Gauge-invariant fermion

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We show that in quantum electrodynamics the Green's functions of the Dirac-dressed fermion coincide, for any value of the gauge parameter in the R_ξ gauge and to all orders in perturbation theory, with those of the undressed fermion in the Landau gauge. The dressed fermion couples only to the transverse photon, and the gauge parameter is confined to a decoupled sector. The dressed vertex then obeys a homogeneous Ward identity, and the dressed field is renormalized multiplicatively with no new counterterm. A partially dressed family realizes the Landau-Khalatnikov-Fradkin relation exactly, and dressings with other kernels reproduce other gauges. Every dressing yields the same pole mass, and assigning the off-shell differences between dressings to the vertex defines a common gauge-invariant mass function.

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I. INTRODUCTION

A gauge-invariant charged field was first constructed by Dirac [1], who attached a "photon cloud" to the bare electron field,

$$\psi_D(x) = e^{ie\chi(x)}\psi(x). \tag{1}$$

Dirac's original choice $\chi = \partial_i A_i/\nabla^2$ creates the electron together with its Coulomb field. In this paper we use the Lorentz-covariant kernel

$$\chi(x) = \frac{\partial_\mu A^\mu}{\partial^2}(x), \tag{2}$$

which is the dressing associated with the Lorenz condition [2–4]. Following common usage, we refer to Eqs. (1) and (2) as the Dirac dressing. Under a gauge transformation $A_\mu \to A_\mu + \partial_\mu\Lambda$, $\psi \to e^{-ie\Lambda}\psi$, the dressing transforms as $\chi \to \chi + \Lambda$, so ψ_D is gauge invariant. We call ψ_D the dressed fermion.

The inverse d'Alembertian makes ψ_D nonlocal in spacetime. This raises the questions whether the dressed field can be treated with the standard tools of perturbative quantum field theory, whether it requires a separate composite-operator renormalization [5] and how it depends on the choice of dressing [3,6]. See also Ref. [7] for a recent operator-algebraic approach to charged states and Ref. [4] for the relation between dressings and gauge fixings.

It is known that a gauge fixing can be used to construct a dressing, and that the dressed field reduces to the undressed one in the corresponding gauge [2,3]. For the Dirac dressing (2), this gauge is the Landau gauge. One-loop propagators of dressed electrons were computed in Refs. [6,8,9], where it was also shown that dressings adapted to the velocity of the charge admit an infrared-finite mass-shell renormalization. Propagators in different covariant gauges are related by the Landau-Khalatnikov-Fradkin (LKF) transformation [10–13]. The propagator of the dressed fermion equals the Landau-gauge propagator, and the LKF transformation follows from a field-dependent gauge transformation that rescales the gauge parameter [14]. In non-Abelian theories, the Green's functions of a suitably dressed quark field are independent of the gauge parameter [14–16].

In this paper we give an explicit all-orders proof of these relations for the Dirac dressing in QED, covering all Green's functions of the dressed fermion and the transverse photon. Our main result is the following statement.

Proposition. In perturbative QED, for any value of ξ in the R_ξ gauge, the Green's functions of ψ_D, $\bar\psi_D$ and the transverse photon A^T_μ coincide with the Green's functions of ψ, $\bar\psi$ and A_μ in the Landau gauge, $\xi = 0$.

The proof in Sec. IV is an exact change of variables in the regularized path integral of local QED. It does not take the nonlocal transformed Lagrangian as a starting point. The consequences are derived in the subsequent sections. The dressed photon vertex obeys a homogeneous Ward identity, and the dressed field is renormalized multiplicatively by the Landau-gauge wave-function renormalization, with no new counterterm and no operator mixing (Sec. V). A one-parameter family of partially dressed fields realizes the

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LKF transformation exactly (Sec. VI). Dressings with other kernels reproduce the Green's functions of other gauges, and a gauge-invariant Wilson-line bilocal is a different object whose contour dependence is physical (Sec. VII). All dressings share the pole mass. Off shell, the mass function of the dressed field has the Landau form. Assigning its difference from the Feynman form to the vertex, as in the pinch technique, yields the gauge-invariant mass function of Refs. [17,18] (Sec. VIII).

We restrict the analysis to QED. The dressing remains nonlocal, and we limit our discussion of ψ_D to its Green's functions.

II. TRANSVERSE COUPLING OF THE DRESSED FIELD

We define the inverse d'Alembertian in Euclidean space, where it is unambiguous for fields that vanish at infinity, as in the Euclidean construction of the gauge-invariant transverse field in Ref. [14]. After continuation to Minkowski space [19–21] this is the Feynman prescription, $1/\partial^2 \to -1/(k^2 + i\epsilon)$ in momentum space, the same prescription that defines the projector in the Landau-gauge photon propagator. A retarded or advanced Green's function would define a different operator, to which the results below need not apply.

Substituting $\psi = e^{-ie\chi}\psi_D$ into the QED Lagrangian and using $i\partial_\mu(e^{-ie\chi}\psi_D) = e^{-ie\chi}(i\partial_\mu + e\partial_\mu\chi)\psi_D$, we obtain

$$\bar\psi(i\not\partial - e\not A - m)\psi = \bar\psi_D(i\not\partial - e\gamma^\mu A^T_\mu - m)\psi_D, \quad (3)$$

where

$$A^T_\mu = P^T_{\mu\nu}A^\nu, \qquad P^T_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}. \quad (4)$$

The dressed fermion couples only to the transverse photon. The projector $P^T_{\mu\nu}$, and with it the interaction in Eq. (3), is nonlocal in spacetime, so Eq. (3) is not a conventional local Lagrangian density. We use it only as an algebraic identity and base the all-orders argument of Sec. IV on local QED.

Reading off the interaction in Eq. (3) in momentum space, where $\partial_\mu \to -ik_\mu$ and $\partial^2 \to -k^2$, the tree-level dressed vertex coupling to A_μ with photon momentum k is $-ie\Gamma^\mu_{D,0}$ with [3,6]

$$\Gamma^\mu_{D,0} \equiv \gamma_\nu P^{T\nu\mu}(k) \equiv \gamma^\mu - \frac{\not k k^\mu}{k^2}. \quad (5)$$

The subscript 0 distinguishes this tree-level vertex from the full one-particle irreducible (1PI) vertex Γ^μ_D that we deal with later. The coupling e is unchanged; the dressing modifies the Lorentz structure, not the coupling strength. The tree-level vertex is transverse,

$$\Gamma^\mu_{D,0}k_\mu = \not k - \frac{\not k k^2}{k^2} = 0. \quad (6)$$

In Sec. V we show that this is the tree-level form of an exact Ward identity.

III. ONE-LOOP SELF-ENERGY

With the tree-level vertex (5), the one-loop self-energy of the dressed fermion is

$$-i\Sigma^{(\psi_D)}(q) = (-ie)^2\int\frac{d^dk}{(2\pi)^d}\Gamma^\mu_{D,0}S_F(q-k)\Gamma^\nu_{D,0}D_{\mu\nu}(k), \quad (7)$$

where $S_F(k) = i/(\not k - m)$ and the R_ξ photon propagator splits into transverse and longitudinal parts,

$$\begin{aligned} D_{\mu\nu}(k) &= D^T_{\mu\nu}(k) + D^L_{\mu\nu}(k),\\ D^T_{\mu\nu}(k) &\equiv -\frac{i}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right),\\ D^L_{\mu\nu}(k) &\equiv -i\xi\frac{k_\mu}{k^2}\frac{k_\nu}{k^2}. \end{aligned} \quad (8)$$

The longitudinal part factorizes into one factor k_μ/k^2 for each end of the photon line.[1] At each end the dressing absorbs the factor k_μ/k^2, $\Gamma^\mu_{D,0}k_\mu/k^2 = 0$ by Eq. (6), so

$$\Gamma^\mu_{D,0}S_F(q-k)\Gamma^\nu_{D,0}D^L_{\mu\nu}(k) = 0. \quad (9)$$

Only the transverse propagator contributes. Since $D^T_{\mu\nu}k^\nu = 0$, the dressing correction $-\not k k^\mu/k^2$ in $\Gamma^\mu_{D,0}$ also drops out, $\Gamma^\mu_{D,0}D^T_{\mu\nu} = \gamma^\mu D^T_{\mu\nu}$. At one loop, the self-energy of the dressed field is therefore

$$\hat\Sigma^{(\psi_D)}(q) = \Sigma_{\xi=0}(q), \quad (10)$$

the conventional self-energy in the Landau gauge. Here and below, a hat denotes a self-energy that does not depend on the gauge parameter. Equation (10) is the result of this explicit one-loop calculation. Its extension to all orders follows separately from the argument of Sec. IV.

The subscript $\xi = 0$ labels the form of the result, not the gauge used in the computation. The calculation is performed in an arbitrary R_ξ gauge, and the transversality of $\Gamma^\mu_{D,0}$ projects the answer onto $\Sigma_{\xi=0}$ regardless of ξ. No gauge condition is imposed, so this is not gauge fixing. The Landau form is an output of the calculation, not an input. Following Ref. [18] we write the conventional one-loop self-energy as

[1]Each factor $1/k^2$ is the inverse d'Alembertian of the dressing (2) under the replacement $\partial^2 \to -k^2$ of Sec. II. Since $k^\mu k^\nu D_{\mu\nu}(k) = -i\xi$ is momentum independent, $D^L_{\mu\nu}$ is the propagator of $\partial_\mu\chi$, with one inverse d'Alembertian from χ at each end of the line (Sec. IV).

$$\Sigma_\xi(q) = \Sigma_{\xi=1}(q) + (1-\xi)(\not{q} - m)\Omega(q). \quad (11)$$

Here $(1-\xi)(\not{q}-m)\Omega(q)$ is the contribution of the term $i(1-\xi)k_\mu k_\nu/(k^2)^2$, by which the R_ξ propagator differs from the Feynman-gauge one. Applying the Ward-Takahashi identity $\not{k} = (\not{q}-m) - (\not{q}-\not{k}-m)$ at both vertices shows that this contribution is proportional to $\not{q}-m$, with the coefficient

$$\Omega(q) = ie^2 \int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2)^2}\left[\frac{1}{\not{q}-\not{k}-m}(\not{q}-m) - 1\right].$$

Thus $\Omega(q)$ is a combination of the unit matrix and $\not{q}$ with coefficients depending on q^2, and it is ultraviolet divergent. Its explicit form in terms of standard one-loop functions is given in Ref. [18]. Equation (11) is a one-loop relation, since beyond one loop the self-energy contains higher powers of ξ, and the same holds for the relations derived from it below. The longitudinal contribution $\Sigma_\xi - \Sigma_{\xi=0} = -\xi(\not{q}-m)\Omega(q)$, which vanishes at the pole, is what the transversality of the tree-level vertex eliminates.

IV. EQUIVALENCE TO ALL ORDERS

The result (10) extends to all orders and to all Green's functions of the dressed fields. We start from local QED in the R_ξ gauge with dimensional regularization and couple sources to the gauge-invariant fields ψ_D, $\bar\psi_D$ and A^T_μ,

$$Z[\eta,\bar\eta,J] = \int \mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}A \times \exp\left\{iS_\xi + i\int d^dx(\bar\eta\psi_D + \bar\psi_D\eta + J^\mu A^T_\mu)\right\}, \quad (12)$$

where

$$S_\xi = \int d^dx\left[\bar\psi(i\not{\partial} - e\not{A} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2\right]. \quad (13)$$

We change variables to

$$A_\mu = A^T_\mu + \partial_\mu\chi, \qquad \psi = e^{-ie\chi}\psi_D, \qquad \bar\psi = \bar\psi_D e^{ie\chi}, \quad (14)$$

with $\partial^\mu A^T_\mu = 0$ and χ given by Eq. (2). The decomposition of the gauge field in Eq. (14) is an identity for every gauge-field configuration that vanishes at infinity, not a gauge condition. Under a gauge transformation, A^T_μ is invariant and $\chi \to \chi + \Lambda$, so A^T_μ labels the gauge orbit and χ the position along it. The path integral still runs over all configurations, and the gauge is fixed only by the R_ξ term, which in these variables weights χ alone. The Landau-gauge configurations $\chi = 0$ are reached only in the limit $\xi \to 0$ of this weight, which we use below to evaluate Green's functions already shown to be independent of ξ. In the language of Ref. [4], A^T_μ is a dressed field rather than a gauge-fixed potential.

The change of the gauge field is linear, with a field-independent Jacobian. The fermionic Jacobians of ψ and $\bar\psi$ cancel, since the transformation is a local vector phase rotation having no anomaly. The field strength depends only on A^T_μ, the gauge-fixing term depends only on $\partial_\mu A^\mu = \partial^2\chi$ and the fermion Lagrangian becomes Eq. (3). The action therefore separates,

$$S_\xi = S_{\rm T}[\psi_D,\bar\psi_D,A^T] - \frac{1}{2\xi}\int d^dx(\partial^2\chi)^2, \quad (15)$$

with

$$S_{\rm T} = \int d^dx\left[\bar\psi_D(i\not{\partial} - e\gamma^\mu A^T_\mu - m)\psi_D - \frac{1}{4}F_{\mu\nu}[A^T]F^{\mu\nu}[A^T]\right]. \quad (16)$$

Neither $S_{\rm T}$ nor the sources in Eq. (12) contain ξ or χ. The integral over χ is Gaussian, factorizes and cancels in normalized Green's functions. Every Green's function of ψ_D, $\bar\psi_D$ and A^T_μ is therefore independent of ξ.

Its value can be read off at $\xi \to 0$, where the Euclidean Gaussian weight enforces $\partial^2\chi = 0$ and hence $\chi = 0$ for fields that vanish at infinity. There $\psi_D = \psi$ and $A^T_\mu = A_\mu$, which proves the Proposition,

$$\langle\psi_D\cdots\bar\psi_D\cdots A^T\cdots\rangle_\xi = \langle\psi\cdots\bar\psi\cdots A\cdots\rangle_{\xi=0}, \quad (17)$$

for regularized Green's functions. In particular, the 1PI self-energy of ψ_D equals the Landau-gauge self-energy to all orders, $\hat\Sigma^{(\psi_D)} = \Sigma_{\xi=0}$, of which Eq. (10) is the one-loop case. Renormalization is discussed in Sec. V.

The separation (15) is the all-orders form of the mechanism of Sec. III. In momentum space, the tree-level coupling to A^T_μ is the projected vertex (5), and the propagator of A^T_μ is $D^T_{\mu\nu}$. The longitudinal part $D^L_{\mu\nu} = -i\xi(k_\mu/k^2)(k_\nu/k^2)$ is the propagator of $\partial_\mu\chi$, which never reaches the dressed sector. Equation (9) is this statement for a single photon line. The change of variables justifies the use of the projected vertex at every vertex and to all orders, starting from local QED.

Three remarks are in order. First, the argument holds equally with a photon-mass regulator, since $A_\mu A^\mu = A^T_\mu A^{T\mu} + \partial_\mu\chi\partial^\mu\chi$ up to a total derivative, so a mass term also separates. Second, the argument does not use the locality of Eq. (3). The regularized theory is local QED, and its renormalization is that of local QED in the Landau gauge. Third, there is no Gribov ambiguity [22,23] in QED,

and the Landau-gauge limit is stable under renormalization, since $\xi_B = Z_3\xi$.

V. WARD IDENTITY, RENORMALIZATION AND NONLOCALITY

A. Homogeneous Ward identity

In the variables (14) the photon enters the dressed-fermion sector only through $A^T_\mu = P^T_{\mu\nu}A^\nu$. Every attachment of an external photon with momentum k therefore carries the projector $P^T(k)$, and by the Proposition the full 1PI photon vertex Γ^μ_D of the dressed field is the transverse part of the full Landau-gauge vertex $\Gamma^\mu_{\rm L}$,

$$\Gamma^\mu_D(p+k,p) = P^{T\mu}{}_\nu(k)\Gamma^\nu_{\rm L}(p+k,p),$$
$$k_\mu\Gamma^\mu_D(p+k,p) = 0, \tag{18}$$

to all orders. At tree level $\Gamma^\nu_{\rm L} = \gamma^\nu$, and Eq. (18) reduces to the tree-level vertex (5) and its transversality (6). The homogeneous Ward identity expresses the local gauge invariance of ψ_D, whose gauge transformation acts only on χ.

The dressed field still carries the global charge, since the conserved current is unchanged, $\bar\psi_D\gamma^\mu\psi_D = \bar\psi\gamma^\mu\psi$. Insertions of this current obey the usual inhomogeneous Ward-Takahashi identity,

$$k_\mu\Lambda^\mu(p+k,p) = S^{-1}_{\rm L}(p+k) - S^{-1}_{\rm L}(p), \tag{19}$$

where Λ^μ is the amputated current vertex and $S^{-1}_{\rm L}(p) = \not{p} - m_B - \Sigma_{\xi=0}(p)$ is the inverse propagator of ψ_D, with m_B the bare mass. Since the mass cancels in the difference, the identity holds equally for the renormalized quantities below, where $Z_1 = Z_2$. The two identities are compatible because the projector in Eq. (18) removes the longitudinal part of the vertex, which is the part fixed by Eq. (19).

B. Renormalization

Landau-gauge QED is renormalizable, with $\psi^{\rm un} = (Z^{\rm L}_2)^{1/2}\psi$, $A^{\rm un}_\mu = Z_3^{1/2}A_\mu$, $m_B = Z_m m$ and $e_B = \mu^\epsilon Z_e e$ in $d = 4-2\epsilon$ dimensions, where the Ward identity gives $Z_e Z_3^{1/2} = 1$. Since χ is linear in the gauge field, $\chi^{\rm un} = Z_3^{1/2}\chi$ and

$$e_B\chi^{\rm un} = \mu^\epsilon e\chi. \tag{20}$$

By the Proposition, the unrenormalized dressed field $\psi^{\rm un}_D = e^{ie_B\chi^{\rm un}}\psi^{\rm un}$ is renormalized multiplicatively as

$$\psi^{\rm un}_D = (Z^{\rm L}_2)^{1/2}\psi_D, \tag{21}$$

in every gauge. Although ψ_D contains arbitrarily many photon fields and inverse d'Alembertians, it requires no new counterterm and does not mix with other operators.

In a gauge $\xi \neq 0$, where $\psi^{\rm un} = (Z^{(\xi)}_2)^{1/2}\psi$, Eq. (21) reads

$$\psi_D = \left(\frac{Z^{(\xi)}_2}{Z^{\rm L}_2}\right)^{1/2} e^{i\mu^\epsilon e\chi}\psi. \tag{22}$$

The product $e^{i\mu^\epsilon e\chi}\psi$ of renormalized fields thus requires the factor $(Z^{(\xi)}_2/Z^{\rm L}_2)^{1/2}$, which is the gauge dependence of the wave-function renormalization determined by the LKF transformation [12]. At one loop in minimal subtraction $Z^{(\xi)}_2 = 1 - \xi\alpha/(4\pi\epsilon)$, so $Z^{\rm L}_2$ is finite at this order, consistent with Eq. (10). This is the only renormalization the dressing requires.

C. Nonlocality

These results do not make ψ_D local. The projector P^T, the interaction in Eq. (3) and the dressed field itself are nonlocal in spacetime. What the Proposition shows is that this nonlocality does not obstruct perturbative calculation or renormalization, because the dressed Green's functions are those of local field theory in the Landau gauge.

The locality of the Landau-gauge field ψ does not transfer to ψ_D as an operator. In a covariant gauge, ψ is local but not gauge invariant, and it acts in a space with indefinite metric. In a positive-metric space where Gauss's law holds, physical charged states cannot be created by local operators [7,24,25], and the nonlocality of a dressing reflects this fact. Equal-time commutation relations, the asymptotic behavior entering the Lehmann-Symanzik-Zimmermann reduction formula, causality and cluster decomposition concern the operator ψ_D itself, and we do not address them here.

VI. PARTIALLY DRESSED FIELDS AND THE LKF TRANSFORMATION

Consider the family

$$\psi_\kappa \equiv e^{-i\kappa e\chi}\psi_D = e^{i(1-\kappa)e\chi}\psi \tag{23}$$

with real κ. Under a gauge transformation $\psi_\kappa \to e^{-i\kappa e\Lambda}\psi_\kappa$, so ψ_κ is gauge invariant only at $\kappa = 0$, and $\kappa = 1$ is the undressed field.

At tree level, ψ_κ couples to the photon through the vertex $\Gamma^\mu_{\kappa,0} = \gamma^\mu - (1-\kappa)\not{k}k^\mu/k^2$. Each end of the longitudinal propagator (8) then contributes $\Gamma^\mu_{\kappa,0}k_\mu/k^2 = \kappa\not{k}/k^2$, so the longitudinal exchange is rescaled by κ^2, one factor κ from each vertex. The partial dressing thus acts as a square root of a change of the gauge parameter. The following factorization makes this exact. In the variables of Sec. IV, χ is a free field decoupled from ψ_D, with propagator $\langle\chi\chi\rangle_\xi = \xi\Delta$,

$$\Delta(x) = \int \frac{d^dk}{(2\pi)^d}\frac{-ie^{-ik\cdot x}}{(k^2+i\epsilon)^2}, \tag{24}$$

so that the two-point function factorizes,

$$\langle\psi_\kappa(x)\bar{\psi}_\kappa(y)\rangle_\xi = \langle\psi_D(x)\bar{\psi}_D(y)\rangle \times e^{-\kappa^2 e^2\xi[\Delta(0)-\Delta(x-y)]}. \tag{25}$$

For $\kappa = 1$, this is the LKF transformation from the Landau gauge [10–13]. Comparing the two, the propagator of ψ_κ in the gauge ξ equals the propagator of ψ in the gauge

$$\xi' = \kappa^2\xi, \tag{26}$$

exactly. The same holds for all Green's functions of ψ_κ, $\bar{\psi}_\kappa$ and A^T_μ, since the Gaussian average of the phases, which contains the propagator of χ between every pair of fermion insertions, depends on κ and ξ only through $\kappa^2\xi$.[2] At one loop, where Σ_ξ is linear in ξ, the self-energy of ψ_κ in the gauge ξ is

$$\begin{aligned}\Sigma^{(\psi_\kappa)}_\xi(q) &= \Sigma_{\xi'}(q) \\ &= \Sigma_{\xi=0}(q) + \frac{\xi'}{\xi}[\Sigma_\xi(q) - \Sigma_{\xi=0}(q)].\end{aligned} \tag{27}$$

A field redefinition and a change of the gauge parameter are different operations. Equation (26) is an identity between Green's functions that relates the two, and it is exact in QED. The value $\kappa = 0$ gives the unique gauge-invariant member of the family, where $\xi' = 0$ for every ξ and the Green's functions no longer depend on the gauge. The map $\xi \to \xi'$ is invertible for $\kappa \neq 0$, while at $\kappa = 0$ it sends every ξ to $\xi' = 0$, so the dressed field carries no memory of the gauge in which it is computed.

VII. OTHER DRESSINGS AND WILSON LINES

A. General kernels

A general linear dressing has the form

$$\chi_V(x) = \int d^4y V^\mu(x-y)A_\mu(y), \tag{28}$$

where V^μ is a nonlocal linear kernel rather than an ordinary vector. Under $A_\mu \to A_\mu + \partial_\mu\Lambda$, with Λ vanishing at infinity, the dressing shifts as $\chi_V \to \chi_V + \Lambda$, so that $\psi_V = e^{ie\chi_V}\psi$ is gauge invariant, provided

$$\frac{\partial}{\partial x^\mu}V^\mu(x-y) = \delta^4(x-y). \tag{29}$$

With the Fourier conventions

$$A_\mu(x) = \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot x}\tilde{A}_\mu(k),$$

$$V^\mu(x) = i\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot x}v^\mu(k),$$

the dressing becomes $\tilde{\chi}_V(k) = iv^\mu(k)\tilde{A}_\mu(k)$, and the condition (29) has the general solution

$$\begin{aligned}&k_\mu v^\mu(k) = 1, \qquad v^\mu(k) = \frac{k^\mu}{k^2} + v^\mu_T(k), \\ &k_\mu v^\mu_T(k) = 0.\end{aligned} \tag{30}$$

Gauge invariance thus fixes the longitudinal part of the kernel, while the transverse part v^μ_T parametrizes the freedom in choosing different dressings. The Dirac dressing (2) has $V^\mu(x-y) = \partial G(x-y)/\partial x_\mu$, with G the Green's function of ∂^2 defined in Sec. II, and $v^\mu_T = 0$. Dirac's original kernel $\chi = \partial_i A_i/\nabla^2$ has $v^\mu = [k^\mu - (u\cdot k)u^\mu]/[k^2 - (u\cdot k)^2]$, with u^μ the unit vector in the time direction, and a straight Wilson line joining x to infinity along a fixed direction n^μ has $v^\mu = n^\mu/(n\cdot k)$.

Since χ_V shifts by Λ, the Green's functions of ψ_V can be evaluated in the gauge $\chi_V = 0$, where $\psi_V = \psi$. They are those of ψ in that gauge, with the pole prescription fixed by the kernel. The tree-level vertex of ψ_V is $\gamma^\mu - \not{k}v^\mu = \Gamma^\mu_{D,0} - \not{k}v^\mu_T$, and the fermion couples to $(g_{\mu\nu} - k_\mu v_\nu)A^\nu$. Since $(g_{\mu\nu} - k_\mu v_\nu)k^\nu = 0$ by Eq. (30), the longitudinal part of the photon propagator drops out for every kernel. For kernels odd in k, $v^\mu(-k) = -v^\mu(k)$, as for the Dirac and Coulomb kernels, the effective photon propagator is

$$\begin{aligned}&-\frac{i}{k^2}(g_{\mu\nu} - k_\mu v_\nu - v_\mu k_\nu + v^2 k_\mu k_\nu) \\ &= D^T_{\mu\nu} + \frac{i}{k^2}(k_\mu v_{T\nu} + v_{T\mu}k_\nu - v^2_T k_\mu k_\nu),\end{aligned} \tag{31}$$

the propagator of the gauge $v\cdot A = 0$. The transverse part of the kernel enters only through terms proportional to k_μ or k_ν. For $v^\mu_T \neq 0$ these terms remain in the off-shell self-energy, and the dressing correction does not drop out as it does in Sec. III. In this way Dirac's original kernel gives the Coulomb gauge, and the Wilson line gives the axial gauge $n\cdot A = 0$, with the pole at $n\cdot k = 0$ prescribed by the orientation of the line.

Different kernels therefore define physically different operators with different off-shell Green's functions. They also differ in their infrared behavior. Dressings adapted to the velocity of the charge admit an infrared-finite mass-shell renormalization [8,9], in the spirit of the asymptotic dressings of Kulish and Faddeev [26], while the Dirac dressing (2) inherits the infrared structure of the Landau gauge. What all dressings share is the position of the mass-shell singularity, because the pole mass is gauge independent [27–31].

[2]Equation (26) is the relation $\alpha' = (1+X)^2\alpha$ obtained in Ref. [14] from a field-dependent gauge transformation, with $1 + X = \kappa$.

B. Wilson-line bilocals

A different gauge-invariant object is the bilocal

$$G_C(x,y) = \left\langle \bar{\psi}(y) e^{-ie\int_C dz^\mu A_\mu(z)} \psi(x) \right\rangle, \tag{32}$$

with a contour C from x to y. It is the propagator of neither ψ nor ψ_D. Using $A_\mu = A^T_\mu + \partial_\mu \chi$ and $\int_C dz^\mu \partial_\mu \chi = \chi(y) - \chi(x)$, one finds

$$G_C(x,y) = \left\langle \bar{\psi}_D(y) e^{-ie\int_C dz^\mu A^T_\mu(z)} \psi_D(x) \right\rangle. \tag{33}$$

The bilocal thus differs from the dressed propagator by a gauge-invariant transverse Wilson factor, which is not unity. Two contours C_1 and C_2 differ by the closed loop

$$e^{-ie\oint_{C_1-C_2} dz^\mu A_\mu} = e^{-\frac{ie}{2}\int_S d\sigma^{\mu\nu} F_{\mu\nu}}, \tag{34}$$

where $\mathcal{S}$ is a surface bounded by $C_1 - C_2$. The contour dependence is a physical flux, not a gauge artifact, and the results for the self-energy of ψ_D do not apply to G_C.

VIII. THE MASS

A. Mass function of the dressed field

The Landau-gauge self-energy (10) contains ultraviolet divergences. The on-shell subtraction

$$m_D(q) = m + \hat{\Sigma}^{(\psi_D)}(q) - \hat{\Sigma}^{(\psi_D)}(m) - (\not{q} - m)\frac{d\hat{\Sigma}^{(\psi_D)}}{d\not{q}}(m) \tag{35}$$

removes them and defines the renormalized mass function of the dressed field [17,18,32]. The mass function $m_D(q)$ is a combination of the unit matrix and $\not{q}$, and the dressed propagator is $i/(\not{q} - m_D(q))$, with $m_D(m) = m$ and unit residue. With a massless photon, the derivative in the last term is infrared divergent in the Landau gauge, as is the conventional on-shell wave-function renormalization. Off shell, $m_D(q)$ is therefore defined at a fixed infrared regulator [18]. The pole mass is infrared finite.

B. Comparison with the pinch technique

For the undressed field, the pinch technique [33–37] rearranges the longitudinal parts of the self-energy, vertex and box graphs of an amplitude and yields the gauge-independent self-energy $\Sigma_{\xi=1}$. This holds at one and two loops [38]. The pinch-technique self-energy is a building block of amplitudes, not the propagator of ψ. By Eq. (11) the dressed and pinch-technique self-energies differ at one loop by

$$\Sigma_{\xi=0}(q) - \Sigma_{\xi=1}(q) = (\not{q} - m)\Omega(q), \tag{36}$$

which vanishes at the pole, so both give the same pole mass. The pinch technique extracts from the vertex and box graphs the longitudinal contributions that the dressing places in the vertex from the start. Writing $D_{\mu\nu} = D^T_{\mu\nu} + \xi D^{(1)}_{\mu\nu}$ with $D^{(1)}_{\mu\nu} = -ik_\mu k_\nu/(k^2)^2$, the pinch technique removes $(\xi - 1)D^{(1)}_{\mu\nu}$ from the self-energy, while the dressing removes $\xi D^{(1)}_{\mu\nu}$.

The difference (36) is kinetic, and in any amplitude it can be assigned to the vertices, where the Ward-Takahashi identity ties it to a compensating shift of the vertex correction. With the pinch-technique convention of Refs. [17,18], which assigns to the vertices every term the Ward-Takahashi identity can act on, the dressing terms in $\Gamma^\mu_{D,0}$ and the $k_\mu k_\nu$ term of $D^T_{\mu\nu}$ are assigned there as well. At one loop every dressing then leads to the same mass function, the one obtained from Eq. (35) with $\Sigma_{\xi=1}$ in place of $\hat{\Sigma}^{(\psi_D)}$. The propagator of ψ_D itself keeps the Landau form, and the two mass functions differ by $(\not{q} - m)[\Omega(q) - \Omega(m)]$, carried by the vertex. Other assignments select other reference gauges [18,39], while the pole and all amplitudes are the same for every choice.

IX. CONCLUSIONS AND OUTLOOK

For the Dirac dressing (2) in QED, the Green's functions of the dressed fermion and the transverse photon coincide, for any value of ξ in the R_ξ gauge and to all orders, with the Landau-gauge Green's functions of the undressed fields. The proof is a change of variables in local QED that confines the gauge parameter to a decoupled sector. It implies a homogeneous Ward identity for the dressed vertex, multiplicative renormalization of the dressed field by the Landau-gauge constant and the exact LKF relation (26) for partially dressed fields. Dressings with other kernels reproduce other gauges. They are physically different operators that share the pole mass. Wilson-line bilocals are distinct objects whose contour dependence is physical. The nonlocality of the dressing remains, but it does not obstruct perturbative calculation and renormalization. The infrared properties of the Dirac dressing (2) are those of the Landau gauge, and an infrared-finite description of charged asymptotic states requires dressings adapted to the velocity of the charge [8,9].

We have restricted the analysis to QED. The Abelian construction does not carry over directly to a non-Abelian theory. For $U = e^{ig\chi^a t^a}$ with $\chi^a = \partial \cdot A^a/\partial^2$, the transformed covariant derivative generates an infinite series of commutator terms, and χ^a does not shift additively under gauge transformations, so the transformed interaction is not the non-Abelian analog of Eq. (3). The appropriate generalization is a dressing $h[A]$, constructed order by order, for which the gauge field A^h is transverse. The Green's functions of $\psi^h = h^\dagger \psi$ are then independent of the gauge parameter and equal to the Landau-gauge ones, and a local

formulation with ψ^h is renormalizable to all orders [14–16]. Nonperturbatively, the Gribov ambiguity [22,23] obstructs a global dressing. An extension of the present analysis along these lines is left for separate work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

[1] P. A. M. Dirac, Can. J. Phys. **33**, 650 (1955).
[2] M. Lavelle and D. McMullan, Phys. Lett. B **371**, 83 (1996).
[3] M. Lavelle and D. McMullan, Phys. Rep. **279**, 1 (1997).
[4] P. Berghofer and J. François, Found. Phys. **54**, 72 (2024).
[5] J. Frohlich, G. Morchio, and F. Strocchi, Nucl. Phys. **B190**, 553 (1981).
[6] E. Bagan, M. Lavelle, and D. McMullan, Ann. Phys. (N.Y.) **282**, 503 (2000).
[7] P. A. Grassi and M. Porrati, J. High Energy Phys. 03 (2025) 175.
[8] E. Bagan, M. Lavelle, and D. McMullan, Phys. Lett. B **370**, 128 (1996).
[9] E. Bagan, M. Lavelle, and D. McMullan, Phys. Rev. D **56**, 3732 (1997).
[10] L. D. Landau and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **29**, 89 (1955); [Sov. Phys. JETP **2**, 69 (1956)].
[11] E. S. Fradkin, Zh. Eksp. Teor. Fiz. **29**, 258 (1955); [Sov. Phys. JETP **2**, 361 (1956)].
[12] K. Johnson and B. Zumino, Phys. Rev. Lett. **3**, 351 (1959).
[13] B. Zumino, J. Math. Phys. (N.Y.) **1**, 1 (1960).
[14] T. De Meerleer, D. Dudal, S. P. Sorella, P. Dall'Olio, and A. Bashir, Phys. Rev. D **97**, 074017 (2018).
[15] T. De Meerleer, D. Dudal, S. P. Sorella, P. Dall'Olio, and A. Bashir, Phys. Rev. D **101**, 085005 (2020).
[16] M. A. L. Capri, S. P. Sorella, and R. C. Terin, Ann. Phys. (Amsterdam) **414**, 168077 (2020).
[17] K.-S. Choi, arXiv:2604.04569.
[18] K.-S. Choi and H. Im, arXiv:2607.08426.
[19] G. C. Wick, Phys. Rev. **96**, 1124 (1954).
[20] K. Osterwalder and R. Schrader, Commun. Math. Phys. **31**, 83 (1973).
[21] K. Osterwalder and R. Schrader, Commun. Math. Phys. **42**, 281 (1975).
[22] V. N. Gribov, Nucl. Phys. **B139**, 1 (1978).
[23] I. M. Singer, Commun. Math. Phys. **60**, 7 (1978).
[24] R. Ferrari, L. E. Picasso, and F. Strocchi, Commun. Math. Phys. **35**, 25 (1974).
[25] D. Buchholz, Phys. Lett. B **174**, 331 (1986).
[26] P. P. Kulish and L. D. Faddeev, Theor. Math. Phys. **4**, 745 (1970).
[27] N. K. Nielsen, Nucl. Phys. **B101**, 173 (1975).
[28] R. Tarrach, Nucl. Phys. **B183**, 384 (1981).
[29] J. C. Breckenridge, M. J. Lavelle, and T. G. Steele, Z. Phys. C **65**, 155 (1995).
[30] A. S. Kronfeld, Phys. Rev. D **58**, 051501 (1998).
[31] P. Gambino and P. A. Grassi, Phys. Rev. D **62**, 076002 (2000).
[32] K.-S. Choi, J. Korean Phys. Soc. **84**, 591 (2024); **86**, 156(E) (2025).
[33] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
[34] J. M. Cornwall and J. Papavassiliou, Phys. Rev. D **40**, 3474 (1989).
[35] J. Papavassiliou, Phys. Rev. D **51**, 856 (1995).
[36] D. Binosi and J. Papavassiliou, Phys. Rep. **479**, 1 (2009).
[37] J. M. Cornwall, J. Papavassiliou, and D. Binosi, *The Pinch Technique and its Applications to Non-Abelian Gauge Theories* (Cambridge University Press, Cambridge, United Kingdom, 2011).
[38] D. Binosi and J. Papavassiliou, Phys. Rev. D **65**, 085003 (2002).
[39] A. Denner, G. Weiglein, and S. Dittmaier, Phys. Lett. B **333**, 420 (1994).