

Dynamics of Z_N domain walls in $SU(N)$ gauge theories

Sayanjit Banerjee^{*,†}, Sanatan Digal[‡], and Sumit Shaw[‡]

*Institute of Mathematical Sciences, Chennai 600113, India Homi Bhabha National Institute,
Training School Complex, Anushaktinagar, Mumbai 400094, India*



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We study collisions of domain walls in $SU(N)$ gauge theories using the Polyakov-loop effective potential models. We find that string junctions play a crucial role in the dynamics of Z_N domain walls. In $SU(3)$ gauge theory, the merger of two nonplanar Z_3 domain walls into a single wall proceeds via the creation of a vortex-antivortex pair in $2 + 1$ dimensions. In $SU(4)$ gauge theory, low-energy collisions of Z_4 walls result in the formation of a single domain wall without the creation of vortices. At higher collision energies, two Z_4 domain walls can either bounce back or scatter into another pair of domain walls through the formation of vortices. The creation of vortex-antivortex pairs generalizes to the creation of string loops in $3 + 1$ dimensions for both gauge theories. These results demonstrate a direct dynamical role for topological strings in the evolution of center-domain-wall networks and reveal a new aspect of defect dynamics in non-Abelian gauge theories.

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I. INTRODUCTION

Topological defects play a central role across a wide range of physical systems, from condensed-matter systems to high-energy quantum field theories and cosmological models of the early Universe. They often arise during phase transitions involving spontaneous symmetry breaking, when the vacuum manifold becomes degenerate [1]. In such situations, nontrivial field configurations interpolating between distinct vacua can form, giving rise to defects such as domain walls, strings, and monopoles [2,3]. The types of defects that can occur are determined by the topology of the vacuum manifold and the dimensionality of space [4]. Understanding the properties and dynamics of these defects is often essential for understanding the behavior of the system in its symmetry-broken phase, including phase ordering kinetics, defect-network evolution, and nonequilibrium dynamics [5,6].

Finite-temperature pure $SU(N)$ gauge theories provide an important example of a system with a rich topological structure. These theories undergo a confinement-deconfinement (CD) transition at high temperatures, with the Polyakov loop serving as an order parameter for the

transition [7–9]. Above a critical temperature T_c , the Polyakov loop acquires a nonzero expectation value, and the global center symmetry, Z_N , is spontaneously broken, leading to N degenerate deconfined vacua related by center transformations [8,10]. Consequently, the deconfined phase supports domain walls separating distinct deconfined vacua [11].

Recently, we demonstrated that the deconfined phase of $SU(N)$ gauge theories also supports topological string defects [12]. These strings arise as junctions of center domain walls and are characterized by nontrivial winding of the Polyakov-loop order parameter around their cores [11–13]. Similar wall-junction configurations have previously appeared in supersymmetric gauge theories [14,15]. The existence of such defects enriches the topological structure of the deconfined phase and suggests a close connection between string and domain-wall dynamics. In particular, since domain walls can terminate on or merge through string junctions, these defects provide a natural mechanism through which domain walls can interact and reorganize their topology.

The static properties of the domain walls in $SU(N)$ gauge theories have been studied extensively using perturbative analyses, effective descriptions, and lattice simulations [11,13,16–19]. Domain-wall tensions, interface profiles and scaling properties are now reasonably well understood [11,19–22]. In particular, studies of interface tensions suggest attractive interactions between elementary walls and wetting phenomena near the deconfinement transition [23–27]. Together, these investigations have yielded important insights into the nonperturbative structure of finite-temperature gauge theories.

*Contact author: sayanjitb@imsc.res.in

†Contact author: digal@imsc.res.in

‡Contact author: sumitshaw@imsc.res.in

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Despite the considerable progress in understanding the static properties of domain walls and the recent identification of topological string defects [11–13,19], much less is known about their interaction dynamics. Although existing studies suggest attractive interactions between elementary domain walls [23,24,26,27], the free-energy landscape governing their approach has not been investigated in detail. To our knowledge, there are no quantitative studies of how the free energy depends on the separation between domain walls, nor of the processes that occur when domain walls collide. Since collisions can naturally lead to the formation of string junctions and other composite topological configurations [12,14,15], understanding these processes is essential for developing a complete picture of defect dynamics in the deconfined phase.

Recently, domain walls arising from the spontaneous breaking of discrete Z_N symmetries beyond the simplest Z_2 case have also been investigated. In particular, the properties of Z_3 domain walls, including their scalar profiles, tensions, thicknesses, collapse dynamics and the resulting gravitational-wave signatures, have been studied in Ref. [28]. A general classification of Abelian Z_N domain walls, including adjacent and nonadjacent walls as well as walls bounded by strings, has subsequently been presented in Ref. [29]. These works illustrate some of the rich structures that can arise in theories with spontaneously broken discrete symmetries.

In this work, we investigate interactions between center domain walls in effective Polyakov-loop theories describing the deconfined phase of $SU(N)$ gauge theories. In particular, we focus on the $SU(3)$ and $SU(4)$ cases. Using real-time numerical simulations in $2+1$ dimensions in the Polyakov-loop effective potential models, we uncover a common and previously unexplored mechanism governing domain-wall interactions. Although our simulations are performed in two spatial dimensions, the mechanism extends naturally to $3+1$ dimensions, and we discuss the corresponding interaction processes in $3+1$ dimensions.

Our results show that, in $SU(3)$, the collision of two domain walls dynamically generates a localized vortex-antivortex pair. As the vortex and antivortex separate, a third domain wall forms between them. A straightforward generalization to three spatial dimensions suggests that, instead of a vortex-antivortex pair, a string loop is created, forming the boundary of the third domain. Thus, the collision proceeds through the creation of the same topological string defect previously identified in lattice simulations of the full gauge theory [12]. For low collision energies, the domain walls merge to form a single domain wall through the vortex-antivortex-mediated process described above. For a different choice of parameters of the Polyakov-loop effective potential and sufficiently high collision energies, the wall formed in the collision subsequently splits back into the original pair through the

creation of vortices in two dimensions and string loops in three dimensions.

The dynamics in $SU(4)$ share some qualitative features with $SU(3)$ but also display important differences. For low collision energies, two domain walls interpolating between nearest-neighbor vacua merge to form a single domain wall connecting next-to-nearest-neighbor vacua in the vacuum manifold. In this process, no vortices or string loops are created. At higher collision energies, however, the walls scatter into a different pair of nearest-neighbor domain walls. This process is mediated by the creation of vortices in two dimensions and string loops in three dimensions. These results indicate that string-mediated wall recombination and scattering are generic features of deconfined $SU(N)$ gauge theories.

The paper is organized as follows. In Sec. II, we review the Polyakov-loop effective potential model that describes the CD transition and spontaneous breaking of Z_N in $SU(N)$ gauge theories. In Sec. III, we discuss topological defects associated with the spontaneous breaking of Z_N symmetry; in particular, we focus on $SU(3)$ and $SU(4)$ gauge theories and obtain their spatial profiles using numerical simulations. Section IV presents our results for the interaction of domain walls via strings for both $SU(3)$ and $SU(4)$. Finally, Sec. V contains our conclusions and outlook.

II. EFFECTIVE POLYAKOV-LOOP MODEL FOR $SU(N)$ GAUGE THEORIES

A. Center symmetry and the Polyakov loop

In pure $SU(N)$ gauge theories, the CD phase transition is associated with the realization of the global center symmetry Z_N . The Polyakov loop serves as an order parameter for this transition [7]. It is defined as,

$$L(\mathbf{x}) = \frac{1}{N} \text{Tr} \mathcal{P} \exp \left(ig \int_0^{1/T} A_0(\mathbf{x}, \tau) d\tau \right), \quad (1)$$

where T denotes the temperature, A_0 is the temporal gauge field, and $\mathcal{P}$ represents path ordering in Euclidean time.

Under a gauge transformation that is periodic only up to an element of the center, Z_N , of $SU(N)$ [7,10], i.e.,

$$\begin{aligned} A_\mu(x) &\longrightarrow UA_\mu(x)U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}, \\ U(\vec{x}, \tau = 0) &= z^{-1}U(\vec{x}, \tau = \beta), \\ z &= e^{2\pi i k/N} \in Z_N, \quad \text{and} \quad k = 0, \dots, N-1, \end{aligned}$$

the Polyakov loop transforms as

$$L(\mathbf{x}) \rightarrow zL(\mathbf{x}). \quad (2)$$

In the confined phase,

$$\langle L \rangle = 0. \quad (3)$$

Thus, the center symmetry remains unbroken [7,10]. Above the critical temperature T_c , the Polyakov loop acquires a nonvanishing expectation value,

$$\langle L \rangle \neq 0. \quad (4)$$

Consequently, the expectation value of the Polyakov loop transforms nontrivially under the center symmetry, signaling spontaneous breaking of the Z_N symmetry [8,9]. The deconfined phase therefore contains N degenerate vacua related by center transformations [8–10]. The emergence of these degenerate vacua plays a central role in the formation of topological defects in the deconfined phase.

B. The Polyakov-loop effective potential

Near the confinement-deconfinement transition, the long-distance dynamics of the $SU(N)$ theory can be described in terms of the Polyakov-loop effective potential for the complex order parameter L , which is similar to Landau-Ginzburg models [8,30,31]. The effective potential is constrained by the underlying Z_N center symmetry. Polyakov-loop effective theories have also been extended to include dynamical quarks and have found widespread application in studies of the quantum chromodynamics (QCD) phase structure and dynamics [32,33].

For $SU(3)$, the effective potential consistent with Z_3 symmetry is

$$V(L) = \frac{b_2}{2} |L|^2 - \frac{b_3}{3} (L^3 + L^{*3}) + \frac{b_4}{4} |L|^4, \quad (5)$$

$$L = L_R + iL_I,$$

where L_R and L_I denote the real and imaginary components of the Polyakov-loop field, respectively. Choosing b_2 to be temperature dependent, while treating b_3 and b_4 as constants, yields a good description of the thermodynamics of pure $SU(3)$ gauge theory as determined from nonperturbative lattice simulations [8,30,31]. The cubic term, which is allowed by the Z_3 symmetry, is also responsible for the first-order nature of the confinement-deconfinement transition in pure $SU(3)$ gauge theory. For $T > T_c$, in the deconfined phase, the potential develops three degenerate minima related by Z_3 transformations.

The above form can be generalized to higher-rank gauge groups as

$$V(L) = a_N |L|^2 + c_N |L|^4 + d_N (L^N + L^{*N}) + \dots \quad (6)$$

for $SU(N)$. Note that higher-order Z_N -invariant operators in Eq. (6), may be required to ensure stability of the potential and to reproduce the correct thermodynamic properties of the underlying gauge theory. In the deconfined phase, the

minima of the effective potential correspond to the distinct center-related vacua.

In the deconfined phase, typical forms of the effective potential for $SU(3)$ and $SU(4)$ are shown in Figs. 1 and 2, respectively. In both cases, the center symmetry is spontaneously broken. In the deconfined phase of $SU(3)$ gauge theory, spontaneous breaking of the Z_3 symmetry leads to three degenerate vacua,

$$L = \eta(1, e^{2\pi i/3}, e^{4\pi i/3}) \equiv (L_1, L_2, L_3), \quad (7)$$

where $\eta = |\langle L \rangle|$ denotes the magnitude of the Polyakov-loop expectation value. In the deconfined phase of $SU(4)$ gauge theory, spontaneous breaking of the Z_4 symmetry leads to four degenerate vacua,

$$L = \eta(1, e^{i\pi/2}, e^{i\pi}, e^{3i\pi/2}) \equiv (L_1, L_2, L_3, L_4), \quad (8)$$

where $\eta = |\langle L \rangle|$ is the magnitude of the Polyakov-loop.

III. TOPOLOGICAL DEFECTS ASSOCIATED WITH BROKEN Z_N SYMMETRY

To study the long-distance properties and spacetime evolution of topological defects associated with the spontaneously broken center symmetry, we employ a Landau-Ginzburg effective action for the Polyakov-loop field,

$$S_{\text{eff}} = \int d^4x [K \partial_\mu L^* \partial^\mu L - V(L)], \quad (9)$$

where K is a temperature-dependent coefficient [30,31,34]. The kinetic term governs the spacetime variations of the Polyakov-loop field, while the effective potential determines the vacuum structure associated with the center symmetry.

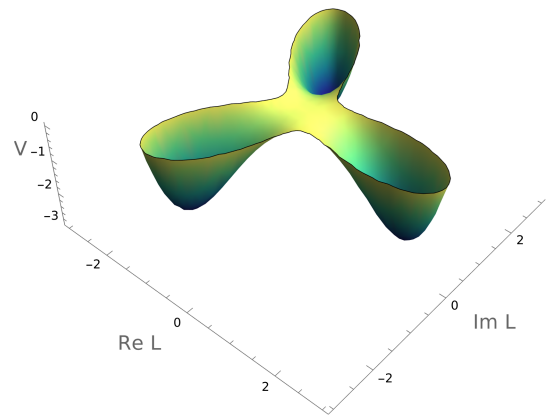


FIG. 1. Polyakov-loop effective potential $V(L)$ in the complex- L plane exhibiting spontaneous breaking of Z_3 symmetry with three degenerate minima.

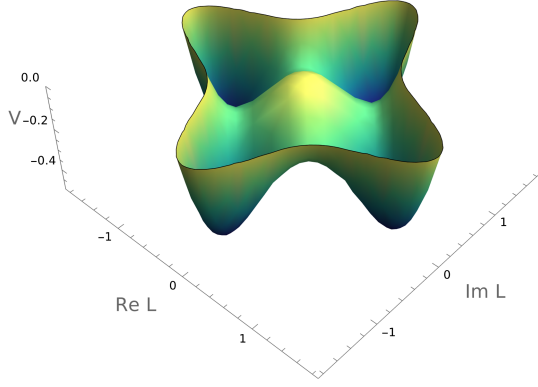


FIG. 2. Polyakov-loop effective potential $V(L)$ in the complex- L plane exhibiting spontaneous breaking of Z_4 symmetry with four degenerate minima.

A. Domain walls and strings in $SU(3)$ gauge theory

In the spontaneously broken Z_3 phase, shown in Fig. 1, there are three degenerate vacua,

$$L_1 = \eta, \quad L_2 = \eta e^{2\pi i/3}, \quad L_3 = \eta e^{4\pi i/3}.$$

Consequently, there are three domain-wall solutions interpolating between pairs of vacua [11]. We denote by W_{ij} the domain wall connecting the vacua L_i and L_j . Across each wall, the phase of the Polyakov loop changes by $2\pi/3$.

A Z_3 string is formed when all three domain walls meet along a common line [12]. As one traverses a closed loop encircling the string, the phase of the Polyakov loop winds by 2π , passing successively through the three Z_3 vacua. Continuity of the field then requires the Polyakov loop to pass through the confining value $L = 0$ at the center of the configuration. Consequently, the string core corresponds to a locally restored confining phase and forms the junction of the three domain walls. Such configurations are topologically stable within the effective Polyakov-loop theory, since the nontrivial winding of the order parameter cannot be continuously unwound without forcing the field through the confining point $L = 0$.

To obtain the domain-wall and string configurations, we numerically solve the equations of motion for the Polyakov-loop field derived from the effective action, Eq. (9), using the parameters $b_2 = -0.27$, $b_3 = 2.0$, $b_4 = 1.0$, and $K = 1.649$ [30,31,34]. The resulting domain walls and strings correspond to static solutions of the field equations. However, instead of solving the static field equations, for simplicity we evolve a suitably chosen initial configuration using time-dependent field equations including a small dissipative term [35]. The field equations with dissipation coefficients are as follows,

$$2K \frac{\partial^2 \phi_i}{\partial t^2} + \xi \frac{\partial \phi_i}{\partial t} - \nabla^2 \phi_i = -\frac{\partial V}{\partial \phi_i}, \quad i = 1, 2,$$

$$\text{where, } \nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}, \quad (10)$$

and ξ is a damping coefficient. The dissipative term removes excess energy from the system while preserving the underlying vacuum structure. As the evolution proceeds, the field velocities and accelerations approach zero, and the configuration relaxes to a stationary defect solution corresponding to a local minimum of the effective action. This relaxation procedure provides an efficient method for obtaining domain-wall and string configurations. The same field equations can also be used to investigate the subsequent dynamical evolution and interactions of these defects.

For the W_{12} domain-wall solution, we impose the boundary conditions

$$\lim_{x \rightarrow -R/2} L = L_1, \quad \lim_{x \rightarrow +R/2} L = L_2, \quad (11)$$

where R denotes the spatial extent of the lattice and is chosen to be much larger than the expected wall thickness. Twisted boundary conditions are imposed along the spatial direction, which solves the problems that arise from boundary effects [26]. Starting from an interpolating field configuration, the system is evolved using Eq. (10) until the residual oscillations in the total energy become negligible. More precisely, the evolution is terminated when the variation in a single time step falls below 10^{-8} , which is the numerical precision of the code. The real and imaginary parts of the Polyakov loop profile of the resulting W_{12} domain-wall is shown in Fig. 3, interpolating between L_1 and L_2 vacua.

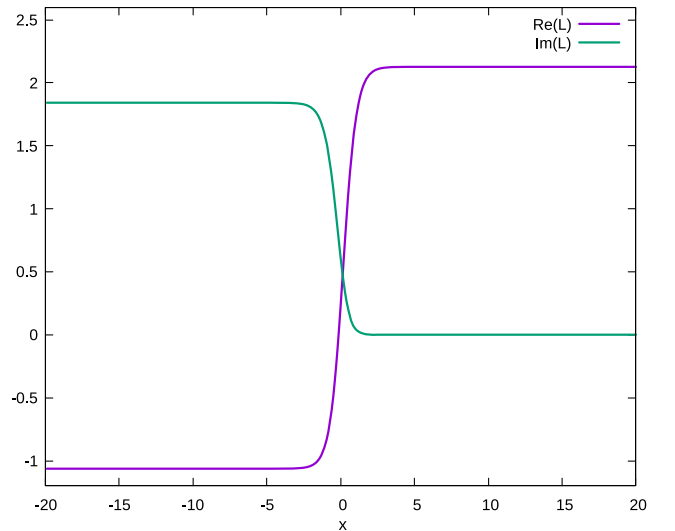


FIG. 3. Real and imaginary components of the stationary W_{12} domain-wall profile in the broken Z_3 phase. The Polyakov-loop field interpolates from L_1 to L_2 , following dissipative relaxation under Eq. (10) with Z_3 potential.

Since the minimum-energy configuration of an isolated string is translationally invariant along its length, it is sufficient to determine the field configuration in a plane perpendicular to the string. The resulting field configuration is effectively that of a vortex (or antivortex) in two spatial dimensions. The initial configuration is therefore chosen to possess the appropriate winding structure around the center of the lattice. We choose an appropriate configuration at the boundary of the lattice that is consistent with a string configuration at the center. Upon relaxation according to Eq. (10), the configuration evolves into a stable Z_3 string connected to three domain walls. The resulting string solution is shown in Fig. 4. Here, the Z_3 string is formed at the junction of three domain walls associated with the three degenerate Z_3 vacua.

B. Domain walls and strings in $SU(4)$ gauge theory

The spontaneously broken Z_4 phase contains four degenerate vacua,

$$L_1 = \eta, \quad L_2 = \eta e^{i\pi/2}, \quad L_3 = \eta e^{i\pi}, \quad L_4 = \eta e^{i3\pi/2},$$

related by center transformations. The corresponding vacuum structure supports both domain walls and string-like defects [11,13]. The construction of these defects is analogous to the $SU(3)$ case. A nontrivial winding of the Polyakov loop around a closed contour forces the field to pass through the confining value $L = 0$ at the defect core, giving rise to a topologically stable Z_4 string.

The four vacua give rise to six distinct domain-wall solutions [29]. We denote by W_{ij} the wall interpolating between the vacua L_i and L_j . Four of these walls connect adjacent vacua in the complex- L plane,

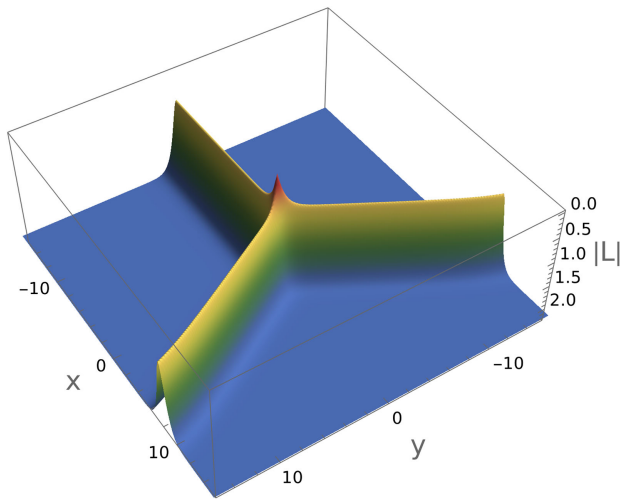


FIG. 4. Spatial profile of the Z_3 string configuration formed at the junction of three domain walls. The string core corresponds to the confining phase where $L \simeq 0$.

$$\begin{aligned} W_{12} &\equiv (L_1, L_2), & W_{23} &\equiv (L_2, L_3), \\ W_{34} &\equiv (L_3, L_4), & W_{14} &\equiv (L_4, L_1), \end{aligned}$$

while the remaining two connect antipodal vacua,

$$W_{13} \equiv (L_1, L_3), \quad W_{24} \equiv (L_2, L_4).$$

Unlike the $SU(3)$ case, the presence of both adjacent-vacuum and antipodal walls introduces multiple channels for wall recombination and scattering. As we will show in Sec. IV, this richer defect spectrum leads to qualitatively new collision dynamics.

Stationary wall and string configurations are obtained using the same relaxation procedure described for the $SU(3)$ theory. For this purpose, we use the parameter values $a_4 = -0.25$, $c_4 = 0.4$, and $d_4 = -0.1$ in the effective potential, Eq. (6). The coefficient K in the effective action, Eq. (9), is taken to be $K = 1$. The Polyakov-loop field is evolved according to Eq. (10), with a small dissipative term, until a stationary configuration corresponding to a local minimum of the effective action is reached.

To obtain the domain-wall solutions, we impose boundary conditions corresponding to the desired pair of vacua. As an example, the W_{12} wall connecting vacua L_1 and L_2 is obtained by imposing

$$\lim_{x \rightarrow -R/2} L = L_1, \quad \lim_{x \rightarrow +R/2} L = L_2, \quad (12)$$

where R denotes the spatial extent of the lattice and is chosen to be much larger than the expected wall thickness. Twisted boundary conditions are imposed along the spatial

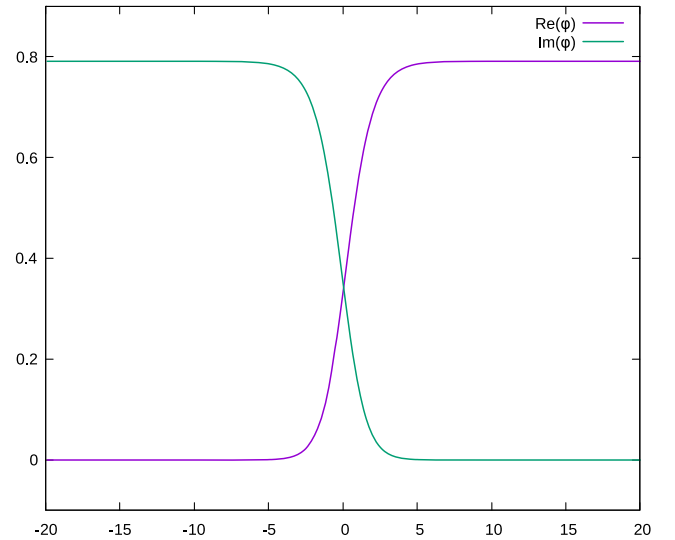


FIG. 5. Real and imaginary components of the stationary W_{12} domain-wall profile in the broken Z_4 phase. The Polyakov-loop field interpolates between adjacent vacua L_1 and L_2 , following dissipative relaxation under Eq. (10) with Z_4 potential.

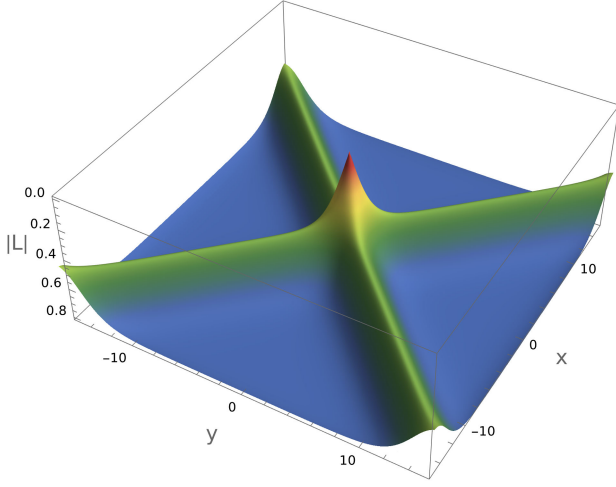


FIG. 6. Spatial profile of the Z_4 string configuration formed at the junction of four domain walls. The string core corresponds to the confining phase where $L \simeq 0$.

direction. Starting from an interpolating field configuration, the system is evolved according to Eq. (10), until the residual oscillations in the total energy become negligible. The evolution is terminated when the amplitude of the energy oscillations falls below 10^{-8} , as previously mentioned. The real and imaginary part of the Polyakov loop, which interpolates between L_1 and L_2 vacua of the Z_4 broken potential, is shown in Fig. 5.

The construction of the Z_4 string proceeds analogously to the SU(3) case. An initial field configuration with the appropriate winding is evolved according to Eq. (10), until it relaxes to a stationary solution. The resulting configuration consists of a string core from which four domain walls emerge with approximate angular separations of 90° . No spontaneous rearrangement into a configuration involving antipodal walls is observed. Thus, the fundamental Z_4 string naturally appears as a junction of four adjacent-vacuum walls. The Fig. 6 shows the Z_4 string formed at the junction of four domain walls separating the four degenerate Z_4 vacua.

IV. INTERACTION OF DOMAIN WALLS VIA STRINGS

A. Interactions of domain walls in SU(3) gauge theory

We now investigate collisions of domain walls in the effective Polyakov-loop model for SU(3) gauge theory. We begin with planar collisions between a W_{12} wall and a W_{23} wall in $(2+1)$ dimensions. Depending on the collision energy and the parameters of the effective potential, the collision can lead either to the formation of a W_{13} wall or to a subsequent reseparation of the original walls.

For the collision studies presented below, we use the same parameter values as those used in Sec. III(A). The initial configuration consists of a W_{12} wall and a W_{23} wall

approaching one another. For low collision energies, the Polyakov-loop field in the interaction region evolves away from the intermediate vacuum L_2 toward the confining point $L = 0$. It subsequently approaches the field value corresponding to the core of a W_{13} wall, resulting in the merger of the two incoming walls into a single W_{13} wall. Representative snapshots before and after the collision are shown in Fig. 7. The solid curve shows the magnitude profile of the Polyakov loop for the two domain walls along the x axis. The dashed curve represents the same for the configuration after the collision.

The merger of W_{12} and W_{23} releases the binding as well as initial kinetic energy into both radial and phase excitations of the Polyakov-loop field, which propagate away from the collision region. The radial modes remain localized in the collision region, as seen in Fig. 7, and as the collision energy is increased, the amplitudes of these excitations become larger and cover a larger area. Immediately after the collision, the field remains close to $L = 0$ for some time. The field in the region does not flip back to the L_2 vacuum.

More interesting phenomena arise in collisions of non-planar domain walls. In this case, different segments of the walls collide at different times. When local segments of the W_{12} and W_{23} walls merge, a corresponding segment of the W_{13} wall is formed. The endpoints of the newly created W_{13} segment are topologically nontrivial and correspond to a vortex-antivortex pair. The spatial profile of the Polyakov loop magnitude in the xy -plane is shown in Fig. 8, localized zeros of $|L|$ correspond to the vortex-antivortex winding, which can be understood from the vector plot of the real and imaginary components of the Polyakov loop of the

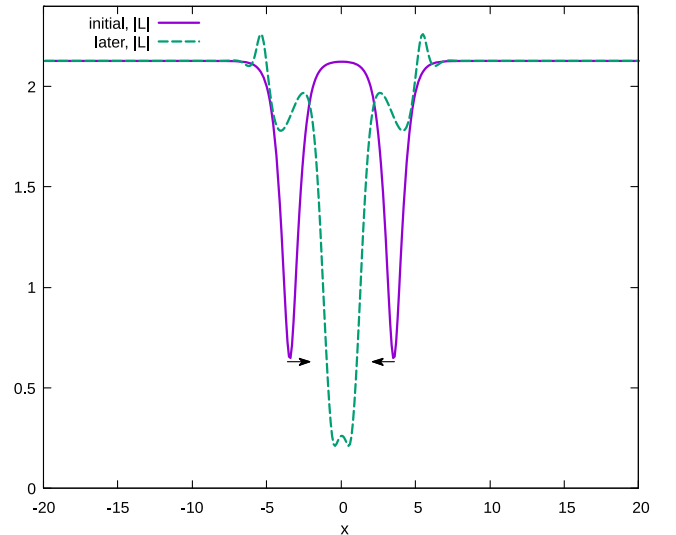


FIG. 7. Initial (solid line) and late-time (dashed) profiles of the Polyakov-loop magnitude during a low-energy planar collision of Z_3 walls (W_{12} and W_{23}) in $(2+1)$ dimensions. The two incoming walls merge to form a single W_{13} wall; the arrows indicate the directions of motion of the incoming walls.

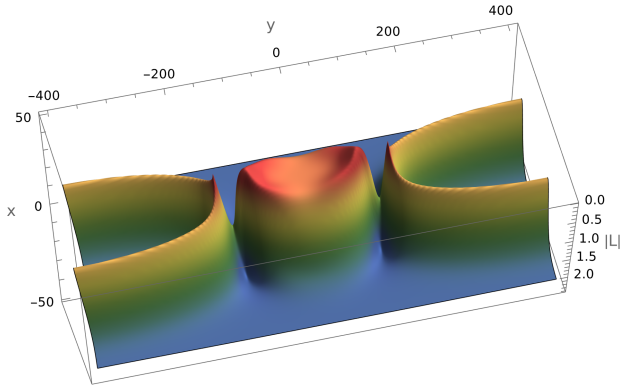


FIG. 8. Collision of nonplanar W_{12} and W_{23} domain walls resulting in formation of a vortex-antivortex pair and large fluctuations of the Polyakov loop in the W_{13} wall at zero damping. The vortices appear at the endpoints of the newly formed W_{13} segment.

same spatial magnitude profile. Thus, the conversion of W_{12} and W_{23} walls into a W_{13} wall proceeds through the dynamical creation of topological defects. As the collision proceeds, the vortex and antivortex move apart along the collision front while additional portions of the incoming walls merge and are converted into the W_{13} wall. The oscillations observed in planar collisions are also visible in the nonplanar collisions along the newly formed wall segment in Fig. 8. With a time-dependent damping, the oscillatory excitations are suppressed, and the topological structure becomes manifest, as seen in Fig. 9. The vortex and antivortex appear as junctions of three domain walls, corresponding to the two-dimensional analogue of the Z_3 string discussed in Sec. III.

At higher collision energies, the radial and phase excitations of the Polyakov loop become more prominent and cover a larger area. Though additional vortices form in these regions, they decay or annihilate before separating

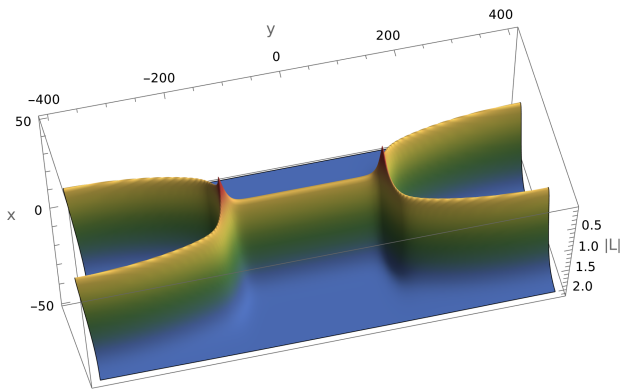


FIG. 9. Formation of a vortex-antivortex pair in the presence of time-dependent damping, which suppresses fluctuations. The spatial profile of $|L(x, y)|$ shows that the formation of a vortex-antivortex pair appears at the endpoints of the newly formed wall segment.

into fully formed vortices. For some choice of potential parameters, i.e., $b_2 = -0.87$, additional vortices do form; however, this value of b_2 is not consistent with pure $SU(3)$ gauge theory dynamics.

The vortex-antivortex pair creation observed in $(2+1)$ dimensions admits a natural interpretation in $(3+1)$ dimensions. In the higher-dimensional theory, vortices are replaced by Z_3 strings and antivortices by anti-strings. The transient splitting of the W_{13} wall therefore corresponds to the nucleation of a closed string loop on the wall, followed by its subsequent contraction and annihilation. These processes reveal the direct dynamical role played by topological strings in domain-wall interactions.

1. Free-energy landscape in deconfined $SU(3)$

While the collision dynamics discussed above reveal the possible outcomes of domain-wall interactions, it is also useful to examine the free-energy landscape of a two-wall system. Although the separation between two walls becomes difficult to define once their profiles begin to overlap, the value of the Polyakov loop at the midpoint between the walls, denoted by L_{mid} , evolves monotonically during the collision process. This suggests that L_{mid} may serve as a convenient parameter for characterizing overlapping two-wall configurations.

To determine the free energy (F_{2W}) as a function of L_{mid} , we consider a configuration consisting of a W_{12} wall connecting the vacua L_1 and L_2 and a W_{23} wall connecting the vacua L_2 and L_3 . We fix the value of L_{mid} and evolve the field configuration using Eq. (10), for the effective action (9), supplemented by a damping term to remove excess energy. Repeating this procedure for different values of L_{mid} along a straight-line path in the complex- L plane connecting L_2 to the core value, $L_{\text{core}} \simeq -0.3L_2$, of the W_{13} wall yields the normalized free-energy profile shown in Fig. 10. In the figure, instead of plotting F_{2W} , we plot the ratio, $F_{2W}/2F_W$, where F_W is the free energy of a single wall. α represents coordinate along a line connecting L_2 and the origin in the complex L -plane. $\alpha = 1, 0$, and -0.3 corresponds to $L_{\text{mid}} = L_2, (0,0)$, and L_{core} respectively. The monotonic decrease of F_{2W} in α , indicates an attractive interaction between the W_{12} and W_{23} walls.

B. Interactions of domain walls in $SU(4)$ gauge theory

In the following, we consider domain-wall dynamics in the effective Polyakov-loop model for $SU(4)$ gauge theory. For this purpose, we use the same parameters in the effective action, Eq. (9), as in Sec. III(B). The presence of both adjacent-vacuum walls ($W_{12}, W_{23}, W_{34}, W_{14}$) and antipodal walls (W_{13}, W_{24}) gives rise to collision channels that are absent in $SU(3)$. We observe three qualitatively distinct outcomes for collisions between W_{12} and W_{23} walls, depending on the collision energy: (i) fusion into a W_{13} wall, (ii) temporary fusion followed by bounce-back into the original wall pair, and (iii) transmutation into a

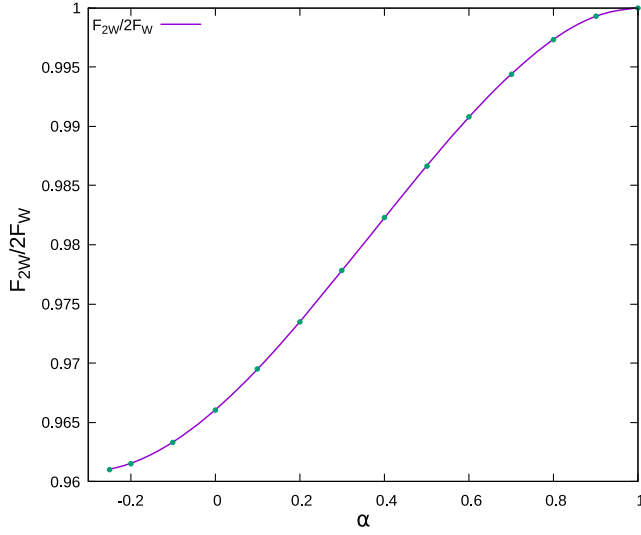


FIG. 10. Normalized free energy, $F_{2W}/(2F_W)$, of a constrained $W_{12} + W_{23}$ configuration in the Z_3 theory as a function of the midpoint coordinate α . Here $\alpha = 1, 0$, and -0.3 correspond, respectively, to $L_{\text{mid}} = L_2$, $L_{\text{mid}} = 0$, and the core value $L_{\text{mid}} \simeq -0.3L_2$ of the resulting W_{13} wall. The decrease of the free energy as the walls overlap demonstrates an attractive interaction.

different wall pair, $W_{14} + W_{43}$. A notable difference from the $SU(3)$ theory is that the low-energy fusion process does not require vortex-antivortex pair creation, even for non-planar collisions.

For sufficiently low collision energies, the collision of the planar W_{12} and W_{23} walls leads to the formation of a single W_{13} wall,

$$W_{12} + W_{23} \longrightarrow W_{13}. \quad (13)$$

During the collision, both radial and phase fluctuations of the Polyakov-loop field are generated in the interaction region. These excitations subsequently propagate away from the collision site, carrying away excess energy. In Fig. 11, the real and imaginary parts of the Polyakov loop are shown at several stages of the evolution. The arrows indicate the directions of motion of the initial wall profiles. At $t = 0$, the configuration consists of two domain walls, W_{12} and W_{23} , approaching each other. At later times, these walls merge to form a single W_{13} wall. Oscillatory excitations generated during the collision cause the field at the center of the profile to fluctuate around $L = 0$. This behavior is most clearly visible in the imaginary part of the Polyakov loop. At $t = 152$, $\text{Im}(L)$ at the wall center is positive, whereas at $t = 177$ it becomes negative, demonstrating the oscillatory motion of the field. Introducing nonzero damping suppresses these fluctuations and allows the configuration to relax to a well-defined W_{13} wall. Here, it can be clearly seen that the fluctuations in the Polyakov loop lead to fluctuations in the width of the wall.

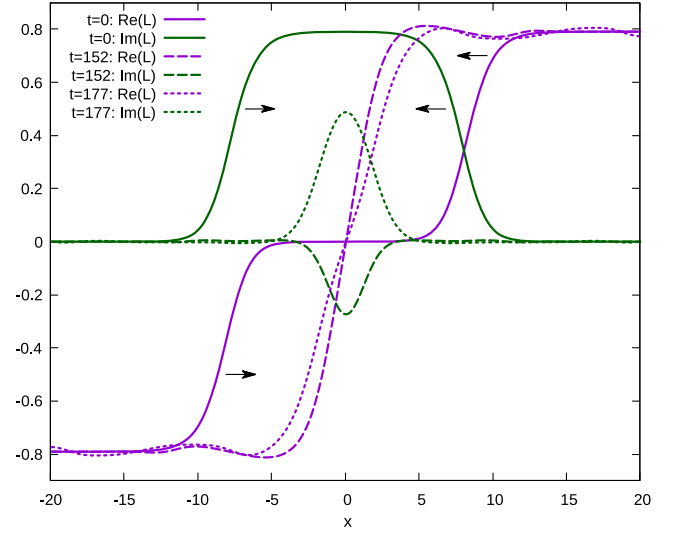


FIG. 11. Evolution of the real and imaginary components of the Polyakov-loop field during a low-energy planar collision of W_{12} and W_{23} walls in the Z_4 theory. The initial (solid line) and late-time (dotted and dashed lines) profiles at the indicated times show the fusion of the two incoming walls into a single W_{13} wall; arrows indicate the wall motion.

A qualitatively different behavior emerges for higher energy collisions. As the two walls merge, the Polyakov-loop field in the overlap region evolves from L_2 toward L_4 and subsequently returns to L_2 . These large oscillations drive the field away from the emerging W_{13} configuration and temporarily restore a region of the L_2 vacuum between the walls. Consequently, the W_{13} wall splits into a W_{12} and W_{23} pair, which subsequently move apart. The real and imaginary parts of the Polyakov loop configuration along the x -direction before and after the collision are shown in Fig. 12, with arrow marks suggesting the direction of the wall movement. At $t = 0$, the initial W_{12} and W_{23} walls move towards each other. After the collision, the two walls separate and propagate away from the collision region. By $t = 502$, their separation is larger than that in the initial configuration. The outgoing walls effectively retrace the trajectories of the incoming walls.

For sufficiently high collision energies, the Polyakov-loop field in the collision region overshoots the confining point $L = 0$ and evolves toward the vacuum value L_4 . As a consequence, the collision no longer results in wall fusion. Instead, the incoming walls are converted into a different pair of walls according to

$$W_{12} + W_{23} \longrightarrow W_{14} + W_{43}. \quad (14)$$

Thus, sufficiently energetic collisions open a new scattering channel that is absent in the $SU(3)$ theory. This behavior can be understood from the trajectory of the Polyakov-loop field in the complex- L plane. While low-energy collisions drive the field toward the confining point

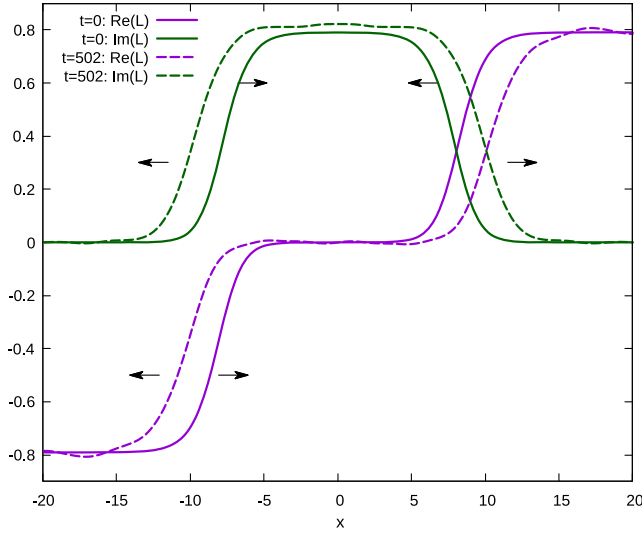


FIG. 12. Evolution of real and imaginary parts of the initial (solid lines) and late-time (dashed lines) Polyakov-loop profiles for an intermediate-energy planar collision of W_{12} and W_{23} walls in the Z_4 theory. After temporarily forming a W_{13} wall, the configuration splits back into the original $W_{12} + W_{23}$ pair, which propagates away from the collision region, as indicated by the arrows.

$L = 0$, higher collision energies allow the field to overshoot this point and reach the basin of attraction of the L_4 vacuum. The resulting wall configuration therefore differs qualitatively from that obtained at lower energies. A representative example of this scattering process is shown in Fig. 13. The figure displays the real and imaginary parts of the Polyakov-loop field before and after the collision, with the arrows indicating the directions of wall motion. The profiles at $t = 0$ represent the initial W_{12} and W_{23} walls. At $t = 125$, the newly formed W_{14} and W_{43} walls move apart, and their separation exceeds that of the initial wall pair.

More intricate phenomena arise in collisions of nonplanar domain walls. A key difference from the $SU(3)$ theory is that the low-energy fusion process,

$$W_{12} + W_{23} \longrightarrow W_{13},$$

proceeds without the creation of vortex-antivortex pairs, even for nonplanar collisions. Thus, wall fusion in $SU(4)$ is not intrinsically tied to the formation of topological defects.

For sufficiently energetic collisions, the Polyakov-loop field in the overlap region can evolve from the intermediate vacuum L_2 to the vacuum L_4 , as in the planar bounce-back and transmutation processes discussed above. The subsequent evolution of this L_4 domain determines whether the walls ultimately transmute or retrace their original trajectories. In nonplanar collisions, the resulting L_4 domain is bounded by W_{14} and W_{43} walls, whose endpoints correspond to a vortex-antivortex pair in $(2 + 1)$ dimensions.

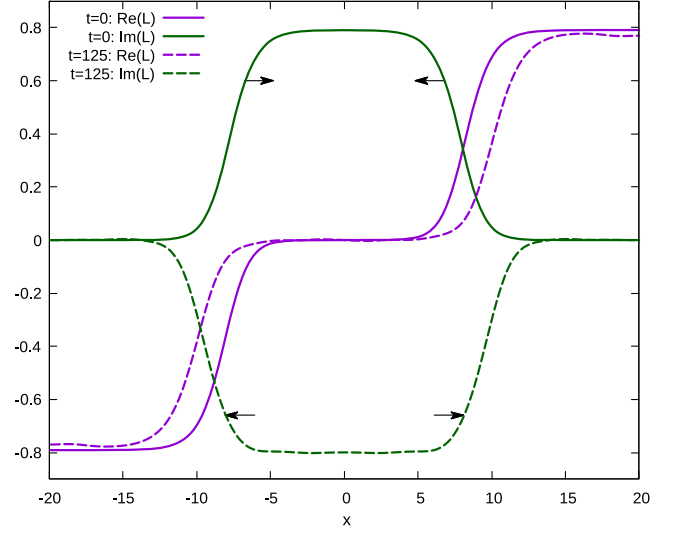


FIG. 13. Evolution of real and imaginary parts of the initial (solid lines) and late-time (dashed lines) Polyakov-loop profiles for a high-energy planar collision in the Z_4 theory. The field in the collision region crosses the confining point and reaches the L_4 vacuum, converting the incoming walls through the scattering channel $W_{12} + W_{23} \rightarrow W_{14} + W_{43}$. Arrows indicate the directions of wall propagation.

Depending on the collision energy and the subsequent evolution of the field, the walls may either scatter into the $W_{14} + W_{43}$ configuration or bounce back and retrace the trajectories of the incoming walls. In Fig. 14, we see the spatial profile of the Polyakov loop magnitude along the xy -plane. Again, the zeros of $|L|$ correspond to the vortex and antivortex, which can be understood from the winding in the vector plot of the same magnitude profile.

This phenomenon is closely analogous to the vortex-mediated wall interactions observed in the $SU(3)$ theory. In $(3 + 1)$ dimensions, the vortex and antivortex are replaced by a Z_4 string. The wall-scattering process therefore

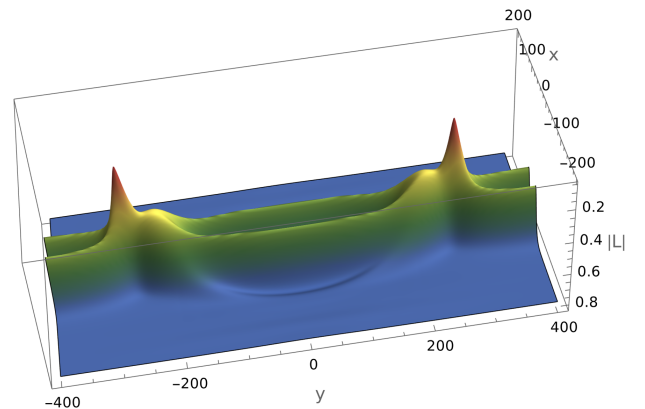


FIG. 14. Formation of a vortex-antivortex pair during the collision of nonplanar W_{12} and W_{23} domain walls in Z_4 theory. Spatial profiles of $|L(x, y)|$ show that the formation of the vortex-antivortex appears at the endpoints of newly formed walls.

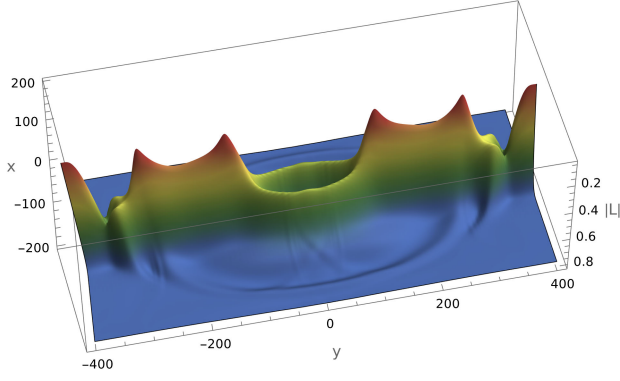


FIG. 15. Formation of multiple vortex-antivortex pairs during a relativistic nonplanar collision of W_{12} and W_{23} walls in the Z_4 theory. Large-amplitude oscillations of the Polyakov-loop field produce multiple vortex-antivortex pairs.

corresponds to the nucleation of a closed string loop. As the scattering front propagates along the colliding walls, the corresponding string loop expands, providing a direct higher-dimensional interpretation of the vortex-mediated dynamics observed in $(2+1)$ dimensions.

For relativistic collisions, i.e., when the wall velocities in the lab frame are $v_x \simeq 0.5c$, the dynamics become even richer. Large oscillations of the Polyakov-loop field are generated in the collision region, leading to repeated excursions of the field through different regions of the effective potential. As a result, additional vortex-antivortex pairs can be nucleated as observed previously in the case of $SU(3)$. As shown in Fig. 15, the spatial profile of the Polyakov-loop magnitude for the high-energy collision process leads to the excitations of the multiple vortex-antivortex pairs. The vortex-antivortex pairs are identified with the zeros of $|L|$ and from the winding of L from the vector plot of the same profile. The corresponding process in $(3+1)$ dimensions is the nucleation and subsequent collapse of additional string loops. Thus, relativistic wall collisions can produce a transient population of string loops before the system relaxes toward its final configuration.

These results demonstrate that the interactions of the $SU(4)$ domain walls are qualitatively richer than those in $SU(3)$. The enlarged spectrum of wall species in the broken Z_4 phase leads to multiple collision outcomes, including fusion, bounce-back, and wall transmutation. While low-energy collisions are governed by the attractive interaction between neighboring walls and result in the formation of a W_{13} wall, higher collision energies allow the Polyakov-loop field to access additional vacuum sectors, giving rise to qualitatively new dynamics. The interplay between domain walls and topological strings revealed by these processes highlights the rich nonequilibrium behavior of center-domain-wall networks in $SU(4)$ gauge theory.

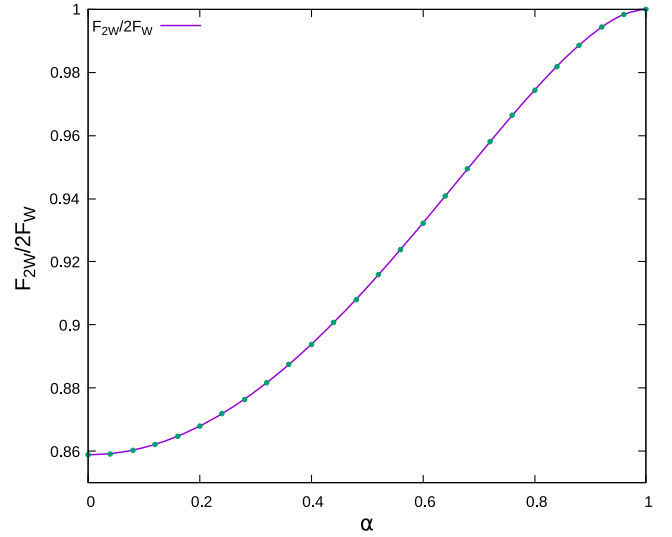


FIG. 16. Normalized free energy $F_{2W}/(2F_W)$ of a constrained $W_{12} + W_{23}$ configuration in the Z_4 theory as a function of the midpoint coordinate α . The decrease of the free energy as the walls overlap demonstrates an attractive interaction.

1. Free-energy landscape in deconfined $SU(4)$

The low-energy collisions show that as the walls W_{12} and W_{23} continue to merge, the value of the Polyakov loop, L_{mid} , at the center between the walls continuously varies from L_2 to $L = 0$. Thus, as in the case of $SU(3)$, we compute the free energy of a two-wall system, F_{2W} , in terms of L_{mid} .

To find the free energy of two walls at various stages of overlap, we consider a configuration consisting of a W_{12} wall connecting the vacua L_1 and L_2 and a W_{23} wall connecting the vacua L_2 and L_3 . To quantify the interaction, we calculate the free energy of the configuration as a function of L_{mid} . The resulting normalized free-energy profile is shown in Fig. 16. The free energy decreases monotonically as the midpoint value moves from L_2 toward $L = 0$. When the midpoint field reaches $L = 0$, the overlap between the two walls becomes complete, and the configuration corresponds to a single wall connecting the vacua L_1 and L_3 . These results indicate that the interaction between the W_{12} and W_{23} walls is attractive. The attractive interaction seen here is responsible for the large oscillations in the Polyakov loop observed even for low-energy collisions.

V. CONCLUSIONS

We have investigated the interaction and collision dynamics of center domain walls in the deconfined phase of $SU(N)$ gauge theories using effective Polyakov-loop models. Building on our earlier identification of topological

string defects in the deconfined phase, we have explored how these defects influence the dynamics of domain-wall interactions. Our analysis combines static free-energy considerations with real-time simulations of domain-wall collisions in $2 + 1$ dimensions, together with their interpretation in $3 + 1$ dimensions.

For both $SU(3)$ and $SU(4)$ gauge theories, we find that neighboring domain walls interact attractively. The free-energy landscapes obtained from constrained two-wall configurations decrease monotonically as the walls approach one another, demonstrating that wall recombination is energetically favored. Real-time simulations reveal the distinct dynamical mechanisms through which this energetically preferred configuration is reached.

In $SU(3)$ gauge theory, the merger of two domain walls proceeds through the creation of a vortex-antivortex pair in $2 + 1$ dimensions. As the vortex and antivortex separate, a segment of a third domain wall is generated between them, providing a microscopic mechanism for wall recombination.

The $SU(4)$ theory exhibits qualitatively different behavior. In contrast to $SU(3)$, low-energy recombination of neighboring domain walls proceeds without the creation of vortex-antivortex pairs, even for nonplanar collisions. Thus, wall fusion in $SU(4)$ is not intrinsically tied to the formation of topological defects. At higher collision energies, however, a new scattering channel becomes accessible in which the incoming walls are converted into a different pair of walls. In nonplanar collisions, this process is accompanied by the creation of vortex-antivortex pairs, indicating that topological defects again play an essential dynamical role.

The vortex-mediated processes observed in $2 + 1$ dimensions admit a natural interpretation in $3 + 1$ dimensions.

Vortices are replaced by topological strings, and vortex-antivortex pair creation corresponds to the nucleation of closed string loops. Domain-wall recombination, wall splitting, and wall scattering therefore become string-mediated processes. Our results demonstrate that topological strings are not merely static junctions of domain walls but actively participate in the nonequilibrium dynamics of defect networks.

These findings reveal a previously unexplored aspect of topological defects dynamics in deconfined non-Abelian gauge theories. The results show that topological strings are not merely static junctions connecting center domain walls but can be dynamically created during wall collisions and subsequently mediate wall recombination, wall splitting, and wall transmutation. The mechanisms identified here suggest that string-mediated interactions may be a generic feature of center-domain-wall networks in theories with spontaneously broken center symmetry. An important next step will be to investigate whether analogous processes occur in lattice simulations of the underlying gauge theories and to explore their consequences for the nonequilibrium evolution of deconfined matter near the confinement-deconfinement transition.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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