

Nonlinear stability of the matter-dominated universe

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We numerically study nonlinear perturbations of the Einstein-de Sitter spacetime as a solution to the Gowdy-symmetric Einstein-Euler system for a polytropic equation of state. The results suggest that the Einstein-de Sitter spacetime is stable for sufficiently small but otherwise generic perturbations. This is in stark contrast to the well-known instability of this spacetime when the matter model is dust. Moreover, this indicates a previously unknown stable regime of the Einstein-Euler equations with direct implications for cosmology.

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Introduction. The matter-dominated epoch is the longest period in the evolution of the Universe, lasting from redshift $z \approx 3500$ until $z \approx 0.4$ [1]. In relativistic cosmology, this regime is typically modeled by the Einstein-de Sitter spacetime [2], also known as the matter-dominated universe, which is the Friedman-Lemaître-Robertson-Walker (FLRW) solution with vanishing spatial curvature and dust as a matter source. In cosmic time it takes the form

$$g_{\text{EdS}} = -dt^2 + t^{\frac{4}{3}}(dx^2 + dy^2 + dz^2) \quad (1)$$

with spatial topology $\mathbb{T}^3$ or $\mathbb{R}^3$. It is well known that (1) is dynamically unstable as a solution to the Einstein-dust system, in the sense that the future asymptotic behavior of solutions arising from arbitrarily small perturbations of (1) is determined by growing density perturbations and hence deviates from (1) [1,3,4]. Indeed, this instability is often invoked as an explanation for the formation of large-scale structures during the matter-dominated epoch [1,3]. On the other hand, any model which is used to describe a significant portion of the cosmological evolution should, in principle, be dynamically stable to allow for small inhomogeneities without strongly deviating from the homogeneous geometry over time. From this perspective, it is natural to look for cosmological models which display homogenizing behavior for small perturbations as is well established for a positive cosmological constant [5–10] or curvature domination [11–13].

Unlike spacetimes which are curvature or Λ -dominated, the expansion of matter-dominated cosmological models is caused by the energy density and pressure of the matter. That is to say, the dynamical properties of the matter model greatly affect the evolution of curvature inhomogeneities in this class of solutions [4,14,15]. Thus, in order for such a model to be an attractor, one requires a stabilizing mechanism intrinsic to the matter dynamics, which homogenizes the matter fields and allows for the coupled dynamics to stabilize. Importantly, dust does *not* have such a mechanism due to the decoupling of energy density and velocity.

In this letter, we demonstrate that adding an arbitrarily small but nonvanishing pressure to the fluid, by considering a polytropic equation of state with minimal polytropic index $n > 3$, stabilizes the matter-dominated universe. In particular, this explains the homogenization of spacetime on large scales as a direct consequence of the nonlinear interaction of the Einstein-fluid dynamics.

The polytropic fluid model is characterized by the equation of state $p = K\rho^{1+\frac{1}{n}}$, where $K > 0$ with the polytropic index $n > 0$. For these fluids, the speed of sound, $\frac{\partial p}{\partial \rho} = (1 + \frac{1}{n})K\rho^{1/n}$, decays as the universe expands. This means polytropic fluids have a dustlike pressure at late times. Moreover, the Einstein-Euler system for a polytropic fluid admits spatially flat FLRW solutions (14) which asymptote toward g_{EdS} for large times.

However, in contrast to dust, the pressure of a polytropic fluid is nonvanishing for a positive energy density. Our results suggest that the coupling of energy density and fluid velocity via the pressure term introduces a balancing mechanism which dominates the fluid dynamics, particularly in the small data regime [cf. (22)]. The late-time asymptotics of the energy density, which assure the matter-dominated expansion rate, in combination with the homogenization due to the nontrivial pressure, makes the

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asymptotic behavior of the fluid compatible with the Einstein-de Sitter asymptotics. This appears to have significant consequences for the dynamics when coupled with the Einstein equations as we demonstrate numerically below.

Specifically, we find convincing evidence that for sufficiently small, but otherwise generic, initial inhomogeneities of the metric and fluid variables, the corresponding solutions to the Einstein-Euler system converge to an Einstein-de Sitter universe. This indicates the orbital stability of the EdS spacetime (1).¹ In particular, this is the first evidence for the existence of a stable, fluid-filled, cosmological model for the matter-dominated epoch and the first stable solution to the Einstein-Euler equations in the decelerated regime. Our results also reveal a mechanism by which cosmological evolution drives the spatial geometry toward a flat metric, offering a gravity-based explanation for the emergence of large-scale spatial flatness [16].

Equations and numerical setup. We numerically study nonlinear perturbations of the FLRW solution to the Einstein-Euler equations for a polytropic fluid in the expanding direction of spacetime. In particular, we restrict our attention to $\mathbb{T}^3$ Gowdy symmetric spacetimes (cf. [17,18]) of the form

$$g = e^{2(\nu-U)}(-dt^2 + e^{-2\alpha}dx^2) + e^{2U}(dy + Adz)^2 + e^{-2U+2t}dz^2$$

where t is an areal time coordinate, see [19] for details. In this case, the metric coefficient functions ν , U , α and A are functions of $t \in [t_0, \infty)$ and $x \in [0, 2\pi)$ alone, and the Einstein-Euler equations reduce to a 1 + 1-dimensional system. In Gowdy symmetry, the normalized fluid four-velocity has only two nonzero components, $u = u_0 dt + u_1 dx$, where u_1 depends on $(t, x) \in [t_0, \infty) \times [0, 2\pi)$ and $(u^0)^2 - e^{-2\alpha}(u^1)^2 = e^{-2(\nu-U)}$.

We introduce three effective variables for the matter, the scalar velocity $v = \frac{u^1}{e^\alpha u^0}$, $|v| \in [0, 1]$, $\Gamma = \frac{1}{\sqrt{1-v^2}}$ and the modified density $\mu = e^{2(\nu-U)}\rho$. The Gowdy symmetric Einstein-Euler equations can be expressed as a system of balance laws, using $\partial_t A = A_0$, $\partial_t U = U_0$,

$$\partial_t(e^{-\alpha}A_0) - \partial_x(A_1) = e^{-\alpha}(4(A_1U_1 - A_0U_0) + A_0), \quad (2)$$

$$\partial_t(e^{-\alpha}A_1) - \partial_x(A_0) = 0, \quad (3)$$

¹The expansion-normalized spatial metric converges to a flat torus near the standard one, which reflects the nontrivial moduli space of flat spatial metrics compatible with (1).

$$\partial_t\left(e^{-\alpha}\left(U_0 - \frac{1}{2}\right)\right) - \partial_x(U_1) = e^{-\alpha}\left(\frac{1}{2}e^{4U-2t}(A_0^2 - A_1^2) + \frac{1}{2} - U_0\right), \quad (4)$$

$$\partial_t(e^{-\alpha}U_1) - \partial_x(U_0) = 0, \quad (5)$$

$$\partial_t\nu = 1 + (\Gamma^2 - 1)\mu + e^{\frac{-2}{n}(\nu-U)}K\Gamma^2\mu^{\frac{n+1}{n}} + e^{2\alpha}U_x^2 + U_t^2 + \frac{1}{4}e^{4U-2t}(e^{2\alpha}A_x^2 + A_t^2), \quad (6)$$

$$\partial_t\alpha = 1 - (\mu - e^{\frac{-2}{n}(\nu-U)}K\mu^{\frac{n+1}{n}}), \quad (7)$$

$$\begin{aligned} &\partial_t(e^{-\alpha}\tau) + \partial_x(S) \\ &= \frac{1}{4}e^{-\alpha-2t}(\mu - Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}}) \\ &\quad \times [e^{4U}(A_1^2 + A_0^2) + 4e^{2t}(U_1^2 + U_0(U_0 - 1))] \\ &\quad - Ke^{\frac{-2}{n}(\nu-U)-\alpha}\mu^{\frac{n+1}{n}}, \end{aligned} \quad (8)$$

$$\begin{aligned} &\partial_t(e^{-\alpha}S) + \partial_x(H) = -\frac{1}{2}e^{-\alpha}(\mu - Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}}) \\ &\quad \times (e^{4U-2t}A_1A_0 + 2U_1(2U_0 - 1)), \end{aligned} \quad (9)$$

and constraints

$$0 = C_{A_1} := A_1 - e^\alpha\partial_x A, \quad (10)$$

$$0 = C_{U_1} := U_1 - e^\alpha\partial_x U, \quad (11)$$

$$\begin{aligned} &\partial_x\nu = -e^{-\alpha}v\Gamma^2(\mu + e^{\frac{-2}{n}(\nu-U)}K\mu^{\frac{n+1}{n}}) \\ &\quad + \frac{1}{2}e^{4U-2t}A_xA_t + 2U_xU_t, \end{aligned} \quad (12)$$

where

$$\begin{aligned} A_1 &:= e^\alpha A_x, & U_1 &:= e^\alpha U_x, \\ \tau &:= (\mu + Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}})\Gamma^2 - Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}}, \\ S &:= (\mu + Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}})\Gamma^2 v, \\ H &:= (\mu + Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}})\Gamma^2 v^2 + Ke^{\frac{-2}{n}(\nu-U)}\mu^{\frac{n+1}{n}}. \end{aligned}$$

The spatial computational domain is taken to be $[0, 2\pi)$ with periodic boundary conditions and N cells. We numerically solve the system (2)–(9) using a local Lax-Friedrichs finite volume discretization with fifth-order WENO reconstruction and fourth-order Runge-Kutta timestepping. For computational simplicity, we have treated point values and cell averages as equivalent in our code, which is a second-order accurate approximation. Thus, our numerical scheme is formally second-order accurate for smooth solutions. Further details on our code, including recovery of the primitive variables and convergence tests, are given in the Supplemental Material [20].

The FLRW solution to (2)–(12) is given by

$$g = \frac{-3}{4} e^{\frac{3t}{2}} (1 - K e^{\frac{-3t}{2n}})^n dt^2 + e^t (dx^2 + dy^2 + dz^2),$$

$$\rho = e^{\frac{-3t}{2}} (1 - K e^{\frac{-3t}{2n}})^{-n}, \quad u = -\sqrt{\frac{3}{4}} e^{\frac{3t}{4}} (1 - K e^{\frac{-3t}{2n}})^{\frac{n}{2}} dt. \quad (13)$$

As discussed above, for large t this asymptotes to the standard Einstein-de Sitter metric (1) in areal coordinates,

$$g_{\text{EdS}} = -\frac{3}{4} e^{\frac{3t}{2}} dt^2 + e^t (dx^2 + dy^2 + dz^2),$$

$$\rho_{\text{EdS}} = e^{\frac{-3t}{2}}, \quad u_{\text{EdS}} = -\left(\frac{3}{4}\right)^{\frac{1}{2}} e^{\frac{3t}{4}} dt. \quad (14)$$

Stable polytropic solutions. Ultimately, we are interested in the asymptotic behavior of small nonlinear perturbations of (13). To this end, we have studied classes of initial data at $t = 0$ characterized by a single parameter $\epsilon > 0$ such that $\epsilon = 0$ corresponds to *exact* FLRW initial data. That is to say, ϵ parametrizes the size, with respect to appropriate Sobolev norms, of our nonlinear perturbations. Independent of the specific class of initial data we observe the same universal dynamical behavior of perturbations of metric and fluid variables away from the background. In this letter, we will focus on the following family of initial data at $t = 0$,

$$\alpha = \log\left(\frac{\sqrt{3}}{2} (1 - K)^{\frac{n}{2}}\right) + \epsilon,$$

$$\nu = \log\left(\frac{\sqrt{3}}{2} (1 - K)^{\frac{n}{2}}\right) + \epsilon \cos(x),$$

$$U = \epsilon \cos(x), \quad U_0 = \frac{1}{2}, \quad U_1 = e^\alpha \partial_x U,$$

$$A = \epsilon \sin(x), \quad A_0 = 0, \quad A_1 = e^\alpha \partial_x A,$$

$$v = 0, \quad \mu = \frac{3}{4} + \epsilon \sin(x). \quad (15)$$

However, we emphasize that the behavior is qualitatively the same for all initial data sets we have studied.

For given fixed ϵ and K , we find there are values of n such that the fluid variables rapidly homogenize and decay to the background FLRW solution. An example of this behavior is shown in Fig. 1, where the $\dot{H}^1$ -norm is defined by $\|f\|_{\dot{H}^1} := \|\partial_x f\|_{L^2}$. Similarly, the metric functions A , U , and ν all homogenize while the function α approaches a fixed spatial profile at late times, see Fig. 2. Although the H^1 norm of α does not decay, this is merely a coordinate effect as we discuss below. To further illustrate that the spacetime does in fact homogenize toward the future, we compute the expansion normalized spatial Kretschmann scalar, displayed in Fig. 3. In particular, the decay of the

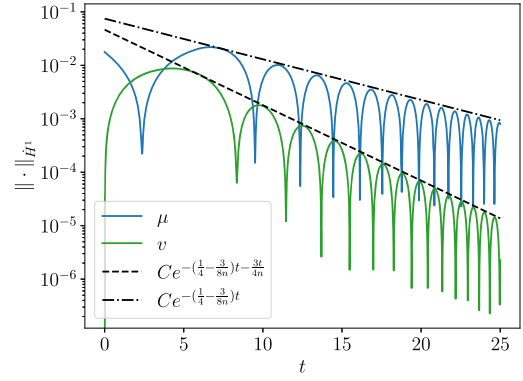


FIG. 1. Asymptotic behavior of the $\dot{H}^1$ norms of μ and v . $n = 5$, $K = 0.2$. The black lines show the theoretical decay rates predicted by the decay estimate (24).

Kretschmann scalar demonstrates the solution evolves toward a flat spatial geometry. In fact, an expansion-normalized spatial limit metric of the form $e^{2(\nu_\infty - U_\infty)} (e^{-2\alpha_\infty(x)} dx^2) + e^{2U_\infty} (dy + A_\infty dz)^2 + e^{-2U_\infty} dz^2$, where $U_\infty, \nu_\infty, A_\infty \in \mathbb{R}$ and $\alpha_\infty(x)$ is a function, is a flat torus and hence homogeneous.

The above numerical results suggest that the fluid pressure has a stabilizing effect on the spacetime. Indeed, for all $K > 0$ and suitably small ϵ , we observe stable asymptotics for the polytropic fluid while dust develops shocks for arbitrarily small initial data, see Fig. 4. This is particularly remarkable given the fact that the pressure of a polytropic fluid asymptotically approaches that of dust toward the future as $\rho \searrow 0$. In particular, this implies that a polytropic fluid with arbitrarily small pressure coupled to Einstein's equations is sufficient to stabilize the matter-dominated universe.

Transition between shock formation and stability. The results of the previous section demonstrate that the FLRW solution is stable for certain fixed values of ϵ and n . In this section, we analyze the transition from stable

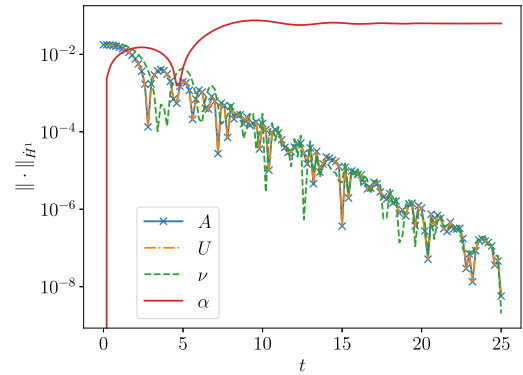


FIG. 2. Asymptotic behavior of the $\dot{H}^1$ norms of the metric variables. $n = 5$, $K = 0.2$, $\epsilon = 0.01$. We have only displayed a subset of timesteps to improve the clarity of the plot.

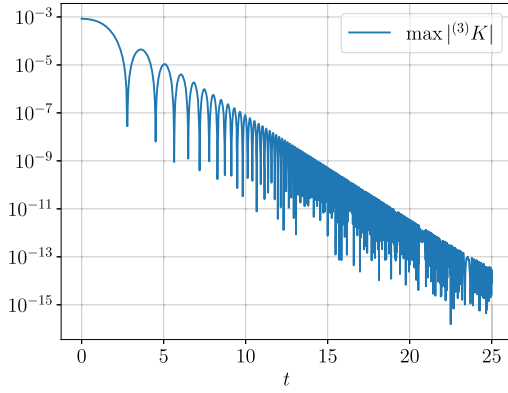


FIG. 3. Maximum of the absolute value of the expansion normalized spatial Kretschmann scalar. $n = 5$, $K = 0.2$, $\epsilon = 0.01$.

to unstable behavior for varying ϵ and n . In the vein of our previous work on isothermal fluids [4,21], we monitor the energy functional

$$\mathcal{H}_k(t) := \|\partial_x^k w(t)\|_{L^2}^2 + \|\partial_x^k v(t)\|_{L^2}^2 \quad (16)$$

where $w := \sqrt{4n(n+1)K\rho^{\frac{1}{2n}}}$ is the so-called Makino density variable. We classify a solution as *stable* if

$$\mathcal{H}_3(t) \leq 10^{-6} \mathcal{H}_3(0) \quad (17)$$

and *unstable* if

$$\left(\frac{\mathcal{H}_3}{\mathcal{H}_0}\right)(t) \geq 10^6 \left(\frac{\mathcal{H}_3}{\mathcal{H}_0}\right)(0). \quad (18)$$

Our numerical results indicate that for suitably small, fixed values of ϵ and K , there exists a critical value n^* of the polytropic index such that solutions with $n > n^*$ display stable behavior and solutions with $n < n^*$ develop shocks. An example of this transition for $\epsilon = 0.01$ is shown in

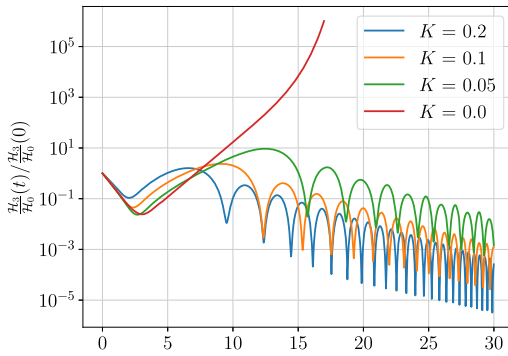


FIG. 4. The behavior of the norm ratio (18) for different values of K . In this example, the norm ratio is computed using μ instead of w since the latter is undefined when $K = 0$. All simulations used $\epsilon = 0.01$, $n = 5$.

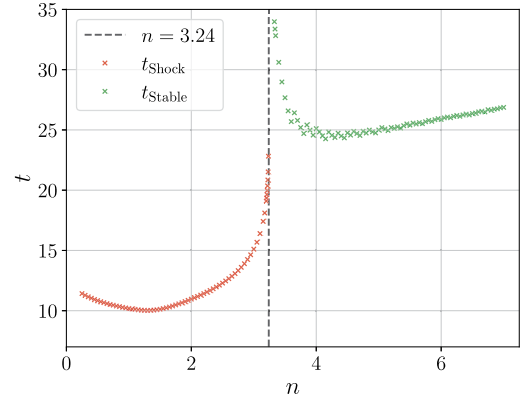


FIG. 5. The transition between stable and shock-forming solutions for $\epsilon = 0.01$, $K = 0.2$. The critical value $n^* \approx 3.24$ was estimated by fitting the data to (19).

Fig. 5. In order to determine the value of n^* , we perform a scaling analysis by varying the polytropic index and measuring the time of reaching the stabilization or shock thresholds (17)–(18). For n suitably close to n^* , we find the shock and stabilization times scale like

$$\begin{aligned} t_{\text{Shock}} &\approx -a_1 \log(n^* - n) + b_1, \\ t_{\text{Stable}} &\approx -a_2 \log(n - n^*) + b_2 \end{aligned} \quad (19)$$

for positive real constants a_i and b_i . For a given fixed ϵ , the corresponding value of n^* is estimated by fitting the threshold times to (19). By repeating this analysis for varying ϵ , we observe that n^* decreases monotonically as $\epsilon \searrow 0$. This is consistent with the rigorous analysis in the following section; as the size of the initial perturbation is decreased the estimated value of n^* approaches approximately 3.10, while the theoretical lower bound for stable solutions is $n > 3$.

Fluid homogenization. We complement the numerics with analytical estimates on a simplified model. A detailed analysis of the full $3 + 1$ -dimensional Einstein-Euler system is given in the paper [22].

A key mechanism for the nonlinear stability is the homogenization of polytropic fluids. The analytical explanation of this effect is based on the existence of a monotone energy for the fluid variables. We consider the respective asymptotic equations ($t \gg t_0$, $|v| \ll 1$) near the homogeneous background spacetime where we allow for spatial inhomogeneities of $\nu = 5t/4 + \tilde{\nu}$ and α , while all manifestly faster decaying terms are set to zero.

$$\partial_t \mu = -e^\alpha (\mu \partial_x v + v \partial_x \mu) + \mu \left(\frac{3}{4} - \mu \right) \quad (20)$$

$$\partial_t v = v/4 - e^\alpha v \partial_x v - \frac{e^\alpha K(1+n)}{n} e^{-\frac{3t}{4}} e^{-\frac{2\tilde{\nu}}{n}} \mu^{\frac{1}{n}-1} \partial_x \mu \quad (21)$$

In line with the numerics we assume decay of the Sobolev norms of $\tilde{v}$ and a decomposition $\alpha = t/4 + \tilde{\alpha}$, where $\tilde{\alpha}(t, x)$ solves the asymptotic equation $\partial_t \tilde{\alpha} = 3/4 - \mu$. As a consequence of this and (20),

$$e^{3t/4} \int e^{-\tilde{\alpha}(t,x)} (3/4 - \mu) dx = \text{const.}$$

For the L^2 -norm of the velocity we invoke the asymptotic momentum constraint, $\partial_x v = -e^{-\alpha} \mu v$, which implies

$$\|e^{-\tilde{\alpha}/2} v\|_{L^2} \leq C \|e^{-\tilde{\alpha}/2} \partial_x v\|_{L^2}.$$

To measure the inhomogeneity of the fluid variables we introduce an energy, using $\nabla := e^{\tilde{\alpha}} \partial_x$, by

$$\begin{aligned} E[\mu, v]_s &:= \sum_{\ell=1}^s \int |\nabla^\ell (\mu^{1/2n})|^2 e^{-\tilde{\alpha}} dx + \frac{e^{3t/2n}}{4K(n+1)n} \\ &\times \int e^{2\tilde{v}/n} |\nabla^\ell v|^2 e^{-\tilde{\alpha}} dx - \frac{(1 + \frac{3}{4n}) e^{3t/2n - t/4}}{2K(n+1)(3/4)^{\frac{1}{2n}}} \\ &\times \int e^{2\tilde{v}/n} \nabla^{\ell-1} v \cdot \nabla^\ell (\mu^{1/2n}) e^{-\tilde{\alpha}} dx. \end{aligned} \quad (22)$$

For sufficiently large times the condition $n > 3$, which is consistent with the numerical results on the critical value of the polytropic index n^* , assures that the energy is equivalent to the Sobolev norm of order $s-1$ of $(\nabla \mu, \nabla v)$.

For $s \geq 2$ we obtain, by a straightforward application of the asymptotic system, the differential inequality

$$\partial_t E[\mu, v]_s = -\left(\frac{1}{2} - \frac{3}{4n}\right) E[\mu, v]_s + e^{\frac{1-3/n}{4}t} E[\mu, v]_s^{3/2} + \dots, \quad (23)$$

where negligible terms are suppressed. This implies decay rates of the order $e^{-(\frac{1}{2} - \frac{3}{4n})t}$ for the energy and, in turn, for the standard Sobolev norms of the form

$$\|\mu\|_{\dot{H}^s} + e^{3t/4n} \|v\|_{\dot{H}^s} \lesssim e^{-(\frac{1}{4} - \frac{3}{8n})t}. \quad (24)$$

This is consistent with both numerical results on the decay of the fluid Sobolev norms (cf. Fig. 1) and on the critical polytropic index (cf. Fig. 5).

Conclusions. The results presented above provide strong numerical evidence that the Einstein–de Sitter (EdS) solution is future dynamically stable when the matter source is a polytropic fluid with polytropic index $n > 3$. This behavior is in marked contrast with the corresponding Einstein–dust evolutions arising from closely comparable initial data.

To the best of our knowledge, this constitutes the first numerical identification of a stable, matter-dominated attractor for the Einstein–matter system based on a physically plausible large-scale matter model relevant to the matter-dominated epoch. In particular, the stability observed here is not the outcome of a finely tuned choice of equation of state or initial data. Indeed, the polytropic model approaches dust at late times, and thus remains close to the standard dust description in the regime in which the asymptotics are assessed. Moreover, polytropic fluids are widely used in cosmological modeling, including for ranges of indices consistent with the condition $n > 3$ [23,24]. We further note that stability is observed for large values of n , in which regime the polytropic law is arbitrarily close to the linear equation of state $p = K\rho$ commonly employed to describe isothermal fluids in cosmology. The analytical reason for the necessity of the lower bound on the polytropic index is that it assures the positivity of the fluid energy (22). Larger values of n mean that the equation of state function is closer to the linear one, while for smaller values of n it is closer to dust. The fluid energy captures the coupling mechanism of energy density and fluid velocity through the pressure term, which enables the homogenization of the fluid (cf. [21]). This mechanism holds for fluids with linear equation of state (under suitable conditions) but fails for dust. In this regard the range of n where stability holds can be explained by sufficient proximity to the linear equation of state. A complete explanation would, however, require an understanding of the mechanism of shock formation for smaller values of n , which we do not provide here. A detailed discussion of the physical implications will be presented in forthcoming work; nevertheless, several consequences are immediate. For sufficiently large n , the polytropic fluid homogenizes and does not form structure: in particular, no Jeans-type instability is present in the expansion-normalized variables, in the sense that the normalized energy density becomes spatially uniform while the fluid velocity decays relative to it and homogenizes. This conclusion does not, however, exclude structure formation in coupled multifluid settings. One may expect the polytropic component to govern the large-scale expansion with scale factor $a(t) = t^{2/3}$ and to drive large-scale homogenization of the geometry, while additional components with linear equations of state may have a comparatively weaker late-time influence on the metric yet still develop nonlinear structures, as suggested, for example, by analyses of shock formation in [21,25]. Finally, the most relevant consequence of the presented results is, in our opinion, the evidence for the stability of the flat FLRW model (1), which is a new, physically relevant nonlinearly stable regime of solutions to the Einstein–Euler equations.

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Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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