

Trimaximal mixing patterns meet the first JUNO result

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The Jiangmen Underground Neutrino Observatory (JUNO) experiment has recently released its first measurement results based on 59.1 days of data, achieving unprecedented precision in measuring the lepton mixing angle θ_{12} . This significant improvement places stringent constraints on certain neutrino mass models and flavor mixing patterns. In this work, we examine the impact of the latest JUNO results on the two trimaximal (i.e., TM1 and TM2) mixing patterns. They are two well-motivated variants of the tribimaximal mixing pattern and predict specific correlations between θ_{12} and θ_{13} . After taking into account the first JUNO results, the TM1 mixing pattern sits on the edge of the experimentally allowed 1σ region, while the TM2 mixing pattern lies outside the 3σ region. To reconcile these TM mixing patterns with the latest experimental data, we further investigate the renormalization group (RG) running effects on them in the both Majorana and Dirac neutrino cases. Our analytical and numerical results show that RG corrections can bring the two TM mixing patterns into excellent agreement with the latest JUNO data if neutrino masses are quasidegenerate. However, the Majorana case faces severe constraints from neutrinoless double beta decay limits, and particularly, the TM2 mixing pattern with Majorana neutrinos has been essentially ruled out. In the Dirac case, the TM1 mixing pattern is fully consistent with current data including beta decay results, whereas the TM2 pattern is strongly constrained by the KATRIN limit and even could be largely ruled out if the KATRIN experiment reaches its final sensitivity without any discovery. Future high-precision measurements of lepton mixing parameters and absolute neutrino masses in both oscillation and nonoscillation experiments will provide decisive tests of these mixing patterns.

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I. INTRODUCTION

Neutrino oscillation experiments have firmly established that lepton flavors are mixed through two large and one small mixing angles, in stark contrast to the small mixing observed in the quark sector [1,2]. This peculiar pattern of lepton flavor mixing has inspired numerous neutrino mass models based on flavor symmetries, many of which predict simple and appealing constant mixing patterns (see, e.g., reviews [3–12] and references therein). One of them is the tribimaximal (TBM) mixing pattern [13–15]:

$$U_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

predicting $\theta_{23} = 45^\circ$, $\theta_{12} = \arctan(\sqrt{2}/2) = 35.26^\circ$, and $\theta_{13} = 0$ under the standard parametrization [1] of the Pontecorvo-Maki-Nakagawa-Sakata matrix [16–18],

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (2)$$

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TABLE I. Central values of neutrino oscillation parameters together with their 1σ uncertainties from the NuFIT6.0/6.1 [19,37] and the latest JUNO results [35]. $\ell = 1$ is for the normal mass ordering (NMO) with $m_1 < m_2 < m_3$, and $\ell = 2$ for the inverted mass ordering (IMO) with $m_3 < m_1 < m_2$.

		$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$	$\delta/^\circ$	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$
NuFIT6.0	NMO	$0.308^{+0.012}_{-0.011}$	$0.02215^{+0.00056}_{-0.00058}$	$0.470^{+0.017}_{-0.013}$	212^{+26}_{-41}	$7.49^{+0.19}_{-0.19}$	$+2.513^{+0.021}_{-0.019}$
	IMO		$0.02231^{+0.00056}_{-0.00056}$	$0.550^{+0.012}_{-0.015}$	274^{+22}_{-25}		$-2.484^{+0.020}_{-0.020}$
NuFIT6.1	NMO	$0.3088^{+0.0067}_{-0.0066}$	$0.02248^{+0.00055}_{-0.00059}$	$0.470^{+0.017}_{-0.014}$	212^{+26}_{-36}	$7.537^{+0.094}_{-0.10}$	$+2.511^{+0.021}_{-0.020}$
	IMO		$0.02262^{+0.00057}_{-0.00056}$	$0.550^{+0.013}_{-0.016}$	274^{+22}_{-25}		$-2.483^{+0.020}_{-0.020}$
JUNO	NMO	$0.3092^{+0.0087}_{-0.0087}$	...	...	...	$7.50^{+0.12}_{-0.12}$	...
	IMO						

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ (for $ij = 12, 13, 23$), and $P_\nu = \text{Diag}\{e^{i\varphi}, e^{i\sigma}, 1\}$ is the Majorana (or unphysical) phase matrix in the Majorana (or Dirac) neutrino case. The former two predictions are very close to the best-fit values of θ_{23} and θ_{12} and lie within their 3σ ranges [19,20]. This success had motivated extensive studies on flavor-symmetry realizations and phenomenological consequences of the TBM mixing pattern. However, θ_{13} turns out to be not as tiny as initially expected [21–24], and the most precise measurement from the Daya bay experiment gives $\sin^2 2\theta_{13} = 0.0851 \pm 0.0024$ [25], which strongly disfavors the TBM mixing pattern and calls for its modifications or alternatives.

The trimaximal (TM) mixing patterns are the simplest and most popular variants of the TBM pattern. They are obtained by introducing a single complex rotation to the right of U_{TBM} , such that one column of U_{TBM} remains untouched. A rotation in the 1-2 plane leaves θ_{13} vanishing, making it phenomenologically uninteresting. In contrast, the rotations in the 2-3 and 1-3 planes introduced to the right of U_{TBM} are able to generate a nonvanishing θ_{13} , depending on the rotation angle θ , and are thus much more attractive. These two variants are usually referred to as the TM1 [26–28] and TM2 [14,27–32] mixing patterns, respectively, and are explicitly given by

$$U_{\text{TM1}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} & \frac{\sin \theta}{\sqrt{3}} e^{-i\varphi} \\ -\frac{1}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} - \frac{\sin \theta}{\sqrt{2}} e^{i\varphi} & \frac{\cos \theta}{\sqrt{2}} + \frac{\sin \theta}{\sqrt{3}} e^{-i\varphi} \\ \frac{1}{\sqrt{6}} & -\frac{\cos \theta}{\sqrt{3}} - \frac{\sin \theta}{\sqrt{2}} e^{i\varphi} & \frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{3}} e^{-i\varphi} \end{pmatrix}, \quad (3)$$

and

$$U_{\text{TM2}} = \begin{pmatrix} \frac{2\cos \theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2\sin \theta}{\sqrt{6}} e^{-i\varphi} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{\sin \theta}{\sqrt{2}} e^{i\varphi} & \frac{1}{\sqrt{3}} & \frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{6}} e^{-i\varphi} \\ \frac{\cos \theta}{\sqrt{6}} - \frac{\sin \theta}{\sqrt{2}} e^{i\varphi} & -\frac{1}{\sqrt{3}} & \frac{\cos \theta}{\sqrt{2}} + \frac{\sin \theta}{\sqrt{6}} e^{-i\varphi} \end{pmatrix}. \quad (4)$$

The mixing angle θ_{13} in these two trimaximal patterns are determined by the rotation angle θ , namely, $\sin \theta_{13}^{\text{TM1}} = \sin \theta / \sqrt{3}$ and $\sin \theta_{13}^{\text{TM2}} = 2 \sin \theta / \sqrt{6}$. Furthermore, an intriguing correlation between θ_{12} and θ_{13} in each pattern can be easily achieved from Eqs. (3) and (4)¹:

$$\sin^2 \theta_{12}^{\text{TM1}} = \frac{1}{3} - \frac{2}{3} \tan^2 \theta_{13}^{\text{TM1}}, \quad \sin^2 \theta_{12}^{\text{TM2}} = \frac{1}{3} + \frac{1}{3} \tan^2 \theta_{13}^{\text{TM2}}. \quad (5)$$

Such simple relations between mixing angles are particularly striking, as they can be exploited for stringent experimental tests and even potential exclusion of the corresponding mixing patterns and their underlying flavor symmetries, especially now that we are entering the era of subpercent precision in neutrino oscillation measurements [20]. The Jiangmen Underground Neutrino Observatory (JUNO) experiment [34] has very recently released its first measurement based on 59.1 days of data [35], and the uncertainties of θ_{12} and Δm_{21}^2 have been remarkably reduced with respect to previous measurements. These new measurement results have been incorporated into the latest global analysis of neutrino oscillation data [36,37], where the JUNO results dominate the global fit for θ_{12} and Δm_{21}^2 . We summarize the latest JUNO measurement and the NuFIT results from a global three-flavor oscillation analysis in Table I, where the NuFIT6.1 includes the JUNO measurement while the NuFIT6.0 does not. For consistency, hereafter we exploit the NuFIT6.0 and 6.1 results to discuss the implications of the first JUNO measurements. With the most precise measurement of θ_{12} from the JUNO experiment, the TM mixing patterns are meeting the most serious challenge through the correlations between θ_{12} and θ_{13} in Eq. (5). To clearly see this point, we plot in Fig. 1 the

¹In a recent paper [33], the correlation between θ_{12} and θ_{13} in the TM1 mixing pattern has been generalized into that among the three elements in the first row of U , namely $|U_{e1}|^2 = 2(|U_{e2}|^2 + |U_{e3}|^2)$. This relation at the matrix element level is remarkably applicable to a nonunitary U . The potential experimental tests of the relation and nonunitarity with the JUNO and Daya Bay measurements have also been discussed.

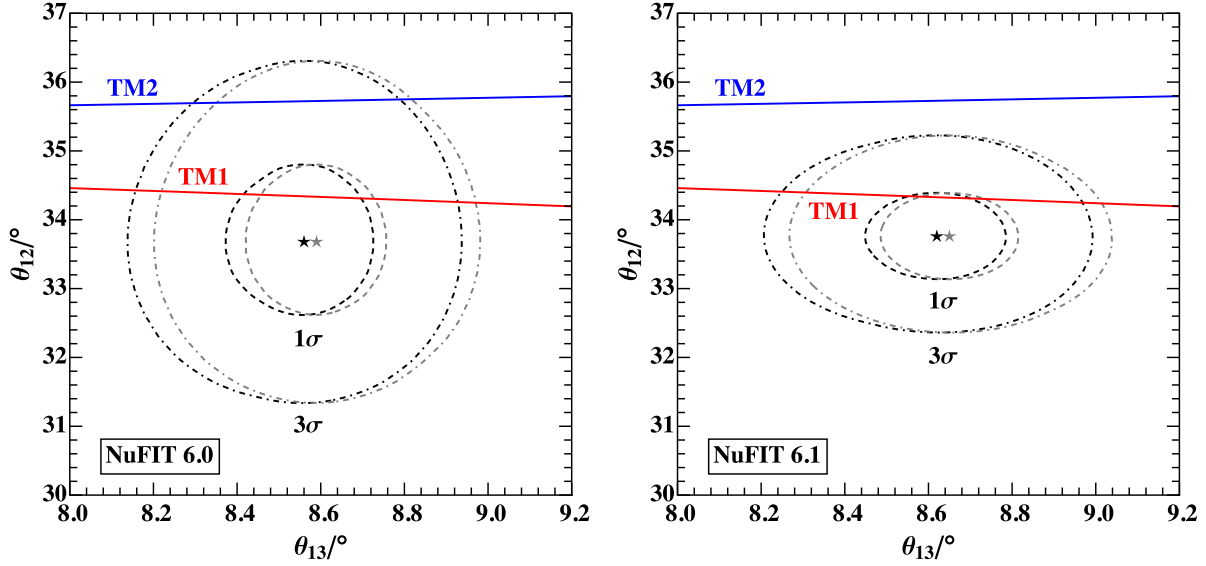


FIG. 1. Correlations between θ_{12} and θ_{13} in the TM1 and TM2 mixing patterns. The black (gray) stars, dashed lines, and dashed-dotted lines stand for the best-fit point, 1σ region, and 3σ region of θ_{12} and θ_{13} , respectively, in the NMO (IMO) case. In the left panel, the central values of θ_{12} and θ_{13} together with their uncertainties are both taken from the NuFIT6.0 results, while those in the right panel are from the NuFIT6.1 results incorporating the latest JUNO measurement.

two TM correlations in Eq. (5) together with the best-fit values, and 1σ and 3σ regions of θ_{12} and θ_{13} directly from their two-dimensional χ^2 projections given by the NuFIT6.0 and 6.1, respectively. It is obvious that with the incorporation of the JUNO result, the experimental data increasingly disfavor the TM mixing patterns. Notably, the TM2 pattern lies substantially outside the 3σ region, and therefore, it is strongly disfavored in light of the improved precision on θ_{12} , whereas the TM1 pattern sits on the edge of the 1σ region. However, it should be noted that these flavor mixing patterns are typically realized through certain flavor symmetries at a superhigh energy scale [32,38–50]. Therefore, when confronting them with low-energy experimental data, one must properly take into account renormalization group (RG) running effects on neutrino parameters (see, e.g., the review [51] and references therein). In general, the TM correlations given in Eq. (5) are not invariants against the RG evolution, and consequently, RG corrections to θ_{12} and θ_{13} can break their correlations and allow θ_{12} to deviate from the values predicted by the exact relations. This suggests that the RG evolution of neutrino parameters between energy scales may render the TM mixing patterns compatible with current experimental data, but it critically depends on the direction and magnitude of the running effects.

In this context, the present work aims to carefully investigate the RG running effects on the TM mixing patterns and to examine whether, or to what extent, these corrections can reconcile the TM predictions with the current and forthcoming precision measurements. We consider both Majorana and Dirac neutrino cases, where the Standard Model is extended with the unique

dimension-five Weinberg operator [52] and with three right-handed neutrinos, respectively, to generate neutrino masses. To remain model-independent and generic, we simply assume that the lepton flavor mixing exhibits the TM mixing patterns at a typical superhigh energy scale, without specifying their possible origins from underlying flavor symmetries.

The remainder of this paper is organized as follows. In Sec. II, we take a closer look at the TM mixing patterns and briefly discuss their predictions in the absence of RG corrections. The RG equations in the Majorana and Dirac cases are introduced in Sec. III, where analytical estimates of the RG running effects on the TM θ_{12} - θ_{13} correlations are also performed. Section IV provides detailed numerical analyses of the RG effects on the TM1 mixing pattern and its compatibility with the latest experimental data, followed by a similar numerical discussion of the TM2 mixing pattern in Sec. V. Finally, we present our summary in Sec. VI.

II. TRIMAXIMAL MIXING PATTERNS

As shown in the right panel of Fig. 1, the TM1 mixing pattern still lies within the 3σ region of θ_{12} and θ_{13} when the latest JUNO results are included, whereas the TM2 pattern is already substantially outside the 3σ region. For this reason, the former appears more appealing and could be more easily brought into better agreement with the experimental data. Therefore, we mainly focus on the TM1 mixing pattern in the following analysis, and then in Sec. V, we will briefly discuss the results for the TM2 mixing pattern.

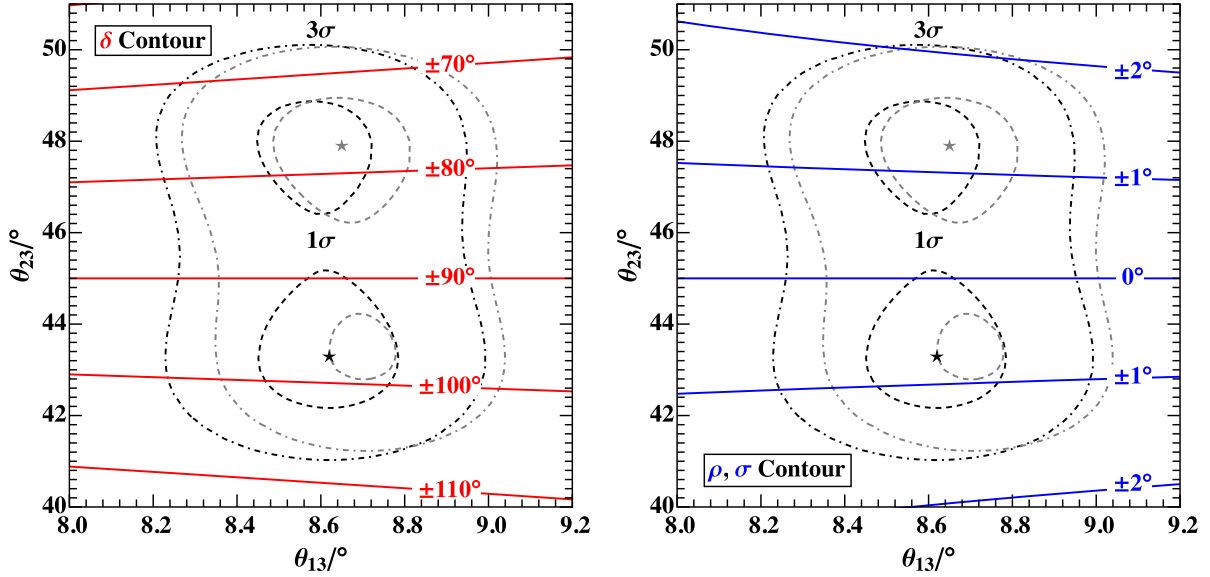


FIG. 2. Contour plots for δ and $\rho(\sigma)$ in the θ_{23} - θ_{13} plane in both NMO and IMO cases. The black (gray) stars, dashed lines, and dashed-dotted lines stand for the best-fit point, 1σ region, and 3σ region of θ_{13} and θ_{23} from the NuFIT6.1 global analysis.

Since the TM1 mixing matrix U_{TM1} in Eq. (3) contains only two free parameters, i.e., the angle θ and phase ϕ , which determine all three mixing angles θ_{ij} (for $ij = 12, 13, 23$), one Dirac CP -violating phase δ , and two Majorana CP -violating phases ρ and σ (present only in the Majorana case and unphysical in the Dirac case) under the standard

parametrization in Eq. (2), it gives rise to one (three) additional correlation(s) among neutrino mixing parameters in the Dirac (Majorana) case, apart from the correlation between θ_{12} and θ_{13} already shown in Eq. (5), and two relations linking the mixing parameters to the free parameters θ and ϕ . These relations are found to be

$$\begin{aligned} \sin \theta_{13} &= \frac{1}{\sqrt{3}} \sin \theta, & \cos 2\theta_{23} &= -2\sqrt{2} \tan \theta_{13} \sqrt{1 - 2\tan^2 \theta_{13} \cos \phi}, & \sin \phi &= \sin 2\theta_{23} \sin \delta, \\ \cos \delta &= -\frac{\cot 2\theta_{23}(1 - 5\sin^2 \theta_{13})}{2\sqrt{2} \sin \theta_{13} \sqrt{1 - 3\sin^2 \theta_{13}}}, & \rho &= \sigma = \phi - \delta. \end{aligned} \quad (6)$$

The last relation for ρ and σ in the above equation is physically meaningful only in the Majorana case, and we keep ϕ and δ explicitly in the relation, instead of substituting their expressions in terms of θ_{13} and θ_{23} , for brevity. The three relations in the first row of Eq. (6) establish connections between the mixing parameters and the two free parameters, and it is convenient to use the first two of them as conversion relations between the (θ, ϕ) coordinate and the $(\theta_{13}, \theta_{23})$ coordinate systems. Consequently, all other mixing parameters can be determined from a given pair of $(\theta_{13}, \theta_{23})$ values through the correlations in Eq. (5) and in the last line of Eq. (6). The dependence of θ_{12} on θ_{13} has been presented in Fig. 1, and here, we show contour plots for δ and $\rho(\sigma)$ in the θ_{23} - θ_{13} plane in Fig. 2, where the best-fit values, 1σ and 3σ regions of $(\theta_{13}, \theta_{23})$ from the NuFIT6.1, are also shown.

As seen from the left panel of Fig. 2, each contour line of δ corresponds to two values with opposite signs due to the

ambiguity in $\cos \delta$, and the quadrant of δ is fully correlated with the octant of θ_{23} . More specifically, the first (second) octant of θ_{23} results in the second or third (first or fourth) quadrant of δ . As the best-fit values of θ_{23} in the NMO and IMO cases are in different octants, different quadrants of δ are preferred in these two cases. Inputting the best-fit values of θ_{13} and θ_{23} from the NuFIT6.1 listed in Table I, one gets $\delta = 97.49^\circ$ or 262.51° in the NMO case while $\delta = 77.47^\circ$ or 282.53° in the IMO case. The latter values in the two cases lie within the current experimental 3σ and 1σ ranges of δ , respectively. Note that $\theta_{23} = 45^\circ$ leads to $\delta = \phi = \pm 90^\circ$, and then the TM1 mixing matrix possesses a μ - τ symmetry, namely $|U_{\mu i}| = |U_{\tau i}|$ (for $i = 1, 2, 3$) [53]. Since the present best-fit value of θ_{23} slightly deviates from 45° , the TM1 mixing pattern with an approximate μ - τ symmetry is favored by the current experimental data. As shown in the right panel of Fig. 2, the contour lines of ρ and σ are symmetric with respect to $\rho, \sigma = 0$,

corresponding to $\theta_{23} = 45^\circ$, and each line also represents two values with opposite signs. Moreover, within the experimental 3σ ranges of θ_{23} and θ_{13} , the values of ρ and σ are very tiny. In fact, ρ and σ capture the breaking of the μ - τ symmetry, and hence their tiny values again indicate that the current experimental data are in favor of an approximate μ - τ symmetry embedded in the TM1 mixing pattern. One can take advantage of the relations in Eq. (6) to understand the aforementioned features of ρ and σ . Taking into account the experimentally allowed ranges of θ_{23} and θ_{13} , the relations in Eq. (6) infer that $\cos \delta / \cos \phi > 0$ and $\sin \delta / \sin \phi > 0$ hold, and thereby, δ and ϕ are always in the same quadrant. As the difference between ϕ and δ , the signs and magnitudes of ρ and σ are governed by the relation $\sin \phi = \sin 2\theta_{23} \sin \delta$, which is symmetric with respect to $\theta_{23} = 45^\circ$. The fact that the experimental measurement of θ_{23} is very close to 45° leads to small values of $\rho = \sigma = \phi - \delta$. In turn, the smallness of ρ and σ quantifies the deviation of θ_{23} from 45° , or equivalently, the extent of μ - τ symmetry breaking. Because of the ambiguous quadrants of ϕ and δ , the signs of ρ and δ remain undetermined: they are positive when ϕ and δ lie in the second or fourth quadrant and negative when ϕ and δ lie in the first or third quadrant.

III. RENORMALIZATION GROUP RUNNING EFFECTS

Since the CP -violating phase(s) has/have not yet been well measured, and the Majorana or Dirac nature of neutrinos even remains an open question, the θ_{12} - θ_{13} correlations in Eq. (5), being independent of the CP -violating phase(s), can serve as a distinctive feature of the TM mixing patterns and be exploited to test them, particularly now that the latest JUNO results have significantly improved the precision of θ_{12} . As clearly shown in Fig. 1, the θ_{12} - θ_{13} correlations predicted by the TM1 and TM2 mixing patterns already lie outside the experimentally allowed 1σ and 3σ region, respectively, after taking into account the latest JUNO results. However, it is necessary to properly include RG corrections to these relations when confronting them with low-energy measurements, as the TM mixing patterns are typically realized through underlying flavor symmetries at a superhigh energy scale. In this section, we introduce the RG equations for the neutrino mass matrix in both Majorana and Dirac neutrino cases and examine whether RG corrections can bring the θ_{12} - θ_{13} correlations into better agreement with current experimental measurements. Here we do not discuss the underlying origin of the TM patterns, but instead simply assume that they are realized at a superhigh energy scale, i.e., $\Lambda = 10^{16}$ GeV.

Considering that neutrino masses in the Majorana and Dirac cases are generated through the Weinberg operator [52] and the Higgs mechanism [54–57], respectively, the

one-loop renormalization group equation (RGE) for the neutrino mass matrix M_ν can be written as²

$$16\pi^2 \mu \frac{dM_\nu}{d\mu} = \alpha_M M_\nu - \frac{3}{2} (Y_l Y_l^\dagger) M_\nu - \frac{3}{2} M_\nu (Y_l Y_l^\dagger)^T \quad (7)$$

in the Majorana case [64–66], and

$$16\pi^2 \mu \frac{dM_\nu}{d\mu} = \alpha_D M_\nu - \frac{3}{2} (Y_l Y_l^\dagger - Y_\nu Y_\nu^\dagger) M_\nu \quad (8)$$

in the Dirac case [67–69], where the flavor-blind factors α_M and α_D are given by

$$\begin{aligned} \alpha_M &= 4\lambda - 3g_2^2 + 2\text{Tr}(Y_l Y_l^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger), \\ \alpha_D &= -\frac{3}{4}(g_1^2 + 3g_2^2) + \text{Tr}(Y_\nu Y_\nu^\dagger + Y_l Y_l^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger), \end{aligned} \quad (9)$$

and μ is the renormalization energy scale. Given the fact that $Y_\nu/Y_l \lesssim 10^{-6}$ holds in the Dirac case, one can safely omit Y_ν in Eq. (8) as well as those in α_D and other couplings' RGEs. This leaves all couplings' RGEs the same in both two cases, except for the ones of M_ν . Working in the basis where Y_l is diagonal, i.e., $Y_l = \text{Diag}\{y_e, y_\mu, y_\tau\}$, at the initial energy scale Λ , Y_l remains diagonal during RG running. In this case, the lepton flavor mixing always arises entirely from the diagonalization of the neutrino mass matrix, namely, $U^\dagger M_\nu U^* = \text{Diag}\{m_1, m_2, m_3\}$ or $U^\dagger H_\nu U = \text{Diag}\{m_1^2, m_2^2, m_3^2\}$ in the Majorana or Dirac case, where m_i (for $i = 1, 2, 3$) denote neutrino masses, and $H_\nu \equiv M_\nu M_\nu^\dagger$ is introduced in the Dirac case. Furthermore, the integral solution to the neutrino mass matrix's RGE turns out to be [70]

$$M_\nu(\mu) = I_M T_l M_\nu(\Lambda) T_l, \quad (10)$$

in the Majorana case, and

$$H_\nu(\mu) = I_D^2 T_l H_\nu(\Lambda) T_l, \quad (11)$$

in the Dirac case, where $T_l = \text{Diag}\{I_e, I_\mu, I_\tau\}$ is the same in two cases, and the scale-dependent running factors $I_{M,D}$ and I_α (for $\alpha = e, \mu, \tau$) are given by

²The two-loop RGE of the Weinberg operators has been derived in Refs. [58,59]. It is unnecessary to include it in the present case due to its relative smallness compared with the one-loop effects. However, in the case where the lightest neutrino is massless, these two-loop RG effects can generate a nonzero mass for this initially massless neutrino [58–60], while the one-loop effects cannot do this job [61–63].

$$I_{M,D}(\mu) = \exp \left[\frac{1}{16\pi^2} \int_{\ln \Lambda}^{\ln \mu} \alpha_{M,D} d \ln \mu' \right],$$

$$I_\alpha(\mu) = \exp \left[-\frac{3}{32\pi^2} \int_{\ln \Lambda}^{\ln \mu} y_\alpha^2 d \ln \mu' \right]. \quad (12)$$

Obviously, I_M and I_D are flavor blind, and they only affect the magnitude of neutrino masses universally, whereas I_α (for $\alpha = e, \mu, \tau$) can change the flavor structure of M_ν or H_ν and hence dominate the running of flavor mixing parameters. Considering $y_e \ll y_\mu \ll y_\tau \ll 1$, we have $T_l \simeq \text{Diag}\{1, 1, 1 + \Delta_\tau\}$ with

$$\Delta_\tau(\mu) = -\frac{3}{32\pi^2} \int_{\ln \Lambda}^{\ln \mu} y_\tau^2 d \ln \mu'. \quad (13)$$

Figure 3 shows the values of running factors $I_{M,D}$ and Δ_τ against the energy scale μ from $\Lambda = 10^{16}$ GeV down to the electroweak scale $\Lambda_{\text{EW}} = 200$ GeV. It manifests that Δ_τ

gains a very tiny value around 3×10^{-5} at Λ_{EW} , and consequently, RG corrections to mixing angles and phases are, in general, very small. Fortunately, RG corrections to the mixing angle θ_{12} can be significantly enhanced by the factor $(m_2 + m_1)/(m_2 - m_1)$ or $(m_2^2 + m_1^2)/(m_2^2 - m_1^2)$ when neutrinos have a nearly degenerate mass spectrum [51,70–76]. Such an enhancement of θ_{12} is highly desirable in the present context, and moreover, a nearly degenerate mass spectrum would be sensitive to neutrino nonoscillation experiments, such as beta decay and neutrinoless double beta decay experiments. However, in addition to the size of RG corrections, an equally crucial factor is their direction, which determines whether these corrections drive θ_{12} toward or away from its experimentally measured value. To demonstrate these essential points, we reproduce here the general expression of the RG correction to θ_{12} , i.e., $\Delta\theta_{12}(\mu) \equiv \theta_{12}(\mu) - \theta_{12}(\Lambda)$, which can be approximately derived from Eq. (10) or (11) [70]:

$$\begin{aligned} \Delta\theta_{12} \simeq \frac{\Delta_\tau}{2} \bigg\{ & \sin 2\theta_{12} \sin^2 \theta_{13} \cos^2 \theta_{23} [\zeta_{32} \sin^2(\delta + \sigma) - \zeta_{31} \sin^2(\delta + \rho) + \zeta_{32}^{-1} \cos^2(\delta + \sigma) \\ & - \zeta_{31}^{-1} \cos^2(\delta + \rho)] - [(\sin^2 \theta_{23} - \sin^2 \theta_{13} \cos^2 \theta_{23}) \sin 2\theta_{12} - \sin \theta_{13} \sin 2\theta_{23} \cos 2\theta_{12} \\ & \times \cos \delta] [\zeta_{21} \sin^2(\rho - \sigma) + \zeta_{21}^{-1} \cos^2(\rho - \sigma)] + \sin 2\theta_{23} \sin \theta_{13} [(\zeta_{31} - \zeta_{31}^{-1}) \sin^2 \theta_{12} \\ & \times \sin(\delta + \rho) \sin \rho + (\zeta_{32} - \zeta_{32}^{-1}) \cos^2 \theta_{12} \sin(\delta + \sigma) \sin \sigma + \frac{1}{2} (\zeta_{21} - \zeta_{21}^{-1}) \sin 2(\rho - \sigma) \\ & \times \sin \delta + (\zeta_{31}^{-1} \sin^2 \theta_{12} + \zeta_{32}^{-1} \cos^2 \theta_{12}) \cos \delta] \bigg\}, \end{aligned} \quad (14)$$

with $\zeta_{ij} = (m_i - m_j)/(m_i + m_j)$ in the Majorana case, and

$$\begin{aligned} \Delta\theta_{12} \simeq \frac{\Delta_\tau}{2} [& \sin \theta_{13} \sin 2\theta_{23} \cos \delta (\xi_{21} \cos 2\theta_{12} + \xi_{31} \sin^2 \theta_{12} + \xi_{32} \cos^2 \theta_{12}) \\ & + \sin 2\theta_{12} \sin^2 \theta_{13} \cos^2 \theta_{23} (\xi_{21} + \xi_{32} - \xi_{31}) - \xi_{21} \sin 2\theta_{12} \sin^2 \theta_{23}], \end{aligned} \quad (15)$$

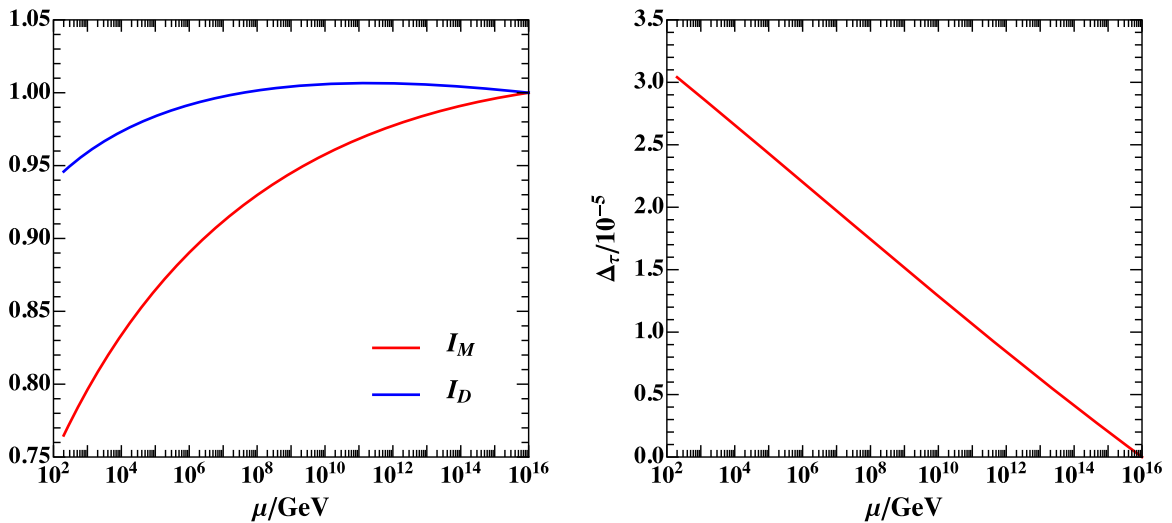


FIG. 3. The values of running factors $I_{M,D}$ and Δ_τ against the energy scale μ from $\Lambda = 10^{16}$ GeV down to the electroweak scale $\Lambda_{\text{EW}} = 200$ GeV.

with $\xi_{ij} = (m_i^2 + m_j^2)/(m_i^2 - m_j^2)$ in the Dirac case. The approximate integral results for neutrino masses and other mixing parameters in both cases can be found in Ref. [70].

Now, let us define a function that quantifies the deviation from the TM1 θ_{12} - θ_{13} correlation in Eq. (5) at the energy scale μ :

$$f(\mu) = \sin^2 \theta_{12}(\mu) + \frac{2}{3} \tan^2 \theta_{13}(\mu) - \frac{1}{3}. \quad (16)$$

One immediately has $f(\Lambda) = 0$ as indicated by the exact θ_{12} - θ_{13} correlation, and at the leading order, $f(\mu)$ can be expanded as

$$\begin{aligned} f(\mu) &\simeq \sin 2\theta_{12}(\Lambda) \cdot \Delta\theta_{12}(\mu) \\ &\quad + \frac{4}{3} \tan \theta_{13}(\Lambda) \sec^2 \theta_{13}(\Lambda) \cdot \Delta\theta_{13}(\mu) \\ &\simeq \sin 2\theta_{12}(\Lambda_{\text{EW}}) \cdot \Delta\theta_{12}(\mu) \\ &\quad + \frac{4}{3} \tan \theta_{13}(\Lambda_{\text{EW}}) \sec^2 \theta_{13}(\Lambda_{\text{EW}}) \cdot \Delta\theta_{13}(\mu), \end{aligned} \quad (17)$$

where $f(\Lambda) = 0$ has been used. Generally, $f(\mu) = 0$ is not protected against RG corrections, and any deviation of $f(\mu)$ from 0 stands for a breaking of the TM1 mixing pattern. If one simply substitutes the latest best-fit values for $\theta_{12} = 33.76^\circ$ and $\theta_{13} = 8.62^\circ$ from NuFIT6.1 into Eq. (16), it gives $f_{\text{exp}}(\Lambda_{\text{EW}}) \simeq -9 \times 10^{-3}$. Therefore, if RG effects could sizeably decrease $f(\mu)$ from $f(\Lambda) = 0$ to $f_{\text{exp}}(\Lambda_{\text{EW}}) \simeq -9 \times 10^{-3}$, the TM1 θ_{12} - θ_{13} correlation incorporating RG corrections would become fully consistent with the latest experimental data. Considering that the factor $\Delta_\tau(\Lambda_{\text{EW}}) \simeq 3 \times 10^{-5}$ governing the overall size of RG corrections to mixing angles is much smaller than 9×10^{-3} , the only possibility of achieving such a large correction in the present context is that neutrino masses are highly degenerate. In this case, $\Delta\theta_{12}$ is largely enhanced by ζ_{12}^{-1} or ξ_{12} while $\Delta\theta_{13}$ is not, and hence the second term involving $\Delta\theta_{13}$ in Eq. (17) can be simply omitted. With the help of Eqs. (14) or (15) and taking into account relations in Eqs. (5) and (6), $f(\Lambda_{\text{EW}})$ in Eq. (17) is approximately given by

$$f(\Lambda_{\text{EW}}) \sim \begin{cases} -\frac{4\Lambda_i}{9} \zeta_{21}^{-1} \sin^2 \theta_{23}, & \text{(Majorana case)} \\ -\frac{4\Lambda_i}{9} \xi_{21} \sin^2 \theta_{23}, & \text{(Dirac case)} \end{cases}. \quad (18)$$

Fortunately, it turns out that the RG corrections in the both Majorana and Dirac cases are in the right direction that leads to a negative value of $f(\Lambda_{\text{EW}})$. Now, requiring $f(\Lambda_{\text{EW}}) = f_{\text{exp}}(\Lambda_{\text{EW}}) \simeq -9 \times 10^{-3}$ and inputting the best-fit values of $\theta_{23} = 43.29^\circ$ and $\Delta m_{21}^2 = 7.537 \times 10^{-5} \text{ eV}^2$, one obtains $m_1 \sim 0.16 \text{ eV}$ in the Majorana case or $m_1 \sim 0.23 \text{ eV}$ in the Dirac case. The latter is twice larger than the former due to $\zeta_{12}^{-1} \sim 2\xi_{21}$ in the almost degenerate case. Therefore, in both

the Majorana and Dirac cases, the RG corrections with nearly degenerate neutrino masses act in the right direction and are sufficiently large to reconcile the TM1 mixing pattern with experimental data, including the latest JUNO results. However, a nearly degenerate mass spectrum in the Majorana case may encounter severe tension with current neutrinoless double beta decay experiments, while the Dirac case remains unaffected. We will discuss these issues in detail when performing more precise and comprehensive numerical analyses in the next section.

IV. NUMERICAL ANALYSES

In the previous section, we have analytically estimated the RG effects on the TM1 θ_{12} - θ_{13} correlation in both Majorana and Dirac cases and found that the TM1 mixing pattern can become fully consistent with the latest JUNO result once RG corrections are included. In this section, we scrutinize these RG effects on the TM1 mixing pattern by performing comprehensive numerical analyses and discuss some constraints from neutrino nonoscillation experiments.

We adopt the χ^2 technique to quantify how well the TM1 mixing pattern incorporating RG effects agrees with the current experimental data and to extract the allowed parameter space. Given the distinct experimental sensitivities, we construct the total χ^2 function as the sum of individual contributions from the solar, reactor, and atmospheric sectors, namely,

$$\begin{aligned} \chi^2 &= \chi_{\text{Solar}}^2[\theta_{12}(\mathbf{p}), \Delta m_{21}^2(\mathbf{p})] + \chi_{\text{Atmos}}^2[\theta_{23}(\mathbf{p}), \Delta m_{3\ell}^2(\mathbf{p})] \\ &\quad + \chi_{\text{Reactor}}^2[\theta_{13}(\mathbf{p})], \end{aligned} \quad (19)$$

where the small correlations among these sectors are neglected, and the χ_i^2 (for $i = \text{Solar}, \text{Atmos}, \text{and Reactor}$) in each sector is taken from the projected χ^2 data provided by NuFIT6.1 [37].³ Each χ_i^2 and the total χ^2 are functions of the initial inputs $\mathbf{p} = \{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$ at $\Lambda = 10^{16} \text{ GeV}$ with $m_{\text{lightest}} = m_1$, $\ell = 1$ for the NMO and $m_{\text{lightest}} = m_3$, $\ell = 2$ for the IMO. We exclude the Dirac CP -violating phase δ from the definition of χ^2 function since its value is still subject to a large uncertainty and even shows a mild tension between the T2K and NOvA measurements in the NMO case [19,77–79]. We take advantage of the gradient descent method [80] to minimize the χ^2 function and obtain the corresponding best-fit initial inputs for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$. Starting with this set of best-fit initial inputs, we then employ the Metropolis algorithm [81] to explore the allowed parameter space.

Besides neutrino masses, mixing angles, and CP -violating phase(s), we also calculate two nonoscillation

³The mass ordering preference is removed from the χ_i data provided by NuFIT6.1 [37]. In other words, we subtract $\Delta\chi_{\text{MO}}^2 = 5.9$ from each χ_i function in the inverted mass ordering.

TABLE II. Initial inputs for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$ in the TM1 mixing pattern at Λ that minimize χ^2 and predictions for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \rho, \sigma, m_\beta, m_{\beta\beta}\}$ at Λ_{EW} together with the minimum value χ_{min}^2 of χ^2 . The third and fourth columns show results in the Majorana case for the normal and inverted neutrino mass orderings, respectively, whereas the fifth and sixth columns present those in the Dirac case, where $\{\rho, \sigma, m_{\beta\beta}\}$ are not physical observables.

		The Majorana case		The Dirac case	
		NMO	IMO	NMO	IMO
Inputs at Λ	$m_{\text{lightest}}/\text{eV}$	0.21703	0.19384	0.24893	0.23943
	$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	12.790	12.761	8.3615	8.3377
	$\Delta m_{3\ell}^2/10^{-3} \text{ eV}^2$	4.2895	4.2475	2.8047	2.7767
	$\theta/^\circ$	15.059	15.093	15.069	15.084
	$\phi(\text{or } 360^\circ - \phi)/^\circ$	98.033	76.387	98.033	76.398
Predictions at Λ_{EW}	$m_{\text{lightest}}/\text{eV}$	0.16599	0.14826	0.23547	0.22648
	$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	7.5334	7.5336	7.5336	7.5361
	$\Delta m_{3\ell}^2/10^{-3} \text{ eV}^2$	2.5105	-2.4846	2.5107	-2.4843
	$\theta_{12}/^\circ$	33.770	33.765	33.766	33.710
	$\theta_{13}/^\circ$	8.6268	8.6464	8.6268	8.6463
	$\theta_{23}/^\circ$	43.281	47.869	43.280	47.870
	$\delta(\text{or } 360^\circ - \delta)/^\circ$	97.295	77.605	96.889	77.230
	$\rho(\text{or } 180^\circ - \rho)/^\circ$	0.74035	178.78	...	...
	$\sigma(\text{or } 180^\circ - \rho)/^\circ$	0.72729	178.79	...	...
	m_β/eV	0.16623	0.15607	0.23563	0.23167
	$m_{\beta\beta}/\text{eV}$	0.15858	0.14974	...	...
	χ_{min}^2	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}	5.9×10^{-3}

observables, namely, the two effective neutrino masses, $m_\beta = \sqrt{\sum_{i=1}^3 m_i^2 |U_{ei}|^2}$ and $m_{\beta\beta} = |\sum_{i=1}^3 m_i U_{ei}^2|$, which can be probed in beta decay and neutrinoless double beta decay experiments, respectively. The most stringent current constraints on m_β and $m_{\beta\beta}$ are provided by KATRIN and KamLAND-Zen experiments, which set upper limits of $m_\beta < 0.45 \text{ eV}$ [82] and $m_{\beta\beta} < 0.028\text{--}0.122 \text{ eV}$ [83], respectively. These upper limits may impose strong constraints on the allowed parameter space, as a sizable lightest neutrino mass is required to enhance the RG effects. However, the constraint from neutrinoless double beta decay experiments is relevant only in the Majorana case, and it is absent in the Dirac case.

A. The Majorana case

We list in Table II the initial inputs for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$ at the high energy scale Λ that minimize χ^2 , together with the predictions for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \rho, \sigma, m_\beta, m_{\beta\beta}\}$ at the electroweak scale Λ_{EW} , and the minimum value χ_{min}^2 of χ^2 . To clearly illustrate the allowed parameter space and correlations among observables, we show in Figs. 4 and 5 several representative pairs of predicted observables at Λ_{EW} , namely $(\theta_{12}, \theta_{13})$, (δ, θ_{23}) , (σ, ρ) , $(\theta_{12}, m_{\text{lightest}})$, $(m_\beta, m_{\text{lightest}})$, and $(m_{\beta\beta}, m_{\text{lightest}})$, for the NMO and IMO cases, respectively. In each plot, the cyan star and the red and blue regions correspond to χ_{min}^2 ,

$\Delta\chi^2 \equiv \chi^2 - \chi_{\text{min}}^2 \leq 2.30$, and $\Delta\chi^2 \leq 11.83$ for two degrees of freedom, respectively. The lighter gray line in the $(m_\beta, m_{\text{lightest}})$ plane and lighter gray band in the $(m_{\beta\beta}, m_{\text{lightest}})$ plane denote the upper limits on m_β and $m_{\beta\beta}$ from KATRIN and KamLAND-Zen, respectively. The darker gray regions in these two planes represent the allowed 3σ regions obtained by varying neutrino oscillation parameters within their 3σ ranges from NuFIT6.1 and taking $\rho, \sigma \in [0, 180^\circ)$. The dashed line in the $(\theta_{12}, \theta_{13})$ plane corresponds to $f(\Lambda) = 0$, i.e., the TM1 θ_{12} - θ_{13} correlation in Eq. (5) and the one in the $(\theta_{12}, m_{\text{lightest}})$ plane indicates the maximal value of θ_{12} predicted by the TM1 θ_{12} - θ_{13} correlation when θ_{12} and θ_{13} vary within their 3σ region from the NuFIT6.1 results. We also draw several dashed gray grid lines (namely, $\delta = 90^\circ, 180^\circ, 270^\circ$ and $\theta_{23} = 45^\circ$) in the (δ, θ_{23}) plane to help visualize the correlation between the quadrant of δ and the octant of θ_{23} . Some comments and discussions on these numerical results are given as blow:

- (i) As indicated in Table II, the RG corrections can render the TM1 mixing pattern perfectly consistent with the current neutrino oscillation data, with the minimum value of χ^2 being around $\mathcal{O}(10^{-2})$ in both the NMO and IMO cases. However, achieving such agreement requires a nearly degenerate mass spectrum with $m_{\text{lightest}} \sim 0.15 \text{ eV}$. This is consistent with the analytical discussions in the previous section, and moreover, the value of m_1 has already been well estimated with the help of Eq. (18).

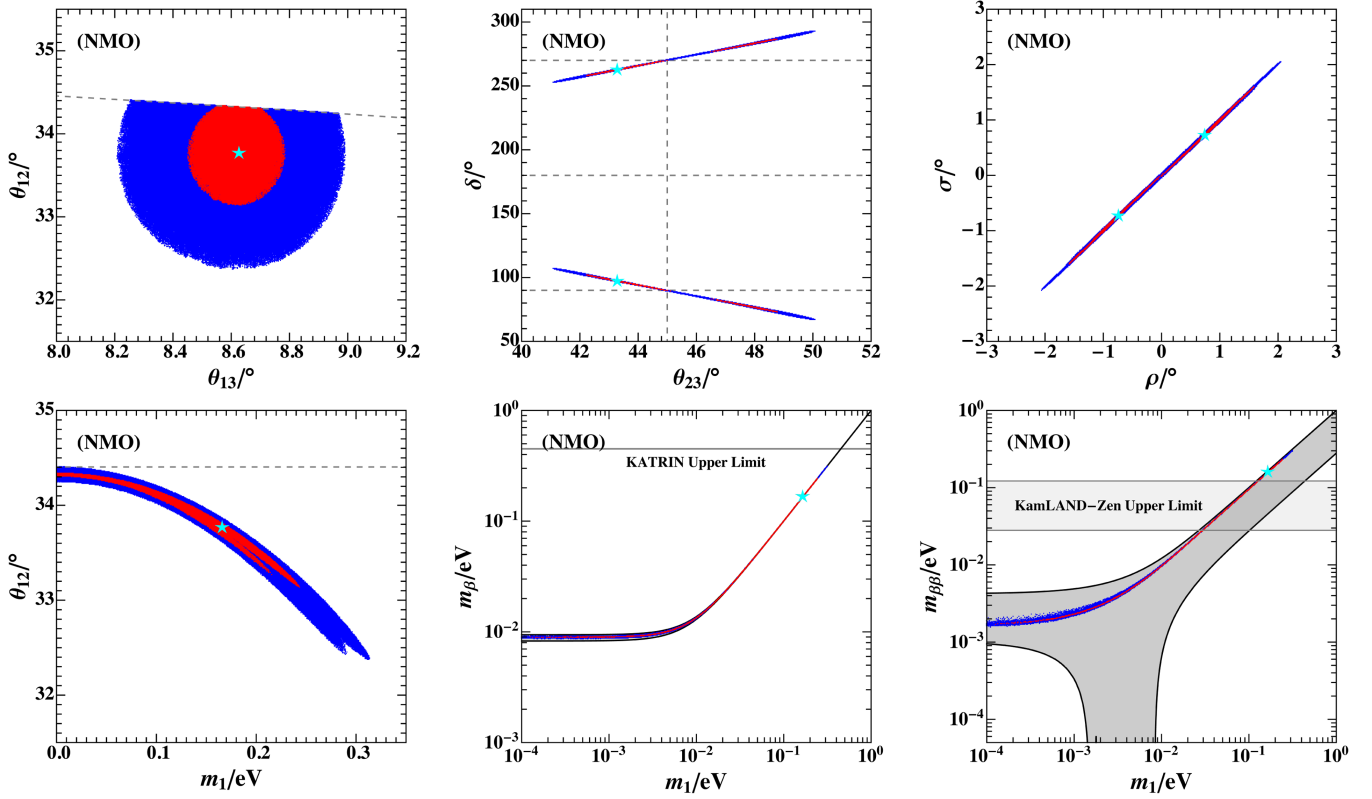


FIG. 4. Correlations among observables at Λ_{EW} in the NMO case with Majorana neutrinos. The cyan star and the red and blue regions correspond to $\chi^2 = \chi^2_{min}$, $\Delta\chi^2 \leq 2.30$, and $\Delta\chi^2 \leq 11.83$, respectively. Further details are provided in the main text.

- (ii) The upper-left panels in Figs. 4 and 5 show that the RG corrections to θ_{12} fortunately decrease its value and are able to make θ_{12} and θ_{13} distributed within their allowed 1σ and 3σ regions below the $f(\Lambda) = 0$ line in both the NMO and IMO cases. Note that a visible gap exists between the $f(\Lambda) = 0$ line and the 3σ region in the IMO case, which does not appear in the NMO case. This feature is attributed to the strong degeneracy between m_1 and m_2 even when $m_3 = 0$ in the IMO case, and hence $f(\Lambda_{EW})$ is always enhanced by ζ_{21}^{-1} and deviates from $f(\Lambda) = 0$. In contrast, in the NMO case with $m_1 = 0$, no such enhancement occurs since $\zeta_{21}^{-1} = 1$, and the contribution from ζ_{32}^{-1} is suppressed by $\sin\theta_{13}$, as indicated in Eq. (14). This behavior is also reflected in the lower-left panels of Figs. 4 and 5, which further indicate that a larger m_1 (m_3) leads to a smaller θ_{12} . As a result, avoiding a too small value of θ_{12} imposes an upper limit on the lightest neutrino mass, i.e., $m_1 \lesssim 0.313$ eV ($m_3 \lesssim 0.304$ eV) in the NMO (IMO) case.
- (iii) The upper-middle panels in Figs. 4 and 5 display two separated regions for the Dirac CP -violating phase δ , whose best-fit values are $\delta = 97.295^\circ$ and 262.70° in the NMO case and $\delta = 77.605^\circ$ and 282.40° in the IMO case. These two separated regions arise from

the fact that both δ and $2\pi - \delta$ can be extracted from $\cos\delta$, and only $\cos\delta$ is determined by θ_{23} and θ_{13} in the TM1 mixing pattern, which can be seen from the relation in Eq. (6). Furthermore, it shows that the quadrant of δ is still tightly correlated with the octant of θ_{23} : the first octant of θ_{23} corresponds to the second and third quadrants of δ , while the second octant of θ_{23} corresponds to the first and fourth quadrants of δ . As shown in the upper-right panels of Figs. 4 and 5, the two Majorana phases ρ and σ also appear in two separated regions around zero. This behavior arises because they are roughly determined by $\rho = \sigma = \phi - \delta$, where the difference between ϕ and δ approximately governed by $\sin\phi = \sin 2\theta_{23} \sin\delta$ is small.

- (iv) As discussed above, a large lightest neutrino mass is required to generate sizable RG corrections to θ_{12} . However, such a large mass also results in large values of both m_β and $m_{\beta\beta}$, which may be strongly constrained by current experimental limits. From the lower-middle and lower-right panels of Figs. 4 and 5, one can see that the predicted 3σ range of m_β is well below the latest KATRIN upper bound in both the NMO and IMO cases. In contrast, the predicted best-fit value and part of the 1σ range of $m_{\beta\beta}$ already exceed the KamLAND-Zen upper bound. Compared

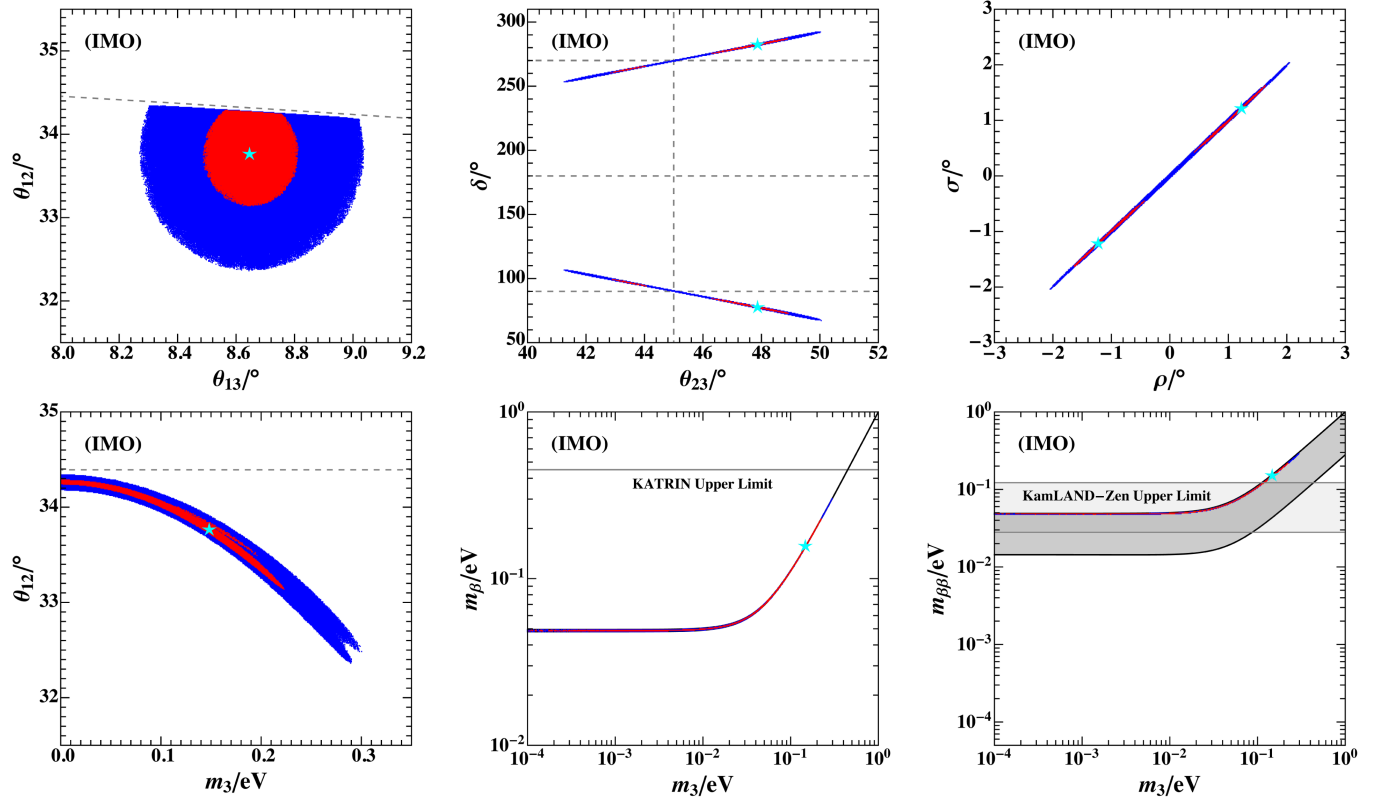


FIG. 5. The same as Fig. 4 but in the IMO case.

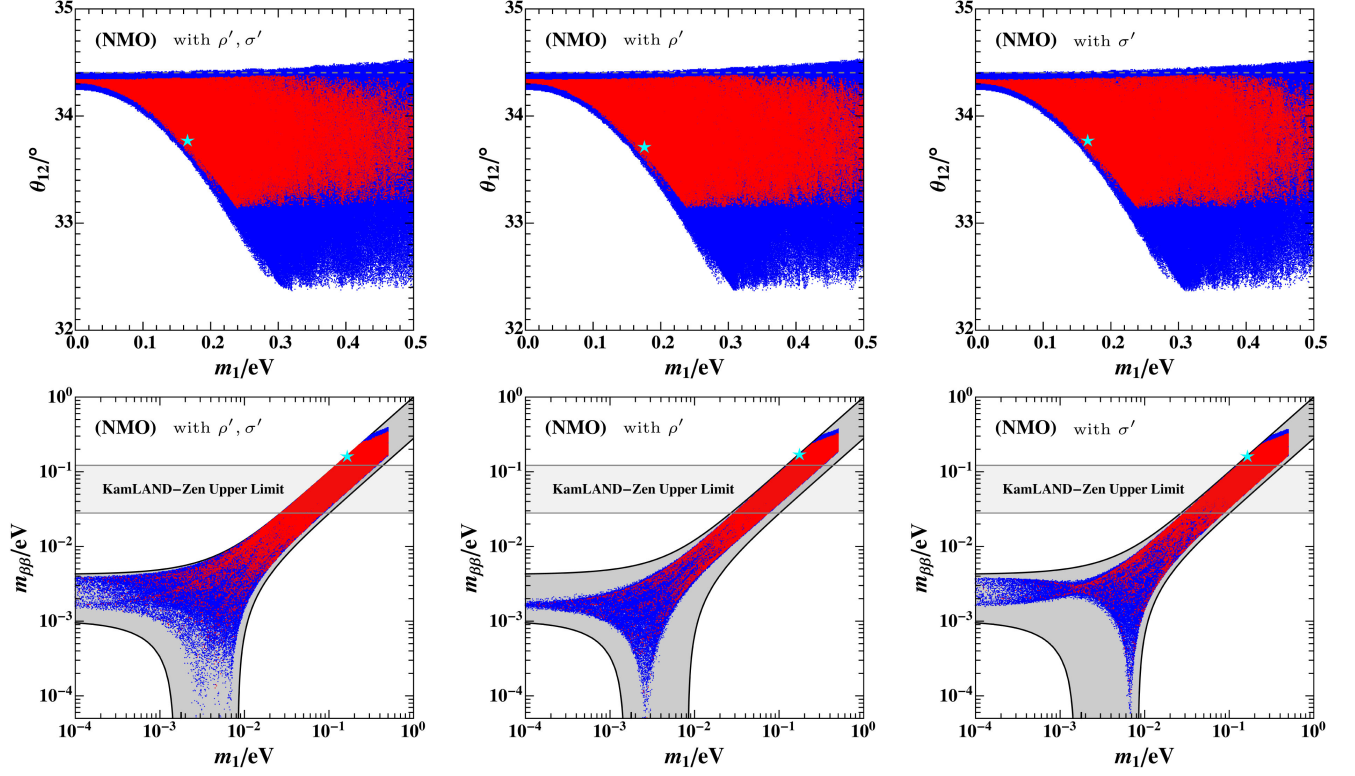
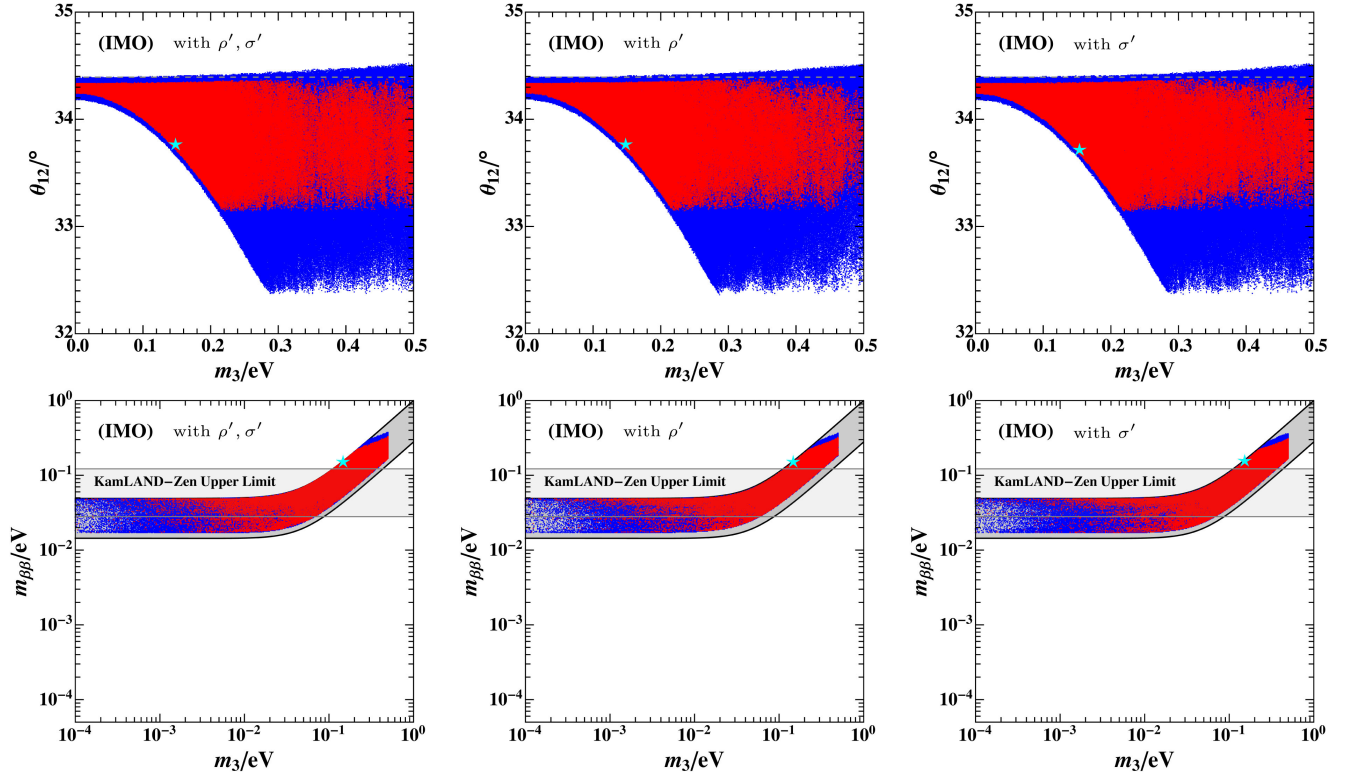
with the NMO case where part of the 1σ range of $m_{\beta\beta}$ still lies below the upper limit, the IMO case faces a stronger challenge from neutrinoless double beta decay experiments, as its remaining 1σ and 3σ ranges lie entirely within the experimental uncertainty.

It is worth noting that all the above analyses are based on the pure TM1 mixing matrix U_{TM1} given in Eq. (3). In general, an additional diagonal phase matrix $P = \text{Diag}\{e^{i\rho'}, e^{i\sigma'}, 1\}$, rephasing Majorana neutrino fields, can be introduced on the right-hand side of U_{TM1} . The resulting mixing matrix $U'_{\text{TM1}} = U_{\text{TM1}}P$ yields the same relations as those derived from U_{TM1} for all mixing parameters except the two Majorana phases, before including the RG corrections. With this new mixing matrix U'_{TM1} , the two Majorana phases become

$$\rho = \phi - \delta + \rho', \quad \sigma = \phi - \delta + \sigma', \quad (20)$$

and now they behave as free parameters and are allowed to vary over the entire range $[0, 180^\circ)$. Consequently, the predicted region of $m_{\beta\beta}$ can be significantly modified, and the RG corrections may also be strongly affected by the widely varying Majorana phases. However, the key question is whether these effects can alleviate the constraint from the neutrinoless double beta decay experiments. To examine this point, we perform the same numerical calculations as before but include two additional input parameters,

ρ' and σ' , besides $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$. It is unnecessary to minimize χ^2 again, and instead, we directly take the previous best-fit initial inputs for $\{m_{\text{lightest}}, \Delta m_{21}^2, \Delta m_{3\ell}^2, \theta, \phi\}$ given in Table II, together with $\rho' = \sigma' = 0$, as the starting point for the Metropolis algorithm to explore the allowed parameter space. During the search process, we further restrict $m_{\text{lightest}}(\Lambda_{\text{EW}}) \lesssim 0.5$ eV to avoid generating excessively large m_{lightest} and a consequent reduction in the sampling efficiency. The results for correlations of θ_{12} and $m_{\beta\beta}$ with m_{lightest} are presented in Figs. 6 and 7 for three cases where both ρ' and σ' , or only one of them is introduced. As expected, the allowed 1σ and 3σ regions for the pair of $m_{\beta\beta}$ and m_{lightest} are enlarged a lot. In particular, part of 1σ and 3σ regions now lie below the KamLAND-Zen upper limit in both the NMO and IMO cases, although the best-fit points remain above the limit. Note that, in specific flavor models leading to the TM mixing patterns [32,39–50], one or two Majorana phases may be correlated with other model parameters and hence are not necessarily free. The number of independent phases depends on the detailed structure of the underlying flavor symmetry and its breaking pattern. Consequently, the model-independent results shown in Figs. 6 and 7 generally overestimate the available parameter space of concrete flavor realizations of the TM1 mixing pattern. Nevertheless, these results provide a useful overview of the qualitative impact of varying Majorana phases.

FIG. 6. The same as Fig. 4 but with ρ' and σ' or with only one of them in the NMO case.FIG. 7. The same as Fig. 4 but with ρ' and σ' or with only one of them in the IMO case.

The upper panels of Figs. 6 and 7 indicate a notable broadening of the allowed 1σ and 3σ regions for the $(\theta_{12}, m_{\text{lightest}})$ pair, which now extend over the entire range of large m_{lightest} values. Furthermore, part of the 3σ region of $(\theta_{12}, m_{\text{lightest}})$ even lies above the maximal θ_{12} value (the dashed gray line) predicted by the TM1 θ_{12} - θ_{13} correlation within the experimental 3σ ranges of θ_{12} and θ_{13} . This means that the direction of the RG corrections to θ_{12} in this region is opposite to that in the others. These behaviors can be well understood with the help of Eq. (14), which shows that the enhancement factor ζ_{21}^{-1} is multiplied by $\cos^2(\rho - \sigma)$ or $\sin 2(\rho - \sigma)$. In the case without ρ' and σ' , $\rho \simeq \sigma$ holds and leads to $\cos^2(\rho - \sigma) \simeq 1$ and $\sin 2(\rho - \sigma) = 0$, so that the enhancement of $\Delta\theta_{12}$ from

large ζ_{21}^{-1} is unsuppressed. In contrast, when ρ' and (or) σ' are introduced, ρ and (or) σ become independent free parameters, and the enhancement from ζ_{21}^{-1} can be largely suppressed by the small or even vanishing values of $\cos^2(\rho - \sigma)$ and $\sin 2(\rho - \sigma)$. When $\rho - \sigma$ approaches $\pm 90^\circ$, a larger ζ_{21}^{-1} (equivalently a larger m_{lightest}) is required to compensate for the suppression by small $\cos^2(\rho - \sigma)$ and $\sin 2(\rho - \sigma)$. This explains why the allowed $\theta_{12} - m_{\text{lightest}}$ region can extend across the entire range of large m_{lightest} values. When $\rho - \sigma$ is extremely close or equal to $\pm 90^\circ$, the terms involving ζ_{12}^{-1} are strongly suppressed or even entirely vanish, and the contributions from ζ_{31}^{-1} and ζ_{32}^{-1} become dominant, namely,

$$\Delta\theta_{12}(\Lambda_{\text{EW}}) \sim \frac{1}{6} \Delta\tau \sin\theta_{13} \sin 2\theta_{23} [\zeta_{31}^{-1} \cos\delta \cos(\delta + 2\rho) + 2\zeta_{32}^{-1} \sin\delta \sin(\delta + 2\rho)], \quad (21)$$

where $\rho - \sigma \sim \pm 90^\circ$ and the correlations in Eqs. (5) and (6) have been exploited. It is evident that $\Delta\theta_{12}(\Lambda_{\text{EW}})$ can be positive for certain values of ρ , e.g., $\rho = 0$. However, the enhancements from ζ_{31}^{-1} and ζ_{32}^{-1} are generally much smaller than the unsuppressed contributions from ζ_{21}^{-1} . Therefore, the allowed θ_{12} - m_{lightest} region can lie above the dashed gray line, particularly, in the large m_{lightest} range, but it does not extend far beyond it.⁴

In summary, introducing the one or two additional phases from the right-hand side of U_{TM1} can extremely relax the constraint from neutrinoless double beta decay experiments, although the best-fit value and part of the 1σ range of $m_{\beta\beta}$ still lie above the current experimental upper limit.

B. The Dirac case

In the previous subsection, we have shown that the TM1 mixing pattern in the Majorana case can fully coincide with

the latest JUNO result once RG corrections are taken into account, although its parameter space receives strong constraints from neutrinoless double beta decay experiments. Given the fact that RG corrections in the Dirac case exhibit similar enhancement and direction to those in the Majorana case [see, e.g., Eq. (18)], it is expected that RG corrections in the Dirac case can not only render the TM1 mixing pattern consistent with the JUNO data but also avoid the strong constraints from neutrinoless double beta decay experiments.

We perform the same numerical analysis in the Dirac case, where the Dirac CP -violating phase δ is the only physical phase, and all other phases are unphysical and do not affect physical observables. The initial inputs at Λ that minimize χ^2 , together with the corresponding predictions for the observables at Λ_{EW} and minimum value χ_{min}^2 , are summarized in Table II. The allowed regions for the parameter pairs $(\theta_{12}, \theta_{13})$, $(\theta_{12}, m_{\text{lightest}})$, and (δ, θ_{23}) are shown in Fig. 8. It shows that the TM1 mixing pattern can indeed coincide with the experimental data very well with $\chi_{\text{min}}^2 \sim \mathcal{O}(10^{-2})$, and the results for the $(\theta_{12}, \theta_{13})$, $(\theta_{12}, m_{\text{lightest}})$, and (δ, θ_{23}) correlations are qualitatively similar to those in the Majorana case without ρ' and σ' . Thus, most discussions in the Majorana case remain applicable here and will not be repeated.

It is worth noting that the lightest neutrino mass in the Dirac case spans a wider range than that in the Majorana case. As indicated in Eq. (18), to reach the same value of $f(\Lambda_{\text{EW}})$ in the Majorana and Dirac cases, one requires $\zeta_{21}^{-1} = \xi_{21}$, implying a larger m_{lightest} in the Dirac case unless $m_1 = 0$ in both cases. Although m_{lightest} in the Dirac case can be larger, the effective neutrino mass $m_{\beta\beta}$ almost remains below the most stringent constraint from the KATRIN experiment. Moreover, the constraint from

⁴This RG running behavior of θ_{12} is reminiscent of RG corrections in the Minimal Supersymmetric Standard Model (MSSM), which can be substantially enhanced by a large $\tan\beta$ and typically proceed in the opposite direction compared with those in the SM [70,73–76,84]. Due to the opposite RG running direction, RG corrections in the MSSM are not able to bring the TM1 mixing pattern closer to the experimental results, but instead drive it further away, when adopting the pure TM1 mixing matrix U_{TM1} . Nevertheless, as indicated by Eq. (21), it is possible to reverse the direction of the RG corrections to θ_{12} by introducing additional phases ρ' and σ' . This implies that, within certain regions of parameter space, RG corrections in the MSSM could act in the right direction to reduce θ_{12} . However, to which extent MSSM RG effects with reasonable values of $\tan\beta$ and m_{lightest} can reconcile the TM1 mixing pattern with experimental data remains to be carefully investigated, which is beyond the scope of the present work.

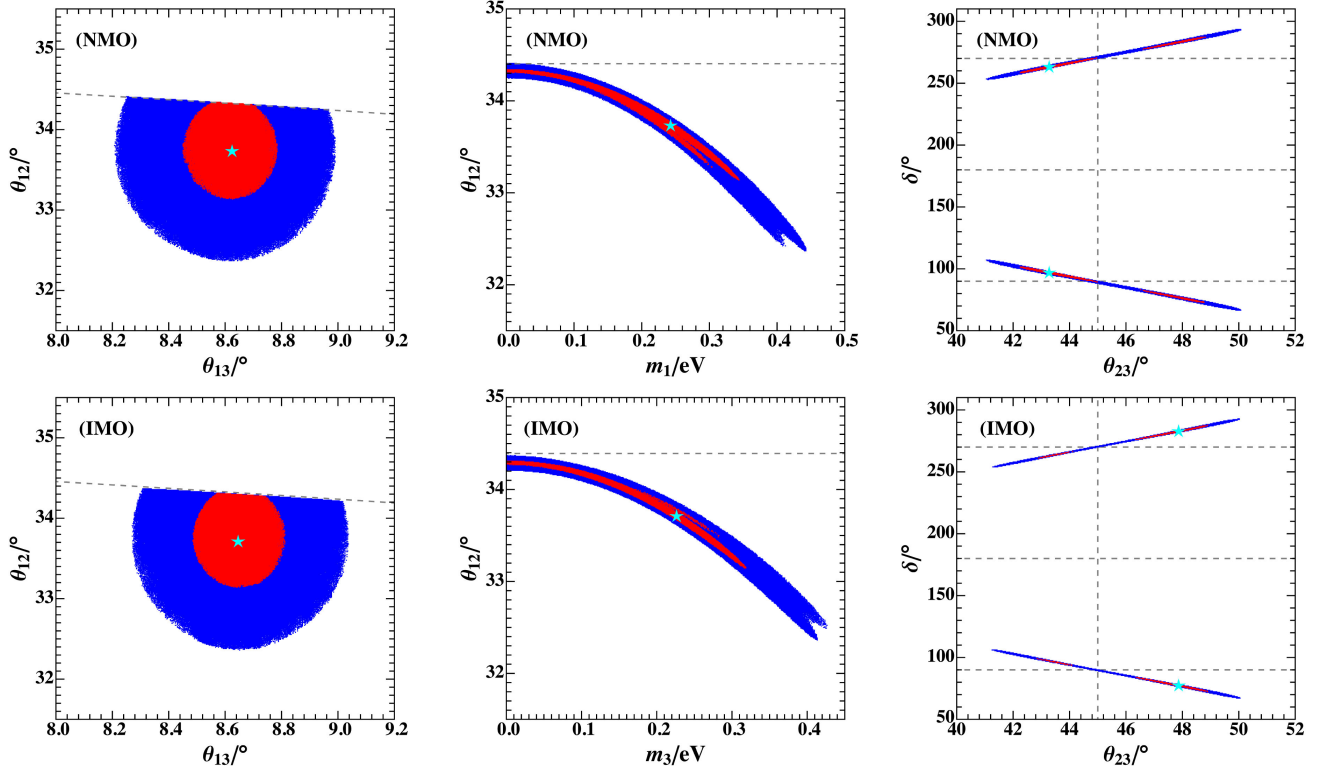


FIG. 8. Correlations between $(\theta_{12}, \theta_{13})$, $(\theta_{12}, m_{\text{lightest}})$, (δ, θ_{23}) pairs at Λ_{EW} in the Dirac case. The upper row is for the NMO case, and the lower one is for the IMO case. The cyan star and red and blue regions correspond to $\chi^2 = \chi^2_{\text{min}}$, $\Delta\chi^2 \leq 2.30$, and $\Delta\chi^2 \leq 11.83$, respectively.

neutrinoless double beta decay experiments is absent in the Dirac case. Therefore, the current experimental data exhibit a stronger preference for the TM1 mixing pattern in the Dirac case than in the Majorana case.

V. BRIEF DISCUSSION ON TM2 MIXING PATTERN

As explicitly illustrated in Fig. 1, the TM2 mixing pattern without RG corrections already lies outside the experimental 3σ region of θ_{12} and θ_{13} once the latest JUNO result is taken into account. Compared with the TM1 mixing pattern, larger RG corrections (or equivalently, a larger m_{lightest}) are required to make the TM2 pattern consistent with experimental data. Consequently, the TM2 mixing pattern in the Majorana case faces even stronger constraints from neutrinoless double beta decay experiments. Indeed, numerical calculations show that the entire allowed range of $m_{\beta\beta}$ for both neutrino mass hierarchies in the Majorana case lies above the KamLAND-Zen upper limit, even if two additional phases ρ' and σ' are introduced. This means that the TM2 mixing pattern in the Majorana case is essentially excluded by current experimental data, even after including RG corrections. In contrast, substantially viable parameter regions still exist in the Dirac case, thanks to the absence

of constraints from neutrinoless double beta decay experiments.

The allowed regions of $(\theta_{12}, \theta_{13})$, (θ_{12}, m_{β}) , and (δ, θ_{23}) parameter pairs in the Dirac case are presented in Fig. 9, where the minimum value χ^2_{min} is 9.4×10^{-3} and 6.6×10^{-3} in the NMO and IMO cases, respectively. Some further discussions are given as below:

- (i) Unlike the results for the TM1 mixing pattern, the allowed regions of θ_{12} and θ_{13} in the two left panels of Fig. 9 cover their entire 1σ and 3σ regions allowed by current experimental data. This behavior originates from the fact that the TM2 θ_{12} - θ_{13} relation without RG corrections lies outside the experimental 3σ region, as shown in Fig. 1. Moreover, the gap between the TM2 θ_{12} - θ_{13} relation and the experimentally allowed region gives rise to a lower limit on the lightest neutrino mass m_{lightest} . Therefore, m_{lightest} is constrained with both upper and lower bounds, namely, $0.21 \text{ eV} \lesssim m_1 \lesssim 0.59 \text{ eV}$ in the NMO case and $0.20 \text{ eV} \lesssim m_3 \lesssim 0.58 \text{ eV}$ in the IMO case. These allowed ranges will be further narrowed as the precision of the JUNO measurement on θ_{12} continues to improve.
- (ii) The two right panels of Fig. 9 display the allowed regions of δ and θ_{23} . It can be seen that the quadrant of δ is inversely correlated with the octant of θ_{23} , in

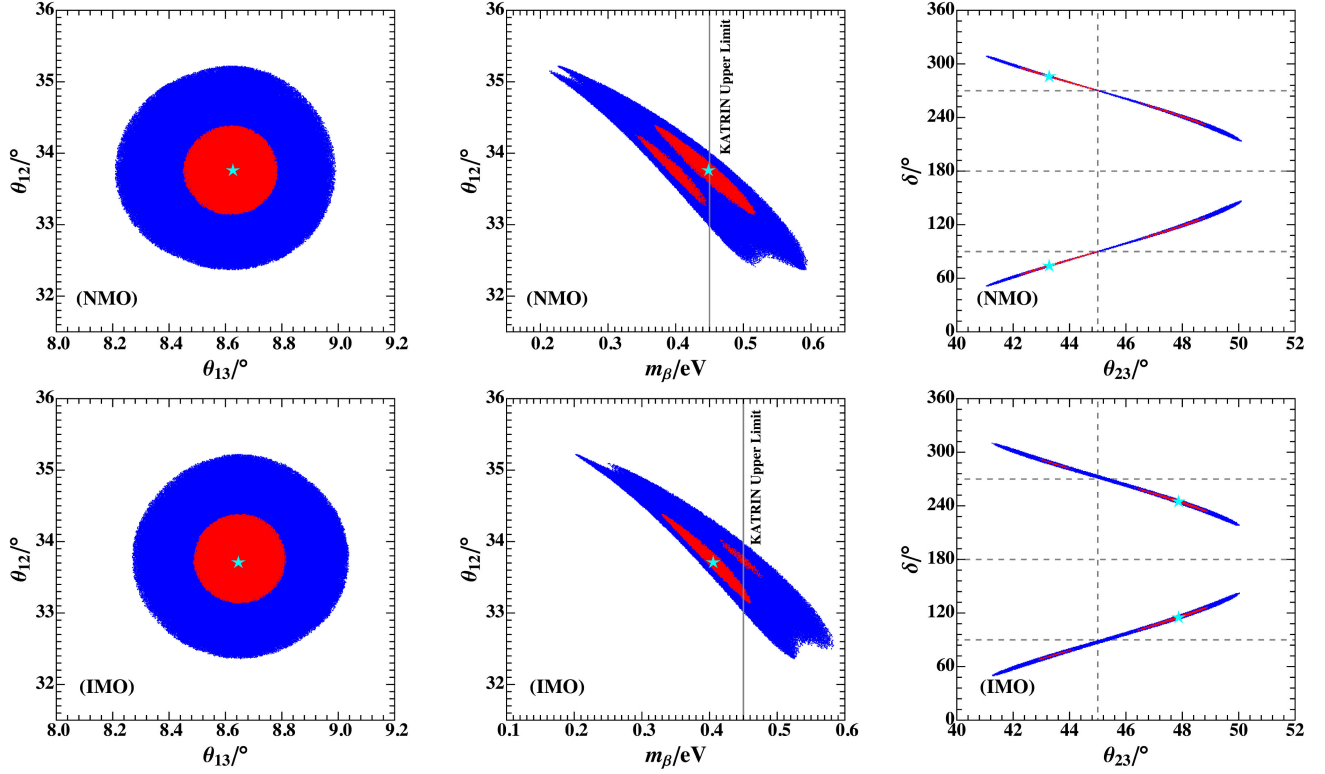


FIG. 9. Correlations between $(\theta_{12}, \theta_{13})$, (θ_{12}, m_β) , (δ, θ_{23}) parameter pairs at Λ_{EW} in the Dirac case with the TM2 mixing pattern. The upper row is for the NMO case, and the lower one is for the IMO case. The cyan star and red and blue regions correspond to $\chi^2 = \chi^2_{\min}$, $\Delta\chi^2 \leq 2.30$, and $\Delta\chi^2 \leq 11.83$, respectively.

contrast to the correlation found in the TM1 mixing pattern. More specifically, the first and fourth quadrants of δ correspond to the first octant of θ_{23} , while the second and third quadrants of δ correspond to the second octant of θ_{23} . This difference between the TM1 and TM2 mixing patterns arises from their distinct δ - θ_{23} relations. The TM1 δ - θ_{23} relation is given in Eq. (6), and the one in the TM2 pattern is found to be

$$\cos \delta = \frac{\cot 2\theta_{23}(1 - 2 \sin^2 \theta_{13})}{\sin \theta_{13} \sqrt{2 - 3 \sin^2 \theta_{13}}}. \quad (22)$$

Obviously, $\cos \delta$ takes opposite signs in the TM1 and TM2 mixing patterns, given that the value of θ_{13} is small.

- (iii) The two middle panels of Fig. 9 show the allowed region of θ_{12} and m_β , together with the KATRIN upper limit. In the NMO case, the best-fit point and nearly half of 1σ and 3σ regions are below the KATRIN bound, and in the IMO case, the best-fit point and most 1σ region remain below the limit. Therefore, the current experimental data show a slight preference for the IMO case over the NMO case. In addition, if the KATRIN experiment

achieves its final sensitivity of sub-300 meV [82] without any discovery, then most of the currently allowed parameter space including the entire 1σ regions will be excluded in both the NMO and IMO cases.

Before ending this section, we briefly comment on cosmological constraints on the sum of neutrino masses. Within the Λ cold dark matter (Λ CDM) + $\sum m_i$ framework, the combination of cosmic microwave background and baryon acoustic oscillation data has yielded an increasingly stringent upper limit of $\sum m_i \lesssim 0.07$ eV at the 95% confidence level [85–87]. Notably, this bound lies below the minimal value $\sum m_i \simeq 0.1$ eV allowed by neutrino oscillation data in the IMO case and is close to the minimal value $\sum m_i \simeq 0.06$ eV in the NMO case. Furthermore, a negative total neutrino mass appears statistically favored when the physical prior $\sum m_i > 0$ is relaxed (see, e.g., Refs. [88–92]). Indeed, these cosmological bounds depend sensitively on datasets included in the analysis, analysis methodology, and cosmological assumptions, and can be relaxed to the sub-eV scale in extended scenarios, such as models with dynamical dark energy [1,20,85–87,93–95]. From this perspective, the TM2 mixing pattern that requires $0.2 \text{ eV} \lesssim m_{\text{lightest}} \lesssim 0.6 \text{ eV}$ is robustly disfavored by current cosmological observations, whereas the TM1 mixing pattern remains consistent with

oscillation data at the 1σ level. Nevertheless, the relatively large mass scale $m_{\text{lightest}} \sim 0.15$ eV (or 0.23 eV) required by the best-fit point of the TM1 mixing pattern in the Majorana (or Dirac) case is in significant tension with the most restrictive cosmological limits within the minimal Λ CDM or its simple extensions. This tension appears difficult to resolve through standard cosmological modifications alone. However, if one takes into account threshold effects potentially enhancing RG corrections to the mixing angles in models with nondegenerate heavy particles [63,75,96], then it is possible to reduce the required neutrino mass scale and thereby alleviate its tension with the cosmological constraints.

VI. SUMMARY

The JUNO experiment has very recently released its first measurement results based on 59.1 days of data and provided a world-leading determination of θ_{12} and Δm_{21}^2 . This improvement in the precision of θ_{12} poses a greater challenge to various neutrino mass models or flavor mixing patterns that predict a specific value of θ_{12} or establish correlations of θ_{12} with other mixing parameters. Against this background, we have investigated the impact of the latest JUNO results on the TM1 and TM2 mixing patterns, which are two popular variants of the tribimaximal mixing pattern and give intriguing correlations between θ_{12} and θ_{13} . These TM1 and TM2 θ_{12} - θ_{13} correlations were consistent with the NuFIT6.0 global analysis of all available experimental data prior to JUNO's results within the 1σ and 3σ levels, respectively, but now the former sits on the edge of the allowed 1σ region, while the latter lies outside of the allowed 3σ region once the latest JUNO measurement is included.

To explore the possibility of reconciling the TM1 and TM2 mixing patterns with experimental data, we have further investigated their RG running effects from a typical superhigh energy scale down to the electroweak scale, assuming no additional physics thresholds in between, in both Majorana and Dirac neutrino cases. Through detailed analytical and numerical analyses, we have demonstrated that RG corrections can significantly modify the θ_{12} - θ_{13} correlations and, fortunately, bring them into perfect agreement with the latest JUNO measurement when neutrino masses are nearly degenerate. However, in the Majorana case, this required nearly degenerate neutrino mass spectrum is strongly constrained by the upper limit on the effective neutrino mass $m_{\beta\beta}$ probed in neutrinoless double beta decay experiments, while in the Dirac case such

constraints are absent. Consequently, the TM mixing patterns with Dirac neutrinos receive more support from both oscillation and nonoscillation experiments than those with Majorana neutrinos. The TM1 mixing pattern in the Majorana case still retains viable parameter space that lies below the KamLAND-Zen upper limit or within its uncertainty for both neutrino mass orderings, particularly after introducing two additional rephasing phases. By contrast, almost the entire parameter space of the TM2 mixing pattern in the Majorana case has been excluded by the KamLAND-Zen limit for both neutrino mass orderings. Furthermore, among the two TM mixing patterns, current experimental data clearly favor TM1 over TM2, regardless of whether RG corrections are included. This point in the Majorana case has been clearly described above. In the Dirac case, although the constraints from neutrinoless double beta decay experiments are absent, the parameter space of the TM2 mixing pattern is partially excluded by the current KATRIN upper limit and could be essentially ruled out once the KATRIN experiment reaches its final sensitivity without any discovery. In contrast, the TM1 mixing pattern remains safely below both the present and projected KATRIN limits.

Finally, we note that our results ignoring small parameter correlations in the defined χ^2 function in Eq. (19) serve as a robust first approximation, as a fully comprehensive analysis incorporating all exact multidimensional parameter correlations is beyond the scope of the present work. Nevertheless, with more forthcoming data expected from JUNO and other oscillation and nonoscillation experiments, the precision of lepton flavor mixing parameters will be further improved, and additional information on the absolute neutrino masses will be obtained. These advances will not only provide decisive tests of the two attractive mixing patterns but also offer valuable insights into the underlying lepton flavor structure.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [97,98].

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