

Nelson-Barr models with vectorlike quark doublets

G. H. S. Alves^{1,*}, C. C. Nishi^{2,†} and L. Vecchi^{3,‡}

¹*Centro de Ciências Naturais e Humanas, Universidade Federal do ABC,
09.210-170, Santo André, São Paulo, Brazil*

²*Centro de Matemática, Computação e Cognição, Universidade Federal do ABC,
09.210-170, Santo André, São Paulo, Brazil*

³*INFN, Sezione di Padova, Via Marzolo 8, Padova, 35131, Italy*



(Received 23 April 2026; accepted 15 June 2026; published 16 July 2026)

We investigate Nelson-Barr (NB) solutions to the strong CP problem in which spontaneous CP violation is transmitted to the Standard Model (SM) through mixing with a vectorlike partner of the SM quark doublet. We show that these constructions constitute compelling and phenomenologically viable alternatives to the more widely studied singlet-based NB models. A key result of our analysis is that an accidental symmetry of the renormalizable theory delays the leading contributions to $\bar{\theta}$ until three loops, naturally suppressing hadronic CP violation. We outline the main phenomenological constraints, including future electric dipole moment experiments, as well as the main differences between these scenarios and generic models with doublet vectorlike quarks.

DOI: [10.1103/sy92-fyjf](https://doi.org/10.1103/sy92-fyjf)

I. INTRODUCTION

The persistent absence of observable CP violation in strong interactions remains one of the most striking puzzles in particle physics. Among the proposed resolutions, the Nelson-Barr (NB) mechanism [1,2] occupies a prominent position as an axionless class of approaches. In this paradigm, CP is imposed as a fundamental symmetry of nature and is subsequently broken spontaneously by the condensate of a (fundamental or composite) scalar field. The resulting CP violation is transmitted to the Standard Model (SM) via fermionic mediators. A Yukawa interaction among the CP -violating order parameter, the SM fermions, and heavy vectorlike mediators ensures that physical CP -violating phases enter the Cabibbo-Kobayashi-Maskawa (CKM) matrix at tree level, while the QCD angle $\bar{\theta}$ is generated only radiatively. The radiative corrections involving the couplings of the CP -violating Higgs sector, which generically arise already at one loop [3–5],¹ are model dependent but can naturally lie well within current bounds, especially in models with a

composite CP -violating scalar [7,8]. There are, however, additional radiative contributions to $\bar{\theta}$ that arise solely from the minimal ingredients of these scenarios—the SM plus the fermionic mediators [9]. These irreducible contributions remain under control provided an appropriate choice of field content is made. Because they are unavoidable, they offer a distinctive signature of such scenarios: NB models necessarily predict a nonzero $\bar{\theta}$, potentially within reach of future electric dipole moment (EDM) experiments.

In this work, we investigate a class of NB models in which CP violation is communicated via mixing between the SM quark doublets and vectorlike quark doublets with the same electroweak charges—a scenario referred to as q -mediated NB models in Ref. [9]. This setup is a variant of the popular d -mediated NB framework (see examples in Refs. [3,4,10–15]), where the mediators are vectorlike quarks with the quantum numbers of the SM quark singlet d . The case of u -mediation is analogously possible. Two important differences distinguish these scenarios from those previously studied in the literature, motivating a dedicated analysis. First, generic realizations of this type of mediation are typically disfavored because they induce large two-loop contributions to $\bar{\theta}$ [16]. However, we show that minimal *renormalizable* NB models with q -mediation are protected from these large corrections by an accidental symmetry. Second, it is well known that NB scenarios require special scrutiny because their coupling structure is so constrained that reproducing the CKM matrix and the observed Yukawa hierarchy becomes highly nontrivial and must be carefully verified. This feature marks a key difference from more common singlet vectorlike quark (VLQ) models, which possess a less restrictive structure to fit the SM.

*Contact author: alves.gustavo@ufabc.edu.br

†Contact author: celso.nishi@ufabc.edu.br

‡Contact author: luca.vecchi@pd.infn.it

¹They can be postponed to two loops using a nonconventional CP symmetry [6].

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

Our analysis is structured as follows: in Sec. II, we review the basic elements of Nelson-Barr models with q -mediation and compare their flavor structure to that of generic VLQ scenarios. Details of the precise parameter mapping are presented in the Appendix A. In Sec. III, we explore in detail the region of parameter space that reproduces the flavor structure and CP violation of the SM. This is a nontrivial task that can unfortunately only be carried out numerically. Our analysis is partially supported by analytical manipulations (collected in Appendixes B and C) and partially guided by basis-independent considerations (see Appendix D). Our scenarios are subject to several constraints, the most important of which are discussed in Sec. IV. These include perturbativity requirements (Sec. IV A), precision electroweak constraints (Sec. IV B), and bounds from flavor-violating observables (Sec. IV C). Special attention is devoted to the nonobservation of hadronic CP violation. In particular, the radiative corrections to the QCD $\bar{\theta}$ parameter are estimated in Sec. V, as these do not decouple when the new physics scale is increased and thus provide a robust signature of NB scenarios with q -mediation. Our conclusions are presented in Sec. VI.

II. DOUBLET VECTORLIKE QUARK OF NELSON-BARR TYPE

Let us introduce a Dirac quark $Q = Q_L + Q_R$ in the representation $(2, 1/6)$ of the electroweak $SU(2)_L \times U(1)_Y$, the same of the SM quark doublets q_{iL} (with $i = 1, 2, 3$ the generation index).

A doublet vectorlike quark Q is of Nelson-Barr type (NB-VLQ) when its Lagrangian is

$$-\mathcal{L} = \bar{q}_{iL} \mathcal{Y}_{ij}^d H d_{jR} + \bar{q}_{iL} \mathcal{Y}_{ij}^u \tilde{H} u_{jR} + \bar{q}_{iL} \mathcal{M}_i^{qQ} Q_R + \bar{Q}_L \mathcal{M}_Q Q_R + \text{H.c.}, \quad (1)$$

where d_{jR} and u_{jR} are the quark singlets; $\mathcal{Y}^d$, $\mathcal{Y}^u$, and $\mathcal{M}_Q$ are *real* parameters; and $\mathcal{M}^{qQ}$ is complex in the same field basis with vanishing θ angle. The former framework is realized assuming that the fundamental theory respects CP invariance as well as a $\mathbb{Z}_2$ symmetry under which Q is odd, and that $CP \times \mathbb{Z}_2$ is softly broken only by $\mathcal{M}^{qQ}$.

The Lagrangian (1) depends on $1(\mathcal{M}_Q) + 3(\mathcal{Y}^u) + 6(\mathcal{Y}^d) + 5(\mathcal{M}^{qQ}) = 15$ parameters, as can be seen in the basis in which $\mathcal{Y}^u = \hat{\mathcal{Y}}^u$ is diagonal and $\mathcal{Y}^d = O_{d_L} \hat{\mathcal{Y}}^d$, where O_{d_L} and $\hat{\mathcal{Y}}^d$ are a real orthogonal matrix and a diagonal matrix with real positive eigenvalues, respectively. As we will see, nontrivial correlations among the Lagrangian parameters are necessary to reproduce the flavor and CP -violating structure of the SM below the mass scale of the exotic fermion. The origin of these correlations will not be investigated here; we will simply

assume that the necessary structure is generated by some more fundamental physics.

In order to derive the effective field theory below the mass of the VLQ, it is convenient to change field basis. Performing a special unitary rotation among the fields (q_{iL}, Q_L) , one can reexpress Eq. (1) in the form

$$-\mathcal{L} = \bar{q}_{iL} Y_{ij}^d H d_{jR} + \bar{q}_{iL} Y_{ij}^u \tilde{H} u_{jR} + \bar{Q}_L Y_j^{Qd} H d_{jR} + \bar{Q}_L Y_j^{Qu} \tilde{H} u_{jR} + \bar{Q}_L M_Q Q_R + \text{H.c.} \quad (2)$$

The explicit mapping is found to be

$$Y^d = (1_3 - w w^\dagger)^{1/2} \mathcal{Y}^d, \quad Y^{Qd} = w^\dagger \mathcal{Y}^d, \quad (3a)$$

$$Y^u = (1_3 - w w^\dagger)^{1/2} \mathcal{Y}^u, \quad Y^{Qu} = w^\dagger \mathcal{Y}^u, \quad (3b)$$

$$M^Q = (1 - |w|^2)^{-1/2} \mathcal{M}_Q, \quad (3c)$$

where the three-vector w is implicitly defined by

$$w = \mathcal{M}^{qQ} M_Q^{-1}, \quad (4)$$

and has a norm in the range

$$0 < |w| < 1. \quad (5)$$

See Appendix A for more details. The utility of this new basis is evidenced by the fact that in this basis, the heavy state does not mix with the SM fermions before electroweak symmetry breaking, so that $Y_{ij}^{u,d}$ can be identified (up to radiative corrections) as the SM Yukawas and M_Q as the dominant source of VLQ mass. We will call the field basis in Eq. (2) the “VLQ basis,” as opposed to the “Nelson-Barr basis” of Eq. (1).

Note that a generic model for a VLQ Q can always be put in the form (2). The key difference is that a generic model depends on $1(M^Q) + 3(Y^u) + 7(Y^d) + 6(Y^{Qd}) + 5(Y^{Qu}) = 22$ parameters,² namely 12 parameters in addition to the SM, whereas our model, which originates from Eq. (1), only has five extra parameters, the same number as in a *singlet* NB-VLQ model.

III. REPRODUCING THE SM YUKAWAS AND THE CKM MATRIX

In this section, we discuss under which conditions the parameters in Eq. (1) can reproduce the SM below the scale M_Q . As will become evident shortly, this turns out to be possible only in the regime $|w| \sim 1$. In that limit, there is no small expansion parameter one can use to parametrize CP

²This can be seen by choosing, for instance, the basis in which $Y^u = \hat{Y}^u$ is diagonal, $Y^d = V \hat{Y}^d$, and one phase in Y^{Qu} is removed by rephasing Q_L, Q_R .

violation. Furthermore, the matrix $\mathbb{1}_3 - ww^\dagger$ becomes approximately rank-2, indicating that some correlation among the entries in $\mathcal{Y}^d$, $\mathcal{Y}^u$ is necessary to recover the quark mass hierarchy. These considerations indicate that the SM flavor structure in NB-VLQ models can be studied reliably only via a numerical analysis. This is what we will do next.

To start, we observe that

$$\begin{aligned} (\mathbb{1}_3 - ww^\dagger)^{1/2} \mathcal{Y}^d \mathcal{Y}^{d\dagger} (\mathbb{1}_3 - ww^\dagger)^{1/2} &= Y^d Y^{d\dagger}, \\ (\mathbb{1}_3 - ww^\dagger)^{1/2} \mathcal{Y}^u \mathcal{Y}^{u\dagger} (\mathbb{1}_3 - ww^\dagger)^{1/2} &= Y^u Y^{u\dagger}. \end{aligned} \quad (6)$$

The rhs of these relations can be diagonalized via independent rotations, as follows:

$$Y^u Y^{u\dagger} = V_{u_L} \text{diag}(y_{u_i}^2) V_{u_L}^\dagger, \quad Y^d Y^{d\dagger} = V_{d_L} \text{diag}(y_{d_i}^2) V_{d_L}^\dagger, \quad (7)$$

where $(y_{u_i}) = (y_u, y_c, y_t)$ correspond to the eigenvalues of the up-quark Yukawas and are determined experimentally. Analogous considerations can be made for y_{d_i} . However, the matrices V_{u_L} , V_{d_L} cannot be determined unambiguously. Only the CKM matrix V is constrained by data, as follows:

$$V_{u_L}^\dagger V_{d_L} = V. \quad (8)$$

We therefore see that our parameters w , $\mathcal{Y}^d$, and $\mathcal{Y}^u$ are constrained by Eq. (6), but unfortunately cannot be fully determined. In the following subsections, we will attempt to extract as much information as possible from Eq. (6). In Secs. III A and III B, we will see how the quark masses and the CKM matrix constrain w , $\mathcal{Y}^d$, $\mathcal{Y}^u$. From these results, the structure of the couplings $Y^{Qu, Qd}$ of Eq. (2) will follow. The latter will be discussed in Sec. III C.

A. SM Yukawas as input

The relation between the SM Y and the CP -conserving $\mathcal{Y}$ in both up and down sectors is given in Eq. (3):

$$\Omega^{1/2} \mathcal{Y} = Y, \quad (9)$$

where we introduced the shorthand

$$\Omega \equiv \mathbb{1} - ww^\dagger. \quad (10)$$

The expression $YY^\dagger$ gives the relations (6), whose mismatch in the diagonalization matrices leads to the CKM mixing. If we take the other Hermitian combination, we obtain

$$\mathcal{Y}^\dagger \Omega \mathcal{Y} = Y^\dagger Y. \quad (11)$$

The procedure to solve this relation for $\mathcal{Y}$, given $Y^\dagger Y$ and w , was discussed in Ref. [10] for a rank-1 w and in Ref. [12] for w with rank-2 or greater. Applying that procedure to the present

scenario allows us to use the SM Yukawas as input parameters to constrain w , $\mathcal{Y}^d$, and $\mathcal{Y}^u$. We summarize the procedure in the following and relegate the details to Appendix B.

We first split the rhs of Eq. (11) as

$$Y^\dagger Y = \text{Re}(Y^\dagger Y) + i \text{Im}(Y^\dagger Y) = X^{1/2} [\mathbb{1}_3 + iZ] X^{1/2}, \quad (12)$$

where $X = \text{Re}(Y^\dagger Y)$ is positive definite and $X^{1/2}$ is its square root. Note that X is real symmetric, while $\text{Im}(Y^\dagger Y)$ and Z , are real antisymmetric. Therefore, Z can be parametrized in terms of a (pseudo) vector $\mathbf{z} = (z_k)$ as

$$Z_{ij} = \epsilon_{ijk} z_k. \quad (13)$$

Although there are two of these vectors, $\mathbf{z}^u$ and $\mathbf{z}^d$, one for the up and one for the down sector, their norm turn out to be the same (see Appendix B), as follows:

$$\mathbf{z}^u \cdot \mathbf{z}^u = \mathbf{z}^d \cdot \mathbf{z}^d = \mu^2, \quad (14)$$

and entirely determined by w . The quantity μ is a measure of CP violation in $Y^{u\dagger} Y^u$, $Y^{d\dagger} Y^d$. It is related to w , as we will see below.

Performing an orthogonal basis transformation on d_R and u_R will not affect the SM input in Eq. (6). We can choose a basis where X is diagonal, as follows:

$$X = \text{Re}(Y^\dagger Y) = \text{diag}(x_1, x_2, x_3). \quad (15)$$

This can be achieved independently for Y^u and Y^d . The eigenvalues x_i scale as y_i^2 , with y_i the SM Yukawas.

Now, for a given $\mathbf{z}$, Eq. (12) can be solved for x_i in each sector in terms of the eigenvalues y_i^2 of $Y^\dagger Y$. One can check that x_i inherit the hierarchy from y_i^2 , and we find

$$y_1^2 < x_1 < y_2^2, \quad y_2^2 \lesssim x_2 < y_3^2, \quad x_3 \sim y_3^2. \quad (16)$$

This can be seen in the points shown in a lighter shade in Fig. 1, where we see the possible values for $\sqrt{x_i}/y_3$ as a function of $1 - \mu$ for each sector. It corresponds to the spectrum of $X^{1/2}/y_3$, while the gray horizontal lines show the values of y_i/y_3 corresponding to the spectrum of $(YY^\dagger)^{1/2}/y_3$. Qualitatively, the eigenvalues of X lie between the largest and smallest eigenvalues of $YY^\dagger$. The gray vertical line in both panels shows the minimum value of $1 - \mu$ that allows solutions for Eq. (12) in the down sector; see Appendix C. For the up sector separately, $1 - \mu$ can in principle reach smaller values; but these are discarded because of Eq. (14). For definiteness, we use running SM Yukawas at 1 TeV [17].³

³More up-to-date values can be obtained in Ref. [18], but they do not lead to significant differences.

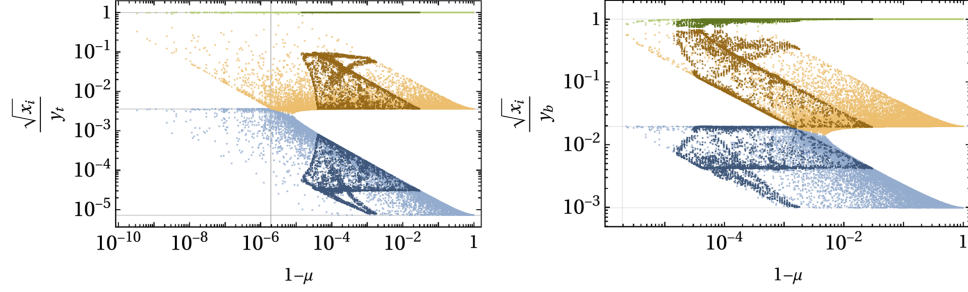


FIG. 1. Distribution of $\sqrt{x_i}/y_3$ against $1 - \mu$ for up (left) and down (right) sectors with $i = 3, 2, 1$ corresponding to green, orange, and blue points. Points shown in a lighter shade only reproduce the eigenvalues of the SM Yukawas, while the points in a darker shade also reproduce the CKM angles and phase. The horizontal lines show y_i/y_3 , while the vertical line refers to $(1 - \mu)_{\min}$ in Eq. (B4).

Our next goal is to see how these considerations can be used to constrain $\mathcal{Y}$. With this in mind, we go back to the lhs of Eq. (11) and write

$$\Omega = \mathbb{1}_3 - ww^\dagger = \Omega_1 + i\Omega_2. \quad (17)$$

Then the real and imaginary parts of Eq. (11) become

$$\begin{aligned} \mathcal{Y}^\dagger \Omega_1 \mathcal{Y} &= X, \\ \mathcal{Y}^\dagger \Omega_2 \mathcal{Y} &= X^{1/2} Z X^{1/2}. \end{aligned} \quad (18)$$

These relations can next be solved for $\mathcal{Y}$ in terms of w , x_i , and $\mathbf{z}$, which in turn depend on the SM Yukawas y_i (y_{u_i} or y_{d_i}) as explained above. The solution for the real part yields

$$\mathcal{Y}^\dagger = X^{1/2} \mathcal{O} \Omega_1^{-1/2} \quad \text{or} \quad \mathcal{Y} = \Omega_1^{-1/2} \mathcal{O}^\dagger X^{1/2}, \quad (19)$$

where $\mathcal{O}$ is an orthogonal matrix determined by the vector $\mathbf{z}$ (see Appendix B).

Choosing the basis where

$$w = (a, ib, 0)^\dagger, \quad (20)$$

the results of Appendix B give [12]

$$\mu = \frac{ab}{\sqrt{1 - a^2} \sqrt{1 - b^2}}, \quad (21)$$

which is the advertised relation between μ and w . We will interpret this as the relation determining a given the pair (b, μ) . Also, observe that nonvanishing μ implies nonvanishing ab . The parameter μ can be also related directly to $Y^\dagger Y$; see Eq. (B8).

Let us summarize what we have achieved so far. We started from the 14 parameters w , $\mathcal{Y}^d$, $\mathcal{Y}^u$ and constrained them via Eq. (6) (the 15th parameter is the mass scale $\mathcal{M}^Q$ and remains undetermined). In this section, we showed how we can trade them for μ , b (two unknown parameters), the SM Yukawas y_{u_i, d_i} (six measured quantities), and the matrices $\mathcal{O}_{u, d}$ (six unknown parameters in total). Of the original 14 unknown parameters, six have thus been

determined. Of the remaining eight, namely

$$\mu, b, \mathcal{O}_{u, d}, \quad (22)$$

μ is forced to be in the range shown in Fig. 1, whereas b and $\mathcal{O}_{u, d}$ are still unconstrained. In the following, we will see how the CKM matrix can be used to constrain them.

B. The CKM as input

We rewrite the relations (6) using Eq. (19) and obtain

$$\begin{aligned} \Omega^{1/2} \Omega_1^{-1/2} \mathcal{O}_u^\dagger X^u \mathcal{O}_u \Omega_1^{-1/2} \Omega^{1/2} &= Y^u Y^{u\dagger}, \\ \Omega^{1/2} \Omega_1^{-1/2} \mathcal{O}_d^\dagger X^d \mathcal{O}_d \Omega_1^{-1/2} \Omega^{1/2} &= Y^d Y^{d\dagger}. \end{aligned} \quad (23)$$

The unitary matrices that diagonalize them determine the CKM matrix according to Eq. (8).

To find the physically relevant parameters, we perform a scan in the eight-dimensional space (22) and select only the points that lead to the correct CKM matrix. We allow a 3σ variation of the CKM parameters at 1 TeV [17]. Our numerical analysis is complemented with an analytic one, which makes use of some flavor invariants as described in the Appendix D.

Some viable points are shown in Fig. 1 in the plane $(1 - \mu, \sqrt{x_i}/y_3)$ for both up (left panel) and down (right panel) sectors. The pale-colored points are compatible with the SM Yukawas, but are not necessarily capable of reproducing the CKM matrix. The darker-colored points instead reproduce both the SM Yukawas as well as the CKM matrix. Note that to reproduce the CKM matrix, we find that the parameter μ must be rather close to unity, as follows:

$$1.4 \times 10^{-5} \lesssim 1 - \mu \lesssim 0.03. \quad (24)$$

In this regime, and writing

$$\frac{w}{|w|} = (\cos \theta, i \sin \theta, 0), \quad \theta \in (0, \pi/2), \quad (25)$$

Eq. (21) can be approximated as

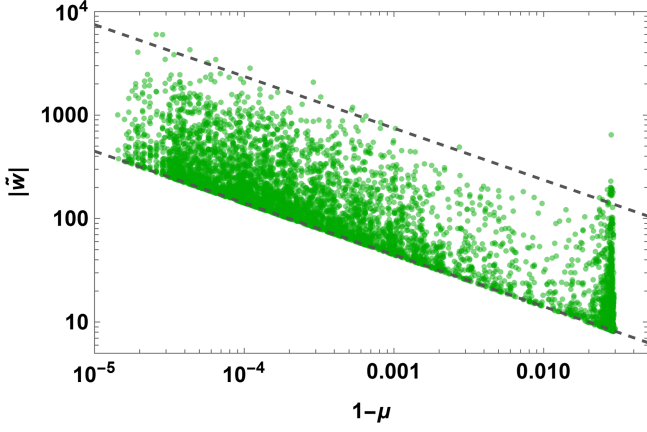


FIG. 2. $|\tilde{w}|$ as a function of $1 - \mu$ for physical points that reproduce the SM. The dashed lines obey $|\tilde{w}| = \sqrt{\frac{2}{1-\mu}}$ and $|\tilde{w}| = \sqrt{\frac{556}{1-\mu}}$, corresponding to $\sin \theta = 1$ and 0.03 in Eq. (26) approximately.

$$1 - \mu \approx \frac{1 - |w|^2}{2s_\theta^2 c_\theta^2} = \frac{1}{2s_\theta^2 c_\theta^2 (1 + |\tilde{w}|^2)}. \quad (26)$$

This implies that

$$|w| \approx 1. \quad (27)$$

We show in Fig. 2 the quantity [see Eq. (A6)]

$$\tilde{w} = \mathcal{M}^{qQ} / \mathcal{M}^Q = \frac{w}{\sqrt{1 - |w|^2}} \quad (28)$$

as a function of $1 - \mu$ and see that Eq. (26) is valid to a good approximation. Our scan only covers the domain $\sin \theta \gtrsim 0.03$.

Equation (27) is the relation we alluded to at the beginning of Sec. III. It is a necessary condition to reproduce the CKM structure, and also the technical reason why our numerical analysis is necessary to identify in which regime Eq. (1) reproduces the SM at low energies.

C. Hierarchies in the VLQ couplings

Having identified the relevant region of parameter space, we can now discuss the resulting flavor structure of the couplings $Y^{Qu, Qd}$ of the VLQ doublet to the SM quarks, as defined in the VLQ basis (2).

Using the convenient notation (28) (see Appendix A) we rewrite the relations (3) in the basis where $Y^u = \hat{Y}^u$ is diagonal:

$$Y^{Qu} = \tilde{w}^\dagger \hat{Y}^u, \quad Y^{Qd} = \tilde{w}^\dagger V \hat{Y}^d, \quad (29)$$

with V the CKM matrix. It is clear that if all the components of $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3)$ are of the same order,

these couplings inherit the hierarchies of the SM Yukawas. In contrast, the overall scale of the couplings are not determined by the SM Yukawas. It is controlled by $|\tilde{w}|$ in Eq. (28) which, as seen in Fig. 2, can be significantly larger than unity for the doublet NB-VLQ (similarly to the singlet versions [12]).

We first inspect the relative size of the components $\tilde{w}_i$, working in the basis $Y^u = \hat{Y}^u$ [rather than the one we adopted in the parametrization (20)]. We show in Fig. 3 the ratios $|\tilde{w}_1/\tilde{w}_3|$ and $|\tilde{w}_2/\tilde{w}_3|$ as a function of $|\tilde{w}|$. We see that within a broad variation, there is an *inverted* hierarchy which roughly follows

$$|\tilde{w}_1| : |\tilde{w}_2| : |\tilde{w}_3| \approx 200 : 10 : 1. \quad (30)$$

The inverted hierarchy for $\tilde{w}_i$ implies that the components Y_i^{Qu} and Y_i^{Qd} exhibit a hierarchy less pronounced than the corresponding Yukawa couplings of the SM. From the rough estimate (30) indeed we expect the components of Eq. (29) to scale as

$$\begin{aligned} |Y_i^{Qu}| &\sim |\tilde{w}| \left(y_u, \frac{1}{20} y_c, \frac{1}{200} y_t \right) \\ &\sim |\tilde{w}| (6 \times 10^{-6}, 1.5 \times 10^{-4}, 4 \times 10^{-3}), \\ |Y_i^{Qd}| &\sim |\tilde{w}| (y_d, 0.22 y_s, 0.01 y_b) \\ &\sim |\tilde{w}| (10^{-5}, 6 \times 10^{-5}, 10^{-4}), \end{aligned} \quad (31)$$

where in the estimation for Y^{Qd} we used

$$\begin{aligned} \tilde{w}^\dagger V &\sim |\tilde{w}| \left(1, |V_{ud}|, |V_{ub}| + \frac{1}{20} |V_{ub}| + \frac{1}{200} |V_{tb}| \right) \\ &\sim |\tilde{w}| (1, 0.22, 0.01). \end{aligned} \quad (32)$$

Note that the hierarchies in Eq. (31) differ from the case of singlet NB-VLQs of up or down type which roughly

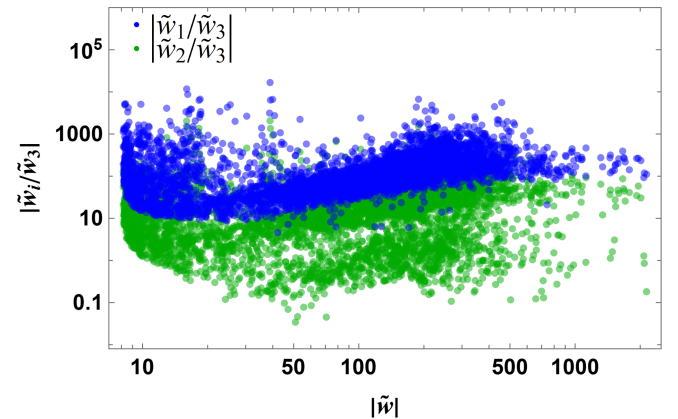


FIG. 3. Ratios $|\tilde{w}_i|/|\tilde{w}_3|$, $i = 1, 2$ against the norm $|\tilde{w}|$. All points reproduce the SM.

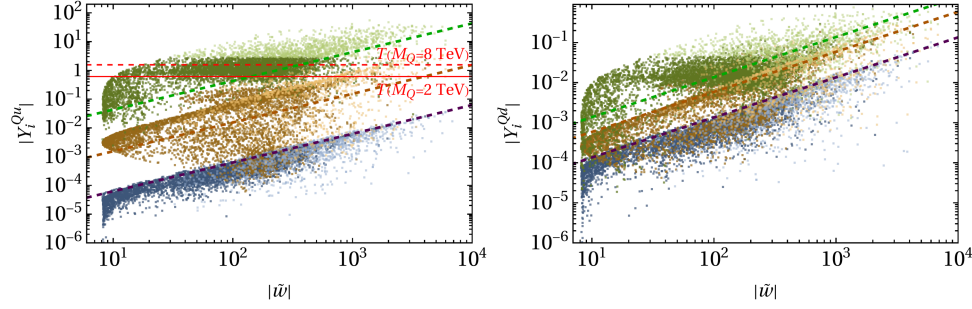


FIG. 4. $|Y_i^{Qu}|$ (left) and $|Y_i^{Qd}|$ (right) as a function of $|\tilde{w}|$ for $i = 3, 2, 1$, respectively, in green, orange, and blue. The lighter green points are excluded by the perturbativity limit (36). The dashed lines with the same color as the points show the approximate functions in Eq. (31). The horizontal continuous (dashed) red line indicate the constraint coming from electroweak precision observables for $M_Q = 2$ TeV ($M_Q = 8$ TeV); see Sec. IV B.

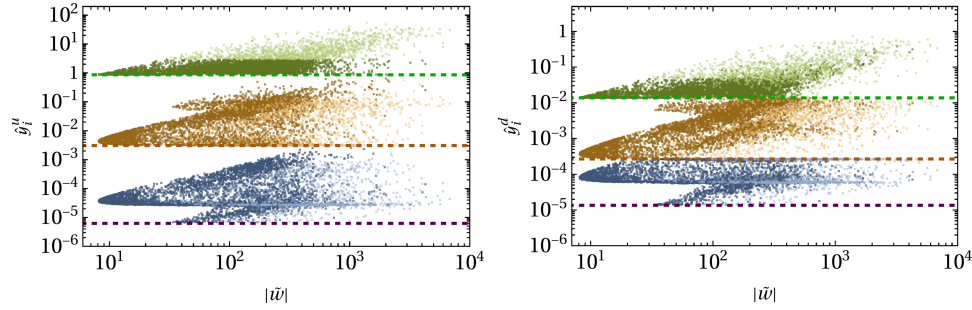


FIG. 5. Eigenvalues $\hat{\mathcal{Y}}_i^u$ (left) and $\hat{\mathcal{Y}}_i^d$ (right) of the CP -conserving Yukawa couplings as functions of $|\tilde{w}|$. The dashed lines correspond to the values of the SM Yukawas at 1 TeV. The color coding is the same as in Fig. 4.

follow the hierarchies of the CKM third row or column, respectively [11].

The qualitative hierarchy in the Yukawas is confirmed by the more accurate numerical analysis shown in Fig. 4. The colored dots show the actual numerical values of $|Y_i^{Qu}|$ and $|Y_i^{Qd}|$ against $|\tilde{w}|$, whereas the rough estimates (31) are shown in the dashed lines. We note that the Yukawa couplings can actually vary over more than 1 order of

magnitude compared to our crude estimate (31). For comparison, we also show in Fig. 5 the behavior of the eigenvalues of the CP -conserving Yukawas $\mathcal{Y}$ against $|\tilde{w}|$. They are always larger than the SM values, as we also emphasize in Sec. IV A.

Finally, Fig. 6 reveals a strong positive correlation between $|Y_3^{Qu}|$ and $|Y_3^{Qd}|$, and of course that $|Y_3^{Qu}| \gg |Y_3^{Qd}|$. A similar correlation is found between $|Y_i^{Qu}|$ and $|Y_i^{Qd}|$, as suggested by the estimate (31). These features qualitatively distinguish scenarios with NB-VLQs from generic doublet VLQs.

IV. PHENOMENOLOGICAL CONSTRAINTS

Here, we review the most relevant constraints on our models. These include collider constraints, perturbativity bounds, precision electroweak constraints, and flavor constraints. Constraints from hadronic CP violation will be discussed in Sec. V.

First, from direct collider searches [19] the VLQ must be heavier than

$$M_Q > 1.5 \text{ TeV}, \quad (33)$$

for a VLQ that couples only to the third SM family. This is approximately the case for NB-VLQs. For a review on the constraints for generic singlet VLQs, see also Ref. [20].

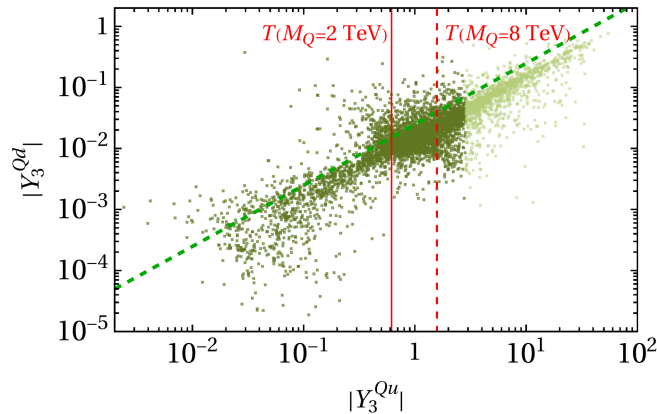


FIG. 6. Correlation between $|Y_3^{Qu}|$ and $|Y_3^{Qd}|$. The darker and lighter green points are as explained in Fig. 4. The green dashed line shows the estimate (31).

Let us next discuss the other bounds in turn.

A. Perturbativity

In order for our models to be predictable, the new couplings must all be small to allow a perturbative expansion. In the basis (1), the new couplings are $\mathcal{Y}$, whereas in the VLQ basis, they are $Y^{Qu,Qd}$. Using the relation in Eq. (3), we find that

$$|Y^{Qu}|^2 \leq \text{tr}[\mathcal{Y}^u \mathcal{Y}^{u\dagger}] |w|^2 < \sum_i (\hat{\mathcal{Y}}_i^u)^2 \approx (\hat{\mathcal{Y}}_3^u)^2, \quad (34)$$

where the first inequality follows from Schwartz inequality and the second one from $|w| < 1$. We thus see that the perturbativity constraint on $\mathcal{Y}^u$ is generally stronger. Constraints arising from $\mathcal{Y}^d$ are obviously less relevant because the down-type Yukawas are smaller; we will therefore restrict our attention to the up-quark sector.

From the relation (3) or (11) between $\mathcal{Y}^u$ and Y^u , we also see that the largest eigenvalue $\hat{\mathcal{Y}}_3^u$ of $\mathcal{Y}^u$ is always larger than the largest eigenvalue y_t of Y^u , i.e., the top Yukawa. This is confirmed by our numerical analysis shown in Fig. 5. An upper bound on $\hat{\mathcal{Y}}_3^u$ can therefore represent an important constraint for our theory, as y_t is already close to unity in the SM.

To provide a rough upper bound on $\hat{\mathcal{Y}}_3^u$, we study its RG evolution and require that a hypothetical Landau pole be

sufficiently heavier than the scale M_Q of the VLQ. From the Lagrangian (1) we see that the running of $\hat{\mathcal{Y}}_3^u$ at one loop is governed by the same renormalization group equations as the top Yukawa in the SM [21]. Retaining only the dominant contributions, we have

$$16\pi^2 \mu \frac{d\hat{\mathcal{Y}}_3^u}{d\mu} = \frac{9}{4} (\hat{\mathcal{Y}}_3^u)^3 - 4g_s^2 \hat{\mathcal{Y}}_3^u, \quad (35)$$

which reveals that the β function is positive whenever $\hat{\mathcal{Y}}_3^u > 4g_s/3 \gtrsim 1.3$. Starting at the scale $\mu = M_Q$ and requiring that the Landau pole stays above $10M_Q$, one arrives at

$$\hat{\mathcal{Y}}_3^u(M_Q) \lesssim 3. \quad (36)$$

This requirement is significantly more constraining than imposing a naive perturbativity bound $\hat{\mathcal{Y}}_3^u < 4\pi$. We will therefore adopt Eq. (36) in the following.

B. Electroweak precision observables

The vectorlike quark doublet induces loop corrections to electroweak precision observables, which can be parametrized by the oblique parameters S and T . The expressions, recalling that Y_3^{Qu} and Y_3^{Qd} dominate over the other Yukawa couplings, are [22,23]

$$\begin{aligned} \Delta S &= S - S_{\text{SM}} \\ &= \frac{3}{18\pi} \frac{v^2}{M_Q^2} \left(|Y_3^{Qu}|^2 \left[-10 + 4 \log \left(\frac{M_Q^2}{m_t^2} \right) \right] + |Y_3^{Qd}|^2 \left[-6 + 2 \log \left(\frac{M_Q^2}{m_b^2} \right) \right] \right), \end{aligned} \quad (37a)$$

$$\begin{aligned} \Delta T &= T - T_{\text{SM}} \\ &= \frac{3v^2}{8\pi s_W^2 M_W^2} \left(\frac{m_t^2 |Y_3^{Qu}|^2}{M_Q^2} \left[-3 + 2 \log \left(\frac{M_Q^2}{m_t^2} \right) \right] + \frac{2}{3} \frac{v^2}{M_Q^2} (|Y_3^{Qu}|^2 - |Y_3^{Qd}|^2)^2 \right), \end{aligned} \quad (37b)$$

where $v = 246$ GeV. The parameters S , T (for $\Delta U = 0$) are currently constrained at 95% CL to be within [24]

$$\Delta S = -0.05 \pm 0.07, \quad \Delta T = 0.00 \pm 0.06, \quad (38)$$

with a correlation $\rho = 0.92$.

In our model, $|\Delta T| \gg |\Delta S|$. We can therefore neglect ΔS . The constraints arising from ΔT are shown in Fig. 4 as a horizontal solid line for $M_Q = 2$ TeV and dashed line for $M_Q = 8$ TeV. To get a qualitative understanding, we note that for $Y_3^{Qu} \lesssim 1.7$ the $Y^2 y_t^2 / M^2$ contribution to ΔT dominates and we estimate the 2σ bound, as follows:

$$\frac{M_Q}{\text{TeV}} \gtrsim 2.5 |Y_3^{Qu}| \sim 10^{-2} |\tilde{w}|, \quad (39)$$

where in the last relation we used the rough approximation (31). For $Y_3^{Qu} \gtrsim 3$, the Y^4 / M^2 term dominates, but that limit is incompatible with our perturbative constraint (36). In the interesting regime $1.7 \lesssim Y_3^{Qu} \lesssim 3$, both terms contribute comparably. In general, for a fixed Y_3^{Qu} the corrections to ΔT decouple as $M_Q \gg 1$ TeV. This can be clearly seen in Fig. 4 by comparing the horizontal solid gray line at $M_Q = 2$ TeV with the dashed one at $M_Q = 8$ TeV. With $M_Q \sim 12$ TeV, the constraint obtained from the T parameter becomes comparable to the perturbativity limit of Eq. (36).

C. Flavor constraints

Generic models of VLQs are significantly constrained by flavor observables. In NB-VLQ scenarios, instead, the Yukawa couplings in Eq. (2) have naturally a hierarchical structure, as seen in Fig. 4. As a consequence, we will argue that flavor constraints are less relevant than the perturbativity bound and the electroweak precision observables discussed earlier.

Doublet VLQs induce the SM EFT operators O_{Hu} , O_{Hd} , O_{Hud} ($\psi^2 H^2 D$), and O_{uH} , O_{dH} ($\psi^2 H^3$) at tree level [25]. We first discuss flavor violation and subsequently flavor-diagonal transitions.

The operators O_{Hu} and O_{Hd} lead to flavor-changing Z couplings of the right-handed quarks, which induce primarily dangerous $\Delta F = 1$ transitions [26,27]. The limits coming from $\Delta F = 1$ transitions $d_i \rightarrow d_j$ and $u \rightarrow c$ are [26]

$$\begin{aligned} \frac{M_Q^2}{\text{TeV}^2} &> 66^2 \times |Y_1^{Qd*} Y_2^{Qd}| \sim (1.6 \times 10^{-3} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &> 100^2 \times |\text{Re}(Y_1^{Qd*} Y_2^{Qd})| \sim (2.4 \times 10^{-3} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &> 30^2 \times |Y_3^{Qd*} Y_1^{Qd}| \sim (9.5 \times 10^{-4} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &> 18^2 \times |Y_3^{Qd*} Y_2^{Qd}| \sim (1.4 \times 10^{-3} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &> 3.9^2 \times |Y_1^{Qu*} Y_2^{Qu}| \sim (1.2 \times 10^{-4} |\tilde{w}|)^2, \end{aligned} \quad (40)$$

where the last approximation uses the estimate (31). Consider, for instance, the fourth constraint, coming from $B_s \rightarrow \mu^+ \mu^-$, which leads to $M_Q/\text{TeV} \gtrsim 1.4 \times 10^{-3} |\tilde{w}|$. This bound, like all the others quoted in Eq. (40), is weaker than the rough lower limit from ΔT in Eq. (39).

The operators O_{Hu} and O_{Hd} also contribute to $\Delta F = 2$ transitions through Z exchange. However, for M_Q above ~ 1 TeV, such processes are dominated by direct one-loop contributions from box diagrams [27]. Actually, for the (sd) sector, the latter are further superseded by the contributions from $\bar{L}L\bar{R}R$ four-fermion operators at the electroweak scale induced by one-loop RG mixing from the tree-level-generated O_{Hd} . Flavor-violating transitions mediated by O_{uH} and O_{dH} are suppressed by the SM Yukawas and can be neglected. In summary, the most relevant $\Delta F = 2$ bounds are⁴

⁴The first, 4th and 5th bounds come from [26] while the rest come from [28]. It seems to us that the $\Delta F = 2$ bounds quoted for the sectors (tu) and (tc) for the doublet Q in Table 1 of Ref. [26] (coupling $\lambda_i^{(2)} = Y_i^{Qu}$) might contain a typo.

$$\begin{aligned} \frac{M_Q^2}{\text{TeV}^2} &> 59^2 \times |\text{Re}(Y_1^{Qd*} Y_2^{Qd})| \sim (1.9 \times 10^{-4} |\tilde{w}|)^4, \\ \frac{M_Q^2}{\text{TeV}^2} &> 10^6 \times |\text{Im}(Y_1^{Qd*} Y_2^{Qd})| \sim (7.7 \times 10^{-4} |\tilde{w}|)^4, \\ \frac{M_Q}{\text{TeV}} &> 44 \times |Y_3^{Qd*} Y_1^{Qd}| \sim (2.1 \times 10^{-4} |\tilde{w}|)^2, \\ \frac{M_Q}{\text{TeV}} &> 9.1 \times |Y_3^{Qd*} Y_2^{Qd}| \sim (2.3 \times 10^{-4} |\tilde{w}|)^2, \\ \frac{M_Q}{\text{TeV}} &> 67 \times |Y_1^{Qu*} Y_2^{Qu}| \sim (2.5 \times 10^{-4} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &> 400^2 |\text{Im}(Y_1^{Qu*} Y_2^{Qu})| \sim (6 \times 10^{-4} |\tilde{w}|)^4, \end{aligned} \quad (41)$$

where the last approximations follow from Eq. (31). We note that the most stringent constraints on $|\tilde{w}|$ are the second and the last ones, corresponding to ϵ_K and D^0 mixing, respectively. The first is stronger than Eq. (39) only when $|\tilde{w}| \gtrsim 1.7 \times 10^4$, which is, however, outside of the physical range of $|\tilde{w}|$; see Fig. 2.

For completeness, we also report the constraint from flavor-changing decays $t \rightarrow u_i Z$ and $t \rightarrow u_i H$ in colliders which are comparable to low-energy constraints from flavor observables [22]:

$$\begin{aligned} \frac{M_Q^2}{\text{TeV}^2} &\gtrsim 3 |Y_1^{Qu} Y_3^{Qu}| \sim (2.7 \times 10^{-4} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &\gtrsim 2.3 \times |Y_2^{Qu} Y_3^{Qu}| \sim (1.1 \times 10^{-3} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &\gtrsim 0.37 \times y_t |Y_1^{Qu} Y_3^{Qu}| \sim (9 \times 10^{-5} |\tilde{w}|)^2, \\ \frac{M_Q^2}{\text{TeV}^2} &\gtrsim 0.32 \times y_t |Y_2^{Qu} Y_3^{Qu}| \sim (4.4 \times 10^{-4} |\tilde{w}|)^2. \end{aligned} \quad (42)$$

These are also weaker than the ΔT bound, though they often apply to different combinations of Y^{Qu} 's as the previous constraints.

It remains to evaluate the impact of the flavor-conserving transitions beyond the SM. The flavor-diagonal components in O_{Hu} , O_{Hd} , O_{uH} , and O_{dH} are not particularly relevant in our models. The reason is that the couplings to the light generations are strongly suppressed by the hierarchy of $Y^{Qu,Qd}$. The operator $O_{Hud} = \bar{u}_R \gamma^\mu d_R \tilde{H}^\dagger i D_\mu H$, generated in our models at tree level with Wilson coefficient $C_{Hud} = Y^{Qu\dagger} Y^{Qd} / M_Q^2$, is more interesting. After the Higgs has acquired its vacuum expectation value, that operator mediates exotic couplings of the $W^\pm$ to the right-handed quarks, which are not present in the SM at tree level. Following the discussion of Ref. [29], these exotic interactions induce a novel contribution $\delta C_7(\mu_{ew}) = -0.777(m_t/m_b)[C_{Hud}]_{33}v^2/2$ to the effective dipole operator O_7 describing $b \rightarrow s + \gamma$. Using the updated experimental average of HFLAG [30] with the theoretical SM

prediction [31], we find at 95% CL, as follows:

$$-0.038 < \text{Re}\delta C_7(\mu_{ew}) < 0.26, \quad (43)$$

which reads

$$\frac{M_Q^2}{\text{TeV}^2} \gtrsim 5.5^2 \times |Y_3^{Qu} Y_3^{Qd}| \sim (3.5 \times 10^{-3} |\tilde{w}|)^2. \quad (44)$$

For a given $|\tilde{w}|$, this bound is again weaker than the one in Eq. (39) from ΔT . Yet, it constrains a different combination of Yukawas than those collected above.

We should emphasize that the limits on M_Q^2/Y^2 and M_Q^2/Y^4 quoted in this section, as well as the one in Eq. (39), are accurate as written. However, the bounds on M_Q shown as a function of $\tilde{w}$ are all inherently approximate because they are based on the rough estimate (31). We checked that the bounds quoted here and in Eq. (39) are generically conservative. In fact, analogously to the singlet case [12], there might be regions in the parameter space where special cancellations take place and the lower bounds on M_Q can be further relaxed.

V. IRREDUCIBLE CONTRIBUTIONS TO $\bar{\theta}$

The main challenge faced by scenarios with spontaneous CP violation is to make sure that corrections to $\bar{\theta}$ remain under control even after spontaneous symmetry breaking. In Nelson-Barr models, there are two classes of radiative corrections. The first class involves the degrees of freedom of the Higgs sector responsible for CP violation. These are rather model dependent, but can be naturally made small if the couplings of the new scalars are sufficiently suppressed, as in Ref. [7]. The second class of corrections to $\bar{\theta}$ arises from loops of the SM and the VLQ, which effectively communicates CP violation to the SM. These corrections are unavoidable in these models and are independent of how CP is broken. They have therefore been dubbed “irreducible” in Ref. [9]. Crucially, corrections to $\bar{\theta}$ do not decouple as the mass scale of the VLQ is increased.

Reference [9] presents a comprehensive discussion of the irreducible corrections to $\bar{\theta}$ in models with electroweak-singlet vectorlike fermions, i.e., models with u -mediation and d -mediation. Scenarios with q -mediation—namely the scenarios considered here—were erroneously dismissed on the grounds of allegedly large radiative corrections to $\bar{\theta}$. That claim originates from the earlier work [16], which provided model-independent two-loop estimates applicable (among others) to *generic* q -mediation frameworks. However, what Ref. [9] overlooked is that, in minimal Nelson-Barr scenarios of q -mediation, an accidental symmetry prevents such two-loop corrections from arising.⁵

⁵L. V. would like to thank Alessandro Valenti, who crucially contributed to the realization of this point.

In this subsection, we clarify this important aspect and argue that the first irreducible corrections to $\bar{\theta}$ in renormalizable Nelson-Barr models for q -mediation (the models discussed in this paper) arise only at three loops, and can be naturally within the current experimental bounds.

The contributions to $\bar{\theta}$ are more conveniently identified in the basis (2) and taking advantage of the spurionic flavor symmetries. In that language, the coupling Y^u can be seen as a spurion transforming under flavor $U(3)_q \times U(3)_u \times U(3)_d \times U(n)_Q$ as $Y^u \rightarrow U_q Y^u U_u^\dagger$, with $U_q \in U(3)_q$ and analogously $U_u \in U(3)_u$. Similar considerations reveal that $Y^d \rightarrow U_q Y^d U_d^\dagger$ whereas $Y^{Qu} \rightarrow U_Q Y^{Qu} U_u^\dagger$ and $Y^{Qd} \rightarrow U_Q Y^{Qd} U_d^\dagger$.

Now, recalling that in the SM,

$$\bar{\theta} = \theta - \text{Arg}[\text{Det}(Y^{u,\text{SM}})\text{Det}(Y^{d,\text{SM}})],$$

we see that the corrections to $\bar{\theta}$ belong to two distinct classes. The first includes direct contributions to θ [32]. They are parametrized by polynomial CP -odd, flavor-invariant combinations of the couplings of the theory. The second class comes from corrections to the Yukawa couplings. The spurionic flavor symmetries and the fact that $Y^{Qu,Qd} \propto Y^{u,d}$ [see Eqs. (A9c) and (A9d)] indicate that in our models these are proportional to the tree-level Yukawas. Specifically, they are of the form $Y^{u,\text{SM}} = F_u Y^u$ and $Y^{d,\text{SM}} = F_d Y^d$, where $F_{u,d} = 1 + \delta F_{u,d}$ with 1 the tree-level contribution and $\delta F_{u,d}$ polynomial combinations of the couplings parametrizing the radiative effects. However, $\text{Arg}[\text{Det}(Y^{u,\text{SM}})\text{Det}(Y^{d,\text{SM}})] = \text{Arg}[\text{Det}(F_u)\text{Det}(F_d)] = \text{Im}[\ln \text{Det}(F_u)] + \text{Im}[\ln \text{Det}(F_d)]$, since NB models by construction do not generate corrections at tree level. So, radiative corrections to the Yukawas are parametrized by structures such as $\text{Im}[\ln \text{Det}(F_{u,d})]$, which are also polynomial CP -odd flavor-invariant combinations of the couplings. An exhaustive analysis of all corrections to $\bar{\theta}$ can therefore be obtained by studying the polynomial flavor invariants. For example, in the case of one singlet VLQ of down or up type, all of the CP -odd invariants can be written as a linear combination of nine basic CP -odd invariants multiplied by CP -even invariants [33].

The basic objects necessary to construct invariants are

$$\begin{aligned} Y^{u\dagger} Y^u, \quad Y^{d\dagger} Y^d, \quad Y^{u\dagger} Y^d, \quad Y^{Qu\dagger} Y^{Qu}, \\ Y^{Qd\dagger} Y^{Qd}, \quad Y^{Qu\dagger} Y^{Qd}. \end{aligned} \quad (45)$$

Corrections beyond the SM are of course parametrized by invariants involving the last three building blocks in Eq. (45). In addition, since Y^u , Y^d , Y^{Qu} , and Y^{Qd} are perturbative, one should look for expressions with the lowest number of coupling insertions. The CP -odd flavor-invariant with the

lowest number of building blocks turns out to be [16]

$$\text{Im Tr}[Y^{u\dagger} Y^d Y^{Qd\dagger} Y^{Qu} - \text{hc}] = \text{Im}[\tilde{w}^\dagger [Y^u Y^{u\dagger}, Y^d Y^{d\dagger}] \tilde{w}]. \quad (46)$$

Now, the crucial point is that radiative corrections proportional to this combination and no other couplings are possible in the generic theories considered in Ref. [16], but cannot be induced by the theory in Eq. (2). Indeed, the latter enjoys an accidental spurious symmetry under which the up and down quarks are exchanged, $H \rightarrow \tilde{H}$, and

$$Y^u \leftrightarrow Y^d, \quad Y^{Qu} \leftrightarrow Y^{Qd}, \quad Y_u \leftrightarrow Y_d, \quad Y_H \rightarrow -Y_H, \quad (47)$$

where $Y_{u,d}$, Y_H are the hypercharges of the right-handed quarks and the Higgs doublet, respectively. The combination (48) is obviously odd under that spurious symmetry and therefore cannot be generated unless an odd number of Y_H appears or contributions proportional to more Yukawa are considered. In the former case, one is forced to include at least a loop involving the hypercharge vector. Adding appropriate powers of $1/16\pi^2$ for each $\hbar$ factor, we estimate that the largest CP -odd invariant with a single hypercharge loop is

$$\begin{aligned} I_1 &= c_1 \frac{g^2}{(16\pi^2)^3} Y_H (Y_u + Y_d) \text{Im Tr}[Y^{u\dagger} Y^d Y^{Qd\dagger} Y^{Qu} - \text{hc}] \\ &= c_1 \frac{g^2}{(16\pi^2)^3} Y_H (Y_u + Y_d) \text{Im}[\tilde{w}^\dagger [Y^u Y^{u\dagger}, Y^d Y^{d\dagger}] \tilde{w}], \end{aligned} \quad (48)$$

up to some calculable real number c_1 . As already anticipated, other potentially important contributions to $\bar{\theta}$ are generated by loops including additional powers of the Yukawa couplings. We find two possibilities with a single additional Y pair. They are

$$\begin{aligned} I_2 &= c_2 \frac{1}{(16\pi^2)^3} \text{Im Tr}[Y^{u\dagger} Y^d Y^{Qd\dagger} Y^{Qu} Y^{u\dagger} Y^u - \text{hc}] \\ &\quad + (u \leftrightarrow d) = c_2 \frac{1}{(16\pi^2)^3} \text{Im}[\tilde{w}^\dagger [(Y^u Y^{u\dagger})^2, Y^d Y^{d\dagger}] \tilde{w}] \\ &\quad + (u \leftrightarrow d), \end{aligned} \quad (49)$$

and

$$\begin{aligned} I_3 &= c_3 \frac{1}{(16\pi^2)^3} \text{Im Tr}[Y^{u\dagger} Y^d Y^{Qd\dagger} Y^{Qu} Y^{Qu\dagger} Y^{Qu} - \text{hc}] \\ &\quad + (u \leftrightarrow d) = c_3 \frac{1}{(16\pi^2)^3} \text{Im}[\tilde{w}^\dagger [Y^u Y^{u\dagger}, Y^d Y^{d\dagger}] \tilde{w}] \\ &\quad \times (\tilde{w}^\dagger Y^u Y^{u\dagger} \tilde{w} - \tilde{w}^\dagger Y^d Y^{d\dagger} \tilde{w}), \end{aligned} \quad (50)$$

where c_2 and c_3 are again calculable real numbers.

Equations (48)–(50) arise from irreducible three-loop contributions involving virtual quarks, Higgses, gauge bosons, and VLQs. They are expected to parametrize the dominant corrections to $\bar{\theta}$ in models of q -mediation according to $\delta\bar{\theta} = I_1 + I_2 + I_3$. Additional contributions to $\bar{\theta}$ involve higher powers of Yukawa and gauge couplings or decouple as the VLQ mass is much higher than the weak scale [9]. In either case, they are numerically smaller than Eqs. (48)–(50).

A careful numerical evaluation of Eqs. (48)–(50)—obtained assuming $c_1 = c_2 = c_3 = 1$ —will be discussed in the subsequent section. However, in order to gauge how strongly q -mediation scenarios are constrained by current data, it is useful to first provide an analytical estimate. To this end, we exploit the flavor-invariant nature of our expressions to go in a field basis with diagonal $Y^u = \hat{Y}^u$, $Y^d = V \hat{Y}^d$, and $\tilde{w} = \hat{\tilde{w}}$. Our approximation consists in assuming that all components of the latter are of the same order, and again taking $c_1 = c_2 = c_3 = 1$. Under such assumptions, one finds that

$$\begin{aligned} I_1 &\sim |\tilde{w}_3 \tilde{w}_2^*| \lambda_C^2 \frac{g^2}{16\pi^2} \frac{y_t^2}{16\pi^2} \frac{y_b^2}{16\pi^2} \sim 5 \times 10^{-12} |\tilde{w}_3 \tilde{w}_2^*| \\ &\sim 1.2 \times 10^{-15} |\tilde{w}|^2 I_2 \sim |\tilde{w}_3 \tilde{w}_2^*| \lambda_C^2 \left(\frac{y_t^2}{16\pi^2} \right)^2 \frac{y_b^2}{16\pi^2} \\ &\sim 3 \times 10^{-13} |\tilde{w}_3 \tilde{w}_2^*| \sim 7.5 \times 10^{-19} |\tilde{w}|^2 I_3 \\ &\sim |\tilde{w}_3 \tilde{w}_2^*| |\tilde{w}_3|^2 \lambda_C^2 \left(\frac{y_t^2}{16\pi^2} \right)^2 \frac{y_b^2}{16\pi^2} \sim 3 \times 10^{-13} |\tilde{w}_3 \tilde{w}_2^*| |\tilde{w}_3|^2 \\ &\sim 1.8 \times 10^{-21} |\tilde{w}|^4, \end{aligned} \quad (51)$$

where $\lambda_C \sim 0.2$ is the Cabibbo angle, and in the final step we used the rough approximation in Eq. (30). We have retained the parametric dependence on the norm of $\tilde{w}$, since such a vector is actually unbounded and usually has a norm much larger than unity (see Sec. III C). Of course, the requirement that the coupling Y^{Qu} stays perturbative also constrains the size of $|\tilde{w}|^2$; see Eq. (3c). Our estimates suggest that I_3 should be more relevant than I_2 , and potentially not far from the current experimental bound. This qualitative behavior is confirmed by the numerical analysis presented below.

A. Numerical estimates

Here we estimate numerically the irreducible contributions due to the three dominant flavor invariants I_1 , I_2 , and I_3 in Eqs. (48)–(50).

The coefficients c_1 , c_2 , and c_3 in front of the invariants are expected to be of order one and can be only determined by a full calculation within a complete model. In the plots, we take $c_1 = c_2 = c_3 = 1$ for simplicity.

In Fig. 7, we show I_1 and I_2 , while in Fig. 8 we show I_3 . The experimental bound

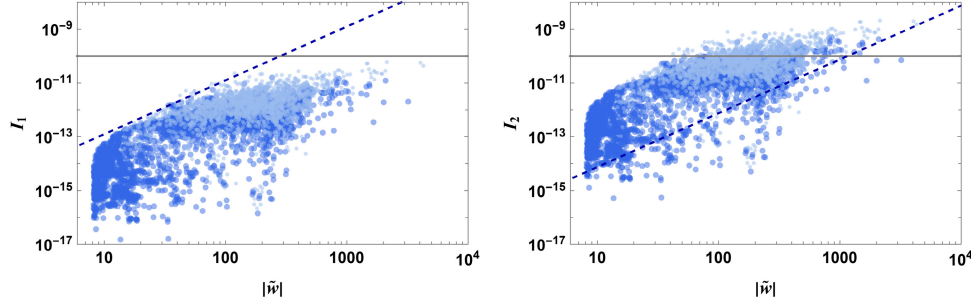


FIG. 7. Contributions to $\bar{\theta}$ as a function of $|\tilde{w}|$ parametrized by the invariants I_1 and I_2 in Eqs. (48) and (49) for $c_1 = c_2 = 1$. The lighter blue points are excluded by the oblique parameter constraints for $M_Q = 8$ TeV, while the blue points are allowed by all other constraints. The dashed blue line represents the estimate in Eq. (51), and the solid gray line denotes the experimental bound (52). All points satisfy the perturbativity bound (36).

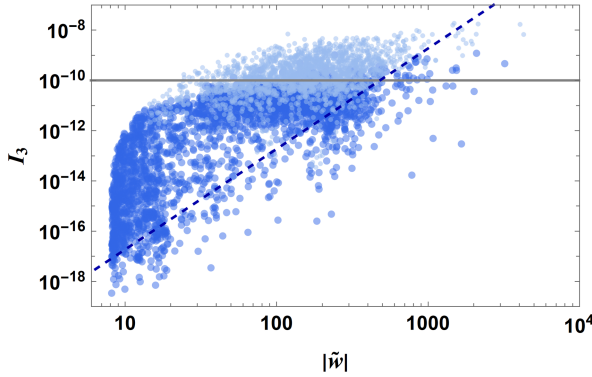


FIG. 8. Contribution to $\bar{\theta}$ as a function of $|\tilde{w}|$ from the invariant I_3 in Eq. (48) for $c_3 = 1$. The rest is as in Fig. 7.

$$|\bar{\theta}| \lesssim 10^{-10} \quad (52)$$

is included as a horizontal gray line. We can see that I_3 likely corresponds to the largest contribution to $\bar{\theta}$. In the plots, we only show points that satisfy the perturbativity bound (36).

It is interesting to compare the impact of Eq. (52) on the parameter space compared to that arising from the electroweak constraints. This is seen in Figs. 7 and 8, where the lighter color denotes the points that are already excluded by the T parameter with $M_Q = 8$ TeV. We see that the latter constraint is comparable to Eq. (52) in the case of TeV-scale VLQs, though of course they should be viewed as complementary. Crucially, however, for $M_Q > 8$ TeV, the oblique parameter constraints become weaker, whereas the one from Eq. (52) does not decouple. For large VLQ masses, that is clearly the most important constraint on our models.

VI. CONCLUSIONS

In this work, we performed a systematic study of Nelson-Barr solutions to the strong CP problem featuring vectorlike partners of the SM quark doublet. These models have received considerably less attention than their analogs with

electroweak-singlet VLQs, yet our analysis shows that this neglect is unwarranted.

We first identified the region of parameter space in which the SM flavor structure—the quark Yukawa couplings and the CKM matrix—is successfully reproduced. Because no small expansion parameter controls this identification, a numerical analysis was necessary.

We then estimated the dominant contributions to $\bar{\theta}$ arising from *irreducible* diagrams involving both the VLQ and the SM fields. A key observation is that these contributions first appear only at three loops, owing to an accidental symmetry of the renormalizable model. As a consequence, in a significant portion of the parameter space, the resulting $|\bar{\theta}|$ lies well below current bounds yet within reach of future experiments, in particular proposed proton EDM measurements [34].

We also examined other relevant experimental constraints on these scenarios: the exotic couplings $Y_i^{Qu, Qd}$ between the VLQ and the SM exhibit, in both the up and down sectors, a hierarchical pattern $|Y_1| \ll |Y_2| \ll |Y_3|$, inherited from—though milder than—the quark-mass hierarchy. This feature distinguishes NB doublet VLQs from generic doublet VLQs and ensures that flavor-violating effects beyond the SM are typically small. Overall, within the perturbative domain, electroweak precision data provide the leading bounds for VLQ masses below a few TeV, while hadronic CP violation induced by $\bar{\theta}$ becomes increasingly important at higher masses. Nevertheless, a broad region of parameter space remains currently viable, offering meaningful opportunities for future exploration.

In conclusion, our results show that doublet NB-VLQs—complementing the better-known singlet NB-VLQs of up or down type—constitute a robust and well-motivated solution to the strong CP problem, with potentially interesting phenomenological signatures.

ACKNOWLEDGMENTS

G. H. S. A. acknowledges financial support by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. C. C. N.

acknowledges partial support by Brazilian CNPq, Grant No. 312866/2022-4. L. V. acknowledges partial support by the Iniziativa Specifica “Physics at the Energy, Intensity, and Astroparticle Frontiers” (APINE) of Istituto Nazionale di Fisica Nucleare (INFN), and the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant agreement No. 860881-HIDDeN.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: CHANGE OF BASIS

The change of basis from Eqs. (1) and (2) can be performed by a unitary transformation on the space of doublets (q_L, Q_L) , as follows:

$$\begin{pmatrix} q_L \\ Q_L \end{pmatrix} \rightarrow W_L \begin{pmatrix} q_L \\ Q_L \end{pmatrix}, \quad (\text{A1})$$

where $W_L \in U(3+n)$. Here, we leave the number n of VLQ doublets general.

The basis change can be performed considering the mass matrices after electroweak symmetry breaking, which read

$$\begin{aligned} \text{NB: } \mathcal{M}_d &= \begin{pmatrix} \frac{v}{\sqrt{2}} \mathcal{Y}^d & \mathcal{M}^{qQ} \\ 0 & \mathcal{M}_Q \end{pmatrix}; \\ \text{generic: } M_d^{q+Q} &= \begin{pmatrix} \frac{v}{\sqrt{2}} Y^d & 0 \\ \frac{v}{\sqrt{2}} Y^{dQ} & M_Q \end{pmatrix}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \text{NB: } \mathcal{M}_u &= \begin{pmatrix} \frac{v}{\sqrt{2}} \mathcal{Y}^u & \mathcal{M}^{qQ} \\ 0 & \mathcal{M}_Q \end{pmatrix}; \\ \text{generic: } M_u^{q+Q} &= \begin{pmatrix} \frac{v}{\sqrt{2}} Y^u & 0 \\ \frac{v}{\sqrt{2}} Y^{uQ} & M_Q \end{pmatrix}. \end{aligned} \quad (\text{A3})$$

The unitary transformation W_L connects the mass matrices in the two basis:

$$M_{d/u}^{q+Q} = W_L^\dagger \mathcal{M}_{d/u}^{q+Q}. \quad (\text{A4})$$

We can write an explicit expression for W_L as

$$\begin{aligned} W_L &= \begin{pmatrix} \mathbb{1}_3 & \tilde{w} \\ -\tilde{w}^\dagger & \mathbb{1}_n \end{pmatrix} \begin{pmatrix} (\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} & 0 \\ 0 & (\mathbb{1}_n + \tilde{w}^\dagger\tilde{w})^{-1/2} \end{pmatrix}, \\ &= \begin{pmatrix} (\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} & \tilde{w}(\mathbb{1}_n + \tilde{w}^\dagger\tilde{w})^{-1/2} \\ -\tilde{w}^\dagger(\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} & (\mathbb{1}_n + \tilde{w}^\dagger\tilde{w})^{-1/2} \end{pmatrix} \end{aligned} \quad (\text{A5})$$

where

$$\tilde{w} = \mathcal{M}^{qQ} \mathcal{M}^{Q-1}. \quad (\text{A6})$$

Note that both matrices in Eq. (A5) are necessary to have a unitary W_L , with the first ensuring the zero in the upper-right block of M^{q+Q} .

It will be useful for us to reexpress all these relations in terms of a different matrix w , defined by

$$w = \tilde{w}(\mathbb{1}_n + \tilde{w}^\dagger\tilde{w})^{-1/2} = (\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2}\tilde{w}. \quad (\text{A7})$$

In this notation, the expressions simplify. In particular,

$$W_L = \begin{pmatrix} (\mathbb{1}_3 - ww^\dagger)^{1/2} & w \\ -w^\dagger & (\mathbb{1}_n - w^\dagger w)^{1/2} \end{pmatrix}, \quad (\text{A8})$$

and

$$Y^u = (\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} \mathcal{Y}^u = (\mathbb{1}_3 - ww^\dagger)^{1/2} \mathcal{Y}^u, \quad (\text{A9a})$$

$$Y^d = (\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} \mathcal{Y}^d = (\mathbb{1}_3 - ww^\dagger)^{1/2} \mathcal{Y}^d, \quad (\text{A9b})$$

$$Y^{Qu} = \tilde{w}^\dagger(\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} \mathcal{Y}^u = w^\dagger \mathcal{Y}^u = \tilde{w}^\dagger Y^u, \quad (\text{A9c})$$

$$Y^{Qd} = \tilde{w}^\dagger(\mathbb{1}_3 + \tilde{w}\tilde{w}^\dagger)^{-1/2} \mathcal{Y}^d = w^\dagger \mathcal{Y}^d = \tilde{w}^\dagger Y^d, \quad (\text{A9d})$$

$$M^Q = (\mathbb{1}_n + \tilde{w}^\dagger\tilde{w})^{1/2} \mathcal{M}^Q = (\mathbb{1}_n - w^\dagger w)^{-1/2} \mathcal{M}^Q. \quad (\text{A9e})$$

In the case $n = 1$ discussed in the main text, $\tilde{w}$ is a vector with norm $|\tilde{w}| \geq 0$. On the other hand, from Eq. (A7) it follows that

$$|w|^2 = \frac{|\tilde{w}|^2}{1 + |\tilde{w}|^2}, \quad (\text{A10})$$

which says that $0 \leq |w| < 1$. CP violation in the SM further excludes $|w| = |\tilde{w}| = 0$.

APPENDIX B: HOW TO INCORPORATE THE SM YUKAWAS

Here, we detail how to solve Eq. (11) in terms of the eigenvalues, i.e., the SM Yukawas. After splitting both Ω and $YY^\dagger$ into their real and imaginary parts, one obtains Eq. (18). The real part can be solved by Eq. (19). Plugging it into the imaginary part yields

$$\Omega_1^{-1/2} \Omega_2 \Omega_1^{-1/2} = \mathcal{O}^\top Z \mathcal{O}, \quad (\text{B1})$$

which specifies the orthogonal matrix $\mathcal{O}$.

Without loss of generality, we can choose the basis (20) for w . In such a basis,

$$\Omega_1 + i\Omega_2 = \text{diag}(1 - a^2, 1 - b^2, 1) + iab \begin{pmatrix} & & 1 \\ -1 & & \\ & & 0 \end{pmatrix}, \quad (\text{B2})$$

and the lhs of Eq. (B1) has the standard form

$$\Omega_1^{-1/2} \Omega_2 \Omega_1^{-1/2} = \mu \begin{pmatrix} & & 1 \\ -1 & & \\ & & 0 \end{pmatrix}, \quad (\text{B3})$$

with μ given by Eq. (21). The rhs of Eq. (B1) tells us that $\mathcal{O}$ is the matrix that transforms Z to the standard form above, and the third column in $\mathcal{O}$ is the eigenvector associated to the zero eigenvalue of Z . The orthogonal matrix $\mathcal{O}$ is defined up to an additional rotation in the (12) space from the right.

Equation (B1) applies to both up and down Yukawas, in general with $\mathcal{O}_u \neq \mathcal{O}_d$ and $\mathbf{z}^u \neq \mathbf{z}^d$. However, since $\text{Tr}[Z^2] = -2|\mathbf{z}|^2$ is determined entirely by Ω , it must be the same in both sectors. The following nontrivial relation $\mu_u = \mu_d = \mu$, where $\mu_u = |\mathbf{z}^u|$ and $\mu_d = |\mathbf{z}^d|$, then holds. This is what we anticipated in Eq. (14). From the definition, we must have $0 \leq \mu \leq 1$ [10]. These upper and lower limiting values are unphysical. The limit $\mu = 0$ corresponds to the CP -conserving case where $Y^{u\dagger} Y^u, Y^{d\dagger} Y^d$ are both real. The other limit, $\mu = 1$, corresponds to massless quarks as Eq. (12) would have vanishing determinant. Reproducing the eigenvalues of the SM Yukawas at 1 TeV requires

$$1 - \mu_u > 1.1 \times 10^{-10}, \quad 1 - \mu_d > 1.9 \times 10^{-6}. \quad (\text{B4})$$

See Appendix C for details.

The vector $\mathbf{z} = (z_i)$ is an eigenvector of Z associated to zero eigenvalue, so the third column of $\mathcal{O}$ is proportional to it. The parameter μ is the norm $|\mathbf{z}|$. Restricted to $\det \mathcal{O} = 1$, we can conventionally choose the third column of $\mathcal{O}$ as $\mathbf{z}/\mu$ directly. Note that $Z \rightarrow OZO^\top$ induces $\mathbf{z} \rightarrow O\mathbf{z}$.

Now, given $\mathbf{z}$, the three parameters x_i in Eq. (15) can be determined from the SM Yukawas y_i (y_{u_i} or y_{d_i}). The relation comes from the characteristic equation for $H_R = Y^\dagger Y$, as follows:

$$\det(H_R - \lambda I) = -[\lambda^3 - \gamma_1(H_R)\lambda^2 - \gamma_2(H_R)\lambda - \gamma_3(H_R)], \quad (\text{B5})$$

where the coefficients are

$$\begin{aligned} \gamma_1(H_R) &= \text{Tr}[H_R] = x_1 + x_2 + x_3, \\ \gamma_2(H_R) &= \frac{1}{2} \text{Tr}[H_R^2 - \gamma_1(H_R)H_R] \\ &= -x_1x_2(1 - z_3^2) - x_2x_3(1 - z_1^2) - x_3x_1(1 - z_2^2), \\ \gamma_3(H_R) &= \frac{1}{3} \text{Tr}[H_R^3 - \gamma_1(H_R)H_R^2 - \gamma_2(H_R)H_R] \\ &= \det(H_R) = x_1x_2x_3(1 - \mu^2). \end{aligned} \quad (\text{B6})$$

The last expression in each line is obtained with the last expression in Eq. (12), where we have used $\mu = |\mathbf{z}|$. On the other hand, each of the $\gamma_k(H_R)$ is fixed from the SM Yukawas $h_i \equiv y_i^2$, as follows:

$$\begin{aligned} \gamma_1(H_R) &= h_1 + h_2 + h_3, \\ \gamma_2(H_R) &= -(h_1h_2 + h_2h_3 + h_3h_1), \\ \gamma_3(H_R) &= h_1h_2h_3. \end{aligned} \quad (\text{B7})$$

Because $\text{Tr}[H_R] = \text{Tr}[\text{Re}(H_R)]$ but the determinant obeys $\det[H_R] < \det[\text{Re}(H_R)]$, the spectrum of $\text{Re}(H_R)$ is squashed compared to the spectrum of H_R . This is confirmed in Fig. 1. Comparison between $\det H_R$ and $\det \text{Re}(H_R)$ also leads to a nice formula for μ depending solely on H_R [12], as follows:

$$\mu = \sqrt{1 - \frac{\det Y^\dagger Y}{\det \text{Re}(Y^\dagger Y)}}. \quad (\text{B8})$$

We now analyze the possibilities for z_i . We first note that the expressions (B6) are insensitive to the sign of z_i . In the basis where X is diagonal, we can still keep the form (12) for H_R if we apply sign flips to d_R (or u_R), which induces sign flips to both sides of H_R . This leaves x_i unchanged, but sign flips are induced on z_i . Since $\mathbf{z}$ is a pseudovector due to Eq. (13), we can at most equate the sign of all components and choose either $(z_1, z_2, z_3) \sim (+++)$ or $(---)$. With the convention adopted for $\mathcal{O} \in SO(3)$, these signs are also the same for the third column of $\mathcal{O}$.

We can end this part by checking the number of parameters. If we name the columns of $\mathcal{O}$ explicitly as

$$\mathcal{O} = \left(\mathbf{r}_1 \mid \mathbf{r}_2 \mid \mathbf{r}_3 \right), \quad (\text{B9})$$

we have $\mathbf{r}_3 = \mathbf{z}/\mu$. So $\mathcal{O}$ has its third column defined by $\mathbf{z}$, while $\mathbf{r}_1, \mathbf{r}_2$ are orthogonal. The direction of $\mathbf{z}$ can be parametrized by two angles, while the two directions in the orthogonal plane depend on one more angle. So $\mathcal{O}$ depends on three angles as usual. The 15 parameters are listed in Eq. (22). The directions of the vectors $\mathbf{z}^u$ and $\mathbf{z}^d$ are defined by the third column of $\mathcal{O}_u, \mathcal{O}_d$ while their norm should coincide with μ . With $\mathbf{z}^u$ and $\mathbf{z}^d$, the eigenvalue equations (B6) determine x_i^u, x_i^d given the Yukawas y_{u_i}, y_{d_i} . This fixes six parameters. Leaving $\mathcal{M}^Q$ aside, there are still

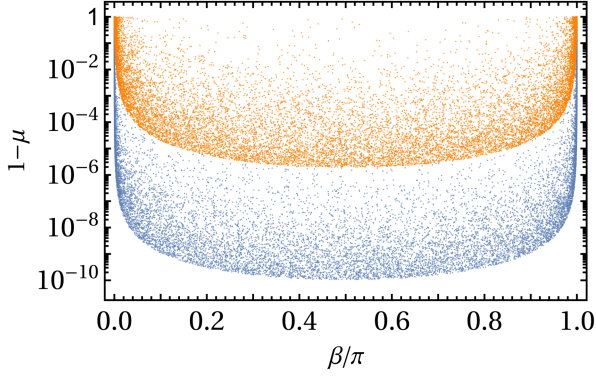


FIG. 9. $1 - \mu$ in Eq. (B8) as a function of β in Eq. (C2) for up (blue) and down (orange) sectors.

eight free parameters in $(\mu, b), \mathcal{O}_u, \mathcal{O}_d$. Four of them should be determined from the CKM structure.

APPENDIX C: CONSTRAINT ON μ

Let us decompose

$$Y^\dagger Y = V_R \hat{Y}^2 V_R^\dagger, \quad (\text{C1})$$

with $\hat{Y}$ being the diagonal matrix, and consider the one-phase parametrization of a unitary matrix with rephasing freedom from the right [35]. After removing an orthogonal matrix from the right, we can parametrize the remaining freedom as

$$V_R^\dagger = R_{23} R_{13} \text{diag}(1, 1, e^{i\beta}). \quad (\text{C2})$$

We can then calculate the value of μ using Eq. (B8). Figure 9 shows the possible values for $1 - \mu$ for the up and down sectors using best fit values for the SM Yukawas at 1 TeV and arbitrary values for the angles and the phase β in Eq. (C2). The nonvanishing of the SM quark masses requires that μ_u and μ_d cannot be arbitrarily close to unity. The figure leads to the constraint (B4) and, to be equal, both μ_u, μ_d need to obey the more strict bound for μ_d .

Notice that these loose bounds follow solely from the nonvanishing quark masses, as V_R for the right-hand fields u_R, d_R are not fixed by SM structure. Imposition of the SM CKM structure including CP violation further restricts μ . For example, the CP -conserving limit $\mu = \mu_u = \mu_d = 0$ is not physical. The physical range restricted additionally by

the CKM is shown in the dark points in Fig. 1 and shown in Eq. (24).

For comparison, for one singlet NB-VLQ [10] of either up type or down type, the constraint on μ also comes from the left-hand field transformation on q_L , which is fixed by the CKM, and then μ is bounded from both sides, as follows:

$$2 \times 10^{-7} \lesssim 1 - \mu_u \lesssim 0.0034 \quad (\text{C3a})$$

$$0.0045 \lesssim 1 - \mu_d \lesssim 0.43. \quad (\text{C3b})$$

We can see that the range in Eq. (24) lies in between the above ones.

APPENDIX D: USING INVARIANTS

The task of enforcing the CKM mixing to the relations (23) is still a difficult task. After enforcing the six SM Yukawas, four parameters in Eq. (22) should be fixed from the CKM, while another four parameters remain free. Although feasible, a scan in an eight-dimensional space is not very efficient. Part of the difficulty in dealing with Eq. (23) is that we cannot work in a basis where one of them is diagonal. One way to extract only the information that is weak basis independent is to use flavor invariants. The use of the Jarlskog invariant to quantify CP violation is a well-known example. Here, we propose a different set of invariants that will prove to be useful.

We first define the shorthands

$$H_u \equiv Y^u Y^{u\dagger}, \quad H_d \equiv Y^d Y^{d\dagger}, \quad (\text{D1})$$

for the SM Yukawas and

$$\mathcal{H}_u \equiv \mathcal{Y}^u \mathcal{Y}^{u\dagger}, \quad \mathcal{H}_d \equiv \mathcal{Y}^d \mathcal{Y}^{d\dagger}, \quad (\text{D2})$$

for real CP -conserving Yukawas. Then, Eq. (6) can be compactly written as

$$\Omega^{1/2} \mathcal{H}_u \Omega^{1/2} = H_u, \quad \Omega^{1/2} \mathcal{H}_d \Omega^{1/2} = H_d, \quad (\text{D3})$$

with the same Ω for both sectors.

It is immediate to see that the following invariants depend on the CKM and are independent of Ω , as follows:

TABLE I. Range for the flavor invariants with Yukawas fixed at best fit and CKM varying within 3σ at 1 TeV.

Invariant	3σ range	Invariant	3σ range
$\tilde{I}_{ud} = \langle \tilde{H}_u \tilde{H}_d^{-1} \rangle$	[72.03, 114.6]	$\tilde{I}_{du} = \langle \tilde{H}_d \tilde{H}_u^{-1} \rangle$	$[5.86, 7.09] \times 10^5$
$\tilde{I}_{udud}$	$[5.12, 13.1] \times 10^3$	$\tilde{I}_{dudu}$	$[3.43, 5.03] \times 10^{11}$
$\tilde{J}_{\text{dim}/i}$	$[1.00, 1.57] \times 10^{12}$		

$$\begin{aligned} I_{ud} &= \langle H_u H_d^{-1} \rangle = \langle \mathcal{H}_u \mathcal{H}_d^{-1} \rangle, \\ I_{du} &= \langle H_d H_u^{-1} \rangle = \langle \mathcal{H}_d \mathcal{H}_u^{-1} \rangle. \end{aligned} \quad (\text{D4})$$

Writing $\mathcal{Y}$ in terms of X using the inversion formula (19), we can write

$$\begin{aligned} I_{ud} &= \sum_{ij} \frac{x_i^u}{x_j^d} O_{ij}^2 = \frac{x_3^u}{x_1^d} O_{31}^2 + \dots, \\ I_{du} &= \sum_{ij} \frac{x_i^d}{x_j^u} O_{ji}^2 = \frac{x_3^d}{x_1^u} O_{13}^2 + \dots, \end{aligned} \quad (\text{D5})$$

where we have defined the real orthogonal matrix

$$O \equiv \mathcal{O}_u \mathcal{O}_d^\top. \quad (\text{D6})$$

In the last equalities in Eq. (D5), we have written the dominant terms assuming x_i^u and x_i^d are hierarchical and O_{13} and O_{31} unsuppressed. Independently of that, given that all the terms in the expression are positive semidefinite, we can write the bounds

$$\begin{aligned} |O_{31}| &< \frac{x_1^d}{x_3^u} \sqrt{I_{ud}}, \\ |O_{13}| &< \frac{x_1^u}{x_3^d} \sqrt{I_{du}}. \end{aligned} \quad (\text{D7})$$

Using the 3σ ranges shown in Table I and taking the ranges (16) for x_i^u and x_i^d , we obtain

$$|O_{31}| \lesssim 0.2. \quad (\text{D8})$$

The bound we obtain for $|O_{13}|$ is larger than unity and hence not useful.

We can equally consider the powers $\langle (H_u H_d^{-1})^2 \rangle$ and $\langle (H_d H_u^{-1})^2 \rangle$. But instead of them, we can consider

$$\begin{aligned} I_{udud} &\equiv \frac{1}{2} I_{ud}^2 - \frac{1}{2} \langle (H_u H_d^{-1})^2 \rangle, \\ I_{dudu} &\equiv \frac{1}{2} I_{du}^2 - \frac{1}{2} \langle (H_d H_u^{-1})^2 \rangle. \end{aligned} \quad (\text{D9})$$

For a CP -violating measure, we can consider instead of the usual Jarlskog,

$$J = \det([H_u, H_d]) = \langle H_u^2 H_d^2 H_u H_d - H_d^2 H_u^2 H_d H_u \rangle, \quad (\text{D10})$$

the dimensionless (when quark masses are considered) version

$$J_{\text{dim}} = \langle H_u H_d H_u^{-1} H_d^{-1} - H_d H_u H_d^{-1} H_u^{-1} \rangle. \quad (\text{D11})$$

Within the SM, they are simply related by

$$J_{\text{dim}} = \frac{J}{y_{u_1}^2 y_{u_2}^2 y_{u_3}^2 y_{d_1}^2 y_{d_2}^2 y_{d_3}^2}. \quad (\text{D12})$$

The dimensionless version is more suitable for writing in terms of $\mathcal{H}_u, \mathcal{H}_d$ as

$$J_{\text{dim}} = \langle \mathcal{H}_u \Omega \mathcal{H}_d \mathcal{H}_u^{-1} \Omega^{-1} \mathcal{H}_d^{-1} \rangle - \text{c.c.}, \quad (\text{D13})$$

where c.c. denotes the complex conjugate. Note that Ω only appear in two places. In terms of X_u and X_d , it becomes

$$\begin{aligned} J_{\text{dim}} &= i\mu \left\langle X_u \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X_d X_u^{-1} \begin{pmatrix} \frac{1}{1-\mu^2} & & \\ & \frac{1}{1-\mu^2} & \\ & & 1 \end{pmatrix} X_d^{-1} \right\rangle \\ &\quad - i \frac{\mu}{1-\mu^2} \left\langle X_u X_d X_u^{-1} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X_d^{-1} \right\rangle - \text{c.c.} \end{aligned} \quad (\text{D14})$$

Here X_u and X_d are not diagonal and absorbed the matrices $\mathcal{O}$ as

$$X_u = \mathcal{O}_u^\top \hat{X}_u \mathcal{O}_u, \quad X_d = \mathcal{O}_d^\top \hat{X}_d \mathcal{O}_d. \quad (\text{D15})$$

We partly use these invariants to guide the search for physical points.

-
- [1] A. E. Nelson, *Phys. Lett.* **136B**, 387 (1984).
 - [2] S. M. Barr, *Phys. Rev. Lett.* **53**, 329 (1984).
 - [3] L. Bento, G. C. Branco, and P. A. Parada, *Phys. Lett. B* **267**, 95 (1991).
 - [4] M. Dine and P. Draper, *J. High Energy Phys.* **08** (2015) 132.
 - [5] G. H. S. Alves and C. C. Nishi, *J. High Energy Phys.* **09** (2025) 162.

- [6] A. L. Cherchiglia and C. C. Nishi, *J. High Energy Phys.* **03** (2019) 040.
- [7] A. Valenti and L. Vecchi, *J. High Energy Phys.* **07** (2021) 152.
- [8] C. Csáki, S. Homiller, and T. Youn, *J. High Energy Phys.* **03** (2026) 220.
- [9] A. Valenti and L. Vecchi, *J. High Energy Phys.* **07** (2021) 203.

- [10] A. L. Cherchiglia and C. C. Nishi, *J. High Energy Phys.* **08** (2020) 104.
- [11] A. L. Cherchiglia, G. De Conto, and C. C. Nishi, *J. High Energy Phys.* **11** (2021) 093.
- [12] G. H. S. Alves, A. L. Cherchiglia, and C. C. Nishi, *Phys. Rev. D* **108**, 035049 (2023).
- [13] P. Asadi, S. Homiller, Q. Lu, and M. Reece, *Phys. Rev. D* **107**, 11 (2023).
- [14] F. Liu, S. Nakagawa, Y. Nakai, and Y. Wang, *J. High Energy Phys.* **04** (2026) 178; C. Murgui and S. Patrone, *J. High Energy Phys.* **09** (2025) 113; K. Murai and K. Nakayama, *J. High Energy Phys.* **11** (2024) 098.
- [15] G. Hiller and M. Schmaltz, *Phys. Lett. B* **514**, 263 (2001).
- [16] L. Vecchi, *J. High Energy Phys.* **04** (2017) 149.
- [17] S. Antusch and V. Maurer, *J. High Energy Phys.* **11** (2013) 115.
- [18] G. y. Huang and S. Zhou, *Phys. Rev. D* **103**, 016010 (2021).
- [19] R. Benbrik, M. Boukidi, M. Ech-chaouy, S. Moretti, K. Salime, and Q. S. Yan, *J. High Energy Phys.* **03** (2025) 020.
- [20] J. M. Alves, G. C. Branco, A. L. Cherchiglia, C. C. Nishi, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo, and J. I. Silva-Marcos, *Phys. Rep.* **1057**, 1 (2024).
- [21] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, *J. High Energy Phys.* **12** (2013) 089.
- [22] B. Belfatto and S. Trifinopoulos, *Phys. Rev. D* **108**, 035022 (2023).
- [23] C. Y. Chen, S. Dawson, and E. Furlan, *Phys. Rev. D* **96**, 015006 (2017).
- [24] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [25] F. del Aguila, M. Perez-Victoria, and J. Santiago, *J. High Energy Phys.* **09** (2000) 011.
- [26] K. Ishiwata, Z. Ligeti, and M. B. Wise, *J. High Energy Phys.* **10** (2015) 027.
- [27] C. Bobeth, A. J. Buras, A. Celis, and M. Jung, *J. High Energy Phys.* **04** (2017) 079.
- [28] A. Glioti, R. Rattazzi, L. Ricci, and L. Vecchi, *SciPost Phys.* **18**, 201 (2025).
- [29] N. Vignaroli, *Phys. Rev. D* **86**, 115011 (2012).
- [30] Y. S. Amhis *et al.* (HFLAV Collaboration), *Phys. Rev. D* **107**, 052008 (2023).
- [31] M. Misiak, A. Rehman, and M. Steinhauser, *J. High Energy Phys.* **06** (2020) 175.
- [32] A. Valenti and L. Vecchi, *J. High Energy Phys.* **01** (2023) 131.
- [33] E. L. F. de Lima and C. C. Nishi, *J. High Energy Phys.* **11** (2024) 157.
- [34] J. Alexander *et al.* (pEDM Collaboration), *arXiv:2205.00830*; *arXiv:2504.12797*.
- [35] C. C. Nishi and J. I. Silva-Marcos, *Phys. Rev. D* **108**, 095031 (2023); J. I. Silva-Marcos, *arXiv:hep-ph/0212089*; D. Emmanuel-Costa, N. R. Agostinho, J. I. Silva-Marcos, and D. Wegman, *Phys. Rev. D* **92**, 013012 (2015).