

Revisiting the model for radiative neutrino masses with dark matter in the $U(1)_{B-L}$ gauge theory

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The radiative seesaw model with gauged $U(1)_{B-L} \times \mathbb{Z}_2$ extension is a well-motivated scenario that gives consistent predictions of active neutrino masses and the abundance of dark matter. Majorana masses of right-handed neutrinos, the lightest of which can be identified as dark matter, are given by the spontaneous breaking of the $U(1)_{B-L}$ gauge symmetry. We revisit this model with the latest constraints from dark-matter searches, neutrino oscillations, flavor experiments, and collider experiments. We examine several benchmark scenarios and identify the valid parameter space of this model, finding that there are still allowed regions under the latest experimental constraints. We present new viable benchmark scenarios for this model, i.e., the right-handed neutrino dark-matter scenario and the scalar dark-matter scenario. We also mention the testability of these benchmark scenarios in future experiments.

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I. INTRODUCTION

Having the discovery of the Higgs boson at the Large Hadron Collider (LHC) in 2012, all the fundamental particles in the Standard Model (SM) of particle physics have been verified by experiments. However, there are some physics phenomena that the SM cannot explain. First, observations of the neutrino oscillation reveal that neutrinos have nonzero masses [1]. Second, the latest observation data provided by the *Planck* satellite [2] shows that approximately one-quarter of the energy density in our Universe is composed of nonbaryonic and nonluminous matter, known as dark matter (DM). If the nature of dark matter is the weakly interacting massive particle (WIMP) [3,4], the observed relic abundance can be satisfied with electroweak mass-scale DM candidates. In addition, the Universe exhibits matter and antimatter asymmetry.

A pioneer in explaining neutrino mass is the so-called seesaw mechanism [5–15], which generates tiny neutrino masses at tree level. For example, the type-I seesaw model [5–9] can explain tiny neutrino masses with very heavy right-handed (RH) neutrinos or very small Yukawa couplings. On the other hand, to explain not only tiny neutrino masses but also dark matter simultaneously, models with

radiatively generated neutrino masses [16–22] have often been studied, in which dark-matter candidates are running in the loop of neutrino masses [19–22]. A representative example is the model proposed by Tao [20] and Ma [21], which can explain observed neutrino masses with TeV scale dark matter. However, this model does not explain the Majorana masses of RH neutrinos. The origin of Majorana masses of RH neutrinos can be explained in the radiative seesaw model with a gauged $U(1)_{B-L} \times \mathbb{Z}_2$ extension [23]. RH neutrinos receive their masses through the spontaneous symmetry breaking (SSB) of the $U(1)_{B-L}$ gauge symmetry [24]. In Ref. [23], the observed DM relic abundance can be realized through the pair annihilation via the s -channel scalar exchange by the mixing between the SM Higgs field and the extra scalar singlet. The stability of DM, which is the lightest RH neutrino, is guaranteed by the unbroken $\mathbb{Z}_2$ symmetry. Tiny neutrino masses are generated radiatively via the one-loop induced operator. Some phenomenology of this model was studied in Refs. [25,26].

In this paper, we revisit the radiative seesaw model with gauged $U(1)_{B-L} \times \mathbb{Z}_2$ extension [23], which was originally proposed before the discovery of the Higgs boson. Since experiments have made great progress in recent years, including flavor experiments [27–31], collider experiments [32–41], neutrino oscillation measurements [42], and dark-matter searches [2,43,44], it would be valuable to study whether there is still allowed parameter region where tiny neutrino masses and dark matter can be explained simultaneously under current constraints. We first discuss theoretical and experimental constraints on this model. We then consider neutrino masses, lepton flavor violation, and DM physics in this model. We show the relic density of DM at

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TABLE I. Particle content and particle quantum numbers.

	Q^i	u_R^i	d_R^i	L^i	e_R^i	Φ	N_α	η	S
$SU(3)_C$	3	3	3	1	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	2	1	2	1
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	0
$U(1)_{B-L}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	-1	0	-1	0	+2
$\mathbb{Z}_2$	+	+	+	+	+	+	-	-	+

our benchmark point. The possible parameter space under current constraints is shown. We then scan the allowed parameter space and give possible mass regions of the DM. Finally, we discuss the testability of this model at future experiments, including flavor experiments, collider experiments, and DM detection experiments.

This paper is organized as follows. In Sec. II, we introduce the content of this model and give predictions of neutrino masses. In Sec. III, we show the theoretical constraints and the latest experimental constraints for this model. In Sec. IV, we give predictions of lepton flavor violation in this model. In Sec. V, we give an analysis of the lightest RH neutrino DM scenario and briefly discuss the lightest inert scalar DM scenario. In Sec. VI, we mention the testability of this model in future experiments. In Sec. VII, we summarize this work and give our conclusions.

II. THE MODEL

The radiative seesaw model with a gauged $U(1)_{B-L} \times \mathbb{Z}_2$ extension [23] is a simple extension of the SM, in which we try to explain radiative generation of the neutrino masses with providing possible dark-matter candidates. This model is an extension of the SM with three $\mathbb{Z}_2$ odd right-handed neutrinos N_α ($\alpha = 1, 2, 3$), a $\mathbb{Z}_2$ odd scalar $SU(2)_L$ doublet η , a scalar singlet S , and an electrically neutral $U(1)_{B-L}$ gauge boson Z' . The Lagrangian is invariant under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ with an unbroken $\mathbb{Z}_2$ discrete symmetry. The particle content is shown in Table I.

The relevant interaction $\mathcal{L}_{\text{int}}$ for our discussion is given by

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{Yukawa}}^{\text{SM}} + \mathcal{L}_N - V(\Phi, \eta, S), \quad (1)$$

where $\mathcal{L}_{\text{Yukawa}}^{\text{SM}}$ is the SM Yukawa interaction and

$$\mathcal{L}_N = \sum_{\alpha=1}^3 \left(- \sum_{i=1}^3 g_{i\alpha} \bar{L}_i \tilde{\eta} N_\alpha - \frac{y_\alpha^R}{2} \bar{N}_\alpha^c S N_\alpha + \text{H.c.} \right), \quad (2)$$

with $\tilde{\eta} = i\tau^2 \eta^*$, where $i, \alpha = 1, 2, 3$ are flavors of the leptons. Without loss of generality, the Yukawa coupling y_α^R of RH neutrinos can be flavor diagonal. The Yukawa coupling among $\bar{L}_i \Phi N_\alpha$ is prohibited due to the unbroken

$\mathbb{Z}_2$ symmetry. The active neutrinos remain massless at tree level and obtain tiny masses at one-loop level.

The scalar potential $V(\Phi, \eta, S)$ in this model is given by

$$\begin{aligned} V(\Phi, \eta, S) = & \mu_1^2 |\Phi|^2 + \mu_2^2 |\eta|^2 + \mu_3^2 |S|^2 + \frac{\lambda_1}{2} |\Phi|^4 \\ & + \frac{\lambda_2}{2} |\eta|^4 + \lambda_3 |S|^4 + \lambda_3 |\Phi|^2 |\eta|^2 + \lambda_4 |\Phi^\dagger \eta|^2 \\ & + \frac{\lambda_5}{2} [(\Phi^\dagger \eta)^2 + \text{H.c.}] + \tilde{\lambda} |\Phi|^2 |S|^2 + \lambda |\eta|^2 |S|^2, \end{aligned} \quad (3)$$

where λ_5 can be taken as real by rephasing the fields. We assume that $\mu_1^2 < 0$, $\mu_3^2 < 0$, and $\mu_2^2 > 0$ so that Φ and S receive nonzero vacuum expectation values (VEVs) by SSB of the $SU(2)_L \times U(1)_Y$ and $U(1)_{B-L}$ gauge symmetries. The VEV of η remains zero.

The $U(1)_{B-L}$ gauge symmetry is assumed to be spontaneously broken above the electroweak scale. The scalar singlet S is parametrized as

$$S = \frac{1}{\sqrt{2}} (v_S + \phi_S + iz_S), \quad (4)$$

where v_S is the VEV of the $U(1)_{B-L}$ symmetry breaking, ϕ_S is a neutral scalar boson, and z_S is the Nambu-Goldstone (NG) boson. The Z' boson obtains its mass as $m_{Z'} = 2g_{B-L} v_S$ when its longitudinal mode absorbs z_S , where g_{B-L} is the gauge coupling for the $U(1)_{B-L}$ symmetry. In the $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge sector, we assume that the kinetic mixing between Z and Z' is negligible. RH neutrinos N_α also receive their masses as

$$m_{N_\alpha} = \frac{y_\alpha^R v_S}{\sqrt{2}}. \quad (5)$$

After the electroweak symmetry breaking (EWSB), the $SU(2)_L$ scalar doublet Φ is parametrized as

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi + iz) \end{pmatrix}, \quad (6)$$

where v ($\simeq 246$ GeV) is the VEV of the electroweak symmetry breaking and G^+ and z are NG bosons absorbed by longitudinal modes of the electroweak gauge bosons, i.e., W and Z bosons, respectively. The stationary conditions give the following relations:

$$\mu_1^2 + \frac{\lambda_1}{2} v^2 + \frac{\tilde{\lambda} v_S^2}{2} = 0, \quad \mu_3^2 + \lambda_3 v_S^2 + \frac{\tilde{\lambda} v^2}{2} = 0. \quad (7)$$

The mixing between ϕ and ϕ_S leads to the following mass terms:

$$\frac{1}{2}(\phi \ \phi_S) \mathcal{M}^2 \begin{pmatrix} \phi \\ \phi_S \end{pmatrix} = \frac{1}{2}(\phi \ \phi_S) \begin{pmatrix} \lambda_1 v^2 & \tilde{\lambda} v v_S \\ \tilde{\lambda} v v_S & 2\lambda_S v_S^2 \end{pmatrix} \begin{pmatrix} \phi \\ \phi_S \end{pmatrix}. \quad (8)$$

The squared mass matrix $\mathcal{M}^2$ can be diagonalized via an orthogonal matrix with the mixing angle α ,

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi \\ \phi_S \end{pmatrix}, \quad (9)$$

where h_1 and h_2 are mass eigenstates of the Higgs bosons

$$m_{h_1}^2 = \lambda_1 v^2 \cos^2 \alpha + 2\lambda_S v_S^2 \sin^2 \alpha - \tilde{\lambda} v v_S \sin 2\alpha, \quad (10)$$

$$m_{h_2}^2 = \lambda_1 v^2 \sin^2 \alpha + 2\lambda_S v_S^2 \cos^2 \alpha + \tilde{\lambda} v v_S \sin 2\alpha, \quad (11)$$

with the constraint

$$\tilde{\lambda} v v_S \cos 2\alpha + \left(\frac{\lambda_1}{2} v^2 - \lambda_S v_S^2 \right) \sin 2\alpha = 0. \quad (12)$$

In this paper, we fix the mass eigenstate h_1 to be the SM-like Higgs boson with $m_{h_1} = 125$ GeV, and h_2 is the additional Higgs boson.

The $\mathbb{Z}_2$ -odd scalar $SU(2)_L$ doublet field η can be parametrized as

$$\eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}. \quad (13)$$

The mass spectrum of $\mathbb{Z}_2$ -odd scalar particles is

$$m_{H^\pm}^2 = \mu_2^2 + \frac{\lambda}{2} v_S^2 + \frac{\lambda_3}{2} v^2, \quad (14)$$

$$m_H^2 = \mu_2^2 + \frac{\lambda}{2} v_S^2 + \frac{\lambda_3 + \lambda_4 + \lambda_5}{2} v^2, \quad (15)$$

$$m_A^2 = \mu_2^2 + \frac{\lambda}{2} v_S^2 + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2} v^2. \quad (16)$$

There are 11 parameters, $\mu_1^2, \mu_2^2, \mu_S^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda, \lambda_S$, and $\tilde{\lambda}$, in the scalar potential in Eq. (3). They can be replaced by nine new physics parameters, $\mu_2^2, m_{h_2}, m_H, m_A, m_{H^\pm}, \alpha, v_S, \lambda_2$, and λ , in addition to the two SM parameters v and m_{h_1} .

A. Neutrino masses

In this model, tiny neutrino masses are generated via the one-loop induced dimension-6 operator $S(\overline{L\Phi})^c L\Phi/\Lambda^2$, where Λ is an energy scale parameter, as shown in Fig. 1.

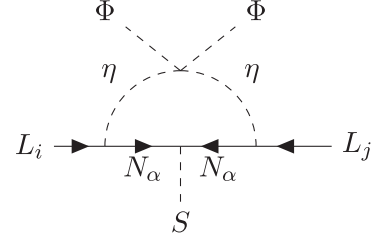


FIG. 1. The one-loop diagram which generates small neutrino masses.

Following the framework of Refs. [20,21], after the EWSB, the mass matrix of light neutrinos at one-loop level is

$$m_\nu^{ij} = \sum_\alpha g_{i\alpha} g_{j\alpha} \Lambda_\alpha \equiv (g\Lambda g^T)^{ij}, \quad (17)$$

where the diagonal matrix Λ is defined by

$$\Lambda_\alpha = \frac{m_{N_\alpha}}{32\pi^2} \left[\frac{m_H^2}{m_{N_\alpha}^2 - m_H^2} \ln \left(\frac{m_{N_\alpha}^2}{m_H^2} \right) - \frac{m_A^2}{m_{N_\alpha}^2 - m_A^2} \ln \left(\frac{m_{N_\alpha}^2}{m_A^2} \right) \right]. \quad (18)$$

To evaluate the Yukawa coupling of neutrinos, we adopt the Casas-Ibarra parametrization [45]. The matrix of the Yukawa coupling can be parametrized as

$$g_{i\alpha} = \left(U_{\text{PMNS}} \sqrt{\mathcal{M}_\nu} R \sqrt{\Lambda^{-1}} \right)_{i\alpha}, \quad (19)$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata matrix [46,47] and the diagonalized mass matrix of active neutrinos is defined as $\mathcal{M}_\nu \equiv \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. The matrix R is an arbitrary complex orthogonal matrix. We take $R = I$ in the following analysis.

III. CONSTRAINTS ON THE MODEL

In this section, we discuss possible theoretical and experimental constraints in this model. We consider perturbativity and vacuum stability as theoretical constraints. We consider experimental constraints from neutrino oscillations, collider experiments, flavor experiments, and dark-matter observations, as well as the electroweak precision tests.

A. Theoretical constraints

In this subsection, we consider theoretical bounds in the model: perturbativity and vacuum stability. Perturbativity requires that all the quartic vertices of scalar fields should satisfy the perturbativity criterion $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_S, \tilde{\lambda}, \lambda < 4\pi$, and the Yukawa couplings should satisfy $(y_1^R)^2, (y_2^R)^2, (y_3^R)^2 < 4\pi$.

Next, vacuum stability at tree level requires the scalar potential to be bounded from below in all directions of the field space [48–57]. Therefore, the vacuum stability requires

$$\begin{aligned} \lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 \pm \lambda_5 \geq -\sqrt{\lambda_1 \lambda_2}, \\ \lambda_5 \geq 0, \quad \lambda \geq -\sqrt{2} \sqrt{\lambda_2 \lambda_5}, \quad \tilde{\lambda} \geq -\sqrt{2} \sqrt{\lambda_1 \lambda_5}. \end{aligned} \quad (20)$$

$$\begin{aligned} \Delta m_{21}^2 &= (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = (2.455 \pm 0.028) \times 10^{-3} \text{ eV}^2, \quad \delta_{CP} = (1.19 \pm 0.22)\pi, \\ \sin^2(\theta_{12}) &= 0.307 \pm 0.013, \quad \sin^2(\theta_{13}) = 0.0219 \pm 0.0007, \quad \sin^2(\theta_{23}) = 0.558 \pm 0.015. \end{aligned} \quad (21)$$

Next, we consider constraints from lepton flavor violation (LFV). In this model, LFV decay processes can be achieved via one-loop diagrams mediated by $H^\pm$ and N_α . We consider the constraints from $\ell_i \rightarrow \ell_j \gamma$ and $\ell_i \rightarrow \ell_j \ell_k \bar{\ell}_k$, as shown in Table II.

Third, this model basically provides two candidates of DM, which are the lightest RH neutrino and the lightest neutral $\mathbb{Z}_2$ odd scalar particle. The relic density of DM candidates should satisfy observation results from the Planck satellite $\Omega h^2 = 0.120 \pm 0.001$ [2]. Direct detection (DD) experiments can also give constraints to this model. We apply limits from the LUX-ZEPLIN (LZ) experiment [43] on the WIMP-nucleon spin independent elastic scattering cross section. A detailed analysis is shown in the next section.

Fourth, collider experiments give rich constraints on this model, including masses of new particles (e.g., Z' , m_{h_2} , and inert scalar particles) and beyond the Standard Model (BSM) parameters (e.g., α and v_S). The LEP II experiment gives the lower limit of the ratio between the mass of the Z' gauge boson and its coupling constant $m_{Z'}/g_{B-L} > 7 \text{ TeV}$ [32,60,61]. Chargino searches at the LEP II experiment can provide a limit of $m_{H^\pm} > 80 \text{ GeV}$ [33]. The LEP II

TABLE II. Current experimental bounds for LFV processes.

LFV processes	Current bounds on branching ratios
$\mu^+ \rightarrow e^+ \gamma$	1.5×10^{-13} [27]
$\mu^+ \rightarrow e^- e^+ e^-$	1.0×10^{-12} [28]
$\tau^+ \rightarrow e^+ \gamma$	3.3×10^{-8} [29]
$\tau^+ \rightarrow \mu^+ \gamma$	4.2×10^{-8} [30]
$\tau^- \rightarrow e^- e^+ e^-$	2.7×10^{-8} [31]
$\tau^- \rightarrow e^+ \mu^- \mu^-$	1.7×10^{-8} [31]
$\tau^- \rightarrow e^- \mu^+ \mu^-$	2.7×10^{-8} [31]
$\tau^- \rightarrow \mu^+ e^- e^-$	1.5×10^{-8} [31]
$\tau^- \rightarrow \mu^- e^+ e^-$	1.8×10^{-8} [31]
$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	2.1×10^{-8} [31]
$\mu^- \text{Ti} \rightarrow e^- \text{Ti}$	4.3×10^{-12} [58]
$\mu^- \text{Au} \rightarrow e^- \text{Au}$	7.0×10^{-13} [59]

B. Constraints from experiments

First, we discuss constraints from the neutrino oscillation. We consider normal mass ordering of neutrinos and adopt the latest data from the Particle Data Group [42], so our analysis is compatible with neutrino oscillation measurements,

experiment also exclude the intersection of the following mass region for inert scalar particles [62]:

$$m_H < 80 \text{ GeV}, \quad m_A < 100 \text{ GeV}, \quad m_A - m_H > 8 \text{ GeV}. \quad (22)$$

With precise measurements of decay widths of W and Z bosons [42], kinematically allowed regions for decay processes $W/Z \rightarrow \eta\eta$ are excluded. The null results from Z' boson searches at ATLAS [34,35] and CMS [37,38] place strong constraints on the mass region $m_{Z'} < 5.15 \text{ TeV}$, requiring the gauge coupling g_{B-L} to be smaller than the electroweak coupling. However, for $m_{Z'} > 5.15 \text{ TeV}$, no experimental constraints currently apply. Precise measurements for the Higgs boson couplings give constraints on the mixing between ϕ and ϕ_S . The scaling factor for the $h_1 ZZ$ coupling is given by

$$\kappa_Z^2 \equiv \frac{\Gamma(h_1 \rightarrow ZZ)}{\Gamma_{\text{SM}}(h_1 \rightarrow ZZ)} = \cos^2 \alpha, \quad (23)$$

where ATLAS [39] and CMS [40] provide

$$\kappa_Z = 1.04 \pm 0.07, \quad (24)$$

from which we obtain $0.90 < \cos \alpha \leq 1.0$ as a criterion of 95% confidence level.¹ The processes $pp \rightarrow H^+ H^- \rightarrow N_1 N_1 l^+ l^-$ constitute characteristic and potentially observable signatures in the N_1 dark-matter scenario. The process $H^\pm \rightarrow N_1 l^\pm$ in our model exhibits a collider signature analogous to that of a slepton decays into a charged lepton and a neutralino in supersymmetric scenarios. Current experimental searches for sleptons yield constraints on the charged inert scalar mass $m_{H^\pm}$ in the framework of this model [36,41].

Finally, we will consider constraints from the electroweak precision test. Electroweak precision measurements

¹The allowed parameter space for m_{h_2} and α is shown in Refs. [63,64] from the direct search results for h_2 .

can provide strong constraints on BSM physics. The oblique parameter T gives the strongest constraint on a multi-Higgs scenario among S , T , and U parameters [65,66]. In this model, inert doublet field and scalar singlet field contribute to deviations of T [67–69],

$$\begin{aligned} \Delta T = & \frac{3\sin^2\alpha}{16\pi s_W^2} \left[f_T\left(\frac{m_{h_2}^2}{m_W^2}\right) - f_T\left(\frac{m_{h_1}^2}{m_W^2}\right) \right. \\ & \left. - \frac{1}{c_W^2} \left(f_T\left(\frac{m_{h_2}^2}{m_Z^2}\right) - f_T\left(\frac{m_{h_1}^2}{m_Z^2}\right) \right) \right] \\ & + \frac{1}{16\pi m_W^2 s_W^2} [F(m_{H^\pm}, m_H) + F(m_{H^\pm}, m_A) \\ & - F(m_H, m_A)], \end{aligned} \quad (25)$$

where $c_W^2 \equiv m_W^2/m_Z^2$ and $s_W^2 = 1 - c_W^2$. The auxiliary functions are defined as

$$F(x, y) = \frac{x^2 + y^2}{2} - \frac{x^2 y^2}{x^2 - y^2} \ln \frac{x^2}{y^2}, \quad f_T(x) = \frac{x \log x}{x - 1}. \quad (26)$$

IV. LEPTON FLAVOR VIOLATION

In this section, we consider constraints from LFV. LFV processes can be enhanced through N_α - $H^\pm$ loop diagrams in this model. The branching ratio for $\ell_i \rightarrow \ell_j \gamma$ is calculated as

$$\text{Br}(\ell_i \rightarrow \ell_j \gamma) = \frac{48\pi^3 \alpha_{\text{em}} |A_D|^2}{G_F^2} \text{Br}(\ell_i \rightarrow \ell_j \bar{\nu}_j \nu_i), \quad (27)$$

with

$$\begin{aligned} m_{N_2} &= m_{N_1} + 2500 \text{ GeV}, & m_{N_3} &= m_{N_1} + 3000 \text{ GeV}, & m_H &= 300 \text{ GeV}, \\ m_{H^\pm} &= m_A, & \lambda_5 &= -10^{-5}, & m_{H^\pm} &= \sqrt{m_H^2 - \lambda_5 v^2}. \end{aligned} \quad (32)$$

LFV constraints on the mass of N_1 are shown in Fig. 2. The solid blue (red) line shows $\text{Br}(\mu \rightarrow e \gamma)$ [$\text{Br}(\mu \rightarrow 3e)$]. The horizontal dashed blue and red lines are the current upper bounds from MEGII [27] and SINDRUM [28], respectively. Future expected bounds from the MEG upgrade [74] and Mu3e [75] are shown as dot-dashed lines. As m_{N_1} increases, the branching ratios of the LFV processes are decreasing, providing the lower bound of m_{N_1} around $m_{N_1} \simeq 0.24 \text{ GeV}$. However, this bound is dependent on the chosen benchmark scenario. Once a specific benchmark is fixed, LFV processes can provide a constraint on m_{N_1} .

V. DARK MATTER

In this section, we consider the freeze-out mechanism for DM candidates. The relic abundance of the DM particle can

$$A_D = \sum_\alpha \frac{ig_{ia}^* g_{ja}}{32\pi^2 m_{H^\pm}^2} F(\xi_\alpha), \quad (28)$$

where ξ_α is defined as $\xi_\alpha \equiv m_{N_\alpha}^2/m_{H^\pm}^2$, α_{em} is the fine structure constant, G_F is the Fermi constant, and the function $F(x)$ is defined as

$$F(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(x - 1)^4}. \quad (29)$$

$\ell_i \rightarrow \ell_j \ell_k \bar{\ell}_k$ processes include photon penguin diagrams and box diagrams. The branching ratios of photon penguin diagrams can have a simple relation with the radiative decay processes [70–72],

$$\text{Br}(\ell_i \rightarrow \ell_j \ell_j \bar{\ell}_j) \simeq \frac{\alpha_{\text{em}}}{3\pi} \left[2 \log \left(\frac{m_{\ell_i}}{m_{\ell_j}} \right) - \frac{11}{4} \right] \text{Br}(\ell_i \rightarrow \ell_j \gamma), \quad (30)$$

$$\begin{aligned} \text{Br}(\ell_i \rightarrow \ell_j \ell_k \bar{\ell}_k) &\simeq \frac{\alpha_{\text{em}}}{3\pi} \left[2 \log \left(\frac{m_{\ell_i}}{m_{\ell_j}} \right) - 3 \right] \\ &\times \text{Br}(\ell_i \rightarrow \ell_j \gamma) (j \neq k). \end{aligned} \quad (31)$$

The box diagrams can be dominant when $\xi \gg 1$ or $\xi \ll 1$ if masses of RH neutrinos are degenerate. $\mu \rightarrow e$ conversion in muonic atoms is also a signature of LFV. However, $\mu \rightarrow e \gamma$ is more stringent than this process [72]. Detailed analysis of LFV can be seen in Refs. [70–73].

Considering the constraints from neutrino oscillations and LFV measurements, we choose the following parameters for later use:

be calculated by solving the Boltzmann equation [76],

$$\frac{dY}{dx} = -\sqrt{\frac{g_* \pi}{45}} \frac{m_{\text{DM}} m_{\text{Pl}}}{x^2} \langle \sigma v \rangle (Y^2 - Y_{\text{EQ}}^2), \quad (33)$$

where $x \equiv m_{\text{DM}}/T$ is an independent variable and $Y \equiv n_{\text{DM}}/s$ is the comoving density of DM. The thermal averaged cross section $\langle \sigma v \rangle$ contains cross sections of all annihilation processes before the DM freezes out. Integrating the Boltzmann equation from $x = 0$ to $x_F = m_{\text{DM}}/T_F$, the relic abundance is given by

$$\Omega_{\text{DM}} h^2 \simeq 2.75 \times 10^{-8} m_{\text{DM}} Y(T_F) \text{GeV}^{-1}, \quad (34)$$

where T_F is the freeze-out temperature of DM.

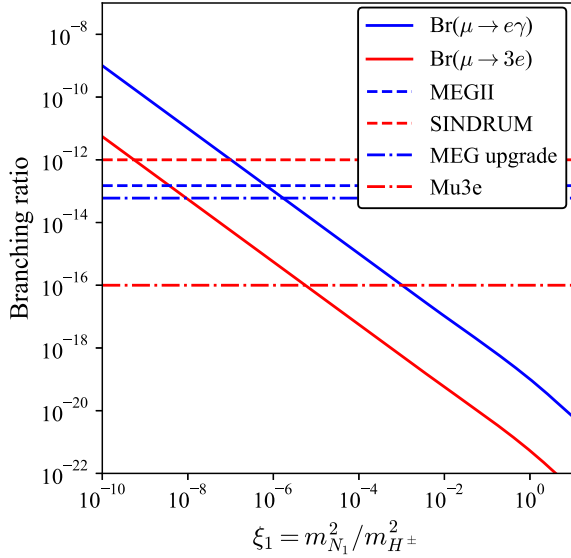


FIG. 2. $\text{Br}(\mu \rightarrow e\gamma)$ (blue lines) and $\text{Br}(\mu \rightarrow 3e)$ (red lines) as a function of $\xi = m_{N_1}^2 / m_{H^\pm}^2$. The horizontal dashed lines show the current upper bounds from the MEGII experiment (blue line) [27] and SINDRUM experiment (red line) [28]. The dot-dashed lines show the future bounds from the MEG upgrade (blue line) [74] and Mu3e (red line) [75].

In this paper, we discuss two scenarios for two DM candidates. We first discuss a fermionic DM scenario where N_1 is the lightest particle in the $\mathbb{Z}_2$ -odd sector and show the allowed parameter spaces under various current experiments. We then briefly discuss the scalar DM scenario in the model in a discussion similar to those in several previous studies [62,77–80]. In the following analysis, we use the numerical analysis package micrOMEGAs 6.0 [81] to calculate the relic abundance of DM.

A. N_1 dark matter

In this scenario, we consider N_1 as the lightest $\mathbb{Z}_2$ -odd particle.

We consider two mass hierarchy scenarios between N_1 and $H^\pm$ that are subject to constraints from slepton searches [36,41],

- (i) Scenario 1: $m_{H^\pm} \simeq m_{N_1} + 50$ GeV, and
- (ii) Scenario 2: $m_{H^\pm} \simeq m_{N_1} + 700$ GeV.

In scenario 1, current experimental constraints from slepton searches imply a lower bound on m_{N_1} of approximately 230 GeV.

$$\begin{aligned}
 \cos \alpha &= 0.99, & v_S &= 30 \text{ TeV}, & m_{h_2} &= 500 \text{ GeV}, & m_{Z'} &= 5.2 \text{ TeV}, \\
 m_{N_2} &= m_{N_1} + 2500 \text{ GeV}, & m_{N_3} &= m_{N_1} + 3000 \text{ GeV}, & \lambda &= 10^{-6}, & \lambda_3 &= 0.1, & \lambda_5 &= -10^{-5}, \\
 m_H &= m_{N_1} + 50 \text{ GeV}, & m_{H^\pm} &= m_A, & m_{H^\pm} &= \sqrt{m_H^2 - \lambda_5 v^2}.
 \end{aligned} \tag{35}$$

The relic abundance of N_1 is reduced significantly around $m_{N_1} \simeq m_{h_1}/2$, $m_{N_1} \simeq m_{h_2}/2$, and $m_{N_1} \simeq m_{Z'}/2$, respectively. Annihilations of RH neutrinos to the SM particles are enhanced resonantly via s -channel exchange of h_1 , h_2 , and Z' .

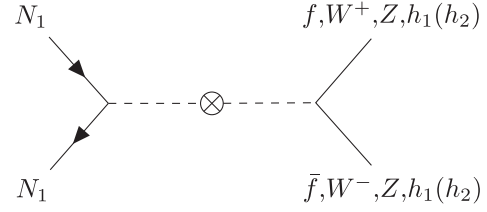


FIG. 3. Annihilation processes $N_1 N_1 \rightarrow h_1(h_2) \rightarrow f\bar{f}$, ZZ , W^+W^- , and $h_1(h_2)h_1(h_2)$ through the mixing between ϕ and ϕ_S .

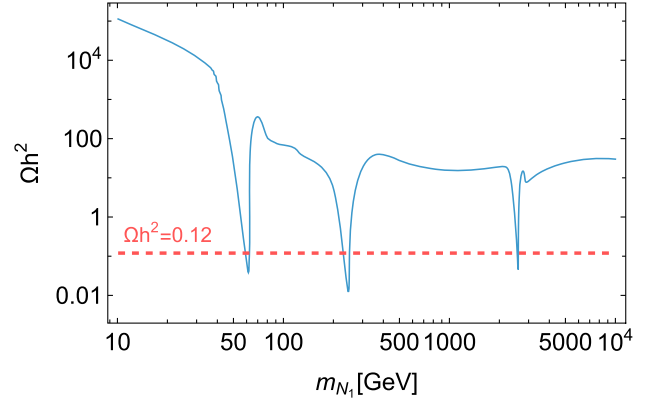


FIG. 4. Relic abundance of N_1 as a function of m_{N_1} . The red dashed line is the current bound of DM relic abundance from the Planck experiment [2].

First, we consider the relic abundance of N_1 dark matter. In the Tao-Ma model [20,21], there are no sufficient annihilation rates for N_1 DM due to strong constraints from LFV experiments [82]. However, it is possible for N_1 to satisfy the DM abundance with the mixing between the Higgs field Φ and the extra scalar singlet S because N_1 can annihilate via $N_1 N_1 \rightarrow h_1(h_2) \rightarrow f\bar{f}$, ZZ , W^+W^- , and $h_1(h_2)h_1(h_2)$ processes [24], as shown in Fig. 3.

In Fig. 4, the relic abundance of N_1 is shown as a function of m_{N_1} . Current LHC experiments place constraints on the $g_{B-L} - m_{Z'}$ plane [34,35,37,38]. For different values of $m_{Z'}$, the corresponding lower bound on v_S varies. A smaller gauge coupling g_{B-L} , compared to the electroweak coupling, allows for a lighter Z' boson but typically requires a larger lower bound of v_S [83,84]. To explore the maximal viable parameter space, we consider the mass region of $m_{Z'} > 5.15$ TeV in our analysis. We use the following parameters to calculate the relic abundance:

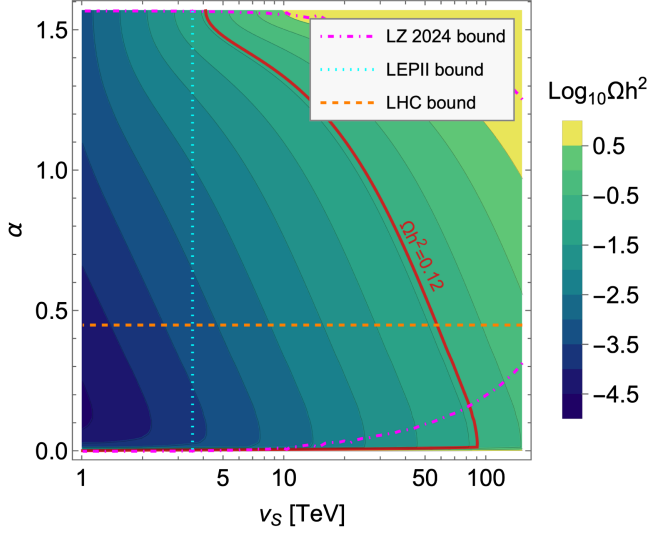


FIG. 5. Parameter space of $m_{h_2} = 500$ GeV and $m_{N_1} = 248$ GeV with constraints from LEP II [32,60], ATLAS [39], CMS [40], and LZ 2024 [44] experiments in scenario 1. The red line is the bound from the Planck experiment [2].

Scenarios 1 and 2 can both accommodate three resonance regions; however, in scenario 1, the SM-like Higgs resonance is not acceptable due to the constraints from the slepton searches [36,41].

Second, we consider the DD of N_1 dark matter. The lightest RH neutrino N_1 can have elastic scattering with nucleons via the Higgs exchange processes. The spin independent (SI) cross section for the proton target is

$$\sigma_{\text{SI}}^p = \frac{4\mu^2}{\pi} f_p^2, \quad (36)$$

where $\mu \equiv \frac{m_{N_1} m_p}{m_{N_1} + m_p}$ is the DM-nucleus reduced mass in the center of mass frame. The hadronic matrix element f_p [85] is given by

$$f_p = \sum_{q=u,d,s} c_q \frac{m_p}{m_q} f_{Tq}^p + \frac{2}{27} f_{TG}^p \sum_{q=c,b,t} c_q \frac{m_p}{m_q}, \quad (37)$$

where f_{Tq}^p and f_{TG}^p express mass contributions to the nucleon from quarks and gluons and m_q is the mass of a quark with a Yukawa coupling y_q . The effective vertex c_q is defined as

$$c_q = y_R^\alpha y_q \left[\left(\frac{\sin \alpha}{\sqrt{2}} \right) \frac{1}{m_{h_1}^2} \left(\frac{\cos \alpha}{\sqrt{2}} \right) - \left(\frac{\sin \alpha}{\sqrt{2}} \right) \frac{1}{m_{h_2}^2} \left(\frac{\cos \alpha}{\sqrt{2}} \right) \right]. \quad (38)$$

The spin independent cross section σ_{SI}^p has a simple relation with BSM parameters in this model,

$$\sigma_{\text{SI}}^p \propto \left(\frac{m_{N_1} \sin(2\alpha)}{v_S} \right)^2 \times \left(\frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2} \right)^2. \quad (39)$$

In Fig. 5, the relic abundance of N_1 DM is shown as the colored contour in the $v_S - \alpha$ plane. We adopt the following parameter set for scenario 1:

$$\begin{aligned} m_{N_1} &= \frac{m_{h_2}}{2} - 2 \text{ GeV}, & m_{h_2} &= 500 \text{ GeV}, & m_{N_2} &= 2500 \text{ GeV}, & m_{N_3} &= 3000 \text{ GeV}, & m_{Z'} &= 5.2 \text{ TeV}, \\ \lambda &= 10^{-6}, & \lambda_3 &= 0.1, & \lambda_5 &= -10^{-5}, & m_H &= 300 \text{ GeV}, & m_{H^\pm} &= m_A, & m_{H^\pm} &= \sqrt{m_H^2 - \lambda_5 v^2}. \end{aligned} \quad (40)$$

The allowed parameter space is the bottom-right region encircled by the LZ 2024 [44] (dot-dashed magenta line) and Planck experiment [2] (solid red line), which is on the right side of the LEP II constraint [32,60] (dotted cyan line). In this benchmark scenario, the dominant annihilation channels are $N_1 N_1 \rightarrow h_2 \rightarrow WW$ and $N_1 N_1 \rightarrow h_2 \rightarrow ZZ$,

with cross sections approximately proportional to $[(m_{N_1}/v_S) \sin(2\alpha)]^2$. As a result, a sufficiently large annihilation cross section can be achieved for small v_S , except in the limiting cases $\alpha = 0$ and $\pi/2$, where the mixing vanishes.

In scenario 2, we adopt the following parameter set:

$$\begin{aligned} m_{h_2} &= 200 \text{ GeV}, & m_{N_2} &= 2500 \text{ GeV}, & m_{N_3} &= 3000 \text{ GeV}, & m_{Z'} &= 5.2 \text{ TeV}, \\ \lambda &= 10^{-6}, & \lambda_3 &= 0.1, & \lambda_5 &= -10^{-5}, & m_H &= 700 \text{ GeV}, & m_{H^\pm} &= m_A, & m_{H^\pm} &= \sqrt{m_H^2 - \lambda_5 v^2}. \end{aligned} \quad (41)$$

We investigate the parameter space of Higgs resonances near the regions $m_{N_1} \simeq m_{h_1}/2$ and $m_{N_1} \simeq m_{h_2}/2$, finding that there are still parameter spaces for N_1 DM, encircled by the LZ 2024 [44] (dot-dashed magenta line), LHC [39,40]

(dashed orange line), and Planck experiments [2] (solid red line), as shown in Fig. 6. In the Z' resonance scenario, the thermally averaged annihilation cross section times the relative velocity $\langle \sigma v_{\text{rel}} \rangle$ is proportional to $1/m_{Z'}^2$, implying

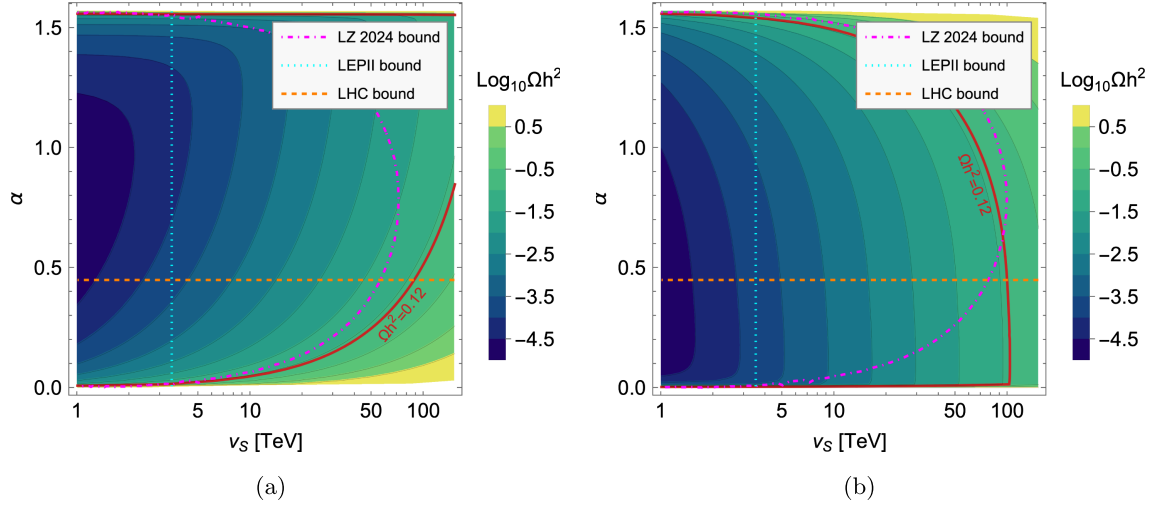


FIG. 6. Parameter space of different Higgs resonances with constraints from LEP II [32,60], ATLAS [39], CMS [40], and LZ 2024 [44] experiments in scenario 2. The red line is the bound from the Planck experiment [2]. (a) $m_{N_1} = 62.4$ GeV. (b) $m_{N_1} = 99.9$ GeV.

that the relic abundance depends primarily on $m_{Z'}$. As a result, the relic density remains nearly constant across the $v_S - \alpha$ plane. Therefore, the observed relic abundance can be satisfied throughout the entire $v_S - \alpha$ parameter space in this scenario.

Third, we explore the allowed mass region of N_1 with the h_2 resonance scenario. We choose the following

parameter space:

$$\cos\alpha \in [0.9, 1], \quad v_S \in [3.5, 80] \text{ TeV}, \quad m_{N_1} \in [10, 10^4] \text{ GeV}, \quad (42)$$

with

$$\begin{aligned} m_{h_2} &= 2m_{N_1} + 0.2 \text{ GeV}, & m_{N_2} &= m_{N_1} + 2500 \text{ GeV}, & m_{N_3} &= m_{N_1} + 3000 \text{ GeV}, & m_{Z'} &= 5.2 \text{ TeV}, \\ \lambda &= 10^{-6}, & \lambda_3 &= 0.1, & \lambda_5 &= -10^{-5}, & m_H &= m_{N_1} + 700 \text{ GeV}, & m_{H^\pm} &= m_A, & m_{H^\pm} &= \sqrt{m_H^2 - \lambda_5 v^2}. \end{aligned} \quad (43)$$

We scan $\cos\alpha$, v_S , and m_{N_1} with $20 \times 20 \times 150$ points, respectively. The allowed parameter space of N_1 is shown

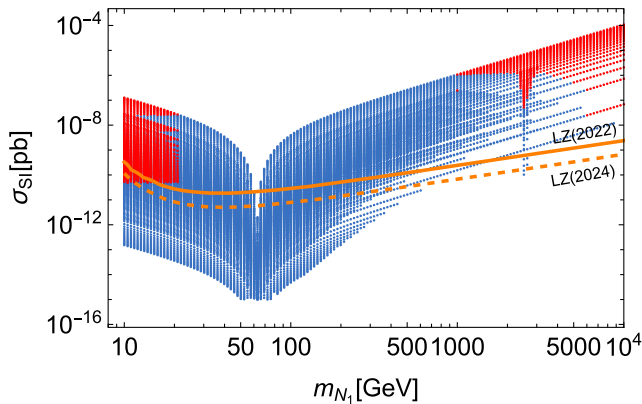


FIG. 7. Possible SI cross section of N_1 as a function of m_{N_1} in our parameter space. The red points are excluded by theoretical constraints. The orange line is the upper bound from the LZ 2022 results [43]. The dashed orange line is the upper bound from the LZ 2024 results [44].

in Fig. 7, in which all points satisfy the constraint $\Omega_{\text{DM}} h^2 < 0.12$. The blue points satisfy all theoretical

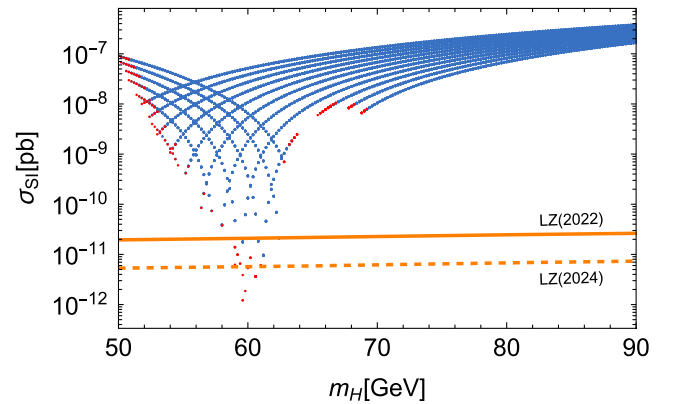


FIG. 8. Possible SI cross section of H as a function of m_H in our parameter space. The red points satisfy $\Omega h^2 \simeq 0.12$. The blue points satisfy $\Omega h^2 < 0.12$. The orange line is the upper bound from the LZ 2022 results [43]. The dashed orange line is the upper bound from the LZ 2024 results [44].

and experimental constraints, while the red points are excluded by theoretical constraints. In the mass region near $m_{N_1} \simeq m_{h_1}/2$, the SI cross section is greatly reduced due to the destructive interference of the two Higgs states. In the mass region $m_{N_1} \gtrsim 80$ GeV, the SI cross section increases as m_{N_1} increases. In the mass region $m_{N_1} > 600$ GeV, the SI cross section exceeds the LZ bound; therefore, current direct detection experiments can give the upper bound on the mass of N_1 . Scenarios 1 and 2 yield very similar results in the parameter scan, as shown in Fig. 7. As previously discussed, scenario 1 features the lower bound on m_{N_1} due to constraints from slepton searches. However, in nonresonant scenarios, such as those dominated by coannihilation processes, the observed relic abundance can still be achieved [25]. In such cases, the upper bound on m_{N_1} is no longer applicable.

B. Scalar dark matter

In this scenario, we consider H to be the lightest $\mathbb{Z}_2$ -odd particle. We briefly examine the key differences between the inert doublet model and our model. Specifically, we highlight additional effects that arise in our model, beyond those present in the inert doublet model, under the chosen benchmark scenario. The H DM scenario has been extensively studied in the inert doublet model [62,77,78,86,87].

Discussions of H DM are similar to the inert doublet model with an additional scalar singlet [79,80].

First, the relic density can be satisfied by the additional Higgs resonance near the mass region $m_H \simeq m_{h_2}/2$. H can annihilate via $HH \rightarrow h_1(h_2) \rightarrow f\bar{f}$, W^+W^- , ZZ , and $h_1(h_2)h_1(h_2)$ processes.

Second, the SI cross section in direct detections can be changed due to the h_2 exchange processes. The SI cross section of H is given by

$$\sigma_{\text{SI}}^H = \frac{\mu_H^2}{4\pi m_H^2} f_p^2, \quad \mu_H \equiv \frac{m_H m_p}{m_H + m_p}, \quad (44)$$

where f_p has the same definition as in Eq. (37) and c_q is defined as²

$$c_q = \frac{y_q}{2\sqrt{2}} \left[\frac{\lambda v_S \sin \alpha \cos \alpha - \lambda_L v \cos^2 \alpha}{m_{h_1}^2} - \frac{\lambda_L v \sin^2 \alpha + \lambda v_S \sin \alpha \cos \alpha}{m_{h_2}^2} \right]. \quad (45)$$

Here we choose the following parameter space,

$$m_H \in [50, 90] \text{ GeV}, \quad \cos \alpha \in [0.9, 1], \quad \mu_2^2 \in [2000, 4000] \text{ GeV}^2, \quad m_{H^\pm} = m_A = m_H + 50 \text{ GeV}, \quad (46)$$

and fix the following parameters for the H DM scenario:

$$\begin{aligned} v_S &= 5000 \text{ GeV}, & m_{h_2} &= 120 \text{ GeV}, & \lambda &= 10^{-4}, \\ m_{N_1} &= 3000 \text{ GeV}, & m_{N_2} &= 3500 \text{ GeV}, & m_{N_3} &= 4000 \text{ GeV}. \end{aligned} \quad (47)$$

The allowed parameter space of H is shown in Fig. 8. The allowed parameter space is near 60 GeV due to the destructive interference of the two Higgs states, indicating that there is still an allowed parameter space for the H DM scenario in this model.

VI. DISCUSSIONS

In this section, we offer some comments on the results shown in Secs. IV and V. In Sec. IV, we showed that experimental results place lower bounds on m_{N_1} . As discussed in Sec. IV, the lower bound is sensitive to the choice of benchmark scenario. Currently, the most stringent constraint arises from the $\mu \rightarrow e\gamma$ process, which sets the strongest lower bound on m_{N_1} within our N_1 DM scenario. In the future, the MEG experiment [74] may improve its sensitivity down to $\text{Br}(\mu \rightarrow e\gamma) \simeq 6 \times 10^{-14}$, and the Mu3e experiment [75] may upgrade the sensitivity to $\text{Br}(\mu \rightarrow 3e) \simeq 1 \times 10^{-16}$.

In Sec. V, we have shown benchmark points and parameter space of DM candidates in this model. Knowing the allowed parameter spaces, we can discuss the testability of new particles at future collider experiments. Here we very briefly mention the prospect for collider searches in each scenario. In the N_1 DM scenario, N_1 could be tested through $H^\pm \rightarrow l^\pm N_1$ signals at LHC and the High-Luminosity Large Hadron Collider (HL-LHC) if it is kinematically allowed, mainly via $pp \rightarrow H^+ H^- \rightarrow N_1 N_1 l^+ l^-$ [77,86,89–92]. In the H DM scenario, inert scalar particles can be produced through monojet processes (e.g., $q\bar{q} \rightarrow Zg \rightarrow H A j$ and $q\bar{q} \rightarrow h_1 g \rightarrow H H g$) in hadron colliders. A and $H^\pm$ can decay to H via weak interactions. Cross sections of processes including the ZHA vertex depend only on m_H and m_A . Cross section of processes

²The SI cross section can be modified depending on the size of λ_2 via loop corrections [88].

including the $h_1 HH$ vertex depend on not only m_H but also the dimensionless parameter $\lambda_3 + \lambda_4 + \lambda_5$. Detailed analyses of inert scalars in the LHC can be found in Refs. [77,90,92]. Inert scalars can also be produced in lepton colliders through $e^+e^- \rightarrow Z \rightarrow AH(H^+H^-)$ processes [86,91].

The existence of the Z' boson is another difference from the Tao-Ma model. Though searches in the LHC give strong constraints on the mass of Z' boson, it can be lighter with smaller gauge couplings. Since hadron colliders (e.g., LHC and HL-LHC) have large backgrounds, it can only reach the gauge coupling g_{B-L} of $\mathcal{O}(10^{-2})$ [93]. However, lepton colliders (e.g., the International Linear Collider) can reach a smaller gauge coupling of $\mathcal{O}(10^{-3})$ [93]. Decays of the Z' boson into SM particles are proportional to $(B-L)^2$. Owing to its large mass, the branching ratios of $Z' \rightarrow q\bar{q}, l^+l^-, \nu_L\bar{\nu}_L, N_1N_1, h_1h_1, h_1h_2$, and h_2h_2 for the parameter set in Eq. (35) are given approximately by 0.20, 0.29, 0.15, 0.15, 6.9×10^{-4} , 4.4×10^{-2} , and 0.17, respectively [23,60].

Direct detection experiments become increasingly stringent for DM candidates. As discussed in Sec. VA, the LZ 2024 results give the upper bound of N_1 DM around 600 GeV in the h_2 resonance scenario. As discussed in Sec. VB, in the low-mass region of m_H , inert scalar dark matter remains viable under direct detection constraints due to destructive interference between the two Higgs states in our model. Future direct detection experiments are expected to impose stronger constraints on both scenarios.

VII. CONCLUSIONS

The radiative seesaw model with a gauged $U(1)_{B-L} \times \mathbb{Z}_2$ extension is a well-motivated scenario which gives

consistent predictions of active neutrino masses and the abundance of dark matter. Majorana masses of right-handed neutrinos, the lightest of which can be identified as dark matter, are given by the spontaneous breaking of the $U(1)_{B-L}$ gauge symmetry. We have revisited this model with the latest constraints from dark-matter searches, neutrino oscillations, flavor experiments, and collider experiments. We have examined several benchmark scenarios and identified the valid parameter space of this model. We have found that there is still an allowed region for this model under the latest experimental constraints. We have presented new viable benchmark scenarios for this model, i.e., the right-handed neutrino dark-matter scenario and the scalar dark matter scenario. We also have mentioned the testability of these benchmark scenarios at future experiments.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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