

# Hadronic tensor in lattice gauge theories by quantum computing

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The hadronic tensor encodes crucial information regarding the internal structure of hadrons, reflecting the nonperturbative features of quantum chromodynamics (QCD). In this work, we directly compute the hadronic tensor within  $(1+1)$ -dimensional  $U(1)$  and  $SU(2)$  gauge theories by evaluating real-time current-current correlation functions. Utilizing quantum algorithms executed on classical hardware, we demonstrate that the hadron form factors for both meson and baryon states can be reliably extracted from the hadronic tensor. Our methodology is validated by strong agreement with both direct calculation and exact diagonalization of the form factors.

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## I. INTRODUCTION

Exploring the internal structure of hadrons remains a central goal in high-energy and nuclear physics. Among the various probes available, lepton-hadron scattering serves as a primary experimental avenue, having pioneered our understanding of hadron structure since the 1970s. In these scattering processes, the cross section is governed by the contraction of the leptonic tensor  $L_{\mu\nu}$  and the hadronic tensor  $W^{\mu\nu}$ , schematically written as  $d\sigma \propto L_{\mu\nu} W^{\mu\nu}$ . For inclusive lepton-hadron scattering, the nonperturbative hadronic information is entirely encoded within this hadronic tensor. In  $D$ -dimensional space-time, it is defined as [1]

$$W^{\mu\nu}(q) = \int d^D z e^{iqz} \langle h | J^\mu(z) J^\nu(0) | h \rangle, \quad (1)$$

where  $|h\rangle$  denotes the initial hadron state and  $q$  is the momentum of the virtual photon. Considering the electromagnetic interaction between the incident lepton and the hadron, the electromagnetic current operator is given by  $J^\mu = e_f \bar{\psi} \gamma^\mu \psi$ , where  $e_f$  is the electric charge of the fermion  $f$ . As schematically illustrated for one-photon exchange in Fig. 1, the two current insertions create and annihilate all allowed intermediate hadronic states  $X$ .

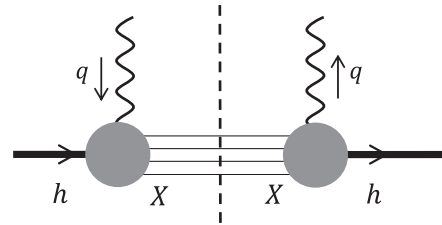


FIG. 1. Schematic representation of the hadronic tensor in inclusive lepton-hadron scattering. The insertion of two electromagnetic currents sums over all allowed intermediate hadronic states  $X$ .

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The resulting current-current correlation function consequently captures the relevant nonperturbative QCD dynamics.

The physical information extracted from the hadronic tensor strongly depends on the kinematic scale of the momentum transfer,  $Q^2 \equiv -q^2$ . When  $Q^2 \gg \Lambda_{\text{QCD}}^2$ , the process enters the deep inelastic scattering (DIS) regime. In this high- $Q^2$  limit, the hadronic tensor factorizes into a convolution of perturbatively calculable hard coefficients and nonperturbative parton distribution functions (PDFs), schematically given by  $W^{\mu\nu}(q) \sim \sum_a c_i^a \otimes f_{a/h}(x)$ . Conversely, in the low-to-intermediate  $Q^2$  region, where  $Q^2 \sim m_h^2$  ( $m_h$  being the hadron mass), the available momentum transfer is restricted. Here, the hadronic tensor predominantly probes hadron form factors, which characterize the spatial distribution of the hadron's constituents and its low-energy excitation dynamics.

Owing to the characteristic hadronic scales involved, the hadronic tensor is inherently nonperturbative and cannot be computed using perturbative QCD alone. While traditional lattice QCD in Euclidean space provides a powerful first principle framework, with extractions of the hadronic tensor from four-point correlation functions already successfully demonstrated [2–4], it faces a severe challenge. The physical Minkowski space response must be reconstructed from Euclidean-time data, an inverse problem that is notoriously ill-posed numerically. Quantum simulation offers a natural alternative through the Hamiltonian framework, where real-time dynamics can be simulated directly at the quantum state level. This capability has driven a surge of quantum-computing studies targeting real-time nonperturbative physics [5–8], such as hadron scattering [9–22], PDFs [23–28], light-cone distribution amplitudes [29], hadronization [30–34], and nonequilibrium physics [35–39].

Quantum algorithms for parton physics and hadronic tensor were initially proposed in Ref. [23], which demonstrated the computation of PDFs and the hadronic tensor using the  $(1+1)$ -dimensional Thirring model on a universal quantum computer. More recently, simulations of the DIS hadronic tensor and PDFs in the Schwinger model was performed using tensor networks [17,27,40–42], enabling the extraction of the longitudinal structure of hadrons. In this work, rather than focusing on DIS structure functions, we investigate the low- $Q^2$  region of the hadronic tensor to extract hadron form factors, thereby providing a crucial complement to these developments. Furthermore, while the Schwinger-model DIS study of Ref. [17] employs staggered fermions, we emphasize the necessity of utilizing Wilson fermions. For the present form factor calculation, the Wilson formulation offers a distinct advantage because it preserves charge conjugation symmetry at finite lattice spacing. This allows the relevant hadronic states and current matrix elements to be classified more directly by their discrete quantum numbers, providing a cleaner setup for isolating elastic contributions and interpreting the form factor signals.

The objective of this work is therefore targeted yet concrete. We present a quantum-computing-oriented framework for evaluating the hadronic tensor from its real-time current-current correlation function in low-dimensional lattice gauge theories. This framework is implemented and benchmarked in both Abelian U(1) and non-Abelian SU(2) gauge theories in  $(1+1)$ -dimensions [43–46]. The U(1) theory serves as a controlled test bed via the Schwinger model, while the SU(2) theory admits gauge-singlet multi-fermion states that act as simple baryonic analogs [47–49]. Although these models do not reproduce the full spin, flavor, and spatial complexity of  $(3+1)$ -dimensional QCD, they share essential features with full gauge theories, including confinement mechanisms [50–52] and chiral condensate formation [48,53,54]. They therefore provide valuable benchmarks for developing and validating quantum algorithms aimed at hadron-structure observables in the real world.

The formal definition of the hadronic tensor is common to both the low- $Q^2$  and DIS regimes, and the same real-time current-current correlator can in principle be evaluated at different momentum transfers. The restriction to low  $Q^2$  in the present work is mainly practical, arising from the limited lattice size, momentum resolution, and real-time evolution that can be simulated efficiently on classical resources. Accessing the DIS regime would require larger spatial volumes, finer lattice spacings, higher momentum resolution, and more accurate long-time real-time evolution. These requirements are beyond the scope of the present classical simulations, but they are natural targets for future fault-tolerant quantum computers. In that setting, the same algorithmic framework may be extended toward high- $Q^2$  hadronic tensors and DIS structure functions. In the present work, we focus on the low- $Q^2$  response and demonstrate that the elastic form factors can be extracted from the computed hadronic tensor. The resulting form factors are compared with exact-diagonalization benchmarks, providing a direct test of the proposed real-time simulation strategy.

The rest of this paper is organized as follows. In Sec. II, we introduce the Hamiltonian lattice formulations used in this work and describe the U(1) and SU(2) models with Wilson fermions. In Sec. III, we review the quantum algorithms for hadron-state preparation and for the simulation of current-current correlation functions. In Sec. IV, we present numerical results for the hadronic tensor and the extracted elastic form factors. Finally, we summarize our conclusions in Sec. V.

## II. THE LATTICE GAUGE THEORY IN $(1+1)$ -DIMENSIONS

To simulate the hadronic tensor, the underlying gauge field theories must first be regularized on a lattice. For this purpose, we employ the Wilson fermion formalism [55],

which resolves the fermion doubling problem by introducing a Wilson term. This term explicitly lifts the unphysical doubler modes to the scale of the ultraviolet cutoff, giving them masses of order  $\mathcal{O}(r/a_l)$ , where  $r$  is the Wilson parameter and  $a_l$  denotes the lattice spacing [44]. In contrast to staggered fermions [56], the Wilson formulation cleanly preserves charge conjugation symmetry at finite lattice spacing. This property enables a rigorous classification of the target hadronic states by their  $C$ -parity [28], which is crucial for interpreting the hadronic tensor where quantum selection rules associated with charge conjugation play a vital role.

In  $(1+1)$ -dimensions, the lattice Hamiltonian of Wilson fermions can be written in general as follows

$$H_W = -\frac{1}{2a_l} \sum_{n=0}^{N-1} [\bar{\psi}_n(r + i\gamma^1) \mathcal{U}_n \psi_{n+1} + \text{H.c.}] + \left(m + \frac{r}{a_l}\right) \sum_{n=0}^{N-1} \bar{\psi}_n \psi_n + \frac{g^2 a_l}{2} \sum_{n=0}^{N-1} \mathbf{L}_n^2, \quad (2)$$

where  $N$  is the number of lattice sites,  $g$  is the bare coupling constant,  $m$  is the bare mass, and  $\psi_n$  is the two-component Dirac spinor at site  $n$ ,  $\psi_n = (\psi_{n,1}, \psi_{n,2})^T$ . Throughout this work, the Wilson parameter is fixed to  $r = 1$ , and we choose  $\gamma^0 = \sigma^1$ ,  $\gamma^1 = -i\sigma^3$  where  $\sigma^1$  and  $\sigma^3$  are Pauli matrices. This convention leads to the Hamiltonian

$$H_W = -\frac{1}{a_l} \sum_{n=0}^{N-1} (\psi_{n,2}^\dagger \mathcal{U}_n \psi_{n+1,1} + \text{H.c.}) + \frac{g^2 a_l}{2} \sum_{n=0}^{N-1} \mathbf{L}_n^2 + \left(m + \frac{1}{a_l}\right) \sum_{n=0}^{N-1} (\psi_{n,1}^\dagger \psi_{n,2} + \text{H.c.}). \quad (3)$$

The gauge field degrees of freedom are encoded in the Wilson link operators  $\mathcal{U}_n$ , which ensure gauge invariance of the lattice Hamiltonian.

Let  $T^a$  be the generators of the Lie group  $G$ , which satisfy

$$[T^a, T^b] = if^{abc} T^c, \quad (4)$$

where  $f^{abc}$  are the structure constants of the group  $G$ . The conjugate momenta of  $\mathcal{U}_n$  are denoted by the left or right electric fields  $L_n^a$  and  $R_n^a$ , which are related by the adjoint representation,  $R_n^a = (\mathcal{U}_n^{adj})_{ab} L_n^b$ , with  $(\mathcal{U}_n^{adj})_{ab} = 2\text{Tr}[\mathcal{U}_n T^a \mathcal{U}_n^\dagger T^b]$  and  $\mathbf{L}_n^2 = \sum_a L_n^a L_n^a = \sum_a R_n^a R_n^a$ . For a given lattice site  $n$ , operators  $L^a$ ,  $R^a$ , and  $\mathcal{U}$  satisfy the following commutation relations:

$$[L^a, L^b] = -if^{abc} L^c, \quad [R^a, R^b] = if^{abc} R^c, \\ [L^a, \mathcal{U}] = T^a \mathcal{U}, \quad [R^a, \mathcal{U}] = \mathcal{U} T^a. \quad (5)$$

Equation (5) implies that the Hilbert space corresponding to the gauge link has infinite dimension.

In  $(1+1)$ -dimensional gauge theories, Gauss's law can be used to express the electric fields in terms of the fermionic charge densities. The lattice version of Gauss's law  $G_n^a = 0$  is given by [57,58]

$$G_n^a \equiv L_n^a - R_{n-1}^a - \mathcal{Q}_n^a = 0, \quad (6)$$

where  $\mathcal{Q}_n^a$  is the charge density on lattice site  $n$ , which only depends on the fermion field  $\psi_n^\dagger$  and  $\psi_n$ . Equation (6) can be solved with a given boundary condition. We use periodic boundary conditions (PBCs) to preserve the translational invariance, hence ensure that hadron momenta are well-defined. In PBCs, Gauss's law determines only the local fluctuation of the electric field, while leaving a global electric flux degree of freedom unfixed [59,60]. The general solution of Gauss's law in PBCs can be written as

$$L_n^a = \mathcal{E}^a + \delta L_n^a = \mathcal{E}^a + \frac{1}{N} \sum_{n'=1}^N (n' - N\theta_{n'>n}) \mathcal{Q}_{n'}^a, \\ \theta_{n'>n} = \begin{cases} 1, & n' > n, \\ 0, & \text{others,} \end{cases} \quad (7)$$

where  $\delta L_n^a$  is the local electric field generated by the fermionic color charges, and  $\mathcal{E}^a = \frac{1}{N} \sum_n L_n^a$  represents the global electric flux, which is conjugate to the gauge-field holonomy around the spatial circle. In this work, we fix  $\mathcal{E}^a = 0$ , which can be understood as an approximation in which the global electric flux is restricted to its lowest-energy sector. The justification for this choice follows from the electric energy associated with a nonzero global flux. In the absence of external background fields, a state in a nontrivial global-flux sector characterized by the representation  $\mathcal{R}$  receives an additional contribution

$$\Delta E_{\mathcal{R}} = \frac{g^2 N a_l}{2} C_2(\mathcal{R}), \quad (8)$$

where  $C_2(\mathcal{R})$  is the quadratic Casimir invariant. For  $U(1)$ ,  $C_2(\ell) = \ell^2$ , while for  $SU(2)$ ,  $C_2(j) = j(j+1)$ . Therefore, the zero-flux sector corresponds to the lowest-energy sector in the absence of external fields, and the nonzero-flux sector has an energy gap proportional to the number of lattice site  $N$ . Since we focus on low-energy color-singlet hadrons and choose the lattice size larger than the hadron correlation length, contributions from nonzero  $\mathcal{E}^a$  sectors are expected to be suppressed. After fixing the global flux sector  $\mathcal{E}^a = 0$  and solving Gauss's law for  $\delta L_n^a$ , the Hamiltonian can be expressed solely in terms of fermionic operators.

The above discussions are general, and the charge operator  $\mathcal{Q}_n^a$  for specific cases such as  $U(1)$  and  $SU(2)$  gauge field theory can be written explicitly. In the Schwinger

model, the only generator of the U(1) group is  $T^1 = 1$ , so the Abelian charge is defined as  $\mathcal{Q}_n = \sum_s \psi_{n,s}^\dagger \psi_{n,s} - I$ . The total charge  $\mathcal{Q}_{\text{tot}} = \sum_{n=0}^{N-1} \mathcal{Q}_n$  is a conserved quantity. Combining with translation invariance of PBCs, we can use three quantum numbers to label the energy eigenstate of the Schwinger model, i.e., energy  $E$ , momentum  $\mathbf{p}$ , and electric charge  $\mathcal{Q}_{\text{tot}}$ .

In the non-Abelian SU(2) case, each spinor of the fermion field consists of red and green components labeled by  $c = 0, 1$ ,  $\psi_{n,s} = (\psi_{n,s}^0, \psi_{n,s}^1)^T$ , so it spans the fundamental representation space of the SU(2) gauge group. In this representation, the three generators are  $T^a = \frac{1}{2}\sigma^a$ , where  $\sigma^a$  represents the three Pauli matrices, and the structure constants are given by the Levi-Civita tensor  $f^{abc} = \epsilon^{abc}$ . The non-Abelian charges are defined as  $\mathcal{Q}_n^a = \sum_s \psi_{n,s}^\dagger T^a \psi_{n,s}$ , and the total charge  $\mathcal{Q}_{\text{tot}}^a = \sum_{n=0}^{N-1} \mathcal{Q}_n^a$  are conserved quantities. In addition to the three non-Abelian charges, the system possesses another conserved quantity, baryon number  $B$ , arising from global U(1) symmetry, which is defined as

$$B = -N + \frac{1}{2} \sum_{n=0}^{N-1} \sum_s (\psi_{n,s}^{0\dagger} \psi_{n,s}^0 + \psi_{n,s}^{1\dagger} \psi_{n,s}^1). \quad (9)$$

Therefore, the good quantum numbers in SU(2) LGT are  $E$ ,  $\mathbf{p}$ ,  $(\mathcal{Q}_{\text{tot}})^2 \equiv \sum_a (\mathcal{Q}_{\text{tot}}^a)^2$ ,  $\mathcal{Q}_{\text{tot}}^3$ , and  $B$ . All of the energy eigenstates in SU(2) LGT can be labeled by those five quantum numbers.

To simulate the system on a quantum computer, the Hamiltonian must be expressed in terms of Pauli operators. This can be achieved by the Jordan-Wigner transformation [61]

$$\phi_k \rightarrow \left( \prod_{n < k} \sigma_n^3 \right) \sigma_k^-, \quad \phi_k^\dagger \rightarrow \left( \prod_{n < k} \sigma_n^3 \right) \sigma_k^+, \quad (10)$$

where  $\sigma_n^i$  represents the Pauli matrices acting on the  $n$ th qubit, and  $\sigma^\pm = (\sigma^1 \pm i\sigma^2)/2$ . This transformation preserves the canonical anticommutation relations of the fermion field.

For the U(1) case, the fermion field  $\phi_k = \psi_{n,s}$  carries a site index  $n$ , and a spinor index  $s = 1, 2$ , is mapped to a qubit with index  $k = 2n + s - 1$ . Following this and the Jordan-Wigner transformation, the Abelian charge  $\mathcal{Q}_n$  is given by

$$\mathcal{Q}_n = \frac{1}{2} (\sigma_{2n}^3 + \sigma_{2n+1}^3). \quad (11)$$

The kinematic, mass, and electric energy terms in the Hamiltonian can be written as

$$\begin{aligned} H_{\text{kin}} &= \sum_{n=0}^{N-1} (\sigma_{2n+1}^+ \sigma_{2n+2}^- + \text{H.c.}), \\ H_{\text{m}} &= -(m+1) \sum_{n=0}^{N-1} (\sigma_{2n}^+ \sigma_{2n+1}^- + \text{H.c.}), \\ H_{\text{e}} &= \frac{g^2 a_l}{2N^2} \sum_{n=0}^{N-1} \left[ \sum_{n'=1}^N (n' - N\theta_{n'>n}) \mathcal{Q}_{n'} \right]^2. \end{aligned} \quad (12)$$

For the SU(2) LGT, the fermion field  $\phi_k = \psi_{n,s}^c$  carries an additional color index  $c$ , and is mapped to a qubit labeled by  $k = 4n + 2(s-1) + c$ . After that, the non-Abelian charges are given by

$$\begin{aligned} \mathcal{Q}_n^1 &= -\frac{1}{2} [(\sigma_{4n}^+ \sigma_{4n+1}^- + \sigma_{4n+2}^+ \sigma_{4n+3}^-) + \text{H.c.}], \\ \mathcal{Q}_n^2 &= \frac{i}{2} [(\sigma_{4n}^+ \sigma_{4n+1}^- + \sigma_{4n+2}^+ \sigma_{4n+3}^-) - \text{H.c.}], \\ \mathcal{Q}_n^3 &= \frac{1}{4} (\sigma_{4n}^3 + \sigma_{4n+2}^3 - \sigma_{4n+1}^3 - \sigma_{4n+3}^3). \end{aligned} \quad (13)$$

The qubit Hamiltonian of SU(2) LGT can be obtained by combining the Eqs. (2), (7), (10), and (13). Then the Hamiltonian is given by  $\bar{H} = \bar{H}_{\text{kin}} + \bar{H}_{\text{m}} + \bar{H}_{\text{e}}$ , with

$$\begin{aligned} \bar{H}_{\text{kin}} &= \sum_{n=0}^{N-1} [(\sigma_{4n+2}^+ \sigma_{4n+3}^3 \sigma_{4n+4}^- + \sigma_{4n+3}^+ \sigma_{4n+4}^3 \sigma_{4n+5}^-) + \text{H.c.}], \\ \bar{H}_{\text{m}} &= -(m+1) \sum_{n=0}^{N-1} [(\sigma_{4n}^+ \sigma_{4n+1}^3 \sigma_{4n+2}^- \\ &\quad + \sigma_{4n+1}^+ \sigma_{4n+2}^3 \sigma_{4n+3}^-) + \text{H.c.}], \\ \bar{H}_{\text{e}} &= \frac{g^2 a_l}{2N^2} \sum_a \sum_{n=0}^{N-1} \left[ \sum_{n'=1}^N (n' - N\theta_{n'>n}) \mathcal{Q}_{n'}^a \right]^2. \end{aligned} \quad (14)$$

The hadronic tensor in  $(1+1)$ -dimensions also needs to be mapped to qubits. This can be done by the Jordan-Wigner transformation

$$W^{\mu\nu}(q^0, q^1) = \int dt \sum_z e^{iq^0 t} e^{-iq^1 z} \mathcal{O}^{\mu\nu}(z, t), \quad (15)$$

where

$$\mathcal{O}^{\mu\nu}(z, t) = \langle h | e^{iHt} J_m^\mu e^{-iHt} J_n^\nu | h \rangle, \quad z = m - n. \quad (16)$$

The current operator  $J_n^\mu$  can then be expressed in terms of Pauli operators as follows. For the U(1) LGT

$$\begin{aligned} J_n^0 &= \frac{e_f}{2} (\sigma_{2n}^3 + \sigma_{2n+1}^3), \\ J_n^1 &= ie_f (\sigma_{2n+1}^+ \sigma_{2n}^- - \sigma_{2n}^+ \sigma_{2n+1}^-), \end{aligned} \quad (17)$$

and for SU(2) LGT

$$J_n^0 = \frac{e_f}{2}(\sigma_{4n}^3 + \sigma_{4n+1}^3 + \sigma_{4n+2}^3 + \sigma_{4n+3}^3),$$

$$J_n^1 = ie_f[(\sigma_{4n+2}^+ \sigma_{4n+1}^3 \sigma_{4n}^- + \sigma_{4n+3}^+ \sigma_{4n+2}^3 \sigma_{4n+1}^-) - \text{H.c.}]. \quad (18)$$

With the above mapping of lattice gauge theory to qubits, we are ready to simulate the hadronic tensor on a quantum computer.

### III. QUANTUM ALGORITHMS

By employing the quantum algorithm proposed in Ref. [26], the simulation of the hadronic tensor on a quantum computer proceeds in two main steps: (1) Preparation of the interacting vacuum state  $|\Omega\rangle$  and the hadron state  $|h\rangle$ ; (2) Evaluation of the current-current correlation functions.

The hadron state can be prepared by a quantum-number-resolving variational quantum eigensolver (VQE). The framework of this method has been discussed in Ref. [10], and we give a brief review here. Consider the target hadron state as the  $k$ th excited state with quantum numbers  $l$ . To prepare the lowest  $k$  states simultaneously. We need  $k$  initial reference states  $|\psi_{l,i}\rangle_{\text{ref}}$ ,  $i = 0, 1, \dots, k-1$ , which have the same quantum number  $l$  as the hadron state. Then, a quantum number preserving ansatz  $U_A(\theta)$  is needed to generate the trial wave functions  $|\psi_{l,i}(\theta)\rangle \equiv U_A(\theta)|\psi_{l,i}\rangle_{\text{ref}}$ , which have the same quantum number as the hadron state.  $U_A(\theta)$  can be constructed by quantum alternating operator ansatz (QAOA). The key point of the QAOA lies in splitting the Hamiltonian into  $d$  ( $d \geq 2$ ) parts,  $H = H_1 + \dots + H_d$ , while satisfying  $[H_i, H_{i+1}] \neq 0$  and  $[H_i, C] = 0$  for every  $H_i$  and conserved charge  $C$ . Then  $U_A(\theta)$  is given by

$$U_A(\theta) = \prod_{i=1}^p \left( \prod_{j=1}^d e^{i\theta_{ij} H_j} \right), \quad (19)$$

where  $p$  is the number of layers for alternating evolution of the Hamiltonian.

It is necessary to minimize the cost function  $E_l(\theta)$  on the classical computer to select the target hadron state from numerous trial states [62],

$$E_l(\theta) = \sum_{i=0}^{k-1} w_{li} \langle \psi_{l,i}(\theta) | H | \psi_{l,i}(\theta) \rangle, \quad (20)$$

where the weights  $w_{li}$  are required to satisfy  $w_{l,0} > w_{l,1} > \dots > w_{l,k-1} > 0$ , so that  $U_A(\theta^*)|\psi_{l,i}\rangle_{\text{ref}}$  are the eigenstates in quantum numbers  $l$  with the optimized parameter  $\theta^*$ . The  $(k-1)$ th excited hadron state  $|h\rangle$  is then prepared as  $|h\rangle = U_A(\theta^*)|\psi_{l,k-1}\rangle$ .

In the case of the U(1), the conserved quantities include charge and momentum, so the  $U_A(\theta)$  needs to preserve those two quantum numbers. The vacuum and the lightest  $q\bar{q}$  bound state are the ground and first excited state in the

$Q_{\text{tot}} = 0$  and  $\mathbf{p} = 0$  subspace. The construction of the reference states and the decomposition of the Hamiltonian for U(1) follow a procedure analogous to that of the SU(2) case discussed below.

As discussed in Sec. II, the quantum number of SU(2) LGT is more complex than U(1) LGT. That means we need to design the reference states and divide the Hamiltonian more carefully. The vacuum and the lightest  $q\bar{q}$  meson state considered here are the ground and first excited states in the  $(Q_{\text{tot}})^2 = Q_{\text{tot}}^3 = 0$ ,  $B = 0$ , and  $\mathbf{p} = 0$  subspace. We are also concerned about the lightest  $qq$  baryon bound state, which is the ground state of the  $(Q_{\text{tot}})^2 = Q_{\text{tot}}^3 = 0$ ,  $B = 1$ , and  $\mathbf{p} = 0$  subspace. Except for baryon number  $B$  and energy  $E$ , the remaining quantum numbers of the above states are the same. To construct suitable reference states within these sectors, we first identify local color-singlet subspaces and then select states with fixed baryon number. We use  $B$  and  $i$  to label the local reference states  $|\psi_{B,i}\rangle_{\text{local ref}}$ :

$$|\psi_{0,0}\rangle_{\text{local ref}} = \frac{1}{2}(|0011\rangle - |0110\rangle + |1001\rangle + |1100\rangle),$$

$$|\psi_{0,1}\rangle_{\text{local ref}} = \frac{1}{\sqrt{2}}(|0011\rangle - |1100\rangle),$$

$$|\psi_{1,0}\rangle_{\text{local ref}} = |0000\rangle, \quad (21)$$

where the states on the right-hand side of the equation are the computational basis, which are defined by

$$|i_0, i_1, \dots, i_{N-1}\rangle \equiv |i_0\rangle \otimes |i_1\rangle \otimes \dots \otimes |i_{N-1}\rangle,$$

$$i_0, \dots, i_{N-1} = 0, 1, \quad (22)$$

where  $\sigma^3|0\rangle \equiv |0\rangle$ ,  $\sigma^3|1\rangle \equiv -|1\rangle$ . Therefore, the reference states are constructed as tensor products of these local reference states

$$|\psi_{B,i}\rangle_{\text{ref}} = \frac{1}{\sqrt{N}} \sum_{n=1}^N |\psi_{0,0}\rangle_{\text{local ref}}^{\otimes(n-1)} \otimes |\psi_{B,i}\rangle_{\text{local ref}} \otimes |\psi_{0,0}\rangle_{\text{local ref}}^{\otimes(N-n)}. \quad (23)$$

It is straightforward to verify that those three reference states have the same quantum number (except energy  $E$ ) of the target vacuum, meson, and baryon state.

To implement this state, we adopt a three-step procedure. First, we initialize a set of  $N$  ancilla qubits in the Dicke state  $|D_1^N\rangle$  [63]. In general, the Dicke state  $|D_l^n\rangle$  is defined as the equal superposition of all  $n$ -qubit states  $|x\rangle$

$$|D_l^n\rangle \equiv \sqrt{\frac{1}{C_n^l}} \sum_{\substack{n_x \in \{0,1\}^n \\ w(x)=l}} |x\rangle, \quad (24)$$

where  $C_n^l = \frac{n!}{l!(n-l)!}$  and  $w(x)$  is the Hamming weight, representing the number of ones in  $|x\rangle$ . Second, we define

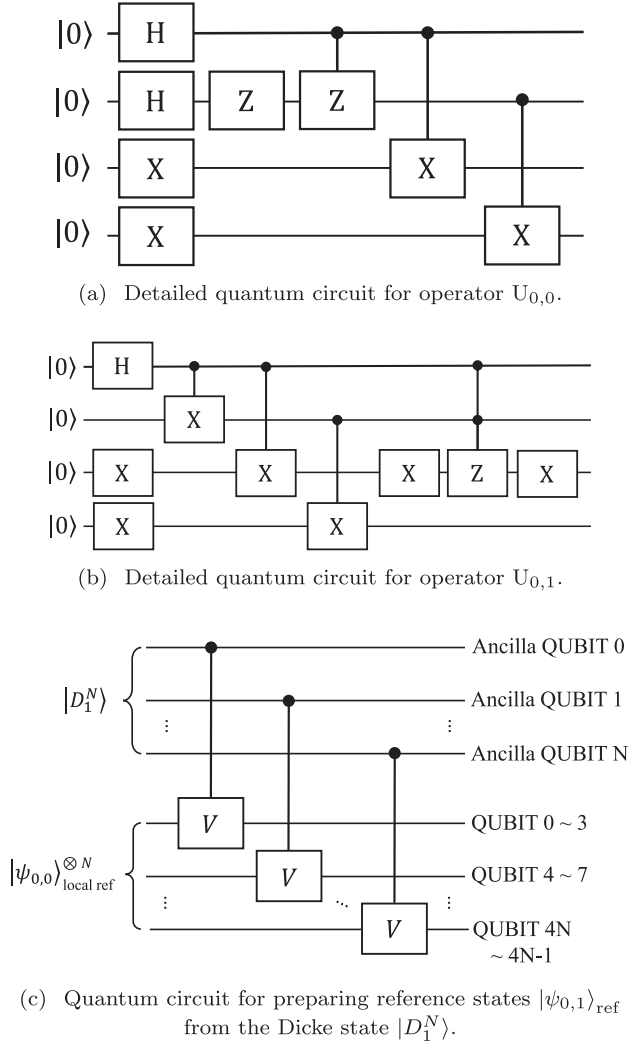


FIG. 2. Quantum circuits for preparing the reference states.

two unitary operators,  $U_{0,0}$  and  $U_{0,1}$ , acting on four qubits, such that  $U_{0,0}|0\rangle^{\otimes 4} = |\psi_{0,0}\rangle_{\text{local ref}}$  and  $U_{0,1}|0\rangle^{\otimes 4} = |\psi_{0,1}\rangle_{\text{local ref}}$ . The detailed quantum circuits for  $U_{0,0}$  and  $U_{0,1}$  are shown in Fig. 2(a) and Fig. 2(b). A transition operator  $V = U_{0,1}U_{0,0}^\dagger$  is then constructed to map the state  $|\psi_{0,0}\rangle_{\text{local ref}}$  to  $|\psi_{0,1}\rangle_{\text{local ref}}$ . The  $U_{0,0}^{\otimes N}$  operator is then applied to prepare the product state  $|\psi_{0,0}\rangle_{\text{local ref}}^{\otimes N}$ . Finally, as shown in Fig. 2(c),  $|\psi_{0,1}\rangle_{\text{ref}}$  can be prepared by applying a controlled-V gate from the  $i$ th ancilla qubit in  $|D_1^N\rangle$  to the corresponding  $i$ th local reference state  $|\psi_{0,0}\rangle_{\text{local ref}}$ . The state  $|\psi_{1,0}\rangle_{\text{ref}}$  can be prepared similarly by replacing all controlled-V gates with controlled- $U_{0,0}^\dagger$  gates.

In addition, the Hamiltonian of  $SU(2)$  can be divided as follows:

$$\begin{aligned} H_1 &= \frac{1}{2} \bar{H}_{\text{kin}}, & H_2 &= \bar{H}_{\text{m}}, \\ H_3 &= \frac{1}{2} \bar{H}_{\text{kin}}, & H_4 &= \bar{H}_{\text{e}}. \end{aligned} \quad (25)$$

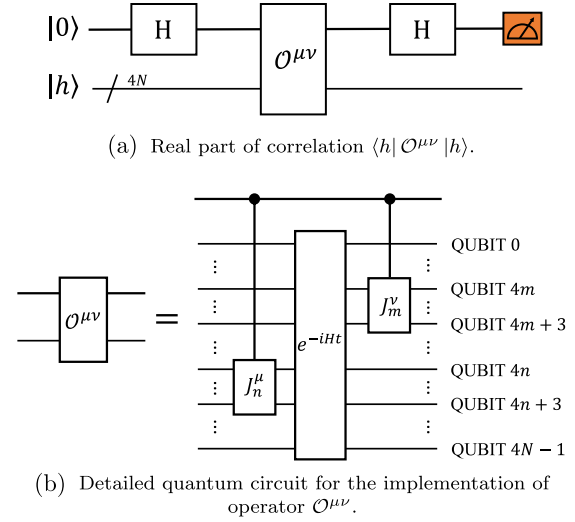


FIG. 3. Quantum algorithm for computing current-current correlation functions.

Note that each term in Eq. (25) satisfies

$$[H_i, H_{i+1}] \neq 0, \quad [\mathcal{C}, H_i] = 0, \quad [T, H_i] = 0, \quad (26)$$

where  $\mathcal{C} = (\mathcal{Q}_{\text{tot}})^2$ ,  $\mathcal{Q}_{\text{tot}}^3$ ,  $B$ , and  $T$  is the translation operator. Thus, the decomposition of the Hamiltonian is feasible. Then we set  $p = N$  in Eq. (19) and  $w_{B,0} = 1$ ,  $w_{B,1} = 0.5$  in Eq. (20) to prepare the state in  $B = 0$  sector. After optimizing the cost function, the vacuum state and the hadron state can be prepared by  $U_A(\theta_B^*)$

$$\begin{aligned} |\Omega\rangle &= U_A(\theta_0^*) |\psi_{0,0}\rangle_{\text{ref}}, \\ |h = q\bar{q}\rangle &= U_A(\theta_0^*) |\psi_{0,1}\rangle_{\text{ref}}, \\ |h = qq\rangle &= U_A(\theta_1^*) |\psi_{1,0}\rangle_{\text{ref}}. \end{aligned} \quad (27)$$

After preparing the hadron state, we perform the quantum algorithm, which is shown in Fig. 3(a), to simulate the real part of current-current correlation functions by using an ancilla qubit [64]. The imaginary part can be obtained by replacing the last Hadamard gate with the  $R_x(\pi/2)$  gate on the ancilla qubit in Fig. 3(a). Furthermore, the time evolution operator in Fig. 3(b) can be implemented on a quantum computer using the standard Trotterization method [65].

Finally, to obtain the hadronic tensor  $W^{\mu\nu}(q)$ , one just need to perform a Fourier transformation on the simulated current-current correlation function, which can be done numerically on a classical computer.

#### IV. RESULTS

We use the open-source package QuSpin [66] to perform the quantum simulation on classical computers. For the  $U(1)$  LGT, we simulate the hadronic tensor of the lowest-lying meson using  $N_q = 22$  qubits, corresponding

TABLE I. Choice of bare parameters and resulting hadron masses.

| Theory | Hadron $h$       | $m$  | $g$ | $e_f$ | $m_h a_l$ |
|--------|------------------|------|-----|-------|-----------|
| U(1)   | meson $q\bar{q}$ | 0.25 | 1   | 1     | 1.04      |
| SU(2)  | meson $q\bar{q}$ | 0.30 | 1   | 1     | 0.83      |
| SU(2)  | baryon $qq$      | 0.30 | 1   | 1     | 0.83      |

to  $N = 11$  lattice sites. For the SU(2) LGT, we simulate both the lowest-lying meson and baryon states using  $N_q = 20$  qubits, corresponding to  $N = 5$  lattice sites. The bare parameters  $m$  and  $g$  are chosen to prepare the zero-momentum hadron state  $|h\rangle$ , whose mass is defined as  $m_h = \langle h|H|h\rangle - \langle \Omega|H|\Omega\rangle$ . The dimensionless parameter  $m_h a_l$  controls discretization effects, while finite-volume effects are governed by the dimensionless lattice size  $Nm_h a_l$ . To suppress finite-volume effects, we require  $Nm_h a_l \gg 1$ , which is satisfied in our lattice setup, i.e.,  $Nm_h a_l = 11.4$  for the U(1) meson and  $Nm_h a_l = 4.2$  for the SU(2) meson and baryon states.

The simulation parameters and the resulting hadron masses are summarized in Table I. For the SU(2) case, the meson and baryon masses are degenerate, which follows from the charge conjugation symmetry of the theory.

### A. Hadronic tensor of $q\bar{q}$ meson state in U(1) LGT

After fixing the parameters  $m$ ,  $g$ , and  $m_h a_l$  in the U(1) LGT, we can simulate the quantum algorithm on a classical device. We show the real part of the hadronic tensor  $W^{00}(q)$  for the lightest  $q\bar{q}$  bound state in U(1) LGT in Fig. 4, as a function of continuous  $q^0 a_l$  and discrete  $q^1 a_l$ . The imaginary part of  $W^{00}$  is nearly zero and not shown in the figure. In the Hamiltonian formulation of LGT, space is discretized while time remains continuous. Consequently, the spatial momentum  $q^1 a_l$  takes discrete values  $q^1 a_l = 2\pi k/N$ , where  $k = 0, 1, \dots, N-1$ . For a given  $q^1 a_l$ , the variable  $q^0 a_l$  remains continuous, as it is conjugate to the continuous time  $t$ .

From Fig. 4, we can see that for a given  $q^1 a_l$ , the hadronic tensor  $W^{00}(q)$  has a series of peaks in different values of  $q^0 a_l$ . This structure arises from the spectral decomposition. By inserting the completeness relation of the Hilbert space into the current-current correlation function, we obtain

$$W^{\mu\nu}(q) = \sum_X \int d\Pi_X (2\pi)^2 \delta^{(2)}(q + p_h - p_X) \times \langle h|J^\mu(0)|X\rangle \langle X|J^\nu(0)|h\rangle, \quad (28)$$

where  $d\Pi_X$  is the phase space of the final states  $|X\rangle$  and the spacetime translation of the matrix element  $\langle h|J^\mu(z)|X\rangle = \langle h|J^\mu(0)|X\rangle e^{i(p_h - p_X)z}$  was used. Equation (28) implies

$W^{\mu\nu}(q)$  should have a nonzero contribution when the delta function is satisfied for  $q = p_X - p_h$ . After choosing the current component  $\mu = \nu = 0$  and isolating the contribution from the one-particle final states  $|h'(p')\rangle$ , we have

$$W^{00}(q) = \sum_{h'} \frac{2\pi}{2E_{p'}} \delta^{(1)}(q^0 + p^0 - p'^0) |_{q^1=p'^1-p^1} \times \langle h|J^0(0)|h'(p')\rangle \langle h'(p')|J^0(0)|h\rangle + \text{others}, \quad (29)$$

where “others” denotes the contributions from multiple-particle final states. As shown in Eq. (29), the terms with final states being one-particle states contribute a sum of delta functions to the hadronic tensor, which appear as sharp peaks whenever the kinematic variables satisfy energy-momentum conservation. In Fig. 4, dashed lines denote the exact energy  $\Delta E = p^0 - p_h^0$  of the lowest-lying pseudoscalar (P) and the scalar (S) meson state of the lattice Schwinger model. Notice that the electromagnetic current

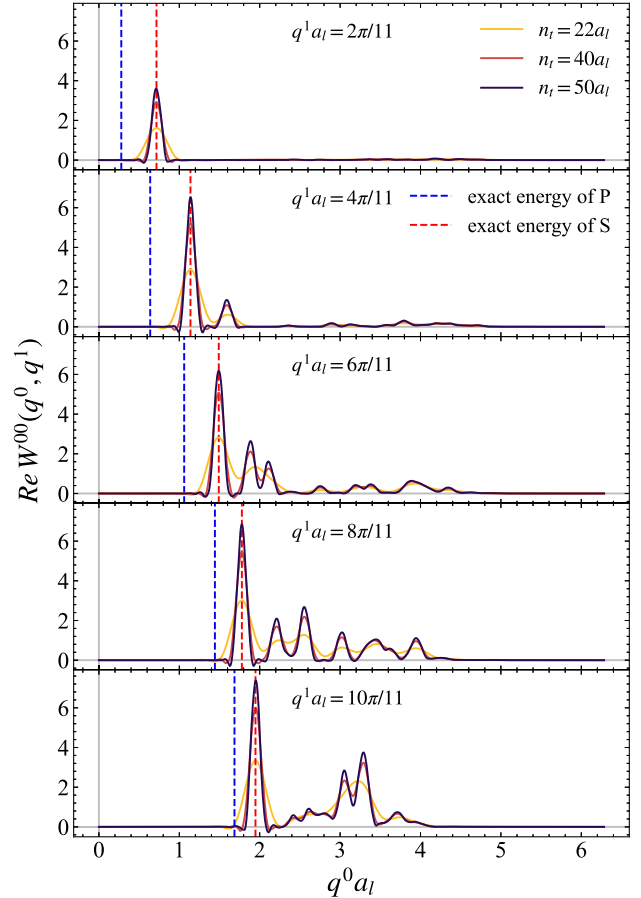


FIG. 4.  $W^{00}(q)$  of U(1) LGT as a function of  $q^0 a_l$  and  $q^1 a_l$ , with  $N_q = 22$ ,  $m = 0.25$ ,  $g = 1$  with the meson mass  $m_h a_l = 1.04$ . Dashed lines denote the exact energy  $\Delta E = p^0 - p_h^0$  of the lowest-lying pseudoscalar (P) and the scalar (S) meson states obtained from the Hamiltonian spectrum.

is odd under charge conjugation,  $CJ^\mu C^{-1} = -J^\mu$ , and the lightest  $q\bar{q}$  P meson bound state is odd under charge conjugation. This implies that only C-even intermediate states can contribute to the hadronic tensor. Consequently, the C-odd final states do not contribute to the hadronic tensor, so the hadronic tensor vanishes until the  $q^0 a_l$  is larger than the energy threshold of producing the C-even S meson. This selection rule relies on charge conjugation symmetry, which is preserved in our Wilson-fermion formulation. While we have checked numerically that this symmetry is violated in staggered fermion formalism. We emphasize that, although the momentum transfer accessible in the present calculations is limited and remains of order  $Q^2 \sim m_h^2$ , all contributions shown in Fig. 4 are inelastic, since the final states  $|X\rangle$  differ from the initial hadron  $|h\rangle$ . Therefore, no elastic contribution ( $X = h$ ) appears in the hadronic tensor in the present calculation. Moreover, one can see that the first peak of  $W^{00}(q^0, q^1)$  moves to larger values of  $q^0 a_l$  as  $q^1 a_l$  increases; this is because the  $p^0 a_l$  of one-particle state increases when it has larger and larger spatial momentum. At higher energies, contributions from multiparticle final states become significant.

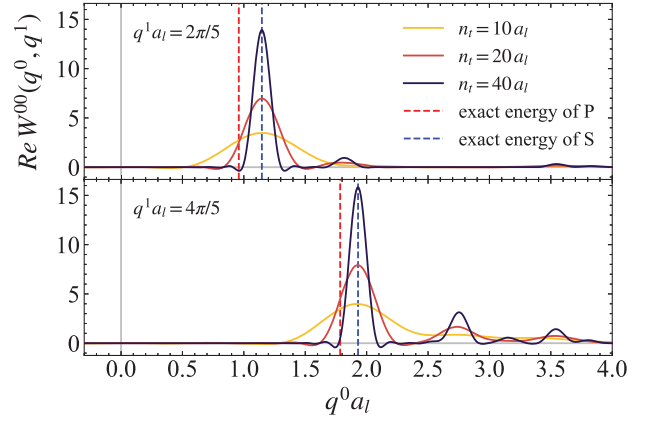
In Eq. (15), the integration over time  $t$  runs from  $-\infty$  to  $\infty$ . For numerical evaluation, a finite cutoff  $(-n_t, n_t)$  in  $t$  is required, which replaces the exact delta function with a broadened peak. We multiply by a Hann window function in the time domain to reduce boundary effects [67,68]. As shown in Fig. 4, the peak width decreases while its height increases as  $n_t$  grows.

### B. Hadronic tensor of $q\bar{q}$ meson and $qq$ baryon states in SU(2) LGT

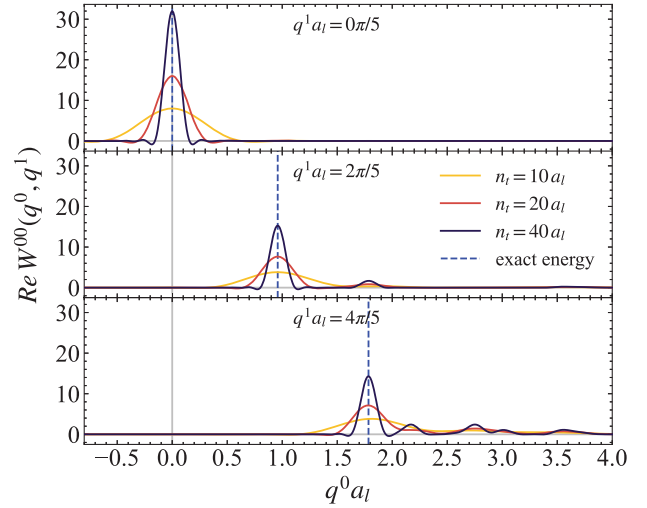
The hadronic tensors of the lowest-lying  $q\bar{q}$  meson and  $qq$  baryon bound states are shown in Figs. 5(a) and 5(b), respectively. We also find a series of sharp peaks in  $W^{00}(q)$  located in the exact hadron energy region, which, as discussed before, originate from one-particle final states  $|h'(p')\rangle$ . To isolate the contribution of each peak, integrate  $W^{00}$  over a small region  $R$  in  $q^0$  that contains  $p^0$  but excludes other peaks

$$\int_R dq^0 W^{00}(q) = \frac{2\pi}{2E_{p'}} |\mathcal{J}^0|^2, \quad (30)$$

where  $|\mathcal{J}^\mu|^2 \equiv |\langle h|J^\mu(0)|h'\rangle|^2$ . The region of integration  $R$  must be chosen large enough to capture the broadened peak but small enough to exclude nearby peaks such that the resulting integral remains approximately invariant. For comparison, the matrix element  $|\mathcal{J}^0|^2$  is also computed directly using exact diagonalization (ED). Figure 6 shows the relative errors  $\varepsilon_{\text{rel}}$  between the hadronic tensor results  $|\mathcal{J}_{\text{HT}}^0|^2$  and the direct calculation  $|\mathcal{J}_{\text{DC}}^0|^2$  for various time truncations  $n_t$ ,



(a)  $W^{00}(q)$  of meson state in SU(2) LGT.



(b)  $W^{00}(q)$  of baryon state in SU(2) LGT.

FIG. 5.  $W^{00}(q)$  of SU(2) LGT as functions of  $q^0$  and  $q^1$ , with  $N_q = 20$ ,  $g = 1$ , and  $m = 0.30$  with  $m_h a_l = 0.83$  for both the meson and baryon states.

$$\varepsilon_{\text{rel}} = \frac{||\mathcal{J}_{\text{HT}}^0|^2 - |\mathcal{J}_{\text{DC}}^0|^2|}{|\mathcal{J}_{\text{DC}}^0|^2}, \quad (31)$$

indicating that the chosen time truncation is sufficient to guarantee the required accuracy.

Also, the matrix element  $\mathcal{J}^\mu$  can be expressed as the form factor through Lorentz symmetry:

$$\langle h(p')|J^\mu(0)|h(p)\rangle = (p + p')^\mu F(Q^2). \quad (32)$$

In Fig. 7, we compare the  $|F_{qq}(Q^2)|^2$  obtained from direct calculation (yellow circles) and extracted from the hadronic tensor (blue crosses). With unit fermion charge, the lowest-lying baryon carries total charge two, and its form factor square thus equals four at  $Q^2 a_l^2 = 0$ . The baryon form factor exhibits a nontrivial  $Q^2 a_l^2$  dependence due to the internal structure of the state [69]. The good agreement

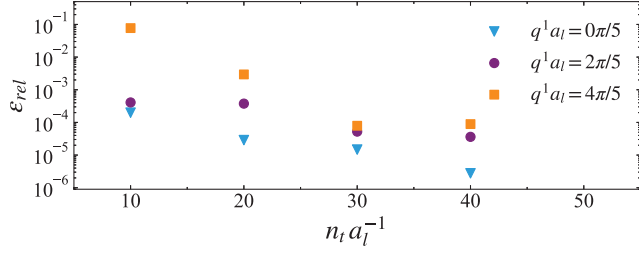


FIG. 6. Relative errors of  $|\mathcal{J}^0|^2$  from hadronic tensor with respect to direct calculation for time truncations  $n_t$  in the SU(2) baryon case.

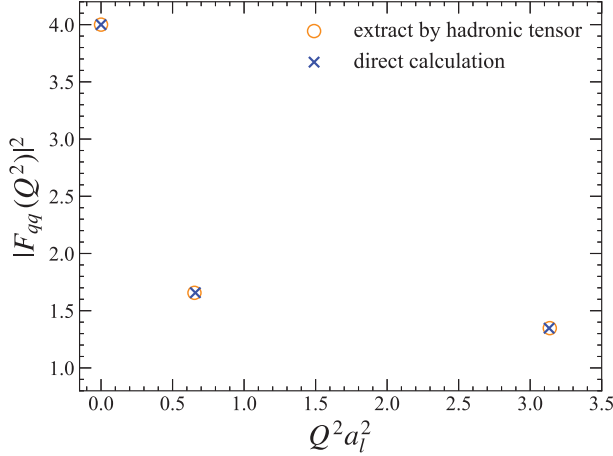


FIG. 7. The  $Q^2 a_t^2$  dependence of  $|F_{qq}(Q^2)|^2$  for the baryon.

between the baryon form factor  $|F_{qq}(Q^2)|^2$  obtained from direct calculation and that extracted from the hadronic tensor shows the correctness of our quantum algorithm as well as the results of the hadronic tensor in SU(2) LGT.

## V. SUMMARY

In this study, we have developed and validated a quantum algorithm for simulating the hadronic tensor in U(1) and SU(2) lattice gauge theories within the Hamiltonian formulation. Using a quantum-number-resolving variational quantum eigensolver, both meson and baryon states can be prepared in well-defined symmetry sectors. The hadronic tensor is then evaluated through real-time current-current correlation functions, which are

accessed by introducing an auxiliary qubit. We apply this algorithm to compute the hadronic tensor of the lightest meson in both U(1) and SU(2) lattice gauge theories, as well as that of the baryon in the SU(2) case, by simulating the quantum circuits on classical computers. The resulting hadronic tensors exhibit clear spectral structures associated with one-particle intermediate states, in agreement with the Hamiltonian spectra. We further demonstrate that charge-conjugation symmetry governs the selection rules of the electromagnetic current, leading to vanishing contributions from C-odd states while allowing nonzero contributions from C-even states.

Furthermore, we extract hadronic form factors from the hadronic tensor. As a benchmark, the same form factors are independently calculated using exact diagonalization. The good agreement between the two approaches validates both the quantum algorithm and the resulting hadronic tensors.

In the future, it is highly desirable to extend this framework to the direct extraction of parton distribution functions from the hadronic tensor. Achieving this goal requires access to the deep inelastic region, where the factorization framework becomes applicable, and consequently demands much larger momentum transfers. On the lattice, this translates into the need for finer momentum resolution and larger lattice sizes, which in turn require significantly more qubits. Preliminary estimates suggest that accessing this kinematic region would require scaling the qubit count by a factor of two to three, surpassing the capabilities of current classical simulators. This makes the hadronic tensor a promising observable for demonstrating the advantages of quantum computing in lattice gauge theory.

## ACKNOWLEDGMENTS

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## DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] J. Collins, *Foundations of Perturbative QCD*, Vol. 32 (Cambridge University Press, Cambridge, England, 2011).
- [2] J. Liang, T. Draper, K.-F. Liu, A. Rothkopf, and Y.-B. Yang (XQCD Collaboration), *Phys. Rev. D* **101**, 114503 (2020).
- [3] K.-F. Liu, *Phys. Rev. D* **62**, 074501 (2000).

- [4] K.-F. Liu and S.-J. Dong, *Phys. Rev. Lett.* **72**, 1790 (1994).
- [5] Y. Fang, C. Gao, Y.-Y. Li, J. Shu, Y. Wu, H. Xing, B. Xu, L. Xu, and C. Zhou, *Sci. China Phys. Mech. Astron.* **68**, 260301 (2025).

- [6] D.-B. Zhang, H. Xing, H. Yan, E. Wang, and S.-L. Zhu, *Chin. Phys. B* **30**, 020306 (2021).
- [7] C. W. Bauer *et al.*, *PRX Quantum* **4**, 027001 (2023).
- [8] C. W. Bauer, Z. Davoudi, N. Klco, and M. J. Savage, *Nat. Rev. Phys.* **5**, 420 (2023).
- [9] S. P. Jordan, K. S. M. Lee, and J. Preskill, *Quantum Inf. Comput.* **14**, 1014 (2014).
- [10] T. Li, W. K. Lai, E. Wang, and H. Xing (QuNu Collaboration), *Phys. Rev. D* **109**, 036025 (2024).
- [11] R. C. Farrell, M. Illa, A. N. Ciavarella, and M. J. Savage, *Phys. Rev. D* **109**, 114510 (2024).
- [12] M. Carena, Y.-Y. Li, T. Ou, and H. Singh, *Phys. Rev. D* **113**, 014502 (2026).
- [13] I. Papaefstathiou, J. Knolle, and M. C. Bañuls, *Phys. Rev. D* **111**, 014504 (2025).
- [14] G.-X. Su, J. J. Osborne, and J. C. Halimeh, *PRX Quantum* **5**, 040310 (2024).
- [15] E. R. Bennewitz *et al.*, *Quantum* **9**, 1773 (2025).
- [16] Y. Chai, Y. Guo, and S. Kühn, *arXiv:2505.21240*.
- [17] K. Ikeda, Z.-B. Kang, D. E. Kharzeev, and W. Qian, *J. High Energy Phys.* **07** (2026) 242.
- [18] R. Joshi *et al.*, *arXiv:2507.12614*.
- [19] J. Schuhmacher *et al.*, *arXiv:2505.20387*.
- [20] Z. Davoudi, C.-C. Hsieh, and S. V. Kadam, *arXiv:2505.20408*.
- [21] R. C. Farrell, N. A. Zemlevskiy, M. Illa, and J. Preskill, *arXiv:2505.03111*.
- [22] J. Barata, J. Hormaza, Z.-B. Kang, and W. Qian, *J. High Energy Phys.* **06** (2026) 168.
- [23] H. Lamm, S. Lawrence, and Y. Yamauchi (NuQS Collaboration), *Phys. Rev. Res.* **2**, 013272 (2020).
- [24] M. Kreshchuk, W. M. Kirby, G. Goldstein, H. Beauchemin, and P. J. Love, *Phys. Rev. A* **105**, 032418 (2022).
- [25] W. Qian, R. Basili, S. Pal, G. Luecke, and J. P. Vary, *Phys. Rev. Res.* **4**, 043193 (2022).
- [26] T. Li, X. Guo, W. K. Lai, X. Liu, E. Wang, H. Xing, D.-B. Zhang, and S.-L. Zhu (QuNu Collaboration), *Phys. Rev. D* **105**, L111502 (2022).
- [27] M. C. Bañuls, K. Cichy, C. J. D. Lin, and M. Schneider, *Phys. Rev. D* **113**, L011502 (2026).
- [28] J.-W. Chen, Y.-T. Chen, and G. Meher, *arXiv:2506.16829*.
- [29] T. Li, X. Guo, W. K. Lai, X. Liu, E. Wang, H. Xing, D.-B. Zhang, and S.-L. Zhu (QuNu Collaboration), *Sci. China Phys. Mech. Astron.* **66**, 281011 (2023).
- [30] K. Bepari, S. Malik, M. Spannowsky, and S. Williams, *Phys. Rev. D* **103**, 076020 (2021).
- [31] A. Florio, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, S. Shi, and K. Yu, *Phys. Rev. Lett.* **131**, 021902 (2023).
- [32] A. Florio, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, S. Shi, and K. Yu, *Phys. Rev. D* **110**, 094029 (2024).
- [33] T. Li, H. Xing, and D.-B. Zhang, *arXiv:2406.05683*.
- [34] J. a. Barata and E. Rico, *Commun. Phys.* **9**, 155 (2026).
- [35] X. Du and W. Qian, *Phys. Rev. D* **109**, 076025 (2024).
- [36] S. Chen, L. Yan, and S. Shi, *arXiv:2412.00662*.
- [37] J.-Q. Gong and J.-C. Yang, *Phys. Rev. D* **112**, 096020 (2025).
- [38] C. Artiago, J. Barata, and E. Rico, *arXiv:2510.16101*.
- [39] M. Will, T. A. Cochran, E. Rosenberg, B. Jobst, N. M. Eassa, P. Roushan, M. Knap, A. Gammon-Smith, and F. Pollmann, *Nature (London)* **645**, 348 (2025).
- [40] Z.-B. Kang, N. Moran, P. Nguyen, and W. Qian, *J. High Energy Phys.* **09** (2025) 176.
- [41] S. Griener, K. Ikeda, and I. Zahed, *Phys. Rev. D* **110**, 076008 (2024).
- [42] S. Griener, J. Montgomery, F. Ringer, and I. Zahed, *Phys. Rev. D* **113**, 074509 (2026).
- [43] C. Muschik, M. Heyl, E. Martinez, T. Monz, P. Schindler, B. Vogell, M. Dalmonte, P. Hauke, R. Blatt, and P. Zoller, *New J. Phys.* **19**, 103020 (2017).
- [44] T. V. Zache, F. Hebenstreit, F. Jendrzejewski, M. K. Oberthaler, J. Berges, and P. Hauke, *Quantum Sci. Technol.* **3**, 034010 (2018).
- [45] N. Klco, J. R. Stryker, and M. J. Savage, *Phys. Rev. D* **101**, 074512 (2020).
- [46] G. Calajò, G. Magnifico, C. Edmunds, M. Ringbauer, S. Montangero, and P. Silvi, *PRX Quantum* **5**, 040309 (2024).
- [47] Y. Y. Atas, J. Zhang, R. Lewis, A. Jahanpour, J. F. Haase, and C. A. Muschik, *Nat. Commun.* **12**, 6499 (2021).
- [48] G. Zhang, X. Guo, E. Wang, and H. Xing, *Phys. Rev. D* **111**, 056031 (2025).
- [49] K. Lee, F. Turro, and X. Yao, *Phys. Rev. D* **111**, 054514 (2025).
- [50] J. H. Lowenstein and J. A. Swieca, *Ann. Phys. (Amsterdam)* **68**, 172 (1971).
- [51] E. Abdalla, M. C. B. Abdalla, and K. D. Rothe, *Non-Perturbative Methods in Two-Dimensional Quantum Field Theory* (World Scientific, Singapore, 1991).
- [52] S. R. Coleman, *Ann. Phys. (Amsterdam)* **101**, 239 (1976).
- [53] T. Byrnes, P. Sriganesh, R. J. Bursill, and C. J. Hamer, *Phys. Rev. D* **66**, 013002 (2002).
- [54] B. Buyens, S. Montangero, J. Haegeman, F. Verstraete, and K. Van Acoleyen, *Phys. Rev. D* **95**, 094509 (2017).
- [55] K. G. Wilson, *Phys. Rev. D* **10**, 2445 (1974).
- [56] J. Kogut and L. Susskind, *Phys. Rev. D* **11**, 395 (1975).
- [57] C. J. Hamer, W.-h. Zheng, and J. Oitmaa, *Phys. Rev. D* **56**, 55 (1997).
- [58] P. Sala, T. Shi, S. Kühn, M. C. Bañuls, E. Demler, and J. I. Cirac, *Phys. Rev. D* **98**, 034505 (2018).
- [59] C. Nagele, J. E. Cejudo, T. Byrnes, and M. Kleban, *Phys. Rev. D* **99**, 094501 (2019).
- [60] E. Arguello Cruz, G. Tarnopolsky, and Y. Xin, *Phys. Rev. D* **112**, 034023 (2025).
- [61] S. Backens, A. Shnirman, and Y. Makhlin, *Sci. Rep.* **9** (2019).
- [62] K. M. Nakanishi, K. Mitarai, and K. Fujii, *Phys. Rev. Res.* **1**, 033062 (2019).
- [63] A. Bärttschi and S. Eidenbenz, *Lect. Notes Comput. Sci.* **11651**, 126 (2019).
- [64] S. Pedernales, J. R. D. Candia, L. Egusquiza, I. J. Casanova, and E. Solano, *Phys. Rev. Lett.* **113**, 020505 (2014).
- [65] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, Cambridge, England, 2010).
- [66] P. Weinberg and M. Bukov, *SciPost Phys.* **2**, 003 (2017).
- [67] F. J. Harris, *Proc. IEEE* **66**, 51 (1978).
- [68] S. R. White and A. E. Feiguin, *Phys. Rev. Lett.* **93**, 076401 (2004).
- [69] B. Kubis and U.-G. Meissner, *Nucl. Phys. A* **679**, 698 (2001).