

Flavor dependence of chiral symmetry breaking and the conformal window

Yi-huai Chen^{1,*}, Yi Lu^{2,3,4,†}, Zhi-wei Wang^{5,‡}, Yu-xin Liu^{2,3,4,§} and Fei Gao^{1,||}

¹*School of Physics, Beijing Institute of Technology, 100081 Beijing, China*

²*Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

³*Center for High Energy Physics, Peking University, 100871 Beijing, China*

⁴*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

⁵*School of Physics, The University of Electronic Science and Technology of China, Chengdu, China*



(Received 9 March 2026; accepted 14 July 2026; published 29 July 2026)

We investigate the phase structure of QCD in the vacuum as a function of quark flavor number N_f within the chiral limit. By self-consistently solving the coupled Dyson-Schwinger equations for the quark and gluon propagators in a minimal QCD scheme, we elucidate the nonperturbative dynamics governing dynamical chiral symmetry breaking. Our calculations determine a critical flavor number of $N_f^c = 6.81$ which marks the chiral symmetry restoration of quarks. Further analysis reveals the critical exponents of the chiral condensate as $-\langle\bar{\psi}\psi\rangle \sim |N_f - N_f^c|^{0.53(9)}$, characterized the second order feature of this phase transition of chiral symmetry. Additionally, we discuss the implications for the walking regime towards the conformal window at larger flavor.

DOI: [10.1103/sbyz-5gl5](https://doi.org/10.1103/sbyz-5gl5)

I. INTRODUCTION

QCD, which governs the strong interaction, exhibits profoundly different behaviors across energy scales. While perturbation theory successfully describes high-energy processes via asymptotic freedom, the low-energy regime remains challenging due to nonperturbative properties in association to confinement, dynamical chiral symmetry breaking (DCSB). The dependence of the quark flavor N_f in QCD is therefore crucial to illustrate the transition of the dynamics between the perturbative and nonperturbative region. As has been widely investigated, the beta function for the coupling becomes positive for $N_f > 16.5$ with $N_c = 3$ in QCD. Just below this critical flavor number where QCD is no longer asymptotic free, an infrared fixed point is also found, i.e., the Caswell-Banks-Zaks (CBZ) fixed point [1,2]. Consequently, there exists the conformal window between $N_f = 16.5$ and the critical flavor number of CBZ fixed point for which the theory is infrared

conformal [1–6]. Moreover, below this conformal window, it may result in a slowly running, i.e., the walking regime, distinct from the situation for lower-flavor numbers which there typically exist the chiral symmetry breaking and confinement [7–10].

While the upper boundary of the conformal window at $N_f = 16.5$ can be accessed through perturbative methods, determining its lower boundary which is signaled by the infrared CBZ fixed point demands nonperturbative analysis [11–13]. The precise determination of this lower boundary remains challenging as the various methods deliver different predictions [8,10,14–17]. The lattice QCD simulations have confirmed the conformality of $N_f = 12$ [18–21], and found that the case of $N_f = 8$ shows the feature that the system is in the pre conformal walking regime [9,22–24]. Notably, in the walking regime, it has been argued that this might result from a symmetric mass generation without the chiral symmetry breaking [9], and hence, it is of great importance to conduct an anatomic analysis on the quark and gluon dynamics separately to really understand the walking regime.

The functional QCD approaches including the Dyson-Schwinger equations (DSEs) approach [8,16] and also the functional renormalization group (fRG) approach [10] opens an access to give such a comprehensive analysis on the flavor dependence of QCD. As the result of quark propagator straightforwardly gives the scale of chiral symmetry breaking, while the gluon propagator can define the mass scale of gluon, potentially originating the symmetric mass generation. Here in specific, we apply the

*Contact author: 3120231486@bit.edu.cn

†Contact author: qwertylou@pku.edu.cn

‡Contact author: zhiwei.wang@uestc.edu.cn

§Contact author: yxliu@pku.edu.cn

||Contact author: fei.gao@bit.edu.cn

DSEs approach which offers a self-consistent framework for investigating conformal dynamics via the infrared behavior of propagators, revealing the emergence of the dynamical mass gap of quark and gluon. We perform a nonperturbative analysis by solving the coupled equations of quark and gluon propagator's Dyson-Schwinger equations in the up to date minimal QCD scheme which quantitatively captures the correct running behavior of the quark, gluon propagator and also quark gluon vertex [25–31]. Such a quantitative calculation helps to classify the actual scenario that happens in the phase transition with varying the quark flavor and also at the finite temperature.

In Sec. II we present the minimal QCD scheme of Dyson-Schwinger equations in particular for the extension to arbitrary flavor number. In Sec. III we first check the consistency of the results at $N_f = 2$ and $N_f = 3$ and then give the results for the critical flavor number where the chiral symmetry is restored. We also discuss the results in association to the relation between this critical point and the conformal window. In Sec. IV, we briefly summarize the main results and provide an outlook on the next steps.

II. FRAMEWORK OF DYSON-SCHWINGER EQUATIONS

Central to our framework is to solve the coupled DSEs for quark and gluon propagator. The quark gap equation serves as the cornerstone in the framework which determines the running of quark masses and order parameter of chiral symmetry, requiring only the gluon propagator and quark-gluon vertex as input. We implement a minimal parametrization of quark gluon vertex based on the Slavnov-Taylor identity and the requirement of renormalization. As demonstrated in Refs. [28–30], this treatment yields results consistent with more elaborate calculations that explicitly solve quark gluon vertex DSEs [27,32]. For the gluon propagator, we adopt the methodology introduced in Refs. [33,34] using the quenched result for $N_f = 0$ obtained from lattice results [35] as a baseline, and calculating the quark loop in the self energy of gluon gap equation. One may also expand the calculation on the gluon propagator at $N_f = 3$ which is also widely studied in functional approaches and lattice QCD simulations. The consistency between these different expansion schemes will be quantitatively demonstrated in the next section.

We begin by briefly deriving the DSEs for QCD from the path integral formalism. The generating functional of the theory is given by

$$Z[J, \eta, \bar{\eta}] = \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} e^{iS + \int JA + \bar{\eta}q + \eta\bar{q}}, \quad (1)$$

where S is the action of QCD,

$$S[\bar{q}, q, A_\mu] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - m)q \right]. \quad (2)$$

The Lagrangian involves the gauge covariant derivative and the non-Abelian field strength tensor $F_{\mu\nu}^a$. To quantize the theory, we employ the Faddeev-Popov procedure in its non-Abelian formulation to fix the gauge. The Becchi-Rouet-Stora (BRS) symmetry of the gauge-fixed action leads to the Slavnov-Taylor identities (STIs) and the non-Abelian generalizations of the Ward-Takahashi identities (WTIs) in QED. This gauge-fixing process necessitates the introduction of Faddeev-Popov ghost fields to ensure that physical observables remain gauge-independent. The generating functional of the theory can be Legendre-transformed to introduce the quantum effective action Γ , whose functional derivatives are the one-particle irreducible (1PI) correlation functions [36,37],

$$\Gamma[\phi^c] = -\ln Z[j] + \phi_i^c j_i, \quad \phi_c^i = \frac{\delta \ln Z[j]}{\delta j_i}. \quad (3)$$

Based on the action principle, the Green's functions of QCD can be derived by performing functional variations on the generating functional, such as the quark propagator,

$$S[x, y, A_\mu] = \left(\frac{\delta^2 \Gamma}{\delta \bar{q}(x) \delta q(y)} \right)_{\bar{q}=q=0}^{-1}. \quad (4)$$

The DSE for quark propagator, i.e., the quark gap equation, can be expressed in momentum space (cf. Fig. 1) as

$$S^{-1}(p) = S_0^{-1}(p) + \Sigma(p), \quad (5)$$

with

$$\Sigma(p) = \frac{4}{3} g^2 \int \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}(p-q) \gamma_\mu S(q) \Gamma_\nu(q, p).$$

The dressed quark propagator $S(p)$ is obtained by solving the DSE depicted in Fig. 1, which involves the bare quark propagator $S_0(p)$ and the quark self-energy $\Sigma(p)$. The interaction kernel for the self-energy is defined by the gluon propagator $D_{\mu\nu}(k)$ and the quark-gluon vertex $\Gamma_\nu(p, q)$. The DSE of the gluon propagator is also employed here for the calculation, but only the self energy of quark loop is involved. The quark gluon vertex $\Gamma_\nu(p, q)$ is determined by the combination of quark and gluon dressing functions. This self-consistent interplay between the quark propagator and the interaction kernel is fundamental to the DSE formalism and drives DCSB. The key components and justification for the quark gap equation are detailed in Sec. II A, while for the calculation of the gluon propagator $D_{\mu\nu}(k)$ is put in Sec. II B.

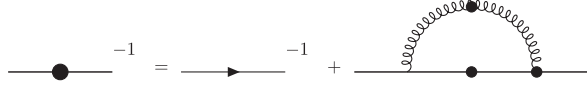


FIG. 1. The Dyson Schwinger equation for quark propagator, i.e., the quark gap equation.

A. The quark gap equation

To solve the nonperturbative gap equation, one can expand the quark and gluon propagator on a complete basis with the Dirac vector and scalar dressing functions, $A(p^2)$ and $B(p^2)$, respectively. In the Landau gauge, with the color indices being factored out directly, the quark and gluon propagators take the following form:

$$S^{-1}(p) = i\not{p}A(p) + B(p) \quad (6)$$

and

$$D_{\mu\nu}(k) = G_A(k)\Pi_{\mu\nu}^\perp(k), \quad \Pi_{\mu\nu}^\perp(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \quad (7)$$

with

$$G_A(k) = Z(k)/k^2, \quad (8)$$

where $Z(k^2)$ is the gluon dressing function. In the Landau gauge the gluon propagator is transverse, $k_\mu D_{\mu\nu}(k) = 0$, so only transverse quark-gluon vertex components contribute to $\Sigma(p)$; longitudinal parts are projected out by $\Pi_{\mu\nu}^\perp(k)$. The last key component is the quark-gluon vertex. In the vacuum, its complete tensor basis consists of twelve independent structures, eight of which comprise the transverse part which is required in the Landau gauge,

$$\Gamma_\mu(q, p) = \sum_{i=1}^{12} T_\mu^i(q, p) \lambda^i(q, p). \quad (9)$$

Continuum and lattice studies further indicate that the leading nonperturbative strength is carried by the Dirac (vector) and Pauli (tensor) structures over a wide momentum range [27,38,39]. The quark gluon vertex can be then simplified as

$$\Gamma_\mu(q, p) = T_\mu^1(q, p) \lambda^1(q, p) + T_\mu^4(q, p) \lambda^4(q, p), \quad (10)$$

with

$$\begin{aligned} T_\mu^{(1)}(q, p) &= -i\gamma_\mu, \\ T_\mu^{(4)}(q, p) &= -i\sigma_{\mu\nu} k^\nu, \quad \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]. \end{aligned}$$

Where k is the gluon propagator momentum. The dressing of the tensor structures is constrained by the STI and the

requirement of renormalization. Since the STI constrains the contracted vertex $k^\mu \Gamma_\mu$ and $k^\mu \sigma_{\mu\nu} k^\nu = 0$, the ghost factor appearing in the STI can only dress the Dirac component in our two-term truncation, which leads to $\lambda^1(q, p) = F(k^2) \Sigma_A(q, p)$ (the remaining STI-induced structures are omitted here). Thus we have

$$\lambda^1(q, p) = F(k^2) \Sigma_A(q, p), \quad \Sigma_A(q, p) = \frac{A(p) + A(q)}{2}. \quad (11)$$

Where $F(k^2)$ is the ghost dressing $k^2 D(k^2)$, $D(k^2)$ is the ghost propagator,

$$\lambda^4(q, p) = \Delta_B(q, p) / \sqrt{Z(k^2)}, \quad \Delta_B(q, p) = \frac{B(p) - B(q)}{p^2 - q^2}. \quad (12)$$

For the ghost dressing function that is required in the λ^1 expansion, the increase of the flavor number produces only subleading corrections to the ghost propagator, as it does not directly appear in the self energy of the ghost propagator and comes only from the quark loop of the gluon propagator. We therefore neglect its flavor dependence and adopt the fitting result [29,30,40] that is based on the fRG calculation from Ref. [26] with the fitting formula being put in Appendix. In the following, we will elaborate how to include the flavor dependence of the gluon propagator.

B. The quark loop in gluon DSE

The gluon propagator satisfies the Dyson-Schwinger equation,

$$D_{\mu\nu}(k)^{-1} = [D_0(k)]_{\mu\nu}^{-1} + \Pi_{\mu\nu}(p)^{gh,gl,qu}, \quad (13)$$

where $\Pi_{\mu\nu}^{gh}$, $\Pi_{\mu\nu}^{gl}$, and $\Pi_{\mu\nu}^{qu}$ denote the ghost-, gluon-, and quark-loop contributions to the gluon self-energy, respectively. The Yang-Mills sector is relatively separable and has been widely studied [35,41–48]. Here we apply a fit formula based on the lattice calculation in pure Yang-Mills theory in Ref. [35], which its specific form is put in the Appendix. In the present setup, the contributions of gluons and ghost loop are absorbed into the quenched gluon propagator introduced above (grey blob in Fig. 2), and we explicitly compute the quark-loop back-reaction $\Pi_{\mu\nu}^{qu}$ below.

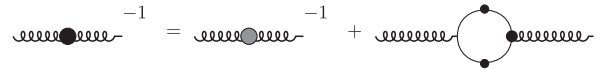


FIG. 2. The Dyson-Schwinger equation for the gluon propagator, with the grey point standing for the quenched gluon propagator calculated in Yang-Mills theory and thus only the quark loop is left aside in this DSE of gluon propagator.

The quark loop's contribution,

$$\Pi_{\mu\nu}^{qu}(p) = \frac{-g^2 N_f}{2} \int \frac{d^4 q}{(2\pi)^4} \text{Tr}[\gamma_\mu S(q) \Gamma_\nu(q, p) S(k)], \quad (14)$$

where p is the external gluon momentum while q and $k = p - q$ denote the internal quark loop momenta. We adopt the transverse tensor structures λ_1 and λ_4 for the quark-gluon vertex, consistent with the gap equation. The Pauli dressing λ_4 is driven by dynamical chiral symmetry breaking; in our STI-inspired construction $\lambda_4 \propto \Delta_B$ and therefore becomes suppressed when the quark mass function $M(p^2) = B(p^2)/A(p^2)$ decreases at large N_f toward the (near-)conformal regime. In fact, the numerical result confirms that λ_4 term does not have impacts yet at small flavor number and can be safely neglected.

In the gluon DSE, the introduction of an ultraviolet cutoff Λ induces spurious quadratic divergences. To properly extract the momentum-dependent gluon dressing function in a gauge-invariant manner, we employ the Brown-Pennington projector [49,50], which is defined as

$$\mathcal{P}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{4p_\mu p_\nu}{p^2}. \quad (15)$$

Note that $P_{\mu\mu}(p) = 0$, i.e., $P_{\mu\nu}(p)$ is traceless, and therefore removes any pure-trace contribution $\propto \delta_{\mu\nu}$ (including the spurious $\sim \delta_{\mu\nu} \Lambda^2$ term induced by a hard cutoff Λ). Contracting the Brown-Pennington projector with the free Lorentz indices of the quark loop, and one gets

$$\begin{aligned} \Pi(p) &= \mathcal{P}_{\mu\nu} \Pi_{\mu\nu}^q \\ &= -\frac{4}{3} g^2 N_f \int \frac{d^4 q}{(2\pi)^4} \lambda_1 A(k) A(q) \left(\frac{4kp \cdot kq}{p^2} - kq \right). \end{aligned} \quad (16)$$

In the Landau gauge, this contraction isolates the transverse (physical) part of the quark polarization tensor that enters the gluon dressing function. After performing the Dirac trace, the scalar dressing B contributes only to the pure-trace part $\propto \delta_{\mu\nu}$ (while the mixed AB terms vanish due to an odd- γ trace), which is removed by the traceless Brown-Pennington projector $\mathcal{P}_{\mu\nu}$; this is why Eq. (16) depends only on $A(q^2)A(k^2)$. Note that a complete dynamical description must account for the back-reaction of quark loops on the gluon propagator. Contributions from quark loops of different flavors collectively determine the dynamics of the gluon propagator. In particular, increasing the number of flavors enhances screening, which dynamically suppresses the infrared strength of the gluon propagator and, in turn, weakens the effective quark-gluon coupling.

This completes the minimal QCD scheme of DSEs with the extension for large flavor numbers. Note, however, that this quenched-baseline subtraction (difference) scheme, in

which the quark loop is inserted into the quenched gluon propagator, should not be pushed to very large N_f . In particular, the fact that it always maintains a mass gap in the gluon propagator indicates that it cannot correctly capture the dynamics once the theory loses the asymptotic freedom at $N_f \geq 16.5$, where large- N_f studies suggest a qualitatively different (possibly UV-safe) RG behavior [51–53].

C. Renormalization scheme

In our scheme, the quantities that require the renormalization are the quark and gluon propagator. We employ the additive renormalization scheme as in Ref. [27]. The bare inverse quark two-point function derived from the renormalized action is

$$S_0^{-1}(p) = iZ_2 \not{p} + Z_{m_q} m_q, \quad (17)$$

with Z_2 and Z_{m_q} the quark wave function and mass renormalization constants, and m_q the bare current mass. The respective renormalization factors would be given by

$$Z_2 = 1 + \frac{\text{tr}[i\not{p}\Sigma(p)]}{p^2} \Big|_{p^2=\mu^2}, \quad Z_{m_q} = 1 - \frac{\text{tr}[\Sigma(p)]}{m_q} \Big|_{p^2=\mu^2}, \quad (18)$$

with the renormalization scale setting as $\mu = 10$ GeV. After including this renormalization constants in the gap equation, one gets

$$A(p = \mu) = 1, \quad B(p = \mu) = m_q.$$

For the gluon propagator, the input data of the quenched gluon propagator has been renormalized at $\mu = 10$ GeV as described in Appendix. Only the difference of the self energy between finite flavor and quenched case requires the renormalization procedure, and thus here we set

$$\Pi(p; \mu) = \Pi(p) - \Pi(p = \mu). \quad (19)$$

Since here we apply the ansatz of quark gluon vertex as in Eqs. (11) and (12) without solving the DSE for the quark gluon vertex, the renormalization constant of the quark gluon vertex Z_1 is underdetermined. Here we apply the additive renormalization scheme and simply apply $Z_1 = 1$ which then leads to

$$Z_1 g^2 \Gamma_\mu(q, p) = g^2(\mu) [\lambda^1 T_\mu^{(1)}(q, p) + \lambda^4 T_\mu^{(4)}(q, p)]. \quad (20)$$

If one applies the QED-like multiplicative renormalization scheme, one can also choose $Z_1 = Z_2$. The different choice of Z_1 comes from the fact that the subtraction scheme differs in different renormalization scheme, and hence, the divergence of the self energy can be attributed to different renormalization constants. Such different choices

of Z_1 can be equivalently absorbed into the definition of the coupling together with the Ansatz of quark gluon vertex as in Eq. (20), and hence the value of the coupling may differ in different renormalization and truncation scheme. Here we apply $\alpha(\mu) = g^2(\mu)/(4\pi) = 0.28$ at renormalization scale $\mu = 10$ GeV together with the Ansatz in Eqs. (11) and (12). This then completes the calculation scheme for the coupled DSEs for quark and gluon propagator. In the next section, we will depict the numerical results of the flavor dependence for both the quark and gluon propagator.

III. NUMERICAL RESULTS

We present our numerical results for theories with different numbers of flavors N_f . Our computation proceeds stepwise along the flavor axis, from $N_f = 0$ to the target flavor number. At each fixed N_f , the quark gap equation and the gluon DSE are iterated self-consistently; the quark-loop contribution updates the gluon propagator, which in turn refines the quark propagator via the gap equation. This alternating iteration continues until both propagators converge at the given N_f . The converged quark propagator then provides the initial guess for the next flavor step. Thus, the coupled system is fully solved at every intermediate N_f , and the backreaction of quark loops on the gluon propagator is self-consistently accounted for at each stage.

A. The benchmark calculation for the gluon and quark propagator

Our calculation starts from the case at $N_f = 0$ with the quenched gluon propagator from lattice QCD being put in and we rigorously solve the Dyson-Schwinger equation (gap equation) to obtain the quark propagator in the minimal QCD scheme as described above. To make sure our calculation captures the correct quark number dependence, we first verify the consistency between the resulting $N_f = 2$ and 3 system and the previous benchmark results of $N_f = 2$ and $N_f = 2 + 1$ from lattice QCD simulation and functional QCD approaches.

We first depict the gluon dressing function for $N_f = 0 + 2$ and $N_f = 0 + 3$ (with $N_f = 0 + N$ denoting a quenched-gluon baseline plus the N -flavor quark-loop backreaction), together with the direct calculation results from lattice QCD and functional QCD approach in Fig. 3. Our results are in good agreement with the previous results from direct calculation, confirming that the difference scheme that inserts the quark loop into the quenched gluon propagator is valid for a large range of flavor number. In particular, due to the consistency of the results at $N_f = 3$, one may equivalently take the $N_f = 3$ result as the baseline and extend to larger N_f by adding the extra-flavor quark-loop backreaction, which further strengthens the reliability of the scheme for large flavor number.

The scalar function $B(p^2)$ of quark propagator for $N_f = 0 + 2$ and $N_f = 0 + 3$ are presented in Fig. 4, which

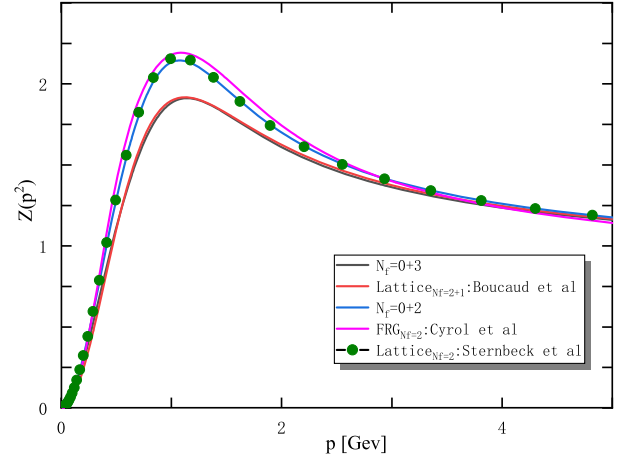


FIG. 3. Gluon propagator dressing function in the current work at $N_f = 0 + 2$ and $0 + 3$, in comparison to the previous results of $N_f = 2$ flavor [26,35] and $2 + 1$ flavor [27,54].

also show good consistency with the previous direct functional-QCD result of Ref. [28], where the DSE calculation is assisted by fRG input at $N_f = 2$, i.e., treating unquenching effects self-consistently rather than using a quenched-gluon baseline with an inserted quark-loop backreaction as in our $N_f = 0 + N$ scheme. The wave function of quark propagator $A(p^2)$ has 5% deviation in comparison to the previous results, due to the neglect of the other Lorentz structures in the quark gluon vertex. Nevertheless, the scalar function of the quark propagator is the central element as the criterion of the chiral symmetry breaking. The consistency of the scalar function of the quark propagator at $N_f = 0 + 2$ and $N_f = 0 + 3$ validate the scheme for the extension to large flavor number.

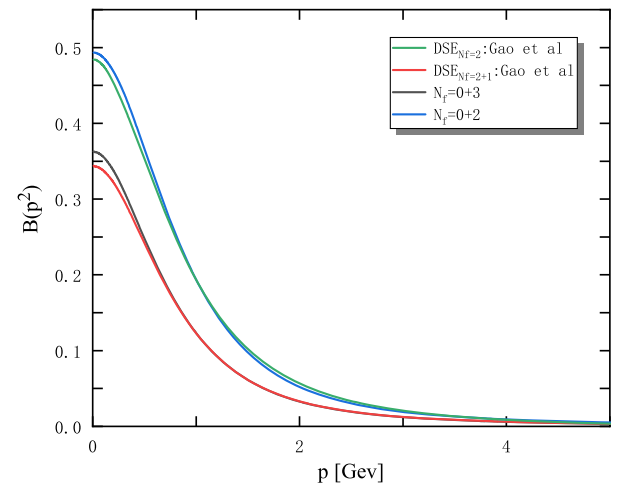


FIG. 4. The obtained scalar function of quark propagator which determines the strength of the chiral symmetry breaking, in comparison with the previous results at $N_f = 2$ and $2 + 1$ flavors [28].

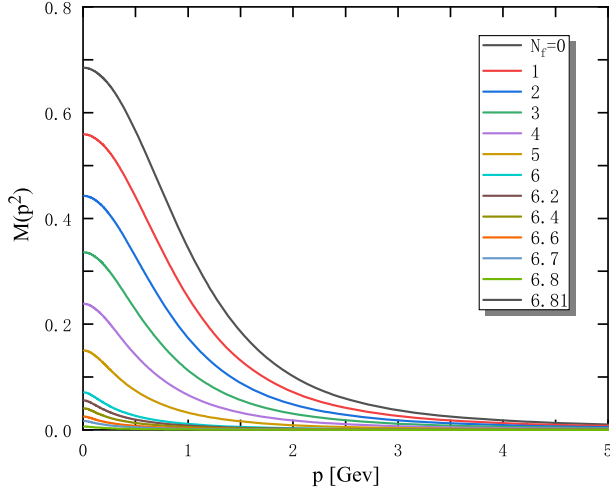


FIG. 5. The evolution of quark mass function along with the change of flavor numbers. At $N_f^c = 6.81$, the quark mass function vanishes.

B. The critical flavor number for chiral symmetry restoration and conformal window

After solving the coupled DSEs, one can study the chiral symmetry directly from the quark propagator. As illustrated in Fig. 5, the quark mass function $M(p^2) = B(p^2)/A(p^2)$ decreases monotonously as the flavor number increases, representing the restoration of the chiral symmetry. At a critical flavor number $N_f^c = 6.81$, the mass function vanishes and the dynamical mass generation ceases entirely.

As usual, we apply the chiral condensate as the order parameter of the chiral symmetry. In chiral limit without the current quark mass, no quadratic divergence is present and the direct trace of the quark propagator gives the correct result up to the renormalization. One can directly define the chiral condensate Δ as

$$\Delta \equiv -\langle \bar{\psi}\psi \rangle = \int \frac{d^4 p}{(2\pi)^4} \text{Tr}[S(p)], \quad (21)$$

where the trace includes the color and Dirac indices. For $N_f = 3$, the chiral condensate at $\mu = 2$ GeV is

$$-\langle \bar{\psi}\psi \rangle^{\frac{1}{3}} = 274 \text{ MeV},$$

which is consistent with the results from lattice QCD and also functional QCD results [27,55].

Further quantitative evidence for the chiral condensate is provided in Fig. 6, which demonstrates a characteristic flavor-induced suppression of the chiral condensate. Moreover, we find that near the critical flavor number, the condensate shows scaling behavior, as it can be described by a power law behavior as

$$\Delta \sim |N_f - N_f^c|^{\frac{1}{\Theta}}, \quad \Theta = 1.88 \pm 0.032. \quad (22)$$

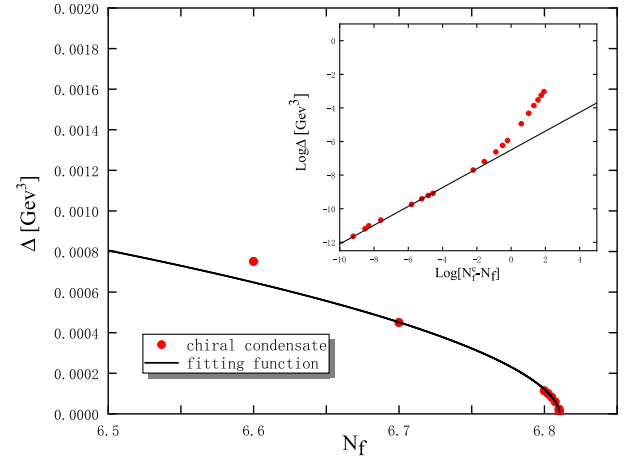


FIG. 6. The chiral condensate Δ as a function of flavor number N_f . Near the critical flavor $N_f^c = 6.81$ where the chiral condensate is zero, it can be fitted by a power law behavior as $|N_f - N_f^c|^{0.53(9)}$.

In the logarithmic plot, the scaling behavior and the scaling region is shown clearly. The scaling region is small as $|1 - N_f^c/N_f| \sim 10^{-3}$. The scaling exponent is consistent with the mean field value $\Theta = 2$ [16].

The previous calculation identifies the critical point at $N_f^c = 6.81$, marking the disappearance of the chiral symmetry breaking. However, it does not yet reach the conformal window, since the gluon propagator still exhibits an infrared mass scale; the gluon dressing function in Fig. 7 shows a clear turnover at $p \sim m_g$, where $m_g^{-2} \equiv G_A(0)$. The mass scale gives a nonzero gluon condensate as suggested in Refs. [56,57], preventing true conformal behavior.

Notably, we observe that as the flavor number increases, the gluon dressing function becomes progressively flatter.

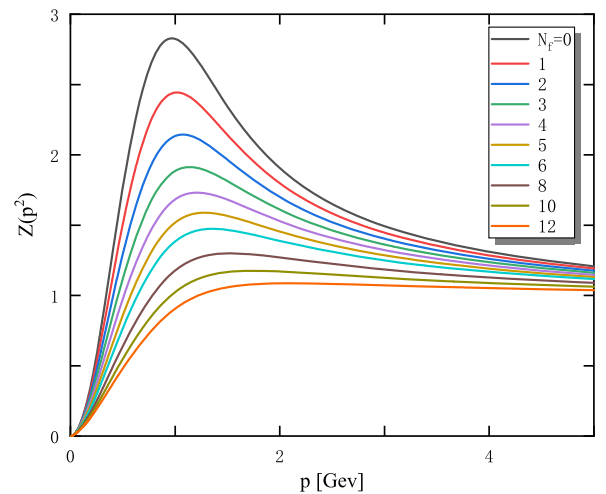


FIG. 7. Gluon propagator dressing function as a function of flavor number. The flattening of the dressing function for large flavor number implies the walking regime.

In our present approximation, the flavor dependence of the ghost dressing function—which arises from ghost-quark scattering and the quark loop in the gluon propagator, both being higher-order corrections in the ghost self-energy—is neglected at leading order. Consequently, the flavor dependence of the coupling is carried entirely by the gluon dressing function, as illustrated by the effective charge or the Taylor coupling [58–60]. The observed flattening of $Z(k^2)$ therefore directly signals the flattening of the running coupling, a smoking gun for the walking regime.

Besides, our results indicate that the walking regime is dominant by the gluon mass generation without chiral symmetry breaking. Such a pattern and the obtained N_f^c for the walking regime is consistent with the previous results in DSE [8]. Note that the current scheme does not include the hadron resonance channel in the quark gluon vertex [61], which has potentially impact on the critical point location and also the critical exponents. Such a subleading channel could make a 10% difference as found in Ref. [62], and one needs to include this channel to make the final conclusion on the exact critical flavor number which is currently in progress. Crucially, if the scale of the theory in such a walking regime comes solely from the gluon condensate, this would support the symmetric mass generation that is proposed to explain the results for lattice QCD simulation with $N_f = 8$ [9,63]. Furthermore, the symmetric mass generation is also applied to explain the emergence of a new symmetry, i.e., the chiral spin symmetry, observed in the QCD matter near above the chiral phase transition temperature [64], where the phase is chiral symmetric but with nonzero gluon condensate. It is therefore interesting to check the chiral spin symmetry among the mesons at $N_f = 8$ to verify the connection of these phenomenons.

It needs to mention that the current minimal QCD scheme employs the difference DSE for the gluon propagator expanding on the quenched propagator, and consequently, it fails to track down the lower boundary of the conformal window, i.e., the location of the CBZ fixed point, where the gluon condensate also vanishes. The incorporation for a full self-consistent determination of gluon and ghost propagator can help to give an estimate for the CBZ fixed point, which we plan to pursue in the future studies.

IV. SUMMARY

This work presents a nonperturbative study of the flavor dependence for chiral symmetry breaking within QCD, employing the Dyson-Schwinger equation approach. A minimal QCD scheme is introduced, which maintains the running behavior of QCD quantitatively, in particular of the quark and gluon propagator. The flavor dependence of the gluon dressing function and the quark mass function is investigated across $N_f \in [0, 12]$. Our results reveal a phase transition marked by the disappearance of the chiral condensate at $N_f^c = 6.81$, with a clear scaling behavior

as $|N_f - N_f^c|^{0.53(9)}$. Beyond this critical point, the flattening of the gluon dressing function indicates that the theory steps into the walking regime where the chiral symmetry is restored while the gluon propagator retains a dynamical mass scale. Limited by the current scheme of calculating the Yang-Mills sector, it is not capable of giving an estimate on the boundary of walking regime and conformal window. Future refinements incorporating self-consistent gluon and ghost propagator calculations will help resolve this question.

Now it is also interesting to consider the mapping between the flavor dependence of QCD and its phase structure at finite temperature. For instance, in QCD with eight massless flavors, a chiral phase transition at finite temperature is not expected, since the theory already lies in the chiral symmetric walking regime at zero temperature with a vanishing chiral condensate. Nevertheless, there may exist a Miransky/BKT-type transition, or a crossover between the walking regime and the conformal limit which can be signaled by the gluon condensate as discussed above. For systems with fewer flavors, this transition may also take place at temperatures above the chiral transition, establishing a possible link between the walking regime and the chiral spin symmetric matter observed in the lattice QCD simulation. Therefore, a systematic investigation for the many-flavor system at finite temperature is in demand to deepen the understanding of the walking regime and to uncover the properties of QCD matter near and above the chiral phase transition temperature.

ACKNOWLEDGMENTS

F.G. and Y.L. thank J. Pawłowski and A. Pastor for very helpful discussions. This work is supported by the National Natural Science Foundation of China under Grants No. 12305134, No. 12247107, No. 12175007, and No. 12475105.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: PARAMETRIZED FORMULA OF THE QUENCHED GLUON PROPAGATOR AND GHOST PROPAGATOR

For the quenched gluon dressing function, we adopt the following parametrization of the lattice data for pure Yang-Mills theory in Ref. [35] similar to Ref. [27],

$$G_A(p) = \frac{Z(p)}{p^2} = \frac{h(a^2 + p^2)/(b^2 + p^2)}{M^2(p^2) + p^2[1 + c \ln(d^2 p^2 + e^2 M^2(p^2))]^r}, \quad (\text{A1})$$

with

$$M^2(p^2) = \frac{f^4}{g^2 + p^2}. \quad (\text{A2})$$

The fitting parameters are given by

$$\begin{aligned} a &= 1 \text{ GeV}, & b &= 0.735 \text{ GeV}, & c &= 0.12, \\ d &= 0.0375 \text{ GeV}^{-1}, & e &= 0.19 \text{ GeV}^{-1}, & f &= 0.4725 \text{ GeV}, \\ g &= 0.56 \text{ GeV}, & h &= 0.8363, & \gamma &= 0.5. \end{aligned}$$

For the ghost dressing function, we adopt the same parametrization as in Ref. [29] which is based on the fRG result from Ref. [26],

$$F(p^2) = \frac{(a_1 + b_1\sqrt{p^2 + p^2})/(c_1 + d_1\sqrt{p^2 + p^2})}{h_1[1 + e_1 \ln(f_1^2 p^2 + g_1^2 M^2(p^2))]^\delta}, \quad (\text{A3})$$

where

$$\begin{aligned} a_1 &= 0.152 \text{ GeV}^2, & b_1 &= 0.697 \text{ GeV}, & c_1 &= 0.0055 \text{ GeV}^2, \\ d_1 &= 0.016 \text{ GeV}, & e_1 &= 0.045, & f_1 &= 0.025 \text{ GeV}^{-1}, \\ g_1 &= 0.0237 \text{ GeV}^{-1}, & h_1 &= 1.10863, & \delta &= 0.27. \end{aligned}$$

$M(p^2)$ is the same function as in quenched gluon fitting function. The renormalization point for both the gluon and ghost dressing functions is set at $\mu = 10 \text{ GeV}$ by rescaling the dressing function to maintain $F(p = \mu), Z(p = \mu) = 1$.

-
- [1] William E. Caswell, Asymptotic behavior of nonAbelian gauge theories to two loop order, *Phys. Rev. Lett.* **33**, 244 (1974).
 - [2] Tom Banks and A. Zaks, On the phase structure of vector-like gauge theories with massless fermions, *Nucl. Phys.* **B196**, 189 (1982).
 - [3] Dennis D. Dietrich and Francesco Sannino, Conformal window of SU(N) gauge theories with fermions in higher dimensional representations, *Phys. Rev. D* **75**, 085018 (2007).
 - [4] Jens Braun, Fermion interactions and universal behavior in strongly interacting theories, *J. Phys. G* **39**, 033001 (2012).
 - [5] Slava Rychkov, EPFL Lectures on Conformal Field Theory in D >= 3 Dimensions *SpringerBriefs in Physics* (2016), 10.1007/978-3-319-43626-5.
 - [6] Frederik F. Hansen, Tadeusz Janowski, Kasper Langæble, Robert B. Mann, Francesco Sannino, Tom G. Steele, and Zhi-Wei Wang, Phase structure of complete asymptotically free SU(N_c) theories with quarks and scalar quarks, *Phys. Rev. D* **97**, 065014 (2018).
 - [7] Kenneth D. Lane and M. V. Ramana, Walking technicolor signatures at hadron colliders, *Phys. Rev. D* **44**, 2678 (1991).
 - [8] Markus Hopfer, Christian S. Fischer, and Reinhard Alkofer, Running coupling in the conformal window of large-Nf QCD, *J. High Energy Phys.* **11** (2014) 035.
 - [9] Anna Hasenfratz, Emergent strongly coupled ultraviolet fixed point in four dimensions with eight Kähler-Dirac fermions, *Phys. Rev. D* **106**, 014513 (2022).
 - [10] Florian Goertz, Álvaro Pastor-Gutiérrez, and Jan M. Pawłowski, Gauge-fermion cartography: From confinement and chiral symmetry breaking to conformality, *Phys. Rev. D* **112**, 034029 (2025).
 - [11] V. A. Miransky and Koichi Yamawaki, Conformal phase transition in gauge theories, *Phys. Rev. D* **55**, 5051 (1997); **56**, 3768(E) (1997).
 - [12] Holger Gies and Joerg Jaeckel, Chiral phase structure of QCD with many flavors, *Eur. Phys. J. C* **46**, 433 (2006).
 - [13] Adi Armoni, The conformal window from the worldline formalism, *Nucl. Phys.* **B826**, 328 (2010).
 - [14] Jens Braun and Holger Gies, Scaling laws near the conformal window of many-flavor QCD, *J. High Energy Phys.* **05** (2010) 060.
 - [15] Claudio Pica and Francesco Sannino, UV and IR zeros of gauge theories at the four loop order and beyond, *Phys. Rev. D* **83**, 035013 (2011).
 - [16] A. Bashir, A. Raya, and J. Rodriguez-Quintero, QCD: Restoration of chiral symmetry and deconfinement for large N_f , *Phys. Rev. D* **88**, 054003 (2013).
 - [17] Jong-Wan Lee, Conformal window from conformal expansion, *Phys. Rev. D* **103**, 076006 (2021).

- [18] Thomas Appelquist, George T. Fleming, and Ethan T. Neil, Lattice study of the conformal window in QCD-like theories, *Phys. Rev. Lett.* **100**, 171607 (2008); **102**, 149902(E) (2009).
- [19] A. Deuzeman, M. P. Lombardo, and E. Pallante, Evidence for a conformal phase in SU(N) gauge theories, *Phys. Rev. D* **82**, 074503 (2010).
- [20] Anna Hasenfratz, Infrared fixed point of the 12-fermion SU(3) gauge model based on 2-lattice MCRG matching, *Phys. Rev. Lett.* **108**, 061601 (2012).
- [21] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki, Low energy spectra in many flavor QCD with $N_f = 12$ and 16, *Proc. Sci. LATTICE2012* (2012) 029.
- [22] Kohtaroh Miura and Maria Paola Lombardo, Lattice Monte-Carlo study of pre-conformal dynamics in strongly flavoured QCD in the light of the chiral phase transition at finite temperature, *Nucl. Phys.* **B871**, 52 (2013).
- [23] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki, Walking signals in $N_f = 8$ QCD on the lattice, *Phys. Rev. D* **87**, 094511 (2013).
- [24] Yasumichi Aoki *et al.*, Light flavor-singlet scalars and walking signals in $N_f = 8$ QCD on the lattice, *Phys. Rev. D* **96**, 014508 (2017).
- [25] Richard Williams, Christian S. Fischer, and Walter Heupel, Light mesons in QCD and unquenching effects from the 3PI effective action, *Phys. Rev. D* **93**, 034026 (2016).
- [26] Anton K. Cyrol, Mario Mitter, Jan M. Pawłowski, and Nils Strodthoff, Nonperturbative quark, gluon, and meson correlators of unquenched QCD, *Phys. Rev. D* **97**, 054006 (2018).
- [27] Fei Gao, Joannis Papavassiliou, and Jan M. Pawłowski, Fully coupled functional equations for the quark sector of QCD, *Phys. Rev. D* **103**, 094013 (2021).
- [28] Fei Gao and Jan M. Pawłowski, QCD phase structure from functional methods, *Phys. Rev. D* **102**, 034027 (2020).
- [29] Yi Lu, Fei Gao, Yu-Xin Liu, and Jan M. Pawłowski, QCD equation of state and thermodynamic observables from computationally minimal Dyson-Schwinger equations, *Phys. Rev. D* **110**, 014036 (2024).
- [30] Yi Lu, Fei Gao, Yu-xin Liu, and Jan M. Pawłowski, Finite density signatures of confining and chiral dynamics in QCD thermodynamics and fluctuations of conserved charges, *Phys. Rev. D* **113**, 054019 (2026).
- [31] Georg Wieland and Reinhard Alkofer, No planar degeneracy for the Landau gauge quark-gluon vertex, 2026.
- [32] Fei Gao and Jan M. Pawłowski, Chiral phase structure and critical end point in QCD, *Phys. Lett. B* **820**, 136584 (2021).
- [33] Christian S. Fischer and Jan Luecker, Propagators and phase structure of $N_f = 2$ and $N_f = 2 + 1$ QCD, *Phys. Lett. B* **718**, 1036 (2013).
- [34] Christian S. Fischer, Jan Luecker, and Christian A. Welzbacher, Phase structure of three and four flavor QCD, *Phys. Rev. D* **90**, 034022 (2014).
- [35] A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, and A. Schiller, Towards the infrared limit in SU(3) Landau gauge lattice gluodynamics, *Phys. Rev. D* **72**, 014507 (2005).
- [36] Reinhard Alkofer and Lorenz von Smekal, The infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states, *Phys. Rep.* **353**, 281 (2001).
- [37] N. Dupuis, L. Canet, A. Eichhorn, W. Metzner, J. M. Pawłowski, M. Tissier, and N. Wschebor, The nonperturbative functional renormalization group and its applications, *Phys. Rep.* **910**, 1 (2021).
- [38] Si-Xue Qin, Lei Chang, Yu-Xin Liu, Craig D. Roberts, and Sebastian M. Schmidt, Practical corollaries of transverse Ward-Green-Takahashi identities, *Phys. Lett. B* **722**, 384 (2013).
- [39] Lei Chang, Yu-Bin Liu, Khépani Raya, J. Rodríguez-Quintero, and Yi-Bo Yang, Linking continuum and lattice quark mass functions via an effective charge, *Phys. Rev. D* **104**, 094509 (2021).
- [40] Hui-wen Zheng, Yi Lu, Fei Gao, Si-xue Qin, and Yu-xin Liu, Effective potential of composite operators in the first order region of the QCD phase transition, *Phys. Rev. D* **109**, 114013 (2024).
- [41] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared, *Phys. Lett. B* **676**, 69 (2009).
- [42] Orlando Oliveira and Paulo J. Silva, The lattice Landau gauge gluon propagator: Lattice spacing and volume dependence, *Phys. Rev. D* **86**, 114513 (2012).
- [43] A. C. Aguilar, D. Binosi, and J. Papavassiliou, Unquenching the gluon propagator with Schwinger-Dyson equations, *Phys. Rev. D* **86**, 014032 (2012).
- [44] Anton K. Cyrol, Leonard Fister, Mario Mitter, Jan M. Pawłowski, and Nils Strodthoff, Landau gauge Yang-Mills correlation functions, *Phys. Rev. D* **94**, 054005 (2016).
- [45] Markus Q. Huber, Nonperturbative properties of Yang-Mills theories, *Phys. Rep.* **879**, 1 (2020).
- [46] Gernot Eichmann, Jan M. Pawłowski, and João M. Silva, Mass generation in Landau-gauge Yang-Mills theory, *Phys. Rev. D* **104**, 114016 (2021).
- [47] A. C. Aguilar, M. N. Ferreira, and J. Papavassiliou, Exploring smoking-gun signals of the Schwinger mechanism in QCD, *Phys. Rev. D* **105**, 014030 (2022).
- [48] M. N. Ferreira and J. Papavassiliou, Gluon mass scale through the Schwinger mechanism, *Prog. Part. Nucl. Phys.* **144**, 104186 (2025).
- [49] Nicholas Brown and M. R. Pennington, Studies of confinement: How quarks and gluons propagate, *Phys. Rev. D* **38**, 2266 (1988).
- [50] Christian S. Fischer and Reinhard Alkofer, Nonperturbative propagators, running coupling and dynamical quark mass of Landau gauge QCD, *Phys. Rev. D* **67**, 094020 (2003).
- [51] Robert Mann, Julia Meffe, Francesco Sannino, Tom Steele, Zhi-Wei Wang, and Chen Zhang, Asymptotically safe standard model via vectorlike fermions, *Phys. Rev. Lett.* **119**, 261802 (2017).
- [52] Oleg Antipin and Francesco Sannino, Conformal Window 2.0: The large N_f safe story, *Phys. Rev. D* **97**, 116007 (2018).
- [53] Oleg Antipin, Nicola Andrea Donni, Francesco Sannino, Anders Eller Thomsen, and Zhi-Wei Wang, Gauge-Yukawa

- theories: Beta functions at large N_f , *Phys. Rev. D* **98**, 016003 (2018).
- [54] Ph. Boucaud, F. De Soto, K. Raya, J. Rodríguez-Quintero, and S. Zafeiropoulos, Discretization effects on renormalized gauge-field Green's functions, scale setting, and the gluon mass, *Phys. Rev. D* **98**, 114515 (2018).
- [55] S. Aoki *et al.*, FLAG review 2019: Flavour lattice averaging group (FLAG), *Eur. Phys. J. C* **80**, 113 (2020).
- [56] John M. Cornwall, Dynamical mass generation in continuum QCD, *Phys. Rev. D* **26**, 1453 (1982).
- [57] Frank R. Graziani, The gluon condensate and the effective gluon mass, *Z. Phys. C* **33**, 397 (1987).
- [58] Philippe Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, and J. Rodríguez-Quintero, Ghost-gluon running coupling, power corrections and the determination of $\Lambda(\overline{\text{MS}})$, *Phys. Rev. D* **79**, 014508 (2009).
- [59] A. C. Aguilar, D. Binosi, and J. Papavassiliou, QCD effective charges from lattice data, *J. High Energy Phys.* **07** (2010) 002.
- [60] S. Zafeiropoulos, P. Boucaud, F. De Soto, J. Rodríguez-Quintero, and J. Segovia, Strong running coupling from the gauge sector of domain wall lattice QCD with physical quark masses, *Phys. Rev. Lett.* **122**, 162002 (2019).
- [61] Fei Gao, Yi Lu, Si-xue Qin, Zhan Bai, Lei Chang, and Yu-xin Liu, Illustrating the liquid gas transition of nuclear matter in QCD, *Phys. Rev. D* **111**, L091503 (2025).
- [62] Pascal J. Gunkel and Christian S. Fischer, Locating the critical endpoint of QCD: Mesonic backcoupling effects, *Phys. Rev. D* **104**, 054022 (2021).
- [63] Oliver Witzel and Anna Hasenfratz, Investigating SU(3) with $N_f = 8$ fundamental fermions at strong renormalized coupling, *Proc. Sci. LATTICE2024* (2025) 146.
- [64] Leonid Ya. Glozman, Owe Philipsen, and Robert D. Pisarski, Chiral spin symmetry and the QCD phase diagram, *Eur. Phys. J. A* **58**, 247 (2022).