

Leptogenesis in the presence of density perturbations

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We point out a new effect on the freeze-out process of heavy particles induced by density perturbations in the early Universe, which we call “acoustically driven freeze-out.” This beyond-linear effect is caused by the exponential decoupling of heavy particles from the thermal bath in the presence of density perturbations, and already at moderately large values $\delta T/\bar{T} = \mathcal{O}(10^{-2})$, it cannot be captured by linear perturbation theory. We illustrate this effect with leptogenesis taking the decay and inverse decay of heavy neutrinos into account and discuss its phenomenological implications. We found that perturbations always enhance the (spatially averaged) values of the final lepton asymmetry, and as a result, constraints on the mass of heavy neutrinos are found to be relaxed in the presence of perturbations.

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I. INTRODUCTION

Inflation provides a successful explanation for the homogeneity, isotropy, and flatness of the Universe [1–5] as well as the origin of the large-scale structure [6–9]. One of its characteristic predictions is that it generates density perturbations over different scales. While density perturbations are well constrained at large scales from the cosmic microwave background (CMB) [10] and large-scale structure observations [11], those at small scales are yet to be explored. These scalar fluctuations reenter the Hubble horizon after inflation, driving acoustic oscillation of density and temperature.

Out-of-equilibrium phenomena play central roles in the history of the Universe. One of the prominent examples is the production of baryon asymmetry, which requires out-of-equilibrium interactions among the Sakharov’s three conditions [12]. Other examples include preheating after inflation [13–15] and freeze-out/freeze-in of dark matter [16,17], as well as Standard Model (SM) processes such as big bang nucleosynthesis (BBN) and recombination. Among these out-of-equilibrium processes, the effect we point out occurs when heavy particles decouple from the thermal bath. In the presence of temperature fluctuation δT ,

the Boltzmann factor for the equilibrium abundance of such particles with mass M is given by

$$\begin{aligned} \exp(-E/T) &= \exp[-E/(\bar{T} + \delta T)] \\ &\simeq \exp(-E/\bar{T}) \exp(E\delta T/\bar{T}^2). \end{aligned} \quad (1)$$

When $E \simeq M$ is much larger than $\bar{T}$, the exponent of the second factor in the right-hand side of Eq. (1) is not necessarily smaller than unity, invalidating truncation of the system at the first order in perturbation. The actual abundance of this particle species also closely follows this equilibrium value around the decoupling time, and hence, the final abundance of this species itself or its decay product cannot also be predicted with truncation at the first order.

Baryogenesis is one of the phenomena in which this beyond-linear effect can potentially play an important role, and in this paper, we illustrate the effect with one the simplest baryogenesis scenarios. *Leptogenesis* [18] is among the most successful scenarios, in which the Sakharov’s three conditions are satisfied by the CP -violating decay of right-handed neutrinos together with the sphaleron process in the SM. These right-handed neutrinos naturally exist in the UV completion of the SM such as grand unification theories and can also naturally explain the masses of light neutrinos through the seesaw mechanism [19–22]. The masses, mixing angles, and CP phase(s) of these light neutrinos are being explored with solar, reactor, atmospheric, and accelerator neutrino oscillation experiments (see, e.g., Ref. [23]), while their Dirac or Majorana nature may be determined

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in the current or future $0\nu\beta\beta$ decay experiments (see, e.g., Ref. [24]).

Density perturbations at small scales, on the other hand, are gaining interest in view of gravitational-wave (GW) observations [25–30]. They give rise to multipole structures in the matter distribution, and for sufficiently large amplitude, they produce secondary GWs [31,32] and lead to the formation of primordial black holes [33–35]. Part of the motivations of the current paper is reassessing the effect of an inhomogeneous universe on particle physics processes, demonstrating the interplay of micro- and macrophysics. In this perspective, we investigate the influence of density perturbations on the lepton and baryon asymmetries within the simplest leptogenesis scenarios. As we see in the following, sound waves induce a sudden freeze-out of heavy particles, resulting in a net enhancement in the particle abundance after freeze-out even after spatial average.¹ We call this process “acoustically driven freeze-out.”

This paper is organized as follows. In Sec. II, we derive the Boltzmann equations for the heavy neutrino N_1 and for the baryon-minus-lepton number $B - L$, in the presence of density perturbations. We also review various parameters related to the neutrino masses. In Sec. III, we discuss the intuitive picture for the acoustically driven freeze-out and then present the time evolution of the right-handed neutrino and $B - L$ as well as the parameter space for successful baryogenesis. Finally in Sec. IV, we discuss the implications of our results and present possible future directions.

II. LEPTOGENESIS IN THE PRESENCE OF COSMOLOGICAL PERTURBATIONS

A. Prerequisites for leptogenesis

Leptogenesis [18] is a scenario in which the CP -violating decays of heavy right-handed neutrinos, together with the sphaleron process in the SM, generate the baryon asymmetry of the Universe. In leptogenesis, the SM is extended with right-handed neutrinos with heavy Majorana masses that violate the lepton number. These right-handed neutrinos have Yukawa interactions with the Higgs and light leptons, whose complex phases give rise to CP violation. Their CP -violating decays produce lepton asymmetry, part of which is then converted into baryon asymmetry through the SM sphaleron process. The Majorana masses of the right-handed neutrinos at the same time

¹The effects of density perturbations on baryon asymmetry are partially studied in Ref. [36], where the authors consider perturbations of the same order of magnitude as CMB and find the effect to be negligibly small. The present analysis is different from theirs in that we calculate the spatial average of the resulting baryon asymmetry, which should be the relevant quantity for the observed baryon asymmetry. Another difference is that we consider relatively large density perturbations, which we will find to have a net effect on the final averaged baryon abundance.

explain the observed light masses for left-handed neutrinos via the seesaw mechanism.

Leptogenesis scenarios can be classified into two classes: thermal and nonthermal (see, e.g., [37,38]). The former assumes thermal abundance for the right-handed neutrinos as one of the initial conditions, while in the latter, the right-handed neutrinos are produced via various nonthermal processes. One of the attractive features of the former class is its robust predictivity: The number of free parameters is significantly smaller, thereby allowing for more robust experimental predictions. For this reason, we focus on thermal leptogenesis in this paper.

We consider a simple model so-called vanilla leptogenesis, in which heavy right-handed neutrinos N_1 , N_2 , and N_3 with masses in the order $M_1 < M_2 < M_3$ are added to the SM. In this model, the Majorana mass terms and Yukawa couplings of the right-handed neutrinos

$$\mathcal{L} = -\frac{M_\alpha}{2}\bar{N}_\alpha N_\alpha - y_{\alpha\beta}\phi^*\bar{l}_\alpha N_\beta + \text{H.c.} \quad (2)$$

are added to the SM Lagrangian together with their kinetic terms. Here, M_α and $y_{\alpha\beta}$ denote the Majorana masses and Yukawa couplings, respectively, while ϕ and l_α are the Higgs and lepton doublets of the SM, respectively. The summation over lepton flavors $\alpha, \beta = 1, 2, 3$ is implicit. The light neutrino masses are generated from the type-I seesaw mechanism [19–22].

The lightest right-handed neutrino N_1 decays into a lepton doublet and Higgs doublet pair or its CP -conjugate:

$$N_1 \rightarrow l_\alpha\phi, \quad N_1 \rightarrow \bar{l}_\alpha\phi^*. \quad (3)$$

In this paper, only the decay and inverse decay of N_1 are included, and scattering processes are ignored. The decays of N_2 and N_3 are assumed to be negligible in the final asymmetry, and they appear only as internal lines in the diagrams of N_1 decay. This simplified model is sufficient to capture the beyond-linear effect we point out in the present study. The $B - L$ asymmetry is generated from the CP -violating decays (3) as

$$\begin{aligned} \frac{\Gamma(N_1 \rightarrow l_\alpha\phi)}{\Gamma(N_1 \rightarrow l_\alpha\phi) + \Gamma(N_1 \rightarrow \bar{l}_\alpha\phi^*)} &= \frac{1 + \varepsilon_1}{2}, \\ \frac{\Gamma(N_1 \rightarrow \bar{l}_\alpha\phi^*)}{\Gamma(N_1 \rightarrow l_\alpha\phi) + \Gamma(N_1 \rightarrow \bar{l}_\alpha\phi^*)} &= \frac{1 - \varepsilon_1}{2}. \end{aligned} \quad (4)$$

Here, the parameter $\varepsilon_1 (< 0)$ describing CP asymmetry is given by (see Appendix D)

$$\begin{aligned} \varepsilon_1 &= \sum_{\gamma=2,3} \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha\gamma}^* y_{\beta\gamma}^*)}{8\pi \sum_\alpha |y_{\alpha 1}|^2} f(x), \quad x \equiv \frac{M_\gamma^2}{M_1^2} \\ f(x) &\equiv \sqrt{x} \left[1 + \frac{1}{1-x} - (1+x) \ln \left(1 + \frac{1}{x} \right) \right]. \end{aligned} \quad (5)$$

In the following, we consider the time evolution of the ratios $N_{N_1} \equiv n_{N_1}/n_\gamma$ and $N_{B-L} \equiv (n_B - n_L)/n_\gamma$, where n_{N_1} , n_γ , n_B , and n_L denote the number densities of right-handed neutrinos, photons, baryons, and leptons, respectively. Their initial conditions are given at sufficiently high temperatures $T \gg M_1$ by

$$N_{N_1}^{\text{eq}}(T \gg M_1) = \frac{n_{N_1}^{\text{eq}}}{n_\gamma} = \frac{3}{4}, \quad N_{B-L}^{\text{eq}}(T \gg M_1) = 0. \quad (6)$$

After the decay of right-handed neutrinos, the negative lepton number L is converted into positive B and negative L through the sphaleron process [39,40]. Since $B-L$ is conserved in this process, the final baryon number B is related to the value of $B-L$ generated by N_1 decay as [41,42]

$$B = \frac{8\nu_f + 4\nu_s}{22\nu_f + 13\nu_s} (B-L) = \frac{28}{79} (B-L), \quad (7)$$

where $\nu_f = 3$ is the number of fermion generations, and $\nu_s = 1$ is the number of Higgs doublets.

B. Prerequisites for cosmological perturbations

In an inhomogeneous universe, the time evolution of the right-handed neutrinos and baryon and/or lepton numbers occurs in the presence of cosmological perturbations. In this paper, we follow Refs. [43,44] for the formulation of cosmological perturbations.

The background metric is given by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2 \delta_{ij} dx^i dx^j, \quad (8)$$

where $g_{\mu\nu}$ is the metric tensor, and $a = a(t)$ is the scale factor of the Universe. We assume that the Universe is radiation dominated in the relevant epoch, and in this case, the Friedmann equation is

$$3M_{\text{P}}^2 H^2 = \rho_r, \quad (9)$$

with

$$H = \frac{1}{2t}, \quad \rho_r = \frac{\pi^2}{30} g_* T^4. \quad (10)$$

Here, $M_{\text{P}} = 1/\sqrt{8\pi G}$ is the reduced Planck mass, $H \equiv \dot{a}/a = a'/a^2$ is the Hubble parameter, and ρ_r is the radiation energy density. During the radiation-dominated epoch, the conformal time η defined by $d\eta = dt/a$ is related to the cosmological time as $\eta \propto t^{1/2}$. In what follows, the dot denotes differentiation with respect to the physical time t , and the prime denotes differentiation with respect to the conformal time η .

The scalar perturbations of the metric tensor $g_{\mu\nu}$ are expressed in terms of four independent functions of time and space, A , B , C , and E , as [44]

$$g_{00} = -a^2(1 + 2A), \quad (11)$$

$$g_{0i} = -a^2 B_{,i}, \quad (12)$$

$$g_{ij} = a^2 \left[(1 + 2C) \delta_{ij} + 2 \left(E_{,ij} - \frac{1}{3} \delta_{ij} \nabla^2 E \right) \right]. \quad (13)$$

On the other hand, perturbation in the matter sector appears in the temperature fluctuation δ_T defined by

$$T = \bar{T} + \delta T, \quad \delta_T \equiv \frac{\delta T}{\bar{T}}, \quad (14)$$

where $\bar{T}$ is the spatially averaged temperature of the Universe. With these definitions, it is useful to introduce new variables:

$$\bar{z} \equiv \frac{M_1}{\bar{T}}, \quad (15)$$

$$z_T \equiv \frac{M_1}{\bar{T} + \delta T} = \frac{\bar{z}}{1 + \delta_T}. \quad (16)$$

In the following sections, we will use $\bar{z}$ and z_T instead of physical time t and physical temperature T , respectively.

C. Boltzmann equation

The evolution of the number density of each particle species in the presence of collision processes is governed by the Boltzmann equation. After integrating over the momentum space, it takes the form (see Appendix B)

$$\mathcal{N}^\mu{}_{;\mu} = \frac{g_{\text{deg}}}{(2\pi)^3} \int \frac{d^3 p}{E} C[f], \quad (17)$$

where $\mathcal{N}^\mu$ is the number density current defined by $\mathcal{N}^\mu \equiv n U^\mu$, with n and U^μ being the number density and the four-velocity, respectively, g_{deg} is the degrees of freedom of the species, f is the distribution function, and $C[f]$ is the collision term. The left-hand side of Eq. (17) can be written as

$$(n U^\mu)_{;\mu} = \bar{z} H (1 - A) \left(\frac{dn}{d\bar{z}} + \frac{3}{\bar{z}} n \right) + \frac{n}{a} (v^i{}_{,i} + 3C'), \quad (18)$$

where v^i is the three-velocity, the latter of which is first order in perturbation. Here, we used

$$U^0 = a^{-1}(1 - A), \quad U^i = a^{-1} v^i, \quad (19)$$

and Eqs. (C1)–(C10).

For the right-handed neutrinos N_1 , we take only the decay and inverse decay into account. In this case, assuming that the right-handed neutrinos are in kinetic equilibrium, the thermal average of the right-hand side of Eq. (17) yields

$$\begin{aligned} & \left\langle \frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} C \right\rangle \\ &= -\frac{4\pi^3 g_l g_\phi |\mathcal{M}|^2}{M_1} \cdot \frac{K_1(z_T)}{K_2(z_T)} \cdot [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)] \\ &\equiv -\Gamma_D(\bar{z} = \infty) \left\langle \frac{1}{\gamma} \right\rangle (z_T) [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)]. \end{aligned} \quad (20)$$

The collision term (20) is invariant under coordinate transformations: Indeed, when the collision term is in the local inertial frame instantaneously at rest with respect to the comoving observer (LIFIRCO), the cross sections have the same expressions as in the Minkowski space, and the collision term contains *no metric fluctuations* [45]. Any metric fluctuation is contained in, and arises from, the Liouville term, i.e., the left-hand side of Eq. (17). As a result, we obtain

$$\begin{aligned} & \left(\frac{dn_{N_1}}{d\bar{z}} + \frac{3}{\bar{z}} n_{N_1} \right) + \frac{n_{N_1}}{d\bar{z}H} (v^i{}_{,i} + 3C') \\ &\simeq -(1+A) \frac{\Gamma_D(\bar{z} = \infty)}{\bar{z}H} \left\langle \frac{1}{\gamma} \right\rangle (z_T) [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)] \\ &= -(1+A) \bar{z} K \left\langle \frac{1}{\gamma} \right\rangle (z_T) [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)], \end{aligned} \quad (21)$$

where $K \equiv \Gamma_D(\bar{z} = \infty)/\bar{z}^2 H$ is the so-called decay parameter. The evolution of the number density ratio $N_{N_1} = n_{N_1}(\bar{z})/n_{N_1}^{\text{eq}}(z_T)$ becomes

$$\frac{dN_{N_1}(\bar{z})}{d\bar{z}} = -(1+A) D(z_T) [N_{N_1}(\bar{z}) - N_{N_1}^{\text{eq}}(z_T)], \quad (22)$$

where

$$D(z_T) \equiv K \bar{z} \left\langle \frac{1}{\gamma} \right\rangle (z_T) = K \bar{z} \frac{K_1(z_T)}{K_2(z_T)}, \quad (23)$$

and

$$N_{N_1}^{\text{eq}}(z_T) = \frac{3}{8} z_T^2 K_2(z_T). \quad (24)$$

For N_{B-L} , the Liouville term is evaluated in the same way as before. The collision term contains the sourcing of lepton asymmetry from the CP asymmetry ε_1 through the difference between the nonequilibrium and equilibrium abundances of right-handed neutrinos. As a result, the

evolution of N_{B-L} is governed by

$$\begin{aligned} \frac{dN_{B-L}(\bar{z})}{d\bar{z}} &= -\varepsilon_1 (1+A) D(z_T) [N_{N_1}(\bar{z}) - N_{N_1}^{\text{eq}}(z_T)] \\ &\quad - (1+A) W_{\text{ID}} N_{B-L}, \end{aligned} \quad (25)$$

where

$$W_{\text{ID}}(z_T) = \frac{1}{4} K \bar{z} z_T^2 K_1(z_T) \quad (26)$$

is the washout factor without scattering [37].

To solve Eqs. (22) and (25), we choose the conformal Newtonian gauge $A = \Psi$, $C = -\Phi$ and $B = E = 0$. The anisotropic stress is assumed to be negligible, and in this case, $\Phi = \Psi$ holds. Equations (22) and (25) then, respectively, become

$$\frac{dN_{N_1}(\bar{z})}{d\bar{z}} = -(1+\Psi) D(z_T) [N_{N_1}(\bar{z}) - N_{N_1}^{\text{eq}}(z_T)], \quad (27)$$

and

$$\begin{aligned} \frac{dN_{B-L}(\bar{z})}{d\bar{z}} &= -\varepsilon_1 (1+\Psi) D(z_T) [N_{N_1}(\bar{z}) - N_{N_1}^{\text{eq}}(z_T)] \\ &\quad - (1+\Psi) W_{\text{ID}} N_{B-L}. \end{aligned} \quad (28)$$

The time evolution of $\Psi = \Phi$ in a radiation dominated universe is given by [44]

$$\Psi(\eta, \mathbf{k}) = 2\mathcal{R}_i \frac{\sin \varphi - \varphi \cos \varphi}{\varphi^3}, \quad (29)$$

where $\mathcal{R}_i$ is the primordial curvature perturbation in wave number space, and $\mathbf{k}$ is the wave number, with k being its absolute value. In a radiation-dominated epoch, φ is expressed as

$$\varphi = \frac{k\eta}{\sqrt{3}} \equiv \frac{\bar{z}}{\bar{z}_H}. \quad (30)$$

The parameter $\bar{z}_H$ defined here can be regarded as specifying the time of horizon entry for each wave number. Since $\Psi(\eta, \mathbf{x})$ is real, the Fourier transform can be written as

$$\begin{aligned} \Psi(\eta, \mathbf{x}) &= \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \Psi(\eta, \mathbf{k}) \\ &= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2} [e^{i\mathbf{k}\cdot\mathbf{x}} \Psi(\eta, \mathbf{k}) + e^{-i\mathbf{k}\cdot\mathbf{x}} \Psi^*(\eta, \mathbf{k})] \\ &= \int \frac{d^3 k}{(2\pi)^3} \left[2|\mathcal{R}_i| \cos \delta \frac{\sin \varphi - \varphi \cos \varphi}{\varphi^3} \right]. \end{aligned} \quad (31)$$

Here, $\delta \equiv \delta' + \mathbf{k} \cdot \mathbf{x}$ is the phase of the perturbation with δ' being the complex phase of $\mathcal{R}_i = |\mathcal{R}_i| e^{i\delta'}$. Written this way,

the time evolution of $\Psi = \Phi$ can be understood in terms of real functions only

$$\Psi(\eta, \mathbf{k}) = 2|\mathcal{R}_i| \cos \delta \frac{\sin \varphi - \varphi \cos \varphi}{\varphi^3}. \quad (32)$$

For each wave number $\mathbf{k}$, different values of the phase parameter δ can be regarded as specifying different spatial points. Since the final lepton asymmetry we observe is the spatial average, in our numerical analysis, we take average of the final asymmetry with respect to the phase δ .

The temperature fluctuation δ_T is related to the fluctuation in the radiation energy density δ_r as

$$\delta_r = \frac{\delta \rho_r}{\bar{\rho}} = 4 \frac{\delta T}{\bar{T}} = 4\delta_T. \quad (33)$$

The evolution of δ_r is given by [44]

$$\delta_r = -\frac{2}{3}(k\eta)^2 \Psi - 2\eta \Psi' - 2\Psi, \quad (34)$$

where we used $\Psi = \Phi$. Substituting Eq. (32) into Eq. (34), we obtain

$$\delta_r = 8|\mathcal{R}_i| \cos \delta \frac{\sin \varphi - \varphi \cos \varphi - \varphi^2 \sin \varphi + \frac{1}{2}\varphi^3 \cos \varphi}{\varphi^3}, \quad (35)$$

and correspondingly

$$\delta_T = 2|\mathcal{R}_i| \cos \delta \frac{\sin \varphi - \varphi \cos \varphi - \varphi^2 \sin \varphi + \frac{1}{2}\varphi^3 \cos \varphi}{\varphi^3}. \quad (36)$$

D. Neutrino mass parameters

In our setup, the amount of baryon asymmetry depends on the heavy neutrino mass M_1 , the decay parameter K , and the degree of CP asymmetry ε_1 . M_1 determines the temperature at which the abundance of N_1 starts to decline. K determines the time (or, the value of $\bar{z}$) when N_1 decouples from the thermal bath. ε_1 controls the efficiency of the generation of $B-L$ via N_1 decay. The latter two parameters K and ε_1 depend on the neutrino masses, Yukawa couplings, and other parameters in the Lagrangian. In this subsection, we introduce various neutrino mass parameters that are useful to interpret the results (see, e.g., Ref. [37] for details).

The masses of light neutrinos are denoted by m_1 , m_2 , and m_3 ($m_1 < m_2 < m_3$). The *effective neutrino mass* is defined by

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1}, \quad (37)$$

where $(m_D)_{\alpha\beta} = y_{\alpha\beta} v$ with $v \simeq 174$ GeV being the Higgs vacuum expectation value (VEV). The decay parameter K is proportional to $\tilde{m}_1$:

$$K \equiv \frac{\tilde{m}_1}{m_*}, \quad m_* = \sqrt{\frac{64\pi^3 g_*}{45}} \frac{v^2}{M_P} \simeq 1.08 \times 10^{-3} \text{ eV}, \quad (38)$$

where $g_* = 106.75$ is the number of degrees of freedom in the SM. For instance, $K = 0.01, 1, 100$ corresponds to $\tilde{m}_1 \simeq 10^{-5}, 10^{-3}, 10^{-1}$ eV, respectively. The two regimes $\tilde{m}_1 > m_*$ and $\tilde{m}_1 < m_*$ are referred to as the *strong washout* and *weak washout* regimes, respectively [37]. Note that $\tilde{m}_1$ must lie in the range of $m_1 \leq \tilde{m}_1 \lesssim m_3$ [46].

The Yukawa coupling has an upper bound from perturbativity, which in turn imposes a corresponding upper limit on the absolute value of the CP asymmetry. The maximal absolute CP asymmetry is given by (see Appendix E)

$$|\varepsilon_1|_{\max} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \left[1 - \frac{m_1}{m_3} \left(1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2} \right)^{1/2} \right]. \quad (39)$$

In this paper, we use the maximum value of the CP asymmetry (39) for $|\varepsilon_1|$ to calculate the maximum baryon-to-photon ratio $\eta_b^{\max}$.

In the subsequent section, we compute the evolution of N_{N_1} and N_{B-L} for given M_1 , K , and ε_1 . Then we map the result to the $(\tilde{m}_1, M_1)$ plane and discuss the allowed region, where the generated $B-L$ asymmetry is large enough to explain the observed baryon asymmetry. Since entropy injection may occur after the production of asymmetry, we only require that the baryon asymmetry exceed the observed value. For the latter, we use the result from the Planck Collaboration [10]

$$\eta_b^{\text{CMB}} = (6.144 \pm 0.038) \times 10^{-10}, \quad (40)$$

and require $\eta_b^{\max} \geq (\eta_b^{\text{CMB}})_{\text{low}}$, where

$$(\eta_b^{\text{CMB}})_{\text{low}} = 6.0 \times 10^{-10} \quad (41)$$

is the 3σ lower limit.

Another useful mass is the *absolute mass scale* defined by

$$\bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2}. \quad (42)$$

As discussed in Ref. [46], the allowed region on the $(\tilde{m}_1, M_1)$ plane shrinks with increasing $\bar{m}$ because the lower limit of $\tilde{m}_1$ increases, as we sketch below.

The value of $\bar{m}$ can be inferred from the mass measurements of neutrino oscillation experiments and cosmological observations. Given the neutrino mass patterns, i.e., either $m_3^2 - m_2^2 > m_2^2 - m_1^2$ or $m_3^2 - m_2^2 < m_2^2 - m_1^2$, the dependence of m_3 on m_1 is fixed. In this paper, we analyze the

normal hierarchy $m_1 < m_2 < m_3$ only. In this case, the relations $m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2$ lead to

$$\begin{aligned} m_3^2 &= m_1^2 + \Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2, \\ m_2^2 &= m_1^2 + \Delta m_{\text{sol}}^2, \\ \bar{m}^2 &= 3m_1^2 + \Delta m_{\text{atm}}^2 + 2\Delta m_{\text{sol}}^2. \end{aligned} \quad (43)$$

These relations can be rewritten as

$$\begin{aligned} m_1^2 &= \frac{1}{3}(\bar{m}^2 - \Delta m_{\text{atm}}^2 - 2\Delta m_{\text{sol}}^2), \\ m_2^2 &= \frac{1}{3}(\bar{m}^2 - \Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2), \\ m_3^2 &= \frac{1}{3}(\bar{m}^2 + 2\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2). \end{aligned} \quad (44)$$

According to the neutrino oscillation experiments (see Ref. [47] and references therein), the mass differences are known to be

$$\begin{aligned} \Delta m_{\text{atm}}^2 &= (2.453 \pm 0.033) \times 10^{-3} \text{ eV}^2, \\ \Delta m_{\text{sol}}^2 &= (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2. \end{aligned} \quad (45)$$

Thus $\bar{m}$ takes its minimum value

$$\bar{m} = \sqrt{\Delta m_{\text{atm}}^2 + 2\Delta m_{\text{sol}}^2} \simeq 0.051 \text{ eV}, \quad (46)$$

for $m_1 = 0$. Furthermore, CMB observations together with baryon acoustic oscillation (BAO) measurements constrain the sum of the mass eigenvalues of light neutrinos from above as [11]

$$m_1 + m_2 + m_3 < 0.13 \text{ eV}, \quad (47)$$

at the 2σ level. Substituting Eq. (44) and using Eq. (45), we find

$$\bar{m} < 0.078 \text{ eV}. \quad (48)$$

This upper limit is more relaxed when CMB alone is used without BAO [10]

$$m_1 + m_2 + m_3 < 0.26 \text{ eV} \quad (49)$$

at the 2σ level. In this case, we obtain

$$\bar{m} < 0.15 \text{ eV}. \quad (50)$$

In the following, we present the allowed regions in the $(\bar{m}_1, M_1)$ plane for the lower limit on $\bar{m}$ (46), the two types of upper limits (48) and (50), and also $\bar{m} = \bar{m}_{\text{max}} \equiv 0.19 \text{ eV}$, the last of which corresponds to the value just

before the allowed region disappears when scattering is taken into account.

III. IMPLICATIONS TO LEPTOGENESIS

A. Intuitive picture

Before proceeding to their numerical results, we describe the qualitative behavior of the evolution of right-handed neutrinos in the presence of density perturbations. When the right-handed neutrinos decouple from the thermal bath at temperature T , the equilibrium abundance of right-handed neutrinos N_1 can be approximated by the Boltzmann distribution

$$N_{N_1}^{\text{eq}} \sim \exp\left(-\frac{M_1}{T}\right). \quad (51)$$

Substituting Eqs. (14) into (51), $N_{N_1}^{\text{eq}}$ behaves as

$$N_{N_1}^{\text{eq}} \sim \exp\left(-\frac{M_1}{\bar{T}}\right) \exp\left(\frac{M_1}{\bar{T}} \delta_T\right). \quad (52)$$

Therefore, if the temperature fluctuation is sufficiently large

$$\delta_T \gtrsim \frac{\bar{T}}{M_1}, \quad (53)$$

the linear expansion of $\exp(M_1 \delta_T / \bar{T})$ causes a significant error. For the strong washout regime, the actual abundance N_{N_1} closely follows this equilibrium abundance, and hence, the linear approximation also breaks down for N_{N_1} . The condition (53) translates to $\delta_T \gtrsim 0.05$ given that the freeze-out of $|N_{B-L}|$ occurs at $\bar{z} = M_1 / \bar{T} \sim 10$ –20.

If we linearize with respect to δ_T , the baryon number would never increase because the average over the phase δ will cancel out the contributions from positive δ_T and those from negative δ_T . Therefore, the increase in the baryon number (Fig. 3) represents a *beyond-linear* effect related to δ_T , which we call “acoustically driven freeze-out.”

Before moving on to numerical results, we comment on the origin of the relatively large amplitude of density perturbations. The most straightforward implementation is inflation: It is generally known that inflation models with features in the potential are capable of generating large amplitude for the primordial curvature perturbation. For instance, models with a potential which exhibits local structures—such as steps or oscillations—as well as models with ultra slow-roll or transient slow-roll violation belong to this category. In addition, density perturbations can also be generated through first-order phase transitions or the formation of topological defects in the early Universe. These perturbations can manifest as acoustic waves and, in some cases, may even evolve into shock waves, potentially leading to the effects discussed in

Sec. IV. Such effects would be an interesting future direction to investigate.

B. Time evolution of N_1 and $B-L$

Figure 1 compares the time evolutions of the abundance of the right-handed neutrino N_{N_1} and the baryon-minus-lepton number N_{B-L} in the presence (thick lines) and absence (thin lines) of perturbations. The decay parameter $K = \tilde{\Gamma}_D / \bar{z}^2 H$ is taken to be 100, 1, and 0.01 from top to bottom. The phase is chosen to $\delta = \pi$ that corresponds to a specific spatial point. Also, a monochromatic wave $\bar{z}_H = 1$ is assumed, corresponding to the horizon reentry somewhat before the freeze-out of $|N_{B-L}|$. For the time evolution with other values of $\bar{z}_H = 1$, see Appendix A. Also, for more realistic density fluctuations described by a superposition of monochromatic waves, see the discussion in Sec. IV.

In the top panel of Fig. 1, we show the time evolution for $K = 100$. The actual and equilibrium abundances of right-handed neutrinos N_{N_1} and $N_{N_1}^{\text{eq}}$ oscillate significantly for $\bar{z} \gtrsim \mathcal{O}(1)$. As a result, $|N_{B-L}|$ also oscillates for $\mathcal{O}(1) \lesssim \bar{z} \lesssim 10$, leading to the increase in the value of $|N_{B-L}|$ after the freeze-out denoted by $|N_{B-L}^f|$. This increase occurs for both initial conditions $N_{N_1} = 3/4$ or $N_{N_1} = 0$. On a closer look, we observe that the freeze-out value $|N_{B-L}^f|$ is determined by the downstroke of the temperature and of the corresponding equilibrium value $N_{N_1}^{\text{eq}}$ around $\bar{z} \simeq 10$. For this parameter point, the freeze-out is dominantly driven by the sudden decrease in the temperature caused by sound waves, not by the slow decrease in the average temperature. Of course, different spatial points have different oscillation phases, and thus, we have to take average over the phase δ . We examine the effect of different values of δ below, in Fig. 2.

In the middle panel of Fig. 1, we change the decay parameter to $K = 1$. We still observe oscillations in the abundance of right-handed neutrinos N_{N_1} and its equilibrium value $N_{N_1}^{\text{eq}}$. The freeze-out value $|N_{B-L}^f|$ is thus enhanced in the same way as the top panel, though the enhancement is smaller. Such enhancement is observed for the strong washout regime $1 \leq K \leq 100$. For the weak washout regime $K < 1$, in contrast, N_{N_1} does not oscillate any more, and the freeze-out value $|N_{B-L}^f|$ is almost identical to the case without perturbations, as seen from the bottom panel of Fig. 1 corresponding to $K = 0.01$. This tendency is simply because N_{N_1} is not tightly coupled with $N_{N_1}^{\text{eq}}$ any more, and thus, the oscillations do not play any role in this parameter range.

To examine the effect of δ on $|N_{B-L}|$, in Fig. 2, we show plots obtained by changing only the initial phases δ under the same conditions as in the top panel of Fig. 1. Among

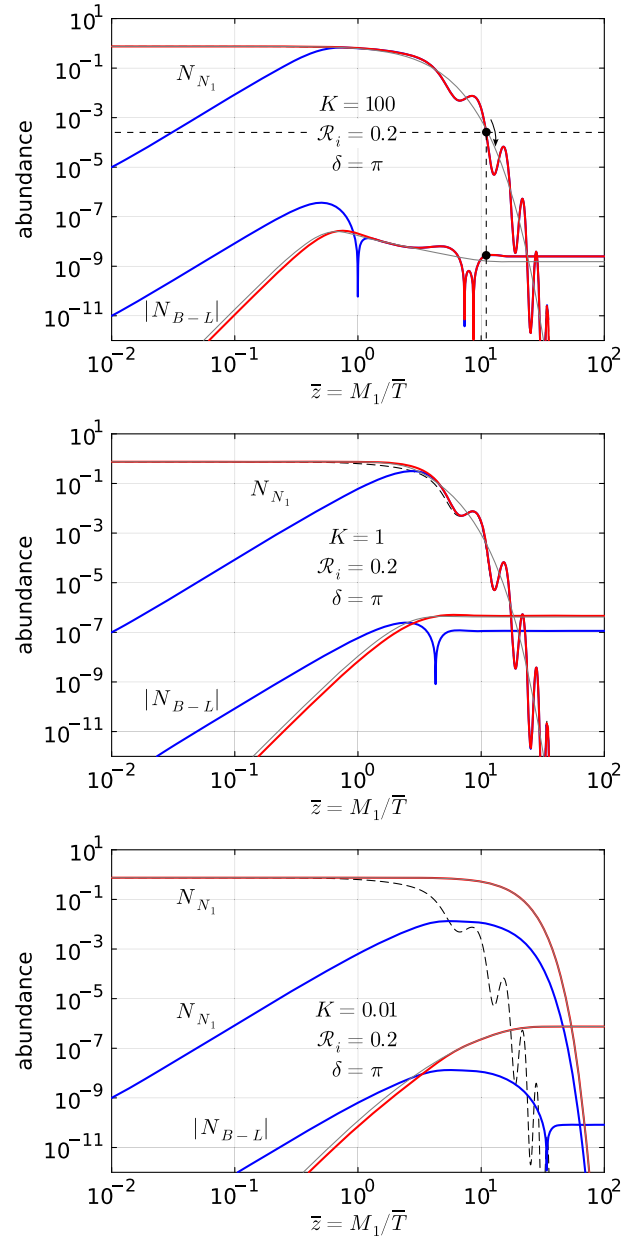


FIG. 1. *Top*: Evolution of the abundance of right-handed neutrinos N_{N_1} and the absolute value of the baryon-minus-lepton number $|N_{B-L}|$ for the decay parameter $K = 100$, the amplitude of primordial curvature perturbation $|\mathcal{R}_i| = 0.2$, and the phase $\delta = \pi$. The red and blue curves show the thermal and zero initial abundances for N_{N_1} , respectively. These curves merge into a single curve at later times. The thin gray curves depict the results for $|\mathcal{R}_i| = 0$ with the thermal initial abundance. The dashed line represents the equilibrium number density of right-handed neutrinos $N_{N_1}^{\text{eq}}$. The black circles indicate the time, when the local temperature equals the freeze-out temperature (see Sec. III B for details). *Middle*: Same as the top panel but for the decay parameter $K = 1$. *Bottom*: Same as the top panel but for the decay parameter $K = 0.01$.

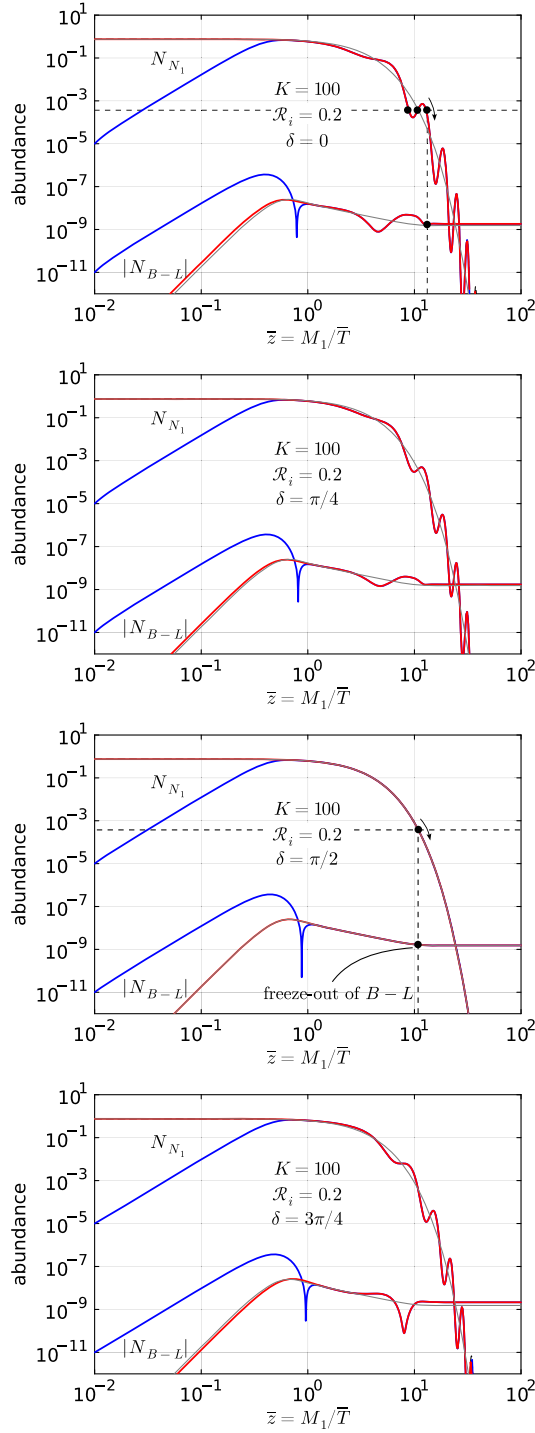


FIG. 2. Time evolutions of the right-handed neutrino and $B-L$ for different values of the initial phase δ . The initial phase is $\delta = 0, \pi/4, \pi/2$, and $3\pi/4$ from the top to the bottom, while the plot for $\delta = \pi$ is shown in Fig. 1. The decay parameter K is chosen as $K = 100$. The evolutions of N_{N_1} and N_{B-L} for $\delta = \pi/2$ are the same as the case without density perturbations since this value of δ corresponds to spatial points at the nodes of standing waves. For the black circles and the black dashed lines, see Sec. III B.

the four panels, the third one shows the evolution with $\delta = \pi/2$, which is essentially equivalent to the absence of density perturbations since this value of phase corresponds to the nodes of standing waves. In this case, one can infer the freeze-out value of $\bar{z}$ simply from the behavior of $|N_{B-L}|$, as indicated by the black circle on the curve of $|N_{B-L}|$ and the vertical dashed line. This value of $\bar{z}$ can be translated into the corresponding temperature $\bar{T}$, and hence, into the abundance N_{N_1} at the time of freeze-out, as indicated by the other black circle and the horizontal dashed line.

From the inferred value of the freeze-out temperature and the corresponding abundance of N_{N_1} at that moment, we can interpret the behavior in the top panel of Fig. 1 as well as the other panels of Fig. 2. Note that the freeze-out value $|N_{B-L}^f|$ is set at the moment when the local temperature reaches the freeze-out temperature, not the global one. Keeping this in mind, in the top panel of Fig. 1, we indicate the intersection between the oscillatory N_{N_1} curve and the freeze-out threshold by the black circle. The vertical dashed line shows the global time $\bar{z}$ at the time of this intersection. In this case, there is a single freeze-out point, and there the local temperature drops much faster than in the unperturbed case due to the fast cooling induced by the sound waves. A faster temperature drop yields a higher freeze-out value of N_{B-L} , and hence, $|N_{B-L}^f|$ for $\delta = \pi$ exceeds that for $\delta = \pi/2$. Similarly, in the first panel of Fig. 2, showing the evolution with $\delta = 0$, we indicate the intersections between N_{N_1} and the freeze-out threshold by the black circles. In this case, there are multiple intersections, but the one that determines the final freeze-out value is the last intersection: Although the first crossing induces the first freeze-out, the system is pushed back to equilibrium again by the second, and finally the third induces the final freeze-out. At this point, the temperature drop is again much faster than in the unperturbed case, and this is why we get larger value of $|N_{B-L}^f|$ than the unperturbed case also for $\delta = 0$. From this observation, we expect that the freeze-out value $|N_{B-L}^f|$ is enhanced for most of the values of δ .

Based on these observations, we compare the spatially averaged freeze-out value, $|N_{B-L}^f(|\mathcal{R}_i| = 0.2)|$, to the corresponding freeze-out value without density perturbations in Fig. 3. To evaluate the effect of density perturbations on the final lepton asymmetry, the freeze-out value $\langle |N_{B-L}^f(|\mathcal{R}_i| = 0.2)| \rangle_{\text{space}}$ must be averaged over different initial phase δ of temperature perturbations. In particular, for our choice of parameters, Fig. 3 shows that

$$\frac{\langle |N_{B-L}^f(|\mathcal{R}_i| = 0.2)| \rangle_{\text{space}}}{|N_{B-L}^f(|\mathcal{R}_i| = 0)|} \sim 1.25. \quad (54)$$

This ratio is greater than unity. Physically, the reason for this enhancement is that the increase of freeze-out value of

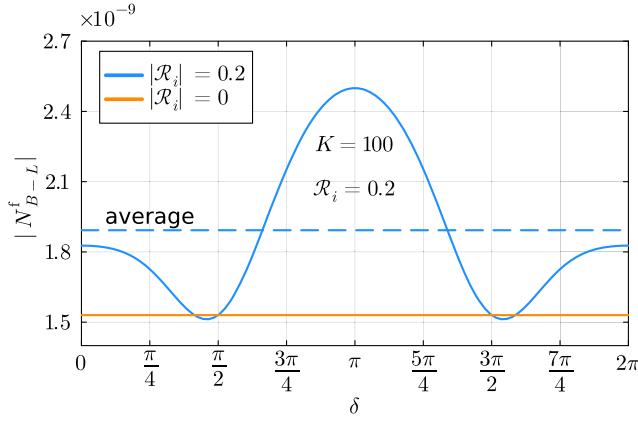


FIG. 3. Baryon-minus-lepton number $|N_{B-L}^f|$ after freeze-out, denoted by $|N_{B-L}^f|$, for different values of δ . The blue solid curve shows the result with temperature fluctuations, while the orange solid line shows that without fluctuations. In the presence of perturbations, temperature oscillations at different spatial locations have different values for the phase δ , resulting in the difference in $|N_{B-L}^f|$ at different spatial points. Averaging this over δ yields the spatial average shown by the blue dashed line, which is found to be greater than the value without perturbations. Here, the zero initial abundance is assumed for N_{N_1} , but the result with the thermal initial abundance is almost the same.

$|N_{B-L}|$ for different initial phase around $\delta \sim \pi$ (where temperature perturbations initially accelerate freeze-out) is not fully canceled by the phase around $\delta \sim 0$ (where temperature perturbations initially delay freeze-out), as explained above. Importantly, we emphasize that this net enhancement arises only when the system is treated beyond the linear approximation in temperature perturbations δ_T .

In Fig. 4, we plot the ratio between the spatially averaged freeze-out value $\langle |N_{B-L}^f(|\mathcal{R}_i|) \rangle_{\text{space}}$ and the freeze-out value without fluctuations as a function of the primordial curvature perturbation $|\mathcal{R}_i|$. We find that the ratio exceeds unity for any $|\mathcal{R}_i| > 0$ and that it increases monotonically

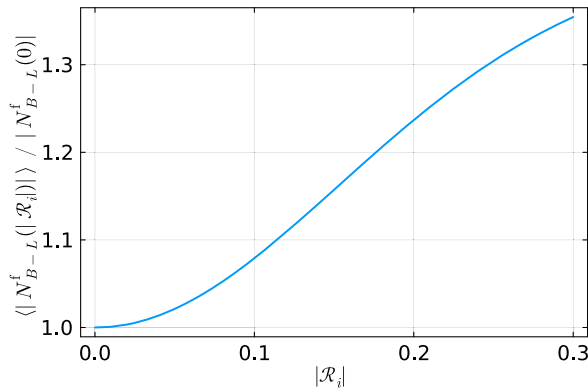


FIG. 4. Ratio between the spatially averaged freeze-out value $\langle |N_{B-L}^f(|\mathcal{R}_i|) \rangle_{\text{space}}$ with and without perturbations, as a function of the amplitude of the primordial curvature perturbation $|\mathcal{R}_i|$.

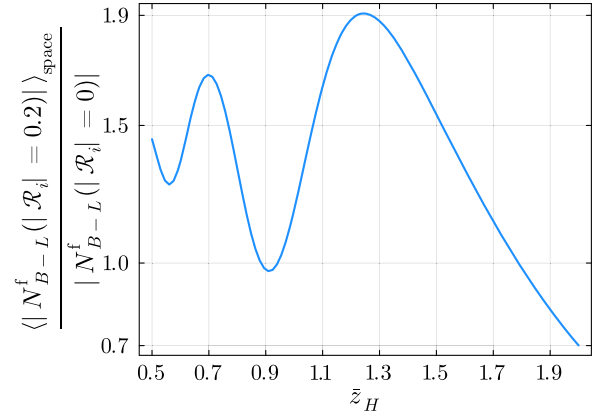


FIG. 5. Plot for the $\bar{z}_H$ -dependence of the normalized $\langle |N_{B-L}^f(|\mathcal{R}_i|) \rangle_{\text{space}}$ for $\delta = 0$, where the amplitude of the primordial curvature perturbation is taken to be $|\mathcal{R}_i| = 0.2$.

for $0 \leq |\mathcal{R}_i| \lesssim 0.2$. Figures with other values of $\bar{z}_H$ are shown in Fig. 5 and in Appendix A.

Figure 5 shows the $\bar{z}_H$ dependence of the ratio for the final lepton asymmetry $\langle |N_{B-L}^f(z)| \rangle_{\text{space}}$ to $|N_{B-L}^f(z)|$ with $K = 100$ and $\delta = 0$ in the range $0.5 \leq \bar{z}_H \leq 2$. The asymmetry exhibits oscillatory behavior with respect to $\bar{z}_H$. The maximum asymmetry occurs around $\bar{z}_H \sim 1.2$, while the minimum appears near $\bar{z}_H \sim 2$. $\bar{z}_H = 0.6, 0.9$ yield local minima. Thus, the timing of the horizon entry $\bar{z} = \bar{z}_H$ significantly affects the local lepton asymmetry, although this oscillatory behavior is expected to be absent in the asymmetry after averaging over the initial phase δ .

Before moving on, we comment on the monochromaticity of the sound waves. In this paper, we assume monochromatic waves for simplicity, but fluctuations are in reality a superposition of different modes with various wave numbers characterized by a power spectrum, and thus, one may wonder about the possible averaging out of the enhancement for nonmonochromatic waves. However, there is a good reason to expect that such cancellation does not occur. Temperature perturbations induce rapid decreases of the local temperatures at the respective time of freeze-out, regardless of the initial phase of the oscillation. Even if nonmonochromatic waves are assumed, local temperature perturbations are still present, although their shape would differ from the simple sine- or cosine waves. These temperature perturbations would still cause a faster drop in the local temperature at the time of freeze-out, similar to the monochromatic case. Thus, we expect that the enhancement of $|N_{B-L}|$ still remains even when waves of different wave numbers are superposed.

C. Parameter regions for successful leptogenesis

In Fig. 6, we show the parameter regions for successful leptogenesis on the $(\tilde{m}_1, M_1)$ plane. For a given value of $\tilde{m}$,

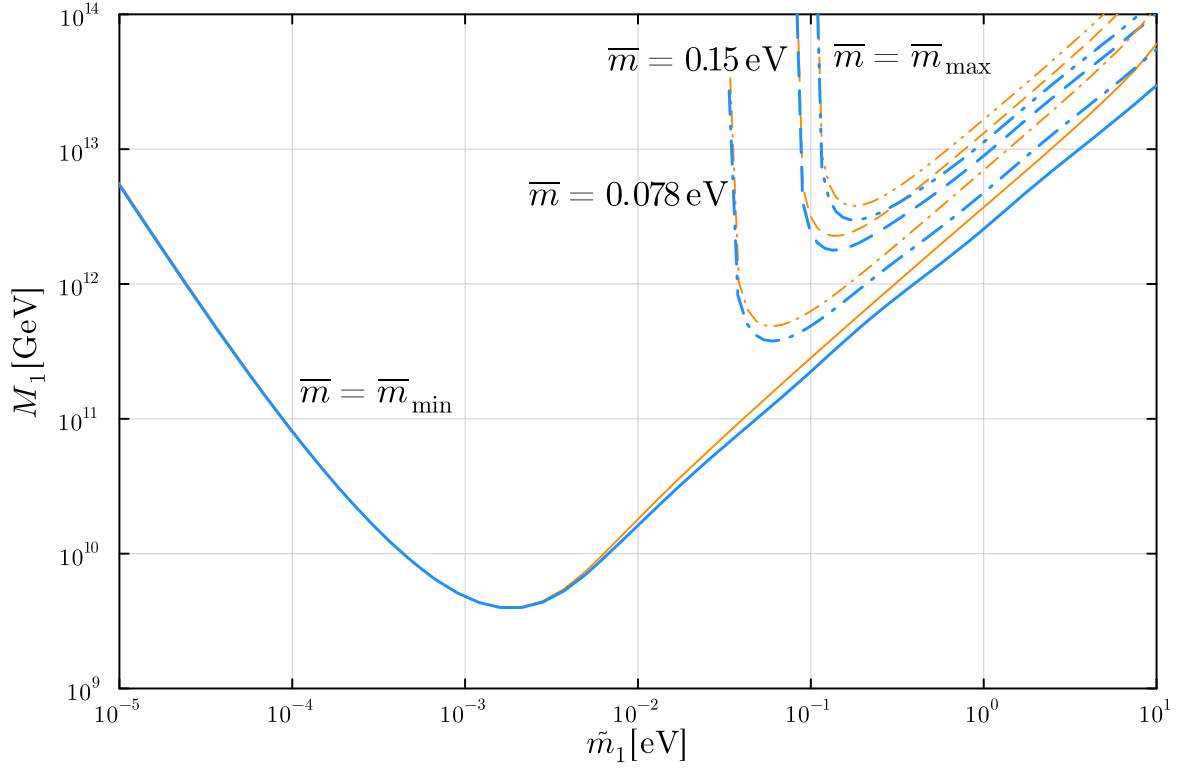


FIG. 6. Allowed regions for successful leptogenesis. The normal hierarchy and zero initial abundance for N_{N_1} are assumed. The curves show $(\tilde{m}_1, M_1)$, where $\eta_b^{\max} = (\eta_b^{\text{CMB}})_{\text{low}} = 6.0 \times 10^{-10}$ is satisfied for $\bar{m} = (0.051, 0.078, 0.15, 0.19)$ eV. For a given $\bar{m}$, the region above the curves is the one for successful leptogenesis. The blue and orange curves represent the case with and without temperature fluctuations δ_T , respectively. The primordial curvature perturbation is taken to be $|\mathcal{R}_i| = 0.2$. We can see that the allowed region in $(\tilde{m}_1, M_1)$ is expanded by the effect of δ_T . The solid lines correspond to $m_1 = 0$ such that $\bar{m} = \bar{m}_{\min} = 0.051$ eV. The dash-dot lines are for $\bar{m} = 0.078$ eV, corresponding to the upper limit from CMB and BAO observations [11]. The dashed lines are for $\bar{m} = 0.15$ eV, corresponding to the upper limit from CMB alone [10]. The dot-dot-dashed lines correspond to $\bar{m} = 0.19$ eV. If scatterings are included, the upper boundary appears, and the allowed region would disappear for $\bar{m} > 0.19$ eV [46].

there are two curves with (blue) and without (orange) density fluctuations. The primordial curvature perturbation is taken to be $|\mathcal{R}_i| = 0.2$. In the region above each curve, the produced amount of baryons exceeds the lower limit given by the CMB data (40). In the previous subsection, we considered the case of $K = 100, 1$, and 0.01 corresponding to $\tilde{m}_1 \simeq 0.1, 10^{-3}$, and 10^{-5} eV, respectively. While the contours are not closed in this plot, it is known that upper limits on M_1 arise once $\Delta L = 2$ scatterings are taken into account (see, e.g., Ref. [37]). The reason we do not take account of these scatterings is simply to illustrate the effect of acoustically driven freeze-out in the simplest setup (see also discussion in Sec. IV).

In the case of $m_1 = 0$, the absolute mass (42) takes its minimum value $\bar{m} = 0.051$ eV. The resulting constraint is shown in the solid lines. In the weak washout regime $\tilde{m}_1 \lesssim 10^{-3}$ eV, the constraint with temperature fluctuations is almost identical to the one without temperature fluctuations. In the strong washout regime $\tilde{m}_1 \gtrsim 10^{-3}$ eV, the allowed region with fluctuations gets slightly extended from that without fluctuations due to the enhancement by the acoustically driven freeze-out.

The allowed parameter space shrinks as $\bar{m}$ increases, as discussed in Ref. [46]. For instance, in the case of $\bar{m} = 0.078$ eV, the region for $\tilde{m}_1 \gtrsim 0.03$ eV is ruled out as shown in the dot-dashed lines. As in the case of $\bar{m} = 0.051$ eV, including fluctuations enlarges the allowed region. Similar behavior is observed for $\bar{m} = 0.15$ eV (dashed lines) and $\bar{m}_{\max} = 0.19$ eV (dot-dot-dashed lines).

We thus conclude that the beyond-linear effect can enlarge the parameter space for successful leptogenesis if the amplitudes of primordial density fluctuations are sufficiently large. The effect of perturbations is important in the strong washout regime

$$8.7 \times 10^{-3} \text{ eV} \lesssim \tilde{m}_1 \lesssim 5.0 \times 10^{-2} \text{ eV}. \quad (55)$$

The dependence on the primordial curvature perturbation $|\mathcal{R}_i|$ can be inferred from Fig. 4. Note that the neutrino oscillation experiments suggest that the masses of light neutrinos likely lie in this range; see Eq. (45). In the future, as neutrinoless double beta decay experiments such as CANDLES [48], CUORE [49], EXO [50], KamLAND-Zen [51], LEGEND [52], NEMO-3, and SuperNEMO

[53,54] (see also Ref. [24] and references therein) achieve better sensitivities, not only will the upper limits on the mass eigenvalues of the light neutrinos improve, but the absolute mass scale $\bar{m}$ may finally be determined. If the determined mass scale falls within the ballpark of $\bar{m} \gtrsim 0.1$ eV, the effect discussed in the present paper turns out to be relevant, as seen from Fig. 6.

D. Fate of sound waves

So far we have focused on the effect of sound waves around the time of freeze-out. These waves subsequently dissipate on subhorizon scales, leading to entropy injection which dilutes the $B - L$ abundance that the sound waves had previously amplified. Here, we estimate the magnitude of this dilution and show that it is too small to offset the enhancement we have identified.

In the subhorizon limit, one has $\varphi \gg 1$. Taking this limit in Eq. (36) yields

$$\delta_T = |\mathcal{R}_i| \cos \delta \cos \varphi. \quad (56)$$

Using $\cos \delta \cos \varphi = [\cos(\delta + \varphi) + \cos(\delta - \varphi)]/2$ and taking both temporal and spatial average, we get [55]

$$\langle \delta_T^2 \rangle = \frac{|\mathcal{R}_i|^2}{4} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{|\mathcal{R}_i|^2}{4}. \quad (57)$$

In the tight-coupling regime, the dipole satisfies $|\Theta_1| \simeq |\Theta_0|/\sqrt{3}$ [56], and hence,

$$\begin{aligned} \langle \Theta^2 \rangle &\equiv \langle \Theta_0^2 + 3\Theta_1^2 \rangle \\ &\simeq \langle \Theta_0^2 \rangle + 3 \left\langle \frac{1}{3} \langle \Theta_0^2 \rangle \right\rangle \\ &= 2 \langle \Theta_0^2 \rangle = 2 \langle \delta_T^2 \rangle = \frac{|\mathcal{R}_i|^2}{2}. \end{aligned} \quad (58)$$

The corresponding fractional change in photon number density after dissipation is then [55]

$$\frac{\Delta N_\gamma}{N_\gamma} = \frac{3}{2} \langle \Theta^2 \rangle = \frac{3}{4} |\mathcal{R}_i|^2. \quad (59)$$

The enhancement induced by the density perturbation is $1.25 - 1 = 0.25$ for $|\mathcal{R}_i| = 0.2$ as seen from Eq. (54). By contrast, the dilution due to the entropy injection is

$$\frac{\Delta N_\gamma}{N_\gamma} = \frac{3}{4} |\mathcal{R}_i|^2 = \frac{3}{4} \times 0.04 = 0.03. \quad (60)$$

Therefore, the dilution effect from the entropy injection is $0.03/0.25 \sim 0.1$ of the effect we point out.

Consequently, although the enhancement is slightly reduced, our result is not significantly affected by the entropy injection.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we studied the effect of primordial density fluctuations on the freeze-out of heavy particles. Primordial density fluctuations within the Hubble horizon propagate as sound waves during the freeze-out of these particles. Sound waves cause temperature oscillation at each spatial point, driving oscillation in the abundance of massive particles with respect to radiation. Using leptogenesis as an illustrative framework, we have derived the governing equations that describe the evolution of these heavy particles in the presence of fluctuations and found that the exponential dependence of the abundance of heavy particles on the local temperature causes a beyond-linear effect in the system; see Eqs. (51) and (52).

As a proof of concept, we applied the formulation to the scenario of vanilla leptogenesis and demonstrated how temperature fluctuations affect the abundance of the right-handed neutrino and the resulting $B - L$ asymmetry. We indeed observed oscillations in the equilibrium abundance of the right-handed neutrino N_1 and in the baryon-minus-lepton number N_{B-L} (Fig. 3). Interestingly, the density fluctuations always increase the final baryon-to-photon ratio compared to the case without fluctuations, even after taking spatial average (Fig. 4). This suggests that the effect of oscillations cannot be captured within linear perturbation. Qualitatively, the equilibrium value of the right-handed neutrino is governed by the Boltzmann factor $\exp(-E/T) \simeq \exp(-E/\bar{T}) \exp(E\delta T/\bar{T}^2)$, and the fast temperature oscillation makes it harder for the nonequilibrium abundance of N_1 to follow this equilibrium value. If the temperature oscillation occurs with the right phase, the freeze-out of N_1 is dominantly driven by the oscillation, not by the slow decrease in the average temperature, and the resulting $|N_{B-L}|$ is enhanced (top panel of Fig. 4, around $\bar{z} \simeq 15$). We call this effect “acoustically driven freeze-out.” Of course, each spatial point has its own oscillation phase, and thus, the enhancement in $|N_{B-L}|$ does not occur uniformly. However, the exponential enhancement of $|N_{B-L}|$ in some regions cannot be fully canceled out by the contributions from other regions. As a result, we get a net enhancement of the baryon asymmetry even after spatial average.

From the consideration above, the allowed parameter space for leptogenesis gets enlarged in the presence of fluctuations. We presented the allowed regions $\eta_b \geq (\eta_b^{\text{CMB}})_{\text{low}}$ in the $(\tilde{m}_1, M_1)$ plane for a given absolute neutrino mass scale $\bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2}$ (Fig. 6).

The effective neutrino mass $\tilde{m}_1$ is linearly proportional to the decay parameter K and thus characterizes the weak and strong washout regimes (Fig. 6, $\tilde{m} \lesssim 10^{-3}$ eV and $\tilde{m} \gtrsim 10^{-3}$ eV, respectively). When temperature fluctuations are included, we indeed found an enlarged allowed regions. The difference arises mainly in the strong washout regime, because in the weak washout regime, the actual N_1 abundance is almost unaffected by its oscillating equilibrium value (bottom panel of Fig. 4). While the quantitative difference is not dramatically large, given that the neutrino oscillation experiments point toward the strong washout regime [47], our findings will be relevant when we find relatively large $\tilde{m}$ from cosmological observations [10,11].

We conclude by mentioning several possible future directions. For the microphysics side, we took only decay and inverse decay into account in this study. However, it is well known that $\Delta L = 2$ scattering imposes an ($\tilde{m}$ -dependent) upper bound on M_1 and thus closes the allowed region in the $(M_1, \tilde{m}_1)$ plane [46]. Such scattering processes will not bring qualitative changes in the behavior of acoustically driven freeze-out since they do not affect the oscillating equilibrium abundance itself, which is the main reason for the net enhancement in the asymmetry. Nevertheless, identifying the allowed parameter space including all the known microphysical effect will be important in preparing for future precision measurements. For the macrophysics side, in this paper, we injected monochromatic sound waves for simplicity. In reality, however, fluctuations are a superposition of different modes with various wave numbers characterized by the power spectrum of the primordial curvature perturbation. Thus, the effect of fluctuations on freeze-out will ultimately be described by the power spectrum, and it would be interesting to construct analytical formulations for it. In addition, it has been pointed out that sound waves can evolve to shock waves in the long run [57]. Shock waves by definition have discontinuities in thermodynamic parameters and hence, are expected to enhance the sudden freeze-out of heavy particles. Incorporating all these aspects will be important and interesting future steps, to which we will come back elsewhere.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: TIME EVOLUTION FOR DIFFERENT VALUES OF $\bar{z}_H$

In this appendix, we plot the time evolution of N_1 and $B - L$, the freeze-out values of $B - L$, and their dependence on $|\mathcal{R}_i|$ for $\bar{z}_H = 0.5$ and 2.

Figures 7 and 8 show the time evolution of N_1 and $B - L$, Figs. 9 and 10 show the freeze-out values of $B - L$,

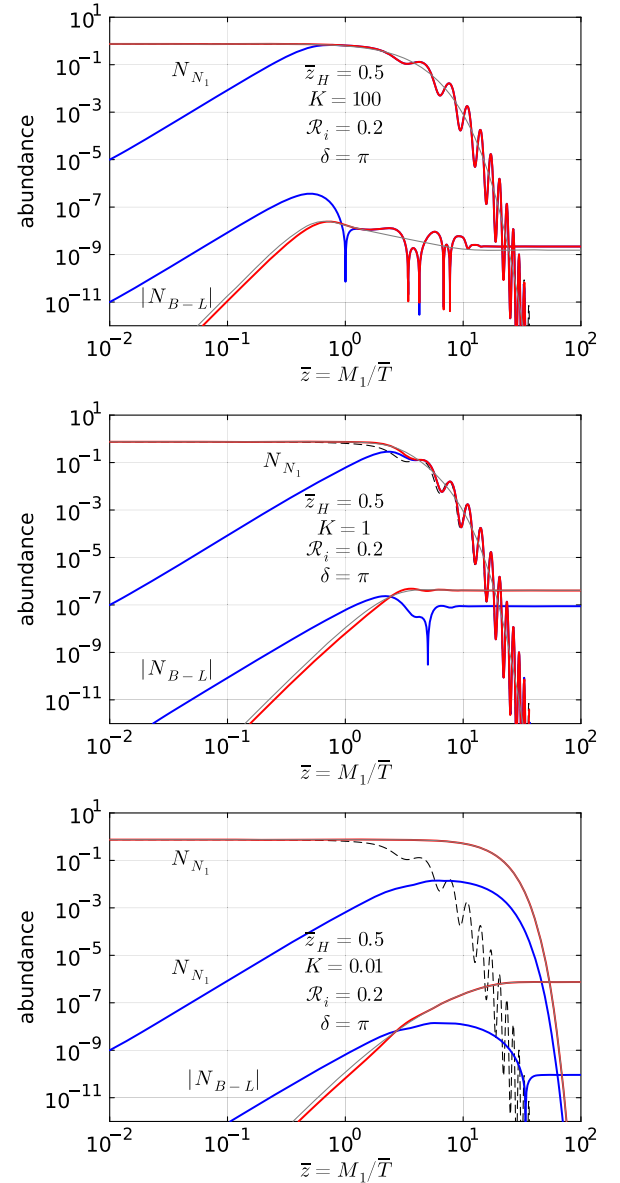


FIG. 7. Time evolution of N_{N_1} and $|N_{B-L}|$. Parameter values are the same as Fig. 1 except for $\bar{z}_H = 0.5$.

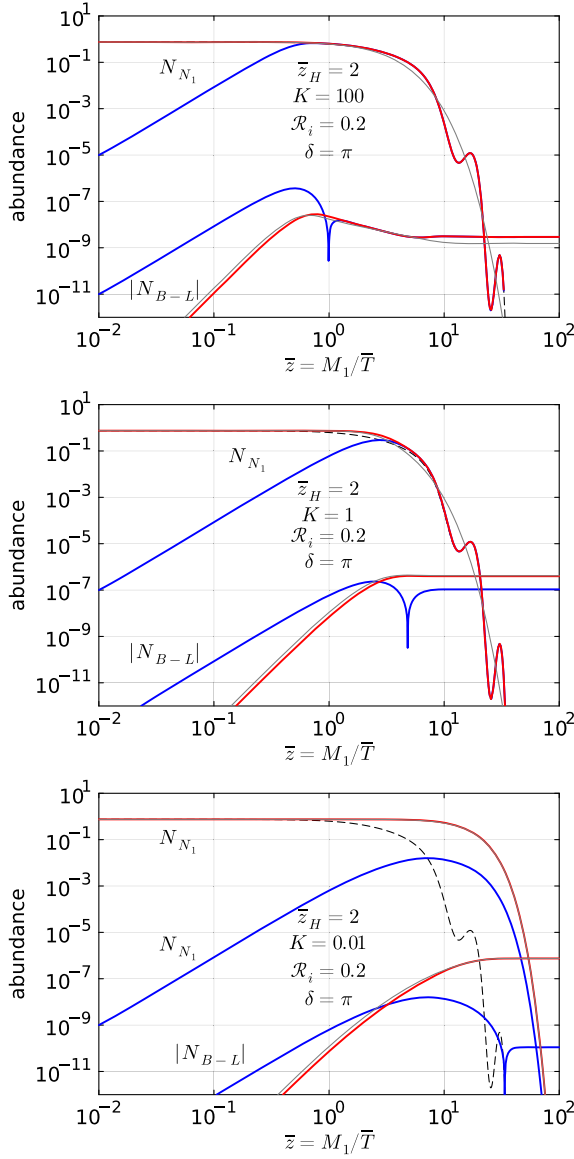


FIG. 8. Time evolution of N_{N_1} and $|N_{B-L}|$. Parameter values are the same as Fig. 1 except for $\bar{z}_H = 2$.

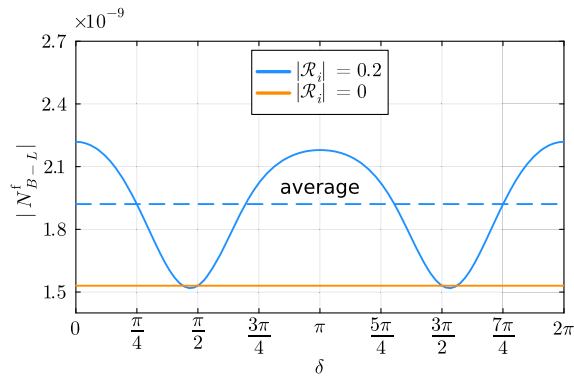


FIG. 9. Freeze-out value of $|N_{B-L}|$ for different values of δ . Parameter values are the same as Fig. 3 except for $\bar{z}_H = 0.5$.

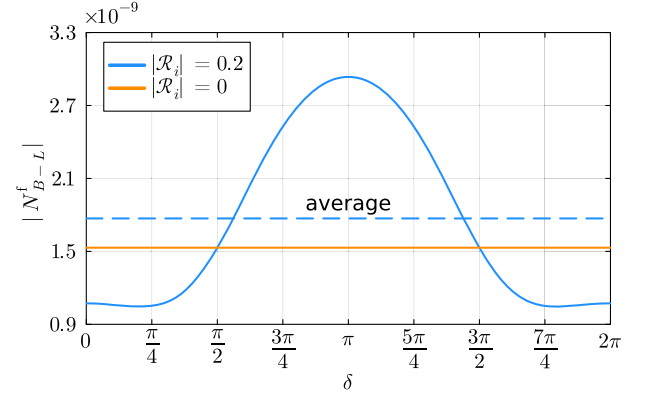


FIG. 10. Freeze-out value of $|N_{B-L}|$ for different values of δ . Parameter values are the same as Fig. 3 except for $\bar{z}_H = 2$.

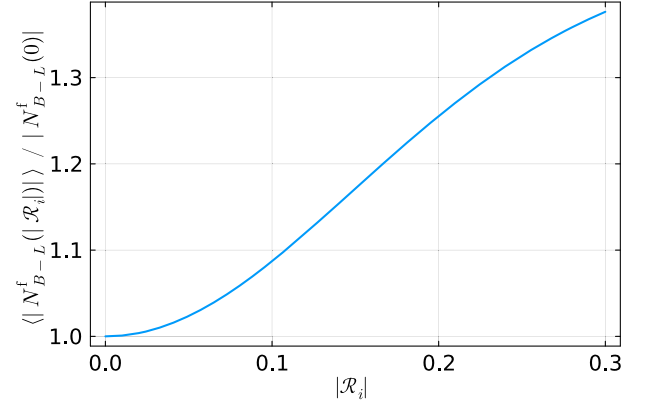


FIG. 11. Ratio between the spatially averaged freeze-out value $\langle |N_{B-L}^f| \rangle_{\text{space}}$ with and without perturbations. Parameter values are the same as Fig. 4 except for $\bar{z}_H = 0.5$.

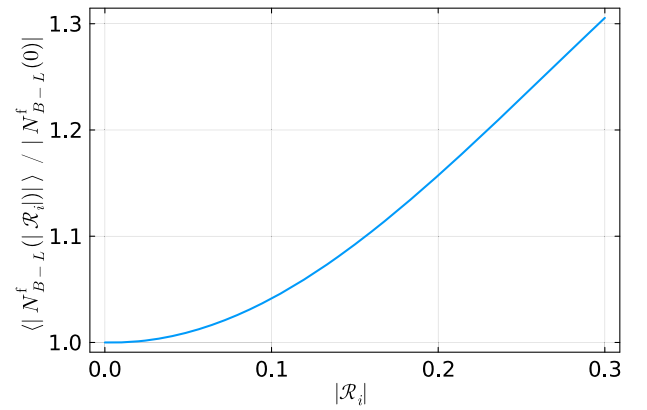


FIG. 12. Ratio between the spatially averaged freeze-out value $\langle |N_{B-L}^f| \rangle_{\text{space}}$ with and without perturbations. Parameter values are the same as Fig. 4 except for $\bar{z}_H = 2$.

and Figs. 11 and 12 show the dependence on $|\mathcal{R}_i|$ for $\bar{z}_H = 0.5$ and 2. These figures correspond to Figs. 1–4 in the main text, in which $\bar{z}_H = 1$ is adopted.

APPENDIX B: DERIVATION OF THE BOLTZMANN EQUATION

1. The Boltzmann equation

We begin with the Boltzmann equation [45]:

$$\frac{df}{d\lambda} = C[f] \Rightarrow P^0 \frac{df}{d\eta} = C[f], \quad (\text{B1})$$

where f is the distribution function, λ is the affine parameter, $C[f]$ is the collision term, and P^0 is the zeroth component of the energy-momentum four-vector P^μ . We integrate the former equation using the integral measure in momentum space defined as

$$\pi = g_{\text{deg}} \frac{d^3 p}{E}, \quad (\text{B2})$$

where g_{deg} represents the internal degrees of freedom of the species, and E and p^i are the energy and three-momentum of the particle evaluated in the local inertial frame, respectively.

2. Equivalence of integral measures

The integral measure (B2) evaluated in an arbitrary coordinate system is given by [45]

$$\pi \equiv g_{\text{deg}} \frac{\sqrt{-g} d^3 P}{P_0} = g_{\text{deg}} \frac{d^3 P}{P^0 \sqrt{-g}}. \quad (\text{B3})$$

To check this, note that the relation between the four-momenta evaluated in the general coordinate system and in the local inertial system is

$$P^\mu = e^\mu{}_{\nu'} p^{\nu'}, \quad (\text{B4})$$

where $e^\mu{}_{\nu'}$ is the tetrad, and $p^0 = -p_0 = E$ is the instantaneous energy of the particle with respect to the local observer. The tetrad satisfies $e^\mu{}_{\alpha'} e^{\nu'}{}_{\beta'} g_{\mu\nu} = \eta_{\alpha'\beta'}$. Using (B4) and $E dE = p_j dp^j$, we may rewrite dP as

$$\begin{aligned} dP^i &= e^i{}_0 \frac{\partial p^0}{\partial p^j} dp^j + e^i{}_k \frac{\partial p^k}{\partial p^j} dp^j \\ &= e^i{}_0 \frac{p_j}{E} dp^j + e^i{}_j dp^j = \left(e^i{}_0 \frac{p_j}{E} + e^i{}_j \right) dp^j. \end{aligned} \quad (\text{B5})$$

Substituting this into dP in (B3) and using

$$P_0 = g_{0\mu} e^\mu{}_{\nu'} p^{\nu'} = \eta_{\nu'\sigma'} (e^{-1})^{\sigma'}{}_0 p^{\nu'}, \quad (\text{B6})$$

the integral measure indeed reduces to

$$\pi = g_{\text{deg}} \frac{d^3 p}{E}. \quad (\text{B7})$$

3. Equivalence of (17) and the integral of (B1)

In this subsection, we show the equivalence of Eq. (17) and the integral of (B1) following Ref. [58]. The momentum integral of the Boltzmann equation (B1) is

$$\frac{g_{\text{deg}}}{(2\pi)^3} \int \frac{\sqrt{-g} d^3 P}{P_0} \frac{df}{d\lambda} = \frac{g_{\text{deg}}}{(2\pi)^3} \int \frac{d^3 p}{E} C[f], \quad (\text{B8})$$

where we used the equivalence of the integral measures (B3) and (B7). We aim to show that this equation is equivalent to Eq. (17). For this purpose, it is sufficient to consider the collisionless Boltzmann equation since the collision terms are already identical. Let f denote the distribution function and consider

$$\frac{df}{d\lambda} = \frac{dx^\mu}{d\lambda} \frac{\partial f}{\partial x^\mu} + \frac{dP^i}{d\lambda} \frac{\partial f}{\partial P^i} = 0, \quad (\text{B9})$$

that is

$$P^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^i{}_{\mu\nu} P^\mu P^\nu \frac{\partial f}{\partial P^i} = 0. \quad (\text{B10})$$

Integrating both sides of Eq. (B10) with the integral measure $(\sqrt{-g^2}/P_0) d^3 P d^4 x$ gives

$$\begin{aligned} &\int \left(P^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^i{}_{\mu\nu} P^\mu P^\nu \frac{\partial f}{\partial P^i} \right) \frac{\sqrt{-g^2}}{P_0} d^3 P d^4 x \\ &= \int \frac{\partial}{\partial x^\mu} \left(P^\mu f \frac{\sqrt{-g^2}}{P_0} \right) d^3 P d^4 x \\ &\quad - \int \frac{\partial}{\partial P^i} \left(\Gamma^i{}_{\mu\nu} P^\mu P^\nu f \frac{\sqrt{-g^2}}{P_0} \right) d^3 P d^4 x = 0. \end{aligned} \quad (\text{B11})$$

The second line of Eq. (B11) can be rewritten as

$$\begin{aligned} &\int \frac{\partial}{\partial x^\mu} \left(P^\mu f \frac{\sqrt{-g^2}}{P_0} \right) d^3 P d^4 x \\ &= \int \left[\frac{\partial}{\partial x^\mu} \left(\int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0} \right) \right. \\ &\quad \left. + \Gamma^\kappa{}_{\mu\kappa} \int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0} \right] \sqrt{-g} d^4 x \\ &= \int \left(\int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0} \right)_{;\mu} \sqrt{-g} d^4 x, \end{aligned} \quad (\text{B12})$$

where we used

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial x^\mu} + \Gamma^\mu{}_{\mu\sigma} A^\sigma \quad (\text{B13})$$

$$= \frac{\partial A^\mu}{\partial x^\mu} + \frac{\partial \ln \sqrt{-g}}{\partial x^\mu} A^\mu = \frac{1}{\sqrt{-g}} \frac{\partial (\sqrt{-g} A^\mu)}{\partial x^\mu}, \quad (\text{B14})$$

which follows from

$$\Gamma^\nu{}_{\mu\nu} = \frac{1}{2} g^{\nu\sigma} \frac{\partial g_{\nu\sigma}}{\partial x^\mu} = \frac{\partial \ln \sqrt{-g}}{\partial x^\mu}. \quad (\text{B15})$$

Also, the left-hand side in the third line of equation (B11) vanishes because the volume integral in momentum space can be rewritten as a surface term at infinity where the distribution function f approaches zero. Thus, Eq. (B11) can be simplified as

$$\int \left(\int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0} \right)_{;\mu} \sqrt{-g} d^4 x = 0. \quad (\text{B16})$$

Since this holds for any integral over spacetime, the integrand must vanish. Thus, we get

$$\left(\int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0} \right)_{;\mu} = 0. \quad (\text{B17})$$

On the other hand, number density current $\mathcal{N}^\mu = n U^\mu$ is defined as

$$\mathcal{N}^\mu \equiv \frac{g_{\text{deg}}}{(2\pi)^3} \int P^\mu f \sqrt{-g} \frac{d^3 P}{P_0}, \quad (\text{B18})$$

and hence, the momentum integral of the collisionless Boltzmann equation (B9) becomes

$$\mathcal{N}^\mu{}_{;\mu} = 0. \quad (\text{B19})$$

In other words, the left-hand side of Eq. (B8) can be replaced with $\mathcal{N}^\mu{}_{;\mu}$, which yields the Boltzmann equation in the form of Eq. (17).

4. Calculation of the covariant derivative of the number density current

In this subsection, we calculate Eq. (18) in detail. In the following, we omit the index N_1 in n_{N_1} . By using Eq. (19) and formulas in Appendix C,

$$\begin{aligned} (nU^\mu)_{;\mu} &= (nU^\mu)_{,\mu} + \Gamma^\mu{}_{\mu\nu} (nU^\nu) \\ &= (nU^0)_{,0} + \Gamma^0{}_{0\nu} (nU^\nu) + (nU^i)_{,i} + \Gamma^i{}_{i\nu} (nU^\nu) \\ &= \left[\frac{n}{a} (1-A) \right]' + \Gamma^0{}_{00} (nU^0) + \Gamma^0{}_{0i} (nU^i) + \left(\frac{n}{a} v^i \right)'_{,i} + \Gamma^i{}_{i0} (nU^0) + \Gamma^i{}_{ij} (nU^j) \\ &= \frac{n'}{a} (1-A) - \frac{a'}{a^2} n (1-A) - \frac{n}{a} A' + \left(\frac{a'}{a} + A' \right) \frac{n}{a} (1-A) - \left(A_{,i} - \frac{a'}{a} B_{,i} \right) \frac{n}{a} v^i + \frac{n}{a} v^i{}_{,i} \\ &\quad + \left[\frac{a'}{a} \delta^i{}_i + C' \delta^i{}_i + \left(E^i{}_{,i} - \frac{1}{3} \delta^i{}_i \nabla^2 E \right)' \right] \frac{n}{a} (1-A) + (\text{1st-order}) \times (\text{1st-order}). \end{aligned} \quad (\text{B20})$$

Dropping the terms of second or higher order in perturbations,

$$\begin{aligned} (nU^\mu)_{;\mu} &\simeq \frac{n'}{a} (1-A) - \frac{a'}{a^2} n (1-A) - \frac{n}{a} A' + \frac{a'}{a} \frac{n}{a} (1-A) + A' \frac{n}{a} + 2 \frac{n}{a} v^i{}_{,i} + 3 \left(\frac{a'}{a} + C' \right) \frac{n}{a} - 3 \frac{a'}{a} A \frac{n}{a} \\ &= \frac{n'}{a} (1-A) + \frac{n}{a} v^i{}_{,i} + 3 \frac{a'}{a} \frac{n}{a} + 3 C' \frac{n}{a} - 3 \frac{a'}{a} A \frac{n}{a} \\ &= (1-A) \left(a^{-1} n' + 3 \frac{a'}{a^2} n \right) + \frac{n}{a} (v^i{}_{,i} + 3 C') \\ &= \bar{z} H (1-A) \left(\frac{dn}{d\bar{z}} + \frac{3}{\bar{z}} n \right) + \frac{n}{a} (v^i{}_{,i} + 3 C'), \end{aligned} \quad (\text{B21})$$

is obtained. Here, we used

$$\frac{d}{d\eta} = a \frac{d}{dt} = a \bar{z} H \frac{d}{d\bar{z}}, \quad (\text{B22})$$

which follows from entropy conservation.

5. Calculation of the collision term

In this subsection, we calculate Eq. (20) in detail. The distribution function of right-handed neutrinos, defined with the normalization constant $\mathcal{C}$, is given by

$$f_{N_1} = \mathcal{C} \exp \left[-\frac{(p_{N_1}^2 + M_1^2)^{1/2} - \mu_{N_1}}{\bar{T} + \delta T} \right]. \quad (\text{B23})$$

By integrating this equation over the entire momentum space, we obtain the particle number density

$$n_{N_1}(\bar{z}) = \frac{g_{N_1} \mathcal{C}}{2\pi^2} \int_0^\infty dp_{N_1} p_{N_1}^2 \exp \left[-\frac{(p_{N_1}^2 + M_1^2)^{1/2} - \mu_{N_1}}{\bar{T} + \delta T} \right]. \quad (\text{B24})$$

Changing the variable to $p_{N_1} = M_1 \sinh \theta$ on the right-hand side, and introducing $z_T(\bar{z}) = \bar{z}/(1 + \delta T)$, the integral reduces to

$$n_{N_1}(\bar{z}) = \frac{g_{N_1} \mathcal{C}}{8\pi^2} M_1^3 \int_0^\infty d\theta (\cosh 3\theta - \cosh \theta) e^{-z_T \cosh \theta}. \quad (\text{B25})$$

Here, note that the modified Bessel function of the second kind K_ν is given by the integral

$$K_\nu(z) = \int_0^\infty d\theta \cosh(\nu\theta) e^{-z \cosh \theta}, \quad (\text{B26})$$

from which follows

$$K_{\nu+1}(z) - K_{\nu-1}(z) = \frac{2\nu}{z} K_\nu(z). \quad (\text{B27})$$

Then the normalization constant is determined to be

$$\mathcal{C} = \frac{2\pi^2 z_T}{g_{N_1} M_1^3 K_2(z_T)} n_{N_1}(\bar{z}) e^{-\mu_{N_1}/(\bar{T} + \delta T)}, \quad (\text{B28})$$

and the distribution function of right-handed neutrinos becomes

$$f_{N_1} = \frac{2\pi^2 z_T n_{N_1}(\bar{z})}{g_{N_1} M_1^3 K_2(z_T)} \exp \left[-\frac{(p_{N_1}^2 + M_1^2)^{1/2}}{\bar{T} + \delta T} \right], \quad (\text{B29})$$

which we rewrite as $f_{N_1} = \mathcal{C}' n_{N_1}(\bar{z}) e^{-E_{N_1}/(\bar{T} + \delta T)}$. On the other hand, assuming that lepton l and Higgs ϕ are in thermal equilibrium and denoting their distribution functions by f_l^{eq} and f_ϕ^{eq} , respectively, the momentum integral of the collision term in the Boltzmann equation for N_1 is

$$\begin{aligned} \frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} \mathcal{C}[f_{N_1}] &= \int d\Pi_{N_1} \int d\Pi_l \int d\Pi_\phi (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_{N_1} - p_l - p_\phi) (f_l^{\text{eq}} f_\phi^{\text{eq}} - f_{N_1}) \\ &= - \int d\Pi_{N_1} \int d\Pi_l \int d\Pi_\phi (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_{N_1} - p_l - p_\phi) \\ &\quad \times \mathcal{C}' \left[n_{N_1}(\bar{z}) e^{-E_{N_1}/(\bar{T} + \delta T)} - n_l^{\text{eq}} n_\phi^{\text{eq}} e^{-(E_l + E_\phi)/(\bar{T} + \delta T)} \right] \\ &\simeq - \int d\Pi_{N_1} \int d\Pi_l \int d\Pi_\phi (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_{N_1} - p_l - p_\phi) \mathcal{C}' e^{-E_{N_1}/(\bar{T} + \delta T)} \left[n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T) \right], \end{aligned} \quad (\text{B30})$$

where we used the energy conservation law $E_{N_1} = E_l + E_\phi$ and detailed balance at high temperatures $n_{N_1}^{\text{eq}} \simeq n_l^{\text{eq}} n_\phi^{\text{eq}}$. Note that the matrix elements for $N_1 \rightarrow l_\alpha \phi$ and $N_1 \rightarrow \bar{l}_\alpha \phi^*$ do not depend on p_{N_1} , p_l , and p_ϕ at the tree level. Let us proceed to perform the integrals over $d\Pi_\phi$, $d\Pi_l$, and $d\Pi_{N_1}$ sequentially. First, for the $d\Pi_\phi$ integral we get

$$\begin{aligned} \frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} \mathcal{C}[f_{N_1}] &= -8\pi^2 |\mathcal{M}|^2 \int \frac{d^3 p_{N_1}}{(2\pi)^3} \frac{g_{N_1}}{2E_{N_1}} \int \frac{d^3 p_l}{(2\pi)^3} \frac{g_l}{2p_l} \frac{g_\phi}{2(E_{N_1} - p_l)} \\ &\quad \times \delta(E_{N_1} - p_l - p_\phi) \mathcal{C}' e^{-E_{N_1}/(\bar{T} + \delta T)} \left[n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T) \right]. \end{aligned} \quad (\text{B31})$$

Here, we approximated the lepton l and Higgs ϕ to be massless, assuming that their masses are sufficiently smaller than that of the right-handed neutrino N_1 . Thus, performing the integral over $d^3 p_l = 4\pi p_l^2 dp_l$, we get

$$\frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} \mathcal{C}[f_{N_1}] = -|\mathcal{M}|^2 \int \frac{d^3 p_{N_1}}{(2\pi)^3} \frac{g_{N_1}}{2E_{N_1}} \frac{g_l g_\phi (E_{N_1} - p_\phi)}{E_{N_1} - p_l} \mathcal{C}' e^{-E_{N_1}/(\bar{T} + \delta T)} \left[n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T) \right]. \quad (\text{B32})$$

Recalling that l and ϕ are in thermal equilibrium, we take thermal average over these species. Since both l and ϕ are massless, we can set $\langle p_l \rangle = \langle p_\phi \rangle$, which yields

$$\begin{aligned} & \left\langle \frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} C[f_{N_1}] \right\rangle \\ &= -g_{N_1} g_l g_\phi |\mathcal{M}|^2 \int \frac{d^3 p_{N_1}}{(2\pi)^3} \\ & \times \frac{C' e^{-E_{N_1}/(\bar{T}+\delta T)}}{2E_{N_1}} [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)]. \end{aligned} \quad (\text{B33})$$

By changing the variable to $p_{N_1} = M_1 \sinh \theta$, Eq. (B33) becomes

$$\begin{aligned} & C' \int_0^\infty \frac{4\pi p_{N_1}^2 dp_{N_1}}{2(p_{N_1}^2 + M_1^2)^{1/2}} e^{-(p_{N_1}^2 + M_1^2)^{1/2}/(\bar{T}+\delta T)} \\ &= 2\pi C' M_1^2 \int_0^\infty d\theta \sinh^2 \theta e^{-z_T \cosh \theta} \\ &= \frac{4\pi^3}{g_{N_1} M_1} \frac{K_1(z_T)}{K_2(z_T)}, \end{aligned} \quad (\text{B34})$$

where we used Eqs. (B26) and (B27). Substituting Eq. (B34) into Eq. (B33), we obtain

$$\begin{aligned} & \left\langle \frac{g_{N_1}}{(2\pi)^3} \int \frac{d^3 p_{N_1}}{E_{N_1}} C \right\rangle \\ &= -\frac{4\pi^3 g_l g_\phi |\mathcal{M}|^2}{M_1} \frac{K_1(z_T)}{K_2(z_T)} [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)] \\ &\equiv -\Gamma_D(\bar{z} = \infty) \left\langle \frac{1}{\gamma} \right\rangle (z_T) [n_{N_1}(\bar{z}) - n_{N_1}^{\text{eq}}(z_T)]. \end{aligned} \quad (\text{B35})$$

6. Derivation of $N_{N_1}^{\text{eq}}$

In this subsection, we discuss several expression of $N_{N_1}^{\text{eq}}(z_T)$, which holds for different ranges of z_T . Approximating with the Boltzmann distribution, the number density $n_{N_1}^{\text{eq}}(z_T)$ can be approximated using the Boltzmann factor as

$$n_{N_1}^{\text{eq}}(z_T) = C_0 \int \frac{d^3 p_{N_1}}{(2\pi)^3} \exp\left(-\frac{E_{N_1}}{\bar{T} + \delta T}\right), \quad (\text{B36})$$

where C_0 is a normalization constant. After changing the variable using $E_{N_1} = (p_{N_1}^2 + M_1^2)^{1/2}$ and $t = E_{N_1}/M_1$, integration by parts yields

$$n_{N_1}^{\text{eq}}(z_T) = C_0 \frac{M_1^3 z_T}{3} \int_1^\infty dt (t^2 - 1)^{3/2} e^{-z_T t}. \quad (\text{B37})$$

By using the integral expression for the modified Bessel function of the second kind

$$K_\nu(z) = \frac{\sqrt{\pi}(z/2)^\nu}{\Gamma(\nu + \frac{1}{2})} \int_1^\infty dt (t^2 - 1)^{\nu-\frac{1}{2}} e^{-zt}, \quad (\text{B38})$$

Eq. (B37) becomes

$$n_{N_1}^{\text{eq}}(z_T) = C_0 \frac{M_1^3 z_T}{3} \cdot \frac{3}{z_T^2} K_2(z_T) = C_0 \frac{M_1^3}{z_T} K_2(z_T). \quad (\text{B39})$$

If the photon number density is normalized $n_\gamma^{\text{eq}}(z_T) \propto (\bar{T} + \delta T)^3$,

$$N_{N_1}^{\text{eq}}(z_T) = \frac{n_{N_1}^{\text{eq}}(z_T)}{n_\gamma^{\text{eq}}(z_T)} = C'_0 z_T^2 K_2(z_T), \quad (\text{B40})$$

with some normalization constant C'_0 . For Eq. (B40) to hold in the relativistic limit $z_T \ll 1$, it must reduce to (6) in this limit. Since

$$K_2(z) \simeq \frac{2}{z} K_1(z) \simeq \frac{2}{z^2}, \quad z \ll 1, \quad (\text{B41})$$

the normalization constant is fixed from

$$\frac{3}{4} = N_{N_1}^{\text{eq}}(z_T \ll 1) \simeq C'_0 z_T^2 \cdot \frac{2}{z_T^2} = 2C'_0, \quad (\text{B42})$$

as $C'_0 = 3/8$. From this and Eq. (B40), we obtain

$$N_{N_1}^{\text{eq}}(z_T) = \frac{3}{8} z_T^2 K_2(z_T). \quad (\text{B43})$$

7. Derivation of the washout factor W_{ID}

In this subsection, we derive the expression for the washout factor W_{ID} . The decay rate of N_1 is given by

$$\Gamma_D(z) = \tilde{\Gamma}_D \left\langle \frac{1}{\gamma} \right\rangle = \tilde{\Gamma}_D \frac{K_1(z_T)}{K_2(z_T)}, \quad (\text{B44})$$

where $\tilde{\Gamma}_D = \Gamma_D(z = \infty)$ from $\lim_{z \rightarrow \infty} K_1(z)/K_2(z) = 1$.

The inverse decay rate Γ_{ID} is related to the decay rate Γ_D through detailed balance

$$\Gamma_{\text{ID}}(z) = \Gamma_D(z) \frac{N_{N_1}^{\text{eq}}(z_T)}{N_l^{\text{eq}} N_\phi^{\text{eq}}}. \quad (\text{B45})$$

The equilibrium number densities of heavy neutrinos, lepton doublets, and Higgs doublets are, as already derived,

$$N_{N_1}^{\text{eq}}(z_T) = \frac{3}{8} z_T^2 K_2(z_T), \quad N_l^{\text{eq}} = \frac{3}{4}, \quad N_\phi^{\text{eq}} = 1. \quad (\text{B46})$$

Hence, the contribution of inverse decays to the washout term W_{ID} is

$$W_{\text{ID}}(z_T) \equiv \frac{1}{2} \frac{\Gamma_{\text{ID}}(z_T)}{\bar{z}H(\bar{z})} = \frac{1}{4} z_T^2 K_2(z_T) \frac{\Gamma_D(z_T)}{\bar{z}H(\bar{z})}. \quad (\text{B47})$$

Substituting Eq. (B44) and $K \equiv \tilde{\Gamma}_D/\bar{z}^2 H(\bar{z})$ into Eq. (B47), the washout factor is found to be

$$\begin{aligned} W_{\text{ID}}(z_T) &= \frac{1}{4} z_T^2 K_2(z_T) \frac{\tilde{\Gamma}_D}{\bar{z}H(\bar{z})} \cdot \frac{K_1(z_T)}{K_2(z_T)} \\ &= \frac{1}{4} K \bar{z} z_T^2 K_1(z_T). \end{aligned} \quad (\text{B48})$$

APPENDIX C: PERTURBATIVE CHRISTOFFEL SYMBOLS

In this appendix, we summarize the metric (11)–(13) and its inverse, together with the Christoffel symbols. The metric and its inverse are given by

$$g_{00} = -a^2(1 + 2A), \quad g^{00} = -a^{-2}(1 - 2A), \quad (\text{C1})$$

$$g_{0i} = -a^2 B_{,i}, \quad g^{0i} = -a^{-2} B^{,i}, \quad (\text{C2})$$

$$g_{ij} = a^2 \left[(1 + 2C) \delta_{ij} + 2 \left(E_{,ij} - \frac{1}{3} \delta_{ij} \nabla^2 E \right) \right], \quad (\text{C3})$$

$$g^{ij} = a^{-2} \left[(1 - 2C) \delta^{ij} - 2 \left(E^{,ij} - \frac{1}{3} \delta^{ij} \nabla^2 E \right) \right]. \quad (\text{C4})$$

The Christoffel symbols up to first order in perturbations are given by

$$\Gamma^0_{00} = \frac{a'}{a} + A', \quad (\text{C5})$$

$$\Gamma^0_{0i} = A_{,i} - \frac{a'}{a} B_{,i}, \quad (\text{C6})$$

$$\begin{aligned} \Gamma^0_{ij} &= \frac{a'}{a} \delta_{ij} + \left[-2 \frac{a'}{a} A + \frac{(a^2 C)'}{a^2} \right] \delta_{ij} \\ &\quad + B_{,ij} + \frac{1}{a^2} \left[a^2 \left(E_{,ij} - \frac{1}{3} \delta_{ij} \nabla^2 E \right) \right]', \end{aligned} \quad (\text{C7})$$

$$\Gamma^i_{00} = A^{,i} - \left(B' + \frac{a'}{a} B \right)^{,i}, \quad (\text{C8})$$

$$\Gamma^i_{0j} = \frac{a'}{a} \delta^i_j + C' \delta^i_j + \left(E^{,i}{}_{,j} - \frac{1}{3} \delta^i_j \nabla^2 E \right)', \quad (\text{C9})$$

$$\begin{aligned} \Gamma^i_{jk} &= \left(C_{,k} \delta^i_j + C_{,j} \delta^i_k - C^i{}_{jk} \right) + \frac{a'}{a} \delta_{jk} B^{,i} \\ &\quad + E^{,i}{}_{,jk} - \frac{1}{3} \nabla^2 \left(\delta^i_j E_{,k} + \delta^i_k E_{,j} - \delta_{jk} E^{,i} \right). \end{aligned} \quad (\text{C10})$$

APPENDIX D: CALCULATION OF CP ASYMMETRY ϵ_1

In this appendix, we calculate the CP asymmetry ϵ_1 from the interference between tree and one-loop diagrams in the decay of N_1 . In order to plot Fig. 6, we should derive the maximum value of the absolute value of CP asymmetry $|\epsilon_1|$ given by Eq. (39). Unlike the usual Davidson-Ibarra bound for type-I seesaw leptogenesis, $|\epsilon_1| \leq \frac{3M_1}{8\pi v^2} (m_3 - m_1)$, Eq. (39) depends on the effective neutrino mass $\tilde{m}_1$, which corresponds to the decay parameter K and is the horizontal axis of the mass window plot. As can be seen from the relation between Eqs. (D41) and (E8), Appendixes D and E are closely related, and combining them yields Eq. (39), thus making our paper self-contained.

1. Prerequisites

The relevant Yukawa terms in the Lagrangian are

$$\mathcal{L} \supset -y_{\alpha\beta}^{(e)} (\bar{l}_L)_\alpha \phi (e_R)_\beta - y_{\alpha\beta}^{(\nu)} [(\bar{l}_L)_\alpha \phi^\dagger] N_\beta + \text{H.c.}, \quad (\text{D1})$$

where $\alpha, \beta = 1, 2, 3$ denote the lepton flavor, and repeated indices are implicitly summed over. In addition, the indices L and R represent the left-chiral and right-chiral eigenvectors of the eigenspace (D46) of chirality (D43).

Left-chiral field l_L and Higgs field ϕ form SU(2) doublets as

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{\text{SU}(2)}, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{\text{SU}(2)}. \quad (\text{D2})$$

The Dirac conjugate of the former is defined as

$$\bar{l}_L = (\bar{\nu}_L \quad \bar{e}_L)_{\text{SU}(2)}, \quad (\text{D3})$$

where ν_L, ν_R, e_L, e_R are all Weyl fields. The Dirac conjugates for them are defined by promoting to Dirac fields and then projecting them onto the eigenspaces of the chirality γ_5 . To do so, we introduce dummy fields ν'_R and e'_R as

$$\nu = \begin{pmatrix} \nu_L \\ \nu'_R \end{pmatrix}_{\text{Dirac}}, \quad e = \begin{pmatrix} e_L \\ e'_R \end{pmatrix}_{\text{Dirac}}, \quad (\text{D4})$$

and then $\bar{\nu}_L$ and $\bar{e}_L$ satisfy

$$\bar{\nu}_L = \overline{P_L \nu} = (P_L \nu)^\dagger \gamma^0 = \nu^\dagger P_L \gamma^0 = \nu^\dagger \gamma^0 P_R, \quad (\text{D5})$$

$$\bar{e}_L = \overline{P_L e} = (P_L e)^\dagger \gamma^0 = e^\dagger P_L \gamma^0 = e^\dagger \gamma^0 P_R, \quad (\text{D6})$$

where P_L and P_R are projection operators of chirality defined in Eq. (D45). These definitions give

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_{\text{SU}(2)} = \begin{pmatrix} \begin{pmatrix} \nu_L \\ \nu'_R \end{pmatrix}_{\text{Dirac}} \\ \begin{pmatrix} e_L \\ e'_R \end{pmatrix}_{\text{Dirac}} \end{pmatrix}_{\text{SU}(2)}$$

$$\Rightarrow l_L = P_L l = \begin{pmatrix} P_L \nu \\ P_L e \end{pmatrix}_{\text{SU}(2)} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{\text{SU}(2)}. \quad (\text{D7})$$

The product $\bar{l}_L \circ \phi^\dagger$ is defined so that it is invariant under SU(2) [note that $i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$]:

$$\begin{aligned} \bar{l}_L \circ \phi^\dagger &= (\bar{\nu}_L \quad \bar{e}_L)_{\text{SU}(2)} \circ \begin{pmatrix} (\phi^+)^\dagger \\ (\phi^0)^\dagger \end{pmatrix}_{\text{SU}(2)} \\ &= (\bar{\nu}_L \quad \bar{e}_L)_{\text{SU}(2)} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} (\phi^+)^\dagger \\ (\phi^0)^\dagger \end{pmatrix}_{\text{SU}(2)} \\ &= \bar{\nu}_L (\phi^0)^\dagger - \bar{e}_L (\phi^+)^\dagger, \end{aligned} \quad (\text{D8})$$

which can be rewritten as

$$\bar{l}_L \circ \phi^\dagger = \overline{P_L l} \circ \phi^\dagger = (\bar{l} P_R) \circ \phi^\dagger = (\bar{l} \circ \phi^\dagger) P_R. \quad (\text{D9})$$

The conjugate of the above gives $\phi \circ l_L$

$$\begin{aligned} \phi \circ l_L &= (\phi^+ \quad \phi^0)_{\text{SU}(2)} \circ \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{\text{SU}(2)} \\ &= (\phi^+ \quad \phi^0)_{\text{SU}(2)} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_{\text{SU}(2)} \\ &= \nu_L \phi^0 - e_L \phi^+, \end{aligned} \quad (\text{D10})$$

which can be rewritten as

$$\phi \circ l_L = P_L (\phi \circ l). \quad (\text{D11})$$

Note that the sign of the operation $\circ$ changes depending on whether the fields are SU(2) fundamental or SU(2) antifundamental.

The Majorana field N is the Majorana representation of the right-chiral Weyl field ν_R given by

$$N = \begin{pmatrix} (\nu_R)^c \\ \nu_R \end{pmatrix}_{\text{Dirac}}, \quad (\text{D12})$$

where the charge conjugation operator c is defined as

$$\begin{aligned} (\text{left chiral})^c &= i\sigma^2 (\text{left chiral})^\dagger, \\ (\text{right chiral})^c &= -i\sigma^2 (\text{right chiral})^\dagger, \end{aligned} \quad (\text{D13})$$

thereby $(\nu_R)^c = -i\sigma^2 \nu_R^\dagger$. Thus, the relevant Yukawa couplings become

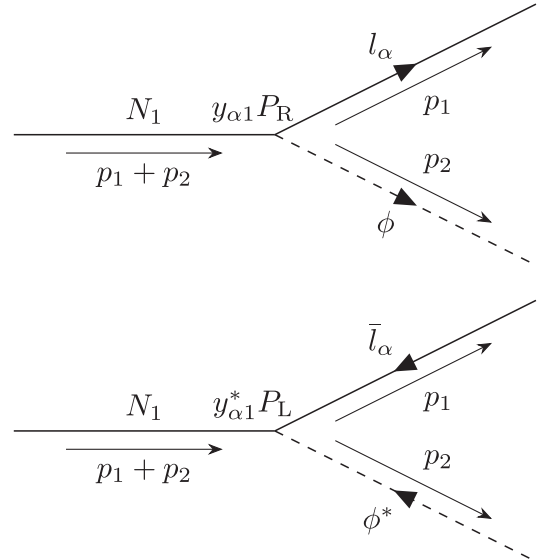
$$\begin{aligned} \mathcal{L}_{\text{int}} &= -y_{\alpha\beta}^{(\nu)} [(\bar{l}_L)_\alpha \circ \phi^\dagger] N_\beta + \text{H.c.} \\ &= -y_{\alpha\beta}^{(\nu)} (\bar{l}_\alpha \circ \phi^\dagger) P_R N_\beta - y_{\alpha\beta}^{(\nu)*} \bar{N}_\beta P_L (\phi \circ l_\alpha). \end{aligned} \quad (\text{D14})$$

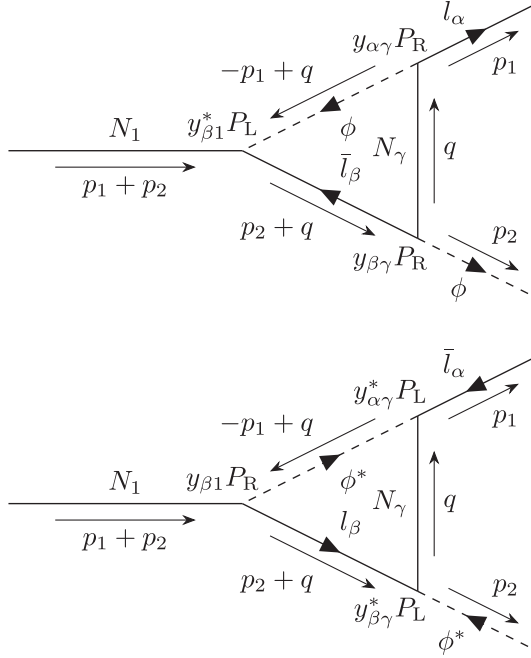
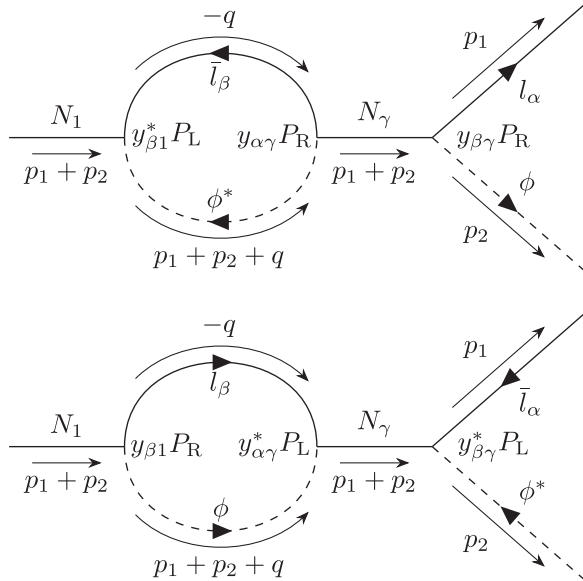
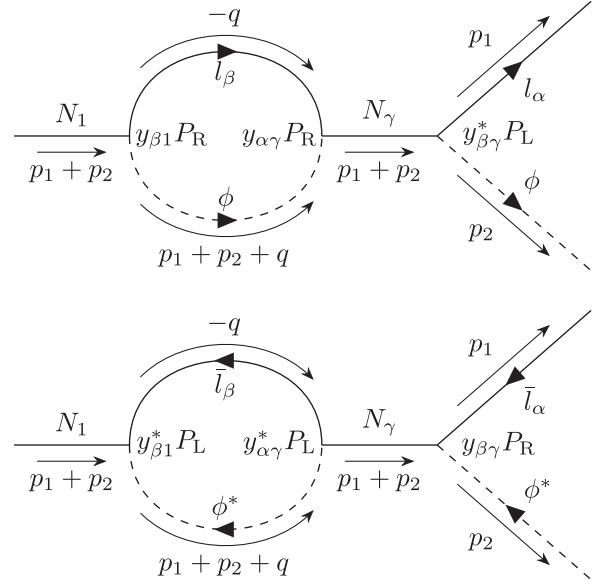
Henceforth, we denote $y_{\alpha\beta}^{(\nu)}$ by $y_{\alpha\beta}$.

2. Feynman diagrams

We list the Feynman diagrams that appear in the one-loop calculation of ε_1 . For the Feynman rules for Majorana fermions, refer to Refs. [59,60].

a. Tree diagrams



b. Vertex diagrams**c. Wave function diagrams I****d. Wave function diagrams II****3. Calculation of tree diagrams**

In this subsection, we calculate the tree diagrams. Hereafter, we assume that the masses of the leptons and the Higgs boson are negligible compared to the mass of the heavy neutrino M_1 . We often use

$$p_1^2 = p_2^2 = 0, \quad p_1 \cdot p_2 = \frac{M_1^2}{2}, \quad (\text{D15})$$

for the momenta of the lepton p_1 and the Higgs boson p_2 . Here, the second equation is derived from $(p_1 + p_2)^2 = M_1^2$.

First we consider $N_1 \rightarrow l_\alpha \phi$. Let s and t be the spins of the initial and final states of N_1 and l_α , respectively. The Lagrangian in Eq. (D14) can be written as

$$\mathcal{L}_{\text{int}} \supset -y_{\alpha\beta} [\bar{\nu}_\alpha(\phi^0)^\dagger - \bar{e}_\alpha(\phi^+)^\dagger] P_R N_\beta. \quad (\text{D16})$$

Thus, there are two possibilities for the final state: $\nu_\alpha \phi^0$ and $e_\alpha \phi^+$, originating from the SU(2) doublet. Here, the matrix element $i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(s,t)}$ is defined for each channel, and hence, the result obtained below must be multiplied by a factor of two when calculating the total decay rate. The total matrix element is the sum of the tree and the one-loop

$$i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree}+1\text{-loop})(s,t)} = i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} + i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(1\text{-loop})(s,t)}. \quad (\text{D17})$$

The squared magnitude of the matrix element at the tree level is obtained by taking the sum over spins s and t and then averaging over the spin s of the initial state

$$\begin{aligned}
 \left| i\bar{\mathcal{M}}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})} \right|^2 &\equiv \frac{1}{2} \sum_{s,t} \left| i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right|^2 \\
 &= \frac{1}{2} \sum_{s,t} |\bar{u}_{l_\alpha}^t(p_1) (-iy_{\alpha 1} P_R) u_{N_1}^s(p_1 + p_2)|^2 \\
 &= \frac{1}{2} |y_{\alpha 1}|^2 \text{tr}[(\not{p}_1 + \not{p}_2 + M_1) P_L \not{p}_1 P_R] \\
 &= |y_{\alpha 1}|^2 (p_1 \cdot p_2) = \frac{1}{2} |y_{\alpha 1}|^2 M_1^2, \quad (\text{D18})
 \end{aligned}$$

where we used the spin sum formula (D64), the trace formulas (D51), (D52), (D55), and (D44), as well as the approximation in Eq. (D15).

Next, for $N_1 \rightarrow \bar{l}_\alpha \phi^*$, a similar calculation yields

$$\begin{aligned}
 \left| i\bar{\mathcal{M}}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(\text{tree})} \right|^2 &\equiv \frac{1}{2} \sum_{s,t} \left| i\mathcal{M}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(\text{tree})(s,t)} \right|^2 \\
 &= \frac{1}{2} \sum_{s,t} |\bar{v}_{l_\alpha}^t(p_1) (-iy_{\alpha 1}^* P_L) u_{N_1}^s(p_1 + p_2)|^2 \\
 &= \frac{1}{2} |y_{\alpha 1}|^2 \text{tr}[(\not{p}_1 + \not{p}_2 - M_1) P_R \not{p}_1 P_L] \\
 &= |y_{\alpha 1}|^2 (p_1 \cdot p_2) = \frac{1}{2} |y_{\alpha 1}|^2 M_1^2. \quad (\text{D19})
 \end{aligned}$$

4. Calculation of vertex diagrams

In this subsection, we calculate vertex diagrams and the resulting interference with the tree-level diagrams. We first consider $N_1 \rightarrow l_\alpha \phi$. The combined contribution of the tree and one-loop diagrams is

$$\begin{aligned}
 \left| i\bar{\mathcal{M}}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree}+1\text{-loop})} \right|^2 &\equiv \frac{1}{2} \sum_{s,t} \left| i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree}+1\text{-loop})(s,t)} \right|^2 \\
 &\simeq \frac{1}{2} \sum_{s,t} \left| i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right|^2 \\
 &\quad + \frac{1}{2} \sum_{s,t} \left[\left(i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right)^* \left(i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(1\text{-loop})(s,t)} \right) + \text{c.c.} \right]. \quad (\text{D20})
 \end{aligned}$$

The interference part is

$$\begin{aligned}
 I_{\text{vertex}} &\equiv \frac{1}{2} \sum_{s,t} \left(i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right)^* \left(i\mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(1\text{-loop})(s,t)} \right) \\
 &= iy_{\alpha 1}^* y_{\beta 1} y_{\alpha \gamma} y_{\beta \gamma} \\
 &\quad \times \int \frac{d^4 q}{(2\pi)^4} \frac{-\frac{1}{2} M_1^3 M_\gamma - M_1 M_\gamma (p_1 \cdot q)}{(q^2 - M_\gamma^2)(-p_2 - q)^2(-p_1 + q)^2} \\
 &\equiv y_{\alpha 1}^* y_{\beta 1} y_{\alpha \gamma} y_{\beta \gamma} D_{\text{vertex}}, \quad (\text{D21})
 \end{aligned}$$

where we used Eq. (D47), the spin sum formula (D64), and the trace formulas (D51), (D52), and (D54). Furthermore, performing the q integration using Feynman parametrization (D67) and the dimensional regularization formulas (D68) and (D69), we get

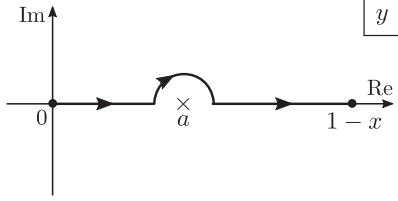
$$\begin{aligned}
 D_{\text{vertex}} &= \frac{1}{2} \int \frac{d^4 q}{i(2\pi)^4} \\
 &\quad \times \frac{M_1^3 M_\gamma + 2M_1 M_\gamma (p_1 \cdot q)}{(q^2 - M_\gamma^2 + i\epsilon)(-p_2 - q + i\epsilon)^2(-p_1 + q + i\epsilon)^2} \\
 &= \frac{M_1^2}{32\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{M_1 M_\gamma (1-y)}{M_\gamma^2(1-x-y) - M_1^2 xy - i\epsilon}, \quad (\text{D22})
 \end{aligned}$$

where $-i\epsilon$ in the denominator comes from the $i\epsilon$ prescription.

We define $m \equiv M_1/M_\gamma$ and perform the integration over y in equation (D22). Considering the complex y plane, there exists a pole at $y = (1-x)/(1+m^2x) \equiv a$ within the range $0 \leq y \leq 1-x$ on the real axis, which we avoid by taking an upper semicircle. We divide the integration range of y into three intervals $[0, a - \epsilon']$, $[a - \epsilon', a + \epsilon']$, and $[a + \epsilon', 1]$. In the middle interval $[a - \epsilon', a + \epsilon']$, we perform the variable transformation $y = a + \epsilon' e^{i\theta}$ (where $\epsilon' > 0$ is a small quantity and $0 \leq \theta \leq \pi$). The semicircular contour in Fig. 13 contributes one half of the residue, which is purely imaginary at the pole a ; consequently, the originally real D_{vertex} acquires this contribution and becomes purely imaginary, $i \text{Im}(D_{\text{vertex}})$. This quantity is then evaluated as

$$\begin{aligned}
 i \text{Im}(D_{\text{vertex}}) &= \frac{M_1^2}{32\pi^2} \int_0^1 dx \int_{a-\epsilon'}^{a+\epsilon'} dy \frac{m(1-y)}{1-x-y-m^2xy-i\epsilon} \\
 &= -\frac{M_1^2}{32\pi^2} \int_0^1 dx \frac{m}{1+m^2x} \int_{a-\epsilon'}^{a+\epsilon'} dy \frac{y-1}{y - \frac{1-x}{1+m^2x} + i\epsilon} \\
 &= -\frac{M_1^2}{32\pi^2} \int_0^1 dx \frac{m}{1+m^2x} \int_\pi^0 i\epsilon' e^{i\theta} d\theta \frac{a-1+\mathcal{O}(\epsilon')}{\epsilon' e^{i\theta}} \\
 &\xrightarrow{\epsilon' \downarrow 0} -\frac{M_1^2}{32\pi^2} \int_0^1 dx (-i\pi) \frac{m}{1+m^2x} \left(\frac{1-x}{1+m^2x} - 1 \right), \quad (\text{D23})
 \end{aligned}$$

where the $+i\epsilon$ in the denominator specifies that the integration contour lies in the upper half-plane.

FIG. 13. Contour of D_{vertex} .

Upon performing the x integration, we find

$$\text{Im}(D_{\text{vertex}}) = \frac{M_1^2}{32\pi} \left\{ \frac{1}{m} \left[1 - \left(1 + \frac{1}{m^2} \right) \ln(1 + m^2) \right] \right\}. \quad (\text{D24})$$

Thus, Eq. (D21) becomes

$$I_{\text{vertex}} = i y_{\alpha 1}^* y_{\beta 1}^* y_{\alpha \gamma} y_{\beta \gamma} \text{Im}(D_{\text{vertex}}). \quad (\text{D25})$$

We next consider $N_1 \rightarrow \bar{l}_\alpha \phi^*$. Similar calculation as above gives

$$\begin{aligned} I'_{\text{vertex}} &\equiv \frac{1}{2} \sum_{s,t} \left(i \mathcal{M}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(\text{tree})(s,t)} \right)^* \left(i \mathcal{M}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(1\text{-loop})(s,t)} \right) \\ &= \dots \\ &= i y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^* \frac{M_1^2}{32\pi} \left\{ \frac{1}{m} \left[1 - \left(1 + \frac{1}{m^2} \right) \ln(1 + m^2) \right] \right\}. \end{aligned} \quad (\text{D26})$$

Regarding the external lines, while the antilepton part changed from $u_{l_\alpha}^t$ to $v_{l_\alpha}^t$, the N_1 part remained as $u_{N_1}^s$. This is due to the Majorana nature of N_1 .

Combining above results, the CP asymmetry $(\varepsilon_1)_{\text{vertex}}$ is given by

$$\begin{aligned} (\varepsilon_1)_{\text{vertex}} &= \frac{\sum_\alpha \left(\left| i \bar{\mathcal{M}}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree}+1\text{-loop})} \right|^2 - \left| i \bar{\mathcal{M}}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(\text{tree}+1\text{-loop})} \right|^2 \right)}{\sum_\alpha \left(\left| i \bar{\mathcal{M}}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})} \right|^2 + \left| i \bar{\mathcal{M}}_{N_1 \rightarrow \bar{l}_\alpha \phi^*}^{(\text{tree})} \right|^2 \right)} \\ &= \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*)}{8\pi \sum_\alpha |y_{\alpha 1}|^2} F(m), \quad m \equiv \frac{M_1}{M_\gamma}, \\ F(m) &\equiv \frac{1}{m} \left[1 - \left(1 + \frac{1}{m^2} \right) \ln(1 + m^2) \right]. \end{aligned} \quad (\text{D27})$$

As seen from this expression, in order for ε_1 to be nonzero, D_{vertex} must have an imaginary part.

5. Calculation of wave function diagrams

Regarding the wave function diagrams, there appears two types of contributions: wave function I and wave function II. However, only the former contributes to the CP asymmetry (this will be verified later). Additionally, be mindful of the factor 2, which appears below. This arises

when two different particle species belonging to the $SU(2)$ group appear in the loop. For example, when fix the decay channel to $N_\alpha \rightarrow \nu_\alpha \phi^0$, the particles appearing in the triangular loop in the vertex diagrams are fixed to ϕ^0 and ν_β since the vertex is connected to the final state. On the other hand, in the wave function diagrams, there is no constraint on the particles entering the loop from the final state, and hence, both $\nu_\alpha \phi^0$ and $e_\alpha \phi^+$ contribute.

The interference term of the wave function diagram I is

$$\begin{aligned} I_{\text{wave}}^{(1)} &= \frac{1}{2} \sum_{s,t} \left(i \mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right)^* \left(i \mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(1\text{-loop})(s,t)} \right) \\ &= i y_{\alpha 1}^* y_{\beta 1}^* y_{\alpha \gamma} y_{\beta \gamma} \frac{2M_1 M_\gamma}{M_1^2 - M_\gamma^2} \int \frac{d^4 q}{(2\pi)^4} \frac{p_1 \cdot q}{q^2 (p_1 + p_2 + q)^2} \\ &\equiv y_{\alpha 1}^* y_{\beta 1}^* y_{\alpha \gamma} y_{\beta \gamma} D_{\text{wave}}^{(1)}, \end{aligned} \quad (\text{D28})$$

where we used (D47), the spin sum formula (D64), and the trace formulas (D51), (D52), and (D55). Furthermore, applying Feynman parametrization (D66) and the dimensional regularization formula (D69), we perform the q integration and extract the imaginary part, obtaining

$$\begin{aligned} D_{\text{wave}}^{(1)} &= -\frac{2M_1 M_\gamma}{M_1^2 - M_\gamma^2} \int \frac{d^4 q}{i(2\pi)^4} \frac{p_1 \cdot q}{q^2 (q + p_1 + p_2)^2} \\ &= -\frac{1}{16\pi^2} \frac{M_1^3 M_\gamma}{M_1^2 - M_\gamma^2} \int_0^1 dx x \log[-M_1^2 x(1-x) - i\epsilon], \end{aligned} \quad (\text{D29})$$

i.e.,

$$\text{Im}(D_{\text{wave}}^{(1)}) = \frac{1}{32\pi} \frac{M_1^3 M_\gamma}{M_1^2 - M_\gamma^2}. \quad (\text{D30})$$

Here, note that $\log[-M_1^2 x(1-x) - i\epsilon]$ is specified with the integration contour lying in the lower half-plane, thus yielding $-i\pi$ instead of $+i\pi$. From this and Eq. (D29), we obtain

$$I_{\text{wave}}^{(1)} = i y_{\alpha 1}^* y_{\beta 1}^* y_{\alpha \gamma} y_{\beta \gamma} \frac{M_1^2}{32\pi} \frac{m}{m^2 - 1}, \quad m \equiv \frac{M_1}{M_\gamma}. \quad (\text{D31})$$

The interference term of the wave function diagram II is

$$\begin{aligned} I_{\text{wave}}^{(2)} &= \frac{1}{2} \sum_{s,t} \left(i \mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(\text{tree})(s,t)} \right)^* \left(i \mathcal{M}_{N_1 \rightarrow l_\alpha \phi}^{(1\text{-loop})(s,t)} \right) \\ &= i y_{\alpha 1}^* y_{\beta 1}^* y_{\alpha \gamma} y_{\beta \gamma} \frac{2M_1^2}{M_1^2 - M_\gamma^2} \int \frac{d^4 q}{(2\pi)^4} \frac{p_2 \cdot q}{q^2 (p_1 + p_2 + q)^2}, \end{aligned} \quad (\text{D32})$$

where we have taken into account Eq. (D47), the spin sum formula (D64), and the trace formulas (D53) and (D56). Applying Feynman parametrization (D66) and the dimensional regularization formula (D69), we perform the q integration in a similar way to those in Eq. (D31), obtaining

$$I_{\text{wave}}^{(2)} = i y_{\alpha 1}^* y_{\beta 1} y_{\alpha \gamma} y_{\beta \gamma}^* \frac{M_1^2}{32\pi m^2 - 1}, \quad m \equiv \frac{M_1}{M_\gamma}. \quad (\text{D33})$$

By summing Eqs. (D31) and (D33), similarly to equation (D27), we find

$$\begin{aligned} (\varepsilon_1)_{\text{wave}} &= \frac{\sum_{\alpha} \left(\left| i \bar{\mathcal{M}}_{N_1 \rightarrow l_a \phi}^{(\text{tree}+1\text{-loop})} \right|^2 - \left| i \bar{\mathcal{M}}_{N_1 \rightarrow \bar{l}_a \phi^*}^{(\text{tree}+1\text{-loop})} \right|^2 \right)}{\sum_{\alpha} \left(\left| i \bar{\mathcal{M}}_{N_1 \rightarrow l_a \phi}^{(\text{tree})} \right|^2 + \left| i \bar{\mathcal{M}}_{N_1 \rightarrow \bar{l}_a \phi^*}^{(\text{tree})} \right|^2 \right)} \\ &= \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*)}{8\pi \sum_{\alpha} |y_{\alpha 1}|^2} \cdot \frac{m}{m^2 - 1} \\ &\quad + \frac{\text{Im}(y_{\alpha 1} y_{\beta 1}^* y_{\alpha \gamma}^* y_{\beta \gamma})}{8\pi \sum_{\alpha} |y_{\alpha 1}|^2} \cdot \frac{m^2}{m^2 - 1}. \end{aligned} \quad (\text{D34})$$

Here, note that

$$\begin{aligned} \text{Im}(y_{\alpha 1} y_{\beta 1}^* y_{\alpha \gamma}^* y_{\beta \gamma}) &= \text{Im} \left[\sum_{\alpha} (y_{\alpha 1} y_{\alpha \gamma}^*) \sum_{\beta} (y_{\beta 1}^* y_{\beta \gamma}) \right] \\ &= \text{Im} \left(\left| \sum_{\alpha} y_{\alpha 1} y_{\alpha \gamma}^* \right|^2 \right) = 0, \end{aligned} \quad (\text{D35})$$

from which follows that the contribution from wave function diagram II is zero. Thus, we find

$$(\varepsilon_1)_{\text{wave}} = \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*)}{8\pi \sum_{\alpha} |y_{\alpha 1}|^2} \cdot \frac{m}{m^2 - 1}, \quad m \equiv \frac{M_1}{M_\gamma}. \quad (\text{D36})$$

6. Final result and approximate expression

By combining Eqs. (D27) and (D36), the expression for the CP asymmetry is obtained as

$$\begin{aligned} \varepsilon_1 &= \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*)}{8\pi \sum_{\alpha} |y_{\alpha 1}|^2} \\ &\quad \times \frac{1}{m} \left[1 + \frac{m^2}{m^2 - 1} - \left(1 + \frac{1}{m^2} \right) \ln(1 + m^2) \right]. \end{aligned} \quad (\text{D37})$$

At this point, we switch to $x \equiv M_\gamma^2/M_1^2$ in place of $m \equiv M_1/M_\gamma$ to match the literature

$$\begin{aligned} \varepsilon_1 &= \frac{\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*)}{8\pi \sum_{\alpha} |y_{\alpha 1}|^2} f(x), \quad x \equiv \frac{M_\gamma^2}{M_1^2}, \\ f(x) &\equiv \sqrt{x} \left[1 + \frac{1}{1-x} - (1+x) \ln \left(1 + \frac{1}{x} \right) \right]. \end{aligned} \quad (\text{D38})$$

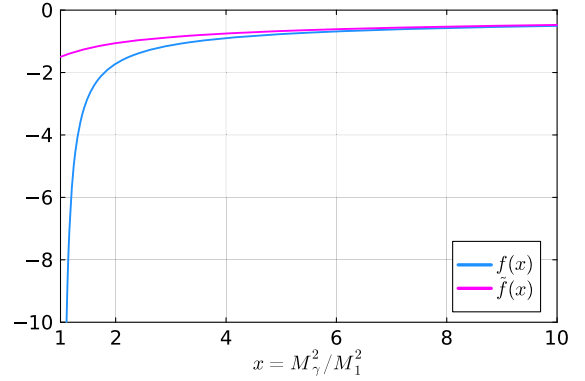


FIG. 14. Functions $f(x)$ (blue) and $\tilde{f}(x) \equiv -3/(2\sqrt{x})$ (magenta).

Note that $\text{Im}(y_{\alpha 1} y_{\beta 1} y_{\alpha \gamma}^* y_{\beta \gamma}^*) = 0$, and hence, $\varepsilon_1 = 0$ for $\gamma = 1$. Thus, the contributions from the second and third generations are necessary to generate nonzero ε_1 .

When the magnitude of the CP asymmetry ε_1 is much smaller than one, $x = M_\gamma^2/M_1^2$ is expected to be much larger than unity. In this case, we perform a Taylor expansion of $f(x)$ for $1/x \ll 1$:

$$f(x) = - \left[\frac{3}{2\sqrt{x}} + \frac{5}{6x^{3/2}} + \mathcal{O}\left(\frac{1}{x^{5/2}}\right) \right], \quad (\text{D39})$$

from which we define $\tilde{f}$ taking the leading term

$$\tilde{f}(x) \equiv -\frac{3}{2\sqrt{x}}. \quad (\text{D40})$$

The plots for $f(x)$ and $\tilde{f}(x)$ are shown in Fig. 14. When approximating $f(x)$ with $\tilde{f}(x)$, the CP asymmetry (D38) is approximated as

$$\varepsilon_1 \simeq -\frac{3M_1}{16\pi \sum_{\alpha} |y_{\alpha 1}|^2} \text{Im} \left(y_{\alpha 1} y_{\alpha \gamma}^* \frac{1}{M_\gamma} y_{\beta 1} y_{\beta \gamma}^* \right). \quad (\text{D41})$$

Now, we list below the formulas used in the above calculations.

7. Formulas for γ matrices

a. Definition and properties of γ matrices

The γ matrices consist of four $n \times n$ matrices γ^μ (where $\mu = 0, 1, 2, 3$) and satisfy the following anticommutation relations:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu} \mathbf{1}_{n \times n}. \quad (\text{D42})$$

Equation (D42) is called the Clifford algebra.

b. Definition and properties of chirality γ_5

We define *chirality* γ_5 as

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3. \quad (\text{D43})$$

γ_5 satisfies

$$(\gamma_5)^2 = \mathbf{1}, \quad (\gamma_5)^\dagger = \gamma_5, \quad \gamma_5\gamma^\mu = -\gamma^\mu\gamma_5, \quad (\text{D44})$$

for $\mu = 0, 1, 2, 3$.

The eigenspaces of γ_5 with eigenvalues ± 1 are referred to as left chiral and right chiral, respectively. Let P_L and P_R denote the projection operators onto the left-chiral and right-chiral eigenspaces, respectively. These satisfy

$$P_R = \frac{\mathbf{1} + \gamma_5}{2}, \quad P_L = \frac{\mathbf{1} - \gamma_5}{2}, \quad (\text{D45})$$

and decompose the Dirac spinor ψ into

$$\psi_R = P_R\psi, \quad \psi_L = P_L\psi. \quad (\text{D46})$$

These projection operators satisfy

$$P_R^2 = P_R, \quad P_L^2 = P_L, \quad P_R P_L = P_L P_R = 0. \quad (\text{D47})$$

c. Feynman slash notation

The contraction between the four-vector a^μ and the γ matrix γ^μ is denoted by

$$\not{a} \equiv \gamma^\mu a_\mu = \gamma_\mu a^\mu, \quad (\text{D48})$$

from which it follows that

$$\not{a}^2 = a^2. \quad (\text{D49})$$

d. Trace formulas

Traces of gamma matrices are given by

$$\text{tr}(\mathbf{1}) = 4, \quad (\text{D50})$$

$$\text{tr}(\not{a}) = \text{tr}(\not{a}\not{b}\not{c}) = \dots = 0, \quad (\text{D51})$$

$$\text{tr}(\not{a}\not{b}) = (a \cdot b)\text{tr}(\mathbf{1}) = 4(a \cdot b), \quad (\text{D52})$$

$$\text{tr}(\not{a}\not{b}\not{c}\not{d}) = 4[(a \cdot b)(c \cdot d) - (a \cdot c)(b \cdot d) + (a \cdot d)(b \cdot c)], \quad (\text{D53})$$

$$\text{tr}(\gamma_5) = 0, \quad (\text{D54})$$

$$\text{tr}(\not{a}\not{b}\gamma_5) = 0, \quad (\text{D55})$$

$$\text{tr}(\not{a}\not{b}\not{c}\not{d}\gamma_5) = -4i\epsilon^{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma. \quad (\text{D56})$$

8. Properties of Dirac fermions

The positive frequency solution $u^s(p)$ of the Dirac equation satisfies

$$(\not{p} - m)u^s(p) = 0, \quad (\text{D57})$$

while the negative frequency solution $v^s(p)$ satisfies

$$(-\not{p} - m)v^s(p) = 0, \quad (\text{D58})$$

where p is the four-momentum, and s denotes the spin eigenvalue. From the normalization condition of the solution $u^s(p)$

$$\bar{u}^s(p)u^{s'}(p) = 2m\delta_{ss'}, \quad (\text{D59})$$

the orthonormality follows

$$\bar{v}^s(p)v^{s'}(p) = -2m\delta_{ss'}, \quad (\text{D60})$$

$$\bar{u}^s(p)v^{s'}(p) = \bar{v}^s(p)u^{s'}(p) = 0, \quad (\text{D61})$$

$$u^{s\dagger}(p)v^{s'}(-p) = v^{s\dagger}(p)u^{s'}(-p) = 0, \quad (\text{D62})$$

$$\bar{u}^s(p)\gamma^\mu u^{s'}(p) = \bar{v}^s(p)\gamma^\mu v^{s'}(p) = 2p^\mu\delta_{ss'}. \quad (\text{D63})$$

Also, $u^{s=\pm}(p)$ and $v^{s=\pm}(p)$ form complete two-dimensional solution spaces satisfying $\not{p} = m$ and $-\not{p} = m$, respectively. The projection operators onto these solution spaces are given by $(\pm\not{p} + m)/2m$, which yield

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \quad (\text{D64})$$

$$\sum_s v^s(p)\bar{v}^s(p) = \not{p} - m. \quad (\text{D65})$$

The proportionality coefficients are determined by taking into account the normalization conditions given in Eqs. (D59) and (D60).

9. Formulas for Feynman integrals

a. Feynman parametrization

Feynman parametrization is given by

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2}, \quad (\text{D66})$$

$$\begin{aligned} \frac{1}{abc} &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{2\delta(x+y+z-1)}{(ax+by+cz)^3} \\ &= \int_0^1 dx \int_0^{1-x} dy \frac{2}{[ax+by+c(1-x-y)]^3}. \end{aligned} \quad (\text{D67})$$

b. Momentum integrals with dimensional regularization

Momentum integrals with dimensional regularization are given by

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{(m^2 + 2k \cdot p - k^2)^\alpha} = \frac{i\Gamma(\alpha - \frac{n}{2})}{(4\pi)^{\frac{n}{2}}\Gamma(\alpha)} \frac{1}{(m^2 + p^2)^{\alpha - \frac{n}{2}}}, \quad (\text{D68})$$

$$\int \frac{d^n k}{(2\pi)^n} \frac{k^\mu}{(m^2 + 2k \cdot p - k^2)^\alpha} = \frac{i\Gamma(\alpha - \frac{n}{2})}{(4\pi)^{\frac{n}{2}}\Gamma(\alpha)} \frac{p^\mu}{(m^2 + p^2)^{\alpha - \frac{n}{2}}}. \quad (\text{D69})$$

Setting $\varepsilon \equiv (4 - n)/2 \ll 1$, the gamma function gives

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + \frac{1}{2} \left(\gamma^2 + \frac{\pi^2}{6} \right) \varepsilon + \mathcal{O}(\varepsilon^2), \quad (\text{D70})$$

where γ is the Euler-Mascheroni constant.

APPENDIX E: DERIVATION OF THE MAXIMAL CP ASYMMETRY $|\varepsilon_1|^{\max}$

From the Lagrangian (2), the neutrino masses are given by [61]

$$\mathcal{L}_m = -\frac{M_{\alpha\beta}}{2} \bar{N}_\alpha^c N_\beta - y_{\alpha\beta} \phi^* \bar{l}_\alpha \bar{N}_\beta + \text{H.c.}, \quad (\text{E1})$$

where M is the heavy neutrino mass matrix. Here, $(m_D)_{\alpha\beta} = y_{\alpha\beta} v$ from the Higgs mechanism, where m_D is the Dirac mass matrix, $y_{\alpha\beta}$ is the Yukawa coupling, and $v = \langle \phi \rangle$ is the VEV of the Higgs field ϕ . We adopt the mass eigenstate basis for the heavy neutrinos, in which M is diagonal with real positive eigenvalues $M_1 \leq M_2 \leq M_3$.

Through the seesaw mechanism, the light neutrino mass matrix m_ν can be expressed in terms of m_D and the inverse of M ,

$$m_\nu = -m_D \frac{1}{M} m_D^T, \quad (\text{E2})$$

where higher order terms in $1/M$ are neglected. m_ν can be diagonalized by a unitary matrix $U^{(\nu)}$

$$U^{(\nu)\dagger} m_\nu U^{(\nu)*} = - \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \equiv -D_m, \quad (\text{E3})$$

with real positive eigenvalues $m_1 \leq m_2 \leq m_3$. Substituting Eq. (E2) into Eq. (E3), we get

$$\begin{aligned} U^{(\nu)\dagger} (-m_D M^{-1} m_D^T) U^{(\nu)*} \\ = -U^{(\nu)\dagger} (y v) D_M^{-1} (y v)^T U^{(\nu)*} \\ = -v^2 U^{(\nu)\dagger} y D_M^{-1} y^T U^{(\nu)*} = -D_m, \end{aligned} \quad (\text{E4})$$

where D_M is the diagonalized heavy mass matrix M . Equation (E4) means that

$$\Omega \equiv v D_M^{-1/2} U^{(\nu)\dagger} y D_M^{-1/2} \quad (\text{E5})$$

is an orthogonal matrix $\Omega \Omega^T = \mathbf{1}$. This implies that $\text{Im}(\Omega^T \Omega)_{11} = 0$, which yields

$$\begin{aligned} 0 &= \text{Im}(\Omega^T \Omega)_{11} \\ &= \text{Im} \left(v D_M^{-1/2} y^T U^{(\nu)*} D_M^{-1/2} \cdot v D_M^{-1/2} U^{(\nu)\dagger} y D_M^{-1/2} \right)_{11} \\ &= v^2 \frac{1}{M_1} \text{Im} \left(D_M^{-1} y^T U^{(\nu)*} \cdot U^{(\nu)\dagger} y \right)_{11} \\ &= v^2 \frac{1}{M_1} \sum_{\alpha=1,2,3} \frac{1}{m_\alpha} \text{Im}(U^{(\nu)\dagger} y)_{\alpha 1}^2; \end{aligned} \quad (\text{E6})$$

that is,

$$\frac{1}{m_1} \text{Im}(U^{(\nu)\dagger} y)_{11}^2 = - \sum_{\alpha \neq 1} \frac{1}{m_\alpha} \text{Im}(U^{(\nu)\dagger} y)_{\alpha 1}^2. \quad (\text{E7})$$

On the other hand, the CP asymmetry ε_1 is obtained from Eq. (D41)

$$\varepsilon_1 \simeq -\frac{3}{16\pi} \frac{M_1}{(y^\dagger y)_{11}} \text{Im} \left(y^\dagger y \frac{1}{M} y^T y^* \right)_{11}, \quad (\text{E8})$$

where M is the heavy neutrino mass matrix. Substituting Eqs. (E3) and (E7) into (E8), and expressing it in terms of

$$\tilde{y} = U^{(\nu)\dagger} y \quad (\text{E9})$$

instead of y , we obtain

$$\begin{aligned} \text{Im} \left(\tilde{y}^\dagger \tilde{y} \frac{1}{M} \tilde{y}^T \tilde{y}^* \right)_{11} &= -\frac{1}{v^2} \text{Im}(\tilde{y}^\dagger U^{(\nu)\dagger} m_\nu U^{(\nu)*} \tilde{y}^*)_{11} \\ &= -\frac{m_1}{v^2} \text{Im}(\tilde{y}_{11}^2) - \sum_{\alpha \neq 1} \frac{m_\alpha}{v^2} \text{Im}(\tilde{y}_{\alpha 1}^2) \\ &= -\frac{1}{v^2} \sum_{\alpha \neq 1} \frac{\Delta m_{\alpha 1}^2}{m_\alpha} \text{Im}(\tilde{y}_{\alpha 1}^2), \end{aligned} \quad (\text{E10})$$

which translates into

$$\varepsilon_1 = \frac{3}{16\pi} \frac{M_1}{v^2} \sum_{\alpha \neq 1} \frac{\Delta m_{\alpha 1}^2}{m_\alpha} \frac{\text{Im}(\tilde{y}_{\alpha 1}^2)}{(\tilde{y}^\dagger \tilde{y})_{11}}, \quad (\text{E11})$$

where $\Delta m_{\alpha 1}^2 \equiv m_\alpha^2 - m_1^2$.

Now, consider the normalized Yukawa couplings [46]

$$z_\alpha = \frac{\tilde{y}_{\alpha 1}^2}{(\tilde{y}^\dagger \tilde{y})_{11}} = X_\alpha + iY_\alpha, \quad (\text{E12})$$

which follow

$$0 \leq |z_\alpha| \leq 1, \quad \sum_\alpha |z_\alpha| = 1. \quad (\text{E13})$$

From the orthogonality condition $(\Omega^\dagger \Omega)_{11} = 1$, the additional constraint is given by

$$\sum_\alpha \frac{\tilde{m}_1}{m_\alpha} z_\alpha = 1, \quad (\text{E14})$$

where $\tilde{m}_1$ is the effective neutrino mass (37). From Eq. (E11), the CP asymmetry reads in the new variables

$$\varepsilon_1 = \frac{3}{16\pi} \frac{M_1}{v^2} \left(\frac{\Delta m_{21}^2}{m_2} Y_2 + \frac{\Delta m_{31}^2}{m_3} Y_3 \right). \quad (\text{E15})$$

Since $m_3 > m_2$,

$$\begin{aligned} \frac{\Delta m_{31}^2}{m_3} - \frac{\Delta m_{21}^2}{m_2} &= \frac{m_3^2 - m_1^2}{m_3} - \frac{m_2^2 - m_1^2}{m_2} \\ &= \frac{(m_3 - m_2)(m_1^2 + m_2 m_3)}{m_2 m_3} > 0 \\ &\Rightarrow \frac{\Delta m_{31}^2}{m_3} > \frac{\Delta m_{21}^2}{m_2}. \end{aligned} \quad (\text{E16})$$

From this, we can presume that the maximal CP asymmetry is achieved when $|Y_3|$ is maximal.

The additional condition (E14) yields

$$\frac{Y_1}{m_1} + \frac{Y_2}{m_2} + \frac{Y_3}{m_3} = 0, \quad (\text{E17})$$

$$\frac{\tilde{m}_1}{m_1} X_1 + \frac{\tilde{m}_1}{m_2} X_2 + \frac{\tilde{m}_1}{m_3} X_3 = 1. \quad (\text{E18})$$

Since $m_3 > m_2 > m_1$, we can set $X_2 = X_3 = Y_2 = 0$ for maximal $|Y_3|$. Then, it follows that

$$Y_1 = -\frac{m_1}{m_3} Y_3, \quad X_1 = \frac{m_1}{\tilde{m}_1}. \quad (\text{E19})$$

The second equation in (E13) gives

$$\sqrt{X_1^2 + Y_1^2} + |Y_3| = 1. \quad (\text{E20})$$

The conditions (E19) and (E20) determine $|Y_3|$ as a function of m_1 , m_3 , and $\tilde{m}_1$:

$$\begin{aligned} \left(1 - \frac{m_1^2}{m_3^2}\right) |Y_3|^2 - 2|Y_3| + 1 - \frac{m_1^2}{\tilde{m}_1^2} &= 0 \\ \Rightarrow |Y_3| &= 1 - \frac{m_1}{m_3} \sqrt{1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2}}, \end{aligned} \quad (\text{E21})$$

where $|Y_3| < 1$ is taken into account. Substituting $Y_2 = 0$ and Eq. (E21) into Eq. (E15) yields

$$|\varepsilon_1|^{\max} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \left[1 - \frac{m_1}{m_3} \left(1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2} \right)^{1/2} \right], \quad (\text{E22})$$

where $m_3^2 - m_1^2 < m_3^2$ is used. This gives the upper bound for $|\varepsilon_1|$.

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