

Coupled-channel study of the three-body DDK and D^*D^*K

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We investigate the three-body DDK system with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$ within a coupled-channel framework that incorporates the DDK , D^*D^*K , $DD_s\eta$ and $DD_s\pi$ channels. The $D^{(*)}D^{(*)}$ interactions are described using the one-boson-exchange model constrained by heavy-quark symmetry and fitted to the pole positions of $X(3872)$, T_{cc}^+ , and $Z_c(3900)$. The $D^{(*)}K$ interaction is from the chiral effective field theory, motivated by the molecular interpretation of $D_{s0}^*(2317)$, and is further constrained by lattice-QCD results for the DK scattering lengths. The resulting three-body problem is solved using the Gaussian expansion method, while the complex scaling method is employed to search for possible resonant states. We find that coupled-channel effects from D^*D^*K are negligible, and the DDK system supports a deeply bound state across a wide range of parameters. Depending on the long-range behavior of the DK interaction, an additional shallow state may emerge near the particle-dimer (D - DK) threshold. The deeply bound state exhibits a compact three-body structure, whereas the shallow state displays characteristic features of a three-body halo configuration. Similar results are obtained for the D^*D^*K system. We also observe that the coupled-channel effects originating from the $DD_s\eta$ channel produce only minor corrections to the DDK bound state and lead to a modest reduction in its binding energy. By contrast, opening the $DD_s\pi$ channel substantially alters the nature of the three-body DDK state and transforms the bound state into a resonance. These findings may provide new insight into few-body dynamics in systems involving charmed mesons and kaons.

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I. INTRODUCTION

The spectroscopy of hadrons containing heavy quarks has undergone rapid development in recent decades. Numerous unconventional states have been observed that cannot be easily accommodated within the conventional quark-antiquark or three-quark classification scheme. These states are commonly referred to as exotic hadrons. The proposed models to interpret them include tetraquarks, pentaquarks, hadronic molecules, etc. Their existence provides valuable insight into the nonperturbative dynamics of quantum chromodynamics (QCD), see reviews [1–12].

One particularly important class of exotic candidates consists of hadronic molecules formed by two or more hadrons interacting through residual strong forces. In the

heavy-quark sector, heavy-quark spin symmetry (HQSS) implies close dynamical connections among systems containing heavy mesons such as D and D^* [13–16]. The systems composed of charmed mesons provide a natural laboratory for exploring molecular configurations. From an experimental perspective, many open/hidden-charm exotic hadrons have been observed since the discovery of $X(3872)$ by the Belle Collaboration in 2003 [17]. For example, the charged charmoniumlike state $Z_c(3900)$ was observed by the BESIII Collaboration [18] and Belle Collaboration [19] in 2013, the doubly charmed tetraquark state T_{cc}^+ was observed by the LHCb Collaboration in 2021 [20,21], and the hidden-charm pentaquark family P_c s were reported by the LHCb Collaboration in 2015 and 2019 [22,23], refer to [1–4,6,8–12,24–30] for comprehensive reviews.

Among the exotic hadrons observed experimentally so far, $D_{s0}^*(2317)$, observed by the BABAR Collaboration [31] and later confirmed by the CLEO [32] and Belle [33] Collaborations is particularly interesting. The unexpectedly low mass of $D_{s0}^*(2317)$ has been interpreted as the evidence for a DK hadronic molecule [34–39]. Its heavy-spin partner, $D_{s1}(2460)$, was observed by several experimental collaborations [32,33,40,41], and exhibits similarly

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anomalous properties of $D_{s0}^*(2317)$, making it a candidate for a D^*K molecular state [42–46] or a multiquark state [47]. Furthermore, the DK interaction has also been investigated by lattice-QCD calculations, in which the DK scattering length and the properties of $D_{s0}^*(2317)$ were extracted [48–54]. These studies indicate that the DK interaction in the isoscalar ($I = 0$) channel is strongly attractive. More analyses further sharpened this near-threshold picture from coupled-channel perspectives [55,56].

Motivated by the strong attraction in the DK channel, three-body systems composed of two charmed mesons and one kaon, such as DDK , have attracted sustained attention. Early studies suggested that such systems may support exotic bound states driven by the underlying two-body dynamics. In particular, the DD^*K system was found within the Born-Oppenheimer framework to admit a bound-state solution associated with a delocalized pion-bond mechanism [57]. A Gaussian expansion method analysis of the DK , DDK , and $DDDK$ systems showed that the strong DK attraction can generate a DDK bound state with $I(J^P) = \frac{1}{2}(0^-)$ and may even support a $DDDK$ four-body state [58]. Besides, within a coupled-channel approach, including channels $DD_s\eta$ and $DD_s\pi$, the DDK system was found to support a bound state [59]. Related few-body studies further emphasized that these systems provide a useful laboratory for understanding how effective two-body interactions give rise to nontrivial three-body dynamics [60]. More recently, the discussion has been extended to nearby analog systems: the $D\bar{D}K$ system was shown in the fixed-center approximation to develop a bound or resonant structure around 4180 MeV [61]. The lattice effective field theory study indicated that the DD^*K system remains bound even in the presence of repulsive three-body forces [62], and subsequent analyses showed that stronger three-body repulsion enlarges the spatial size of the $DD^{(*)}K$ molecule, eventually leading to a configuration resembling a $D^{(*)}K$ cluster plus a distant D meson, whereas the DDK system tends to preserve an isosceles-triangle geometry [63]. In addition, the recently predicted $J^{PC} = 0^{--} \bar{D}_s DK$ bound state suggests that such three-body molecules may provide further insight into the nature of the $D_{s0}^*(2317)$ and enrich the spectrum of exotic multihadron states [64]. Refer to [6,65] for interesting studies on the three-body $D^{(*)}D^{(*)}K$ -like systems. Furthermore, BK , as the heavy-flavor partner of DK , has also been the subject of several dedicated investigations [45,66–69].

Despite previous investigations, the structure of the DDK system remains incompletely understood. In particular, the role of coupled-channel dynamics involving the D^*D^*K configuration and the sensitivity of the three-body spectrum to the underlying two-body interactions require further clarification. The strong attraction in the DK

subsystem may generate nontrivial three-body dynamics, potentially leading to bound states or even resonances.

In this work, we perform a coupled-channel study of the DDK system incorporating both DDK and D^*D^*K configurations with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$. The two-body interactions are constructed using a hybrid approach that combines the one-boson-exchange (OBE) model for the $D^{(*)}D^{(*)}$ sector with chiral-motivated interactions for the $D^{(*)}K$ system. For the $D^{(*)}K$ interaction, we adopt the framework of Ref. [35]. In this model, once two cutoff parameters and a short-range repulsive coupling are specified, the remaining running coupling is fixed by reproducing the $D_{s0}^*(2317)$ pole position. To further reduce model ambiguity, we add one more constraint from the lattice-QCD results of the DK scattering length [48–50].

Regarding the $D^{(*)}D^{(*)}$ interaction, we adopt the results from the OBE model, which is widely applied to systems containing charmed mesons. In OBE model, a form factor with momentum cutoff is generally applied to regularize the interaction to remove the singularity at very short range but keep the long-range structure unchanged. By varying the cutoff, one performs different regularization to the short-range interaction, and thus different final interactions are obtained. Complementary lattice-QCD and coupled-channel studies of the doubly charmed sector provide useful constraints on the near-threshold DD^* dynamics [70–74]. In Refs. [75–77], a new approach was proposed to determine the $D^{(*)}D^{(*)}$ interaction, with which the coupling constants are redetermined by fitting the experimental information of $Z_c(3900)$, $X(3872)$, and T_{cc}^+ . Since with this new approach, the interaction is determined directly from the experimental data, but not indirectly from other processes as the generally used OBE potential, this new interaction is believed to be less model-dependent than the conventional one.

In the present work, the three-body Schrödinger equation is solved using the Gaussian expansion method (GEM) [78], which has proven highly efficient for few-body systems. Possible resonance structures are searched for using the complex scaling method (CSM) [79,80]. The main goals of this work include

- (i) to determine whether the DDK system supports bound or resonant states applying the two-body interactions constrained by the experimental data and lattice-QCD calculations,
- (ii) to analyze the internal structures of the resulting states, and
- (iii) to clarify the role of coupled-channel effects from D^*D^*K , $DD_s\eta$ and $DD_s\pi$.

The paper is organized as follows. In Sec. II, we introduce the two-body interactions used in the calculation. In Sec. III, we present the three-body formalism and numerical methods. Results are discussed in Sec. IV, and a summary is given in Sec. V.

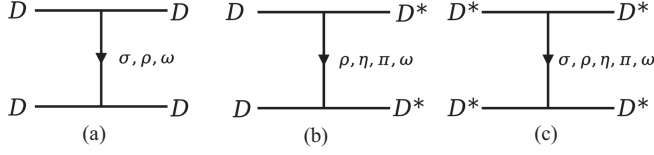


FIG. 1. Tree-level Feynman diagrams used to derive the OBE potentials: (a) $DD \rightarrow DD$, (b) $DD \rightarrow D^*D^*$, and (c) $D^*D^* \rightarrow D^*D^*$.

II. TWO-BODY INTERACTIONS

In the present work, we use the two-body $D^{(*)}K$ and $D^{(*)}D^{(*)}$ interactions to rebuild the interactions for the coupled channels DDK and D^*D^*K . In Fig. 1, the schematic tree-level Feynman diagrams are presented.

A. $D^{(*)}D^{(*)}$ interaction

We derive the $D^{(*)}D^{(*)}$ interaction within the OBE model. We start from the Lagrangians in heavy quark effective field theory, which are constructed based on heavy-quark symmetry (including heavy-quark flavor symmetry and heavy-quark spin symmetry), chiral symmetry, and SU(2) isospin symmetry. Under heavy-quark spin symmetry, the pseudoscalar D meson and vector D^* meson assemble into a superfield $\mathcal{H}$ [14,81]

$$\mathcal{H} = \frac{1 + \not{v}}{2} (P_\mu \gamma^\mu - P \gamma_5), \quad (1)$$

in which $P = (D^0, D^+)$ and $P_\mu^* = (D^{*0}, D^{*+})$. v is the velocity of the charmed meson D or D^* , and in the heavy-quark mass limit, $m_Q \rightarrow \infty$, the static limit $v^\mu = (1, \vec{0})$ is adopted.

The Lagrangian for the charmed meson read

$$\mathcal{L} = i\beta \text{Tr}[\mathcal{H} v_\mu (\mathcal{V}^\mu - \rho^\mu) \tilde{\mathcal{H}}] + i\lambda \text{Tr}[\mathcal{H} \sigma_{\mu\nu} F^{\mu\nu} \tilde{\mathcal{H}}] + g_s \text{Tr}[\mathcal{H} \sigma \tilde{\mathcal{H}}] + i g_a \text{Tr}[\mathcal{H} \gamma_\mu \gamma_5 \mathcal{A}^\mu \tilde{\mathcal{H}}], \quad (2)$$

where the conjugate field is defined as

$$\tilde{\mathcal{H}} \equiv \gamma^0 H^\dagger \gamma^0. \quad (3)$$

The field strength tensor of vector mesons is denoted by $F^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu - [\rho^\mu, \rho^\nu]$, while $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$ are the vector and axial currents respectively, for pseudoscalar mesons,

$$\mathcal{V}^\mu = \frac{1}{2} [\xi^\dagger, \partial_\mu \xi], \quad \mathcal{A}^\mu = \frac{1}{2} \{ \xi^\dagger, \partial_\mu \xi \}, \quad \xi = \exp(i\mathbb{P}/f_\pi). \quad (4)$$

The vector and pseudoscalar meson matrices are respective

$$\rho^\mu = \frac{ig_V}{\sqrt{2}} \begin{bmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ \\ \rho^- & \frac{-\rho^0 + \omega}{\sqrt{2}} \end{bmatrix}^\mu, \quad \mathbb{P} = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \end{bmatrix}. \quad (5)$$

Using the Lagrangians presented in Eq. (2), we derive the scattering amplitude by evaluating the Feynman diagrams shown in Fig. 1. The effective potential between two charmed mesons in momentum space is then obtained via the Breit approximation. The specific expressions are given in Table I. The effective potential has the following general form,

$$V(\mathbf{k}) = C_{\text{coupling}} \times \mathcal{O}_{r,s} \times \mathcal{O}_{\text{iso}}. \quad (6)$$

However, the effective potentials associated with light pseudoscalar meson exchange and vector meson exchange

TABLE I. Effective momentum-space potentials for scattering processes $DD \rightarrow DD$, $D^*D^* \rightarrow D^*D^*$, and $DD \rightarrow D^*D^*$.

V	C_{coupling}	$\mathcal{O}_{r,s}$	$\mathcal{O}_{\text{iso}}$		
			DD	D^*D^*	$DD \rightarrow D^*D^*$
V_σ^S	g_s^2	$-\frac{\mathbf{a} \cdot \mathbf{b}}{u^2 + k^2}$	$\mathbf{I}$	$\mathbf{I}$	0
V_ρ^V	$\frac{\beta^2 g_V^2}{2}$	$-\frac{\mathbf{a} \cdot \mathbf{b}}{u^2 + k^2}$	$-\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	$-\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	0
V_ω^V	$\frac{\beta^2 g_V^2}{2}$	$-\frac{\mathbf{a} \cdot \mathbf{b}}{u^2 + k^2}$	$-\frac{1}{2} \mathbf{I}$	$-\frac{1}{2} \mathbf{I}$	0
V_π^P	$\frac{g_a^2}{f_\pi^2}$	$\frac{(\mathbf{k} \cdot \mathbf{a})(\mathbf{k} \cdot \mathbf{b})}{u^2 + k^2}$	0	$\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	$\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$
V_η^P	$\frac{g_a^2}{f_\pi^2}$	$\frac{(\mathbf{k} \cdot \mathbf{a})(\mathbf{k} \cdot \mathbf{b})}{u^2 + k^2}$	0	$\frac{1}{6} \mathbf{I}$	$\frac{1}{6} \mathbf{I}$
V_ρ^T	$2\lambda^2 g_V^2$	$\frac{(\mathbf{k} \cdot \mathbf{a})(\mathbf{k} \cdot \mathbf{b}) - k^2(\mathbf{a} \cdot \mathbf{b})}{u^2 + k^2}$	0	$-\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$	$-\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{2}$
V_ω^T	$2\lambda^2 g_V^2$	$\frac{(\mathbf{k} \cdot \mathbf{a})(\mathbf{k} \cdot \mathbf{b}) - k^2(\mathbf{a} \cdot \mathbf{b})}{u^2 + k^2}$	0	$-\frac{1}{2} \mathbf{I}$	$-\frac{1}{2} \mathbf{I}$

Notes: Superscripts S and P denote the effective potentials for σ -exchange and pseudoscalar-exchange, respectively while V and T mean the respective vector- and tensor-type potential for the vector meson exchange. $\mathcal{O}_{r,s}$ and $\mathcal{O}_{\text{iso}}$ represent spin-related and isospin operators, respectively. $\mathbf{k} = \mathbf{p}' - \mathbf{p}$ is the momentum of the exchanged light meson. For DD systems, $\mathbf{a} \cdot \mathbf{b} \rightarrow 1$, for the transition $DD \rightarrow D^*D^*$ system, the substitution $\mathbf{a} \rightarrow \mathbf{e}_3^\dagger, \mathbf{b} \rightarrow \mathbf{e}_4^\dagger$ is required; and for D^*D^* system, the substitution $\mathbf{a} \rightarrow \mathbf{e}_1 \times \mathbf{e}_3^\dagger, \mathbf{b} \rightarrow \mathbf{e}_2 \times \mathbf{e}_4^\dagger$ is required.

exhibit short-range singularities, where the two charmed mesons overlap strongly and the OBE model becomes unreliable. To address this issue, a form factor with a momentum cutoff is commonly introduced to regularize the high-momentum behavior of the potential. The coordinate-space effective potential $V(r)$ is then derived via Fourier transform:

$$V^\Lambda(r) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} V(\mathbf{k}, \mu) \mathcal{F}^2(k^2, \Lambda), \quad (7)$$

where the monopole form factor adopted in the present work is given by

$$\mathcal{F}(k^2, \Lambda) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + k^2}. \quad (8)$$

By varying Λ , different regularization procedures are applied to the short-range potential, yielding Λ -dependent effective potentials.

In Refs. [75–77], a novel approach was employed to determine the effective interaction between two charmed mesons. Specifically, the coupling constants were extracted by fitting to the experimental data for $Z_c(3900)$, $X(3872)$, and T_{cc}^+ .

Following Ref. [75], we treat the scalar coupling constant as the sole free parameter for the Z_c channel and fix it using the Z_c pole position. This is motivated by the expectation that the vector ρ -exchange and ω -exchange contributions largely cancel each other, such that the σ -exchange interaction dominates in the $Z_c(3900)$ system. The vector coupling constants are determined from $X(3872)$ and T_{cc}^+ . For simplicity, we introduce the following scaling factors:

$$\lambda \rightarrow \lambda R_\lambda, \quad \beta \rightarrow \beta R_\beta, \quad g_s \rightarrow g_s R_s. \quad (9)$$

The baseline coupling constants and meson masses are summarized in Table II.

B. $D^{(*)}K$ interaction

In covariant chiral perturbation theory, the leading-order (LO) DK interaction is dominated by the Weinberg-Tomozawa term, which in the nonrelativistic limit can be treated as a constant potential,

TABLE II. Baseline coupling constants and the masses of the mesons [82]. Masses and f_π are given in units of GeV, while λ is expressed in GeV^{-1} .

Coupling	f_π	g_a	g_V	β	λ	g_s	
	0.13025	0.57	5.8	0.9	0.56	0.76	
Mass	m_D	m_{D^*}	m_π	m_η	m_ρ	m_ω	m_σ
	1.867	2.009	0.137	0.548	0.775	0.783	0.600

$$V_{DK}(\vec{q}) = -\frac{C_W(I)}{2f_\pi^2}, \quad (10)$$

where $C_W(0) = 2$ and $C_W(1) = 0$, that is, the $I = 1$ channel is noninteracting at LO. Performing a Fourier transform on $V_{DK}(\vec{q})$ and introducing a local Gaussian regulator yields

$$V_{DK}(r) = -\frac{C_W(I)}{2f_\pi^2} \frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3}. \quad (11)$$

The next-to-leading-order (NLO) correction to the $I = 0$ DK S-wave interaction is repulsive [83]. A repulsive core is therefore included to improve the description of the interaction. On this basis, the following form of V_{DK} is adopted [58]:

$$V_{DK}(\vec{r}, R_c) = C_S e^{-(r/R_s)^2} + C_L e^{-(r/R_c)^2}, \quad (12)$$

where the first term accounts for the short-range repulsive core with $C_S \geq 0$ and R_s fixed to 0.5 fm, while the second term provides the dominant long-range attraction.

For V_{D^*K} , the corresponding LO and NLO potentials can be derived from the same covariant chiral Lagrangian [83]:

$$\begin{aligned} V_{\text{LO(NLO)}}(D^*(p_1)K(p_2) \rightarrow D^*(p_3)K(p_4)) \\ = -\epsilon_3^* \cdot \epsilon_1 V_{\text{LO(NLO)}}(D(p_1)K(p_2) \rightarrow D(p_3)K(p_4)). \end{aligned} \quad (13)$$

Under the infinite heavy-quark limit, $m_Q \rightarrow \infty$, this reduces to

$$\begin{aligned} V_{\text{LO(NLO)}}(D^*(p_1)K(p_2) \rightarrow D^*(p_3)K(p_4)) \\ = V_{\text{LO(NLO)}}(D(p_1)K(p_2) \rightarrow D(p_3)K(p_4)). \end{aligned} \quad (14)$$

Assuming that $D_{s0}^*(2317)$ is a DK bound state, the model parameters can be determined by reproducing its pole position. However, since the potential contains four parameters, a fit to a single pole is underconstrained. To obtain a more robust interaction, we also compute the scattering length by solving the Lippmann-Schwinger (LS) equation and compare these results with lattice QCD simulations.

III. THREE-BODY FORMALISM

A. Wave function

We consider the $D^{(*)}D^{(*)}K$ system with $I(J^P) = \frac{1}{2}(0^-)$. Since orbital excitations are difficult to treat accurately in GEM, we restrict our analysis to S-wave configurations. The wave function is written as

$$\Psi \propto S \otimes I \otimes R \quad (15)$$

where S , I , and R are the spin, isospin, and spatial wave functions, respectively. Their explicit forms are given in the following subsections.

B. Gaussian expansion method

We employ the Gaussian expansion method (GEM) [78,84] to solve the three-body Schrödinger equation:

$$\hat{H}\Psi_{JM} = E\Psi_{JM}, \quad (16)$$

where $\hat{H}$ denotes the total Hamiltonian and is given by

$$\begin{aligned} \hat{H} &= \sum_{i=1}^3 \frac{p_i^2}{2m_i} - T_{\text{c.m.}} + \sum_{1 \leq i < j \leq 3} V(r_{ij}) \\ &= -\frac{\hbar^2}{2\mu_r} \nabla_{\vec{r}}^2 - \frac{\hbar^2}{2\mu_R} \nabla_{\vec{R}}^2 + \sum_{1 \leq i < j \leq 3} V(r_{ij}) \end{aligned} \quad (17)$$

with $T_{\text{c.m.}}$ being the center-of-mass (CM) energy. $\vec{r}$ and $\vec{R}$ represent the Jacobi coordinates.

In GEM, the total three-body state is expanded as

$$\Psi_{JM} = \sum_{\alpha,c} C_{\alpha,c} \Psi_{JM,\alpha}^c(\mathbf{r}_c, \mathbf{R}_c), \quad (18)$$

where $\alpha = \{nN, lL\Lambda, tT\}$ denotes all the quantum numbers. Here l and L are the orbital-angular momentum quantum numbers for the two Jacobi coordinates respectively, and Λ is the total orbital momentum quantum number. t and T denote the isospin of two-body subsystem and the total isospin, respectively. n and N are the indices of Gaussian basis for the two Jacobi coordinates. In the present work, due to the numerical calculation issue, we restrict $l = 0$ and $L = 0$, which will be explained in detail later. Therefore, Λ is fixed to 0. Now, the individual basis states are defined as

$$\Psi_{JM,\alpha}^c(\mathbf{r}_c, \mathbf{R}_c) = I_{T,t}^c \otimes [S_{JM} \otimes \Phi_{lL,\Lambda}^c]_{JM}. \quad (19)$$

Here, $I_{T,t}^c$ and $\Phi_{lL,\Lambda}^c$ represent the isospin and spatial wave functions, respectively. In our calculations, we consider different isospin-coupling schemes for the three-body system, where the isospin wave function is given by

$$\begin{aligned} I_{T,t}^{c=1} &= \left[\left[D_1 \left(\frac{1}{2} \right) D_2 \left(\frac{1}{2} \right) \right]_1 K \left(\frac{1}{2} \right) \right]_{\frac{1}{2}}, \\ I_{T,t}^{c=2} &= \left[\left[D_2 \left(\frac{1}{2} \right) K \left(\frac{1}{2} \right) \right]_t D_1 \left(\frac{1}{2} \right) \right]_{\frac{1}{2}}, \\ I_{T,t}^{c=3} &= \left[\left[D_1 \left(\frac{1}{2} \right) K \left(\frac{1}{2} \right) \right]_t D_2 \left(\frac{1}{2} \right) \right]_{\frac{1}{2}}, \end{aligned} \quad (20)$$

with $X(\frac{1}{2})$ denoting the isospin of particle X is $\frac{1}{2}$. Due to Bose-Einstein statistics, the isospin for the subsystem of two identity D mesons can be only 1. In the expression, we have already set the total isospin to $T = \frac{1}{2}$.

Considering the spin quantum numbers: $J(D) = 0$ and $J(K) = 0$, the spin part S_{JM} of the DDK system is trivial. For $J(D^*) = 1$, the spin part S_{JM} of the D^*D^*K system can be expressed as

$$S_{JM} = S_{00} = \frac{1}{\sqrt{3}} |1-1\rangle + \frac{1}{\sqrt{3}} |-11\rangle - \frac{1}{\sqrt{3}} |00\rangle, \quad (21)$$

where the coefficients in front of the states are Clebsch-Gordan (CG) coefficients.

In the CMS frame, we rewrite the Schrödinger equation in terms of Jacobi coordinates and expand the coordinate-space wave function using two sets of Gaussian basis functions, $\phi_{nlm}^G(\mathbf{r}_c)$ and $\psi_{NLM}^G(\mathbf{R}_c)$, corresponding two Jacobi coordinates:

$$\begin{aligned} \Phi_{lL,\Lambda}^c(\mathbf{r}_c, \mathbf{R}_c) &= [\phi_{nlm}^G(\mathbf{r}_c) \psi_{NLM}^G(\mathbf{R}_c)]_{\Lambda}, \\ \phi_{nlm}^G(\mathbf{r}) &= N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \\ \psi_{NLM}^G(\mathbf{R}) &= N_{NL} R^L e^{-\lambda_N R^2} Y_{LM}(\hat{\mathbf{R}}), \\ N_{nl} &= \left(\frac{2^{l+2} (2\nu_n)^{l+\frac{1}{2}}}{\sqrt{\pi} (2l+1)!!} \right)^{\frac{1}{2}}, \end{aligned} \quad (22)$$

where Y_{LM} is the spherical harmonic function. Three sets of Jacobi coordinates are considered in our system, as two of them illustrated in Fig. 2. It is clear that the basis involves powers of coordinates for nonzero l and L , which introduces considerable numerical challenges in the computation of matrix elements. Since the contribution from higher partial waves is expected to be much smaller than that of the S-wave, in this work, we only consider the ground state of the three-body system.

To maximize the numerical accuracy, the Gaussian parameters are chosen in geometric progression:

$$\begin{aligned} \lambda_N &= 1/R_N^2, \quad R_N = R_{\min} A^{N-1} \quad (N = 1, 2, \dots, N_{\max}), \\ \nu_n &= 1/r_n^2, \quad r_n = r_{\min} a^{n-1} \quad (n = 1, 2, \dots, n_{\max}). \end{aligned} \quad (23)$$

In the present work, the values of the size parameters are taken as

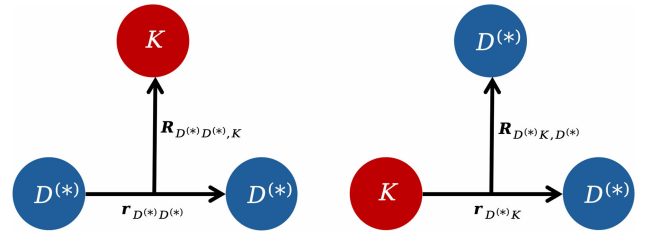


FIG. 2. Two sets of Jacobi coordinates corresponding to different spatial configurations, where blue disks indicate the K meson and the red disk denotes the $D^{(*)}$ meson. The third set of Jacobi coordinates, obtained by exchanging the two $D^{(*)}$ in panel (b), is not shown explicitly, but is included in the calculation.

$$r_1 = 0.1 \text{ fm}, \quad r_{\max} = 10 \text{ fm}, \quad n_{\max} = 12. \quad (24)$$

To account for the coupled channels DDK and D^*D^*K , we solve the coupled-channel Schrödinger equation:

$$\left[\left(-\frac{\hbar^2}{2\mu_r} \nabla_r^2 - \frac{\hbar^2}{2\mu_R} \nabla_R^2 \right) \delta_{ij} + V_{ij} \right] \psi_j = E \psi_i, \quad (25)$$

where the potential matrix V is given by

$$V = \begin{pmatrix} V_{DDK \rightarrow DDK} & V_{DDK \rightarrow D^*D^*K} \\ V_{D^*D^*K \rightarrow DDK} & V_{D^*D^*K \rightarrow D^*D^*K} \end{pmatrix}. \quad (26)$$

C. Complex scaling method

The GEM provides an efficient real-space expansion for locating bound states, but it is not directly applicable to resonant states with complex energies. To address this, we employ the complex scaling method (CSM). In this approach, the Schrödinger equation is analytically continued via a coordinate rotation in the complex plane,

$$\mathbf{r} \rightarrow \mathbf{r}e^{i\theta}, \quad \mathbf{p} \rightarrow \mathbf{p}e^{-i\theta},$$

$$\hat{H}(\theta) = \sum_{k=1}^3 \frac{p_k^2}{2m_k} e^{-2i\theta} - T_{\text{c.m.}} + \sum_{1 \leq k < l \leq 3} V(r_{kl}e^{i\theta}). \quad (27)$$

The complex-scaled Hamiltonian is then diagonalized within the same Gaussian basis used in the GEM. The resulting eigenvalue spectrum can be visualized in the complex-energy plane: bound states appear on the negative real axis, while resonance poles are isolated from the rotated continuum branches. Most eigenvalues lie along rays emanating from thresholds and correspond to scattering states.

The root-mean-square (rms) radius provides a valuable observable for characterizing the internal structure of hadronic bound states, and is often used to distinguish compact multiquark configurations from extended molecular-type systems. It is defined as [85,86]

$$r_{ij}^{\text{rms}} \equiv \sqrt{\frac{\langle \Psi^{IJ} | r_{ij}^2 | \Psi^{IJ} \rangle}{\langle \Psi^{IJ} | \Psi^{IJ} \rangle}}. \quad (28)$$

IV. RESULTS AND DISCUSSION

A. Two-body DK and D^*K

For concreteness, we fix $R_S = 0.5 \text{ fm}$ and scan the remaining model parameters. The long-range coupling constant C_L is fixed by reproducing the pole position of $D_{s0}^*(2317)$. Once this condition is satisfied, the $D_{s1}(2460)$ state is reproduced approximately in the corresponding D^*K channel, as expected by the heavy-quark symmetry.

For each parameter set, we solve the Lippmann-Schwinger (LS) equation for the two-body DK system [87–89]:

$$\mathbf{T} = \mathbf{V} + \mathbf{VGT}, \quad (29)$$

whose partial-wave representation is given by

$$T_\ell(p, q) = V_\ell(p, q) + \frac{\mu}{4\pi^3} \int_0^\infty dk k^2 V_\ell(p, k) G(k) T_\ell(k, q), \quad (30)$$

where $G(k) = 1/(E - \frac{k^2}{2\mu} + i\epsilon)$ denotes the two-body propagator, and the partial-wave potential $V_\ell(p, q)$ is defined as

$$V_\ell(p, q) = 2\pi \int_{-1}^1 dx P_\ell(x) \int d^3r V(r) e^{-i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}}, \quad (31)$$

with $x = \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}$ being the dot product of the momentum unit vectors.

For the partial-wave scattering amplitude f_ℓ , scattering theory yields [90]

$$\frac{f_\ell(k)}{k} = -\frac{\mu}{8\pi^2} T_\ell(k, k; E) = \left(-a^{-1} + \frac{1}{2}rk^2 - ik \right)^{-1}, \quad (32)$$

where $E = \frac{k^2}{2\mu}$ is the two-body energy, and a and r are the scattering length and effective range, respectively.

We calculate the scattering length for $R_C = 1.0\text{--}2.0 \text{ fm}$ and $C_S = 0\text{--}3000 \text{ MeV}$, with the corresponding scattering lengths shown in Fig. 3. We also examined the case with $R_S = 0.5 \text{ fm}$ and $R_C = 3.0 \text{ fm}$, which yields scattering lengths smaller than -15 fm and is therefore not adopted in the main analysis. As shown in Fig. 3(a), for each fixed R_C , the scattering length decreases monotonically with increasing C_S and gradually approaches an asymptotic value. This behavior indicates that the low-energy DK observables become less sensitive to the short-range repulsive core once C_S is sufficiently large. By fitting these curves, we extract the asymptotic values displayed in Fig. 3(b). It can be further seen that the fitted asymptotic scattering length exhibits a nonmonotonic dependence on R_C , reaching its most negative value around $R_C \sim 1.5 \text{ fm}$. This behavior suggests that the long-range structure of the DK interaction plays a nontrivial role in determining the low-energy scattering properties, which must therefore be constrained before proceeding to the three-body calculation.

According to recent lattice QCD simulations [50], the predicted S-wave scattering length for the DK system at physical meson masses is $a_{DK} = -1.87 \text{ fm}$ (central value), while an alternative central value of $a_{DK} = -1.51 \text{ fm}$ is also reported. The sizable discrepancy between these two values arises from systematic error. Using the exponential

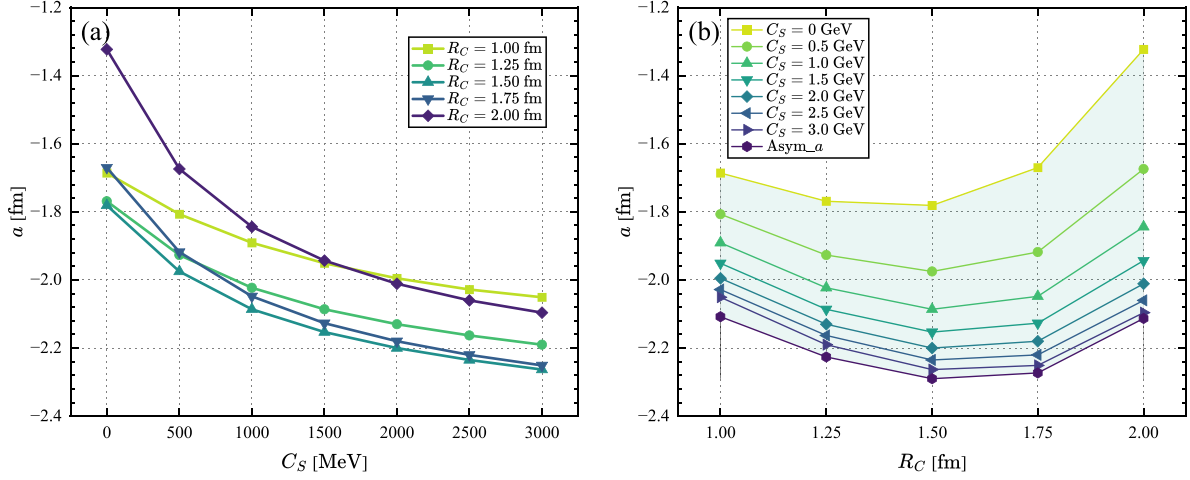


FIG. 3. (a) shows the scattering lengths for different R_C values versus coupling constant C_S , and (b) shows the scattering lengths for different coupling constants C_S versus R_C . Asym_a represents the asymptotic scattering lengths of different R_C , the shaded region between curves represents the allowed range of scattering lengths.

TABLE III. Scattering lengths [50] and their corresponding DK parameters.

a (fm)	R_C (fm)	C_S (MeV)	C_L (MeV)
-1.87	1.00	862.3	-535.3
-1.87	1.25	304.9	-298.1
-1.87	1.50	206.3	-225.0
-1.87	1.75	404.1	-202.9
-1.87	2.00	1089.0	-197.3
-1.51	2.00	243.2	-165.8

fits described above, we extract the C_S values corresponding to these lattice scattering lengths, and then determine C_L by reproducing the $D_{s0}^*(2317)$ pole. The resulting parameter sets are listed in Table III, which will be primarily used in our subsequent three-body calculations.

B. DDK spectrum

We now turn to the three-body DDK system with $I(J^P) = \frac{1}{2}(0^-)$ within a coupled-channel framework that incorporates both DDK and D^*D^*K configurations. The DK interaction is modeled by a two-range Gaussian potential, whose parameters are constrained by the pole position of $D_{s0}^*(2317)$ and the DK scattering length from lattice QCD calculations. Since there are three free parameters but only two constraint conditions from experiment and lattice simulations, we select two representative parameter sets based on the DK scattering length predicted by lattice QCD.

For the $D^{(*)}D^{(*)}$ interaction within the OBE model, the coupling constant for σ -exchange is constrained by the $Z_c(3900)$ pole, whereas those for vector-meson exchange are constrained by the poles of $X(3872)$ and T_{cc}^+ .

Although the near-threshold nature of $Z_c(3900)$ remains under debate, many existing studies favor a virtual-state interpretation. For instance, a refit to experimental data using exactly unitarized amplitudes yields two possible solutions: either a resonance or a virtual state located 30–40 MeV below threshold [91]. A three-coupled-channel analysis identifies $Z_c(3900)$ as a virtual-state pole lying 80 MeV below threshold [92], and a global coupled-channel analysis of $e^+e^- \rightarrow c\bar{c}$ processes also supports a virtual-state solution with the pole positioned 30–40 MeV below threshold [93]. Nevertheless, a resonance solution is reported in Ref. [94]. It should be noted that different investigations adopting the OBE model employed in the present work all favor a virtual-state picture for $Z_c(3900)$, differing only in the concrete pole positions of the virtual state. While a more sophisticated investigation encompassing various pole parametrizations would be worthwhile, carrying out such an elaborate treatment would require modifications to our present dynamical OBE model. This task lies beyond the scope of the present work and is deferred to future studies. Accordingly, we treat $Z_c(3900)$ as a virtual pole in our numerical calculations.

Specifically, the poles of T_{cc}^+ and $X(3872)$ are fixed at -4 MeV relative to their respective thresholds, while the pole position of $Z_c(3900)$ is varied over the range from -5 to -35 MeV.

The resulting three-body spectra are displayed in Fig. 4. For both DK parameter sets, the DDK system supports bound states over a wide range of DD interaction strengths. As shown in Fig. 4, the binding energies exhibit only a mild dependence on the momentum cutoff in the range $\Lambda = 1.1$ –1.3 GeV, indicating that the results are reasonably stable against short-range regularization of the $D^{(*)}D^{(*)}$ interaction. This weak cutoff dependence suggests that the existence of bound-state solutions is not a

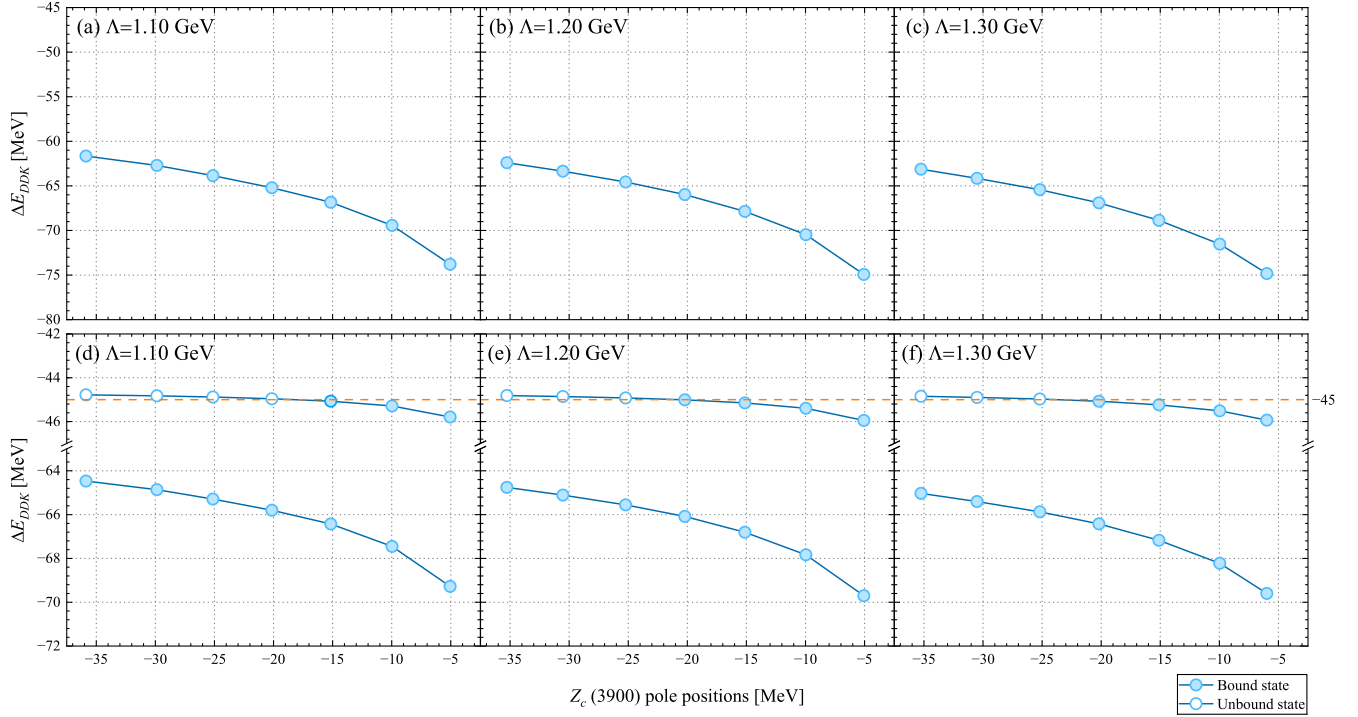


FIG. 4. The cutoff dependence of the DDK bound states. The open (filled) circles represent unbound (bound) states. The orange line indicates the particle-dimer ($D - DK$) threshold. (a)–(c): $R_C = 1.0$ fm, $C_S = 0$, $C_L = 320.1$ MeV. (d)–(f): $R_C = 2.0$ fm, $C_S = 500$ MeV, $C_L = 178.4$ MeV.

numerical artifact of a specific regulator choice, but rather reflects a robust feature of the underlying two- and three-body dynamics within the present framework.

Depending on the DK parameter choice, two scenarios emerge. For the first scenario, which typically corresponds to a smaller R_C , only a single bound state is found. Its binding energy lies in the range of approximately -60 to -75 MeV relative to the three-body threshold and changes smoothly with the input $Z_c(3900)$ pole position. This is consistent with the result of [58], which is around -70 MeV and varies slightly as using different potential parameters. In contrast, for the second scenario with a larger R_C , two bound states emerge in most cases: a deeply bound state and a shallow state located close to the particle-dimer ($D - DK$) threshold. This demonstrates that the long-range structure of the DK interaction plays a crucial role in generating additional three-body solutions. We further analyze the role of coupled-channel dynamics by examining the probability of the D^*D^*K component in the total wave function. It is found that, in all cases considered, the D^*D^*K fraction is extremely small, remaining below 0.1%. This indicates that the contribution from the D^*D^*K channel is negligible, and the structure of the system is overwhelmingly dominated by the DDK component. This justifies, *a posteriori*, the use of a simplified model space when discussing the dominant structure and spatial properties of the system.

Fixing the cutoff at $\Lambda = 1.2$ GeV, we adopt the DK parameters filtered according to the scattering lengths presented in Table III. The corresponding binding energies are plotted in Fig. 5. As shown in the figure, once the DK interaction is constrained by the lattice-derived scattering length of DK , the binding energies are confined to a narrower range. Meanwhile, the dependence on R_C is pretty mild.

C. Internal structure of DDK

To clarify the internal structure of the bound states, we analyze the root-mean-square (rms) radii shown in Figs. 6(a) and 6(b). We find

$$r_{DD} \sim r_{DK} \sim 1 - 2 \text{ fm}. \quad (33)$$

This indicates that all three constituents are within a typical hadronic distance, with no clear separation of scales among the interparticle distances. The wave function is therefore dominated by genuine three-body correlations rather than a two-body cluster structure. In particular, neither a $(DD) + K$ nor a $(DK) + D$ configuration is favored. We thus identify these states as a compact three-body bound state, whose binding is collectively generated by the interplay of all pairwise interactions.

In Figs. 5(d)–5(f), an additional shallow bound state emerges close to the threshold. A key feature of this state is its large spatial extent, as illustrated in Fig. 6(c):

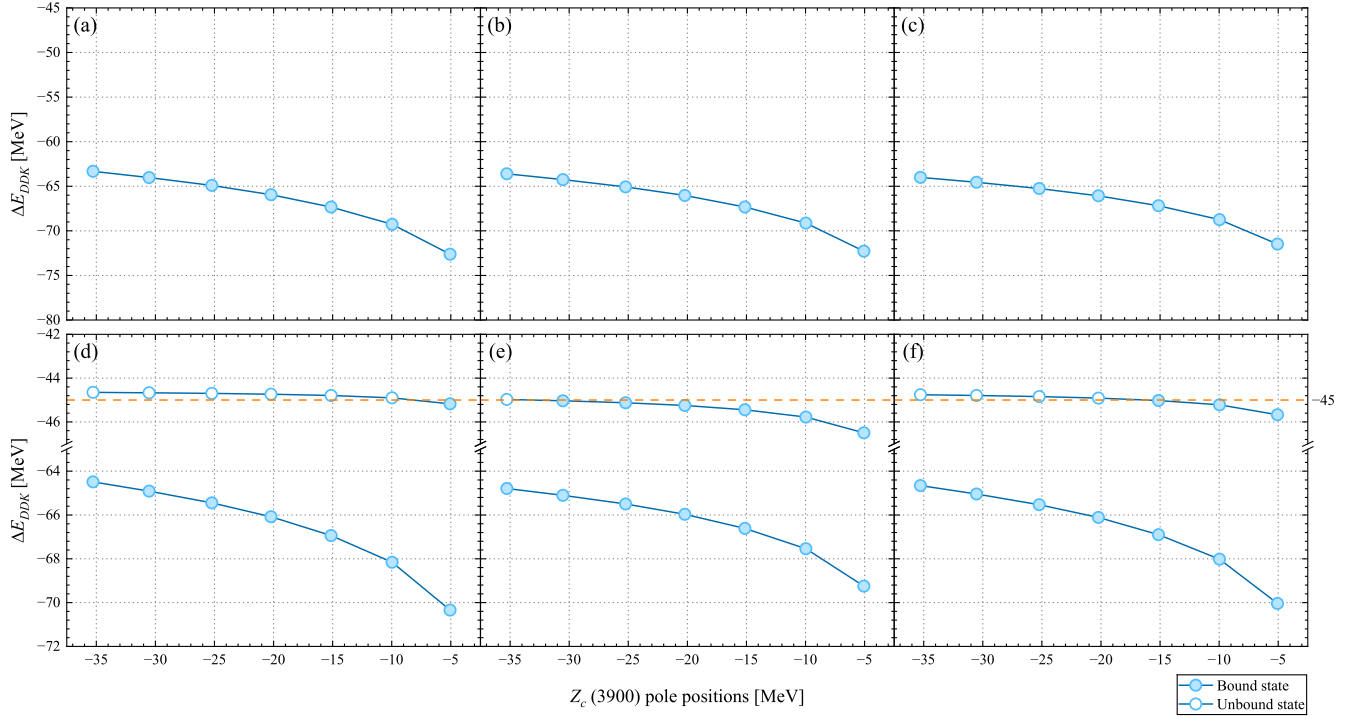


FIG. 5. The DDK bound states vary with the $Z_c(3900)$ virtual state pole position, given that the DK interaction is determined by fitting the lattice-calculated scattering length of DK . The momentum cutoff for the DD interaction is fixed to $\Lambda = 1.2$ GeV. Panels (a)–(e) correspond to $a_{DK} = -1.87$ fm with $R_C = 1.00, 1.25, 1.50, 1.75$, and 2.00 fm, respectively; panel (f) corresponds to $a_{DK} = -1.51$ fm with $R_C = 2.00$ fm.

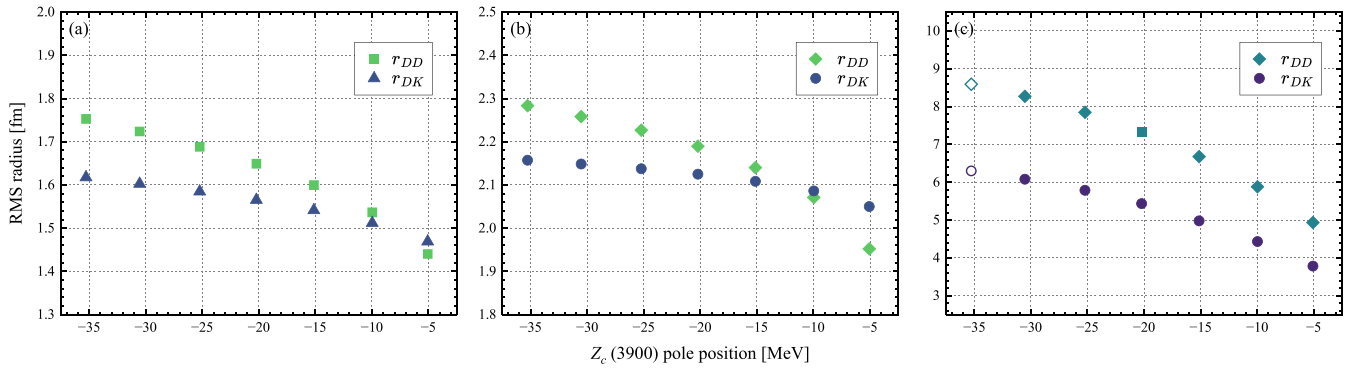


FIG. 6. The root-mean-square (rms) radii of the two-body subsystems DD and DK in the three-body DDK system, where the DK interaction is determined by fitting the lattice-calculated scattering length of the DK system, $a_{DK} = -1.87$ fm. Panels (a)–(c) are defined as follows: (a) corresponds to $R_C = 1.00$ fm; (b) shows the deep bound state for $R_C = 2.00$ fm; (c) shows the shallow bound state for $R_C = 2.00$ fm. Moreover, open shapes denote unbound states, while filled shapes denote bound states.

$$r_{DD} \sim 5 - 9 \text{ fm}, \quad r_{DK} \sim 4 - 7 \text{ fm}. \quad (34)$$

All interparticle distances are significantly larger than the typical interaction range (~ 1 fm), and importantly,

$$r_{DD} \sim r_{DK}. \quad (35)$$

These features clearly distinguish the shallow bound state from a conventional hadron. For a $(DK) + D$

configuration, one would expect a pronounced scale separation: $r_{DK} \ll r_{DD}$, whereas for a $(DD) + K$ structure, one would expect $r_{DD} \ll r_{DK}$. Neither pattern is observed in our results. Instead, the comparable and large values of all rms radii indicate that the three particles are weakly bound and spatially extended, without forming a dominant two-body subcluster. This is a characteristic signature of a three-body “halo” state [95], in which the wave function has substantial support at distances far beyond the range of

the pairwise interactions. The halo interpretation is further supported by the trend that the spatial extension increases as the binding energy approaches the threshold, consistent with the expected scaling behavior

$$r \propto \frac{1}{\sqrt{E_B}}. \quad (36)$$

D. Coupled-channel effects from $DD_s\eta$ and $DD_s\pi$

So far, in our analyses the $D_{s0}^*(2317)$ resonance has been treated as a pure DK molecular state. Nevertheless, its proximity to the $D_s\eta$ threshold suggests the presence of nontrivial DK - $D_s\eta$ coupled-channel effects. Lattice QCD analyses have confirmed the dominant molecular nature of $D_{s0}^*(2317)$. In Ref. [96], the DK molecular component is found to contribute approximately 70% to the internal structure of $D_{s0}^*(2317)$. In Ref. [48], a full lattice QCD calculation supports the interpretation of $D_{s0}^*(2317)$ as a DK molecule. A more recent investigation further shows that the combined DK and $D_s\eta$ molecular components can reach up to 88% [97]. In addition, lattice QCD calculations incorporating both $c\bar{s}$ and DK -type operators in the isospin-0 channel demonstrate that the DK molecular component dominates the bound-state structure [54].

Motivated by these observations, we reconstruct the $D_{s0}^*(2317)$ pole within a coupled DK - $D_s\eta$ framework and systematically explore the energy spectrum of the three-body DDK - $DD_s\eta$ coupled-channel system. Furthermore, considering that $D_{s0}^*(2317)$ is unstable and predominantly decays into $D_s\pi$ through isospin violation, we further incorporate the $DD_s\pi$ channel into our three-body coupled-channel calculations.

It should be noted that the interactions between charmed mesons and Goldstone bosons are nontrivial and involve various renormalization- and subtraction-related procedures, which cannot be fully reproduced within a potential-model framework. Accordingly, we carry out a qualitative study in the present work based on chiral perturbation theory (ChPT) in the heavy-meson limit combined with the on-shell approximation [97,98]. The LO and NLO potentials read

$$\begin{aligned} V_{\text{LO}}(\mathbf{q}) &= \frac{C_{\text{LO}}}{2f_\pi^2}, \\ V_{\text{NLO}}(\mathbf{q}) &= -\frac{2}{f_\pi^2} \left(2C_{24}c_{24} \frac{m_L}{m_H} + C_{35}c_{35} \frac{m_L}{m_H} \right. \\ &\quad \left. + 2C_0c_0 \frac{1}{m_H m_L} - C_1c_1 \frac{1}{m_H m_L} \right). \end{aligned} \quad (37)$$

Here m_L and m_H denote the masses of light and heavy mesons, respectively. The constants $C_{\text{LO}}, \dots$ can be found in [97,98]. $V_{D_s\eta}$ and $V_{D\eta}$ are obtained via Fourier transformation and regularized with a Gaussian regulator

$$g(\mathbf{r}) = \frac{e^{-(r/R_0)^2}}{\pi^{3/2} R_0^3}. \quad (38)$$

The DK interaction is given in Eq. (12), and the transition potentials $V_{D\pi}$, $V_{DK \rightarrow D_s\eta}$, and $V_{DK \rightarrow D_s\pi}$ are given from the following relations,

$$\begin{aligned} V_{DK \rightarrow D_s\eta} &= \frac{\sqrt{3}}{2} V_{DK}, \\ V_{D\pi} &= V_{DK}, \\ V_{DK \rightarrow D_s\pi} &= -\frac{1}{2} V_{DK}. \end{aligned} \quad (39)$$

We first consider the truncated channel space consisting only of the DDK and $DD_s\eta$ channels, and the corresponding numerical results are presented in Fig. 7.

The parameters of the DD interaction are calibrated to reproduce the pole position of $Z_c(3900)$ at -5 MeV. In contrast to the pure single-channel results, the coupled-channel poles show only weak sensitivity to the long-range cutoff R_C . The obtained pole energies lie steadily in the range of -60 to -75 MeV. As R_C increases, a shallow bound state emerges slightly below the dimer-particle threshold at -45 MeV, which is qualitatively consistent with the single-channel predictions. Meanwhile, the $DD_s\eta$ component decreases gradually from approximately 5% to 2% as C_S varies from 0 to 3 GeV.

We further include the $DD_s\pi$ channel into our coupled-channel framework, and the corresponding results are shown in Fig. 8. Points 1 to 7 correspond to calculations with $C_S = 0$ –3 GeV, respectively.

Numerical calculations reveal that the bound state obtained in the DDK - $DD_s\eta$ system evolves into a resonance once the $DD_s\pi$ channel is opened. As C_S varies from 0 to 3 GeV, the $DD_s\eta$ component decreases from approximately 5% to 2%, whereas the $DD_s\pi$ component drops from about 3% to 0.5%. We therefore conclude that the coupled-channel effects originating from $D_{s0}^*(2317)$ yield only minor corrections to the DDK bound state and induce merely a small reduction of its binding energy. In contrast, the inclusion of the $DD_s\pi$ channel substantially changes the nature of the three-body DDK state and converts the bound state into a resonance. This feature can be naturally explained: the opening of the $DD_s\pi$ channel provides a dynamical decay channel for the physical state.

E. D^*D^*K system

We also conduct a dedicated investigation of the D^*D^*K system, independent of the previously analyzed DDK system, to comprehensively understand its bound state characteristics and dynamic behavior. The numerical results obtained using the parameter set listed in Table III are presented in Figs. 9 and 10, which clearly

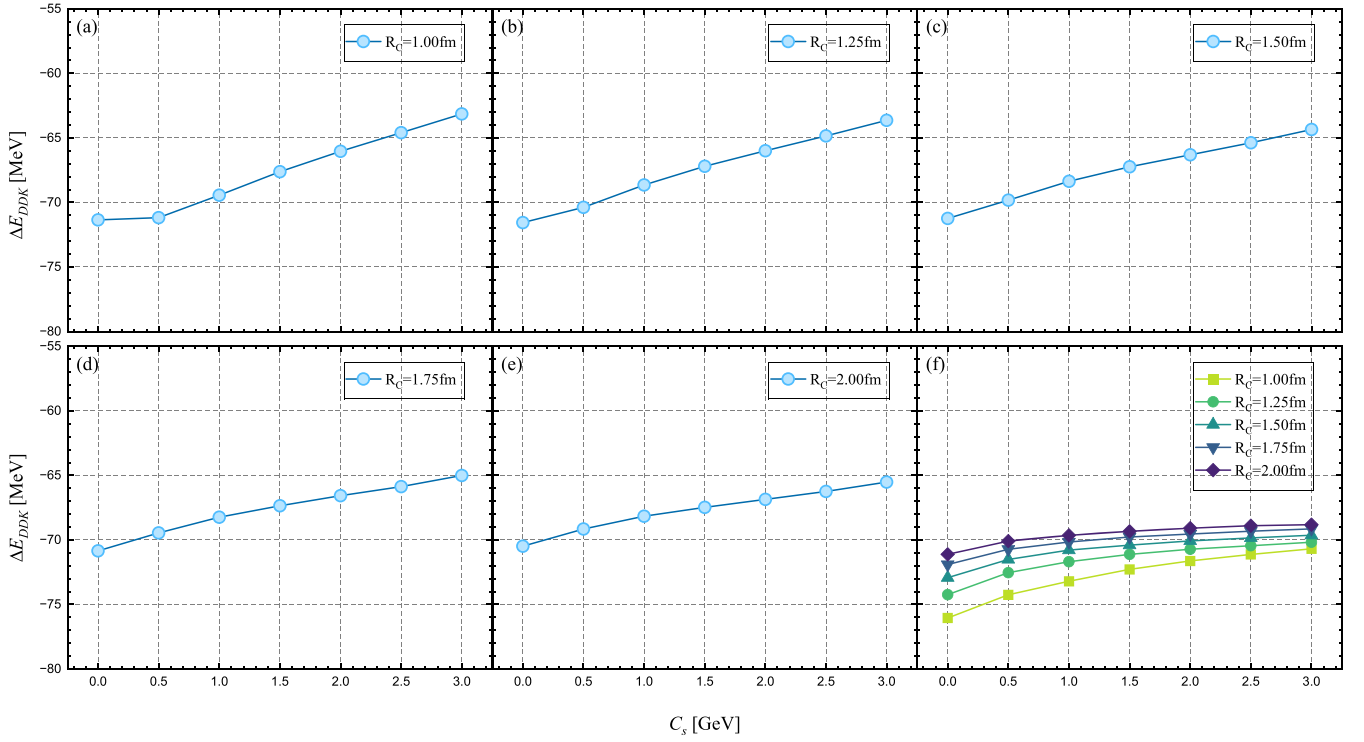


FIG. 7. Pole structures of the $DDK-DD_s\eta$ coupled-channel system and the pure DDK single-channel system. Panels (a)–(e) show the coupled-channel poles obtained with different choices of the cutoff parameters R_C and C_s , while panel (f) displays the corresponding single-channel DDK poles.

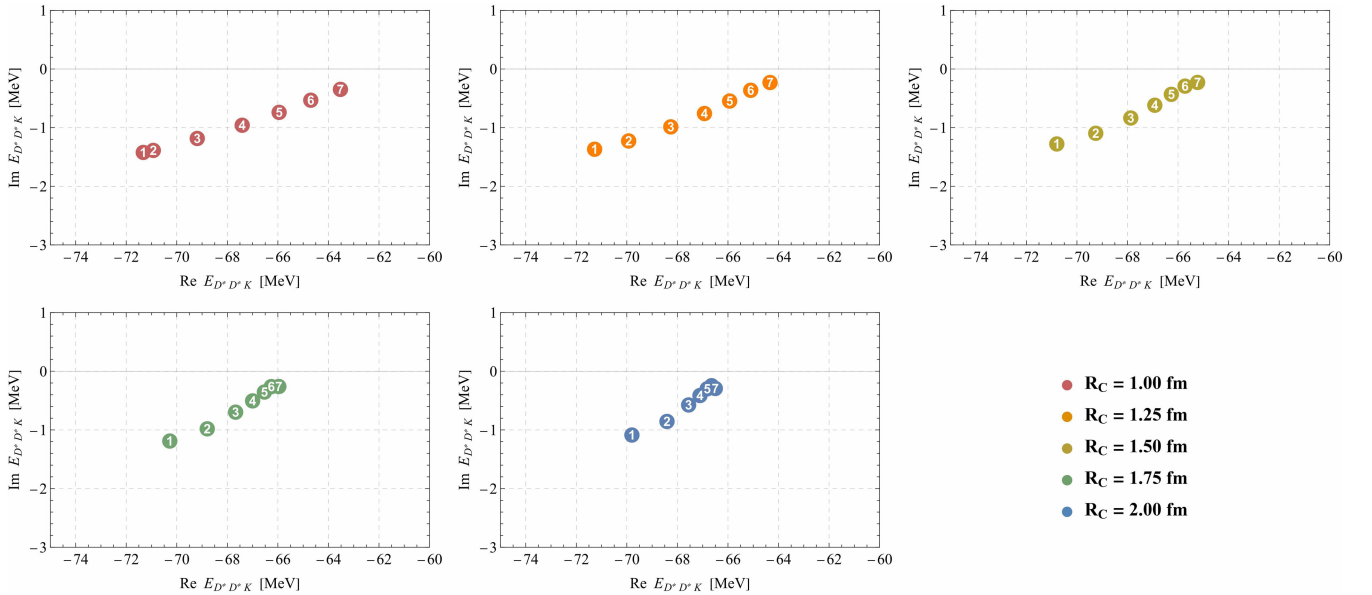


FIG. 8. Pole positions of the $DDK-DD_s\eta-DD_s\pi$ coupled-channel system computed with different values of R_C and C_s .

illustrate the binding energy and spatial properties for the subsystems D^*D^* and D^*K . The bound states of the D^*D^*K system exhibit a behavioral pattern very similar to that of the DDK system: the deeply bound state lies in the range of approximately -60 to -70 MeV, while the

shallow bound state is positioned just below the particle-dimer ($D^* - D^*K$) threshold. Regarding the internal structure of the three-body D^*D^*K system, the deeply bound state is identified as a compact three-body bound state, with the interparticle distances $r_{D^*D^*}$ and r_{D^*K} both in the range

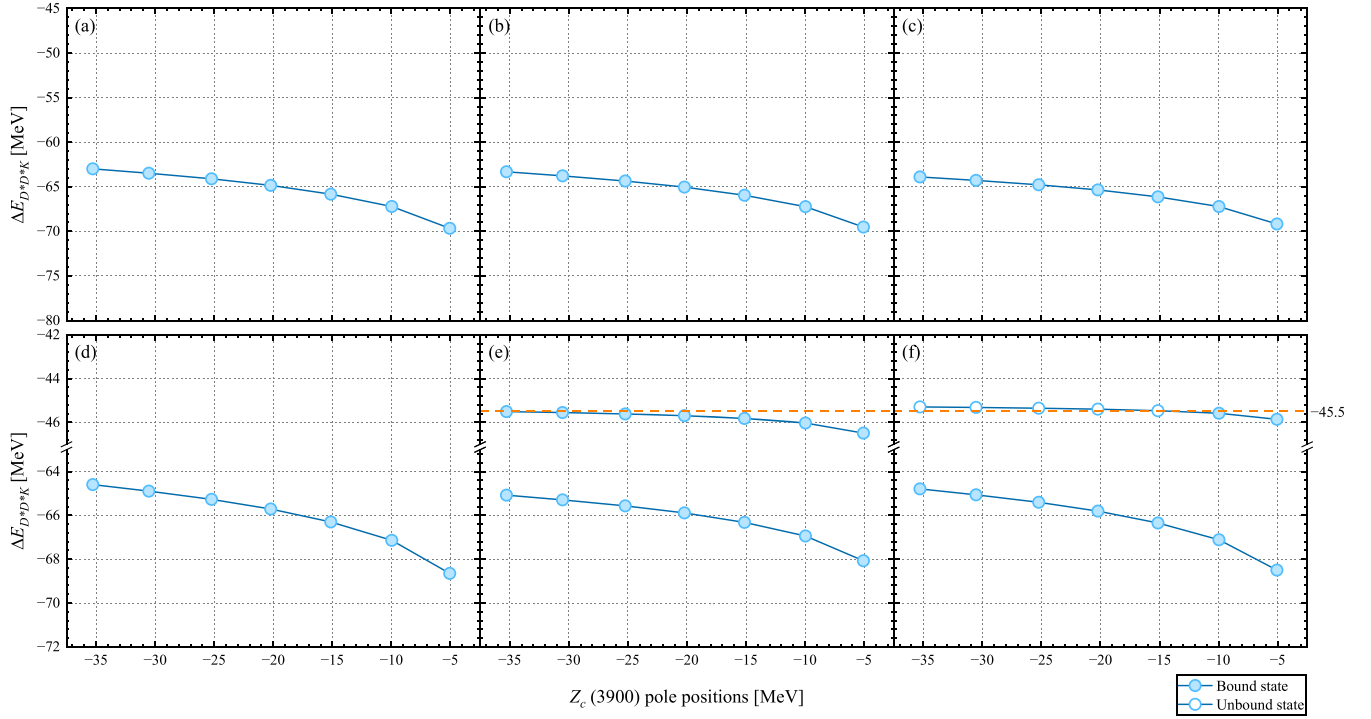


FIG. 9. The three-body D^*D^*K bound states versus the $Z_c(3900)$ virtual state pole position. The parameter settings of (a)–(f) are the same as those in Fig. 5.

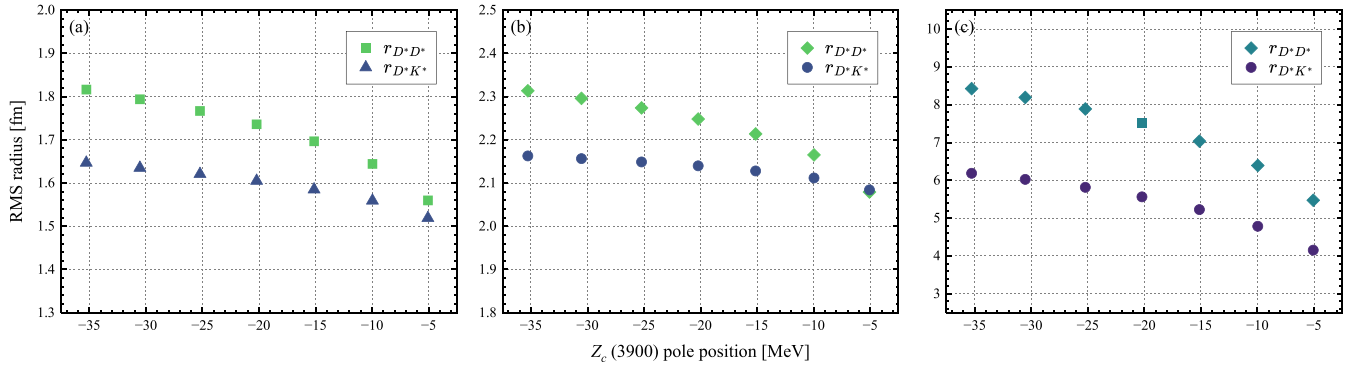


FIG. 10. The rms radii of the two-body subsystems in D^*D^*K system versus the $Z_c(3900)$ virtual-state pole position. The D^*K interaction is determined by fitting the lattice-calculated scattering length of DK , $a_{DK} = -1.87$ fm. Panels (a)–(c) are defined as follows: (a) corresponds to $R_C = 1.00$ fm; (b) shows the deep bound state for $R_C = 2.00$ fm; (c) shows the shallow bound state for $R_C = 2.00$ fm.

of 1–2 fm, indicating a tight and compact spatial configuration. In contrast, the shallow bound state is identified as a three-body halo state, which is characterized by relatively large rms radii for both $r_{D^*D^*}$ and $r_{D^*K^*}$, reflecting a loose and extended spatial distribution.

To further verify the nature of the bound states, we perform a detailed CSM analysis, with Fig. 11 serving as a typical example. The CSM results clearly show that there are no resonance poles in the complex energy plane for either the DDK or D^*D^*K system. Specifically, the bound

states of both systems appear as discrete eigenvalues located on the negative real axis of the complex energy plane, and these eigenvalues remain invariant under the complex rotation operation, consistent with the fundamental characteristics of genuine bound states. Meanwhile, the continuum states of the systems rotate in the complex energy plane as expected, which is a typical feature of nonresonant continuum states. Additionally, no states that remain invariant under the complex rotation operation are found in the third and fourth quadrants of the complex

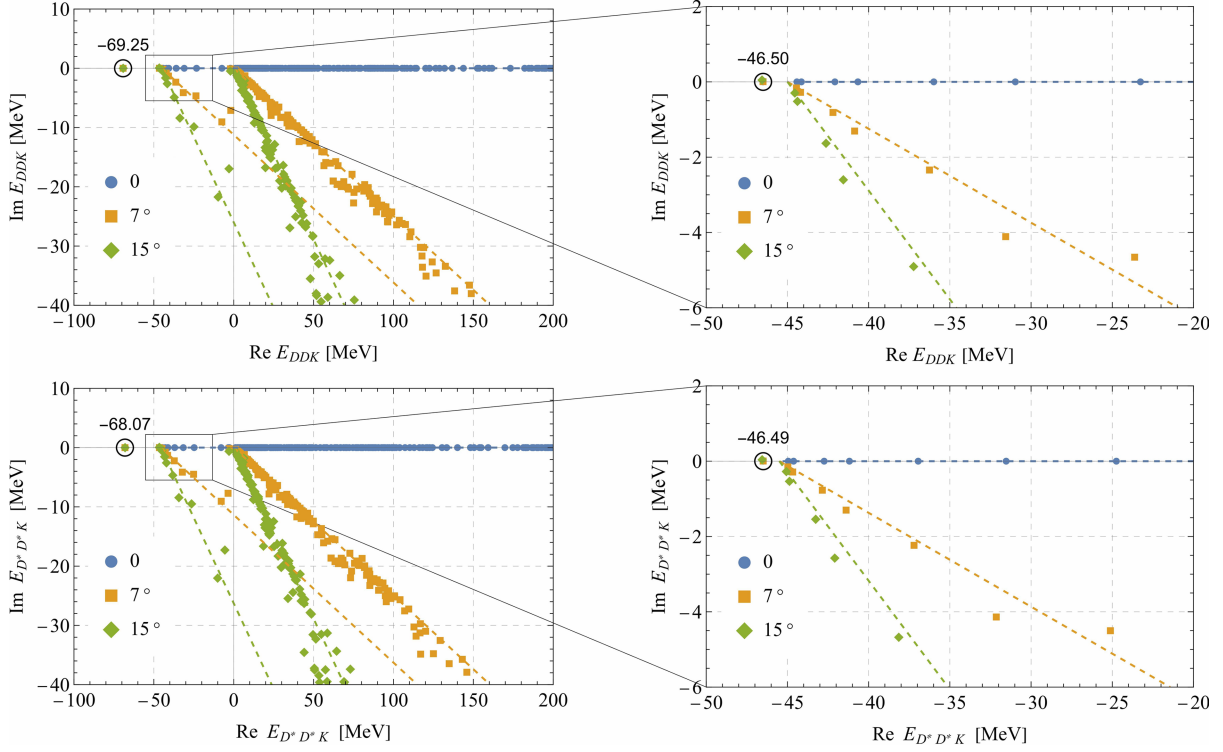


FIG. 11. Complex-energy results of DDK (upper panel) and D^*D^*K (lower panel) as the DK interaction is determined from the lattice-calculated scattering length of DK , $a_{DK} = -1.87$ fm and $R_C = 2.00$ fm. Bound states are marked by circles.

energy plane, further confirming the absence of resonant states. These results collectively demonstrate that the bound states identified in our work are genuine three-body bound states, rather than resonant states or numerical artifacts, and the framework we adopted is reliable and effective for analyzing the three-body systems.

V. SUMMARY

Although several investigations have been conducted so far, the structure of the DDK system remains incompletely understood. In this work, we investigate the $I(J^P) = \frac{1}{2}(0^-)$ DDK system within a coupled-channel framework that includes the DDK , D^*D^*K , $DD_s\eta$ and $DD_s\pi$ channels. In particular, the DD interaction is constrained by the experimental data associated with $Z_c(3900)$, $X(3872)$, and T_{cc}^+ , while the DK interaction is fixed by the pole position of $D_{s0}^*(2317)$ together with DK scattering length from lattice-QCD calculations. Within this constrained theoretical setup, we solve the three-body problem using GEM and searched for possible resonant states via the CSM.

We find that the DDK system supports a deeply bound state over a broad range of parameter choices, with a typical binding energy of approximately 70 MeV below the three-body threshold. The binding energy of this state exhibits only weak dependence on the momentum cutoff in the range $\Lambda = 1.1\text{--}1.3$ GeV, indicating that this state is a

robust prediction within the present framework. In contrast, an additional shallow bound state emerges only for a subset of the parameter space, specifically, when the DK interaction contains a short-range repulsive core alongside a longer-range attractive component, and may disappear into the continuum as the DD interaction strength is varied. Moreover, an analogous study was applied to the D^*D^*K system, in which we found that its bound-state behavior is similar to that of the DDK system.

Analysis of the rms radii indicates that the deeply bound state possesses a compact three-body structure, whereas the shallow state is a candidate of a three-body halo state. This halo state is characterized by large and comparable values of $r_{D^{(*)}D^{(*)}}$ and $r_{D^{(*)}K}$, as well as the absence of a dominant two-body subcluster. We further find that the D^*D^*K admixture is negligible, implying that coupled-channel effect from D^*D^*K plays only a minor role in the formation mechanism of these states. In addition, we observe that coupled-channel effects arising from the $DD_s\eta$ channel yield only minor corrections to the DDK bound state and give rise to a modest reduction in its binding energy. By contrast, opening the $DD_s\pi$ channel substantially alters the nature of the three-body DDK state and transforms the bound state into a resonance. The current study is expected to provide valuable information for understanding the rich structures of the three-body DDK and D^*D^*K systems, as well as for the future experimental search for such exotic

hadronic molecules at facilities such as BESIII, LHCb and Belle II.

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DATA AVAILABILITY

The data that support the findings of this study are openly available in the Zenodo repository [99].

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- [1] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Phys. Rep.* **639**, 1 (2016).
 - [2] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
 - [3] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2023).
 - [4] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
 - [5] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, *Rep. Prog. Phys.* **80**, 076201 (2017).
 - [6] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, *Phys. Rep.* **1108**, 1 (2025).
 - [7] F.-K. Guo, X.-H. Liu, and S. Sakai, *Prog. Part. Nucl. Phys.* **112**, 103757 (2020).
 - [8] Z.-G. Wang, *Front. Phys. (Beijing)* **21**, 016300 (2026).
 - [9] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, *Phys. Rep.* **873**, 1 (2020).
 - [10] L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, *Phys. Rep.* **1019**, 1 (2023).
 - [11] A. Esposito *et al.*, *Phys. Rep.* **668**, 1 (2017).
 - [12] R. Lebed, R. E. Mitchell, and E. S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
 - [13] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, and H.-L. Yu, *Phys. Rev. D* **46**, 1148 (1992); **55**, 5851(E) (1997).
 - [14] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, *Phys. Rep.* **281**, 145 (1997).
 - [15] H. Georgi, *Phys. Lett. B* **240**, 447 (1990).
 - [16] A. V. Manohar and M. B. Wise, *Heavy Quark Physics*, Vol. 10 (Cambridge University Press, Cambridge, 2000).
 - [17] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
 - [18] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **110**, 252001 (2013).
 - [19] Z. Q. Liu *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **110**, 252002 (2013); **111**, 019901(E) (2013).
 - [20] R. Aaij *et al.* (LHCb Collaboration), *Nat. Phys.* **18**, 751 (2022).
 - [21] R. Aaij *et al.* (LHCb Collaboration), *Nat. Commun.* **13**, 3351 (2022).
 - [22] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 072001 (2015).
 - [23] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **122**, 222001 (2019).
 - [24] E. S. Swanson, *Phys. Rep.* **429**, 243 (2006).
 - [25] M. B. Voloshin, *Prog. Part. Nucl. Phys.* **61**, 455 (2008).
 - [26] N. Brambilla *et al.*, *Eur. Phys. J. C* **71**, 1534 (2011).
 - [27] N. Brambilla *et al.*, *Eur. Phys. J. C* **74**, 2981 (2014).
 - [28] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, *Prog. Theor. Exp. Phys.* **2016**, 062C01 (2016).
 - [29] A. Ali, J. S. Lange, and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
 - [30] S. L. Olsen, T. Skwarnicki, and D. Zieminska, *Rev. Mod. Phys.* **90**, 015003 (2018).
 - [31] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **90**, 242001 (2003).
 - [32] D. Besson *et al.* (CLEO Collaboration), *Phys. Rev. D* **68**, 032002 (2003); **75**, 119908(E) (2007).
 - [33] P. Krokovny *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262002 (2003).
 - [34] E. E. Kolomeitsev and M. F. M. Lutz, *Phys. Lett. B* **582**, 39 (2004).
 - [35] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, *Phys. Lett. B* **641**, 278 (2006).
 - [36] T. Barnes, F. E. Close, and H. J. Lipkin, *Phys. Rev. D* **68**, 054006 (2003).
 - [37] E. van Beveren and G. Rupp, *Phys. Rev. Lett.* **91**, 012003 (2003).
 - [38] J. Hofmann and M. F. M. Lutz, *Nucl. Phys. A* **733**, 142 (2004).
 - [39] D. Gamermann, O. Romanets, R. Molina, E. Oset, and M. J. Vicente Vacas, *Phys. Rev. D* **76**, 074016 (2007).
 - [40] B. Aubert, R. Barate, D. Boutigny, J.-M. Gaillard, A. Hicheur, Y. Karyotakis, J. Lees, P. Robbe, V. Tisserand, A. Zghiche *et al.*, *Phys. Rev. D* **69**, 031101 (2004).
 - [41] R. Aaij, A. Abdelmotteleb *et al.*, *Sci. Bull.* **70**, 1432 (2025).
 - [42] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, *Phys. Rev. D* **76**, 114008 (2007).
 - [43] M. Albaladejo, P. Fernandez-Soler, J. Nieves, and P. G. Ortega, *Eur. Phys. J. C* **78**, 722 (2018).
 - [44] c.-J. Xiao, D.-Y. Chen, and Y.-L. Ma, *Phys. Rev. D* **93**, 094011 (2016).
 - [45] H.-L. Fu, H. W. Grießhammer, F.-K. Guo, C. Hanhart, and U.-G. Meißner, *Eur. Phys. J. A* **58**, 70 (2022).
 - [46] W.-H. Liang, Z.-R. Hu, and E. Oset, *Eur. Phys. J. A* **62**, 104 (2026).
 - [47] Y. Tan, Y. Wu, Y. Yang, and J. Ping, *Phys. Rev. D* **111**, 096023 (2025).
 - [48] L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meißner, *Phys. Rev. D* **87**, 014508 (2013).

- [49] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, *Phys. Rev. Lett.* **111**, 222001 (2013).
- [50] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, *Eur. Phys. J. C* **79**, 13 (2019).
- [51] G. S. Bali, S. Collins, A. Cox, and A. Schäfer (RQCD Collaboration), *Phys. Rev. D* **96**, 074501 (2017).
- [52] J. D. E. Yeo, C. E. Thomas, and D. J. Wilson, *J. High Energy Phys.* **07** (2024) 012.
- [53] C. Alexandrou, J. Berlin, J. Finkenrath, T. Leontiou, and M. Wagner, *Phys. Rev. D* **101**, 034502 (2020).
- [54] G. K. C. Cheung, C. E. Thomas, D. J. Wilson, G. Moir, M. Peardon, and S. M. Ryan (Hadron Spectrum Collaboration), *J. High Energy Phys.* **02** (2021) 100.
- [55] C. B. Lang, L. Leskovec, D. Mohler, and S. Prelovsek, *Phys. Rev. D* **90**, 034510 (2014).
- [56] Z.-Y. Yang, Z.-F. Wang, L.-S. Guo, J.-J. Wang, F.-K. Guo, and A. V. Nefediev, *Phys. Rev. Lett.* **129**, 112001 (2022).
- [57] L. Ma, Q. Wang, and U.-G. Meißner, *Chin. Phys. C* **43**, 014102 (2019).
- [58] T.-W. Wu, M.-Z. Liu, L.-S. Geng, E. Hiyama, and M. P. Valderrama, *Phys. Rev. D* **100**, 034029 (2019).
- [59] A. Martinez Torres, K. P. Khemchandani, and L. S. Geng, *Phys. Rev. D* **99**, 076017 (2019).
- [60] A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, *Few Body Syst.* **61**, 35 (2020).
- [61] X. Wei, Q.-H. Shen, and J.-J. Xie, *Eur. Phys. J. C* **82**, 718 (2022).
- [62] Z. Zhang, X.-Y. Hu, G. He, J. Liu, J.-A. Shi, B.-N. Lu, and Q. Wang, *Phys. Rev. D* **111**, 036002 (2025).
- [63] Y.-W. Pan, J.-X. Lu, E. Hiyama, L.-S. Geng, and A. Hosaka, *Phys. Rev. D* **111**, 114006 (2025).
- [64] T.-W. Wu, M.-Z. Liu, and L.-S. Geng, *Phys. Rev. Lett.* **135**, 031902 (2025).
- [65] X.-L. Ren, K. P. Khemchandani, and A. Martínez Torres, *Eur. Phys. J. C* **84**, 1297 (2024).
- [66] U. Özdem, *Phys. Rev. D* **112**, 114013 (2025).
- [67] Z. Zhou, G.-L. Yu, Z.-G. Wang, and J. Lu, *Eur. Phys. J. C* **86**, 968 (2026).
- [68] C.-J. Xiao and D.-Y. Chen, *Eur. Phys. J. A* **53**, 127 (2017).
- [69] Y.-K. Chen, L. Meng, J.-Z. Wang, and S.-L. Zhu, *Phys. Rev. D* **114**, 014048 (2026).
- [70] S.-Y. Chen *et al.*, *Phys. Lett. B* **830**, 137391 (2022).
- [71] M. Padmanath and S. Prelovsek, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [72] M. L. Du, M. Albaladejo, and F. K. Guo, *Phys. Lett. B* **833**, 137290 (2022).
- [73] Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, *Phys. Rev. Lett.* **131**, 161901 (2023).
- [74] T. Whyte, D. J. Wilson, and C. E. Thomas, *Phys. Rev. D* **111**, 034511 (2025).
- [75] H.-X. Zhu, L. Meng, Y. Ma, N. Li, W. Chen, and S.-L. Zhu, *Phys. Rev. D* **111**, 094022 (2025).
- [76] R. Xu, L. Meng, H.-X. Zhu, N. Li, and W. Chen, *Phys. Rev. D* **111**, 094015 (2025).
- [77] S.-Y. Chen, F.-Y. Chen, X.-L. Chen, L. Meng, N. Li, and W. Chen, *Phys. Rev. D* **113**, 116026 (2026).
- [78] E. Hiyama, Y. Kino, and M. Kamimura, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
- [79] J. Aguilar and J. M. Combes, *Commun. Math. Phys.* **22**, 269 (1971).
- [80] E. Balslev and J. M. Combes, *Commun. Math. Phys.* **22**, 280 (1971).
- [81] X. Liu, Z.-G. Luo, Y.-R. Liu, and S.-L. Zhu, *Eur. Phys. J. C* **61**, 411 (2009).
- [82] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [83] M. Altenbuchinger, L.-S. Geng, and W. Weise, *Phys. Rev. D* **89**, 014026 (2014).
- [84] L. Happ, P. Naidon, and E. Hiyama, *Few-Body Syst.* **65**, 38 (2024).
- [85] G. Papadimitriou, *Few-Body Syst.* **57**, 1239 (2016).
- [86] T. Berggren, *Phys. Lett. B* **373**, 1 (1996).
- [87] B. A. Lippmann and J. Schwinger, *Phys. Rev.* **79**, 469 (1950).
- [88] Y. Koike, *Prog. Theor. Phys.* **87**, 775 (1992).
- [89] S. Petschauer, N. Kaiser, J. Haidenbauer, U.-G. Meißner, and W. Weise, *Front. Phys.* **8**, 12 (2020).
- [90] H. A. Bethe, *Phys. Rev.* **76**, 38 (1949).
- [91] M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque, and J. Nieves, *Phys. Lett. B* **755**, 337 (2016).
- [92] K. Yu, G.-J. Wang, J.-J. Wu, and Z. Yang, *Phys. Rev. D* **110**, 114029 (2024).
- [93] S. X. Nakamura, X. H. Li, H. P. Peng, Z. T. Sun, and X. R. Zhou, *Phys. Rev. D* **112**, 054027 (2025).
- [94] Y.-H. Chen, M.-L. Du, and F.-K. Guo, *Sci. China Phys. Mech. Astron.* **67**, 291011 (2024).
- [95] A. S. Jensen, K. Riisager, D. V. Fedorov, and E. Garrido, *Rev. Mod. Phys.* **76**, 215 (2004).
- [96] A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos, *J. High Energy Phys.* **05** (2015) 153.
- [97] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, *Eur. Phys. J. C* **79**, 13 (2019).
- [98] M. Altenbuchinger, L. S. Geng, and W. Weise, *Phys. Rev. D* **89**, 014026 (2014).
- [99] H. Xie, S.-Y. Chen, N. Li, and W. Chen, 10.5281/zenodo.22314046 (2026).