

Spectral function for pions in a magnetic field

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This study examines the spectral functions of neutral (π_0) and charged ($\pi_{\pm}$) pions under a uniform magnetic field using the SU(2) Nambu–Jona-Lasinio model with the Ritus method. The analysis highlights the complex interplay of magnetic field effects, thermal influences, and chiral symmetry on meson properties in extreme QCD environments. For π_0 , whose properties are governed by the behavior of its constituent quarks, magnetic field-induced Landau levels lead to a multipeak structure in its spectral function, reflecting stable and resonance solutions that evolve with temperature, showing shifts and critical enhancements near chiral restoration. For $\pi_{\pm}$, cross terms that come from the asymmetry between the constituent quarks introduce Landau cuts alongside unitary cuts, indicating damping effects, with decay widths narrowing at higher temperatures, suggesting increased stability.

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I. INTRODUCTION

In the early stage of peripheral heavy-ion collision, an intense but short-lived magnetic field is created [1–5]. Its intensity can reach up to $1 - 10m_{\pi}^2$, which leads to a variety of non-negligible phenomenon [6–10]. For nonequilibrium states, one of the most popular subjects is the study of anomalous transport processes, such as the chiral magnetic effect [11] and chiral magnetic wave [12,13]. For equilibrium states, the effect of a uniform magnetic field on the QCD phase structure is a recent hot topic. For instance, the inverse magnetic catalysis effect, where the chiral condensates for quarks drop with increasing magnetic field, has been extensively studied for the past decade [14–47]. To fully understand these behaviors, a profound knowledge about the interplay between QCD and the electromagnetic field is necessary.

The response of neutral and charged pions on magnetic field is also closely related to the research on phase structure, since they are the (pseudo-)Goldstone bosons under chiral symmetry and their masses' change can be

regarded as the signal of chiral symmetry restoration [48,49]. When a uniform magnetic field is turned on, the isospin symmetry is broken, making $\pi_{\pm}$ no longer Goldstone bosons [4,42,50]. Normally there are two different definitions for meson mass: pole mass and screening mass [51,52]. By definition, the pole mass of a particle refers to the mass extracted from the position of the pole in its propagator in momentum space, which corresponds to the physical state of the particle when it is free, whereas the screening mass is associated with the behavior of it in a medium, regarded as the mass “renormalized” by the environment [53]. In vacuum, where the temperature is zero, pole and screening masses share the same value.

For the screening mass (or pole mass at zero temperature), extensive studies have been performed in both lattice simulation [53–55] and theoretical calculation [36,51,56,57]. For pole mass at finite temperature and magnetic field, however, for now there are only results from effective model calculation, which exhibits a quite peculiar behavior. After including a finite magnetic field, the pole equation starts to possess multiple solutions, and the positions of these solutions are separated by the mass sum of constituent quark mass in different Landau levels. As the temperature or chemical potential increases, the physical mass may jump from one solution to another, which is understood as a kind of Mott transition [51,58–62]. It should be noticed that this behavior can only be observed in quark-level theories like the Nambu–Jona-Lasinio (NJL) model. Driven by such a strange phenomena, in this work we will explore the spectral function of both neutral and charged pions at finite temperature and magnetic field.

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Spectral functions provide a more detailed picture beyond mass spectra. They encode information on the dispersion, width, and possible decay channels of mesonic excitations and serve as key inputs for evaluating transport coefficients such as shear viscosity [63–66]. The spectral function of mesons with finite temperature and chemical potential has been vastly studied in the past decades, and recently, the system under rotation and with chiral imbalance has also been taken under consideration [67–70]. In hot and magnetized QCD matter, these studies also offer insights into mass pole, Landau damping [71], and critical behavior [72–74]. Additionally, the spectral functions and screening masses of pseudoscalar and scalar mesons in the high-temperature deconfined phase have been analytically calculated at the lowest order of perturbation theory [75]. While previous studies have examined vector meson spectral functions [76,77], the full analytic structure of pseudoscalar mesons, particularly charged pions, has not been systematically explored.

In this work, we study the spectral functions of both neutral and charged pions at finite temperature in a magnetic field using the SU(2) NJL model with the Ritus method. For π^0 , we demonstrate how Landau-level quantization of the constituent quarks leads to a multipeak structure in the spectral function. For $\pi^\pm$, we show that cross terms arising from isospin asymmetry in its constituent quarks generate additional Landau cuts, reflecting medium-induced damping processes. Our results highlight the rich analytic structure of mesonic excitations in magnetized media and reveal nontrivial temperature and magnetic field dependence of pion stability and decay width.

The article is arranged as follows: In Sec. II, we will illustrate how to calculate the mass and spectral function of neutral and charged pions in the frame of the SU(2) NJL model in a magnetic field. In Sec. III, we will give the numerical results of the spectral structure for neutral and charged pions separately and analyze each of the peak structure. In Sec. IV, we will summarize the key findings in this article.

II. METHOD

We start with the Lagrangian of the SU(2) NJL model in the presence of a uniform magnetic field,

$$\mathcal{L} = \bar{\psi}(i\mathcal{D} - \hat{m})\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]. \quad (1)$$

Here, ψ is two-flavor quark field $\psi = (u, d)^T$, $\hat{m}$ is the quark current mass matrix $\hat{m} = \text{diag}(m_u, m_d)$ with $m_u = m_d = m_0$, which explicitly breaks the chiral symmetry, and τ_i is i th component of Pauli matrices in flavor space. G is the coupling constant in the scalar and pseudoscalar channels. The covariant derivative $D_\mu = \partial_\mu - iQA_\mu$ coupling quarks with electric charge $Q = \text{diag}(q_u, q_d) = \text{diag}(2/3e, -1/3e)$ to a gauge field $\mathbf{B} = \nabla \times \mathbf{A}$. In this

article, we choose the Landau gauge and the potential $A_\mu = (0, 0, Bx_1, 0)$.

In the presence of a uniform magnetic field, the translational invariance of quark in coordinate space is broken, and 4-momenta are no longer “good” quantum numbers to the system. Thus, if we straightforwardly apply the Fourier transformation to the quark propagator from coordinate space into momentum space, it will introduce a Schwinger phase that compensates for the broken translational and gauge invariance, as discussed in [78]. A reasonable solution to this situation is to replace the 4-momentum with a new set of good quantum numbers, which gives rise to the conserved Ritus momentum $\vec{p} = (p_0, 0, -s_f\sqrt{2n|q_f B|}, p_3)$ with n being the quark Landau level in magnetic field and $s_f = \text{sign}(q_f B)$ the quark sign factor. In this Ritus scheme, the quark propagator with flavor f in coordinate space can be expressed as

$$\begin{aligned} S_f(x, y) &= \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i\vec{p}\cdot(x-y)} P_n(x_1, p_2) \\ &\quad \times D_f(\vec{p}) P_n(y_1, p_2) \\ P_n(x_1, p_2) &= \frac{1}{2} [g_n^{s_f}(x_1, p_2) + I_n g_{n-1}^{s_f}(x_1, p_2)] \\ &\quad + \frac{is_f}{2} [g_n^{s_f}(x_1, p_2) - I_n g_{n-1}^{s_f}(x_1, p_2)] \gamma_1 \gamma_2 \\ D_f(\vec{p}) &= \frac{1}{\gamma \cdot \vec{p} - m_q}, \end{aligned} \quad (2)$$

where $\vec{p} = (p_0, 0, p_2, p_3)$ is the Fourier transformed momentum and $I_n = 1 - \delta_{n0}$ is governed by the Landau level. The magnetic-field-dependent function $g_n^{s_f}(x_1, p_2) = \phi_n(x_1 - s_f p_2 / |q_f B|)$ is controlled by the Hermite polynomial $H_n(\zeta)$ through

$$\phi_n(\zeta) = (2^n n! \sqrt{\pi} |q_f B|^{-1/2})^{-1/2} e^{-\zeta^2 |q_f B|/2} H_n(\zeta \sqrt{|q_f B|}). \quad (3)$$

Here, m_q is the constituent mass for both u and d quarks, which is determined by chiral condensates with $m_q = m_0 - 2G\langle\bar{\psi}\psi\rangle$ in the NJL model.

In mean field approximation, the thermodynamic potential of the system at finite temperature and under external magnetic field B takes the form of

$$\Omega_{\text{MF}}(T, B) = \frac{(m_q - m_0)^2}{4G} + \Omega_q(T, B), \quad (4)$$

with the contribution from quarks

$$\Omega_q(T, B) = \text{Tr}_{\{c,f,s,x\}} \ln \left(\frac{1}{T} S^{-1}(x, x) \right). \quad (5)$$

The trace operation is carried out over color (c), flavor (f), and spinor (s) degrees of freedom, as well as over the four-dimensional coordinate (x). $S^{-1}(x, x)$ is the inverse of quark the propagator in coordinate space. After taking the trace in the above spaces, we have

$$\Omega_q = -N_c \sum_{f=u,d} \frac{|Q_f B|}{2\pi} \times \sum_n \alpha_n \int \frac{dp_3}{2\pi} \left[E_f + 2T \ln \left(1 + e^{-\frac{E_f}{T}} \right) \right], \quad (6)$$

with the quark energy $E_f = \sqrt{p_z^2 + 2n|Q_f B| + m_q^2}$ of flavor $f = u, d$, longitudinal momentum p_3 and Landau level l , and the degeneracy of Landau levels $\alpha_n = 2 - \delta_{n0}$.

To determine the effective quark mass, we have to find the ground state by locating the global minimum of the thermodynamic potential

$$\begin{aligned} \frac{\partial \Omega_{\text{MF}}(T, B)}{\partial m_q} &= 0, \\ \frac{\partial^2 \Omega_{\text{MF}}(T, B)}{\partial m_q^2} &\geq 0. \end{aligned} \quad (7)$$

In the NJL model, mesons emerge dynamically as collective excitations of quark-antiquark pairs. In the spirit of the random phase approximation method, the meson propagator is given by

$$U_M(q) = \frac{2G}{1 - 2G\Pi_M(q)}, \quad (8)$$

with polarization function

$$\Pi_M(x, y) = i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_{\{c,f,s\}} [\mathcal{V}_M^* S(x, y) \mathcal{V}_M S(y, x)]. \quad (9)$$

The meson vertex is given by

$$\mathcal{V}_M = \begin{cases} 1, & M = \sigma \\ i\gamma_5 \tau_+, & M = \pi_+ \\ i\gamma_5 \tau_-, & M = \pi_- \\ i\gamma_5 \tau_3, & M = \pi_0. \end{cases} \quad (10)$$

There is one step missing between Eqs. (8) and (9), which is the transformation of the polarization function between coordinate and momentum space. At finite magnetic field, this transformation is nontrivial. For neutral mesons that do not interact with the magnetic field as a whole object, the momentum $q = (\omega, \vec{q})$ itself is conserved, and transformation from coordinate space to momentum space is carried out by just the plane wave,

which is the solution of the Klein-Gordon equation. For charged mesons, though, the transformation kernel from coordinate space to Ritus momentum space, which is the solution of the magnetized Klein-Gordon equation, is $e^{-i\vec{p}\cdot x} g_l^{s_M}(x_1, p_2)$ with l being the mesonic Landau level and $\vec{p}$ is conserved.

Spectral functions provide a powerful tool to study the properties of the above mesons, especially in extreme environments like high temperature and intense magnetic field. It encodes the distribution of states with ω and p .

For mesons, the spectral function is related to the imaginary part of the retarded propagator,

$$\begin{aligned} \xi(q) &\equiv \frac{1}{\pi} \text{Im} U_M(q) \\ &= \frac{1}{\pi} \frac{4G^2 \text{Im} \Pi_M(q)}{(1 - 2G\text{Re} \Pi_M(q))^2 + (2G \text{Im} \Pi_M(q))^2}. \end{aligned} \quad (11)$$

In this work, we set the external momentum $\vec{q} = \vec{0}$ for π_0 and $q_3 = n = 0$ for $\pi_{\pm}$. This choice focuses on the rest-frame properties and isolates the essential physics of Landau-level quantization of constituent quarks, while avoiding complications from spatial anisotropy induced by the magnetic field. The decay width of a meson can be extracted from its spectral function by measuring the width of peak. In our case, the inclusion of multiple unitary cuts and Landau cuts (which will be introduced officially in next subsection) will change the shape of the peak structures in spectral functions, making their shape deviate from the standard Breit-Wigner structure. An alternative method to calculate the decay width is to replace the meson mass m_M by $m_M - i\Gamma_M/2$, and the pole equation takes the complex form [79]

$$1 - 2G\Pi_M(m_M - i\Gamma_M/2, \mathbf{0}) = 0. \quad (12)$$

In practice, the calculation of Eq. (12) usually involves the small-decay-width approximation, which enables us to calculate the mass and decay width separately. Details of this technique will be shown in the following subsections.

A. π_0 case

Starting from the quark propagator in Eq. (8) and substituting into the polarization function defined in Eq. (9), the trace over color, flavor, and spinor degrees of freedom can be carried out analytically. For π_0 , since it is electrically neutral, its center-of-mass momentum is conserved and the standard Fourier transformation applies. Performing the Matsubara summation at finite temperature, the polarization function reduces to

$$\Pi_{\pi_0}(\omega) = \sum_{f=u,d} J_1^{(f)} + \omega^2 J_2^{(f)}(\omega) \quad (13)$$

with

$$\begin{aligned}
 J_1^{(f)} &= 3 \sum_n \alpha_n \frac{|Q_f B|}{2\pi} \int \frac{dp_3}{2\pi} \frac{1 - 2f(E_f)}{E_f}, \\
 J_2^{(f)}(\omega) &= 3 \sum_n \alpha_n \frac{|Q_f B|}{2\pi} \mathcal{P} \int \frac{dp_3}{2\pi} \frac{1 - 2f(E_f)}{E_f(4E_f^2 - \omega^2)} \\
 &\quad + 3i \sum_n \alpha_n \frac{|Q_f B|}{4\pi} \frac{1 - 2f\left(\frac{\omega}{2}\right)}{4\omega \sqrt{\frac{\omega^2}{4} - m_q^2 - 2n|Q_f B|}} \\
 &\quad \times \Theta(\omega - 2\sqrt{m_q^2 + 2n|Q_f B|}). \tag{14}
 \end{aligned}$$

Here, $\mathcal{P}$ denotes the Cauchy principal integral, and $\Theta(k)$ is the Heaviside function. The Fermi-Dirac distribution function is $f(E) = 1/(e^{E/T} + 1)$. If we look into the form of the real part of $J_2^{(f)}(\omega)$, we will notice that each time when $\omega - \min E_{f,n} = \omega - 2\sqrt{m_q^2 + 2n|Q_f B|} \rightarrow 0^-$, the function diverges. However, when $\omega - \min E_{f,n} = \omega - 2\sqrt{m_q^2 + 2n|Q_f B|} \rightarrow 0^+$ the function gives a finite value because the principal integral starts to work. This means that the real part of inverse propagator is discontinuous at $\omega - 2\sqrt{m_q^2 + 2n|Q_f B|} \rightarrow 0$. For the imaginary part, similar behaviors can be noticed. It diverges each time when it approaches the boundary $\omega - 2\sqrt{m_q^2 + 2n|Q_f B|} \rightarrow 0^+$. In another perspective, under a constant magnetic field, π_0 is composed by a series of one-dimensional quarks with different effective masses $m_f^{(n)} = \sqrt{m_q^2 + 2n|Q_f B|}$. The one-dimensionality causes the divergence at $\omega \rightarrow 2m_f^{(n)} + 0^-$, which leads to the possible solution of the pole equation for p_{i0} . This feature will cause the pole

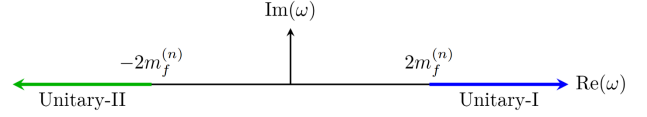


FIG. 1. Analytic structure of self-energy $\Pi(\omega, 0)$ for π_0 in a complex plane. Here is the unitary cut at n th Landau level of quark with flavor f .

equation to have multiple solutions of ω . Following the method in [70,71], branch cuts can be applied at the point of discontinuity and separate the function into several continuous segments. The branch cuts here are referred to as unitary cut I and II at $(-\infty, -2m_f^{(n)})$ and $[2m_f^{(n)}, \infty)$, and the cut structure is shown in Fig. 1.

After taking the approximation of small decay width compared with the mass, mass and width in Eq. (12) get decoupled and we can get the mass and width from

$$1 - 2G\text{Re}\Pi_{\pi_0}(m_{\pi_0}, \mathbf{0}) = 0,$$

$$\Gamma_{\pi_0} \simeq \frac{1}{m_{\pi_0}} \text{Im} \frac{1 - 2G \sum_{f=u,d} J_1^{(f)}}{2G \sum_{f=u,d} \text{Re} J_2^{(f)}(m_{\pi_0} - i\epsilon)}. \tag{15}$$

B. $\pi_{\pm}$ case

For $\pi_{\pm}$, the charged meson couples to the external magnetic field as a whole, breaking translational invariance. Following the approach of [78,80], the Schwinger phase is absorbed into a modified Fourier kernel involving the mesonic Landau level. After performing the trace operations and Matsubara summation, the polarization function acquires additional cross terms (CTs) arising from the asymmetry between the u and d quark Landau levels, and the polarization function $\Pi_{\pi_{\pm}}$ is given by

$$\begin{aligned}
 \Pi_{\pi_{\pm}}(\omega, 0, -\sqrt{|eB|}, 0) &= \sum_{f=u,d} J_1^{(f)} + \sum_{n,n'} j_{n,n'}(\omega^2) J_3^{n,n'}(\omega^2), \\
 J_3^{n,n'}(\omega^2) &= p \frac{dp_3}{2\pi} \frac{1}{4E_n E_{n'}} \left[\frac{f(-E_{n'}) - f(E_n)}{\omega + E_n + E_{n'}} + \frac{f(E_{n'}) - f(-E_n)}{\omega - E_n - E_{n'}} - \frac{f(E_{n'}) - f(E_n)}{\omega - E_n + E_{n'}} - \frac{f(-E_{n'}) - f(E_n)}{\omega - E_n + E_{n'}} \right] \\
 &\quad + \frac{i}{2} \times \sum_{p_3^*} \{ F_1(p_3^*) \Theta(\omega - E_{n'}(0) - E_n(0)) + [F_2(p_3^*) \Theta(n' - 2n) + F_3(p_3^*) \Theta(-n' + 2n)] \\
 &\quad \times \Theta(-\omega^2 - [E_{n'}(0) - E_n(0)]^2) \}, \tag{16}
 \end{aligned}$$

where the imaginary part is

$$\begin{aligned}
F_1(p_3^*) &= -\frac{1}{4\omega^2} \frac{f(-E_{n'}(p_3^*)) - f(E_n(p_3^*))}{4E_n(p_3^*)E_{n'}(p_3^*)} \frac{(\omega + E_n(p_3^*) + E_{n'}(p_3^*))[\omega^2 - (E_{n'}(p_3^*) - E_n(p_3^*))^2]}{2p_3^*}, \\
F_2(p_3^*) &= \frac{1}{4\omega^2} \frac{f(-E_{n'}(p_3^*)) - f(-E_n(p_3^*))}{4E_n(p_3^*)E_{n'}(p_3^*)} \frac{(\omega - E_n(p_3^*) + E_{n'}(p_3^*))[\omega^2 - (E_{n'}(p_3^*) + E_n(p_3^*))^2]}{2p_3^*}, \\
F_3(p_3^*) &= \frac{1}{4\omega^2} \frac{f(E_{n'}(p_3^*)) - f(E_n(p_3^*))}{4E_n(p_3^*)E_{n'}(p_3^*)} \frac{(\omega + E_n(p_3^*) - E_{n'}(p_3^*))[\omega^2 - (E_{n'}(p_3^*) + E_n(p_3^*))^2]}{2p_3^*}, \\
p_3^* &= \pm \sqrt{\omega^4 + (2n'|Q_u B| + 2n|Q_d B|)^2 - 4\omega^2(m^2 + n'|Q_u B| + n|Q_d B|)/(2\omega)}, \tag{17}
\end{aligned}$$

with

$$\begin{aligned}
j_{n,n'}(\omega^2) &= \left(\frac{\omega^2}{2} - n'|Q_u B| - n|Q_d B| \right) j_{n,n'}^+ - 2\sqrt{n'|Q_u B|n|Q_d B|} j_{n,n'}^-, \\
j_{n,n'}^\pm &= 8N_c \int dx_1 dy_1 dp_2 / (2\pi) g_0^+(x_1, k_2) g_0^+(y_1, k_2) A_\pm(x_1, y_1, k_2), \\
A_\pm(x_1, y_1, k_2) &= \alpha_+(x_1, k_2) \alpha_+(y_1, k_2) \pm \alpha_-(x_1, k_2) \alpha_-(y_1, k_2), \\
\alpha_\pm(z, k_2) &= \frac{1}{2} [I_n g_{n-1}^{s_d}(z, p_2) g_{n'}^{s_u}(z, q_2) \pm u \leftrightarrow d]. \tag{18}
\end{aligned}$$

Here the quark energies for different flavors are given by $E_{n'} = \sqrt{p_3^2 + 2n'|Q_u B| + m_q^2}$ and $E_n = \sqrt{p_3^2 + 2n|Q_d B| + m_q^2}$. In the imaginary part, p_3^* corresponds to the singularities that occur in the momentum integral, and the two Heaviside functions guarantee that the imaginary parts activate only when singularities appear in the integral range.

Unlike the neutral pion case, if one looks into the polarization function for charged pions, it is noticeable that, when a uniform magnetic field is turned on, the orthogonality between the two constituent quarks in different Landau levels disappears, and instead there is a coefficient between them named $j_{n,n'}(\omega^2)$. Compared with Eq. (14), in Eq. (16) there are two more CTs, which are the last two terms in $J_3^{n,n'}(\omega^2)$, because of the different quark component between π_0 and $\pi_\pm$. If we look into the polarization function of $\pi_\pm$, it is noticed that, apart from the discontinuities and divergences at $\omega - \sqrt{m_u^2 + 2n'|Q_u B|} - \sqrt{m_d^2 + 2n|Q_d B|} \rightarrow 0$ as is discussed in the π_0 case, more singularities at $\omega - \sqrt{m_u^2 + 2n'|Q_u B|} + \sqrt{m_d^2 + 2n|Q_d B|} \rightarrow 0$ and $\omega + \sqrt{m_u^2 + 2n'|Q_u B|} - \sqrt{m_d^2 + 2n|Q_d B|} \rightarrow 0$ also can be found, which is referred to as the Landau cut. It usually corresponds to the situation of Landau damping, where one particle is emitted from the thermal medium and another is absorbed [81]. The analytical structures of the unitary cut and Landau cut with n' th Landau level of u quark and n th Landau level of d quark are separately shown in the upper and lower parts of Fig. 2.

In practice, the Landau levels of constituent quarks run from 0 to infinite, creating an infinite number of

branch cuts in complex plane. However, thanks to the suppression of Fermi-Dirac distribution, only a few of them will have visible influence, making it possible to observe a set of continuous lines in the real and imaginary part of the inverse propagator, as one can see in the next section.

For $\pi_\pm$, we also can take the small-decay-width approximation and get the width from

$$\begin{aligned}
1 - 2G\text{Re}\Pi_{\pi_\pm}(m_{\pi_\pm}, 0, -\sqrt{|eB|}, 0) &= 0, \\
\Gamma_{\pi_\pm} &\simeq -\frac{2}{m_{\pi_\pm}} \\
&\times \text{Im} \frac{1 - 2G \left[\sum_{f=u,d} J_1^{(f)} - \sum_{n,n'} k_{n,n'} J_3^{n,n'} ((m_{\pi_0} - i\epsilon)^2) \right]}{2G \sum_{n,n'} j_{n,n'}^+ J_3^{n,n'} ((m_{\pi_0} - i\epsilon)^2)}, \tag{19}
\end{aligned}$$

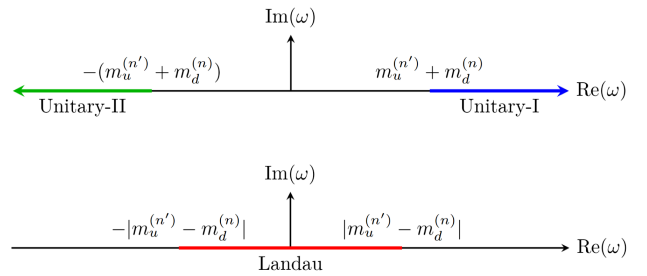


FIG. 2. Analytic structure of self-energy $\Pi(\omega, 0)$ for $\pi_\pm$ in complex plane. Here is the unitary cut (upper) and Landau cut (lower) with n' th Landau level of u quark and n th Landau level of d quark.

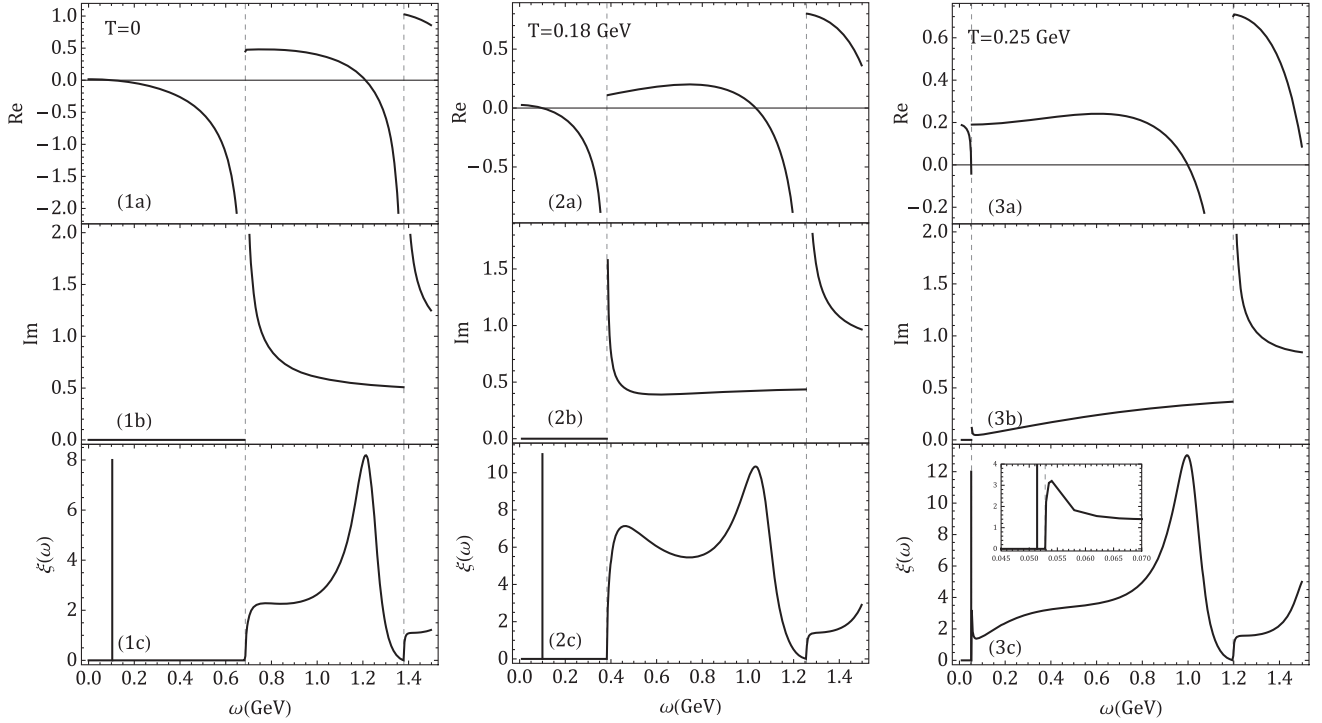


FIG. 3. (1a),(2a),(3a) Real part of inverse propagator $1 - 2G\text{Re}\Pi(\omega)$ as a function of ω . (1b),(2b),(3b) Imaginary part of inverse propagator $2G\text{Im}\Pi(\omega)$ as a function of ω . (1c),(2c),(3c) Spectral function of π_0 as a function of ω . The vertical line represents the δ function, which corresponds to the pole of the propagator. The magnetic field strength is set to be $eB = 30m_\pi^2$, with temperature $T = 0$ (first column), $T = 0.18$ (second column), $T = 0.25$ GeV (third column). The vertical dashed lines represent the mass threshold $2m_f^{(n)}$ with $n = 0, 1, 2, \dots$ being the Landau level for constituent quarks.

with

$$k_{n,n'} = (n'|Q_u B| + n|Q_d B|)j_{n,n'}^+ + 2\sqrt{n'|Q_u B|n|Q_d B|}j_{n,n'}^- \quad (20)$$

III. NUMERICAL RESULTS

In this work, we apply the gauge-invariant Pauli-Villars regularization, and by fitting the physical quantities, i.e., the chiral condensate $\langle \bar{\psi}\psi \rangle = (-0.25 \text{ GeV})^3$ and pion decay constant $f_\pi = 0.093 \text{ GeV}$ in vacuum, we fix the current mass of light quarks $m_0 = 0.005 \text{ GeV}$ and obtain the coupling constant $G = 3.44 \text{ GeV}^{-2}$ and energy scale $\Lambda = 1.127 \text{ GeV}$. In the following numerical calculations, we fix the magnetic field strength to $eB = 30m_\pi^2$ throughout this study, with $m_\pi = 0.137 \text{ GeV}$. The physics behind these peak structures does not depend on the choice of magnetic field strength. We choose this strong magnetic field to clearly expose the analytic structures such as the discrete multipeak features and well-separated Landau cuts. If a lower magnetic field intensity were considered, the Landau levels would become densely packed. This would cause the distinct resonance peaks to heavily overlap and smear out, making it difficult to isolate and trace the individual mass solutions.

A. π_0 case

Given Eq. (11), we can start calculating the spectral function of the pions. Here we start with a simpler case with π_0 . In Fig. 3, we demonstrate the spectral function for π_0 with different temperatures, as well as the real and imaginary part of the inverse propagator $[1 - 2G\Pi(\omega)]$. In the left column, the temperature is set to be zero. Figure 3(1a) shows $1 - 2G\text{Re}\Pi(\omega)$ as a function of ω . The curve crosses zero twice within the displayed range, indicating two solutions of the pole equation for the π_0 mass. The region where the function $1 - 2G\text{Re}\Pi(\omega)$ diverges corresponds to the boundaries where $\omega = 2m_q^{(n)}$, with $m_q^{(n)} = \sqrt{m_q^2 + 2n|q_f B|}$ denoting the effective mass of quarks in n th Landau level. In Fig. 3(1b), we demonstrate the imaginary part of inverse propagator $2G\text{Im}\Pi(\omega)$. Unlike the case with zero magnetic field where the imaginary part increases continuously around the boundary of $\omega = 2m_q$ [70], in the presence of a uniform magnetic field, multiple jumps can be noticed at the boundaries, followed by a fast monotonous decreasing, as a consequence of dimension reduction of constituent quarks. Figure 3(1c) demonstrates the spectral function of π_0 . In the shown range, we can identify two peak structure. One of them is a Dirac δ function, representing the stable

particle solution, while the other has a finite width similar to a Breit-Wigner function, denoting the resonance state solution. Unlike in the absent magnetic field case, where the spectral function usually possesses a standard Breit-Wigner structure, here a “bump” at around 0.8 GeV can be noticed. Numerically speaking, this structure is believed to result from the fast decline of the imaginary part of the inverse propagator in the vicinity of the discontinuous region. The multipeak structure of the π_0 meson spectral function can be understood as follows: although π_0 is a neutral particle, its constituent quarks, u and d , carry electric charge. Their transverse momentum becomes quantized into discrete Landau levels under a magnetic field. This multipeak behavior is physically consistent with the sawtooth structures previously observed in perturbative quark gluon plasma calculations [75], where the emergence of multiple thresholds and singularities is primarily attributed to the dimensional reduction of constituent quarks in a strong magnetic field. Consequently, constituent quarks from different Landau levels may form bound states corresponding to different physical solutions of the Bethe-Salpeter equation. It is generally believed that the first δ -function solution corresponds to the stable excitation of π_0 , whose position is primarily governed by chiral condensate. The subsequent solutions correspond to excited states of the π_0 , with masses affected by both the chiral condensate and the strength of the magnetic field. Another important feature worth noticing is that, at the left-hand side of the threshold of the mass sum of the constituent quark, the real part of the inverse propagator diverges, while the imaginary part diverges at the right-hand side. These two divergences guarantee that the spectral function equals exactly zero at both sides of the threshold, and it is continuous in the whole region.

In the middle column of Fig. 3, the temperature is set to 0.18 GeV, where the system’s chiral symmetry is still broken, but it is already in the vicinity of the critical temperature $T_c = 0.19$ GeV. From the overall trend, Figs. 3(2a) and 3(2b) panel are very similar to the $T = 0$ case, except for a slight nonmonotonic behavior in the curves after the first threshold at $\omega = 0.39$ GeV. Taking the imaginary part of the inverse propagator in the middle panel as an example, it decreases sharply just above the threshold, followed by a slower decline and a slight upward turn, until it reaches the jump caused by the next threshold. In Fig. 3(2c), the previously existing δ function shows almost no shift compared with $T = 0$ case. Meanwhile, the previously observed bump structure “grows” into a new peak. However, this newly emerged peak is not accompanied by a corresponding zero in the real part of the inverse propagator, suggesting that it does not represent a genuine mode excitation.

In the third column, the temperature is set to $T = 0.25$ GeV, where the system is in the chiral restoration

phase. In Fig. 3(3a), it can be seen that the pole equation’s first solution is getting very close to the first threshold at around $\omega \simeq 0.05$ GeV, and the solution becomes almost indistinguishable from the threshold. However, the negative divergence at the left side of threshold indicates that, as long as the real part of inverse propagator at $\omega = 2m_f^n$ is positive, there always will be a solution at $(2m_f^n, 2m_f^{n+1})$. In Fig. 3(3b), the nonmonotonicity of the imaginary part of the inverse propagator becomes even more obvious. In Fig. 3(3c), after the first threshold in the spectral function, a rapid increase followed by a decrease can be observed, forming a sharp peak structure around the threshold $\omega = 2m_q \simeq 0.14$ GeV can be observed. Considering the singular behavior of the imaginary part of the inverse propagator at this point, this peak corresponds to the critical enhancement effect, reflecting the enhancement of the $\pi_0 \rightarrow q\bar{q}$ decay channel.

In Fig. 4, we demonstrate the first two solutions of the pole equation for π_0 . The black line stands for the stable particle solution, which corresponds to the δ function in spectral function. It can be noticed that this solution shows a nonmonotonic behavior: it keeps increasing in the low-temperature region and starts to decline at around $T = 0.5$ GeV. When temperature gets higher than $T = 0.23$ GeV, the first solution for pion masses becomes almost indistinguishable from the threshold $2m_q$. The upper blue line stands for the first resonance solution and comes with a finite width Γ calculated from Eq. (15). The mass keeps decreasing with temperature at $T < 0.21$ GeV and then starts to increase, showing a nonmonotonic behavior. From the decay width, which is stable and small in the low-temperature region and gets large at around critical temperature, it is evident that this solution remains relatively stable for temperatures below 0.2 GeV, but becomes increasingly unstable beyond this point.

In the calculation of the meson mass spectrum, previous studies usually only present a single set of physical π_0 masses, without simultaneously displaying multiple solutions. Therefore, due to different judgment criteria, the reported results vary. In some works, the authors consider the stable particle solution to correspond to the ground state of π_0 and thus display the complete results of this stable solution until it is no longer accessible. Afterward, the mass is allowed to jump to the next solution with the lowest mass [32,51]. In another series of works, however, the authors argue that the bound state solution of π_0 should satisfy the Goldstone theorem, meaning that, as the chiral condensate decreases, the mass of the stable particle solution should monotonically increase [62,80]. Therefore, any decreasing part of this solution is regarded as unphysical, and the meson mass jumps to the next solution at the maximum value. These two criteria lead to significantly different Mott transition temperatures T_M . Using the critical temperature T_c determined by the maximum variation of the order parameter as a reference, T_M obtained by the former

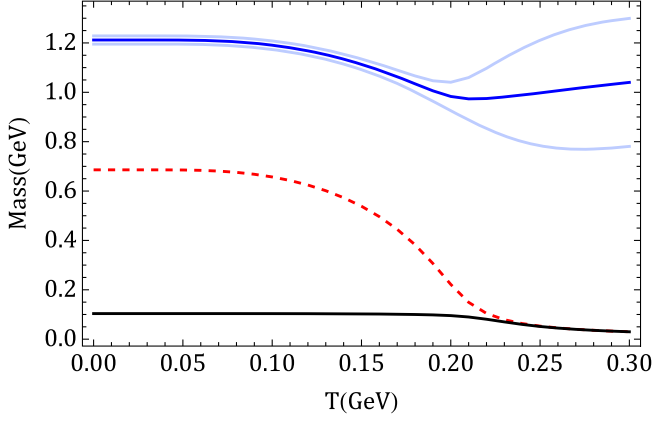


FIG. 4. Pole mass of π_0 as a function of T with magnetic field set to $eB = 30m_\pi^2$. Two solutions of the pole equation are both shown in the figure (black and blue solid lines). Decay width Γ for resonance solution is also shown in the figure by $m_{\pi_0} + \frac{\Gamma}{2}$ and $m_{\pi_0} - \frac{\Gamma}{2}$ (light blue lines). For comparison, the mass sum of the two constituent quarks is shown with the red dashed line.

criterion is larger than T_c , while that from the latter is significantly smaller than T_c . In the present work, we have shown in Figs. 3 and 4 that, for π_0 , the stable solution always exists, since the real part of the inverse propagator

exhibits negative divergence at $\omega = 2m$ and positive at $\omega = 0$.

B. $\pi_\pm$ case

Now we move to results of $\pi_\pm$. In Fig. 5, we demonstrate the real and imaginary part of the inverse propagator with polarization function defined in Eq. (16), as well as the spectral function of $\pi_\pm$, with temperature $T = 0, 0.18, 0.25$ GeV. To fully understand the effect of CTs in Eq. (16), we also plot the spectral function without considering CTs in red color for comparison. For the spectral function, it is observed that the CT has no effect at zero temperature since it cancels with itself. As discussed above, the CT contributes to the Landau cut, and it appears only in medium and arises from scattering of the external particle with particles in medium, namely, the Landau damping effect, as discussed in [71]. At zero temperature, we notice that more discontinuity is found compared with the π_0 case, since there is no orthogonality between the Landau levels of two constituent quarks, and the unitary cut thresholds occur at $\omega = m_u^{(n')} + m_d^{(n)}$ for $n, n' = 0, 1, 2, \dots$. As for the peak structure, the spectral function possesses four peaks, but the pole equation only has three solutions in the shown range, as one can see in the upper panel.

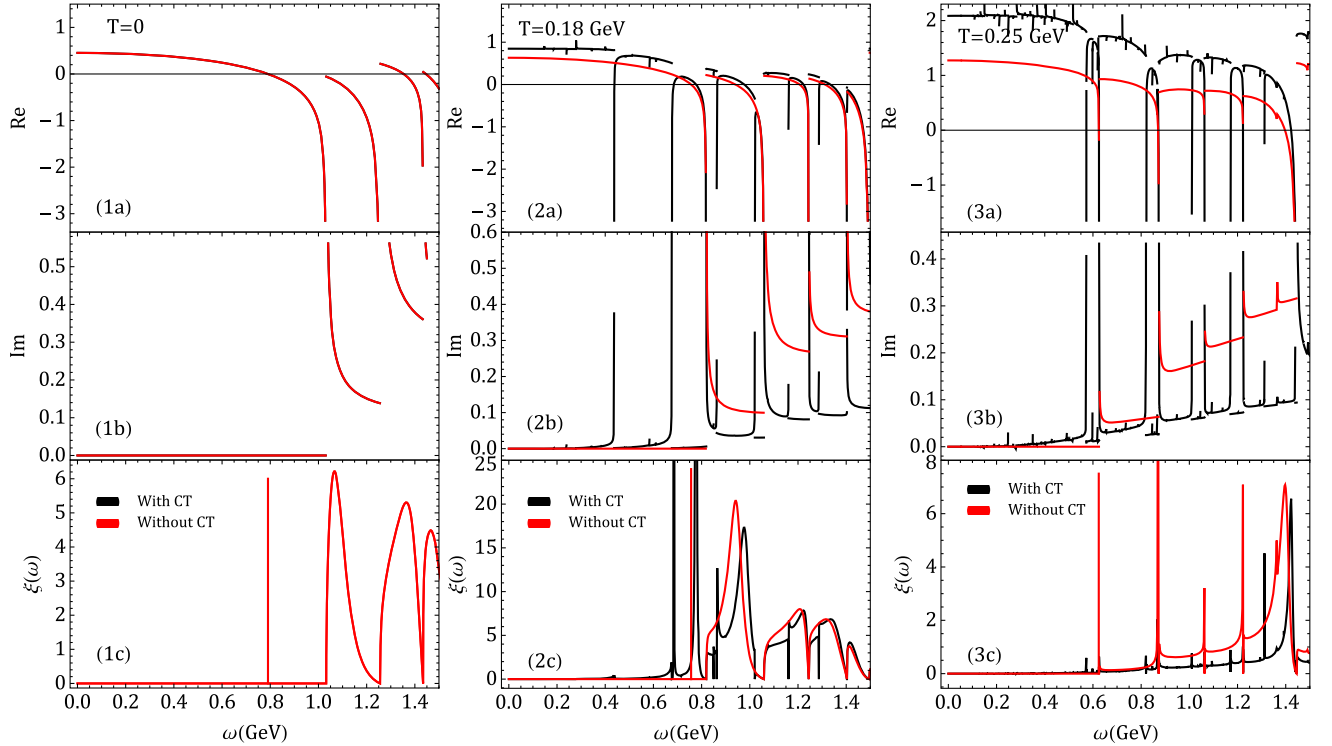


FIG. 5. (1a),(2a),(3a) Real part of inverse propagator $1 - 2G\text{Re}\Pi(\omega)$ for $\pi_\pm$ as a function of ω . (1b),(2b),(3b) Imaginary part of inverse propagator $2G\text{Im}\Pi(\omega)$ as a function of ω . (1c),(2c),(3c) Spectral function of $\pi_\pm$ as a function of ω . The vertical line represents the δ function, which corresponds to the pole of the propagator. The magnetic field strength is set to be $eB = 30m_\pi^2$, with temperature $T = 0$ (first column), $T = 0.18$ GeV (second column), $T = 0.25$ GeV (third column). For comparison, the spectral function without considering cross terms is also shown in red color.

At $\omega = 1.07$ GeV, the inverse of the propagator reaches a local minimum, which is the origin of the peak.

As the temperature increases to $T = 0.18$ GeV, we begin to observe the effects of Landau cuts. For the real part in the upper panel, apart from zeros that can be observed in the case without CTs, each branch is separated into multiple pieces. At the left-hand side of these CT-induced branches, the results for the real part of the inverse propagator will become divergent, giving a lot more zeros. This means that more solutions for the pole equation will be founded. However, as can be seen in Fig. 5(2a), these added solutions all possess a positive slope in $1 - 2G\text{Re}\Pi(\omega)$. As discussed in [82], the Yukawa coupling is given by the relationship

$$g_{\pi q\bar{q}}^2 = (\partial\Pi/\partial k^2)^{-1}|_{k^2=m_\pi^2}. \quad (21)$$

The CT-induced solution, which comes with an imaginary Yukawa coupling, can be regarded as an “unphysical” mesonic excitation [70]. As for the imaginary part, a significant number of new discontinuities also arise, where each continuous segment exhibits a characteristic non-monotonic U-shaped profile. The Landau cut is located at $\omega = |m_u^{(n')} - m_d^{(n)}|$, $n, n' = 0, 1, 2, \dots$. Unlike the finite number of unitary cuts within a given energy interval, the number of Landau cuts in a given range is theoretically infinite. However, due to the suppression from the Fermi-Dirac distribution and the parameter $j_{n,n'}(\omega^2)$, only low Landau levels of quarks, particularly those satisfying $n \approx 2n'$, contribute significantly to the Landau cut effect. As the temperature rises, more quark Landau levels need to be taken into account, resulting in an increasing number of discontinuities in the high-temperature region. For the thresholds created by the Landau cut, the real part of the inverse propagator diverges at the right-hand side of the threshold, while the imaginary part diverges at the left-hand side. As for the spectral function, it also is continuous at the whole range and goes to zero at the threshold.

For the spectral function at the finite-temperature region, the inclusion of CT starts to possess an observable effect. A large number of additional small peak structures appear, which correspond to Landau damping processes. When the temperature further increases to $T = 0.25$ GeV, the effects of Landau cuts become even more pronounced, leading to an even greater number of additional peaks in the spectral function. These effects, together with the critical enhancement phenomenon previously discussed for the π_0 meson, contribute to a rich spectral function structure.

As temperature increases, the peak structure in the spectral function keeps getting sharper. Another significant feature at $T = 0.25$ GeV is that, despite the increasing number of resonance states, their widths tend to decrease with rising temperature. It indicates that, under a strong

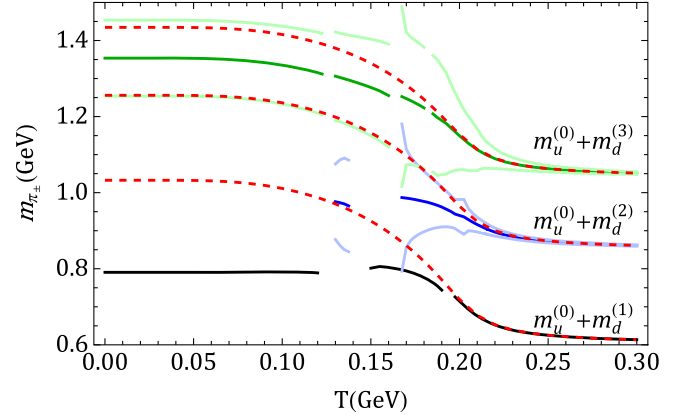


FIG. 6. Ground state energy of $\pi_{\pm}$ as a function of T with magnetic field set to $eB = 30m_\pi^2$. Three lowest solutions of the pole equation are shown in the figure (black, blue, and green solid lines). Decay width Γ for resonance solution is also shown in the figure by $m_{\pi_{\pm}} \pm \frac{\Gamma}{2}$ (light blue and light green lines). For comparison, the mass sums of the two constituent quarks in different Landau levels are shown in red dashed lines.

magnetic field, these modes of the $\pi_{\pm}$ mesons become more stable at high temperature. As one can see from Eq. (11), the width of a peak not only relates to the imaginary part of the inverse propagator, but is also affected by the real part. When the solution of the pole equation approaches the mass threshold $m_u^{(n')} + m_d^{(n)}$, the value of the real part changes fast in the vicinity of the solution of the pole equation, contributing to a sharp peak structure.

In Fig. 6, we show the lowest three physical solutions of the pole equation for $\pi_{\pm}$ with the corresponding decay width. The lowest black line, which is the bound state solution, decreases continuously in the whole temperature region. For the middle blue line, the pole equation does not yield a strict zero-crossing solution at low temperatures. Instead, as shown in the first row of Fig. 5 at $T = 0$, the real part of the inverse propagator only reaches a local maximum (around $\omega = 1.07$ GeV) without crossing the zero axis. A genuine pole solution only emerges when the temperature exceeds $T = 0.14$ GeV due to thermal modifications of the self-energy. Once it appears, this solution shares the same monotonically decreasing behavior with the ground state. Its decay width, in general, gradually becomes narrower as the temperature increases, consistent with the analysis on the spectral function in Fig. 5. Sudden changes in the decay width are observed at $T = 0.15$ and $T = 0.20$ GeV, which can be attributed to the meson mass crossing the Landau cut threshold, thereby inducing a nonzero imaginary part in the polarization function at these temperatures. Usually only the Landau cut effect with relatively low Landau levels will have visible influence on the decay width. As for the upper green line, it has the similar behavior as the bound state solution. Its width, also as discussed before, generally decreases with increasing

temperature, though several discontinuous jumps are observed.

It is worth noting that all three solutions exhibit gaps in certain narrow temperature intervals where no solution can be found. The emergence of “gaps” in the mass spectra of $\pi_{\pm}$ can be understood through the interplay between different types of branch cuts in the complex plane. Typically, a mesonic pole is formed when the real part of the inverse propagator, $1 - 2\text{GRe}\Pi$, decreases monotonically within a branch of the unitary cut and crosses the x axis. However, at finite temperature, the Landau cuts (arising from cross terms in the polarization function) introduce additional discontinuities and local divergences. If a Landau-cut-induced divergence appears at a location that overlaps with the expected zero point of the unitary cut branch, the zero point is effectively “bypassed” or “swallowed.” Consequently, within these narrow temperature intervals, the pole equation fails to find a valid solution, resulting in the observed spectral gaps.

IV. SUMMARY

In summary, within the framework of the two-flavor Nambu–Jona-Lasinio model, we have systematically investigated the spectral function structures of neutral and charged pions (π_0 and $\pi_{\pm}$) in the presence of a uniform magnetic field. By employing the Ritus method, we incorporate the breaking of translational invariance into a modified Fourier transformation.

For the neutral pion π_0 at zero temperature, the spectral function exhibits a distinct multipeak structure. Some of the peaks correspond to multiple solutions of the pole equation, as indicated by the zeros of the real part of the inverse propagator. The first solution, typically featuring a Dirac δ -function-like peak, represents a stable ground state excitation, while the subsequent solutions are resonance states associated with finite-width Breit-Wigner-type structures. These multiple peaks arise due to the quantization of transverse momentum into Landau levels for the charged constituent quarks, which leads to a sequence of effective thresholds at $\omega = 2m_q^{(n)} = 2\sqrt{m_q^2 + 2n|Q_f B|}$, separating different excitation modes. However, it should also be noticed that not all peak structures are related to a physical particle.

As the temperature increases, the spectral function structure undergoes significant changes. At moderate temperatures below the chiral restoration point, the stable peak slightly shifts nonmonotonically. Meanwhile, secondary peaks originating from resonances become wider or shift in position. Near and above the chiral critical temperature T_c , a sharp enhancement in the spectral function occurs at threshold regions, indicating the onset of critical phenomena like $\pi_0 \rightarrow q\bar{q}$ channel openings. As the temperature gets higher, the mass of the stable π_0

solution decreases and approaches the quark-antiquark threshold, making it challenging to distinguish from the continuum. The first resonance solution, in contrast, exhibits a nonmonotonic temperature dependence and increasing decay width near T_c , signaling its diminishing stability.

In comparison, the spectral properties of the charged pions $\pi_{\pm}$ are more intricate. In addition to the unitary cuts (arising from meson decay into quark-antiquark pairs), their analytic structure also includes Landau cuts due to cross terms in the polarization function. The Landau cuts correspond to thermal scattering processes between mesons and medium constituents, i.e., Landau damping. At zero temperature, these terms vanish due to cancellation, and the results with and without mixing terms coincide. At finite temperatures, however, Landau damping manifests through nontrivial structures in the imaginary part of the inverse propagator, especially a characteristic U-shaped nonmonotonicity after each threshold and a proliferation of small peaks in the spectral function.

The unitary cuts appear at $\omega = m_u^{(n')} + m_d^{(n)}$ for $n, n' = 0, 1, 2, \dots$ and increase monotonically with Landau-level index, yielding only a finite number of thresholds within a given energy window. By contrast, Landau cuts occur at $\omega = |m_u^{(n')} - m_d^{(n)}|$, potentially leading to an infinite number of discontinuities. However, due to the Fermi-Dirac suppression and the structure of the overlap integral $j_{n,n'}(\omega^2)$, only a limited subset—primarily from low Landau levels with $n \approx 2n'$ —exert observable influence.

Regarding the meson mass spectrum, all solutions for $\pi_{\pm}$ exhibit a monotonic decrease with increasing temperature under strong magnetic fields. Their decay widths display sudden threshold-induced jumps when the meson mass crosses Landau cut boundaries. Aside from these threshold effects, the widths generally decrease with temperature, implying that the resonance modes of $\pi_{\pm}$ become increasingly stable at high temperatures, in contrast to the expected thermal broadening in the absence of a magnetic field.

The spectral functions obtained in this work can provide input for understanding various transport phenomena and response functions in magnetized QCD matter. In particular, the detailed analytic structure—including the positions and widths of peaks, as well as the threshold behaviors associated with unitary and Landau cuts—directly determines the dilepton production rate, photon emission, and thermal medium modifications of hadron properties. Future extensions of this work could employ these spectral functions to calculate transport coefficients such as shear viscosity, electrical conductivity, and thermal conductivity in the presence of strong magnetic fields. Additionally, the multipeak structure and temperature evolution of the spectral functions may have observable consequences in heavy-ion collision

experiments, where the system traverses different temperature regimes.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [83].

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