

Effective running coupling constant and jet quenching parameter in a spinning background from holography

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In this work, we study the effective running coupling constant of heavy quark pair and jet quenching parameter in the spinning background. Ultralocally, the boosted fluid is described by the boosted parameter and dual to a globally rotating system. Our results show that the angular momentum suppresses the effective running coupling constant and reduces its maximum value. The results demonstrate that the angular momentum promotes the dissociation of quarkonium and has a stronger effect on the effective running coupling constant when the axis of $Q\bar{Q}$ is transverse to the direction of the angular momentum. We also find that the angular momentum enhances the jet quenching parameter and has a stronger effect when the jet moves transversely to the direction of the angular momentum, namely $\hat{q}_\perp > \hat{q}_\parallel$. We discuss the dependence of the jet quenching parameter on the η/s at strong coupling in the presence of the angular momentum.

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I. INTRODUCTION

Extensive experimental results from heavy-ion collisions have established compelling evidence for quark-gluon plasma (QGP) formation under extreme conditions [1–4]. The observed suppression of heavy quarkonium serves as one critical signature for QGP generation [5]. The melting of heavy quarkonium provides important insights into the nonequilibrium dynamics of the QGP. The effective running coupling constant describing the interaction between a heavy quark and antiquark is fundamental to understanding the quarkonium dissociation. Researchers have extensively studied the effective running coupling constant in different theories [6–14]. Another important characteristic of the QGP is jet quenching, denoting the energy loss undergone by energetic partons passing through the hot, dense medium. Quantitatively, jet quenching parameter $\hat{q}$ is used to characterize this energy loss [15–19].

The experimental results reveal that the quark-gluon plasma (QGP) generated in heavy-ion collisions behaves as a strongly coupled fluid [11]. Anti-de Sitter/conformal field theory (AdS/CFT) correspondence [20–22] provides a nonperturbative framework for probing strongly coupled systems. The quark-antiquark pairs correspond to U-shaped strings connecting boundary endpoints from holography. The effects of chemical potential and magnetic field on the effective running coupling constant within the holographic model have been discussed in [23]. Recent studies explore the effective running coupling constant in rotating backgrounds [24] and flavor-dependent systems [25]. Seminal work by Liu *et al.* [26] investigated the jet quenching parameter $\hat{q}$ in $\mathcal{N} = 4$ SYM plasma within the holographic framework. The jet quenching parameter is derived from the minimal world-sheet surface bounded by a lightlike Wilson loop. Motivated by [26], significant research has extended the investigation of the jet quenching parameter in different holographic models. The effect of finite density on $\hat{q}$ has been studied in [27–29]. The jet quenching parameter in the magnetized background has been discussed in [30–34]. Moreover, the jet quenching parameter in Kerr-AdS₅ black hole background has been investigated in [35]. Other related analyses can see [36–40].

In this work, we explore the effective running coupling constant of heavy quark pair and jet quenching parameter in the spinning background from holography [41–45]. The motivation is from the generation of nonzero angular momentum in noncentral heavy-ion collisions [46–50].

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The angular momentum induces relativistic rotation of QGP, motivating the effect of rotation on the effective running coupling constant and jet quenching parameter. We consider the Myers-Perry black holes background, in which the compact boundary of the spacetime supports a gauge theory defined on $S^3 \times \mathbb{R}$ [44]. Myers-Perry black holes are solutions to the vacuum Einstein equations in higher dimensions that describe rotating black holes. The connection between these objects and quantum chromodynamics (QCD) is not a direct physical one, but rather a holographic duality. In the framework of AdS/CFT correspondence, the Myers-Perry black hole can be reduced to a Schwarzschild black brane boosted along the x_3 direction with boost parameter a in the large black hole (high temperature) limit. This boost parameter is directly related to the vorticity of the dual fluid. The relation is motivated by the need to understand strongly coupled systems with rotation. Myers-Perry black holes serve as a theoretical laboratory where calculations are tractable on the gravity side, providing insights into the complex, nonperturbative dynamics of rotating quark-gluon plasmas in QCD. Projecting this fluid flow onto flat spacetime presents a

fluid with nontrivial vorticity and expansion profiles. In order to analyze the effect of rotation, one can consider zooming in on a small patch and find that the fluid exhibits uniform streaming motion. This ultralocal description corresponds to a globally rotating fluid, with the planar black brane providing the essential holographic framework for studying QGP dynamics in the dual field theory. It should be mentioned that the heavy quark dynamics in the rotating background have been explored in [51,52]. The thermodynamics of heavy quarkonium in the spinning black hole have been studied in [53–55].

The paper is organized as follows. In Sec. II, we briefly review the Myers-Perry black hole background. In Sec. III, we discuss the effective running coupling constant of heavy quark pair and jet quenching parameter in the spinning background. In Sec. IV, we give the conclusion and discussion.

II. SPINNING MYERS-PERRY BLACK HOLE BACKGROUND

We first briefly review the spinning black hole background studied by Hawking *et al.* The metric is given by [41]

$$\begin{aligned}
 ds^2 = & -\frac{\Delta}{\rho^2} \left(dt_H - \frac{a \sin^2 \theta_H}{\Xi_a} d\phi_H - \frac{b \cos^2 \theta_H}{\Xi_b} d\psi_H \right)^2 \\
 & + \frac{\Delta_{\theta_H} \sin^2 \theta_H}{\rho^2} \left(a dt_H - \frac{r_H^2 + a^2}{\Xi_a} d\phi_H \right)^2 + \frac{\Delta_{\theta_H} \cos^2 \theta_H}{\rho^2} \left(b dt_H - \frac{r_H^2 + b^2}{\Xi_b} d\psi_H \right)^2 + \frac{\rho^2}{\Delta} dr_H^2 \\
 & - \frac{\rho^2}{\Delta_{\theta_H}} d\theta_H^2 + \frac{1 + \frac{r_H^2}{R^2}}{r_H^2 \rho^2} \left(ab dt_H - \frac{b(r^2 + a^2) \sin^2 \theta_H}{\Xi_a} d\phi_H - \frac{a(r^2 + b^2) \cos^2 \theta_H}{\Xi_b} d\psi_H \right)^2,
 \end{aligned} \tag{1}$$

with

$$\begin{aligned}
 \Delta &= \frac{1}{r_H^2} (r_H^2 + a^2)(r_H^2 + b^2) \left(1 + \frac{r_H^2}{R^2} \right) - 2M, \\
 \Delta_{\theta_H} &= 1 - \frac{a^2}{R^2} \cos^2 \theta_H - \frac{b^2}{R^2} \sin^2 \theta_H, \\
 \rho &= r_H^2 + a^2 \cos^2 \theta_H + b^2 \sin^2 \theta_H, \\
 \Xi_a &= 1 - \frac{a^2}{R^2}, \\
 \Xi_b &= 1 - \frac{b^2}{R^2},
 \end{aligned} \tag{2}$$

where ϕ_H , ψ_H , and θ_H represent Hopf angular coordinates. t_H , R , and r_H represent time, AdS radius, and radial coordinate, respectively. a , b are angular momentum parameters. We consider the $a = b$ case, corresponding to the spinning Myers-Perry black hole [42,43].

One can simplify the calculations by using the following coordinates [56]

$$\begin{aligned}
 t &= t_H, \\
 r^2 &= \frac{a^2 + r_H^2}{1 - \frac{a^2}{R^2}}, \\
 \theta &= 2\theta_H, \\
 \phi &= \phi_H - \psi_H, \\
 \psi &= -\frac{2at_H}{R^2} + \phi_H + \psi_H, \\
 b &= a, \\
 \mu &= \frac{M}{(R^2 - a^2)^3}.
 \end{aligned} \tag{3}$$

The metric (1) can be rewritten as

$$\begin{aligned}
 ds^2 = & -\left(1 + \frac{r^2}{R^2} \right) dt^2 + \frac{dt^2}{G(r)} + \frac{r^2}{4} ((\sigma^1)^2 + (\sigma^2)^2 \\
 & + (\sigma^3)^2) + \frac{2\mu}{r^2} \left(dt + \frac{a}{2} \sigma^3 \right)^2,
 \end{aligned} \tag{4}$$

with

$$\begin{aligned}
 G(r) &= 1 + \frac{r^2}{R^2} - \frac{2\mu(1 - \frac{a^2}{R^2})}{r^2} + \frac{2\mu a^2}{r^4}, \\
 \mu &= \frac{r_h^4(R^2 + r_h^2)}{2R^2 r_h^2 - 2a^2(R^2 + r_h^2)}, \\
 \sigma^1 &= -\sin\psi dt d\theta + \cos\psi \sin\theta d\phi, \\
 \sigma^2 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi, \\
 \sigma^3 &= d\psi + \cos\theta d\phi,
 \end{aligned} \tag{5}$$

where

$$\begin{aligned}
 -\infty < t < \infty, \quad r_h < r < \infty, \\
 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq \psi \leq 4\pi.
 \end{aligned} \tag{6}$$

Applying a coordinate transformation yields the planar black brane [44]

$$\begin{aligned}
 t &= \tau, \\
 \frac{R}{2}(\phi - \pi) &= x_1, \\
 \frac{R}{2}\tan\left(\theta - \frac{\pi}{2}\right) &= x_2, \\
 \frac{R}{2}(\psi - 2\pi) &= x_3, \\
 r &= \tilde{r},
 \end{aligned} \tag{7}$$

where $(\tau, \tilde{r}, x_1, x_2, x_3)$ denote the new coordinates. One can rescale these coordinates by β

$$\begin{aligned}
 \tau &\rightarrow \beta^{-1}\tau, \\
 x_1 &\rightarrow \beta^{-1}x_1, \\
 x_2 &\rightarrow \beta^{-1}x_2, \\
 x_3 &\rightarrow \beta^{-1}x_3, \\
 \tilde{r} &\rightarrow \beta\tilde{r}, \\
 \tilde{r}_h &\rightarrow \beta\tilde{r}_h. (\beta \rightarrow \infty)
 \end{aligned} \tag{8}$$

The boosted Schwarzschild black brane metric in the $\tau - x_3$ plane is given by [44]

$$\begin{aligned}
 ds^2 &= \frac{r^2}{R^2} \left(-d\tau^2 + dx_1^2 + dx_2^2 + dx_3^2 \right. \\
 &\quad \left. + \frac{r_h^4}{r^4(1 - \frac{a^2}{R^2})} \left(d\tau + \frac{a}{R} dx_3 \right)^2 \right) + \frac{R^2 r^2}{r^4 - r_h^4} dr^2,
 \end{aligned} \tag{9}$$

where a is the boost parameter. The Schwarzschild black brane is recovered when parameter $a = 0$.

We employ z ($r = 1/z$) as the holographic fifth coordinate.

$$\begin{aligned}
 ds^2 &= \frac{1}{z^2 R^2} \left[-d\tau^2 + dx_1^2 + dx_2^2 + dx_3^2 \right. \\
 &\quad \left. + \frac{z^4}{z_h^4(1 - \frac{a^2}{R^2})} \left(d\tau + \frac{a}{R} dx_3 \right)^2 \right] + \frac{R^2 z_h^4}{z^2(z_h^4 - z^4)} dz^2.
 \end{aligned} \tag{10}$$

The expression of temperature is [44]

$$T = \frac{\sqrt{1 - a^2}}{z_h \pi R^3}, \tag{11}$$

where z_h denotes the horizon. It should be mentioned that the angular velocity $\Omega = a/R^2$ in the limitation of the large black hole [44].

III. EFFECTIVE RUNNING COUPLING CONSTANT AND JET QUENCHING PARAMETER IN THE SPINNING BLACK HOLE BACKGROUND

A. Effective running coupling constant in the spinning black hole background

In this subsection, we explore the effective running coupling constant of heavy quark pairs in the spinning Myers-Perry black holes background. The Nambu-Goto action of $Q\bar{Q}$ is

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}}, \tag{12}$$

where $g_{\alpha\beta}$ represent the determinant of induced metric.

The metric (10) implies rotation about the x_3 axis. Therefore, we examine the effective running coupling constant in the transverse and parallel cases. The $Q\bar{Q}$ pair axis lies in the $x_1 - x_2$ plane for the transverse case, and along the x_3 axis for parallel case.

We discuss the transverse case first. The coordinates are parametrized by

$$\tau = \xi, \quad x_1 = \eta, \quad x_2 = 0, \quad x_3 = 0, \quad z = z(\eta), \tag{13}$$

The expression of Lagrangian density is

$$\mathcal{L} = \sqrt{A(z) + B(z)\dot{z}^2}, \tag{14}$$

with

$$A(z) = A(z_\perp) = \frac{1}{z^4 R^4} - \frac{1}{R^4 z_h^4 (1 - \frac{a^2}{R^2})},$$

$$B(z) = B(z_\perp) = \frac{z_h^4}{z^4 (z_h^4 - z^4)} - \frac{1}{(z_h^4 - z^4) (1 - \frac{a^2}{R^2})}. \quad (15)$$

For the parallel case, one can parametrize coordinates using

$$\tau = \xi, \quad x_1 = 0, \quad x_2 = 0, \quad x_3 = \eta, \quad z = z(\eta). \quad (16)$$

The expressions of $A(z)$ and $B(z)$ are

$$A(z) = A(z_\parallel) = \frac{z_h^4 - z^4}{z^4 z_h^4 R^4},$$

$$B(z) = B(z_\parallel) = \frac{z_h^4}{z^4 (z_h^4 - z^4)} - \frac{1}{(z_h^4 - z^4) (1 - \frac{a^2}{R^2})}. \quad (17)$$

The interquark distance L of $Q\bar{Q}$ is

$$L = 2 \int_0^{z_c} \sqrt{\frac{A(z_c)B(z)}{A(z)^2 - A(z)A(z_c)}} dz. \quad (18)$$

The free energy of $Q\bar{Q}$ is

$$\frac{\pi F_{Q\bar{Q}}}{\sqrt{\lambda}} = \int_0^{z_c} \left(\sqrt{\frac{A(z)B(z)}{A(z) - A(z_c)}} - \sqrt{A(z \rightarrow 0)} \right) dz$$

$$- \int_{z_c}^\infty \sqrt{A(z \rightarrow 0)} dz, \quad (19)$$

where $\sqrt{\lambda} = \frac{L^2}{\alpha}$ denote the 't Hooft coupling. We take $\lambda = 1$ in the following calculations. To eliminate free energy divergences, one can use minimum subtraction [57].

The free energy of a single quark is

$$\frac{F_Q}{\sqrt{\lambda}} = \frac{1}{2\pi} \left[\int_0^{z_h} \left(\sqrt{B(z)} - \frac{1}{z^2} \right) dz - \frac{1}{z_h} \right]. \quad (20)$$

The potential energy $V_{Q\bar{Q}}$ is $V_{Q\bar{Q}} = F_{Q\bar{Q}} - 2F_Q$ [57].

The expression of effective running coupling constant of quark-antiquark pair can be obtained from lattice QCD [11]

$$\alpha = \frac{3L^2}{4} \frac{dV_{Q\bar{Q}}}{dL}. \quad (21)$$

The temperature dependence of α at short and large distances has been studied from lattice QCD [11]. In the short-distance limit, α exhibits the weakening with decreasing distance that is typical of the perturbative regime. At large distances, the behavior of the coupling is suppressed due to color screening. As a consequence of these two opposing trends, α reaches a maximum at an intermediate distance scale. It is important to note that while color

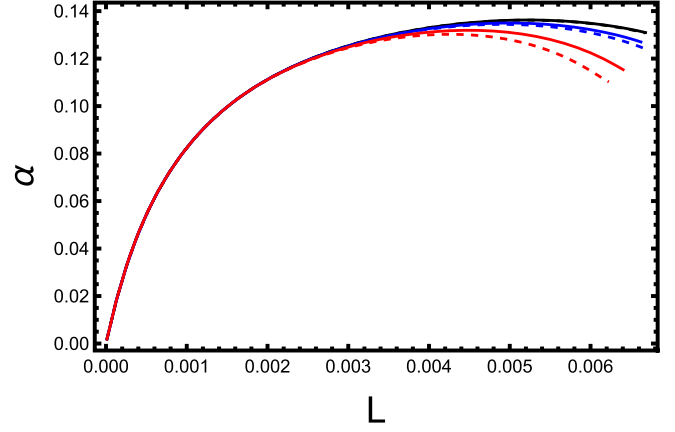


FIG. 1. Effective running coupling constant of $Q\bar{Q}$ versus interquark distance L with different boost parameter a . From top to bottom $a = 0.1, 0.2, 0.3$, respectively. The solid line (dashed line) represents the parallel (transverse) case.

screening suppresses the coupling in the large distance regime, nonperturbative effects still play a dominant role in determining its magnitude [11].

We study the effect of the boost parameter on the effective running coupling constant of $Q\bar{Q}$ in the spinning black hole background, which is dual to a rigidly rotating fluid. In the following calculations, we set $R = 1$. Following [44], we consider the planar black brane and take $T = 100/\pi$, and take the boost parameter within the stable regime $a < 0.75R$.

In Fig. 1, we depict the effective running coupling constant of $Q\bar{Q}$ versus interquark distance L with different boost parameter a . The effective coupling constant characterizes the interaction strength between the quark-antiquark pair in the medium. A large coupling indicates strong interactions where quark interactions dominate. However, a small coupling implies quarks behave nearly independently. We observed that the effective coupling constant initially increases with interquark distance L until it reaches a maximum value. It implies that the interaction force between quarks intensifies until reaching a maximum value, after which the interaction of heavy quark-antiquark pair becomes weak. Moreover, one can find that the boost parameter suppresses the effective running coupling constant and reduces its maximum value. From the results, the boost parameter promotes the dissociation of quarkonium. The angular velocity $\Omega = a/R^2$ [44]. Therefore, the angular momentum promotes the dissociation of quarkonium.

This result can be explained from the potential energy of heavy quarkonium. The authors of Ref. [53] investigated the potential energy of heavy quarkonium in this background and found that it rapidly approaches zero as the angular momentum increases. This indicates that heavy quarkonium becomes more unstable under stronger rotation. In the same spinning black hole background, the authors of Ref. [54] studied the spectral function of heavy

quarkonium and found that an increasing angular momentum facilitates its dissociation. Our results are consistent with the results of Refs. [53,54]. Furthermore, the angular momentum has a stronger effect on the effective running coupling constant when the axis of $Q\bar{Q}$ is transverse to the direction of the angular momentum.

B. Jet quenching parameter in the spinning black hole background

In this subsection, we study the jet quenching parameter $\hat{q}$ in the spinning Myers-Perry black holes background. Following the holographic approach established by Liu *et al.* [26], the jet quenching parameter $\hat{q}$ is extracted from the expectation value of a lightlike Wilson loop. In this framework, $\hat{q}$ characterizes the transverse momentum broadening of a fast parton traversing the strongly coupled medium. The jet quenching parameter can be extracted from [26]

$$\langle W^A[C] \rangle \approx \exp \left[-\frac{1}{4\sqrt{2}} \hat{q} L^- L_k^2 \right], \quad (22)$$

where $\langle W^A[C] \rangle \approx \langle W^F[C] \rangle^2$. $\langle W^F[C] \rangle$ denotes the expectation value in the fundamental representation from AdS/CFT correspondence. One can get [26]

$$\langle W^F[C] \rangle \approx \exp[-S_I]. \quad (23)$$

Therefore, the expression for the jet quenching parameter in $\mathcal{N} = 4$ SYM plasma is [26]

$$\hat{q} = 8\sqrt{2} \frac{S_I}{L^- L_k^2}, \quad (24)$$

where $S_I = S - S_0$. S denote the total energy and S_0 is the self-energy. L_k and L^- are the size of the nulllike rectangular contour.

Due to the rotation direction being along x_3 axis. We can consider the transverse and parallel cases. In the transverse case, the jets propagate transverse to the rotation axis, while momentum broadening could occur both transverse and parallel to the rotation axis. First, the jets propagate along x_1 with momentum broadening along x_2 , characterized by $\hat{q}_{(\perp,\perp)}$. Second, jets propagate along x_1 with momentum broadening along x_3 , characterized by $\hat{q}_{(\perp,\parallel)}$. We first calculate the $\hat{q}_{(\perp,\perp)}$.

By using the light-cone coordinates

$$d\tau = \frac{dx^+ + dx^-}{\sqrt{2}}, \quad dx_1 = \frac{dx^+ - dx^-}{\sqrt{2}}. \quad (25)$$

One can rewrite the metric (10) as

$$ds^2 = \frac{1}{z^2} \left[dx_2^2 + dx_3^2 - 2dx^+ dx^- + \frac{z^4}{z_h^4(1-a^2)} \left(\frac{dx^+ + dx^-}{\sqrt{2}} + adx_3 \right)^2 \right] + \frac{z_h^4}{z^2(z_h^4 - z^4)} dz^2. \quad (26)$$

The ansatz for the string configuration is

$$x^- = \xi, \quad x_2 = \sigma, \quad x^+ = x_3 = \text{const}, \quad z = z(\sigma). \quad (27)$$

Then Eq. (26) becomes

$$ds^2 = \frac{1}{2} \frac{z^2}{z_h^4(1-a^2)} d\xi^2 + \left(\frac{1}{z^2} + \frac{z_h^4}{z^2(z_h^4 - z^4)} \dot{z}^2 \right) d\sigma^2, \quad (28)$$

with $\dot{z} = \frac{dz}{d\sigma}$.

The Nambu-Goto action is

$$S = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_0^{L_k} d\sigma \sqrt{\frac{1}{1-a^2} \left(\frac{1}{z_h^4} + \frac{1}{z_h^4 - z^4} \dot{z}^2 \right)}. \quad (29)$$

The Lagrangian density is σ independent, implying a conserved Hamiltonian, namely $\mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{z}} \dot{z} = E$. Then one can get

$$\dot{z} = \frac{1}{E^2} \sqrt{\frac{z_h^4 - z^4}{z_h^4}} \sqrt{\frac{1}{z_h^4(1-a^2)} - E^2}. \quad (30)$$

In the low energy limit ($E \rightarrow 0$), integrating Eq. (30) to leading order of E^2 yields

$$L_k = 2E \int_0^{z_h} dz \sqrt{\frac{z_h^8(1-a^2)}{z_h^4 - z^4}}. \quad (31)$$

By substituting (30) into (29) and expanding to leading order of E^2

$$S = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_0^{z_h} dz \sqrt{\frac{1}{(z_h^4 - z^4)(1-a^2)}} \left(1 + \frac{E^2 z_h^4(1-a^2)}{2} \right). \quad (32)$$

The self-energy is

$$S_0 = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_0^{z_h} dz \sqrt{\frac{1}{(z_h^4 - z^4)(1-a^2)}}. \quad (33)$$

The action $S_I = S - S_0$ is

$$S_I = \frac{\sqrt{2}L^- E^2}{4\pi\alpha'} \int_0^{z_h} dz \sqrt{\frac{z_h^8(1-a^2)}{z_h^4 - z^4}}. \quad (34)$$

Then one can drive the jet quenching parameter as

$$\hat{q}_{(\perp,\perp)} = \frac{1}{\pi\alpha'} \frac{1}{I_{(\perp,\perp)}}, \quad (35)$$

where

$$I_{(\perp,\perp)} = \int_0^{z_h} dz \sqrt{\frac{z_h^8(1-a^2)}{z_h^4 - z^4}}. \quad (36)$$

Then we calculate the $\hat{q}_{(\perp,\parallel)}$. In this case, we use the following ansatz

$$x^- = \xi, \quad x_3 = \sigma, \quad x^+ = x_2 = \text{const}, \quad z = z(\sigma). \quad (37)$$

Then Eq. (26) becomes

$$ds^2 = \frac{1}{2z_h^4(1-a^2)} d\xi^2 + \left(\frac{1}{z^2} + \frac{z^2 a^2}{z_h^4(1-a^2)} + \frac{z_h^4}{z^2(z_h^4 - z^4)} \dot{z}^2 \right) d\sigma^2 + \frac{\sqrt{2}az^2}{z_h^4(1-a^2)} d\xi d\sigma. \quad (38)$$

The expression of Nambu-Goto action is

$$S = \frac{\sqrt{2}L^-}{2\pi\alpha'} \int_0^{L_k} d\sigma \sqrt{\frac{1}{1-a^2} \left(\frac{1}{z_h^4} + \frac{1}{z_h^4 - z^4} \dot{z}^2 \right)}. \quad (39)$$

One can find that the Nambu-Goto action of $\hat{q}_{(\perp,\parallel)}$ [Eq. (39)] is equal to the Nambu-Goto action of $\hat{q}_{(\perp,\perp)}$ [Eq. (29)], which yields $\hat{q}_{(\perp,\parallel)} = \hat{q}_{(\perp,\perp)}$. Thus we denote $\hat{q}_\perp \equiv \hat{q}_{(\perp,\parallel)} = \hat{q}_{(\perp,\perp)}$.

In parallel case, the jets propagate along the x_3 and the momentum broadening along the x_1 , namely $\hat{q}_{(\parallel,\perp)}$. The light-cone coordinates are

$$d\tau = \frac{dx^+ + dx^-}{\sqrt{2}}, \quad dx_3 = \frac{dx^+ - dx^-}{\sqrt{2}}. \quad (40)$$

Then one can rewrite the metric (10)

$$ds^2 = \frac{1}{z^2} \left[dx_1^2 + dx_2^2 - 2dx^+ dx^- + \frac{z^4}{z_h^4(1-a^2)} \left(\frac{dx^+ + dx^-}{\sqrt{2}} + a \frac{dx^+ - dx^-}{\sqrt{2}} \right)^2 \right] + \frac{z_h^4}{z^2(z_h^4 - z^4)} dz^2. \quad (41)$$

The ansatz can be

$$x^- = \xi, \quad x_1 = \sigma, \quad x^+ = x_2 = \text{const}, \quad z = z(\sigma). \quad (42)$$

Then one can obtain Eq. (41) as

$$ds^2 = \frac{z^2(1-a)}{z_h^4(1+a)} d\xi^2 + \left(\frac{1}{z^2} + \frac{z_h^4}{z^2(z_h^4 - z^4)} \dot{z}^2 \right) d\sigma^2. \quad (43)$$

The jet quenching parameter in the parallel case is

$$\hat{q}_{(\parallel,\perp)} = \frac{1}{\pi\alpha'} \frac{1}{I_{(\parallel,\perp)}}, \quad (44)$$

where

$$I_{(\parallel,\perp)} = \int_0^{z_h} dz \frac{1}{z_h^4} \sqrt{\frac{1-a^2}{z_h^4 - z^4}}. \quad (45)$$

For simplicity, we denote $\hat{q}_\parallel \equiv \hat{q}_{(\parallel,\perp)}$. Therefore, we study the effect of the boost parameter on $\hat{q}_\perp$ and $\hat{q}_\parallel$. In

Fig. 2, we plot the jet quenching parameter $\hat{q}/T^3$ versus boost parameter a . One can observe that the boost parameter enhances the jet quenching parameter, suggesting the jet will lose more energy in the spinning black hole background. The angular velocity is proportional to the boost parameter [44]. Thus, the angular momentum favors

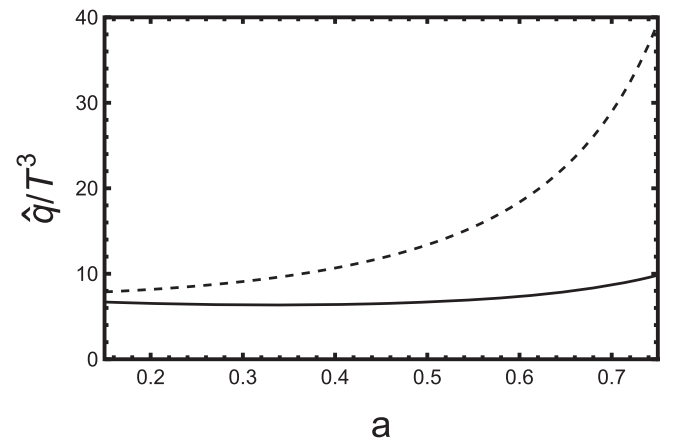


FIG. 2. Jet quenching parameter $\hat{q}/T^3$ versus boost parameter a . The solid line (dashed line) represents the $\hat{q}_\parallel$ ($\hat{q}_\perp$).

the jet quenching parameter. This finding is consistent with the result in the Kerr-AdS₅ black hole background [35]. The enhanced energy loss of jets in a rotating medium can be explained by the centrifugal force, which increases their interaction with the medium. This result agrees with Ref. [58], where rotation was shown to amplify heavy quark energy loss via centrifugal effects. It should be mentioned that the authors of Ref. [59] discuss the jet quenching parameter in the moving plasma. Furthermore, the angular momentum has a slight effect on $\hat{q}_{\parallel}$, while significantly enhancing $\hat{q}_{\perp}$. It implies that the jet may lose more energy when moving transverse to the direction of the angular momentum.

We find that the angular momentum enhances the jet quenching parameter and has a stronger effect when the jet moves transversely to the direction of the angular momentum, namely $\hat{q}_{\perp} > \hat{q}_{\parallel}$. These findings reveal an estimation of the jet quenching parameter's dependence on η/s at strong coupling. As demonstrated in [44], $\eta_{\perp}/s$ maintains unchanged with the angular momentum, whereas $\eta_{\parallel}/s$ decreases as the angular momentum increases. We refrain from commenting on why only one shear viscosity component saturates the KSS bound while the other violates it which is also observed in the anisotropic backgrounds [60–62]. We focus on the $\eta_{\parallel}/s$, which decreases as the angular momentum increases. This suggests that the fluid is closer to the perfect fluid when the angular momentum grows. We find that the angular momentum favors the jet quenching parameter, suggesting the jet quenching parameter increases as η/s decreases in the strongly coupled system.

It should be mentioned that the precise definition of $\hat{q}$ varies depending of the framework and, in the most sound approaches, it is not really a parameter but a function of the transverse momentum. While the jet quenching parameter $\hat{q}$ is a widely used phenomenological measure of medium interaction, the reaction rate is more fundamental from a theoretical physics perspective, which can be related to the expectation value of lightlike Wilson lines.

In heavy-ion collision physics, $\hat{q}$ and the reaction rate are two core quantities that characterize the energy loss of high-energy partons in the QGP, each with distinct advantages and limitations. While $\hat{q}$ offers a more intuitive physical picture and is defined as the mean squared transverse momentum gain per unit path length. The value of $\hat{q}$ can be extracted by analyzing the suppression of high transverse momentum hadron spectra (R_{AA}). However, extracting $\hat{q}$ from experimental data depends strongly on assumptions about the jet energy loss mechanism and the medium evolution model. Moreover, in the moderate transverse momentum region or near the critical temperature T_c , nonperturbative effects may become significant, limiting the reliability of perturbative QCD calculations. In contrast, the jet reaction rate explicitly gives the probability for a parton to lose energy through a specific process. It is

grounded in microscopic interaction cross sections, offering a solid physical foundation. Nevertheless, compared with $\hat{q}$, the interaction rate is more difficult to extract directly from a single experimental observable and typically requires constraints from complex global fits.

The reaction rate and the jet quenching parameter are not mutually exclusive but complementary physical quantities. Within the holographic framework, both originate from the nonperturbative dynamics of Wilson lines, collectively depicting the overall picture of parton energy loss in strongly coupled plasmas. Our current work focuses on $\hat{q}$, which serves as a reasonable baseline observable because it is directly linked to experimentally measurable jet quenching phenomena, such as R_{AA} . Furthermore, it is worth noting that a potentially significant future work is to explore the synergistic use of both the jet quenching parameter and the reaction rate. This can not only describe jet quenching dynamics more accurately but also serve as versatile probes for revealing deeper static properties, dynamical evolution, and even quantum features of the QGP.

IV. CONCLUSION AND DISCUSSION

In this work, we explore the effective running coupling constant of heavy quark pair and jet quenching parameter in the spinning background from holography.

It is found that the effective running coupling constant initially increases with interquark distance L until it reaches a maximum value and then starts to decrease. We also observe that the angular momentum suppresses the effective running coupling constant and reduces its maximum value, indicating the angular momentum promotes the dissociation of quarkonium. Moreover, the angular momentum has a stronger effect on the effective running coupling constant when the axis of $Q\bar{Q}$ is transverse to the direction of the angular momentum. The authors of Ref. [24] study the effective running coupling constant in the local rotating background. They find that angular momentum promotes dissociation and our results agree with Ref. [24]. We also find that the angular momentum enhances the jet quenching parameter and has a stronger effect when the jet moves transversely to the direction of the angular momentum. Although a direct quantitative mapping between the holographic boost parameter a and the angular momentum of the QGP created in heavy-ion collisions is not straightforward, our results demonstrate that within the physically stable regime of the model ($a < 0.75R$), the influence of rotation on the effective coupling strength and the jet quenching parameter is significant and exhibits clear anisotropy. These findings provide qualitative insights into how rotational motion may affect quarkonium dissociation and jet energy loss in a rotating QGP.

The effective coupling α inherently intertwines the running of the coupling and screening effects. One can

focus directly on the behavior of potential and cleanly separate vacuum physics from medium-induced effects. It should be mentioned that the potential of quarkonium in the spinning medium has been studied in our previous work (Ref. [53]). We aim to further explore the interaction strength between the quark-antiquark pair and calculate the effective running coupling constant in the spinning medium.

It may be an interesting investigation to discuss the dissociation of quarkonium and jet quenching in the charged Kerr-AdS₅ background [63]. We expect explore this work in subsequent studies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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