

On the holographic dual of a symmetry operator at finite temperature

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Topological symmetry operators of holographic large N CFT_D 's are dual to dynamical branes in the gravity dual AdS_{D+1} . We use this correspondence to establish a dictionary between thermal expectation values of symmetry operators in the Euclidean CFT_D and the evaluation of gravitational saddles in the presence of a dynamical brane. Expectation values of 0-form symmetry operators in the CFT_D are then related to branes wrapped on volume minimizing cycles in the bulk, i.e., the Euclidean continuation of a black hole horizon. We illustrate with some representative examples, including gravity in AdS_3 , duality/triality defects in four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory, and the dual of R-symmetry operators probing five-dimensional Bogomol'nyi–Prasad–Sommerfield black holes.

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I. INTRODUCTION

Symmetries lead to important constraints on many quantum systems. Recent work points to deep topological structures connected with global symmetries in quantum field theories [1]. In gravity, one expects symmetries to be gauged or broken. It is thus of interest to study symmetries in holographic systems such as the AdS/CFT (conformal field theory) correspondence [2] where global symmetries of the boundary CFT_D correspond to gauge symmetries in the bulk AdS_{D+1} . Recent arguments have established both in top down [3–6] and bottom up [7] holographic setups that the topological symmetry operators of the boundary theory correspond to dynamical branes in the bulk, namely branes that couple to local metric fluctuations.

This has led to a fruitful line of development in two directions. On the one hand, one can start from a known brane in the bulk, thus producing a symmetry operator and its accompanying fusion rules in the boundary theory. On the other hand, given a known symmetry of the boundary theory, holographic considerations *predict* the existence of a corresponding dynamical brane.

In this regard, a fundamental quantity of interest is the overall tension of these bulk branes, i.e., their coupling to gravity in a probe approximation. More broadly, of course, these branes can, in principle, source nontrivial backreaction of the bulk spacetime geometry.

Our aim in this note will be to track the effects of these branes in the bulk by showing how the on-shell gravitational action is deformed by the presence of this brane. In the boundary CFT, we propose that this corresponds to calculating the expectation value of a symmetry operator wrapped on a nontrivial cycle.

To establish this correspondence, it is helpful to recall some basic entries in the AdS/CFT dictionary. Recall that the gravitational path integral with prescribed boundary conditions $\phi_{\text{bulk}} \sim J_{\text{CFT}}$ corresponds to a choice of background sources [8], as follows:

$$Z_{\text{grav}}[J] = Z_{\text{CFT}}[J]. \quad (1.1)$$

This is especially natural in the case of symmetries, where the presence of a source in the CFT simply corresponds to switching on a background chemical potential. In the bulk, we simply demand a particular boundary condition for our bulk fields.

To keep things concrete, we primarily focus on D -dimensional CFTs on the manifold $M_D = S^1 \times S^{D-1}$. We shall be interested in wrapping a zero-form symmetry operator $\mathcal{U}$ on the S^{D-1} factor of M_D and evaluating the deformed partition function:

$$Z_{\text{CFT}}(\beta, \mathcal{U}) = \text{Tr}(e^{-\beta \hat{H}} \mathcal{U}), \quad (1.2)$$

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in the obvious notation. For invertible symmetries, there is a natural interpretation of operator insertions such as $\mathcal{U} = \exp(i\mu\hat{Q})$ as switching on a complexified chemical potential for a charge operator $\hat{Q}$. In the bulk, we interpret this as evaluating the on-shell action with prescribed boundary conditions in the presence of a dynamical brane which we denote by $\tilde{\mathcal{U}}$.

Our plan in the following sections will be to set up this correspondence in more detail. In particular, given a known dynamical brane in the bulk, we can determine the thermal expectation value for symmetry operators in the boundary theory. Conversely, knowing a thermal expectation value allows us to determine the backreaction of the dynamical brane on the bulk. In the limit where the backreaction is small (i.e., in the probe approximation), we can use this to extract the tension of the brane.

II. FREE ENERGIES

We consider a large N holographic CFT_D on the background $S^1 \times S^{D-1}$ in which we wrap a zero-form symmetry operator on S^{D-1} and place it at a particular point on the Euclidean circle. We shall focus on the case of antiperiodic boundary conditions for our fermions, but it is also of interest to consider the case of periodic boundary conditions.

To frame the discussion to follow, consider a Lorentzian signature theory on $\mathbb{R} \times S^{D-1}$ with Hamiltonian density $\mathcal{H}$ and an invertible symmetry with corresponding charge density operator $\mathcal{Q}$. Integrating both densities over the S^{D-1} results in operators $\hat{H}$ and $\hat{Q}$. We can then label states $|E, Q, d\rangle$ by their energy E , charge Q , and an implicit degeneracy factor d . A natural quantity to evaluate is the partition function

$$\text{Tr}(e^{-\beta\hat{H}-\mu\hat{Q}}) = \sum_{E,Q} N_{E,Q} e^{-\beta E - \mu Q}. \quad (2.1)$$

This is a bit awkward in the case of generalized symmetries since the corresponding “chemical potential” is often purely imaginary, when the symmetry is invertible, and sometimes quantized, when the symmetry is discrete. Moreover, generalized symmetry operators often support nontrivial topological field theories on the worldvolumes, making a formal analytic continuation in parameters more subtle. Further, “exponentiating infinitesimal noninvertible symmetries” is rife with possible ambiguities that must be treated with care.

For all these reasons, it will be more convenient to consider the evaluation of the CFT_D with a zero-form symmetry operator inserted, as follows¹:

¹The evaluation of this free energy can, in principle, be scheme dependent. This will not end up being an issue since we evaluate expectation values.

$$Z_{\text{CFT}}(\beta, \mathcal{U}) \equiv e^{-\mathcal{F}_{\text{CFT}}(\beta, \mathcal{U})} \equiv \text{Tr}(e^{-\beta\hat{H}}\mathcal{U}), \quad (2.2)$$

where $\mathcal{F}_{\text{CFT}}$ specifies a free energy. For invertible symmetries of the form $\mathcal{U} = \exp(i\nu\hat{Q})$ one can implicitly analytically continue in the chemical potential $\mu = -i\nu$. For ease of exposition, in what follows, we shall mostly assume the existence of a charge conjugation symmetry such that $\text{Tr}(e^{-\beta\hat{H}}\mathcal{U}) = \text{Tr}(e^{-\beta\hat{H}}\mathcal{U}^\dagger)$.² We also consider charged backgrounds, and explain how our considerations extend in this case as well. More broadly, one should also consider complexified saddle point configurations in the gravity dual, a topic we leave for future work.

We claim that the free energy $\mathcal{F}_{\text{CFT}}$ has a natural interpretation in the Euclidean bulk as the on-shell action in the presence of a dynamical brane. The main point is that the insertion of a symmetry operator in the boundary theory has already been interpreted in the “extrapolate dictionary” (see Ref. [9]) as obtained by pushing a dynamical brane in the bulk all the way to the boundary, or vice versa.³ In the bulk, then, we are interested in evaluating the gravitational path integral in the presence of this Euclidean brane, denoted $\tilde{\mathcal{U}}$.

To frame the discussion, let us briefly review what happens when no brane is inserted. In the case of a CFT_D on a thermal circle (i.e., antiperiodic boundary conditions for fermions), there are two dominant saddles that contribute, as given by the thermal AdS solution and the AdS-Schwarzschild solution. Depending on the size of the thermal circle, one or the other will dominate, and this is specified by the Hawking-Page transition [10,11].⁴ The metric for thermal AdS is

$$ds_{\text{th}}^2 = \left(\frac{r^2}{\ell^2} + 1\right) d\tau^2 + \left(\frac{r^2}{\ell^2} + 1\right)^{-1} dr^2 + r^2 d\Omega_{D-1}^2, \quad (2.3)$$

where ℓ is the AdS radius and $d\Omega_{D-1}^2$ is the metric for a unit radius S^{D-1} . The radial coordinate $r \in [0, \infty)$ with the conformal boundary at $r \rightarrow \infty$. Note that the Euclidean time τ can take any period here.

²We expect the on-shell action to have complex phases when topological terms in the bulk evaluate to a nonzero value. This is expected in the case of some noninvertible symmetries, but for the most part, we can bypass this subtlety in the present work.

³For non-Abelian symmetries the branes which can fully detach from the boundary are labeled by conjugacy classes, otherwise they attach via a flux tube back to the boundary [7]. In this note we assume the brane in question can fully detach from the boundary, leaving other situations for future work.

⁴To get a phase transition in the dual CFT one has to scale the size of S^{D-1} to infinite volume [11]. For the generalization of the Hawking-Page transition to R-charged black holes, see Ref. [12].

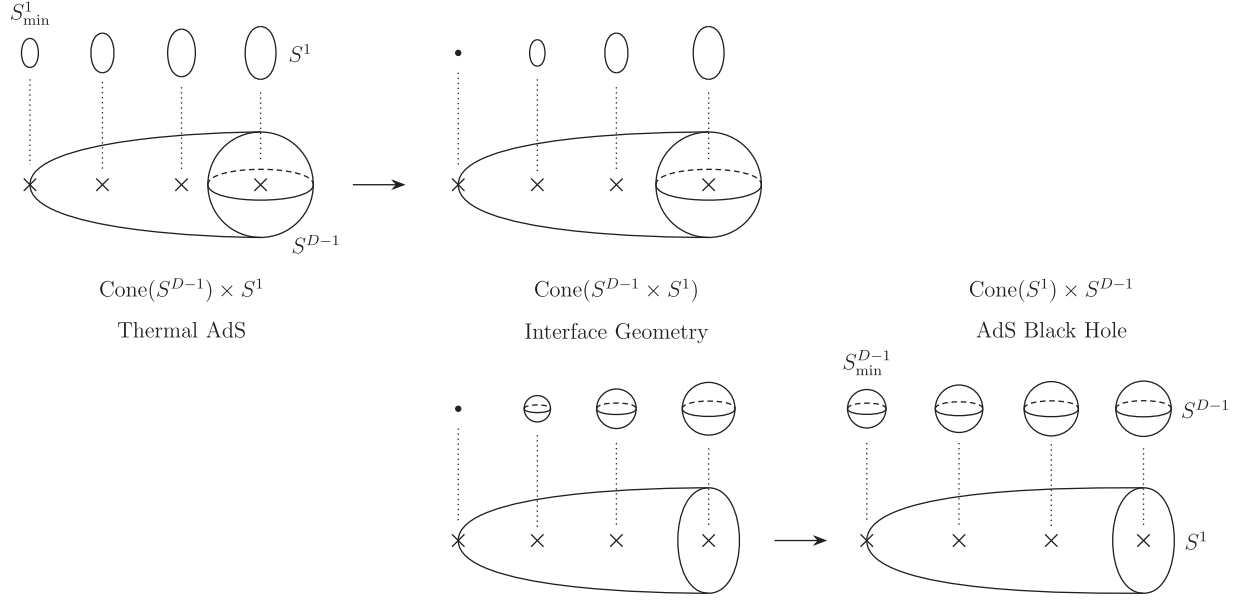


FIG. 1. Sketch of the topological transition associated with the Hawking-Page transition. We label the topological models associated with the two phases and their interface. From left: transition from thermal AdS, through an interface geometry, to an AdS black hole background. The two central figures are distinctly presented, yet equivalent, and related by an interchange $S^1 \leftrightarrow S^{D-1}$. The conformal boundary to all geometries is $M_D = S^1 \times S^{D-1}$. Thermal AdS contains a minimal dimension-1 bulk cycle $S^1_{\min}$, in contrast, the AdS black hole background contains a minimal codimension-2 bulk cycle $\gamma = S^{D-1}_{\min}$.

The AdS-Schwarzschild solution is

$$ds_{\text{bh}}^2 = \left(\frac{r^2}{\ell^2} + 1 - \frac{\alpha M}{r^{D-2}} \right) d\tau^2 + \left(\frac{r^2}{\ell^2} + 1 - \frac{\alpha M}{r^{D-2}} \right)^{-1} dr^2 + r^2 d\Omega_{D-1}^2, \quad (2.4)$$

where M is the mass of the black hole and

$$\alpha = \frac{16\pi G_N}{(D-1)\text{Vol}(\Omega_{D-1})}. \quad (2.5)$$

In this coordinate system, the radial coordinate $r \in [r_+, \infty)$ where r_+ is the largest solution to the equation

$$\frac{r^2}{\ell^2} + 1 - \frac{\alpha M}{r^{D-2}} = 0. \quad (2.6)$$

This geometry is smooth and completely regular only if the periodicity of τ is given by

$$\beta = \frac{4\pi\ell^2 r_+}{Dr_+^2 + (D-2)\ell^2}. \quad (2.7)$$

By abuse of terminology, we shall refer to $r = r_+$ as “the horizon” even though we work in Euclidean signature.⁵

⁵We anticipate there is more to say in Lorentzian signature.

Evaluating the on-shell actions requires regulating each background with an IR cutoff $r = L$. That being said, the difference in the actions is finite and given by

$$\Delta I \equiv I_{\text{th}} - I_{\text{bh}} = \frac{\text{Vol}(\Omega_{D-1})r_+^{D-1}(r_+^2 - \ell^2)}{4G_N(Dr_+^2 + (D-2)\ell^2)}. \quad (2.8)$$

Suppose we now pull our dynamical brane into the bulk. There are two saddle point configurations and thus, two backgrounds probed by the brane. While the worldvolume theory of the branes is often nontrivial, one expects on general grounds that in a saddle point configuration, the brane will wrap a minimal volume cycle of the bulk. The branes we will consider couple only to the geometric background, and their stable bulk configurations are therefore purely determined by their tension minimizing energy. In the case of thermal AdS, there is no such cycle to wrap; the S^{D-1} can pinch off in the interior. On the other hand, in the AdS-Schwarzschild background, there is a minimal volume cycle γ given by wrapping the brane on the S^{D-1} at $r = r_+$. See Fig. 1. This minimal cycle has “area”

$$A = \text{Vol}(\Omega_{D-1})r_+^{D-1} = 4G_N S_{\text{BH}}, \quad (2.9)$$

where in the last equality we have used the relation between the area of the outer horizon and the Bekenstein-Hawking (BH) entropy [13–15].

Putting this together, we learn that the gravitational path integral with the dynamical brane inserted only affects the

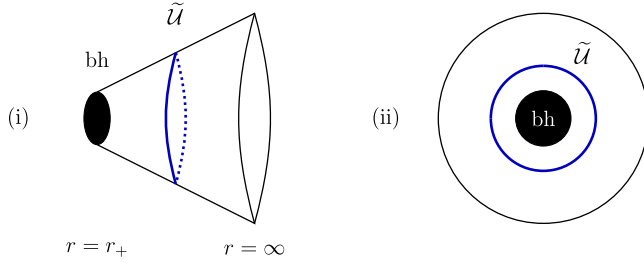


FIG. 2. Sketch of the Schwarzschild AdS black hole with Euclidean brane $\tilde{\mathcal{U}}$ inserted (blue). In (i) we show a profile view, (ii) gives the same geometry head on. In both cases we sketch the S^{D-1} as a circle and do not display S^1_β , where the brane sits at a point. The brane has finite tension and contracts until it stabilizes, wrapping the black hole horizon γ at AdS radius $r = r_+$.

black hole saddle, as follows⁶:

$$Z_{\text{grav}}[\tilde{\mathcal{U}}] = \exp(-I_{\text{th}}) + \exp(-I_{\text{bh}}[\tilde{\mathcal{U}}]). \quad (2.10)$$

The thermal expectation of $\mathcal{U}$ is given by the ratio between the two path integrals:

$$\langle \mathcal{U} \rangle_{\beta, \text{CFT}} = \frac{Z_{\text{grav}}[\tilde{\mathcal{U}}]}{Z_{\text{grav}}} = \frac{\exp(-I_{\text{th}}) + \exp(-I_{\text{bh}}[\tilde{\mathcal{U}}])}{\exp(-I_{\text{th}}) + \exp(-I_{\text{bh}})}. \quad (2.11)$$

At this level of generality, this is as far as we can go. However, there are a number of well-motivated approximations that are natural to consider, leading to more precise estimates.

A. Probe approximation

To proceed further, we now assume that the dynamical brane in the bulk produces negligible backreaction. In this case, the contribution to the on-shell action splits as

$$I_{\text{bh}}[\tilde{\mathcal{U}}] = I_{\text{bh}} + I_B[\gamma], \quad (2.12)$$

where $I_B[\gamma]$ is the on-shell action for the brane wrapped on the minimal volume S^{D-1} (the horizon, see Fig. 2). There can, in principle, be nontrivial dynamics associated with the worldvolume fields of the brane, but all of this can be absorbed into a convention for the overall tension of the brane, as follows⁷:

⁶More precisely, the value of the action I_{th} is unaffected. The boundary 0-form symmetry operator $\mathcal{U}$ is dual in thermal AdS to a flat holonomy along the S^1 factor. Further, I_{th} being unaffected can be anticipated from the CFT dual perspective simply by noting that thermal AdS corresponds to the confined phase, where one expects the degrees of freedom charged under $\mathcal{U}$ to be confined away, suppressing contributions to the leading order as captured by the saddle point approximation.

⁷In many cases $\exp(-I_B[\gamma])$ exhibits nontrivial phases since the brane can support topological terms on its worldvolume. Here we focus on the absolute value which quantifies nontopological features of $\tilde{\mathcal{U}}$.

$$\text{Re}(I_B[\gamma]) = T_B \times A[\gamma], \quad (2.13)$$

where T_B is the tension of the brane in question and $A[\gamma] = \text{Vol}(\Omega_{D-1})r_+^{D-1}$.

As we are dealing with a codimension-2 object, it sources a conical deficit angle χ with $0 < \chi < 2\pi$. In the limit where we can isolate the effective tension of the brane, we have

$$T_B = \frac{\chi}{8\pi G_N}, \quad (2.14)$$

so the on-shell effective action for the brane is

$$I_B[\gamma] = \frac{\chi}{2\pi} \times S_{\text{BH}} + \dots, \quad (2.15)$$

where the “...” indicates possible topological terms resulting in imaginary contributions to the on-shell brane action.

Said differently, the change in the on-shell action from including the brane is proportional to the entropy of the black hole. Let us comment that in related contexts, it is known that Hubeny-Rangamani-Ryu-Takayanagi (HRRT) [16] surfaces can detach from a boundary CFT and end up wrapping a minimal cycle in the bulk [17]. It is thus tempting to speculate that the bulk duals of the symmetry operators considered here are related to HRRT surfaces.

Since we are working in the probe approximation, we implicitly have that $I_B[\gamma]$ is a small correction to the bulk gravitational action. As such, we can relate the thermal expectation value and the brane tension. When the black hole saddle dominates, we have

$$\begin{aligned} \langle \mathcal{U} \rangle_{\beta, \text{CFT}} &= \exp(-T_B \times A[\gamma] + \dots) \\ &= \exp\left(-\frac{\chi}{2\pi} \times S_{\text{BH}} + \dots\right). \end{aligned} \quad (2.16)$$

In contrast, when the thermal AdS saddle dominates, the expression (2.11) evaluates to 1.

Beyond this leading order approximation, we can expect additional subleading saddle point configurations to contribute to the gravitational path integral. In the dual CFT, there are also nonperturbative corrections in $N^2 \sim \ell^D/G_N$ that must be included.

III. ILLUSTRATIVE EXAMPLES

Let us give a few illustrative examples. There are of course many further situations which would be exciting to consider, some of which we collect in Appendix B.

A. Conical deficit in AdS₃

Many of these considerations become especially tractable in the case of AdS₃/CFT₂. With this in mind, consider the boundary CFT on a rectangular torus (no angular momentum) described by the coordinates (τ, x) with

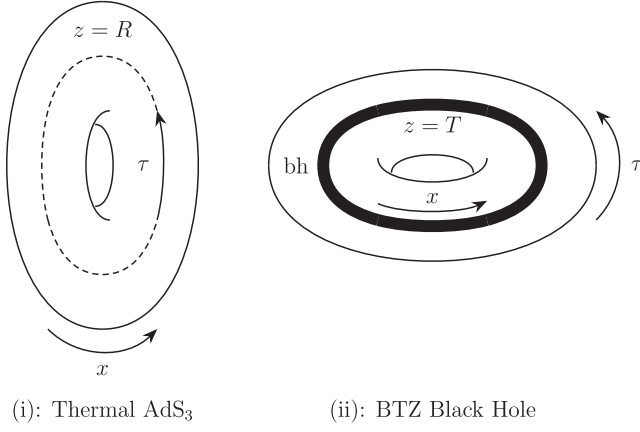


FIG. 3. Metrics of thermal AdS_3 and the BTZ black hole parametrize solid tori. We sketch 2-tori with coordinates x , τ , and radial coordinate z . For thermal AdS_3 and the BTZ black hole the circles parametrized by x and τ collapse at $z = R$ and $z = T$, leaving a single circle parametrized by τ and x respectively (dashed line/thick line).

periodicities $\tau \sim \tau + 2\pi T$ and $x \sim x + 2\pi R$.⁸ As in the general case, we have two bulk saddles—thermal AdS_3 and the Bañados, Teitelboim, Zanelli (BTZ) black hole.⁹

The metric for thermal AdS_3 is

$$ds^2 = \frac{\ell^2}{z^2} \left(d\tau^2 + h(z)dx^2 + \frac{dz^2}{h(z)} \right) \quad \text{with} \quad h(z) = 1 - \frac{z^2}{R^2}, \quad (3.1)$$

where $z \in (0, R]$. For the sake of convenience, we have used a different coordinate choice here, with z being the radial coordinate. The conformal boundary is located at $z = 0$. In this geometry, the x -circle is contractible while the τ -circle is not. See (i) of Fig. 3.

The metric for the BTZ black hole is

$$ds^2 = \frac{\ell^2}{z^2} \left(f(z)d\tau^2 + dx^2 + \frac{dz^2}{f(z)} \right) \quad \text{with} \quad f(z) = 1 - \frac{z^2}{T^2}, \quad (3.2)$$

where $z \in (0, T]$. In this geometry, the τ -circle is contractible while the x -circle is not. See (ii) of Fig. 3. After regularization, the Euclidean action of the thermal AdS_3 and the BTZ black hole are

$$I_{\text{th}} = -\frac{\pi\ell T}{2G_N R}, \quad I_{\text{bh}} = -\frac{\pi\ell R}{2G_N T}. \quad (3.3)$$

Let us now compute the action when we include the backreaction. The Euclidean action of the black hole changes to

$$I_{\text{bh}}[\tilde{\mathcal{U}}] = -\frac{\pi\ell R}{2G_N T} \left(1 - \frac{8\pi G_N m}{2\pi} \right) = -\frac{\pi\ell R}{2G_N T} + \frac{2\pi\ell R m}{T}. \quad (3.4)$$

Taking the difference with I_{bh} , we observe that we recover the expected tension for a conical defect. By inspection, we can now interpret the parameter m as the tension of the codimension-2 defect, i.e., the mass.

B. Duality/triality defects

We now estimate the expectation value of a duality/triality defect of four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory. Bottom-up constructions of these defects were given in Refs. [19,20], and a top-down construction was provided in Refs. [6,21]. See also Ref. [22] for related analyses. In what follows, we assume that the complexified gauge coupling τ has been tuned, and the global structure of the gauge group has been fixed so that a duality/triality operation returns us to the same quantum field theory. In terms of top-down data, this requires a specific choice of the axiodilaton in the boundary CFT, as well as a choice of background two-form potentials compatible with this assumption.

The top-down implementation of duality defects arises from a 7-brane with fixed τ_{IIB} wrapped on the S^5 factor of $\text{AdS}_5 \times S^5$. One can arrange a constant axiodilaton profile via an appropriate bound state of $[p, q]$ 7-branes.¹⁰ In particular, the ones of interest for duality/triality defects are the 7-branes associated with specific monodromies/Kodaira fiber types. We list the relevant 7-branes, including the induced deficit angles¹¹:

τ_{IIB}	Lie algebra	Kodaira fiber	7-branes	χ
$\exp(2\pi i/6)$	$\mathfrak{e}_8$	II^*	$A^7 BC^2$	$10 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{e}_7$	III^*	$A^6 BC^2$	$9 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{e}_6$	IV^*	$A^5 BC^2$	$8 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_3$	IV	$A^3 C$	$4 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{su}_2$	III	$A^2 C$	$3 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_1$	II	AC	$2 \times \frac{2\pi}{12}$

⁸The inverse temperature is related to T by $2\pi T = \beta$. The clash of notation is unfortunate, but standard.

⁹One expects additional, but subdominant saddle point configurations. For example, taking a sum over orbits of $SL(2, \mathbb{Z})$ leads to additional contributions, even in pure gravity [18]. It would be interesting to study such contributions here, but we defer this to future work.

¹⁰See Refs. [23,24].

¹¹Here we formally continue the pattern, allowing “ $\mathfrak{su}_1$,” a nonperturbative bound state of 7-branes that corresponds to a nontrivial Kodaira fiber, but with trivial Lie algebra.

From this, we conclude that the induced deficit angle from a duality defect leads to an order one contribution to the free energy, so in this sense one might view this as a potentially significant backreaction on the background geometry. On the other hand, we expect that this is still small compared to the contribution from a massive black hole, so we expect a controlled expansion even in this limit. Assuming this approximation holds, we can read off the expectation value of the duality defects, as per Eq. (2.16). As a final comment, observe that there is more than one bound state of 7-branes which produces the same topological duality defect in the dual CFT. This amounts to stacking the minimal topological field theory by a non-minimal one. In the gravity dual, this additional stacking leads to an increase in the tension.

C. BPS black holes in AdS_5

As a cross-check on the general set of ideas presented here, we now turn to the case of black holes in five-dimensional $\mathcal{N} = 8$ gauged supergravity¹² in AdS_5 , as obtained from consistent truncation of gravity on $\text{AdS}_5 \times S^5$. See e.g. Refs. [25–29] for foundational work on this class of solutions. These are characterized by three conserved R-charges associated with the angular momenta on the S^5 , as well as two conserved angular momenta in AdS_5 . We shall be interested in Bogomol'nyi–Prasad–Sommerfield (BPS) solutions, namely they are extremal (i.e., zero temperature) as well as supersymmetric. This requires all five charges to be switched on. An additional comment is that in Euclidean signature, the condition of supersymmetry requires a Killing spinor which has antiperiodic boundary conditions on the asymptotic boundary $S^1 \times S^3$ [30]. Since the “thermal circle” at infinity can still be of finite size, we shall, by abuse of terminology, still refer to this as a finite temperature calculation in the dual CFT.

An important recent development is that the black hole entropy can be reproduced from a corresponding superconformal index for the CFT on $\mathbb{R} \times S^3$, provided we work with complexified chemical potentials [30–33] (see also Refs. [34–36]). In Euclidean signature, this involves the index for $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $SU(N)$ on a spatial S^3 , as follows:

$$\mathcal{I}(p, q, y_1, y_2) = \text{Tr}((-1)^F e^{-\beta\{Q, \tilde{Q}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2}), \quad (3.5)$$

where here, Q refers to the supersymmetry generator of a canonical $\mathcal{N} = 1$ subalgebra, and p, q, y_1, y_2 are complex fugacities for the various conserved charges. These complex parameters implicitly specify complexified chemical

potentials.¹³ In the gravity dual, the black hole entropy follows from the Legendre transform of the on-shell action [29], as follows¹⁴:

$$\Gamma = I - 2\pi i \left(\sum_{a=1}^3 Q_a \varphi_a + \sum_{j=1}^2 J_j \omega_j \right) - 2\pi i \Lambda \left(\sum_{a=1}^3 \varphi_a - \sum_{j=1}^2 \omega_j + 1 \right), \quad (3.6)$$

where we have introduced the chemical potentials φ_a and ω_j for the R-charges and angular momenta, Λ is a Lagrange multiplier enforcing a supersymmetry constraint, and I is the on-shell action for a fixed choice of chemical potentials:

$$I = -2\pi i \nu \frac{\varphi_1 \varphi_2 \varphi_3}{\omega_1 \omega_2}, \quad \text{with} \quad \nu = \frac{N^2}{2} = \frac{\pi \ell^3}{4G_N}. \quad (3.7)$$

As found in Refs. [30–33], the logarithm of the superconformal index is, to leading order in a $1/N$ expansion, simply I . A helpful observation is that

$$\varphi_a \frac{\partial \Gamma}{\partial \varphi_a} + \omega_j \frac{\partial \Gamma}{\partial \omega_j} = \Gamma + 2\pi i \Lambda, \quad (3.8)$$

so at the critical point, the Lagrange multiplier is proportional to the entropy, as follows:

$$S_{\text{BH}} = \Gamma_{\text{crit}} = -2\pi i \Lambda_{\text{crit}}. \quad (3.9)$$

We now evaluate the expectation value for a $U(1)$ R-charge topological symmetry operator wrapped on the S^3 , namely

$$\mathcal{U}_\alpha = \exp(2\pi i \Sigma_a \alpha_a \hat{Q}_a). \quad (3.10)$$

This can be viewed as changing the complexified chemical potential to another value. In the gravity dual, it suffices to track the change in the on-shell action after this change. Letting φ_a^{crit} denote the critical point used to extremize Γ , we have that the change in the on-shell action due to inserting the dynamical brane dual to the symmetry operator is

$$I(\varphi_a^{\text{crit}} + \delta\varphi_a) - I(\varphi_a^{\text{crit}}) = \delta\varphi_a \left. \frac{\partial I}{\partial \varphi_a} \right|_{\text{crit}}. \quad (3.11)$$

On the other hand, we also observe that

¹²Namely, 32 real supercharges. We follow the conventions of the R-charged black hole literature.

¹³As explained in Ref. [30], the antiperiodicity condition for the fermions can be accounted for by a modification of the fugacity p .

¹⁴See also Refs. [37–40].

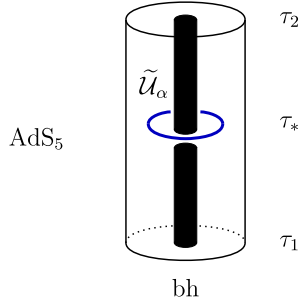


FIG. 4. BPS black hole in AdS_5 with global time coordinate τ . The spatial slice at time τ_* contains a Euclidean brane $\tilde{\mathcal{U}}_\alpha$ linking the world line of the black hole. In particular, the topological worldvolume terms of this brane measure the black hole charges.

$$\delta\varphi_a \frac{\partial I}{\partial \varphi_a} \Big|_{\text{crit}} = \delta\varphi_a \left(\frac{\partial \Gamma}{\partial \varphi_a} + 2\pi i Q_a + 2\pi i \Lambda \right) \Big|_{\text{crit}}. \quad (3.12)$$

Since we are evaluating at a critical point, we have

$$I(\varphi_a^{\text{crit}} + \delta\varphi_a) - I(\varphi_a^{\text{crit}}) = \sum_a \delta\varphi_a (2\pi i Q_a - S_{\text{BH}}). \quad (3.13)$$

To round out the calculation, we observe that on the CFT side of the calculation, $\delta\varphi_a = -\alpha_a$, so we get

$$\Delta I = -\sum_a \alpha_a (2\pi i Q_a - S_{\text{BH}}). \quad (3.14)$$

The CFT thus yields the expectation value

$$\exp \left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3) S_{\text{BH}} \right) = \langle \mathcal{U}_\alpha \rangle_{\text{Index}}, \quad (3.15)$$

where we have introduced the formal “expectation value” obtained from the index:

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \frac{\text{Tr}((-1)^F e^{-\beta\{\mathcal{Q}, \tilde{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2} \mathcal{U}_\alpha)}{\text{Tr}((-1)^F e^{-\beta\{\mathcal{Q}, \tilde{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2})}. \quad (3.16)$$

Turning to the gravity dual, this is exactly the answer we should have expected! The corresponding dynamical brane wraps the outer horizon at $r = r_+$. In particular, in the approximation where backreaction can be neglected, the brane sources a deficit angle¹⁵

¹⁵This result follows from Ref. [41], using results obtained in Ref. [42]. Note that we are working in the limit where α_j takes values in $[0, 1)$ and is implicitly being treated as small; this accounts for the fact that our expression is not a periodic function of the α_j .

$$\chi = 2\pi(\alpha_1 + \alpha_2 + \alpha_3). \quad (3.17)$$

So, we get the expected contribution to the real part of the on-shell action. Additionally, the imaginary part follows from the dynamical brane linking with the charged black hole (viewed as a heavy line operator in the bulk, see Fig. 4). The black hole carries charges Q_a , sourcing a background which is picked up by the brane according to Eq. (3.15) whose imaginary part is associated with the usual phase rotation of the symmetry action. All told, we get an exact match!

IV. CONCLUSIONS AND FUTURE DIRECTIONS

In this note, we have provided some further evidence for the correspondence between bulk dynamical branes and boundary theory topological symmetry operators. In particular, we have related the gravity dual on-shell action in the presence of this dynamical brane to the thermal expectation value of the dual symmetry operator in the boundary CFT. This also leads to a number of natural follow-up questions that would be interesting to explore in the future.

An intriguing feature of the bulk calculation is the appearance of a brane wrapping a volume minimizing cycle in the bulk. This is quite reminiscent of similar structures found in the bulk dual calculation of entanglement entropies in two-sided systems (see e.g., Ref. [17]). It would be interesting to see whether there is a deeper reason for this “phenomenological coincidence.”

There is now some evidence for Hawking-Page transitions in cases where the boundary CFT_D is placed on backgrounds of the form $S^m \times S^n$ with $n + m = D$ (see e.g., Ref. [43]). Wrapping a p -form symmetry operator on S^m (with $p = D - m - 1$) then provides a natural extension of the present case, where now the dynamical brane in the bulk wraps a minimal volume S^m . Similar considerations likely apply to more general boundary topologies.

The results here have focused on the leading order saddle point configurations in the gravity dual. It would be interesting to systematically include subleading corrections. While this is a challenging problem, in asymptotically AdS_3 backgrounds, it should be possible to say more.

Our discussion has centered on Euclidean signature computations. It would be interesting to study the Lorentzian signature case further. For example, the AdS -Schwarzschild geometry has two boundaries. Our current analysis can be viewed as tracking the fate of a single symmetry operator inserted on just one boundary, and the effect on an initial state prepared via the Euclidean path

integral.¹⁶ It would also be interesting to consider more general multipartite systems with symmetry operators inserted (see e.g., Ref. [45]).

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: BRIEF REVIEW OF THE HAWKING-PAGE TRANSITION

In this Appendix we briefly review the Hawking-Page transition. See Refs. [10,11] for further discussion.¹⁷ The on-shell Euclidean action for an AdS_{D+1} solution simplifies to

$$I_E = \frac{D}{8\pi G_N \ell^2} \int d^{D+1}x \sqrt{g}. \quad (\text{A1})$$

¹⁶Recall that the thermofield double for a pair of CFTs $\text{CFT}_L \otimes \text{CFT}_R$ is believed to be dual to the Lorentzian AdS -Schwarzschild solution [44]. This is given by the pure state

$$|\Psi_\beta\rangle = \frac{1}{\sqrt{Z_\beta}} \sum_{E=E_L=E_R} e^{-\beta E/2} |E_L\rangle \otimes |E_R\rangle. \quad (\text{4.1})$$

We can insert a symmetry operator $\mathcal{U}_R$ that acts only on the CFT_R . Then, our thermal expectation value calculation amounts to the vacuum expectation value

$$\langle \Psi_\beta | \mathcal{U}_R | \Psi_\beta \rangle = \frac{\text{Tr}(e^{-\beta H} \mathcal{U}_R)}{\text{Tr}(e^{-\beta H})}. \quad (\text{4.2})$$

In the gravity dual, the Euclidean signature path integral is preparing a preferred state at some initial time $t = 0$. Including the brane wrapped on the horizon amounts to changing this initial wave function.

¹⁷For a discussion of the Hawking-Page transition for R -charged black holes, see Ref. [12].

Thus, for the two geometries given by Eqs. (2.3) and (2.4), respectively, we have

$$I_{\text{th}} = \frac{D}{8\pi G_N \ell^2} \text{Vol}(\Omega_{D-1}) \beta_{\text{th}} \frac{L^D}{D}, \quad (\text{A2})$$

$$I_{\text{bh}} = \frac{D}{8\pi G_N \ell^2} \text{Vol}(\Omega_{D-1}) \beta_{\text{bh}} \frac{L^D - r_+^D}{D}. \quad (\text{A3})$$

Here L is the large-distance cutoff and ℓ the AdS radius. Note that the τ -periodicities are different in the two cases because we need the geometries of the hypersurface at $r = L$ to match. This is achieved by setting

$$\beta_{\text{th}} \sqrt{\frac{L^2}{\ell^2} + 1} = \beta_{\text{bh}} \sqrt{\frac{L^2}{\ell^2} + 1} - \frac{\alpha M}{L^{D-2}}. \quad (\text{A4})$$

In the $L \rightarrow \infty$ limit, the Euclidean actions diverge individually but their difference is finite and given by

$$I_{\text{th}} - I_{\text{bh}} = \frac{\text{Vol}(\Omega_{D-1}) r_+^{D-1} (r_+^2 - \ell^2)}{4G_N (Dr_+^2 + (D-2)\ell^2)}. \quad (\text{A5})$$

Clearly, this difference is positive if $r_+ > \ell$, or equivalently, $\beta < \frac{2\pi\ell}{D-1}$, and the black hole geometry dominates. Otherwise, if $r_+ < \ell$, the thermal geometry dominates. This is the Hawking-Page phase transition [10].

APPENDIX B: FURTHER EXAMPLES

In this Appendix we present some further examples involving symmetry operators of the boundary CFT and their counterparts in the gravity dual.

1. D-brane symmetry operators

Another well-understood string construction, complementary to the duality/triadity defects constructed from 7-branes and discussed in Sec. III B, are settings in which symmetry operators are realized by wrapped D-branes [3–6]. The tension $T_p^{(s)}$ of a Dirichlet p -brane, in string frame, is given by

$$T_p^{(s)} = \frac{1}{g_s (2\pi)^p \ell_s^{p+1}}, \quad (\text{B1})$$

with g_s the string coupling and ℓ_s the string length scale. As such, we can readily compute the CFT quantity $\langle \mathcal{U} \rangle_{\beta, \text{CFT}}$ through bulk computation as in Eqs. (2.12) and (2.13).

Let us consider concretely the near horizon limit of a stack of D3-branes probing the Calabi-Yau orbifold $\mathbb{C}^3/\mathbb{Z}_3$ where the $\mathbb{Z}_3$ acts equally on all complex coordinates by a rotation with $\exp(2\pi i/3)$. The near horizon geometry is $\text{AdS}_5 \times S^5/\mathbb{Z}_3$. Placing the four-dimensional worldvolume theory on $S_\beta^1 \times S^3$ we replace AdS_5 with thermal AdS (2.3)

or the AdS Schwarzschild black hole (2.4) to realize the two saddles dominant for small and large β .

The CFT is characterized at weak 't Hooft coupling by a four-dimensional quiver gauge theory (see e.g., Refs. [46–48]), with a $\mathbb{Z}_3$ zero-form symmetry [49]. The symmetry operator to this discrete symmetry is constructed from a D3-brane wrapped on a generator σ_1 of $H_1(S^5/\mathbb{Z}_3)$ [6]. Therefore, for the AdS-Schwarzschild black hole where the $S^5/\mathbb{Z}_3$ does not collapse in the bulk, we also have that σ_1 remains of finite volume throughout the bulk, namely $\text{Vol}(\sigma_1) = 2\pi R/3$ in a convention where the S^5 has radius R .

Then, making similar approximations as leading up to Eq. (2.13) we have

$$T_B[\sigma_1] = \frac{2\pi\ell}{3g_s(2\pi)^3\ell_s^4}, \quad (\text{B2})$$

where we have set the radius $R = \ell$ the AdS radius. This is compatible, upon making a five-dimensional approximation, with an effective deficit angle χ , following Eq. (2.14), of

$$\chi = 8\pi G_N \times \frac{2\pi\ell}{3g_s(2\pi)^3\ell_s^4}. \quad (\text{B3})$$

Overall, noting that length scales relate as $\ell^4 = 4\pi N g_s \ell_s^4$, we compute from Eq. (2.16)

$$\begin{aligned} \langle \mathcal{U} \rangle_{\beta, \text{CFT}} &= \exp\left(-\frac{2\pi\ell \text{Vol}[\gamma]}{3(2\pi)^3 g_s \ell_s^4}\right) = \exp\left(-\frac{N}{3\pi\ell^3} \text{Vol}[\gamma]\right) \\ &= \exp\left(-\frac{4NG_N}{3\pi\ell^3} S_{\text{BH}}\right), \\ \chi &= \frac{8NG_N}{3\ell^3} \end{aligned} \quad (\text{B4})$$

where γ is the minimal volume bulk S^3 and where lastly we used Eqs. (2.9) and (2.16). Next, we rewrite the deficit angle expressing the gravitational constant in other parameters of the problem. For this, recall that the ten-dimensional Newton constant is $G_N^{(10)} = 8\pi^6 g_s^2 \ell_s^8$. The five-dimensional Newton constant is then given by

$$G_N = \frac{G_N^{(10)}}{\text{Vol}(S^5/\mathbb{Z}_3)} = \frac{24\pi^6 g_s^2 \ell_s^8}{4\pi^3 r_+^5} = \frac{3\pi\ell^3}{8N^2} \frac{\ell^5}{r_+^5}, \quad (\text{B5})$$

and therefore, with the dimensionless ratio $c = \ell^5/r_+^5$, we have

$$\chi = \frac{c\pi}{N}. \quad (\text{B6})$$

2. Nearly extremal R-charged black holes

As another example, we consider nearly extremal zero angular momentum R-charged black hole solutions of five-dimensional $\mathcal{N} = 8$ gauged supergravity in asymptotic AdS_5 . See e.g., Refs. [12, 50–56] for various properties of these solutions.¹⁸ We take the asymptotic boundary to be $\mathbb{R} \times S^3$. Our discussion follows the summary presented in Ref. [56].

The metric is [50]

$$ds^2 = -H^{-2/3} f dt^2 + H^{1/3} (f^{-1} dr^2 + r^2 d\Omega^2), \quad (\text{B7})$$

$$H = \prod_{a=1}^3 H_a = \prod_{a=1}^3 \left(1 + \frac{q_a}{r^2}\right), \quad (\text{B8})$$

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{\ell^2} H, \quad (\text{B9})$$

where ℓ is the AdS radius, μ parametrizes the deviation from extremality and the q_a are related to the three internal angular momentum conserved charges $\tilde{q}_a$ via $\tilde{q}_a = \sqrt{q_a(q_a + \mu)}$. The vector potentials for the three $U(1)$'s and the corresponding three real scalars are

$$A_t^a = \frac{\tilde{q}_a}{q_a + r^2}, \quad X^a = H_a^{-1} H^{1/3}. \quad (\text{B10})$$

Assuming we are near extremality, the Bekenstein-Hawking entropy and inverse temperature are given by

$$S_{\text{BH}} = \frac{A}{4G} \simeq \frac{2\pi^2}{4G} \sqrt{q_1 q_2 q_3} \quad \text{and} \quad \beta = \frac{\pi}{\ell} \sqrt{\frac{q_1 q_2 q_3}{\mu - \mu_{\text{crit}}}}, \quad (\text{B11})$$

where $\mu = \mu_{\text{crit}}$ corresponds to the case where $f(r)$ in line (B9) develops a double zero (extremality).

Let us now turn to the dual CFT. Working in the grand canonical ensemble, we can specify this class of configurations by switching on appropriate chemical potentials. In this case, the unnormalized density matrix is given by $\rho(\beta, \varphi_a) = \exp(-\beta \hat{H} - \Sigma_a \varphi_a \hat{Q}_a)$, where the $\hat{Q}_a$ denote the $U(1)^3$ R-charge operators, and we treat β and φ_a as fixed. We shall be interested in evaluating the expectation value for a symmetry operator associated with a $U(1)$ R-charge symmetry operator wrapped on the S^3 , namely $\mathcal{U}_a = \exp(2\pi i \Sigma_a \alpha_a \hat{Q}_a)$, as follows:

¹⁸See in particular Ref. [12] for the construction of R-charged black holes in $\text{AdS}_4/\text{AdS}_5/\text{AdS}_7$ in terms of scaled solutions of rotating M5/D3/M2-branes on the internal sphere directions.

$$\langle \mathcal{U}_a \rangle_{\beta, \text{CFT}} = \frac{\text{Tr}(\exp(-\beta \hat{H} - \Sigma_a \varphi_a \hat{Q}_a) \mathcal{U}_a)}{\text{Tr}(\exp(-\beta \hat{H} - \Sigma_a \varphi_a \hat{Q}_a))}. \quad (\text{B12})$$

In the holographic dual, this amounts to evaluating the on-shell Euclidean action in the presence of the R-charged black hole. We expect that the corresponding dynamical brane wraps the outer horizon at $r = r_+$. We also anticipate that since the R-charged black hole nontrivially links with this symmetry operator, we can also expect a nontrivial complex phase to be induced. The change in the action is thus

$$\begin{aligned} \Delta I &= -2\pi i \sum_a \alpha_a Q_a + \frac{\chi}{2\pi} S_{\text{BH}} \\ &= -2\pi i \sum_a \alpha_a Q_a + (\alpha_1 + \alpha_2 + \alpha_3) S_{\text{BH}}, \end{aligned} \quad (\text{B13})$$

where in the last equality we used the calculation of the brane tension in Ref. [41], using results obtained in Ref. [42]. Summarizing, we find that the thermal expectation value is

$$\exp \left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3) S_{\text{BH}} \right) = \langle \mathcal{U}_a \rangle_{\beta, \text{CFT}}. \quad (\text{B14})$$

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