corresponds to ϵ^{DI} .

in order to reproduce the experimentally observed baryon asymmetry as

$$\epsilon = \frac{679}{76 \times 3} \frac{Y_B^{\text{exp}}}{Y_N^0 \eta}. \quad (6.13)$$

The results for case B are shown in Fig. 28 where we show density plots of ϵ in the g_N - $\tilde{m}$ plane (left), $\epsilon(\tilde{m})$ (middle) and $\epsilon(g_N)$ (right) for $g_N = [0.1, 0.7, 1]$ and $\tilde{m} = [5 \times 10^{-5}, 10^{-3}, 10^{-1}]$ eV, respectively. For comparison, we also show $\epsilon^{DI}(\tilde{m})$ and $\epsilon^{DI}(g_N)$. We can immediately see that even in the best case scenario of $g_N = 0.1$ and $\tilde{m} \approx 10^{-3}$ eV, the CP violation necessary to explain experimental data is about one order of magnitude larger than ϵ^{DI} . The reason for this large difference is quite simple; the efficiency η_B is maximal for small g_N while the DI bound ϵ^{DI} decreases with $M_N = g_N / \sqrt{2}f$. This is in contrast to VL where the efficiency is independent from M_N and thus in principle, fulfilling the DI bound only requires a sufficiently large neutrino mass. Nevertheless, we stress that this does not *per se* exclude the viability of Majoron plus triplet leptogenesis.

First, recall that η depends on the VEV f . In Fig. 29, we show $\eta_B(f)$ with f ranging from 10^6 GeV to 10^{12} GeV for $g_N = 0.1$ and $\tilde{m} = 10^{-3}$ eV. We also translate the DI bound into a lower limit on the efficiency necessary to reproduce experimental data as

$$\eta^{DI}(M_N) \equiv \frac{679}{3 \times 76} \frac{Y_B^{\text{exp}}}{Y_N^0 \epsilon^{DI}(M_N)}. \quad (6.14)$$

From Fig. 29 it is apparent that a higher breaking scale f results in a larger efficiency while a smaller VEV diminishes the efficiency. Additionally, we have $\eta^{DI} \sim 1/f$ and we can therefore conclude that a larger VEV can close the gap between ϵ and ϵ^{DI} from both ends while in the same manner, a smaller breaking scale would worsen the situation.

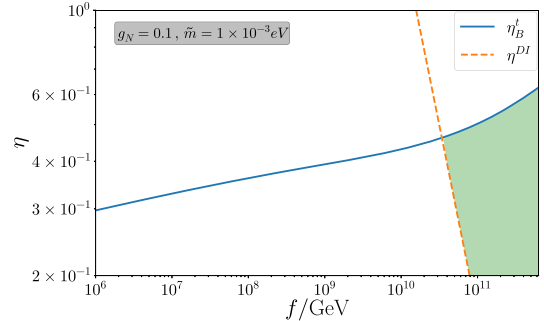


FIG. 29. Efficiency η_B^t for $g_N = 0.1$ and $\tilde{m} = 10^{-3}$ eV as a function of the VEV f , compared to η^{DI} as defined in (6.14). In the green region, η_B^t is sizable enough to reproduce the experimentally observed baryon asymmetry with a CP violation ϵ that fulfills the DI bound.

Additionally, it has been shown that the DI bound can be violated if the hierarchy between M_N and $M_{N_{2,3}}$ is not too strong [34]. In fact, as the masses of $N_{2,3}$ arise via the same VEV as the mass of N_1 and taking into account perturbativity of $g_{N_{2,3}}$, it is even highly plausible that the hierarchy between M_N and $M_{N_{2,3}}$ is rather weak which could easily enhance ϵ past the DI bound. Finally, we neglected flavor effects in the Boltzmann equations. In the type III leptogenesis scenario, flavor effects have been shown to enhance the efficiency in the regime where WO processes dominate the efficiency [32]. We expect a similar enhancement in the WO regime while in the $\overline{\text{WO}}$ regime, the efficiency is dominated by flavor-independent scattering processes and we thus expect only a weak effect on the efficiency.

VII. SUMMARY

We have studied the implications of leptogenesis in the Majoron model extended by an additional right-handed triplet with a Z_2 symmetry. The existence of the triplet changes the $[SU(2)_L]^2 \times U(1)_{L'}$ anomaly factor and hence affects how sphaleron processes convert a lepton asymmetry to a baryon asymmetry.

In particular, we solved the Boltzmann equations relevant for the particle evolutions under two different assumptions: First, we considered a simplified scenario where only neutrinos deviate from thermal equilibrium before considering a scenario where all additional particles in the Majoron plus triplet model can deviate from thermal equilibrium. Moreover, we have considered various different assignments for the parameters of the model and explored the relevance of the initial particle abundances. For certain parameter sets, solving only the Boltzmann equation for the neutrino evolution drastically underestimates the efficiency compared to the scenario where all Boltzmann equations are considered. For $\tilde{m} \rightarrow 10^{-1}$ eV, i.e., the strong WO regime in VL, we generally find that inverse neutrino decays dominate so that the same efficiencies as in VL are reached while for small $\tilde{m}$, the scattering processes with the new particles dominate the neutrino evolution. In the small $\tilde{m}$ regime, we find that sizable efficiencies can be generated if the Yukawa coupling of the heavy neutrino is small, $g_N \sim 0.1$. Moreover, we found that the initial abundances are mostly irrelevant for the final efficiency and only play a role in a very restricted subset of the considered parameters. Additionally, we found that the triplet does not effect the efficiency if the corresponding Yukawa coupling is small, $g_T \lesssim 10^{-3}$, thus restricting the relevance of the triplet to the effects on the conversion of the lepton asymmetry to the baryon asymmetry.

Due to the Z_2 symmetry, the triplet T does not decay and obtains a relic density, qualifying it as a DM candidate. For a VEV of $f = 10^{10}$ eV, the Yukawa

coupling of the triplet is limited from above with $g_T \lesssim 2.29 \times 10^{-7}$ in order to avoid overproduction of DM. We also considered the possibility of Majoron DM, assuming that the Majoron obtains a mass by an unspecified mechanism. We found that the upper limit on the experimentally observed DM relic density requires a Majoron mass too small to render the Majoron cold at structure formation, in contrast to observations. We note however that those bounds may drastically change, depending on the details of the mechanism that generates the Majoron mass.

We compared the results for the efficiency in the scenario where the triplet Yukawa coupling is small, $g_T \sim 10^{-7}$, with the experimentally observed baryon asymmetry, taking into account that the sphaleron conversion rate is different from the SM. In order to reproduce experimental data, in many cases a CP violation ϵ that exceeds the DI bound ϵ^{DI} is necessary. We note however that this bound can easily be avoided if resonance effects are taken into account. Moreover, we briefly examined the effects of the VEV f on the efficiency. While we performed our analysis with $f = 10^{10}$ GeV, a larger VEV could enhance the efficiency that can be reached while additionally enhancing the DI bound. In this work, we neglected flavor effects in the Boltzmann equations. It should be stressed that while we expect that flavor effects are subdominant in the regimes where scattering processes dominate, they can have significant effects on the efficiencies if $\tilde{m}$ is large.

In summary, we find that successful leptogenesis in the Majoron plus triplet model is in principle viable. For future works, a dedicated analysis of the leptogenesis scenario taking e.g., flavor effects or the resonant enhancement of the CP asymmetry into account seems interesting. Moreover, we expect that the inclusion of the mixing term between the triplet and the Higgs which was omitted in this work can have interesting effects on the creation of the lepton asymmetry which are worth examining in future works.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX

1. General formulas

For scattering processes, we compute the thermal rates as

$$\begin{aligned}\gamma_{abij} &\equiv \gamma(ab \leftrightarrow ij) = \frac{T}{64\pi^4} \int_{s_{\min}}^{\infty} ds \sqrt{s} \hat{\sigma}(s) K_1\left(\frac{\sqrt{s}}{T}\right) \\ &= \frac{M_N M_X^3}{64\pi^4} \int_{x_{\min}}^{\infty} dx_X \frac{\sqrt{x_X}}{z_N} \hat{\sigma}(x_X) K_1\left(\frac{M_X}{M_N} \sqrt{x_X z_N}\right),\end{aligned}\quad (\text{A1})$$

where

$$z_N = \frac{M_N}{T}, \quad x_X = \frac{s}{M_X^2}, \quad (\text{A2})$$

$$s_{\min} = \max[(M_a + M_b)^2, (M_i + M_j)^2], \quad (\text{A3})$$

$$x_{\min} = \max\left[\left(\frac{M_a + M_b}{M_X}\right)^2, \left(\frac{M_i + M_j}{M_X}\right)^2\right]. \quad (\text{A4})$$

Moreover, $\hat{\sigma}(s)$ is the reduced cross section defined as

$$\begin{aligned}\hat{\sigma}(s) &= 2s\lambda\left(1, \frac{M_a^2}{s}, \frac{M_b^2}{s}\right)\sigma(s), \\ \sigma(s) &= \int_{-1}^1 d\cos\theta \frac{1}{32\pi s} |\bar{M}|^2,\end{aligned}\quad (\text{A5})$$

where σ is the total cross section summed over initial and final spins and

$$\lambda(a, b, c) = (a - b - c)^2. \quad (\text{A6})$$

For decays, we can write the thermal rate as

$$\gamma_{a,ij} \equiv \gamma(a \leftrightarrow ij) = n_a^{eq} \frac{K_1(z)}{K_2(z)} \Gamma_a, \quad (\text{A7})$$

where Γ_a is the decay rate, $K_n(z)$ is the modified Bessel function of the second kind and

$$\begin{aligned}n_a^{eq} &= g_a \int \frac{d^3p}{(2\pi)^3} f = \frac{g_a M_a T}{2\pi^2} K_1\left(\frac{M_a}{T}\right) \\ &= \begin{cases} \frac{g_a T^3}{\pi^2}, & T \gg M_a, \\ g_a \left(\frac{M_a T}{2\pi}\right)^{3/2} e^{-M_a/T}, & T \ll M_a, \end{cases}\end{aligned}\quad (\text{A8})$$

is the number density of particle a in thermal equilibrium at a temperature T with g_a degrees of freedom and f is approximated as a Maxwell-Boltzmann distribution,

$f \approx e^{-\frac{E_p}{T}}$. Note that unless CP -violating effects are present, the thermal rates for direct and inverse processes are the same.

The evolution of the number density n_a of a particle a in an expanding universe can be written as [42]

$$\dot{n}_a + 3Hn_a = -\sum_{b,i,j} [ab \leftrightarrow ij], \quad (\text{A9})$$

where H is the Hubble parameter,

$$H = \sqrt{\frac{8}{\pi m_{pl}^2} g_s} \frac{1}{T^4}, \quad (\text{A10})$$

$g_s = 106.75$ is the effective number of degrees of freedom, m_{pl} is the Planck mass and

$$[ab \leftrightarrow ij] = (\delta_a \delta_b - \delta_i \delta_j) \gamma(ab \leftrightarrow ij), \quad (\text{A11})$$

with

$$\delta_a = \frac{n_a}{n_a^{eq}}. \quad (\text{A12})$$

It is convenient to normalize the number density n_a to the entropy density s , given by

$$s(z, m) = \frac{4m^3}{z^3 \pi^2} g_s, \quad (\text{A13})$$

yielding the abundance of a as

$$Y_a = \frac{n_a}{s}. \quad (\text{A14})$$

Using

$$3H = -\frac{\dot{s}}{s}, \quad (\text{A15})$$

casting the Boltzmann equation in terms of Y_a allows us to reabsorb the $3H$ term, i.e.,

$$sH z_N \frac{dY_a}{dz_N} = -\sum_{b,i,j} [ab \leftrightarrow ij]. \quad (\text{A16})$$

One subtlety that needs to be addressed is the subtraction of on shell processes. For example, let us consider the Boltzmann equation for a particle X that interacts via a scattering process $XX \leftrightarrow YY$ with s-channel exchange of a particle Z . Considering the scattering process alone, the Boltzmann equation reads,

$$sH z \frac{dY_X}{dz} = -2(\delta_X^2 - \delta_Y^2) \gamma(XX \leftrightarrow YY). \quad (\text{A17})$$

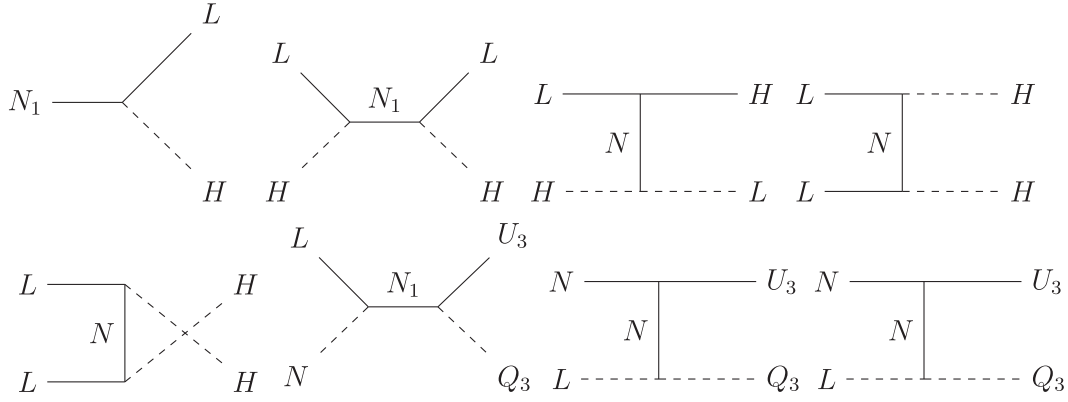


FIG. 30. Feynman diagrams contributing both to VL and to leptogenesis in the Majoron plus triplet model.

Moreover, if decays of Z to XX, YY are kinematically allowed, this would account for a term

$$-2(\delta_X^2 - \delta_Z)\gamma(Z \leftrightarrow XX). \quad (\text{A18})$$

However, the on shell contribution is also accounted for in (A17) and we therefore need to replace $\gamma(XX \leftrightarrow YY)$ with the off shell contribution, i.e.,

$$\begin{aligned} \gamma^{\text{off}}(XX \leftrightarrow YY) &= \gamma(XX \leftrightarrow YY) - \text{Br}_{Z \leftrightarrow YY}\gamma(Z \leftrightarrow XX) \\ &= \gamma(XX \leftrightarrow YY) - \text{Br}_{Z \leftrightarrow XX}\gamma(Z \leftrightarrow YY), \end{aligned} \quad (\text{A19})$$

where

$$\text{Br}_{Z \leftrightarrow YY} = \frac{\Gamma(Z \rightarrow YY)}{\Gamma(Z \rightarrow \text{everything})} \quad (\text{A20})$$

is the branching ratio and

$$\text{Br}_{Z \leftrightarrow YY}\gamma(Z \leftrightarrow XX) = \text{Br}_{Z \leftrightarrow XX}\gamma(Z \leftrightarrow YY). \quad (\text{A21})$$

Consequently, the resulting Boltzmann equation is given by

$$\begin{aligned} sHz \frac{dY_X}{dz} &= -2(\delta_X^2 - \delta_Y^2)\gamma^{\text{off}}(XX \leftrightarrow YY) \\ &\quad - 2(\delta_X^2 - \delta_Z)\gamma(Z \leftrightarrow XX). \end{aligned} \quad (\text{A22})$$

2. Feynman diagrams

In this section, we present the Feynman diagrams relevant for the discussed leptogenesis scenarios.

In Fig. 30, we show the Feynman diagrams for interactions that appear already in the VL scenario and are hence also relevant for leptogenesis in the Majoron plus triplet model. Note that we neglect scatterings involving gauge bosons.

In Fig. 31, we present Feynman diagrams for interactions with neutrinos in the initial or final state that appear in the Majoron plus triplet model. Those interactions are particularly relevant for leptogenesis as they change the neutrino abundance.

In Fig. 32, we show Feynman diagrams for the remaining scattering processes in the Majoron plus triplet model that change the abundances of T , σ , and J . Note that these interactions are only relevant in cases A, B and do not affect leptogenesis in the simplified scenarios $\hat{A}, \hat{B}$.

3. Cross sections and matrix elements

The reduced cross sections for neutrino interactions involving third-generation quarks can e.g., be found in [39] and are given by

$$\sigma_{Q_3 U_3 N L} = \frac{3y_t^2 \tilde{m}_1 M_N}{4\pi v^2} \left(\frac{x_N - 1}{x_N} \right)^2, \quad (\text{A23})$$

$$\sigma_{L \bar{Q}_3 N \bar{U}_3} = \sigma_{L \bar{U}_3 N \bar{Q}_3} = \frac{3y_t^2 M_N \tilde{m}_1}{4\pi v^2} \frac{x - 1}{x} \left(\frac{x - 2 + 2a_r}{x - 1 + a_r} + \frac{1 - 2a_r}{x - 1} \log \left(\frac{x_N - 1 + a_r}{a_r} \right) \right), \quad (\text{A24})$$

where $a_r \equiv \frac{m_H}{M_N}$.

The reduced cross section for scatterings of the triplet T involving gauge bosons, $TT \leftrightarrow AA, L\bar{L}, Q\bar{Q}, H\bar{H}$, is the same as in type III leptogenesis and given by [33–35,43]

$$\hat{\sigma}_{T,\text{gauge}} = \frac{6g_2^4}{72\pi} \left(\frac{45}{2}\beta - \frac{27}{2}\beta^3 - \left(9(\beta^2 - 2)^2 + 18(\beta^2 - 1)^2 \right) \log \left[\frac{1+\beta}{1-\beta} \right] \right), \quad (\text{A25})$$

where $\beta = \sqrt{1 - \frac{4}{x_T}}$.

Next, we present the cross sections and matrix elements that appear in the Majoron plus triplet model. For convenience, we introduce the following abbreviations:

$$\bar{x} = x - 1, \quad \hat{x} = x + 1, \quad \tilde{x} = (x^2 - 1), \quad (\text{A26})$$

$$A = \sqrt{\frac{(M_N^2 \bar{x} + m_\sigma^2)^2}{M_N^2 x}}, \quad B = 4m_J^2 - M_N^2 x, \quad (\text{A27})$$

$$C = 4m_\sigma^2 - M_N^2 x, \quad D = 4M_T^2 - M_N^2 x, \quad (\text{A28})$$

$$E = m_J^2 - M_N^2 x, \quad F = m_\sigma^2 - M_N^2 x, \quad (\text{A29})$$

$$G = M_T^2 - M_N^2 x, \quad H = \sqrt{\frac{M_N^4 \tilde{x}^2 - 2M_N^2 m_\sigma^2 \hat{x} + m_\sigma^4}{M_N^2 x}}, \quad (\text{A30})$$

$$O = \sqrt{\frac{m_J^4 - 2m_J^2 M_N^2 \hat{x} + M_N^4 \tilde{x}^2}{M_N^2 x}}, \quad Q = (-m_J^2 + M_N^2 x + m_\sigma^2)^2, \quad (\text{A31})$$

$$R = M_N^2(x - 4), \quad S = (M_N^2 x + m_\sigma^2), \quad (\text{A32})$$

$$T = (x - c_\beta^2(x - 4)), \quad U = (F^2 + \Gamma_\sigma^2 m_\sigma^2). \quad (\text{A33})$$

The reduced cross sections for $NN \rightarrow TT$ can then be written as

$$\hat{\sigma}(NN \leftrightarrow TT) = \frac{3\sqrt{-D}g_T^2 g_N^2 \sqrt{R} \left(\frac{M_N^4 x^2}{E^2} - \frac{DR}{F^2 + \Gamma_\sigma^2 m_\sigma^2} \right)}{8\pi M_N^2 x}. \quad (\text{A34})$$

For brevity, we only present the squared matrix elements (summed over initial and final spins) for the remaining interactions,

$$|\bar{M}(NN \leftrightarrow JJ)|^2 = \frac{g_N^2 R}{2} \times M_{NN}^{JJ}, \quad (\text{A35})$$

$$|\bar{M}(NTT \leftrightarrow JJ)|^2 = 3|\bar{M}(NN \leftrightarrow JJ)|^2|_{g_N \rightarrow g_T, M_N \rightarrow M_T}, \quad (\text{A36})$$

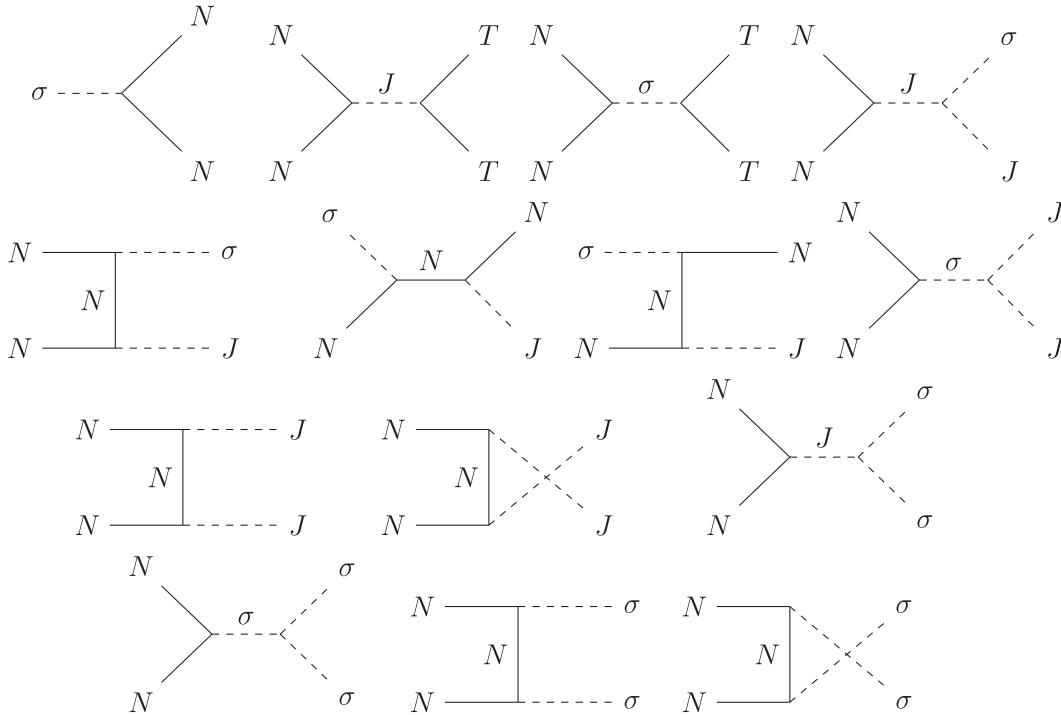


FIG. 31. Feynman diagrams for interactions that change the neutrino abundance in the Majoron plus triplet model.

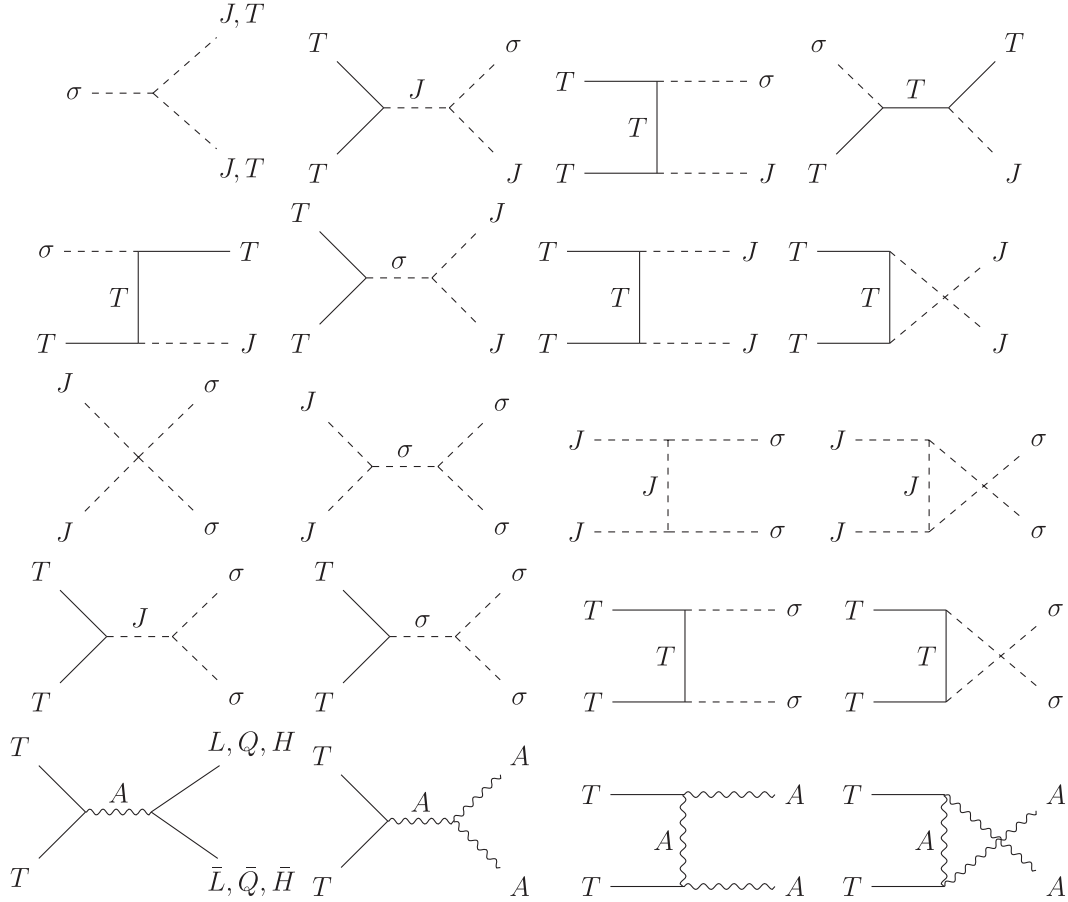


FIG. 32. Feynman diagrams for interactions that change the abundances of T , σ , and J in the Majoron plus triplet model. Here, $A = \{B, W_{1,2,3}\}$ corresponds to the gauge bosons.

$$|\bar{M}(NN \leftrightarrow \sigma J)|^2 = \frac{g_N^2}{E^2 M_N^2 x \left(c_\theta \sqrt{R} \sqrt{\frac{F^2 + m_J^4 - 2m_\sigma^2 S}{M_N^2 x}} + \sqrt{Q} - 2m_\sigma^2 \right)^2} \times M_{NN}^{\sigma J}, \quad (\text{A37})$$

$$|\bar{M}(TT \leftrightarrow \sigma J)|^2 = 3|\bar{M}(NN \leftrightarrow \sigma J)|^2|_{g_N \rightarrow g_T, M_N \rightarrow M_T}, \quad (\text{A38})$$

$$|\bar{M}(\sigma N \leftrightarrow JN)|^2 = \frac{g_N^4 x (M_N^2 x)^{-7/2} \times (Ax M_N^2 M_{\sigma N,1}^{JN} + \sqrt{M_N^2 x} M_{\sigma N,2}^{JN})}{(A(M_N^2 \hat{x} - m_J^2) + (c_\theta H O - 2m_\sigma^2) \sqrt{M_N^2 x})^2 (\bar{x}^2 M_N^2 + \Gamma_N^2)}, \quad (\text{A39})$$

$$|\bar{M}(\sigma T \leftrightarrow JT)|^2 = 3|\bar{M}(\sigma N \leftrightarrow JN)|^2|_{g_N \rightarrow g_T, M_N \rightarrow M_T}, \quad (\text{A40})$$

$$|\bar{M}(NN \leftrightarrow \sigma\sigma)|^2 = \frac{g_N^2 R}{2U(Cc_\theta^2 R + (M_N^2 x - 2m_\sigma^2)^2)} \times M_{NN}^{\sigma\sigma}, \quad (\text{A41})$$

$$|\bar{M}(TT \leftrightarrow \sigma\sigma)|^2 = 3|\bar{M}(NN \leftrightarrow \sigma\sigma)|^2|_{g_N \rightarrow g_T, M_N \rightarrow M_T}, \quad (\text{A42})$$

$$|\bar{M}(\sigma\sigma \leftrightarrow JJ)|^2 = \frac{k_\sigma^4}{m_\sigma^4 U((M_N^2 x - 2m_\sigma^2)^2 - BCc_\theta^2)^2} \times M_{\sigma\sigma}^{JJ}, \quad (\text{A43})$$

where

$$\begin{aligned}
M_{NN}^{JJ} &\equiv \frac{k_\sigma^2}{U} + \frac{4Bc_\theta^2 g_N M_N \left(Bg_N M_N T + \frac{2Fk_\sigma(4m_J^2 M_N^2 T - M_N^4 x T - 4m_J^4)}{U} \right)}{(-4m_J^2 M_N^2 T + M_N^4 x T + 4m_J^4)^2}, \\
M_{NN}^{\sigma J} &\equiv -c_\theta^2 R(E^2 g_N^2 - k_\sigma^2 M_N^2 x)(F^2 + m_J^4 - 2m_J^2 S) \\
&\quad - 2c_\theta k_\sigma M_N^3 \sqrt{R} x \sqrt{\frac{F^2 + m_J^4 - 2m_J^2 S}{M_N^2 x}} (2Eg_N(\sqrt{Q} - 2M_N^2 x) + k_\sigma M_N x(2m_\sigma^2 - \sqrt{Q})) \\
&\quad + M_N^2 x \left\{ E^2 g_N^2 (m_J^4 - 2m_J^2 S + M_N^4 x(x + 16) - 2M_N^2(8\sqrt{Q} + m_\sigma^2(x - 8)) + m_\sigma^4) \right. \\
&\quad \left. - 4Eg_N k_\sigma M_N \left\{ m_J^4 - 2m_J^2 S + M_N^4 x^2 - 2m_\sigma^2 \sqrt{Q} + 6M_N^2 m_\sigma^2 x - 2(M_N^2 x)^{3/2} \sqrt{\frac{Q}{M_N^2 x}} + m_\sigma^4 \right\} \right. \\
&\quad \left. + k_\sigma^2 M_N^2 x(m_J^4 - 2m_J^2 S + M_N^4 x^2 + m_\sigma^2(2M_N^2 x - 4\sqrt{Q}) + 5m_\sigma^4) \right\}, \tag{A44}
\end{aligned}$$

$$\begin{aligned}
M_{NN}^{\sigma\sigma} &\equiv 2Cc_\theta^2(-2g_N^2(M_N^4(x - 16)x - 4M_N^2 m_\sigma^2(x - 8))U + 4g_N k_\sigma M_N(M_N^4 x^2 - 3M_N^2 m_\sigma^2 x + 2m_\sigma^4)(M_N^2(8 - 3x) + 2m_\sigma^2) \\
&\quad + k_\sigma^2 R(M_N^2 x - 2m_\sigma^2)^2) + (M_N^2 x - 2m_\sigma^2)^2(64\Gamma_\sigma^2 g_N^2 M_N^2 m_\sigma^2 + (M_N^2 x(k_\sigma - 8g_N M_N) - 2m_\sigma^2(k_\sigma - 4g_N M_N))^2) \\
&\quad - (c_\theta^4(x - 4)(M_N^3 x - 4M_N m_\sigma^2)^2(-8Fg_N k_\sigma M_N + 4g_N^2 U - k_\sigma^2 R)), \tag{A45}
\end{aligned}$$

$$\begin{aligned}
M_{\sigma\sigma}^{JJ} &\equiv 8m_\sigma^6(\Gamma_\sigma^2(2B^2c_\theta^4 + 11Bc_\theta^2 M_N^2 x + 11M_N^4 x^2) - 2M_N^4 x^2(9Bc_\theta^2 + 17M_N^2 x)) \\
&\quad + 8M_N^2 m_\sigma^4 x(2M_N^2 x(B^2c_\theta^4 + 6Bc_\theta^2 M_N^2 x + 6M_N^4 x^2) - \Gamma_\sigma^2(Bc_\theta^2 + M_N^2 x)(Bc_\theta^2 + 2M_N^2 x)) \\
&\quad + M_N^4 m_\sigma^2 x^2((c_\theta^2 - 1)M_N^2 x - 4c_\theta^2 m_J^2)(8M_N^2 x(Bc_\theta^2 + 2M_N^2 x) - \Gamma_\sigma^2(Bc_\theta^2 + M_N^2 x)) \\
&\quad + 32m_\sigma^8(2M_N^2 x(Bc_\theta^2 + 6M_N^2 x) - 3\Gamma_\sigma^2(Bc_\theta^2 + 2M_N^2 x)) \\
&\quad + M_N^8 x^4((c_\theta^2 - 1)M_N^2 x - 4c_\theta^2 m_J^2)^2 + 16m_\sigma^{10}(9\Gamma_\sigma^2 - 16M_N^2 x) + 64m_\sigma^{12}, \tag{A46}
\end{aligned}$$

$$\begin{aligned}
M_{\sigma N,1}^{JN} &\equiv (-(3c_\theta^2 + 1)\bar{x}^3 M_N^6) + m_\sigma^2 \bar{x}((7x + 5)c_\theta^2 - x + 1)M_N^4 + m_\sigma^4((1 - 5x)c_\theta^2 + x - 1)M_N^2 + (c_\theta^2 + 1)m_\sigma^6 m_J^6 \\
&\quad + M_N^2(3(3c_\theta^2 + 1)\bar{x}^3 \hat{x} M_N^6 - m_\sigma^2 \bar{x}(3(7x + 5)c_\theta^2 + x + 3)M_N^4 + m_\sigma^2(m_\sigma^2(3\hat{x}(5x - 1)c_\theta^2 + 5x^2 + 3) - 2c_\theta H O x \hat{x})M_N^2 \\
&\quad + 2c_\theta H m_\sigma^4 O x - m_\sigma^6(3\hat{x}c_\theta^2 + 7x + 3))m_J^4 + M_N^4(-\bar{x}^2 \bar{x}((9x^2 + 6x + 9)c_\theta^2 + x(7x + 6) + 3) \\
&\quad \times M_N^6 + (\bar{x}((7x + 5)(x(3x + 2) + 3)c_\theta^2 + x(17x\hat{x} + 11) + 3)m_\sigma^2 - 4x^2(x + 3)\Gamma_N^2 - 8c_\theta H O x^2 \bar{x})M_N^4 \\
&\quad - m_\sigma^2(((5x - 1)(x(3x + 2) + 3)c_\theta^2 + x(25x + 17) + 3) + 3)m_\sigma^2 - 4x^2\Gamma_N^2 - 4c_\theta H O x(4x^2 + x + 1))M_N^2 \\
&\quad - 4c_\theta H m_\sigma^4 O x(2x + 1) + m_\sigma^6((x(3x + 2) + 3)c_\theta^2 + (3x + 1)(5x + 3)))m_J^2 + M_N^6 \bar{x}^3(3\bar{x}^2 c_\theta^2 + x(5x + 2) + 1)M_N^6 \\
&\quad + (4x^2((x + 3)\Gamma_N^2 + 2c_\theta H O \bar{x}) - m_\sigma^2 \bar{x}(\bar{x}^2(7x + 5)c_\theta^2 + (3x + 1)(x(5x + 2) + 1))))M_N^4 \\
&\quad + m_\sigma^2(m_\sigma^2(\bar{x}^2(5x - 1)c_\theta^2 + x + x^2(19x + 11) + 1) - 2x(2x\Gamma_N^2 + c_\theta H O(7x^2 + 1)))M_N^2 \\
&\quad + 2c_\theta H m_\sigma^4 O x(3x + 1) - m_\sigma^6(\bar{x}^2 c_\theta^2 + (3x + 1)^2)),
\end{aligned}$$

$$\begin{aligned}
M_{\sigma N}^{JN,2} &\equiv m_\sigma^2(M_N^2 \hat{x} - m_\sigma^2)((c_\theta^2 + 1)\bar{x}^2 M_N^4 - 2m_\sigma^2(\hat{x}c_\theta^2 - x + 1)M_N^2 + (c_\theta^2 + 1)m_\sigma^4)m_J^6 + M_N^2(4(c_\theta^2 + 1)\bar{x}^3 x \hat{x} M_N^8 \\
&\quad - \bar{x}(m_\sigma^2(3\bar{x}^3 + c_\theta^2(5x(3x\hat{x} + 1) - 3)) - c_\theta(c_\theta^2 + 3)H O \bar{x}^2 x)M_N^6 + m_\sigma^2(m_\sigma^2(\hat{x}(x(21x + 2) + 9)c_\theta^2 \\
&\quad - 5x + x^2(7 - 11x) + 9) - 2c_\theta H O \bar{x}x(\hat{x}c_\theta^2 - 2x + 2))M_N^4 - m_\sigma^4(m_\sigma^2((x(13x + 18) + 9)c_\theta^2 + x(10 - 3x) + 9) \\
&\quad - c_\theta(c_\theta^2 - 1)H O \bar{x}x)M_N^2 - 2c_\theta H m_\sigma^6 O x + m_\sigma^8(3\hat{x}c_\theta^2 + 7x + 3))m_J^4 - M_N^4(8\bar{x}^2 x \hat{x}(\bar{x}c_\theta^2 + x(x + 2) - 1)M_N^8 \\
&\quad + (2x\hat{x}(c_\theta(c_\theta^2 + 3)H O \bar{x}^3 + 8x\Gamma_N^2) - m_\sigma^2 \bar{x}(\hat{x}(x(27x + 23) + 17) - 3)c_\theta^2 + \bar{x}^2(x(7x + 12) - 3)))M_N^6 \\
&\quad + m_\sigma^2(((33x^4 + 52x^3 + 30x^2 + 4x + 9)c_\theta^2 - 4x + x^2(6 - x(7x + 4)) + 9)m_\sigma^2 - 4x^2(x + 5)\Gamma_N^2
\end{aligned}$$

$$\begin{aligned}
& -4c_g H O x \tilde{x} (\hat{x} c_g^2 - x + 2)) M_N^4 - m_\sigma^4 ((\hat{x}(x(17x+6)+9)c_g^2 + x(x(9x+7)+23)+9)m_\sigma^2 - 4x^2 \Gamma_N^2 \\
& - 2c_g H O x (\tilde{x} c_g^2 + 3x^2 + 1)) M_N^2 - 4c_g H m_\sigma^6 O x (2x+1) + m_\sigma^8 ((x(3x+2)+3)c_g^2 + (3x+1)(5x+3))) \\
& \times m_J^2 + M_N^6 (4(\tilde{x}^2 c_g^2 + \tilde{x}^2) \tilde{x}^3 x \hat{x} M_N^8 - \tilde{x} ((\tilde{x}^2 (x(13x\hat{x}+7)-1)c_g^2 + \tilde{x}(x(x(5x-32)+2)-8)+1)) m_\sigma^2 \\
& + x(16x\hat{x}\Gamma_N^2 - c_g H O \tilde{x}(c_g^2 \tilde{x}^3 + x(x(7x-17)-3)-3))) M_N^6 + (\hat{x}(\tilde{x}^2 (x(15x-2)+3)c_g^2 + \tilde{x}^2 (x(3x+2)+3)) m_\sigma^4 \\
& + 2x(-2x(x(x+2)+5)\Gamma_N^2 - c_g H O \tilde{x}(c_g^2 \tilde{x}^2 + 2(2x^2+x+1))) m_\sigma^2 + 4c_g H O \tilde{x} x^3 \Gamma_N^2) M_N^4 \\
& + m_\sigma^4 (x(4x\hat{x}\Gamma_N^2 + c_g H O (c_g^2 \tilde{x}^3 + x + x^2(11x+19)+1)) - m_\sigma^2 (\tilde{x}^2 (x(7x+6)+3)c_g^2 \\
& + 11x^4 + 28x^3 + 10x^2 + 12x + 3)) M_N^2 + m_\sigma^6 \hat{x} (m_\sigma^2 (\tilde{x}^2 c_g^2 + (3x+1)^2) - 2c_g H O x (3x+1))).
\end{aligned}$$

4. Boltzmann equation

In order to obtain the correct Boltzmann equations for the evolution of the particle abundances, we need to subtract on shell contributions from $2 \leftrightarrow 2$ processes as discussed in Appendix A 1. In the Majoron plus triplet model, the on shell exchange of σ needs to be subtracted in the thermal rates for γ_{NNTT} , γ_{NNJJ} , and γ_{TTJJ} , resulting in

$$\gamma_{NNTT}^{\text{off}} = \gamma_{TTNN}^{\text{off}} = \gamma_{NNTT} - \text{Br}_{\sigma, TT} \gamma_{\sigma, NN}, \quad (\text{A47})$$

$$\gamma_{NNJJ}^{\text{off}} = \gamma_{JJNN}^{\text{off}} = \gamma_{NNJJ} - \text{Br}_{\sigma, JJ} \gamma_{\sigma, NN}, \quad (\text{A48})$$

$$\gamma_{TTJJ}^{\text{off}} = \gamma_{JJTT}^{\text{off}} = \gamma_{TTJJ} - \text{Br}_{\sigma, JJ} \gamma_{\sigma, TT}. \quad (\text{A49})$$

With this in mind, the relevant processes for the neutrino evolution are given by

$$[N \leftrightarrow LH] = (\delta_N - 1) \gamma_D, \quad [NL \leftrightarrow Q_3 U_3] = 2(\delta_N - 1) \gamma_{NLQ_3 U_3}, \quad (\text{A50})$$

$$[N \overline{U}_3 \leftrightarrow L \overline{Q}_3] = 2(\delta_N - 1) \gamma_{N \overline{U}_3 L \overline{Q}_3}, \quad [N \overline{Q}_3 \leftrightarrow L \overline{U}_3] = 2(\delta_N - 1) \gamma_{N \overline{Q}_3 L \overline{U}_3}, \quad (\text{A51})$$

$$[NN \leftrightarrow \sigma] = 2(\delta_N^2 - \delta_\sigma) \gamma_{\sigma, NN}, \quad [NN \leftrightarrow TT] = 2(\delta_N^2 - \delta_T^2) \gamma_{NNTT}^{\text{off}}, \quad (\text{A52})$$

$$[NN \leftrightarrow \sigma\sigma] = 2(\delta_N^2 - \delta_\sigma^2) \gamma_{NN\sigma\sigma}, \quad [NN \leftrightarrow JJ] = 2(\delta_N^2 - \delta_J^2) \gamma_{NNJJ}^{\text{off}}, \quad (\text{A53})$$

$$[NN \leftrightarrow \sigma J] = 2(\delta_N^2 - \delta_\sigma \delta_J) \gamma_{NN\sigma J}, \quad (\text{A54})$$

where the scattering processes in the first two lines also appear in VL and we assumed that H , A , Q_3 and U_3 are in thermal equilibrium, $Y_{H,A,Q_3,U_3} = Y_{H,A,Q_3,U_3}^{eq}$. Next, the relevant scattering processes for the evolution of T are given by

$$[TT \leftrightarrow \sigma] = 2(\delta_T^2 - \delta_\sigma) \gamma_{\sigma, TT}, \quad [TT \leftrightarrow NN] = 2(\delta_T^2 - \delta_N^2) \gamma_{TTNN}^{\text{off}}, \quad (\text{A55})$$

$$[TT \leftrightarrow \sigma\sigma] = 2(\delta_T^2 - \delta_\sigma^2) \gamma_{TT\sigma\sigma}, \quad [TT \leftrightarrow JJ] = 2(\delta_T^2 - \delta_J^2) \gamma_{TTJJ}^{\text{off}}, \quad (\text{A56})$$

$$[TT \leftrightarrow \sigma J] = 2(\delta_T^2 - \delta_\sigma \delta_J) \gamma_{TT\sigma J}, \quad [TT \leftrightarrow AA] = 2(\delta_T^2 - 1) T, \text{ gauge} \quad (\text{A57})$$

while for J , we have

$$[JJ \leftrightarrow \sigma\sigma] = 2(\delta_J^2 - \delta_\sigma^2) \gamma_{JJ\sigma\sigma}, \quad [JJ \leftrightarrow \sigma] = 2(\delta_J^2 - \delta_\sigma) \gamma_{\sigma, JJ}, \quad (\text{A58})$$

$$[JJ \leftrightarrow NN] = 2(\delta_J^2 - \delta_N^2) \gamma_{JJNN}^{\text{off}}, \quad [JJ \leftrightarrow TT] = 2(\delta_J^2 - \delta_T^2) \gamma_{JJTT}^{\text{off}}, \quad (\text{A59})$$

$$[\sigma J \leftrightarrow NN] = (\delta_J \delta_\sigma - \delta_N^2) \gamma_{\sigma J NN}, \quad [\sigma J \leftrightarrow TT] = (\delta_J \delta_\sigma - \delta_T^2) \gamma_{\sigma J TT}, \quad (\text{A60})$$

$$[JN \leftrightarrow \sigma N] = (\delta_J \delta_N - \delta_\sigma \delta_N) \gamma_{JN\sigma N}, \quad [JT \leftrightarrow \sigma T] = (\delta_J \delta_T - \delta_\sigma \delta_T) \gamma_{JT\sigma T}. \quad (\text{A61})$$

Finally, the relevant scattering processes for the evolution of σ are

$$[\sigma \leftrightarrow JJ] = (\delta_\sigma - \delta_J^2)\gamma_{\sigma JJ}, \quad [\sigma \leftrightarrow NN] = (\delta_\sigma - \delta_N^2)\gamma_{\sigma NN}, \quad (\text{A62})$$

$$[\sigma \leftrightarrow TT] = (\delta_\sigma - \delta_T^2)\gamma_{\sigma TT}, \quad [\sigma\sigma \leftrightarrow JJ] = 2(\delta_\sigma^2 - \delta_J^2)\gamma_{\sigma\sigma JJ}, \quad (\text{A63})$$

$$[\sigma\sigma \leftrightarrow NN] = 2(\delta_\sigma^2 - \delta_N^2)\gamma_{\sigma\sigma NN}, \quad [\sigma\sigma \leftrightarrow TT] = 2(\delta_\sigma^2 - \delta_T^2)\gamma_{\sigma\sigma TT}, \quad (\text{A64})$$

$$[\sigma J \leftrightarrow NN] = (\delta_\sigma \delta_J - \delta_N^2)\gamma_{\sigma J NN}, \quad [\sigma J \leftrightarrow TT] = (\delta_\sigma \delta_J - \delta_T^2)\gamma_{\sigma J TT}, \quad (\text{A65})$$

$$[\sigma N \leftrightarrow JN] = (\delta_\sigma \delta_N - \delta_J \delta_N)\gamma_{\sigma N JN}, \quad [\sigma T \leftrightarrow JT] = (\delta_\sigma \delta_T - \delta_J \delta_T)\gamma_{\sigma T JT}. \quad (\text{A66})$$

Summing the respective processes according to (A16) and using $\text{Br}_{\sigma NN} + \text{Br}_{\sigma TT} + \text{Br}_{\sigma JJ} = 1$, we obtain (3.12)–(3.15) and (3.18).

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