

Transverse force tomography inside a proton from basis light-front quantization

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The twist-3 transverse spin-dependent nucleon structure function g_2 arises in high-energy processes involving a transversely polarized nucleon. Its connection to quark-gluon correlations allows for an interpretation in terms of the average transverse color Lorentz force acting on unpolarized quarks inside a transversely polarized nucleon. In this work, we investigate this force using light-front wave functions obtained by diagonalizing the light-front Hamiltonian with quantum chromodynamics inputs within the basis light-front quantization approach. We evolve our results to a common scale of 5 GeV^2 and present the corresponding form factors in momentum space as well as the transverse force components in impact-parameter space. These distributions provide a complementary perspective on the Sivers asymmetry in transversely polarized deep-inelastic scattering. In the forward limit, we extract the twist-3 reduced matrix element d_2 , and our results are found to be comparable with those from other theoretical calculations and experimental determinations.

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I. INTRODUCTION

In modern nuclear and particle physics, high-energy scattering processes serve as a cornerstone for unraveling the structure of hadrons, testing quantum chromodynamics (QCD), and exploring the fundamental interactions of matter. As an important process among them, deep inelastic scattering (DIS) probes the internal structure of nucleons and nuclei, enabling the extraction of parton distribution functions (PDFs)—a set of fundamental quantities that quantify the probability of finding a parton with a given fraction of the nucleon's momentum. To analyze the

invariant matrix element of the scattering process, the operator product expansion (OPE) and twist analysis are employed [1], where twist is defined as the dimension of the operator minus its spin.

The parton model captures only leading-order (twist-2) contributions and neglects higher-twist effects. In certain cases, however, partonic intuition can be extended to higher twist, such as relating twist-three operators to quark-gluon correlations [2], which admit a phenomenologically appealing interpretation in terms of a color-Lorentz force [3–6]. In this work, we compute the transverse impact-parameter-space distribution of this color-Lorentz force directly from light-front wave functions (LFWFs) within the basic light-front quantization (BLFQ) framework [7,8]. This requires extending the twist-three forward matrix element d_2 to off-forward kinematics and introducing three form factors (FFs) that encode the force distributions [3,5,6]. We show that these distributions provide a consistent and intuitive framework for understanding single-spin asymmetries in semi-inclusive deep-inelastic scattering (SIDIS).

The matrix element associated with the axial-vector current is of particular interest, which is defined as [9]

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$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma^\mu \gamma_5 \psi(\lambda n) | PS \rangle = 2[g_1(x) p^\mu (S \cdot n) + g_T(x) S_\perp^\mu + M^2 g_3(x) (S \cdot n) n^\mu], \quad (1)$$

where p^μ and n^μ are lightlike vectors along the $+$ and $-$ light cone directions with $p \cdot n = 1$, and S^μ denotes the proton polarization vector. These parton correlation functions contribute to hard processes at the orders of $g_1(x)$, $g_T(x)/Q$, and $g_3(x)/Q^2$, and thus correspond to twist-2, twist-3, and twist-4, respectively. Here, x is the Bjorken scaling variable, and Q^2 is the momentum transfer from the virtual photon.

The presence of $S_\perp^\mu$ implies that the twist-3 structure function $g_T(x)$ can be cleanly accessed in transversely polarized protons. This is particularly significant because $g_T(x)$ is sensitive to quark-gluon correlations [10,11], which are in turn connected to transverse spin asymmetries and the transverse color Lorentz force [3,4]. However, it should be noted that $g_T(x)$ does not admit a simple single-particle density interpretation.

Experimental data exist for the chirally even spin-dependent twist-3 parton distribution $g_2(x) = g_T^q(x) - g_1^q(x)$, including results from polarized DIS experiments at SLAC (E142 [12], E143 [13,14], E154 [15], E155 [16,17]), HERMES [18], JLab (JAM [19], Hall A [20,21], RSS [22]), and CERN (SMC [23]) (alongside virtual photon asymmetries). Additionally, the reduced twist-3 matrix element d_2 has been measured at HERMES [18], JLab [21], SANE [24].

On the theoretical side, the twist-3 matrix element d_2 has been computed directly in lattice QCD [25–27]. Significant attention has also been devoted to the transverse color Lorentz force, which provides an intuitive physical interpretation of twist-3 effects in terms of quark-gluon interactions. In particular, the QCDSF collaboration [26] and studies based on the instanton liquid model [27] have computed the spatial distribution of this force in impact-parameter space. These developments are highly relevant for future precision studies at the planned Electron-Ion Collider (EIC) in the United States [28] and the proposed Electron-Ion Collider in China (EicC) [29].

The structure of this work is as follows. In Sec. II, we briefly introduce the BLFQ framework and present the parameters employed in this calculation. In Sec. III, we elaborate on the definition of the transverse color Lorentz force, its connection to the twist-3 spin-dependent structure function g_2 , as well as their overlap representations adopted for the numerical calculations. In Sec. IV, we present our numerical results, and discussions are provided in Sec. V.

II. BASIS LIGHT FRONT QUANTIZATION

BLFQ is a nonperturbative method for solving the eigenvalue problem of light-front (LF) QCD, which incorporates LF QCD interactions via an effective LF

Hamiltonian. The core eigenvalue equation in BLFQ is

$$P^- P^+ |\psi\rangle = M^2 |\psi\rangle, \quad (2)$$

where $P^\pm = P^0 \pm P^3$. Here, P^+ denotes the total longitudinal momentum of the system, P^- is the light-front Hamiltonian, M^2 is the invariant mass squared of the eigenstate, and $|\psi\rangle$ is the corresponding light-front eigenstate. For the proton state, the light-front eigenstate $|\psi\rangle$ is expanded as a superposition of Fock states,

$$|\psi_{\text{proton}}\rangle = \Phi^{qqq} |qqq\rangle + \Phi^{qqqg} |qqqg\rangle + \dots, \quad (3)$$

where Φ^{qqq} and Φ^{qqqg} are the LFWFs corresponding to the $|uud\rangle$ and $|uudg\rangle$ Fock sectors, respectively, and the ellipsis ($\dots$) denotes higher Fock components of the proton. In this work, the Fock space is truncated to these two lowest sectors.

The LF Hamiltonian adopted in this work is $P^- = P_{\text{QCD}}^- + P_{\text{C}}^-$, where P_{QCD}^- contains the relevant QCD interaction terms, and P_{C}^- is the confinement term that models the nonperturbative confinement effect [8]. In the LF gauge $A^+ = 0$, the LF QCD Hamiltonian including one dynamical gluon is given by [8]

$$\begin{aligned} P_{\text{QCD}}^- = & \int d^2x^\perp dx^- \left\{ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^+)^2}{i\partial^+} \psi \right. \\ & + \frac{1}{2} A_a^i [m_g^2 + (i\partial^+)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi \\ & \left. + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right\}, \end{aligned} \quad (4)$$

where: ψ is the quark field operator, m_0 is the bare quark mass, m_g is the bare gluon mass, and g_s is the strong coupling constant. Specifically, the first/second term in the Hamiltonian accounts for the kinetic energy of quarks/gluons, and the last two terms represent the vertex and instantaneous interaction. In the present BLFQ framework, an effective gluon mass m_g is introduced as a phenomenological parameter. It serves to regulate infrared singularities arising from the truncated basis and to effectively mimic confinement-scale gluonic dynamics, in analogy with functional and Hamiltonian approaches [30].

The confinement term in the $|qqq\rangle$ sector is implemented following Ref. [31] as

$$P_{\text{C}}^- P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left[\vec{r}_{ij\perp}^2 - \frac{\partial_{x_i} (x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \right], \quad (5)$$

where $\vec{r}_{ij\perp}$ is the transverse separation, and κ is the confinement strength parameter. For the $|qqqg\rangle$ sector, no explicit confinement term is introduced; instead, the essential confinement behavior is captured by the restricted transverse basis and the inclusion of a massive gluon.

The additional confinement term is introduced to compensate for the incomplete dynamics arising from the truncation of the Fock space. Since the interaction in Eq. (4) represents only a subset of the full QCD Hamiltonian, this term effectively simulates missing non-perturbative gluonic dynamics and leads to a realistic description of hadronic bound states [7,8,31]. Recent studies indicate that, as higher Fock sectors are added, such an explicit confinement term can be omitted [32–34].

Within the BLFQ framework [7], the proton state is expanded in a basis consisting of quarks and a gluon that reside in: (i) longitudinal plane waves (defined in a box of length $2L$ with antiperiodic boundary conditions for quarks and periodic boundary conditions for gluons), (ii) two-dimensional harmonic oscillator (2D-HO) functions $\Phi_{nm}(\vec{p}_\perp; b)$ (with $\vec{p}_\perp$ being the transverse momentum, b the 2D-HO scale parameter, and n, m the 2D-HO quantum numbers) [35], and (iii) light-cone helicity spinors. As a result of these choices, the single-parton states in the basis are labeled by $\alpha_i = \{k_i, n_i, m_i, \lambda_i\}$, where k is the longitudinal momentum quantum number (half-integer for quarks, integer for gluons, excluding the zero mode), and λ is the helicity. For multiparton sectors with more than one color-singlet configuration (e.g., $|qqqg\rangle$), an additional color label is required.

The longitudinal box of length $2L$ is a standard element in the BLFQ basis construction that is adopted from DLCQ [36]. It enables a discrete plane-wave expansion and momentum quantization in the longitudinal direction, providing a well-defined box normalization. This is purely a computational device for constructing the basis and does not affect the underlying QCD dynamics or its symmetries in the continuum limit. An alternative choice for the longitudinal basis is the Jacobi polynomial basis, which has also been employed within the BLFQ framework for the leading Fock sector of mesons [31,37–40]. However, for multiparticle systems this basis becomes computationally less efficient and increasingly challenging to implement.

The basis expansion is truncated by two parameters: $N_{\max}$ and K . $N_{\max}$ imposes the transverse truncation condition $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$, and K fixes the longitudinal resolution with $x_i = k_i/K$, where $K = \sum_i k_i$. The parameters $b, N_{\max}$ and K determine the IR and UV scales of the calculation [35]. Diagonalizing the LF Hamiltonian yields the proton LFWFs with helicity Λ , which can be expressed as

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{N, \Lambda} = \sum_{\{n_i, m_i\}} \psi^N(\{\alpha_i\}) \prod_{i=1}^N \Phi_{n_i m_i}(\vec{p}_{\perp i}, b), \quad (6)$$

where N is the number of partons in the Fock sector ($N = 3/4$ for $|uud\rangle/|uudg\rangle$), and $\psi^N(\{\alpha_i\})$ are the eigenvector components corresponding to the N -parton Fock sectors.

TABLE I. Summary of the model parameters [8]. All parameters have units of GeV except for g_s .

m_u	m_d	m_g	κ	m_f	g_s	b	b_{inst}
0.31	0.25	0.50	0.54	1.80	2.40	0.70	3.00

The Hamiltonian parameters employed in this work are summarized in Table I, with the truncation parameters fixed as $\{N_{\max}, K\} = \{9, 16.5\}$. These parameters are calibrated to reproduce the proton mass ($M = 0.956$ GeV) and its key electromagnetic properties [8]. At the model scale, the proton state has a probability of approximately 44% in the $|qqq\rangle$ sector and 56% in the $|qqqg\rangle$ sector. The resulting LFWFs correspond to a low-resolution scale of $\mu_0^2 \sim 0.24 \pm 0.01$ GeV² [8] and have been successfully applied to describe a wide range of proton observables, including electromagnetic and gravitational FFs, charge and mechanical radii, PDFs, generalized parton distributions (GPDs), transverse momentum-dependent PDFs (TMDs), as well as spin and orbital angular momentum distributions [8,30,41–47].

III. TRANSVERSE COLOR LORENTZ FORCE AND OVERLAP REPRESENTATION

We start with the general matrix element containing the $\bar{q}Gq$ correlator, defined as [3]

$$W_{\Lambda'\Lambda}^{\mu,\nu\rho}(P', P) = \langle P', \Lambda' | \bar{\psi}(0) \gamma^\mu i g G^{\nu\rho}(0) \psi(0) | P, \Lambda \rangle, \quad (7)$$

where $\psi(x)$ is the quark field operator, $G^{\nu\rho}$ is the gluon field strength tensor, and $|P, \Lambda\rangle$ denotes the proton state with four-momentum P and helicity Λ . This matrix element can be parametrized into eight FFs. To extract a physical quantity, we take $\{\mu, \nu, \rho\} = \{+, +, j\}$, leading to the reduced twist-3 matrix element [3,4]

$$d_2^q = - \frac{\langle P, \Lambda | \bar{\psi}(0) \gamma^+ g G^{+y}(0) \psi(0) | P, \Lambda \rangle}{2M(P^+)^2 S^x}, \quad (8)$$

where M is the proton mass and S^x is the x -component of the proton polarization vector. This implies considering a proton transversely polarized along the x -direction, which is a superposition of opposite helicity states [48]

$$|P_x\rangle = \frac{1}{\sqrt{2}}(|P, \uparrow\rangle + |P, \downarrow\rangle). \quad (9)$$

The quantity d_2^q is of particular interest because it does not only measure the response of the color electric and magnetic field to the polarization of the nucleon through the gluon field strength tensor [4,49]

$$G^{+y} \sim (E^y - B^x) = (\vec{E} + \vec{v} \times \vec{B})^y, \quad (10)$$

but also rigorously related to the nucleon's second spin-dependent structure function g_2 via [3]

$$\int_{-1}^1 dx x^2 \bar{g}_2^q(x) = \frac{d_2^q}{3}, \quad (11)$$

with $g_2^q(x)$ defined as [50]

$$g_2^q(x) = g_T^q(x) - g_1^q(x) = g_2^{q,WW}(x) + \bar{g}_2^q(x), \quad (12)$$

$$g_2^{q,WW}(x) = -g_1^q(x) + \int_x^1 \frac{dy}{y} g_1^q(y),$$

where $g_1(x)$ is the twist-2 helicity PDF and $g_T(x)$ is the twist-3 spin-dependent PDF.

Considering the above constraints, the matrix element $W_{\Lambda'\Lambda}^{+,+j}(p', p)$ reduces to five FFs. The parametrization reads [3]

$$W_{\Lambda'\Lambda}^{+,+j}(P', P) = \bar{u}(P', \Lambda') \left\{ \frac{1}{M^2} [\bar{P}^+ \Delta_\perp^j - \bar{P}^\perp \Delta^+] \gamma^+ \Phi_1(t) \right. \\ \left. + \frac{\bar{P}^+}{M} i \sigma^{+j} \Phi_2(t) + \frac{1}{M^3} i \sigma^{+\Delta} [\bar{P}^+ \Delta_\perp^j \Phi_3(t) \right. \\ \left. - \bar{P}^\perp \Delta^+ \Phi_4(t)] + \frac{\bar{P}^+ \Delta^+}{M^3} i \sigma^{j\Delta} \Phi_5(t) \right\} u(P, \Lambda), \quad (13)$$

where $\bar{P}^\mu = (P^\mu + P'^\mu)/2$ is the average four-momentum of the proton, and $-t = \Delta^2 = (P' - P)^2$ is the momentum transfer between the initial and final proton states. Since we are working under a symmetric frame, $\bar{P}^\perp = 0$. For a impact parameter space interpretation [48], we take the limit $\xi \equiv -\Delta^+/(2\bar{P}^+) = 0$ (i.e., $\Delta^+ = 0$), in which only three terms in the above equation survive.

The transverse color Lorentz force is defined as

$$F_{\Lambda'\Lambda}^j(t) = \frac{i}{\sqrt{2}\bar{P}^+} W_{\Lambda'\Lambda}^{+,+j}(P', P), \quad (14)$$

and its transverse-plane distribution is obtained via the Fourier transform of Eq. (14)

$$\mathcal{F}_{\Lambda'\Lambda}^j(\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F_{\Lambda'\Lambda}^j(\Delta_\perp). \quad (15)$$

We present the overlap representations of all relevant FFs below. For brevity, we adopt the following notations

$$[dx]_n = \prod_{i=1}^n \frac{dx_i d^2\vec{k}_i}{(16\pi^3)^n} 16\pi^3 \delta\left(1 - \sum x_i\right) \\ \times \delta^2\left(\sum \vec{k}_i\right) \delta(x - x_1), \quad (16)$$

where the subscript 1 labels the struck quark. We will now employ ψ_n^Λ to denote the LFWF $\psi_{i=1,\dots,n}^\Lambda(x_i, p_i, \lambda_i)$ with

proton helicity Λ and parton helicities λ_i ; $[\Psi_j^{\Lambda'\Lambda}] = \epsilon_j \psi_3^{\Lambda'\star} \psi_4^\Lambda - \epsilon_j^\star \psi_4^{\Lambda'\star} \psi_3^\Lambda$; and $[\gamma^+] = \bar{u}(P', \Lambda') \gamma^+ u(P, \Lambda)$ to encapsulate the helicity combinations of the struck quark, with ϵ_μ representing the gluon polarization vector. Here, the constraint $\delta_{\lambda_2}^{\lambda_2'} \delta_{\lambda_3}^{\lambda_3'} \delta_{\lambda_4}^{\lambda_4'}$ for spectator partons will be implicitly assumed, and $\Delta = \Delta_1 + i\Delta_2$ will refer to the two-dimensional complex representation.

The momentum-space overlap representations of the FFs can then be written

$$\Phi_1^j(t) = \int [dx]_4 \frac{M^2[\gamma^+]}{\sqrt{2}(\bar{P}^+)^2 \Delta_j} [\Psi_j^{\uparrow\uparrow}], \quad (17)$$

$$\Phi_2^j(t) = \int [dx]_4 \frac{\sqrt{2}iM[\gamma^+]}{4(\bar{P}^+)^2 \epsilon_\perp^{kj} \Delta_k} (\Delta[\Psi_j^{\uparrow\downarrow}] + \Delta^\star[\Psi_j^{\downarrow\uparrow}]), \quad (18)$$

$$\Phi_3^j(t) = \int [dx]_4 \frac{\sqrt{2}iM^3[\gamma^+]}{8(\bar{P}^+)^2 \Delta_1 \Delta_2} ((-)^{j+1} [\Psi_j^{\uparrow\downarrow}] + [\Psi_j^{\downarrow\uparrow}]), \quad (19)$$

where $\epsilon_\perp^{kj}$ is the transverse Levi-Civita symbol, and j takes values 1 or 2 [the choice of j does not affect $\Phi_k(t)$]. The repeated index j is not summed over. The impact parameter space components of the transverse color Lorentz force distribution are derived as

$$\mathcal{F}_{\Lambda'\Lambda,1}^j(\mathbf{b}_\perp) = \frac{i}{\sqrt{2}\bar{P}^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ \times \bar{u}(P', \Lambda') \left[\frac{\bar{P}^+ \Delta_\perp^j}{M^2} \gamma^+ \Phi_1(t) \right] u(P, \Lambda) \\ = -\frac{\bar{P}^+}{M^2} \frac{\partial}{\partial b_j} \tilde{\Phi}_1(b^2), \quad (20)$$

$$\mathcal{F}_{\Lambda'\Lambda,2}^j(\mathbf{b}_\perp) = \frac{i}{\sqrt{2}\bar{P}^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ \times \bar{u}(P', \Lambda') \left[\frac{\bar{P}^+}{M} i \sigma^{+j} \Phi_2(t) \right] u(P, \Lambda) \\ = \frac{2\bar{P}^+}{M} \tilde{\Phi}_2(b^2) \delta_2^j, \quad (21)$$

$$\mathcal{F}_{\Lambda'\Lambda,3}^j(\mathbf{b}_\perp) = \frac{i}{\sqrt{2}\bar{P}^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\ \times \bar{u}(P', \Lambda') \left[\frac{\bar{P}^+ \Delta_\perp^j}{M^3} i \sigma^{+\Delta} \Phi_3(t) \right] u(P, \Lambda) \\ = -\frac{4\bar{P}^+}{M^3} \left(\frac{\partial^2}{\partial b_1 \partial b_2} \delta_1^j + \frac{\partial^2}{\partial b_2^2} \delta_2^j \right) \tilde{\Phi}_3(b^2), \quad (22)$$

where $\tilde{\Phi}_i$ are the Fourier transforms of the corresponding momentum-space FFs.

Notably, Φ_4 and Φ_5 do not contribute due to $\Delta^+ = 0$. From Eq. (17), Φ_1 contributes only for identical initial and final nucleon helicities, implying that the first force component, $\mathcal{F}_1$, is insensitive to the nucleon polarization.

In contrast, Eqs. (18)–(19) show that Φ_2 and Φ_3 require a nucleon helicity flip, and consequently $\mathcal{F}_2$ and $\mathcal{F}_3$ are polarization dependent. It has been shown that $\mathcal{F}_2$ corresponds to the spatial distribution of the Sivers force [51].

IV. NUMERICAL RESULTS

In Fig. 1, we present the FFs Φ_1 , Φ_2 , and Φ_3 for u and d quarks in the proton as functions of the momentum transfer $-t$, ranging from 0 to 2.25 GeV^2 . To facilitate comparison at a common scale, we use the SNOWFLAKE package [52] to evolve our results to 5 GeV^2 . We adopt an approximate evolution scheme since a fully rigorous treatment of off-forward twist-3 quark-gluon-quark matrix elements is not available in the current release of the SNOWFLAKE package. Specifically, for each fixed value of t , we evolve the underlying twist-3- x -dependent distributions from the initial scale μ_0^2 to 5 GeV^2 using SNOWFLAKE in the forward (diagonal) approximation, and then reconstruct the force form factors via the same x -integration as in the original definition. This omits genuine off-diagonal contributions to

the evolution kernel, but analogous approximation strategies have been adopted for related quantities such as the gravitational form factor D -term [46,53,54]. Since these form factors correspond to Mellin moments of zero-skewness GPDs (local operator matrix elements), their scale dependence is governed by the same anomalous dimensions as PDF moments. The associated uncertainty bands arise from the variation of the initial scale, $\mu_0^2 = 0.24 \pm 0.01 \text{ GeV}^2$.

For Φ_1 [panel (a)], both quark flavors exhibit negative distributions, characterized by a rapid increase in magnitude at small $-t$, followed by a gradual saturation at larger $-t$. The magnitudes of the u - and d -quark contributions are similar. In contrast, Φ_2 [panel (b)] and Φ_3 [panel (c)] display markedly different behaviors: the u - and d -quark contributions have opposite signs, not only between the two quark flavors but also between the two FFs. The magnitude of Φ_1 for the u quark is slightly larger than that for the d quark, whereas in Φ_2 the u -quark contribution is approximately twice as large as that of the d quark, and in Φ_3 the two magnitudes are comparable. The evolved

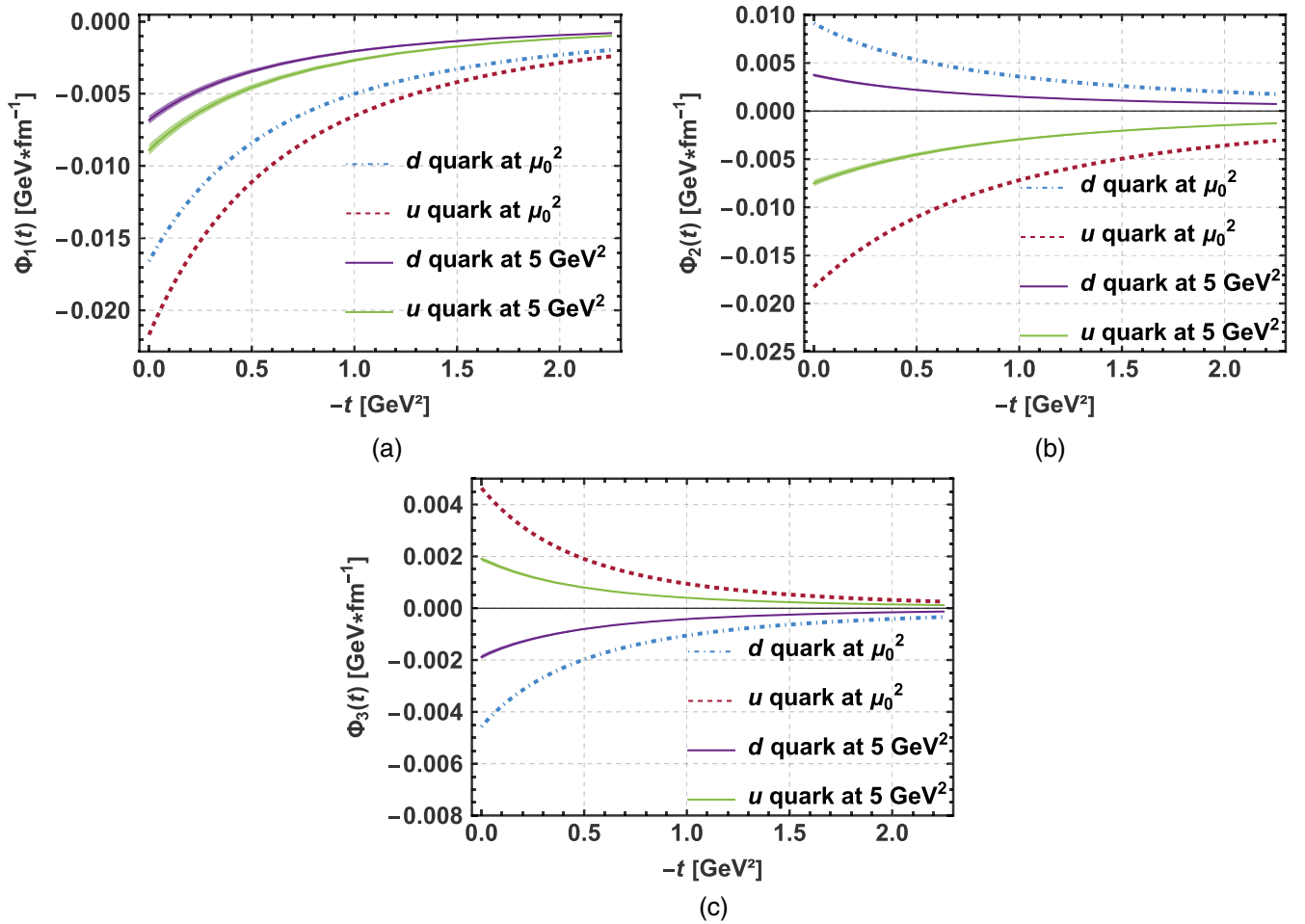


FIG. 1. Plots of the FFs Φ_1 , Φ_2 , and Φ_3 as a function of $-t$ for quarks inside a proton. The blue dash-dotted line stands for d quark at the initial scale and purple solid line stands at 5 GeV^2 . The red dashed line represents results for u quark at the initial scale and the green solid line at 5 GeV^2 .

TABLE II. The results contain d_2^p (for proton) and d_2^n (for neutron) compared here are at 5 GeV² except for ILM and QCDSF which are evaluated at 4 GeV². The Hall A [21] gives $-0.00035(83)(69)(7)$ for neutron at 4.3 GeV².

	BLFQ	RQCD [25]	LQCD [55]	HERMES [18]	c.m. bag [56]	Global fit [57]	ILM [27]	QCDSF [26]
$Q^2(\text{GeV}^2)$	5.0	5.0	5.0	5.0	5.0	5.0	4.0	4.0
d_2^p	0.00319	0.0105(68)	0.004(5)	0.0148(96)(48)	0.0174	0.00347(137)	0.00550	0.046(7)(16)
d_2^n	-0.000906	-0.0009(70)	-0.001(3)	...	-0.00253	-0.00217(63)	0.000198	0.023(5)(8)

results preserve the qualitative features of the unevaluated distributions.

Taking the forward limit $\Delta \rightarrow 0$, one obtains the closely related twist-3 quantity d_2^q ,

$$d_2^q = -\frac{F(0)}{M^2} = -\frac{\Phi_2(0)}{M^2}. \quad (23)$$

Using the light-front wave functions obtained within our BLFQ framework, we find $d_2^d = -0.00996$ ($d_2^u = 0.0200$) at the initial scale, and $d_2^d = -0.00409$ ($d_2^u = 0.00820$) at 5 GeV². The corresponding nucleon results are $d_2^p = 0.00778$ ($d_2^n = -0.00221$) at the initial scale, and $d_2^p = 0.00319$ ($d_2^n = -0.000906$) at 5 GeV². Similar values have been obtained in various other model calculations, as summarized in Table II. Using the extracted d_2 values and $M^2 = 4.643$ GeV/fm (equivalently $M^2 = 0.914$ GeV²), the magnitude of the color-Lorentz force is estimated to be $F_u \approx -0.04$ GeV/fm and $F_d \approx 0.02$ GeV/fm at 5 GeV².

In Fig. 2, we present the results of the force component distributions in impact parameter space, overlaid on the quark density plots. From top to bottom, the panels correspond to $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$, with d -quark distributions on the left and u -quark distributions on the right. For the first component ($\mathcal{F}_1$), both u and d quarks exhibit radially inward attractive forces toward the center. Notably, this is not a smooth increase; instead, the force reaches a maximum value at around $|\mathbf{b}_\perp| \sim 0.25$ fm and tends to 0 near the central ($|\mathbf{b}_\perp| = 0$). For the second component ($\mathcal{F}_2$), both u and d quarks exhibit pronounced alignment along the b_y axis, with their maximum magnitudes concentrated in the central region and opposite signs between the two quark flavors. For the third component ($\mathcal{F}_3$), a dipole structure is observed with maximum values gathered at the center: for d -quark, there is a repulsive pole at $(b_x, b_y) \sim (0, 0.27)$ and an attractive pole at $(b_x, b_y) \sim (0, -0.27)$, leading to a force direction from the upper region to the lower region; the u -quark exhibits the opposite sign.

The two-dimensional Fourier transforms of these FFs Φ_1 , Φ_2 , and Φ_3 reveal strong, localized forces acting on the struck quark. For an unpolarized proton, the force distribution exhibits a central restoring force, whereas for a transversely polarized proton it shows a pronounced downward force in the region of highest quark density. From a phenomenological perspective, these results offer a

complementary interpretation of the Sivers asymmetry observed in transversely polarized SIDIS experiments. Overall, the two-dimensional force distributions provide an intuitive and transparent visualization of the underlying mechanisms responsible for single-spin asymmetries.

The background in these plots represents the quark density, in units of fm⁻², obtained by integrating over the longitudinal momentum fraction of the Fourier transforms of the twist-2 GPDs $H(x, \mathbf{b})$ and $E(x, \mathbf{b})$, or equivalently of the Dirac and Pauli form factors [58–61]. These densities correspond to a proton polarized along the x direction. By normalizing the force density with respect to the quark density, one obtains the average local force strength in a given region. The force distributions shown in Fig. 2 are in good agreement with the results reported in Ref. [3].

V. SUMMARY AND CONCLUSIONS

This study systematically investigates the transverse color Lorentz force and its intrinsic connection to the nucleon twist-3 spin-dependent structure function g_2 within the BLFQ framework. First, we compute the momentum-space force form factors Φ_1 , Φ_2 , and Φ_3 for u and d quarks in a transversely polarized proton. The results reveal clear flavor-dependent features: Φ_1 is negative for both quark flavors, with its magnitude saturating at large $-t$, while Φ_2 and Φ_3 exhibit opposite sign patterns for u and d quarks. Moreover, the magnitudes of Φ_1 , Φ_2 , and Φ_3 for u quarks are approximately 1.5, 2, and 1 times larger, respectively, than those for d quarks.

Second, we extract the twist-3 matrix element d_2^q from the forward limit and obtain numerical estimates for both quark flavors. Since d_2 is predicted to be intrinsically small, its experimental determination will likely be subject to sizable uncertainties. Nevertheless, our theoretical predictions are consistent with existing lattice QCD results and phenomenological extractions.

Third, the force distributions in impact-parameter space exhibit distinct spatial structures for each component. The component $\mathcal{F}_1$ corresponds to a radially attractive force that increases from the periphery toward intermediate distances and then decreases to zero near the center. The component $\mathcal{F}_2$ is oriented along the b_x direction, with its sign depending on the quark flavor, while $\mathcal{F}_3$ displays a dipolelike pattern characterized by repulsive and attractive

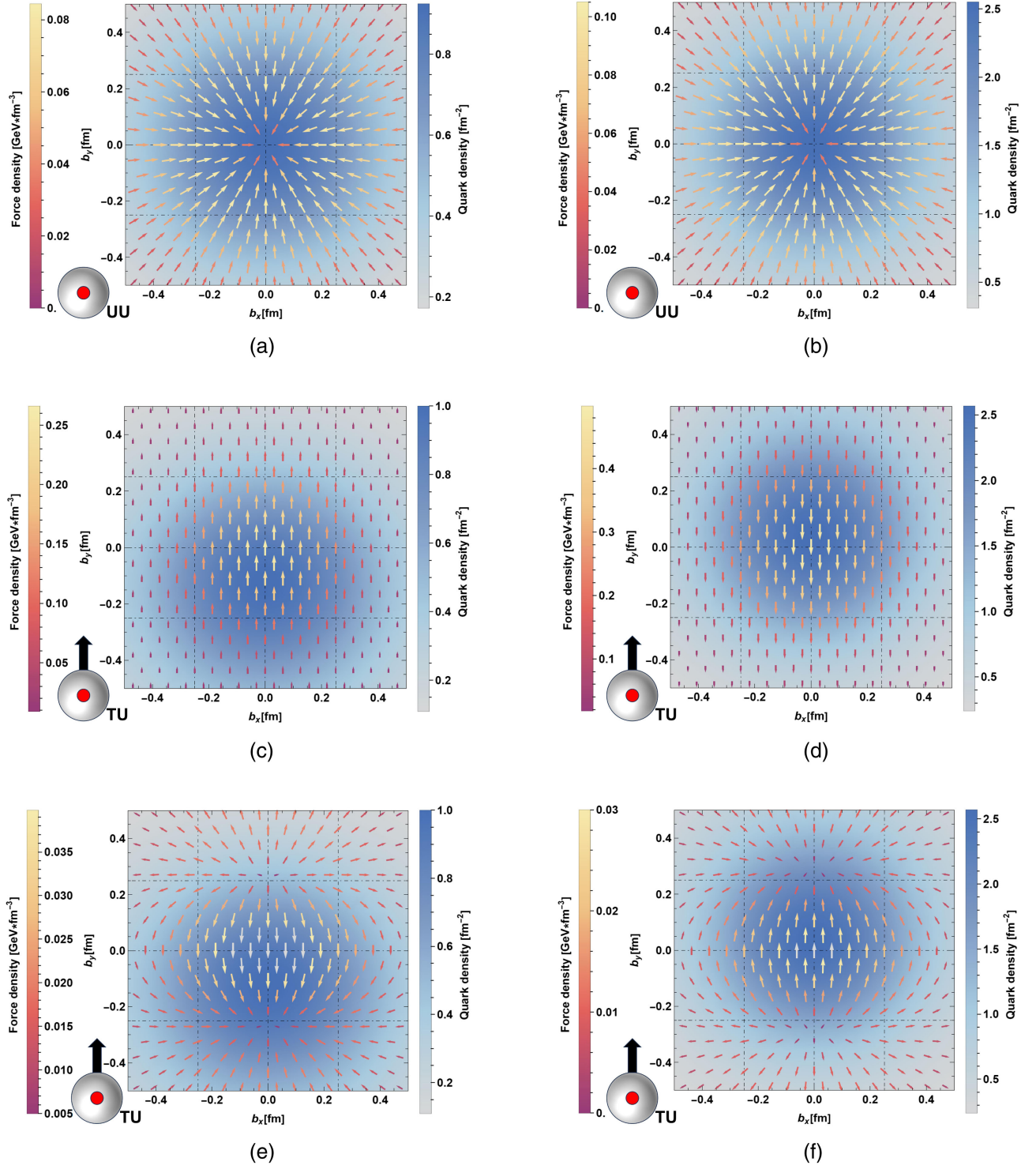


FIG. 2. Plots of the force components $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$ distributions in coordinate space for quarks together with quark density distributions. Left panel for down quark and right panel for up quark. From top to down they are $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$, respectively. The gray sphere at lower left is the proton, and the black arrow marks its polarization direction. In two-letter labels, the first character describes proton polarization and the second quark polarization: T = transverse polarization, U = unpolarized. The red dot corresponds to an unpolarized quark.

poles and also shows flavor dependence. Accordingly, $\mathcal{F}_2$ and $\mathcal{F}_3$ require a proton helicity flip, whereas $\mathcal{F}_1$ is insensitive to nucleon polarization.

Future work will focus on expanding the number of Fock-sectors and increasing the basis truncation parameters ($N_{\max}$ and K) to achieve higher numerical precision. We also plan to extend this framework to other hadron and nuclear systems and to investigate the color Lorentz force acting on transversely polarized quarks [4]. It should be noted, however, that experimental access to this latter observable is expected to be more challenging compared to twist-3 spin-dependent structure functions.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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- [1] Robert L. Jaffe, H. Meyer, and G. Piller, Spin, twist and hadron structure in deep inelastic processes, in *Lectures on QCD: Applications* (Springer, New York, 2007), pp. 178–249.
 - [2] Edward V. Shuryak and A. I. Vainshtein, Theory of power corrections to deep inelastic scattering in quantum chromodynamics. 1: Q^2 effects, *Nucl. Phys.* **B201**, 141 (1982).
 - [3] Fatma P. Aslan, Matthias Burkardt, and Marc Schlegel, Transverse force tomography, *Phys. Rev. D* **100**, 096021 (2019).
 - [4] Matthias Burkardt, Transverse force on quarks in deep-inelastic scattering, *Phys. Rev. D* **88**, 114502 (2013).
 - [5] J. A. Crawford, K. U. Can, R. Horsley, P. E. L. Rakow, G. Schierholz, H. Stüben, R. D. Young, and J. M. Zanotti, Transverse Force Distributions in the Proton from Lattice QCD, *Phys. Rev. Lett.* **134**, 071901 (2025).
 - [6] Wei-Yang Liu, Edward Shuryak, and Ismail Zahed, The color force acting on a quark in the pion and nucleon, 11, 2025.
 - [7] J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, and C. Yang, Hamiltonian light-front field theory in a basis function approach, *Phys. Rev. C* **81**, 035205 (2010).
 - [8] Siqi Xu, Chandan Mondal, Xingbo Zhao, Yang Li, and James P. Vary, Quark and gluon spin and orbital angular momentum in the proton, *Phys. Rev. D* **108**, 094002 (2023).
 - [9] Robert L. Jaffe and Xiangdong Ji, Chiral-odd parton distributions and Drell-Yan processes, *Nucl. Phys.* **B375**, 527 (1992).
 - [10] Robert L. Jaffe and Xiangdong Ji, Studies of the transverse spin-dependent structure function $g_2(x, q^2)$, *Phys. Rev. D* **43**, 724 (1991).
 - [11] Edward V. Shuryak and A. I. Vainshtein, Theory of power corrections to deep inelastic scattering in quantum chromodynamics. 2: Q_4 effects: Polarized target, *Nucl. Phys.* **B201**, 141 (1982).
 - [12] P. L. Anthony, R. G. Arnold, H. R. Band, H. Borel, Peter E. Bosted, Vincent Breton, G. D. Cates, T. E. Chupp, F. S. Dietrich, J. Dunne *et al.*, Deep inelastic scattering of polarized electrons by polarized He 3 and the study of the neutron spin structure, *Phys. Rev. D* **54**, 6620 (1996).
 - [13] K. Abe, T. Akagi, P. L. Anthony, R. Antonov, R. G. Arnold, T. Averett, H. R. Band, J. M. Bauer, H. Borel, Peter E. Bosted *et al.*, Measurements of the proton and deuteron spin structure function g_2 and asymmetry A_2 , *Phys. Rev. Lett.* **76**, 587 (1996).
 - [14] K. Abe, T. Akagi, P. L. Anthony, R. Antonov, R. G. Arnold, T. Averett, H. R. Band, J. M. Bauer, H. Borel, Peter E. Bosted *et al.*, Measurements of the proton and deuteron spin structure functions g_1 and g_2 , *Phys. Rev. D* **58**, 112003 (1998).
 - [15] K. Abe, T. Akagi, B. D. Anderson, P. L. Anthony, R. G. Arnold, T. Averett, H. R. Band, C. M. Berisso, P. Bogorad, H. Borel *et al.*, Measurement of the neutron spin structure function gn_2 and asymmetry An_2 , *Phys. Lett. B* **404**, 377 (1997).
 - [16] R. G. Arnold, P. E. Bosted, P. DePietro, D. Reyna, S. E. Rock, L. Sorrell, Z. Szalata, T. Toole, I. Sick, M. Zeier *et al.*, Measurement of the proton and deuteron spin structure functions g_2 and asymmetry A_2 , *Phys. Lett. B* **458**, 529 (1999).
 - [17] P. L. Anthony, R. G. Arnold, T. Averett, H. R. Band, N. Benmouna, W. Boeglin, H. Borel, P. E. Bosted, Stephen L.

- Bueltmann, G. R. Court *et al.*, Precision measurement of the proton and deuteron spin structure functions g_2 and asymmetries a_2 , *Phys. Lett. B* **553**, 18 (2003).
- [18] A. Airapetian, N. Akopov, Z. Akopov, E. C. Aschenauer, W. Augustyniak, R. Avakian, A. Avetissian, E. Avetisyan, S. Belostotski *et al.* (HERMES Collaboration), Measurement of the virtual-photon asymmetry a_2 and the spin-structure function g_2 of the proton, *Eur. Phys. J. C* **72**, 1921 (2012).
- [19] Nobuo Sato, Wally Melnitchouk, Sebastian E. Kuhn, Jacob J. Ethier, Alberto Accardi, and (Jefferson Lab Angular Momentum Collaboration), Iterative monte carlo analysis of spin-dependent parton distributions, *Phys. Rev. D* **93**, 074005 (2016).
- [20] Xiaochao Zheng, Konrad Aniol, D. S. Armstrong, T. D. Averett, William Bertozzi, Sebastien Binet, Etienne Burtin, Emmanuel Busato, Cornel Butuceanu, John Calarco *et al.*, Precision measurement of the neutron spin asymmetries and spin-dependent structure functions in the valence quark region, *Phys. Rev. C—Nucl. Phys.* **70**, 065207 (2004).
- [21] D. Flay, M. Posik, D. S. Parno, K. Allada, W. R. Armstrong, T. Averett, F. Benmokhtar, W. Bertozzi, A. Camsonne, M. Canan *et al.*, Measurements of d_2 and a_1 : Probing the neutron spin structure, *Phys. Rev. D* **94**, 052003 (2016).
- [22] Frank R. Wesselmann, K. Slifer, S. Tajima, A. Aghalaryan, A. Ahmidouch, R. Asaturyan, F. Bloch, W. Boeglin, P. Bosted, C. Carasco *et al.*, Proton spin structure in the resonance region, *Phys. Rev. Lett.* **98**, 132003 (2007).
- [23] M. K. Ballintijn, T. Cuhadar, G. van Middelkoop, J. Pretz, and E. P. Sichterman, Spin structure of the proton from polarized inclusive deep-inelastic scattering, *Phys. Rev. D* **56**, 5330 (1997).
- [24] W. Armstrong, H. Kang, A. Liyanage, J. Maxwell, J. Mulholland, L. Ndukum, A. Ahmidouch, I. Albayrak, A. Asaturyan, O. Ates *et al.*, Revealing color forces with transverse polarized electron scattering, *Phys. Rev. Lett.* **122**, 022002 (2019).
- [25] S. Bürger, T. Wurm, M. Löffler, M. Göckeler, G. Bali, S. Collins, Andreas Schäfer, and A. Sternbeck (RQCD Collaboration), Lattice results for the longitudinal spin structure and color forces on quarks in a nucleon, *Phys. Rev. D* **105**, 054504 (2022).
- [26] J. A. Crawford, K. U. Can, R. Horsley, P. E. L. Rakow, G. Schierholz, H. Stüben, R. D. Young, and J. M. Zanotti (QCDSF Collaboration), Transverse force distributions in the proton from lattice QCD, *Phys. Rev. Lett.* **134**, 071901 (2025).
- [27] Wei-Yang Liu, Edward Shuryak, and Ismail Zahed, The color force acting on a quark in the pion and nucleon, *Phys. Rev. D* **113**, 114007 (2026).
- [28] R. Abdul Khalek, A. Accardi, J. Adam, D. Adamiak, W. Akers, M. Albaladejo, A. Al-Bataineh, M. G. Alexeev, F. Ameli, P. Antonioli *et al.*, Science requirements and detector concepts for the electron-ion collider: EIC yellow report, *Nucl. Phys. A* **1026**, 122447 (2022).
- [29] Daniele P. Anderle, Valerio Bertone, Xu Cao, Lei Chang, Ningbo Chang, Gu Chen, Xurong Chen, Zhuojun Chen, Zhufang Cui, Lingyun Dai *et al.*, Electron-ion collider in China, *Front. Phys.* **16**, 1 (2021).
- [30] Pengxiang Zhang, Yiping Liu, Siqi Xu, Chandan Mondal, Xingbo Zhao, and James P. Vary, Gluon skewed generalized parton distributions of proton from a light-front Hamiltonian approach, *Phys. Lett. B* **866**, 139584 (2025).
- [31] Yang Li, Pieter Maris, Xingbo Zhao, and James P. Vary, Heavy quarkonium in a holographic basis, *Phys. Lett. B* **758**, 118 (2016).
- [32] James P. Vary, Chandan Mondal, Siqi Xu, Xingbo Zhao, and Yang Li, Nucleon structure from basis light-front quantization: status and prospects, *Eur. Phys. J. Special Topics* **235**, 1557 (2026).
- [33] Siqi Xu, Yiping Liu, Chandan Mondal, Jiangshan Lan, Xingbo Zhao, Yang Li, and James P. Vary, Towards a first principles light-front Hamiltonian for the nucleon, *Phys. Lett. B* **867**, 139599 (2025).
- [34] Yiping Liu, Siqi Xu, Chandan Mondal, Xingbo Zhao, and James P. Vary, Generalized parton distributions of valence, sea, and gluon components of the proton, [arXiv:2604.18355](https://arxiv.org/abs/2604.18355).
- [35] Xingbo Zhao, Heli Honkanen, Pieter Maris, James P. Vary, and Stanley J. Brodsky, Electron g_2 in light-front quantization, *Phys. Lett. B* **737**, 65 (2014).
- [36] Stanley J. Brodsky, Hans-Christian Pauli, and Stephen S. Pinsky, Quantum chromodynamics and other field theories on the light cone, *Phys. Rep.* **301**, 299 (1998).
- [37] Shaoyang Jia and James P. Vary, Basis light front quantization for the charged light mesons with color singlet Nambu–Jona-Lasinio interactions, *Phys. Rev. C* **99**, 035206 (2019).
- [38] Jiangshan Lan, Chandan Mondal, Shaoyang Jia, Xingbo Zhao, and James P. Vary, Parton distribution functions from a light front Hamiltonian and QCD evolution for light mesons, *Phys. Rev. Lett.* **122**, 172001 (2019).
- [39] Lekha Adhikari, Chandan Mondal, Sreeraj Nair, Siqi Xu, Shaoyang Jia, Xingbo Zhao, and James P. Vary, Generalized parton distributions and spin structures of light mesons from a light-front Hamiltonian approach, *Phys. Rev. D* **104**, 114019 (2021).
- [40] Yang Li, Pieter Maris, and James P. Vary, Quarkonium as a relativistic bound state on the light front, *Phys. Rev. D* **96**, 016022 (2017).
- [41] Hongyao Yu, Zhi Hu, Siqi Xu, Chandan Mondal, Xingbo Zhao, and James P. Vary, Transverse-momentum-dependent gluon distributions of proton within basis light-front quantization, *Phys. Lett. B* **855**, 138831 (2024).
- [42] Zhimin Zhu, Siqi Xu, Jiatong Wu, Hongyao Yu, Zhi Hu, Jiangshan Lan, Chandan Mondal, Xingbo Zhao, and James P. Vary, Transverse structure of the proton beyond leading twist: A light-front Hamiltonian approach, *Phys. Lett. B* **855**, 138829 (2024).
- [43] Bolang Lin, Sreeraj Nair, Chandan Mondal, Siqi Xu, Zhi Hu, Pengxiang Zhang, Xingbo Zhao, and James P. Vary, Chiral-odd gluon generalized parton distributions in the proton: A light-front quantization approach, *Phys. Lett. B* **860**, 139153 (2025).
- [44] Bolang Lin, Sreeraj Nair, Siqi Xu, Zhi Hu, Chandan Mondal, Xingbo Zhao, and James P. Vary, Generalized parton distributions of gluon in proton: A light-front quantization approach, *Phys. Lett. B* **847**, 138305 (2023).
- [45] Ziqi Zhang, Zhi Hu, Siqi Xu, Chandan Mondal, Xingbo Zhao, James P. Vary, and (BLFQ Collaboration), Twist-3

- generalized parton distribution for the proton from basis light-front quantization, *Phys. Rev. D* **109**, 034031 (2024).
- [46] Sreeraj Nair, Chandan Mondal, Siqi Xu, Xingbo Zhao, and James P. Vary, Proton gravitational structure and mass decomposition on the light front, *Phys. Rev. D* **112**, 114001 (2025).
- [47] Zhimin Zhu, Jiangshan Lan, Chandan Mondal, Xingbo Zhao, and James P. Vary, Unified description of Sivers and Boer-Mulders asymmetries from twist-3 correlations, *arXiv:2603.23138*.
- [48] Matthias Burkardt, Impact parameter dependent parton distributions and transverse single spin asymmetries, *Phys. Rev. D* **66**, 114005 (2002).
- [49] B. W. Filippone and Xiang-Dong Ji, The spin structure of the nucleon, *Adv. Nucl. Phys.* **26**, 1 (2001).
- [50] S. Wandzura and Frank Wilczek, Sum rules for spin-dependent electroproduction-test of relativistic constituent quarks, *Phys. Lett.* **72B**, 195 (1977).
- [51] Dennis Sivers, Hard-scattering scaling laws for single-spin production asymmetries, *Phys. Rev. D* **43**, 261 (1991).
- [52] Simone Rodini, Lorenzo Rossi, and Alexey Vladimirov, Numerical implementation of evolution equations for twist-3 collinear Pdfs, *Eur. Phys. J. C* **84**, 732 (2024).
- [53] Sreeraj Nair, Chandan Mondal, Siqi Xu, Xingbo Zhao, Asmita Mukherjee, and James P. Vary, Gravitational form factors and mechanical properties of quarks in protons: A basis light-front quantization approach, *Phys. Rev. D* **110**, 056027 (2024).
- [54] Dipankar Chakrabarti, Chandan Mondal, Asmita Mukherjee, Sreeraj Nair, and Xingbo Zhao, Gravitational form factors and mechanical properties of proton in a light-front quark-diquark model, *Phys. Rev. D* **102**, 113011 (2020).
- [55] M. Göckeler, R. Horsley, D. Pleiter, Paul EL Rakow, A. Schäfer, G. Schierholz, H. Stüben, and J.M. Zanotti, Investigation of the second moment of the nucleon's g_1 and g_2 structure functions in two-flavor lattice QCD, *Phys. Rev. D* **72**, 054507 (2005).
- [56] X. Song, Polarized structure function g_2 in the cm bag model, *Phys. Rev. D* **54**, 1955 (1996).
- [57] Alexey Vladimirov, Guillermo Portela, and Simone Rodini, Determination of quark-gluon-quark interference within the proton, *Phys. Rev. Lett.* **137**, 061902 (2026).
- [58] Matthias Burkardt, Impact parameter space interpretation for generalized parton distributions, *Int. J. Mod. Phys. A* **18**, 173 (2003).
- [59] B. Pasquini and S. Boffi, Nucleon spin densities in a light-front constituent quark model, *Phys. Lett. B* **653**, 23 (2007).
- [60] Tanmay Maji, Chandan Mondal, and Dipankar Chakrabarti, Leading twist generalized parton distributions and spin densities in a proton, *Phys. Rev. D* **96**, 013006 (2017).
- [61] Chandan Mondal and Dipankar Chakrabarti, Generalized parton distributions and transverse densities in a light-front quark-diquark model for the nucleons, *Eur. Phys. J. C* **75**, 261 (2015).