

Spontaneous breaking of the Peccei-Quinn symmetry during warm inflation

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We show that, for values of the axion decay constant parametrically close to the grand unified theory scale, the Peccei-Quinn phase transition may naturally occur during warm inflation. This results from interactions between the Peccei-Quinn scalar field and the ambient thermal bath, which is sustained by the inflaton field through dissipative effects. It is therefore possible for the axion field to appear as a dynamical degree of freedom only after observable cosmic microwave background scales have become superhorizon, thus avoiding the large-scale axion isocurvature perturbations that typically plague such models. This nevertheless yields a nearly scale-invariant spectrum of axion isocurvature perturbations on small scales, with a density contrast of up to a few percent, which may have a significant impact on the formation of gravitationally bound axion structures such as miniclusters.

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I. INTRODUCTION

The QCD axion is one of the most promising dark matter candidates, appearing naturally within the Peccei-Quinn (PQ) solution to the strong CP problem [1–3]. In canonical models, the axion appears as the pseudo-Nambu-Goldstone boson associated with the spontaneous breaking of a global $U(1)$ PQ symmetry under which Standard Model quarks, and possibly other fields, are charged. It has a naturally small mass due to nonperturbative QCD instantons, $m_a \sim \Lambda_{\text{QCD}}^2/f_a$, where the axion decay constant f_a corresponds to the PQ symmetry-breaking scale. Astrophysical constraints yield $f_a \gtrsim 10^8$ GeV, such that $m_a \lesssim 0.1$ eV. In this low-mass range, this “invisible” axion decays into photon pairs with a lifetime exceeding the age of the Universe, making it a natural dark matter candidate.

The main cosmological mechanism for axion production is, in most scenarios, the so-called misalignment mechanism, where the axion field oscillates coherently about the minimum of its instanton-induced potential, mimicking a pressureless fluid. The dark matter abundance depends on the axion field’s displacement from the minimum at the onset of

oscillations just after the QCD phase transition, which is proportional to the axion decay constant. If the latter is close to the grand unification scale, 10^{15} – 10^{16} GeV, as typical of, e.g., string theory constructions and grand unified theories (GUTs), this requires a fine-tuning of the initial misalignment angle to avoid an overabundance of dark matter [4].

Even if such a fine-tuning can be attributed to anthropic selection in the string landscape or other dynamical mechanisms, axion models with large decay constants are, at least within the simplest scenarios, in tension with observational data. It is typically assumed in such models that the PQ symmetry remains broken throughout inflation and is not restored during the reheating process. The axion field thus remains light during inflation and acquires significant fluctuations on large scales that later become cold dark matter (CDM) isocurvature modes after the onset of field oscillations [4,5]. Such modes are uncorrelated with the main adiabatic curvature perturbations resulting from inflaton fluctuations and are now severely constrained by the accurate measurements of cosmic microwave background (CMB) anisotropies made by the *Planck* satellite (see [6] for a recent take on the subject). Overall, scenarios with large axion decay constants seem to only be viable within low-scale inflationary models, which predict values of the tensor-to-scalar ratio well below the reach of any CMB observations in the foreseeable future [7] (see also [8–11] and references therein).

An interesting alternative scenario that can prevent the generation of large axion isocurvature perturbations is one where the spontaneous breaking of the PQ symmetry

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occurs during the inflationary phase and is not later restored in the radiation era. This was first proposed by Linde [4] in a model where the complex scalar field S that breaks the global $U(1)$ PQ symmetry is coupled to the inflaton field ϕ , with a scalar potential of the form

$$V(S, \phi) = \lambda \left(|S|^2 - \frac{f_a^2}{2} \right)^2 + \nu \phi^2 |S|^2, \quad (1)$$

in addition to the inflaton potential. Note that the mass of the complex field receives a contribution from the inflaton. If $\nu > 0$, the bilinear interaction acts as a stabilizer, preventing the transition from the symmetric minimum at $|S| = 0$ to the broken minimum at $|S| = f_a \sqrt{1 - (\phi/\phi_c)^2}$ until $\phi < \phi_c = \sqrt{\lambda/\nu} f_a$. The inflaton field thus plays the role of the temperature in a thermal crossover, and it may be possible for the critical field value to be reached only after observable CMB scales have become superhorizon during inflation. Since the axion, corresponding to the phase of the complex S field in this scenario, only becomes an independent dynamical degree of freedom after the PQ phase transition, this then prevents the growth of the troublesome axion isocurvature perturbations on large scales. In addition, even if large axion fluctuations arise on small scales, any potentially associated topological defects [12] can still be inflated away if the PQ phase transition occurs sufficiently long before inflation ends.

In such a scenario, the coupling ν , besides the requirement of a positive sign, must be sufficiently small to ensure that the inflaton field remains light after the PQ phase transition, with the inflaton squared mass shifting by $\Delta m_\phi^2 \sim \nu f_a^2 \ll H^2$. It cannot, however, be too small; otherwise, the critical inflaton field value could be reached before the last 50–60 e-folds of inflation.

Although Linde's scenario is certainly parametrically viable, we argue in this work that warm inflation [13] offers an alternative possibility for the PQ phase transition to occur during inflation. In warm inflation, the inflaton sustains a thermal radiation bath through dissipative effects, with a slowly evolving temperature typically just below the GUT scale. The latter may include some of the Standard Model fields (see, e.g., [14]) that are charged under the PQ symmetry, thus leading to a thermal contribution to the mass of the PQ scalar S . Hence, we may naturally expect a thermal second order PQ phase transition to occur during inflation for axion decay constants around the GUT scale. We will explore in detail the dynamics of such a phase transition and its impact on the spectrum of axion isocurvature perturbations.

This work is organized as follows. Section II summarizes the main features of warm inflation and presents the relevant dynamical quantities for the thermal PQ phase transition, analyzed in Sec. III. Section IV summarizes our main results and conclusions, as well as prospects for future developments.

II. WARM INFLATION

Warm inflation accounts for interactions between the inflaton scalar field ϕ and other degrees of freedom, in a nearly thermal radiation bath, during inflation [13,15–18]. The cosmological evolution of the inflaton-radiation system is thus given by two equations for the inflaton and radiation fluids, alongside the Friedmann equation determining the Hubble expansion rate,

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + \partial_\phi V(\phi) = 0, \quad (2)$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2, \quad (3)$$

$$H^2 = \frac{\rho_\phi + \rho_r}{3M_p^2}, \quad (4)$$

where $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ is the energy density of the inflaton, $\rho_r \equiv (\pi^2/30)g_*T^4$ is the radiation energy density, with g_* denoting the number of relativistic degrees of freedom, and $M_p \equiv \sqrt{\hbar c/(8\pi G)} \simeq 2.44 \times 10^{18}$ GeV is the reduced Planck mass. The dissipation coefficient $\Upsilon = \Upsilon(\phi, T)$ can generally be computed using standard nonequilibrium quantum field theory techniques once the interactions between the inflaton and other fields are specified [16,17,19–24]. Among the several interesting differences between conventional and warm inflation are the latter's smooth transition between the inflationary stage and a radiation-dominated era if $\Upsilon/(3H) \gtrsim 1$ close to the end of inflation, and the thermal nature of inflaton fluctuations, leading to a modified primordial curvature power spectrum [25–28].

For concreteness, we will focus on the warm inflation realization of [14], which constitutes the first implementation of warm inflation within an extension of the Standard Model. This model is based on the “warm little inflaton” scenario proposed in [29], the first to overcome the challenges in realizing warm inflation within a simple quantum field theory model. In the concrete scenario of [14], the inflaton dissipates its energy through its interactions with two of the right-handed neutrinos, sustaining a thermal bath that involves the latter and the Standard Model Higgs and lepton fields. The same interactions also generate light neutrino masses through the seesaw mechanism, and the third right-handed neutrino may generate the observed baryon asymmetry through thermal leptogenesis [30] just after the end of inflation. The underlying symmetries also ensure that a stable inflaton remnant remains until the present day, possibly accounting for dark matter [31] or dark energy [32].

We note, however, that the results of the next sections should be fairly independent of this particular choice of scenario since (i) most of the dynamics occurs during weak dissipation slow-roll dynamics, and (ii) the results below are essentially contingent on a decreasing temperature during

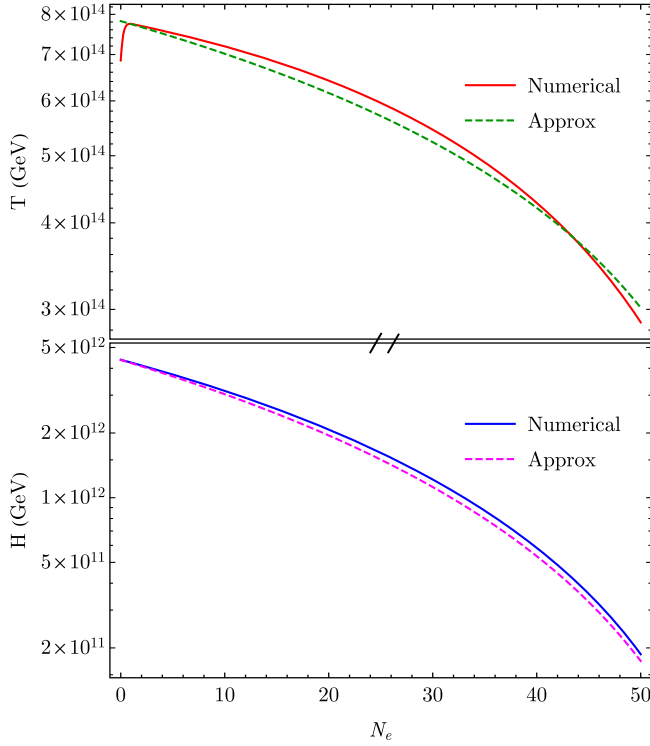


FIG. 1. Numerical evolution of the temperature T and Hubble parameter H (solid lines) as a function of the number of e-folds N_e , in the scenario of [14] for a particular choice of model parameters yielding a consistent warm inflation model. The dashed lines show the corresponding analytical approximations in Eqs. (5) and (6), which deviate less than 10% from the numerical results, which is sufficiently accurate for our purposes.

warm inflation, which is guaranteed for $\Upsilon/(3H) < 1$ and $\epsilon_\phi \equiv (M_p^2/2)(\partial_\phi V/V)^2 < 1$. For more general information on warm inflation and a detailed analysis of the model's inflationary dynamics, we refer the reader to [14] and references therein.

The only quantities that affect the dynamics of the PQ scalar field are the temperature of the thermal bath T and the Hubble expansion rate H during warm inflation. In Fig. 1 we illustrate the evolution of these quantities during warm inflation obtained by solving numerically the system in Eqs. (2)–(4) for a particular choice of model parameters, corresponding to the representative example shown in Fig. 1 of [14]. These are sufficiently well described by

$$T(N_e) = T_* \left(1 - \frac{N_e}{N_e^f}\right)^{13/20}, \quad (5)$$

$$H(N_e) = H_* \left(1 - \frac{N_e}{N_e^f}\right)^{11/5}, \quad (6)$$

where $T_* \simeq 7.8 \times 10^{14}$ GeV, $H_* \simeq 4.4 \times 10^{12}$ GeV, and $N_e^f = 65$. The quality of these approximations will be sufficient for our purposes as we aim to provide an overall

description of the phase transition during warm inflation, the above expressions yielding a typical example of how the temperature and the Hubble parameter vary slowly during a period of warm inflation.

III. PECCEI-QUINN PHASE TRANSITION

As argued above, we will assume that the complex scalar field S that spontaneously breaks the PQ symmetry is coupled to the particles in the radiation bath during warm inflation through an effective coupling α , which may be the result of interactions with different species in the thermal plasma. The leading correction to the PQ scalar potential is then a thermal mass correction,

$$V(|S|) = \lambda \left(|S|^2 - \frac{f_a^2}{2} \right)^2 + \alpha^2 T^2 |S|^2, \quad (7)$$

where T evolves during inflation according to Eq. (5) above in our working example. Here we will assume that the coupling between the PQ and inflaton fields is sufficiently small to be ignored, so that the PQ transition has a purely thermal origin, bearing in mind that there may be situations where both the inflaton coupling and the thermal correction contribute significantly to the PQ field's mass.

We note that there are subleading corrections to the scalar potential from its coupling to the thermal bath particles and its quartic self-coupling, from both zero- and finite-temperature effects. These include Coleman-Weinberg terms that lead to a temperature-dependent quartic coupling $\lambda(T)$, but since the temperature dependence is only logarithmic and, moreover, the temperature varies slowly during warm inflation, it is a sufficiently good approximation to consider a fixed value for this coupling. In addition, scalar self-interactions induce an additional term $\sim \lambda^{3/2} |S|^3 T$ (up to numerical loop factors) from the resummation of “daisy diagrams” [33].¹ While this term could play an important role, and, in fact, potentially change the nature of the phase transition, it is suppressed by a factor $(\lambda/\alpha)^{3/2}$ relative to the leading thermal mass term in the high-temperature regime $T \gg \alpha|S|$ relevant for our discussion. Since it is technically natural to consider $\alpha^4 \lesssim \lambda \ll \alpha$ in our setup, we may safely neglect the effects of this term in our analysis.

The complex scalar field S may be either decomposed into its radial and angular components, $\sqrt{2}S = \sigma e^{i\theta}$, or its real and imaginary part, $\sqrt{2}S = \sigma_R + i\sigma_I$, with $\sigma^2 = \sigma_R^2 + \sigma_I^2$. With the former decomposition, the action reads

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{\sigma^2}{2} \partial_\mu \theta \partial^\mu \theta - V(\sigma) \right], \quad (8)$$

¹Only bosonic degrees of freedom induce this type of correction and in our setup we will assume that only fermion fields in the thermal bath couple to the PQ field.

with

$$V(\sigma) = \frac{\lambda}{4}(\sigma^2 - f_a^2)^2 + \frac{1}{2}\alpha^2 T^2 \sigma^2. \quad (9)$$

At large temperatures, the expectation value of the radial field vanishes, $\langle \sigma \rangle = 0$, and the angular field θ , corresponding to the axion, has no kinetic term and is hence non-dynamical. This is a consequence of θ being ill defined at the U(1)-symmetric minimum. This means that above the critical temperature, $T_c \equiv \sqrt{\lambda} f_a / \alpha$, when the PQ symmetry is restored, it is more appropriate to describe the complex field in terms of its real and imaginary parts. The equation of motion for each of these fields in a flat Friedmann-Lemaître-Robertson-Walker universe are then given by

$$\ddot{\sigma}_R + 3H\dot{\sigma}_R - \frac{\nabla^2 \sigma_R}{a^2} + m_\sigma^2(\sigma) \sigma_R = 0, \quad (10)$$

$$\ddot{\sigma}_I + 3H\dot{\sigma}_I - \frac{\nabla^2 \sigma_I}{a^2} + m_\sigma^2(\sigma) \sigma_I = 0, \quad (11)$$

where the PQ field's effective mass squared is $m_\sigma^2(\sigma) = \lambda(\sigma^2 - f_a^2) + \alpha^2 T^2$. We assume that the energy density in the PQ field is much smaller than that of the inflaton,² $\rho_S \ll \rho_\phi$, so that the Hubble parameter H follows from Eq. (6).

Below the critical temperature, the origin becomes a maximum and the minima lie at $\sigma = f_a \sqrt{1 - (T/T_c)^2}$, which asymptotes to the axion decay constant f_a at late times when $T \ll T_c$. The PQ phase transition will then occur during the last 50–60 e-folds of warm inflation if the critical temperature lies in the range 10^{14} – 10^{15} GeV, which is typical of warm inflation models and not a special feature of our working example. This is natural for axion decay constants parametrically close to the GUT scale, and in Fig. 2 we show the regions in the (λ, α) plane for which the PQ transition occurs within the last ~ 50 e-folds of inflation, assuming the largest observable CMB scales exit the horizon ~ 60 e-folds before radiation becomes dominant.

We thus see that, if neither the self-coupling of the PQ field, nor its effective coupling to the particles in the thermal bath are very suppressed, a thermal PQ phase transition will naturally occur during warm inflation for large values of the axion decay constant $f_a \sim 10^{14}$ – 10^{16} GeV. We note that in this regime we always have $f_a > H/2\pi$ during inflation; nevertheless, the PQ symmetry remains restored until the temperature falls below the critical value, unlike in cold inflation scenarios [6].

Since during warm inflation $T \gg H$, as can be seen in the example of Fig. 1, and we are also interested in the regime of large axion decay constant $f_a \gg H$, the PQ field effective mass will generically exceed the Hubble rate and

²This remains true throughout inflation when the initial amplitude of the radial field guarantees that $V(\sigma)/V(\phi) \ll 1$.

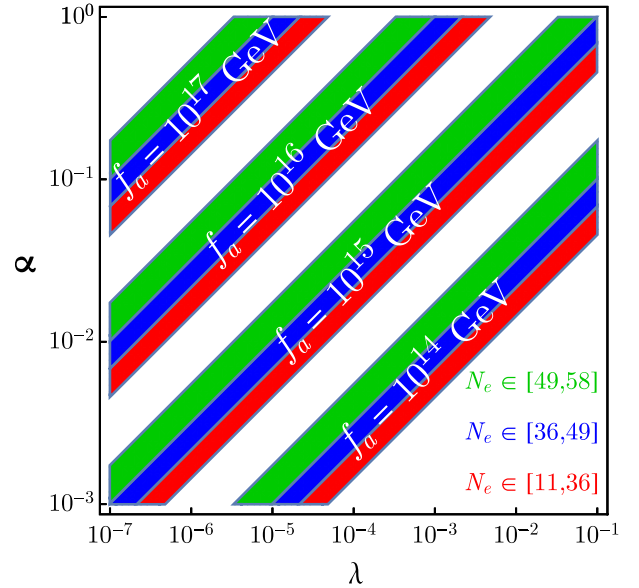


FIG. 2. Regions in the (λ, α) plane for which the PQ phase transition occurs during warm inflation, considering different values of the axion decay constant f_a around the GUT scale. For each value of f_a , the three colored bands given the interval in number of e-folds of inflation where the transition occurs.

both its real and imaginary parts oscillate about the origin with an exponentially damped amplitude for $T > T_c$. When the temperature decreases below the critical value, a tachyonic instability develops driving the radial field toward the broken-symmetry minimum, about which it oscillates thereafter. It is during the instability phase when the field develops a nontrivial expectation value that the axion field becomes a dynamical degree of freedom. This simple picture is, however, incomplete, since fluctuation-dissipation effects are crucial in determining the field value at the onset of the instability, as we will now analyze in detail.

A. Fluctuation-dissipation effects

The coupling between the complex field and the radiation bath generates, in addition to the thermal mass correction, dissipative and thermal noise terms in the equations of motion, much like the analogous effects giving rise to warm inflation itself. The former causes the transfer of energy from the scalar fields σ_R and σ_I to the thermal bath, while the latter prevents the amplitude of the fields from decreasing arbitrarily as a consequence of the Hubble friction. The thermal fluctuations acting on the fields drive their evolution, much like the stochastic collisions of water molecules with a large static pollen particle generate Brownian motion.

Since the energy density of the PQ field is much smaller than that of the inflaton field, $\rho_S \ll \rho_\phi$, we may generically neglect the radiation produced by the PQ field through dissipative effects, with the temperature of the thermal bath being essentially controlled by the inflaton field as in Eq. (5).

Adding the fluctuation-dissipation terms to the equations of motion yields effective Langevin equations for the real and imaginary parts of the PQ field, given by (see, e.g., [17])

$$\ddot{\sigma}_i + (3H + \Gamma)\dot{\sigma}_i + \lambda\sigma_i(\sigma^2 - f_a^2) + \alpha^2 T^2 \sigma_i = \xi, \quad (12)$$

with $i = R, I$, and where Γ is the dissipation coefficient and ξ is the stochastic thermal noise, both associated with the coupling between the PQ field and radiation bath. Note that the noise terms for the real and imaginary parts of the PQ field are independent random variables, but since they have analogous statistical properties we denote them both by ξ . For simplicity, we consider only a generic Yukawa coupling between the PQ field and fermions in the radiation bath of the form

$$\mathcal{L}_{\text{Yukawa}} = -gS \sum_i \bar{\psi}_i \psi'_i, \quad (13)$$

where ψ_i and ψ'_i denote generic light fermions in the thermal bath. The specific PQ charges of these fermions will not be relevant to our study of the phase transition and will be model dependent (see, e.g., the Kim-Shifman-Vainshtein-Zakharov axion model [34,35]). In this case, we have $\alpha^2 \equiv g^2 N_F / 6$, where N_F is the number of fermion species. The dissipation coefficient for a scalar field oscillating about the minimum of its potential coincides with its finite-temperature decay width [22], which in this case is well approximated by [36]

$$\Gamma(T) \simeq \frac{3\alpha^2 m_\sigma^2}{16\pi T} \quad (14)$$

for momentum modes below the temperature and $m_\sigma = \alpha(T - T_c) \lesssim T$. Note that this is valid only while the fields are oscillating about the symmetric minimum, $\langle \sigma_R \rangle = \langle \sigma_I \rangle = 0$, for $T > T_c$. Comparing this dissipation rate with the Hubble friction yields

$$\frac{\Gamma}{H} \simeq \frac{3\lambda\alpha^2}{16\pi} \left(\frac{f_a}{T} \right) \left(\frac{f_a}{H} \right) \left[\left(\frac{T}{T_c} \right)^2 - 1 \right], \quad (15)$$

where the field mass is evaluated at $\sigma = 0$. Although $f_a \gg H$, since $T \sim T_c \sim f_a$ and $\lambda, \alpha < 1$, we find that $\Gamma \lesssim H$ in most of the relevant parameter space.

We assume a Gaussian white noise term, which is a good approximation for momentum modes below the temperature of the thermal bath (see, e.g., [28,37]), with a variance given by the fluctuation-dissipation relation (see, e.g., [16,17])

$$\langle \xi_{\mathbf{k}}(t_1) \xi_{\mathbf{k}'}(t_2) \rangle = 2\Gamma(T)T \frac{(2\pi)^3}{a^3} \delta(\mathbf{k} + \mathbf{k}') \delta(t_1 - t_2), \quad (16)$$

and $\langle \xi_k(t) \rangle = 0$, where the angle brackets represent a thermal ensemble average. Again, we stress that the dynamics of σ_R and σ_I is driven by independent noise terms, but their common decay width implies that the above fluctuation-dissipation relation is identical for both field components.

B. Perturbation evolution

While Hubble expansion drives the PQ field toward the minimum at the origin for $T > T_c$, the thermal noise induces thermal field fluctuations with a nonzero variance that affect the subsequent tachyonic instability. We may then expand the fields σ_R and σ_I about the symmetric minimum to yield the equation of motion for linear field perturbations about the origin, in momentum space,

$$\ddot{\sigma}_k + (3H + \Gamma)\dot{\sigma}_k + \left(\frac{k^2}{a^2} + m_\sigma^2 \right) \sigma_k = \xi_k, \quad (17)$$

where for simplicity we have dropped the index i since the equations for the mode perturbations of σ_R and σ_I are formally identical. Our goal is to solve this equation and evaluate the full field variance on superhorizon scales, which will yield the typical oscillation amplitude of the radial field at the onset of the phase transition.

As a second order differential equation, Eq. (17) admits two homogeneous solutions, $\sigma_k^{\text{hom}}(t, k) = a_k Y_{1k}(t) + b_k Y_{2k}(t)$, where a_k and b_k are coefficients depending on initial conditions. An analytical approximation to these can be found using the WKB method. In the regime where $\dot{H}/H^2 \ll 1$ and $\dot{\Gamma}/(H\Gamma) \ll 1$, one may write

$$\sigma_k^{\text{hom}}(t) = \exp(-(3H + \Gamma)t/2) u_k(t), \quad (18)$$

so that Eq. (17) becomes

$$H^2 u_k'' + \left(\frac{k^2}{a^2} + m_\sigma^2 - \frac{(3H + \Gamma)^2}{4} \right) u_k \simeq 0, \quad (19)$$

where we changed the integration variable to the number of e-folds, $dN_e = H dt$, $u' = du/dN_e$. Let us denote the terms inside the brackets by $f_k(N_e)$. Since $H \ll m_\sigma$ and H is a slowly varying function, u_k admits a power-series solution,

$$u_k = \exp \left(\frac{1}{H} \sum_{n=0}^{\infty} H^n S_n \right). \quad (20)$$

We may then keep only the first two terms of the series and, reverting to the time variable ($t \simeq H^{-1} N_e$), the two independent homogeneous solutions are

$$Y_{1k}(t) \simeq \frac{A e^{-(3H+\Gamma)t/2}}{f(t)^{1/4}} \cos \left(\int_{T_*}^t dt' f_k(t')^{1/2} \right), \quad (21)$$

$$Y_{2k}(t) \simeq \frac{B e^{-(3H+\Gamma)t/2}}{f(t)^{1/4}} \sin \left(\int_{T_*}^t dt' f_k(t')^{1/2} \right), \quad (22)$$

whose amplitude is also exponentially suppressed. The inhomogeneous solution resulting from the noise term ξ can be determined using Green's function method,

$$\sigma_k^{\text{inh}}(t, k) = \int_0^t d\bar{t} G_k(\bar{t}, t) \xi_k(\bar{t}), \quad (23)$$

where $G_k(\bar{t}, t)$ is the kernel function,

$$G_k(\bar{t}, t) = \left[\frac{Y_{1k}(\bar{t})Y_{2k}(t) - Y_{2k}(\bar{t})Y_{1k}(t)}{W(Y_{1k}(\bar{t}), Y_{2k}(\bar{t}))} \right], \quad (24)$$

which vanishes for $\bar{t} = t$. $W(Y_{1k}(t), Y_{2k}(t))$ is the determinant of the Wronskian matrix,

$$W(Y_{1k}(t), Y_{2k}(t)) = Y_{1k}(t)\partial_t Y_{2k}(t) - Y_{2k}(t)\partial_t Y_{1k}(t). \quad (25)$$

The structure of (24) guarantees independence from initial conditions, and so one can compute $Y_{1k}(t)$ and $Y_{2k}(t)$ numerically using random initial conditions and then plug these into Eq. (23) to find the inhomogeneous solution. Alternatively, one can find approximate analytical solutions using the WKB approximation above.

Note that, by definition, the inhomogeneous solution does not decay with expansion [36]. This can be seen by using the decomposition (18) in Eqs. (23) and (24),

$$\sigma_k^{\text{inh}}(t, k) = e^{-(3H+\Gamma)t/2} \int_0^t d\bar{t} e^{(3H+\Gamma)\bar{t}/2} F_k(\bar{t}, t) \xi_k(\bar{t}), \quad (26)$$

where the exponential factors cancel after the integration and $F_k(\bar{t}, t)$ is defined as

$$F_k(\bar{t}, t) = \left[\frac{u_{1k}(\bar{t})u_{2k}(t) - u_{2k}(\bar{t})u_{1k}(t)}{W(u_{1k}(\bar{t}), u_{2k}(\bar{t}))} \right]. \quad (27)$$

As a result, σ_k^{inh} becomes the dominant contribution to the perturbations for $(3H + \Gamma)t \gg 1$. The combined effect of the superhorizon perturbations will then determine the evolution of the average field $\bar{\sigma}$. For this reason, we shall denote the inhomogeneous solution simply as $\sigma_k(t)$.

C. Thermal field variance

Even if the PQ field has a nonvanishing expectation value at the start of inflation, it will quickly be driven to the origin by the exponentially fast expansion. There is, therefore, a point where the contribution of the superhorizon modes to the average real and imaginary components of the field becomes dominant and the evolution thereafter is described by their combined behavior, which is captured in the two-point correlation function and field variance. With (23), we can determine the former,

$$\begin{aligned} \langle \sigma_k(t) \sigma_{k'}(t') \rangle &= \int_0^t d\bar{t} \int_0^{t'} d\bar{t}' G_k(\bar{t}, t) G_{k'}(\bar{t}', t') \\ &\times \langle \xi_k(\bar{t}) \xi_{k'}(\bar{t}') \rangle. \end{aligned} \quad (28)$$

Using the fluctuation-dissipation relation (16) and selecting $t' = t$, we obtain the equal-time correlator

$$\begin{aligned} \langle \sigma_k(t) \sigma_{k'}(t) \rangle &= (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \int_0^t d\bar{t} G_k(\bar{t}, t)^2 \\ &\times \frac{2\Gamma(T)T}{a^3}. \end{aligned} \quad (29)$$

This can be determined analytically by using Eqs. (21) and (22). The kernel function is

$$G_k(\bar{t}, t) \simeq \frac{e^{-(3H+\Gamma)(t-\bar{t})/2}}{(f_k(t)f_k(\bar{t}))^{1/4}} \sin\left(\int_{\bar{t}}^t dt' f_k(t')^{1/2}\right), \quad (30)$$

so that the two-point function becomes

$$\begin{aligned} \langle \sigma_k(t) \sigma_{k'}(t) \rangle &\simeq \delta(k) \frac{e^{-\Gamma t}}{f_k(t)^{1/2}} \int_0^t d\bar{t} \frac{2\Gamma(\bar{t})T(\bar{t})}{f_k(\bar{t})^{1/2}} \\ &\times e^{\Gamma \bar{t}} \sin^2\left(\int_{\bar{t}}^t dt' f_k(t')^{1/2}\right), \end{aligned} \quad (31)$$

where $\delta(k)$ is defined as

$$\delta(k) = \frac{(2\pi)^3}{a(t)^3} \delta(\mathbf{k} + \mathbf{k}'). \quad (32)$$

Since we are interested in the contribution of superhorizon modes, $k < aH$, and $H \ll m_\sigma$, $f_k(t)$ can be approximated as

$$f_k(t) \equiv \frac{k^2}{a^2} + m_\sigma^2 - \frac{(3H + \Gamma)^2}{4} \simeq m_\sigma^2, \quad (33)$$

such that

$$\begin{aligned} \langle \sigma_k(t) \sigma_{k'}(t) \rangle &\simeq \frac{3\alpha^2}{16\pi} \delta(k) \frac{e^{-\Gamma t}}{m_\sigma(t)} \int_0^t d\bar{t} m_\sigma(\bar{t}) e^{\Gamma \bar{t}} \\ &\times \left(1 - \cos\left(2 \int_{\bar{t}}^t dt' m_\sigma(t')\right) \right), \end{aligned} \quad (34)$$

where we used (14) and $\sin^2(x) = (1 - \cos(2x))/2$. Noting again that $H \ll m_\sigma$, it follows that the argument of the trigonometric function in the integral is rapidly oscillating, so that this contribution has a negligible average value. In the limit where $\dot{m}_\sigma/(Hm_\sigma) \ll 1$,

$$\langle \sigma_k(t) \sigma_{k'}(t) \rangle \simeq \frac{(2\pi)^3}{a(t)^3} \delta(\mathbf{k} + \mathbf{k}') \frac{T}{m_\sigma(t)^2} (1 - e^{-\Gamma t}), \quad (35)$$

where we used (14). This has mass dimension -4 , as expected.

We can now compute the variance of the field on superhorizon scales (the “classical” field component) by integrating over the contribution of all superhorizon modes,

$$\langle \sigma_c^2 \rangle = \int \frac{d^3 k d^3 k'}{(2\pi)^6} \langle \sigma_k(t) \sigma_{k'}(t) \rangle e^{i(\mathbf{k}+\mathbf{k}') \cdot \mathbf{x}}, \quad (36)$$

and using the result (35) we then obtain

$$\langle \sigma_c^2 \rangle \simeq \frac{(1 - e^{-\Gamma t})}{(2\pi)^3 a(t)^3} \frac{T}{m_\sigma(t)^2} \int d^3 k. \quad (37)$$

The comoving momentum integral in (37) is given by

$$\int d^3 k = \frac{4\pi}{3} (a(t)H)^3 \left(1 - \left(\frac{a_0}{a(t)} \right)^3 \right), \quad (38)$$

so that the variance (37) becomes

$$\langle \sigma_c^2 \rangle \simeq \frac{1}{6\pi^2} \frac{TH^3}{m_\sigma^2} (1 - e^{-\Gamma t}), \quad (39)$$

where we have neglected $(a_0/a)^3$, valid for $Ht \gg 1$, i.e., after only a few e-folds of inflation.

For $\Gamma t \ll 1$, this can be approximated as

$$\langle \sigma_c^2 \rangle \simeq \frac{\alpha^2}{32\pi^3} H^3 t \simeq \frac{\alpha^2}{32\pi^3} H^2 N_e, \quad (40)$$

where we have used (14). The growing variance results from the fact that the number of superhorizon modes is increasing while their individual amplitude is not yet suppressed. This is very similar to the variance of a light quantum field in a de Sitter stage [38], although fluctuations are thermal in nature.

For $\Gamma t \gtrsim 1$, the variance can be approximated by

$$\langle \sigma_c^2 \rangle \simeq \frac{1}{6\pi^2} \frac{TH^3}{m_\sigma^2}, \quad (41)$$

becoming approximately constant because the growing number of superhorizon modes cannot overcome the decaying amplitude of each mode. This signals that the PQ field reaches thermal equilibrium with the ambient heat bath upon decaying significantly, as displayed by the temperature dependence and the absence of an explicit time dependence (i.e., the process became memoryless). However, this is only attained for $(\Gamma/H)N_e \gg 1$, which may require a substantial number of e-folds of inflation before the phase transition as typically $\Gamma/H \ll 1$.

The typical value of the field amplitude prior to the tachyonic instability is therefore given by the square root of the variance (39),

$$\sqrt{\langle \sigma_c^2 \rangle} \simeq \frac{1}{\sqrt{6\pi}} H \left(\frac{T}{H} \right)^{1/2} \frac{H}{m_\sigma} (1 - e^{-\Gamma t})^{1/2}. \quad (42)$$

We note that, although this approximate expression diverges at the critical temperature where $m_\sigma(\sigma = 0) = 0$, this is only

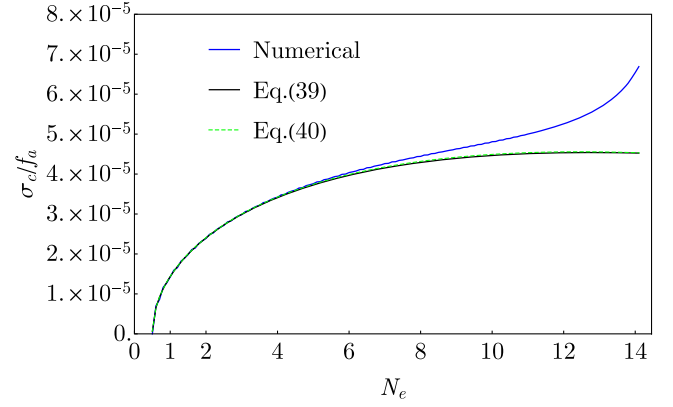


FIG. 3. Comparison between the numerical result for the square root of the variance (36) determined with the numerical (blue) and the analytical approximations (39) (black) and (40) (dashed green). The difference between the numerical and analytical results increases as $T \rightarrow T_c$ and $\dot{m}_\sigma/(Hm_\sigma) \simeq 1$. The microphysical parameters are $f_a = 1.2 \times 10^{15}$ GeV, self-coupling $\lambda = 0.01$, and effective coupling to the thermal bath $\alpha = 0.2$, such that T_c is reached at $N_e \simeq 15$.

due to the approximation used in Eq. (33), while the numerically evaluated value is finite, as can be seen in Fig. 3.

Superhorizon thermal field fluctuations thus play an important role in the dynamics of the phase transition, since the PQ field never becomes too localized at the symmetric minimum and, as we will see, this will determine the duration of the subsequent period of tachyonic instability, i.e., how long the field takes to settle at the symmetry-breaking minimum.

We recall that the above results are valid for both the real and imaginary components of the PQ field, which exhibit independent thermal fluctuations about the origin, albeit with the same average amplitude, given that they interact in the same way with the fields in the thermal bath. The radial field variance is thus $\langle \sigma^2 \rangle = \langle \sigma_R^2 \rangle + \langle \sigma_I^2 \rangle = 2\langle \sigma_R^2 \rangle$.

We also stress that thermal field fluctuations at fixed comoving scale k do not freeze out upon becoming superhorizon and are, in fact, damped by expansion. The constant or linearly growing variance is, as mentioned above, only a consequence of the growing number of modes that become superhorizon as inflation proceeds. This is important since independent fluctuations in σ_R and σ_I lead to fluctuations in the phase θ of the complex PQ field that later becomes the axion. These fluctuations are, however, damped by inflationary expansion until the critical temperature is reached, so that in this scenario there are no significant θ fluctuations on the CMB scales that leave the horizon well before the phase transition.

D. Tachyonic instability

Once the minimum of the scalar potential becomes nontrivial for $T < T_c$, the dissipation coefficient is no longer given by its finite-temperature decay width, which

vanishes at T_c and becomes ill defined in the tachyonic phase. However, in the latter period, the field dynamics will mainly be driven by the negative sign of the field's effective squared mass, so we do not expect fluctuation-dissipation effects to play an important role.

We will, instead, take the superhorizon rms field amplitude computed above at the onset of the phase transition as the initial condition for the evolution of the homogeneous field component, discarding any subsequent fluctuation-dissipation effects. We are interested in determining, in particular, the point at which the field starts oscillating about the nontrivial minimum, so that the PQ symmetry is effectively broken and the angular field θ corresponding to the axion becomes an independent dynamical degree of freedom.

The relevant equation of motion for the homogeneous radial field component, including all superhorizon modes, is then given by

$$\ddot{\sigma} + 3H\dot{\sigma} + \lambda\sigma^3 + \lambda f_a^2 \left(\frac{T^2}{T_c^2} - 1 \right) \sigma = 0. \quad (43)$$

We noted above that, for $\sigma < (T/T_c)f_a$, the thermal mass contribution dominates. As (42) satisfies this condition, the quartic term in the potential remains negligible while the field oscillates about the origin due to the quadratic term. For $T < T_c$, a tachyonic instability develops and the field amplitude σ grows exponentially with time. Eventually, σ becomes large enough for the quartic term to shut down the instability: the radial field executes damped oscillations about $\sigma = f_a \sqrt{1 - (T/T_c)^2}$ thereafter. This is illustrated in the numerical example in Fig. 4.

It is possible to obtain an approximate analytical solution during the tachyonic regime by carrying out the following procedure. First, we neglect the quartic contribution to the potential. Second, taking into account that $\dot{H}/H^2 \ll 1$, we may change variables via $\sigma = \exp(-3Ht/2)u$,

$$\ddot{u} + \left(\lambda f_a^2 \left(\frac{T^2}{T_c^2} - 1 \right) - \frac{9}{4} H^2 \right) u \simeq 0. \quad (44)$$

Third, we may also change the integration variable to the number of e-folds, $dN_e = Hdt$, so that

$$u'' + \left(\frac{\lambda f_a^2}{H^2} \left(\frac{T^2}{T_c^2} - 1 \right) - \frac{9}{4} \right) u \simeq 0. \quad (45)$$

Finally, since T varies slowly during warm inflation, we may expand T^2 around T_c^2 as a function of the number of e-folds,

$$\frac{T^2}{T_c^2} \simeq 1 - \gamma(N_e - N_e^c), \quad (46)$$

where $\gamma > 0$ and N_e^c denotes the e-fold at which $T/T_c = 1$. Plugging T^2/T_c^2 into (45),

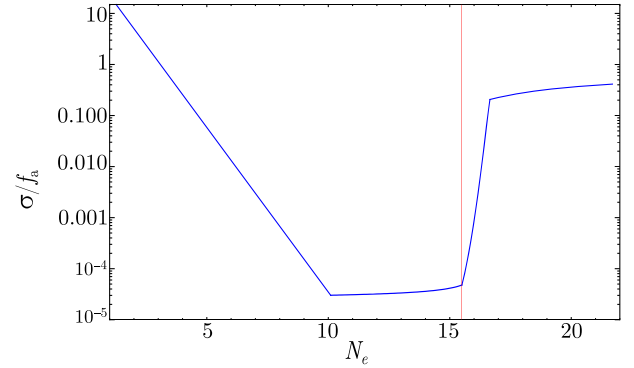


FIG. 4. Numerical evolution of the Peccei-Quinn field's average amplitude during warm inflation in a representative example with $f_a = 1.2 \times 10^{15}$ GeV, self-coupling $\lambda = 0.01$, and effective coupling to the thermal bath $\alpha = 0.2$. The initial amplitude of the field is $\simeq 10f_a$, such that $V(\sigma_i)/V(\phi) \sim 10^{-3}$. The field first oscillates about the origin with an exponentially decreasing amplitude, but its amplitude cannot decrease arbitrarily due to its thermal fluctuations, which dominate the amplitude from around $N_e \simeq 10$ e-folds in this example until the critical temperature is reached around $N_e \simeq 15$ e-folds (vertical red line). At this point, the tachyonic instability drives the field away from the origin and toward the temperature-dependent symmetry-breaking minimum, about which the field begins oscillating at $N_e^c \simeq 17$ e-folds.

$$u'' - \left(\frac{\lambda \gamma f_a^2}{H^2} (N_e - N_e^c) + \frac{9}{4} \right) u \simeq 0. \quad (47)$$

Shifting the integration variable, $x \equiv N_e - N_e^c$,

$$\frac{d^2 u}{dx^2} - \left(qx + \frac{9}{4} \right) u \simeq 0, \quad (48)$$

which is simply an Airy equation with $q \equiv \lambda \gamma f_a^2 / H^2 > 0$, with solutions

$$u(x) = AAi \left[q^{-2/3} \left(qx + \frac{9}{4} \right) \right] + BBi \left[q^{-2/3} \left(qx + \frac{9}{4} \right) \right], \quad (49)$$

where $Ai(z)$ and $Bi(z)$ are the Airy functions of the first and second kind, respectively, exponentially decreasing and exponentially increasing for $x > 0$. For $x < 0$, the two functions oscillate with varying frequency and decreasing amplitude as $x \rightarrow -\infty$. The constants A and B are fixed by the value of (σ, σ') at $N_e \simeq N_e^c$. After doing so, we can neglect the exponentially decreasing term, yielding

$$\sigma \simeq B e^{-3x/2} Bi \left[q^{-2/3} \left(qx + \frac{9}{4} \right) \right]. \quad (50)$$

We can now determine the e-fold number at which the homogeneous field “catches up” with the potential minimum

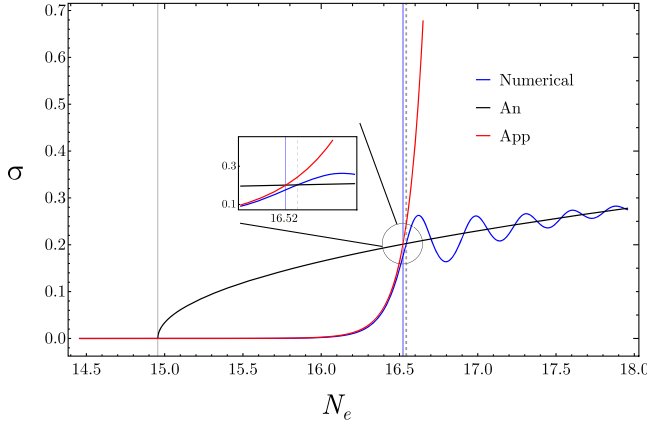


FIG. 5. Comparison between the evolution obtained numerically for the homogeneous PQ field (blue) and the analytical approximation in Eq. (50) (red). The black curve shows the evolution of the minimum of the scalar potential, $f_a \sqrt{1 - (T/T_c)^2}$. The leftmost black vertical line represents $T = T_c$, at $N_e \simeq 15$. The blue (black) vertical (dashed) line represents the e-fold at which the analytical (numerical) result matches with the classical solution, Eq. (51). Their relative difference is about 0.1% (see inset for enlargement). The instability period lasts $x_a \simeq 1.5$ e-folds for the values of Fig. 4.

$f_a \sqrt{1 - (T/T_c)^2}$, N_e^a (see Fig. 5). Using (46) and (50),

$$f_a \sqrt{\gamma x_a} \simeq B e^{-3x_a/2} \text{Bi} \left[q^{-2/3} \left(q x_a + \frac{9}{4} \right) \right], \quad (51)$$

where $x_a \equiv N_e^a - N_e^c$. This equation determines the duration of the instability period x_a , defining how long it takes for the radial PQ field to catch up with its new minimum. We have also checked that this coincides approximately with the point at which the evolution of the radial field becomes once more adiabatic on Hubble timescales. From this we infer that x_a exhibits an approximately logarithmic dependence on $B/f_a \sim \sqrt{\langle \sigma_c^2 \rangle / f_a}$, with a lower field variance at the critical point yielding a longer instability period, as expected. Having a sufficiently accurate determination of the duration of this instability period is essential to ascertain when symmetry breaking effectively takes place and the axion becomes an independent dynamical degree of freedom. In our numerical simulations, as shown in the example in Fig. 5, we observed generically that, at $N_e = N_e^a$, the minimum of the scalar potential lies at $\sigma/f_a = \mathcal{O}(0.1)$, with the tachyonic period lasting for $x_a \sim 1$ –2 e-folds.

It is interesting to contrast this phase transition with its equivalent during the radiation-dominated era after inflation where $a(t) \propto t^{1/2}$ and $H = 1/2t$, rather than $H \simeq \text{const}$. In this case, the solutions of (43) in the unstable phase are given by Bessel functions $J_m(-it)$ and $Y_m(-it)$, where m is a mass parameter that can be read from the equation of motion. Since, however, $a(t) \propto t^{1/2}$ and $N_e = \log(a/a_0) \propto \log(t)$, the elapsed time is exponential with the number of e-folds.

Therefore, the unstable solution in the radiation epoch grows as an exponential of an exponential of N_e , rapidly catching up with the minimum of the potential at $\sigma/f_a = \sqrt{1 - (T/T_c)^2}$. As a result, $N_e^a \simeq N_e^c$ and the radial field reaches the minimum of the potential for $\sigma \ll f_a$, leading to much larger fluctuations in the phase of the field than in warm inflation as we discuss below.

After the instability, the radial field executes damped oscillations about the symmetry-breaking minimum, since its mass is much larger than the Hubble friction. As, however, $m_\sigma < T$, dissipation might still play a role on the evolution of the radial field. Describing this behavior is, however, beyond the scope of this work.

E. Axion isocurvature spectrum

The nontrivial duration of the tachyonic instability period during warm inflation has an important consequence, since the axion field is only truly “born” after the radial field settles at the symmetry-breaking minimum. In particular, during the instability period, which as we have seen lasts for 1–2 e-folds, the Hubble-averaged value of the radial field varies in a nonadiabatic way. Since this determines the normalization of the angular kinetic term, the dynamics of θ and σ is only effectively decoupled after N_e^a , when $\sigma/f_a = \sqrt{1 - (T/T_c)^2}$.

In standard scenarios, the θ field will not interact significantly with the thermal bath, but it will nevertheless exhibit quasi-de Sitter quantum fluctuations, as generic for any light scalar field during inflation [4],

$$\delta\theta = \frac{H}{2\pi f_a \sqrt{1 - (T/T_c)^2}}. \quad (52)$$

We stress that this is only valid after the radial field has reached the broken-symmetry minimum, so that the divergence of this expression at the critical temperature does not really yield arbitrarily large θ fluctuations. As mentioned above, we find, in general, that $\sqrt{1 - (T/T_c)^2} \gtrsim 0.1$ in this regime, with the largest fluctuations corresponding to scales that become superhorizon around N_e^a . Since both the Hubble parameter and the temperature are slowly decreasing functions of the number of e-folds, $\delta\theta$ acquires a small-scale dependence. This will then result in a nearly scale-invariant spectrum of isocurvature density perturbations in the dark matter, once the axion field begins oscillating about its QCD-instanton-induced potential in the radiation era.

In order to determine whether these perturbations are significant, one needs to compute, for a given value of f_a ,³ the value of the initial misalignment angle θ_i for which the axion field accounts for the cosmological density of dark

³This yields a QCD axion mass $m_a \simeq 6 \times 10^{-6} / (f_a / 10^{12} \text{ GeV}) \text{ eV}$ [4], with $m_a \simeq 5 \times 10^{-9} \text{ eV}$ in the example above.

matter.⁴ For the QCD axion, this is given by [7]

$$\Omega_A h^2 = 0.7 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \left(\frac{\theta_i}{\pi} \right)^2. \quad (53)$$

Using $\Omega_{\text{CDM}} h^2 = 0.12$ [39] yields

$$\theta_i \simeq 0.02 \left(\frac{f_a}{10^{15} \text{ GeV}} \right)^{-7/12}, \quad (54)$$

such that $\delta\theta_{\text{max}}/\theta_i \propto f_a^{-5/12}$ is given by the ratio of (52) and (54) evaluated at $N_e = N_e^a$. For the parameters of the representative example above, $\theta_i \simeq 0.02$ and $\delta\theta_{\text{max}}/\theta_i \simeq 0.06$. Since the latter value is small, the contribution of angular perturbations to the homogeneous field amplitude θ_i can be safely neglected.

We note that, in addition to the inverse dependence on the axion decay constant given explicitly in Eq. (52), the factor $\sqrt{1 - (T/T_c)^2}$ also depends on f_a and on the couplings λ and α , albeit in a nontrivial way, since these parameters affect the duration of the instability period x_a according to (51). We have found numerically that $\delta\rho_a/\rho_a = 2\delta\theta/\theta \sim \mathcal{O}(10^{-2}-10^{-1})$ for small-scale axion isocurvature perturbations.

We illustrate in Fig. 6 the spectrum of these perturbations as a function of the comoving wave number and the corresponding number of e-folds at which they become superhorizon, with $N_e = 0$ denoting the point at which the CMB pivot scale exits the horizon during warm inflation. We note that the axion field remains massless until around the time of the QCD phase transition, when QCD instantons generate its periodic potential. Hence, fluctuations in the θ field only generate dark matter fluctuations when the temperature falls below ~ 1 GeV in the radiation-dominated epoch. Fluctuations that reenter the horizon prior to this will be smoothed out by expansion, so that our scenario only predicts significant axion isocurvature perturbations in a range of scales, the largest scale corresponding to the scale that exits the horizon during inflation when symmetry breaking effectively occurs at $N_e = N_e^a$ and the smallest scale to the one reentering the horizon at the onset of axion field oscillations. In this range of scales, the spectrum is nearly scale invariant, which differs significantly from the white noise spectrum associated with PQ symmetry breaking after inflation (see, e.g., [40]).

This will likely affect the formation of gravitationally bound axion structures like miniclusters and their resulting mass spectrum in a nontrivial way, particularly since the scenario of PQ symmetry breaking during warm inflation

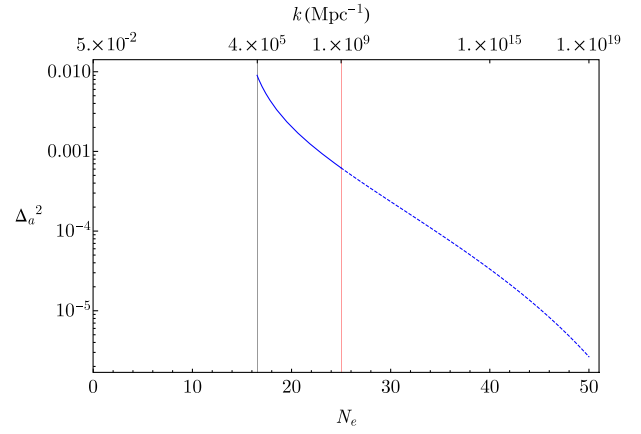


FIG. 6. Axion density fluctuation dimensionless power spectrum $\Delta_a^2 \equiv 4(\delta\theta/\theta_i)^2$ for the example shown in Fig. 4. The vertical black and red lines correspond to the scale that leaves the horizon during inflation at N_e^a (when the radial PQ field reaches the broken-symmetry minimum) and the scale that reenters the horizon at the time of the first axion oscillations in the radiation epoch, at $T \simeq 1$ GeV. Perturbations on scales beyond the red line (dashed curve) become subhorizon before the first axion oscillations and are, hence, damped away.

generates axion density fluctuations on superhorizon scales, as opposed to a phase transition occurring after inflation has ended. A detailed analysis of axion structure formation processes is, however, beyond the goals of this work and will be left for a dedicated future study. Nevertheless, it is clear that a PQ symmetry breaking occurring during warm inflation may have a nontrivial observational impact that may allow one to probe this hypothesis.

We must note that PQ field fluctuations on scales exiting the horizon around T_c may be significant, since they are not damped before the onset of the tachyonic instability, although they cannot be described as independent axion isocurvature fluctuations using Eq. (52). They nevertheless lead to fluctuations in the phase of the PQ field, since its real and imaginary parts fluctuate independently, on the scale of the horizon when the phase transition occurs. If significant, these fluctuations would be incompatible with the assumption of a single initial phase θ_i within the whole observable Universe, potentially leading to an overproduction of axion dark matter.

This may be overcome if thermal fluctuations are small compared to the classical PQ field value, $\sigma_c = \sigma_* e^{-\frac{3}{2}N_e}$ when the critical temperature is reached. Using the thermal field variance computed in Eq. (39) and the evolution of the Hubble parameter in Eq. (6), this yields the following upper bound on the number of e-folds of inflation at which the phase transition occurs (after horizon crossing of CMB scales):

$$N_e^c \lesssim 12 - \frac{2}{3} \left[\log \alpha + \log \left(\frac{H_*}{10^{12} \text{ GeV}} \right) - \log \left(\frac{\sigma_*}{M_p} \right) \right]. \quad (55)$$

⁴While in [14] the inflaton could account for all the dark matter, here we consider an alternative point of view where the axion accounts for most of the dark matter and the inflaton only to a subdominant component, which would correspond to, e.g., lower values of the inflaton mass compared to those considered in [14].

While this may seem somewhat restrictive, it is nevertheless easily compatible with the assumption that PQ symmetry breaking occurs after all observable CMB scales exit the horizon during warm inflation, provided that σ_* is sufficiently large. Note also that, in scenarios where the PQ scalar is very weakly coupled to the thermal bath, $\alpha \ll 1$, the critical temperature may be attained quite late in the inflationary evolution.

In scenarios where thermal fluctuations come to dominate over the classical expectation value above T_c , an overabundant axion field may nevertheless be diluted away, e.g., by late-time entropy production [41–43]. Moreover, topological defects like domain walls, which may form as a result of large fluctuations in the PQ field phase around T_c , are inflated away if the PQ symmetry breaking occurs well before the end of inflation as in the example given.

It is worth emphasizing that PQ field fluctuations are not an intrinsic feature of warm inflation only and will have a similar impact in the cold inflation scenario discussed by Linde in [4]. Fluctuations would, of course, be quantum rather than thermal in nature and suppressed but nevertheless finite for $m_\sigma \gg H$ (see, e.g., [44]). Hence, also in these scenarios there will be an upper bound on the number of e-folds at which spontaneous symmetry breaking occurs to avoid axion overproduction.

IV. SUMMARY AND CONCLUSIONS

In this work, we have studied a scenario where a thermal PQ phase transition, leading to the spontaneous breaking of the global U(1) symmetry proposed to solve the strong CP problem, occurs during warm inflation. This shares several similarities with the scenario proposed in [4] in cold inflation where the PQ field is coupled to the inflaton field itself and the U(1) PQ symmetry is broken below a critical inflaton field value. In warm inflation, however, the external parameter is the (slowly evolving) temperature of the thermal bath sourced by the inflaton field through dissipative effects. In several implementations of the PQ mechanism, some of the Standard Model fields are charged under the U(1) symmetry and, provided that these are excited in the warm thermal bath (like the Higgs and lepton fields in the concrete scenario developed in [14] that we have chosen as our working example), the complex field breaking the PQ symmetry naturally acquires thermal mass corrections that restore the symmetry above a critical temperature.

Since the temperature during warm inflation typically varies in the range 10^{14} – 10^{15} GeV and the critical temperature is parametrically close to the symmetry-breaking scale, a thermal crossover will naturally occur during the last 50–60 e-folds of warm inflation for values of the axion decay constant not far from the grand unification scale. This is arguably more natural than the cold inflation scenario of [4], where the coupling between the PQ and the inflaton fields must be tuned to yield a critical field value just below

or even above the Planck scale as typical of slow-roll inflation models. Irrespective of these “naturalness” considerations, warm inflation certainly offers an alternative way to break the Peccei-Quinn symmetry during the last 50–60 e-folds of inflation, particularly after the large CMB scales have become superhorizon.

Our analysis took into account not only the thermal backreaction on the PQ scalar potential but also fluctuation-dissipation effects that naturally result from the coupling between the PQ field and the light degrees of freedom in the thermal bath. We have shown that, although dissipative friction is generically subleading compared to Hubble expansion in terms of damping the field’s oscillations about the symmetric minimum, the associated thermal fluctuations prevent the field amplitude from becoming arbitrarily small. It is then the variance of the radial PQ field at the critical point that determines its subsequent evolution. We have computed this variance both numerically and through approximate analytical methods and used it to determine the duration of the period of tachyonic instability driving the field to the symmetry-breaking minimum, showing that it typically takes 1–2 e-folds of accelerated expansion. This means that the temperature of the thermal bath when the field reaches the minimum is already slightly below the critical temperature, as opposed to what happens in a postinflationary phase transition (crossover) where the tachyonic phase occurs essentially at the critical temperature, which results in large angular fluctuations as given by Eq. (52) for $T \simeq T_c$.

An important consequence of the non-negligible expansion of the Universe during the tachyonic phase is the fact that the axion field, i.e., the phase of the PQ field, only becomes a truly dynamical independent scalar field at a temperature slightly below the critical value. This limits the amplitude of its quasi-de Sitter quantum fluctuations on superhorizon scales. Moreover, if the phase transition only occurs after the relevant CMB scales have left the horizon, axion isocurvature perturbations are only generated on small scales. Thermal fluctuations in the complex PQ field on scales exiting the horizon above the critical temperature are, in our scenario, damped by the inflationary expansion. The generic prediction is then a nearly scale-invariant spectrum of isocurvature perturbations in an interval of scales which (i) leave the horizon during warm inflation after the field reaches the nontrivial minimum ($N_e > N_e^a$) and (ii) reenter the horizon during the radiation era after the onset of axion oscillations ($T \lesssim 1$ GeV). In contrast, if a thermal PQ phase transition only occurs after inflation, no superhorizon fluctuations are generated and the temperature remains essentially constant during the tachyonic instability period, making the initial seeds for gravitationally bound structures in axion dark matter very different in these two scenarios, with potentially observable consequences.

The formalism that we have employed does not allow for a rigorous description of the axion isocurvature spectrum

for scales that leave the horizon during the period of tachyonic instability, where radial and angular fluctuations in the complex S field are not independent. While these may be significant and potentially lead to axion topological defects, they are inflated away if symmetry breaking occurs well before the end of inflation.

Although our study was focused on the QCD axion and the underlying PQ model, our analysis holds for other axionlike fields. In fact, it applies to any crossoverlike phase transition driven by a complex scalar field that is coupled to the thermal bath during warm inflation, being the first analysis of this kind⁵ (to the best of our knowledge). We hope that our results motivate further exploration of this topic.

⁵See also [45,46] for applications of fluctuation-dissipation dynamics to cosmological phase transitions.

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