

Robust determination of antinuclei production from dark matter via weakly decaying beauty hadrons

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Recently, the Alpha Magnetic Spectrometer (AMS-02) Collaboration presented tentative evidence for the detection of cosmic antihelion-3 ($\overline{^3\text{He}}$) events, alongside a comparable number of antideuterons ($\overline{\text{D}}$). If confirmed, these observations could revolutionize our understanding of cosmic-ray production and propagation and/or serve as compelling indirect evidence for dark matter. Given that the detection of cosmic $\overline{\text{D}}$ is already at the limit of AMS-02 sensitivity, explaining the observation of $\overline{^3\text{He}}$ even within the standard coalescence framework poses a significant challenge. It has recently been shown that a previously overlooked mechanism within the Standard Model of particle physics—namely, the production of antihelion via the displaced-vertex decay of $\overline{\Lambda}_b^0$ baryons—could substantially enhance the $\overline{^3\text{He}}$ flux arising from dark matter-induced processes. In light of these challenges, we present a tuning of Pythia that is consistent with LEP data on the fragmentation function of b quarks into b hadrons—a critical factor for determining the $\overline{\Lambda}_b^0$ multiplicity—and with ALICE and ALEPH data for the $\overline{\text{D}}$ and $\overline{^3\text{He}}$ spectra, which we employ to determine our coalescence model. Our refined Pythia tuning, in conjunction with our coalescence model, results in a predicted branching ratio for the production of $\overline{^3\text{He}}$ from $\overline{\Lambda}_b^0$ decays that is consistent with the recent upper limit measured by LHCb. Furthermore, our prediction indicates that the contribution of $\overline{\text{D}}$ and $\overline{^3\text{He}}$ from beauty-hadron decays is negligible relative to the direct production from hadronization.

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I. INTRODUCTION

The particle nature of dark matter (DM) remains unknown despite decades of extensive theoretical and experimental efforts. Well-motivated DM models have spurred a comprehensive search program that includes indirect detection, direct detection, and collider experiments [1]. Indirect detection seeks to identify DM signals through the flux of cosmic messengers—such as positrons, antiprotons, γ rays, and neutrinos [2,3]—although these fluxes are frequently dominated by astrophysical sources, complicating the identification of any DM contribution (see, e.g., Refs. [4–9]).

Cosmic antinuclei from DM annihilation or decay offer a promising alternative. In particular, antideuterons ($\overline{\text{D}}$) [10] and, to a lesser extent, antihelions ($\overline{^3\text{He}}$) [11,12] are attractive search channels because their astrophysical backgrounds are significantly suppressed for kinetic energies (K) below 1 GeV/nucleon. In scenarios where weakly interacting massive particles (WIMPs) annihilate in the Galactic halo, the predicted $\overline{\text{D}}$ flux at $K = 0.1$ –1 GeV/nucleon exceeds that from secondary production by at least 1 order of magnitude (see, e.g., Refs. [10,13–18]). Thus, the observation of even a few low-energy cosmic $\overline{\text{D}}$ events could serve as a compelling DM signature [19].

So far, no firm detection of cosmic $\overline{\text{D}}$ has been reported. The strongest limit comes from the BESS experiment, which sets an upper limit of $6.7 \times 10^{-5} \text{ (m}^2 \text{ s sr GeV/n)}^{-1}$ for $K = 0.163$ –1.100 GeV/n [20]. Nevertheless, experiments such as the Alpha Magnetic Spectrometer (AMS-02) aboard the International Space Station [21] and the upcoming General AntiParticle Spectrometer (GAPS) mission [22] are expected to significantly improve antinuclei detection sensitivity, reaching levels as low as about $2 \times 10^{-6} \text{ (m}^2 \text{ s sr GeV/n)}^{-1}$ for $K < 1$ GeV/n [19].

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AMS-02 has tentatively detected about a dozen $\overline{^3\text{He}}$ and ^4He events, as well as a few candidates with masses consistent with $\bar{D}$ [23–25]. The observation of roughly similar fluxes for $\bar{D}$, $\overline{^3\text{He}}$, and ^4He is unexpected since kinematic constraints imply that the formation probability drops drastically for each additional antinucleon added in the nucleus. For instance, using our coalescence model implemented in Pythia 8, we obtain for a 50 GeV DM candidate annihilating into $b\bar{b}$ the following multiplicity ratios:

$$\bar{p}:\bar{D}:\overline{^3\text{He}} \sim 1:1.4 \times 10^{-4}:3.4 \times 10^{-8}. \quad (1)$$

The authors of Ref. [26] have proposed that a significant fraction of the $\overline{^3\text{He}}$ flux from DM annihilation could result from the decays of $\bar{\Lambda}_b^0$ baryons, which are predominantly produced in DM annihilation channels involving $b\bar{b}$ quarks. Their decays efficiently produce multi-antinucleon states with small relative momenta, favoring the formation of antideuterons and, in particular, antihelions. In addition to $\bar{\Lambda}_b^0$, other weakly decaying b baryons—such as Σ_b^0 , $\Sigma_b^\pm$, Ξ_b^0 , $\Xi_b^\pm$, and Ω_b^- —are produced in e^+e^- and pp collisions with multiplicities that are approximately a factor of 10 smaller with respect to the $\bar{\Lambda}_b^0$.

In Ref. [26], a specific tune of the Monte Carlo (MC) particle generator Pythia (hereafter the WL21 tune) was used to match the fragmentation function $f(b \rightarrow \Lambda_b^0)$ measured at LEP. Concerning our analysis, this parameter, together with the decay channels of the $\bar{\Lambda}_b^0$, is paramount for accurately modeling the rate of antinuclei production from the decays of b baryons. Their approach leads to an increased yield of $\overline{^3\text{He}}$ from weakly decaying b baryons by about a factor of 10 compared to the default Pythia tune, suggesting an Earth-bound $\overline{^3\text{He}}$ flux that could potentially be detectable by AMS-02.

In this paper, we provide a systematic assessment of antinuclei production rates from the decays of beauty baryons and mesons. We adopt a phenomenologically viable coalescence model from Ref. [27] that successfully explains the ALEPH measurement of $\bar{D}$ multiplicity [28] and the ALICE energy spectrum of $\overline{^3\text{He}}$ [29]. By constructing a dedicated Pythia tuning that adjusts hadronization parameters, flavor selection parameters, and the unmeasured decay branching ratios of $\bar{\Lambda}_b^0$, we achieve $\overline{^3\text{He}}$ production rates that comply with the recent LHCb upper limit. Taking all these factors into account, we demonstrate that the enhancement of antinuclei production via b -baryon decays is negligible compared to prompt production.

II. CURRENT SITUATION

The analysis presented in Ref. [26] relies on a set of assumptions that may compromise the robustness of its

conclusions and therefore merit critical reassessment. Some of the critical points have also been pointed out in Ref. [30].¹

- (i) The increase of the Pythia `StringFlav:probQQtoQ` parameter (hereafter `probQQtoQ`)² by a factor of 3 relative to its default value results in an unrealistic overproduction of protons, antiprotons, and other baryons. It even affects important QCD observables at LEP, such as the Thrust and C parameters. In particular, the WL21 tune predicts a multiplicity for $\bar{p} + p$ at the Z resonance of 2.36, which is about 40σ above the updated combination reported by the PDG, $\langle n_{p+\bar{p}} \rangle = 1.050 \pm 0.032$ [32]. This discrepancy would dramatically increase the predicted rates of antinuclei from the direct production of antinucleons generated in hadronization or resonance decays (also known as the prompt mechanism). In order to compensate for this effect, the authors of Ref. [31] have significantly decreased the coalescence momentum from a typically used value of 0.18–0.20 GeV [27] to 0.124 GeV.
- (ii) The WL21 tune predicts a multiplicity of $\Lambda_b^0 + \bar{\Lambda}_b^0$ (denoted as $\langle n_{\Lambda_b^0} \rangle$) at the Z resonance of 0.044, while the corresponding DELPHI measurement is 0.031 ± 0.016 [33]. In contrast, the default Monash tune [34] of Pythia 8 predicts $\langle n_{\Lambda_b^0} \rangle = 0.016$. Both the WL21 and the default Monash tunes are compatible within 1σ with the LEP measurement while they are a factor of almost 3 different. This suggests that given the LEP data, a significant increase of `probQQtoQ` with respect to the Pythia 8 default seems not to be statistically required by the b -baryon multiplicity measurements.
- (iii) Reference [26] used an estimate of the b -quark fragmentation function into b -baryons of $0.1^{+0.04}_{-0.03}$ taken from a 1998 version of the Review of Particle Physics [35], which was mainly tuned to LEP data. A much more recent and precise estimate from the HFLAV Collaboration, also based on LEP data [36],³ gives

¹The authors of Ref. [26] have replied to those comments in Ref. [31].

²This parameter models the suppression of diquark production relative to quark production and its default value is 0.081. It affects not only the rates of Λ_b but also those of all baryons, such as p , n , Λ_b^0 , etc. The rate of baryon production in Pythia is directly proportional to the value of this parameter, i.e., a larger value implies a higher baryon production rate.

³A larger value of $f(b \rightarrow \Lambda_b) = 0.198 \pm 0.046$ is found when using only Tevatron data [36]. The difference relative to the LEP measurement is likely due to the more complex QCD environment in hadronic collisions, as opposed to the cleaner e^+e^- production. In the remainder of the paper, we use the estimate obtained from LEP data because the DM annihilation process in the WIMP scenario is expected to resemble the electroweak processes at LEP.

$$f(b \rightarrow \Lambda_b) = 0.089 \pm 0.012.$$

The WL21 tune predicts $f(b \rightarrow \Lambda_b^0) \approx 0.12$, which is about 2.6σ higher than this recent estimate. Again, an increase by a factor of 3 for `probQQtoQ` does not seem necessary.

- (iv) The model in Ref. [26] uses the default Pythia 8 assumptions for the $\bar{\Lambda}_b^0$ decay modes. This leads to a branching ratio

$$\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{^3\text{He}} X) \simeq 1.5 \times 10^{-6},$$

which is severely excluded by the LHCb Collaboration, who measured a 90% CL upper limit of 6.3×10^{-8} for the same process [37].

III. TUNING OF THE HADRONIZATION MODEL

The default Pythia 8 setup reproduces the DELPHI mean multiplicity within 1σ [33] but predicts a b -baryon multiplicity that is 3σ too low compared to the $f(b \rightarrow \Lambda_b)$ measurement [36]. Although increasing `probQQtoQ` can partly address this, it predicts too high baryon multiplicities and it affects meson multiplicities as well.⁴ Instead of compensating for this effect with a reduced coalescence parameter, as done in [26], we propose to first tune Pythia 8 to match baryon production measured at LEP, then tune the coalescence model based on antinuclei yields, ensuring compatibility across all datasets.

We therefore perform a comprehensive tune of Pythia 8 parameters related directly to both hadronization and flavor selection, using all the relevant measurements from LEP, SLD, PDG, and HFLAV. For this task, we use Pythia 8.311 [38] to generate MC samples, Rivet 3.1.6 [39] for the experimental analyses at the particle level, and Professor 2.4.0 [40] for the tuning of the parameters. The total number of measurement bins is 4185. The tuning setup is similar to that used in earlier studies (see, e.g., Refs. [41–44]). The Professor toolkit is based on a method which permits the optimization of all the parameters using analytical expressions that are cast as polynomials whose coefficients are determined by fitting MC simulated predictions generated for a set of randomly selected parameter points. The optimal values of the parameters are then obtained with the help of a standard χ^2

⁴It is worth mentioning that increasing the value of `probQQtoQ` does not increase both the differential and total rates of mesons. For instance, increasing `probQQtoQ` by a factor of 3, while leading to an increase of $p/\bar{p}$ and Λ^0 multiplicities by about 120%, it decreases the multiplicities of, e.g., $\pi^\pm$ and π^0 by about 11% and 13%, respectively. The average multiplicity of charged and neutral hadrons slightly decreases since it is dominated by the contribution of mesons. We must note that these changes on the multiplicities and differential spectra caused by the change on `probQQtoQ` are not in agreement with the LEP results.

TABLE I. Pythia 8 parameters subject to this tune which can be split into hadronization function parameters (those starting with `StringZ:`) and flavor selection parameters (those starting with `StringFlav:`). We show their default values obtained in the Monash tune and the results of this study along with their MIGRAD errors.

Parameter	Monash	Had. tune
<code>StringZ:aLund</code>	0.68	0.7832 ± 0.0123
<code>StringZ:bLund</code>	0.98	1.1729 ± 0.0100
<code>StringZ:aExtraDiquark</code>	0.97	0.9251 ± 0.0175
<code>StringFlav:ProbStoUD</code>	0.217	0.2265 ± 0.0016
<code>StringFlav:mesonUDvector</code>	0.50	0.6655 ± 0.0152
<code>StringFlav:mesonSvector</code>	0.55	0.5842 ± 0.0177
<code>StringFlav:etaSup</code>	0.60	0.6499 ± 0.0005
<code>StringFlav:etaPrimeSup</code>	0.12	0.1778 ± 0.0037
<code>StringFlav:probQQtoQ</code>	0.081	0.1112 ± 0.0008
<code>StringFlav:probSQtoQQ</code>	0.915	0.9791 ± 0.0061
<code>StringFlav:probQQ1toQQ0</code>	0.0275	0.8761 ± 0.0171
<code>StringFlav:popcornSpair</code>	0.50	0.6108 ± 0.0241
<code>StringFlav:popcornSmeson</code>	0.90	0.8306 ± 0.0231
<code>StringFlav:popcornRate</code>	0.50	0.4117 ± 0.0055

minimization using Minuit [45]. In this study, we tune 14 parameters listed in Table I and thus we generated MC samples for 6000 random points in the parameter space. The best-fit parameters are determined by minimizing the following χ^2 function:

$$\chi^2 = \sum_{i,j} (\mathcal{O}_i - \mathcal{O}_i^{\text{exp}}) \frac{1}{\sigma_{ij}^2} (\mathcal{O}_j - \mathcal{O}_j^{\text{exp}}), \quad (2)$$

with $\mathcal{O}_i$ being the MC prediction for the observable i which is cast here as a third order polynomial in the parameters $\mathbf{x} = \{x_1, \dots, x_{14}\}$; $\mathcal{O}_i \equiv \sum_{i+j+k \leq 3} c_{ijk} x_\alpha^i x_\beta^j x_\gamma^k$, and the sum is over all the measurements being included. In Eq. (2), σ_{ij} represents the covariance matrix which is assumed to be diagonal, i.e., $\sigma_{ij} = \sigma_i \delta_{ij}$ where σ_i is the total error on the observable $\mathcal{O}_i$ (see, e.g., Ref. [43] for more details).

To build a good hadronization model we include all the possible measurements of event shapes, charged multiplicities, spectra of baryons and mesons, multiplicities of identified particles including vector mesons, and angular correlations between Λ baryons. In this work, we use measurements performed at the Z-pole by ALEPH [46–52], DELPHI [33,53–63], HFLAV [36], L3 [64–68], OPAL [69–80], PDG [32], and SLD [81,82] totaling 4185 bins. The results of the tuning are shown in Table I where we can see that the best-fit value of `probQQtoQ` is 0.111 which yields a deviation of $1 - 1.5\sigma$ from the HFLAV estimation of the b -baryon fragmentation function but only 1σ away from the DELPHI measurement of $\langle n_{\Lambda_b^0} \rangle$. The corresponding goodness-of-fit per degrees of freedom at the minimum is $\chi^2/N_{\text{df}} = 4943.16/4171 \approx 1.18$. We also estimate the uncertainties on the tuned parameters using the eigentunes

method, which is based on the diagonalization of the Hessian matrix near the minimum (see Ref. [83]). In essence, the uncertainties are estimated by diagonalizing the covariance matrix near the minimum, best-fit region. The second-order term of the expansion of $\Delta\chi^2$ distribution, $H_{ij} = \partial^2\chi^2/\partial x_i\partial x_j$ is called the Hessian matrix whose diagonal form leads to the principal directions or eigenvectors and the corresponding poles are the eigenvalues. Therefore, for a set of N_{params} we get $2 \times N_{\text{params}}$ variations, which are 28 variations in one our case. The size of the variations is penalized by a constraint on the corresponding hypersphere of $2 \times N_{\text{params}}$ dimensions called the tolerance (see Ref. [43] for a detailed discussion). A one-sigma uncertainty (eigentunes) is found for $\Delta\chi^2/N_{\text{df}} = 1$ while a 2σ eigentunes corresponds to $\Delta\chi^2 = 4$ where $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$. The comparison of our results for the mean multiplicities of Λ , Λ_b^0 , $p/\bar{p}$, and $\pi^\pm$ with experimental measurements is shown in Fig. 1. We observe that the best-fit value for $f(b \rightarrow \Lambda_b)$ obtained from our tune is about 2σ smaller than the HFLAV estimate, while all the other predictions are fully compatible with the LEP data. The difference is only at 1σ if we consider also the error on $f(b \rightarrow \Lambda_b)$ coming from the fit we perform (orange band in Fig. 1). For the multiplicity of $\Lambda_b^0 + \bar{\Lambda}_b^0$, we obtain

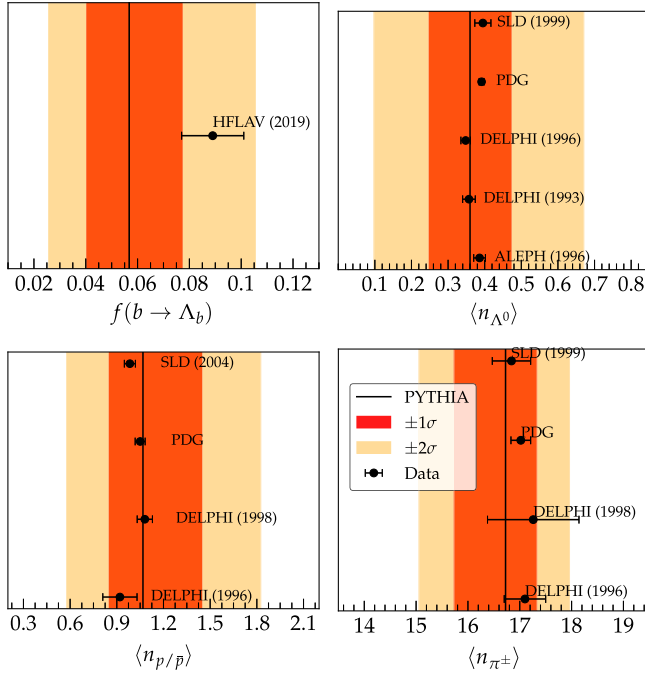


FIG. 1. Comparison between the theory predictions obtained at the best-fit parameter point of Pythia 8 with the Had.tune and the experimental values for $f(b \rightarrow \Lambda_b)$ (upper left), the multiplicity of Λ^0 (upper right), the multiplicity of $p/\bar{p}$ (lower left), and the multiplicity of $\pi^\pm$ (lower right). The 1σ and 2σ theory uncertainties are shown as red and orange bands, respectively. Here Λ_b refers to all the b -baryons and not only Λ_b^0 . Data are taken from Refs. [32,33,36,48,54,61,81,82].

0.021, which is fully compatible with the DELPHI measurement, 0.031 ± 0.016 [33]. For the remainder of the paper, this tune will be referred to as Had.tune.

IV. COALESCENCE MODEL IMPLEMENTED

IN Pythia

A. Monte Carlo setup

In Ref. [27], some of the authors of this paper implemented four different coalescence models that consistently reproduce the results of ALEPH for the $\bar{D}$ multiplicity in Z-resonance hadronic decays [84]. Two of these models are very simple, taking into account either only the difference in momentum or both the difference in momentum and the spatial separation between antinucleons. The other two models are more sophisticated, employing the Wigner formalism with either a Gaussian or Argonne wave function to account for possible antinucleon correlations in space and momentum. The model we use, inspired by Ref. [27], is a fully MC-based approach implemented in Pythia 8.311. For each event, we select all the pairs of $\bar{n}$ and $\bar{p}$ present in the event list and determine their difference of momenta Δp and of distance Δr calculated in the reference frame of the $\bar{n}$ and $\bar{p}$ pair. Then, we apply the coalescence criteria discussed in the manuscript main text, i.e., we test if $\Delta r < 3$ fm and $\Delta p < p_c$, where p_c is the coalescence momentum. If the coalescence criterion is satisfied for a pair of $\bar{n}$ and $\bar{p}$, we assume that the $\bar{D}$ is formed and we calculate its kinetic energy in the center-of-mass reference frame. Once the simulations are finished, we calculate the spectrum from DM annihilations as

$$\frac{dN_{\text{DM}}}{dK_i} = \frac{N_i(K \in [K_i, K_i + \Delta K])}{\Delta K}, \quad (3)$$

where dN/dK_i represents the spectrum evaluated for the i th bin with kinetic energy between $[K_i, K_i + \Delta K]$. Instead, for pp collisions we select only the events detected at midrapidity, i.e., with $|y| < 0.5$ (following Ref. [29]), and evaluate the spectra as

$$\frac{dN_{\text{pp}}}{p_{T,i}} = \frac{N_i(K \in [p_{T,i}, p_{T,i} + \Delta p_T])}{\Delta p_T}, \quad (4)$$

where $dN/dp_{T,i}$ represents the spectrum evaluated for the i th bin with kinetic energy between $[p_{T,i}, p_{T,i} + \Delta p_T]$.

As a proof of principle, we adopt in this paper a simple coalescence model that imposes criteria on the momentum difference ($\Delta p < p_c$) and the spatial separation (Δr) in the center-of-mass frame of the $\bar{p}$ - $\bar{n}$ system.⁵ Including the

⁵Reference [27] shows that very similar $\bar{D}$ spectra are obtained when using different coalescence models that are properly tuned on ALEPH data [28]. Therefore, the conclusions of this paper will not change if we consider the Wigner formalism.

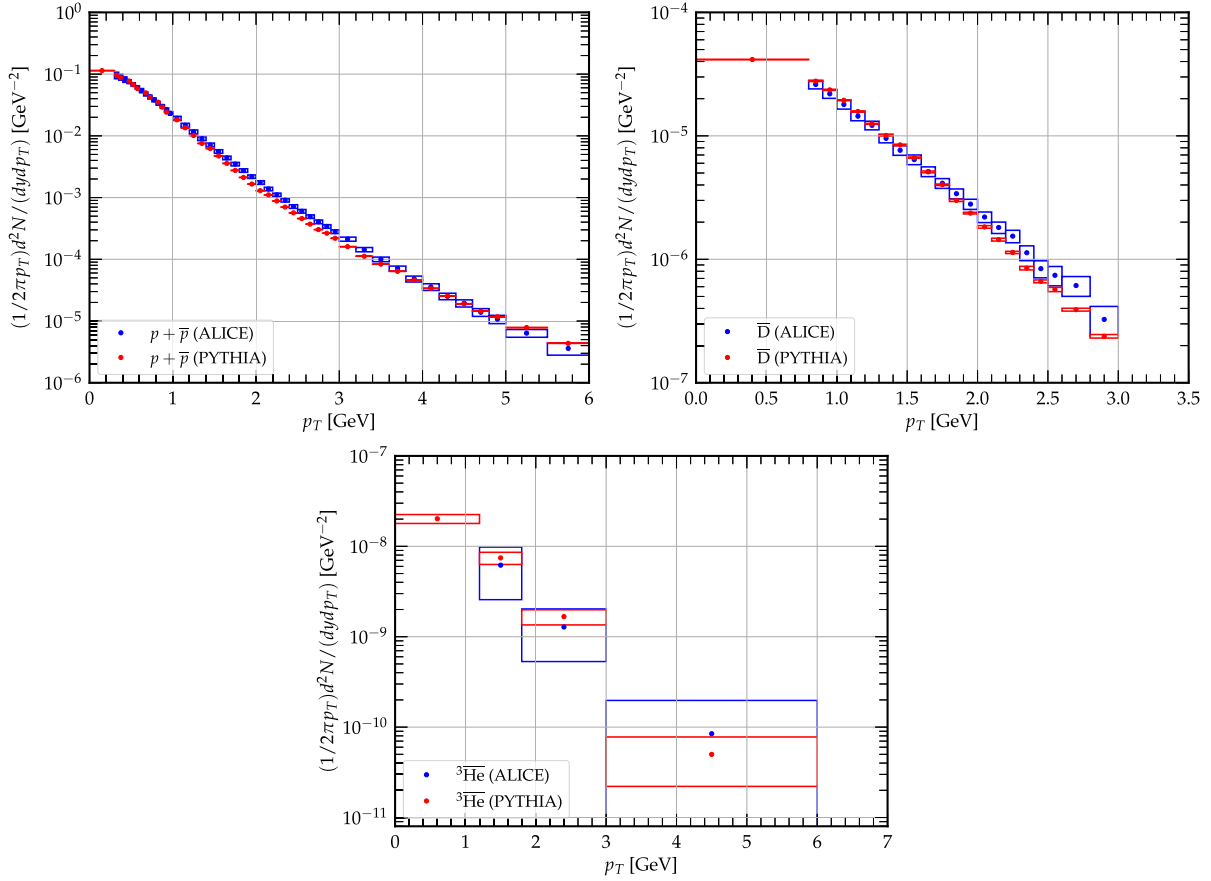


FIG. 2. Source spectra for the production of $p + \bar{p}$ (top left), $\bar{D}$ (top right panel), and $\bar{^3\text{He}}$ (bottom panel) from the pp collision at $\sqrt{s} = 7$ TeV obtained with our tune of Pythia and our coalescence model, for our best-fit value of the coalescence momentum $p_c = 0.20$ GeV. We also show the ALICE data taken at the same center-of-mass energy from [29].

criterion related to the $\bar{n}$ - $\bar{p}$ distance is particularly important for antinuclei generated from off-vertex particle decays.

B. Tuning of the coalescence model for $\bar{^3\text{He}}$ production

One of the main goals of the paper is to predict the spectrum of $\bar{^3\text{He}}$ and the possible contribution of the b -baryon weak decays to its yield. However, we cannot use $e^\pm$ data, which are relevant for predictions of particle produced from DM annihilations, because no $\bar{^3\text{He}}$ event has ever been measured by any LEP experiment. Therefore, we decide to calibrate the coalescence model for the rate of $\bar{^3\text{He}}$ by using the measurement of ALICE in pp collisions at $\sqrt{s} = 7$ TeV [29] and check *a posteriori* if the coalescence model found is compatible with the one found in Ref. [27] for $\bar{D}$ production at ALEPH.

The Pythia tunes available for the production of baryons from pp collisions are different from the one generated for $e^\pm$. We thus have to use for pp collisions a Pythia tune that is different from the one we found by fitting LEP data.

We thus derive first a specific hadronization model setup that properly predicts the proton yield measured by ALICE. We find that by using the default Monash tune—Tune:pp=4—and the following parameter choices: StringZ:aLund=0.360, StringZ:bLund=0.560, and StringFlav:ProbStoUD=0.200, the predicted proton multiplicity is 1σ compatible with the ALICE measurement [29]. In the top left panel of Fig. 2 we show the comparison of our ALICE tune and the data for $p + \bar{p}$.

Once we calibrate the antiprotons spectra, we can refine our coalescence model. In order to do this, we fit the ALICE $\bar{^3\text{He}}$ yield finding that for $\Delta r < 3$ fm the best-fit coalescence momentum is 0.20 ± 0.01 GeV. Note that this value is compatible within 1σ with the value of the coalescence momentum we obtained using the ALEPH data [27] that was 0.21 ± 0.02 GeV. We show in Fig. 2 the comparison of the $\bar{D}$ and $\bar{^3\text{He}}$ spectra with ALICE data. It is quite promising that the coalescence model used to fit the $\bar{^3\text{He}}$ data also works remarkably well to predict the right yield $\bar{D}$. The value of $p_c = 0.20$ GeV found with the fit to the $\bar{^3\text{He}}$ spectra from ALICE data is used in the paper.

V. b -BARYON AND B -MESON FRAGMENTATION FUNCTIONS

As we will show in the results section, the production of antideuterons from weakly decaying B mesons is roughly as important as that from b baryons. B mesons—namely, B^0 and $B^\pm$ particles—are produced with a multiplicity that is 2 to 3 times higher than that of b baryons (see, e.g., Ref. [85]). Therefore, we take into account not only the b -baryon fragmentation function, $f(b \rightarrow \Lambda_b)$, but also the B -meson fragmentation functions, $f(b \rightarrow B^0, B_s^0)$, as reported by the HFLAV Collaboration [36].

When comparing the Had.tune model predictions for the fragmentation functions into weakly decaying b hadrons with the estimates from HFLAV [36], we find a discrepancy at the 2σ level when considering the tune prediction at the phase level and only the error coming from HFLAV estimate (see the top left panel in Fig. 1). This discrepancy can lead to an imprecise prediction for the $\bar{D}/^3\text{He}$ spectra from weakly decaying b hadrons. Therefore, we refine the value of `probQQtoQ` to best match the measured $f(b \rightarrow \Lambda_b, B^0, B_s^0)$ specifically for antinuclei generated from b hadrons.⁶ This further refinement of `probQQtoQ` effectively just influence the number of b hadrons produced and their decays into nucleons, which is what matters the most for our scope. To do this, we perform a simple χ^2 analysis comparing the predictions from Had.tune for the fragmentation functions with the HFLAV estimates. In the Had.tune model, we fix all parameters except for `probQQtoQ` and obtain a best-fit value of 0.19 ± 0.03 . Thanks to this refinement, our model becomes compatible with all b -hadron fragmentation functions within the 1σ uncertainties (see Fig. 3). In the rest of the Paper, we refer to this tuning as Had.tune + QQ.

VI. TUNING THE DECAY BRANCHING RATIOS OF $\bar{\Lambda}_b^0$

The models Had.tune and Had.tune+QQ predict branching ratios for $\text{BR}(\bar{\Lambda}_b^0 \rightarrow ^3\bar{\text{He}}X)$ of $(2.2 \pm 0.3) \times 10^{-7}$ and $(5.6 \pm 0.4) \times 10^{-7}$, respectively—about factors of 3 and 8 above the upper limit of 6.3×10^{-8} reported by LHCb [37]. These differences are less dramatic than those obtained in Ref. [26], which reports $\text{BR}(\bar{\Lambda}_b^0 \rightarrow ^3\bar{\text{He}}X) \simeq 1.5 \times 10^{-6}$.

The prediction of this branching ratio depends mainly on the coalescence model, the `probQQtoQ` value and the decay modes of the Λ_b^0 . The former two ingredients have been fixed already. Therefore, to improve our model and ensure consistency with the LHCb upper limit, we modify the tabulated branching ratios of Λ_b^0 into diquark modes in Pythia. In particular, Pythia assigns the main decay channel to a mode involving three quarks plus one diquark state.

⁶For the prompt production, we retain the Had.tune, which yields the correct hadron yields.

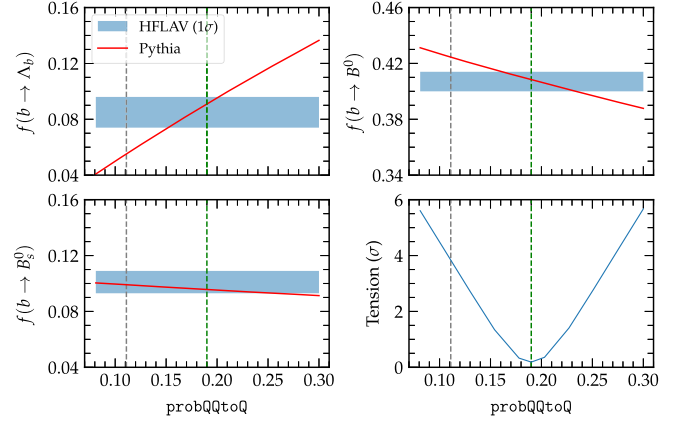


FIG. 3. Comparison between the fragmentation functions for Λ_b^0 (top left panel), B^0 (top right panel), and B_s^0 (bottom left panel) obtained with Pythia (red line) and those reported by HFLAV [36] (blue band, 1σ error) as a function of `probQQtoQ`. The bottom right panel shows the overall tension between Pythia and HFLAV (combining all considered fragmentation functions) with respect to `probQQtoQ`. The best-fit value of `probQQtoQ` from our hadronization tune is indicated by a gray dashed line, and the value that best fits the fragmentation functions is shown as a green dashed line.

The channel with the highest branching ratio is

$$\text{BR}(\Lambda_b^0 \rightarrow \bar{u}dc(ud)_0), \quad (5)$$

fixed at approximately 53%. In contrast, the channel most relevant for antinuclei production is

$$\text{BR}(\Lambda_b^0 \rightarrow \bar{u}du(ud)_0), \quad (6)$$

which occurs at around 1.2%. This difference is largely related to the magnitudes of the Cabibbo-Kobayashi-Maskawa matrix elements, $|V_{ub}|$ and $|V_{cb}|$, which control the transitions $\bar{b} \rightarrow \bar{u}$ and $\bar{b} \rightarrow \bar{c}$, respectively. In particular, $|V_{ub}|^2/|V_{cb}|^2 \approx 0.01$, about a factor of 2 smaller than the ratio $1.2/53 \approx 0.022$ implemented in Pythia. This difference is compensated by the fact that the allowed phase space prefers a higher multiplicity in $\bar{b} \rightarrow \bar{u}$ than in $\bar{b} \rightarrow \bar{c}$. The branching ratios in Eqs. (5) and (6) are however not measured by any experiment yet but can be *thought* as external input parameters in Pythia whose values are just guessed and thus subject to large theoretical uncertainties.

The second most important decay channel involves the production of leptons and charmed baryons via $\bar{\Lambda}_b^0 \rightarrow \Lambda_c^- \ell^+ \nu_\ell$. This decay is measured to occur with a branching fraction of $(6.2_{-1.3}^{+1.4})\%$ [86], while Pythia predicts a value of 5.96%, which is compatible.

The modifications we have applied to the Λ_b^0 decay modes have been dictated by the LHCb upper limit for the branching ratio of the process $\bar{\Lambda}_b^0 \rightarrow ^3\bar{\text{He}}X$, which is 6.3×10^{-8} . In Table II we report the default and the

TABLE II. Tune of the branching ratios of Λ_b^0 into diquark states. For the latter, the remaining branching ratios have been set to 0.

Λ_b^0 decay mode	Default Pythia	Tuned Pythia
$\bar{u}du(ud)_0$	0.012	0.001
$\bar{u}dc(ud)_0$	0.5321147	0.5321147
$\bar{c}su(ud)_0$	0.012	0
$\bar{c}sc(ud)_0$	0.08	0.103

modified Pythia values of the Λ_b^0 decay channels into diquark states. We can see that we only slightly modified the ones for the Λ_b^0 . The main change we applied is a reduction of the $\bar{u}du(ud)_0$ from the default 0.12 to 0.001. This is indeed the decay mode that contributes the most to the antineutrons production.

We show in Table III the most relevant Λ_b^0 decay channels with the measurements reported in the PDG [86] and the predictions from the default Pythia and our modified branching ratios. We can see that Pythia matches well the measurements except for the channel $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \Lambda_c^- p \bar{p} \pi^+)$ for which it predicts a very large value. As for the Λ_c^+ the default Pythia does not predict the right inclusive $\bar{p}/\bar{n}$ decays while our tune model for the branching ratios into quark-diquark states matches the data remarkably well. With the tuned values of the decay channels, shown in Table II, we find

$$\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{3}\text{He}X) = (7 \pm 1) \times 10^{-8}, \quad (7)$$

which is compatible at 1σ with the LHCb upper limit. We show in Fig. 4 the comparison of the theoretical values for $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{3}\text{He}X)$ obtained with our models compared with the LHCb upper limit. In particular, we see that the effect of increasing the `probQQtoQQ` parameter from 0.11 to 0.19, i.e., between the models Had.tune and Had.tune + QQ, is an higher branching ratio by a factor of almost 3.

Overall, our model yields predictions for the main decay modes of Λ_b^0 that are compatible with observations.

TABLE III. Branching ratios (BR) of $\bar{\Lambda}_b^0$ into diverse modes. The first column reports their measurement, provided by PDG [86] and LHCb [37]. The results were obtained using 10^{10} events in Pythia.

$\bar{\Lambda}_b^0$ decay mode	Measured BR	WL21 tune	Had. tune	Benchmark
$\Lambda_c^- \ell^+ \nu_\ell$	$(6.2^{+1.4}_{-1.3})\%$	$(5.9569 \pm 0.0009)\%$	$(5.956 \pm 0.002)\%$	$(5.957 \pm 0.001)\%$
$\bar{p}\bar{D}^0\pi^+$	$(6.2 \pm 0.6) \times 10^{-4}$	$(7.13 \pm 0.01) \times 10^{-4}$	$(6.11 \pm 0.02) \times 10^{-4}$	$(6.64 \pm 0.01) \times 10^{-4}$
$\bar{p}\bar{D}^-\pi^-\pi^+$	$(2.7 \pm 0.4) \times 10^{-4}$	$(2.544 \pm 0.006) \times 10^{-4}$	$(2.154 \pm 0.008) \times 10^{-4}$	$(2.215 \pm 0.006) \times 10^{-4}$
$\bar{p}\pi^+$	$(4.6 \pm 0.8) \times 10^{-6}$	$(1.219 \pm 0.005) \times 10^{-4}$	$(1.035 \pm 0.006) \times 10^{-5}$	$(1.41 \pm 0.02) \times 10^{-5}$
$\Lambda_c^- K^+ K^- \pi^+$	$(1.02 \pm 0.11) \times 10^{-3}$	$(1.342 \pm 0.002) \times 10^{-3}$	$(1.391 \pm 0.003) \times 10^{-3}$	$(1.354 \pm 0.002) \times 10^{-3}$
$\Lambda_c^- p \bar{p} \pi^+$	$(2.63 \pm 0.23) \times 10^{-4}$	$(3.285 \pm 0.003) \times 10^{-3}$	$(2.677 \pm 0.009) \times 10^{-3}$	$(4.615 \pm 0.009) \times 10^{-4}$
$\bar{p}\bar{n}X$	...	$(5.337 \pm 0.003) \times 10^{-3}$	$(7.32 \pm 0.02) \times 10^{-4}$	$(6.56 \pm 0.01) \times 10^{-4}$
$\bar{p}\bar{p}\bar{n}X$	...	$(7.64 \pm 0.04) \times 10^{-5}$	$(1.04 \pm 0.06) \times 10^{-6}$	$(2.4 \pm 0.3) \times 10^{-7}$
$\bar{3}\text{He}X$	$< 6.3 \times 10^{-8}$	$(5.32 \pm 0.08) \times 10^{-6}$	$(2.2 \pm 0.3) \times 10^{-7}$	$(7 \pm 1) \times 10^{-8}$

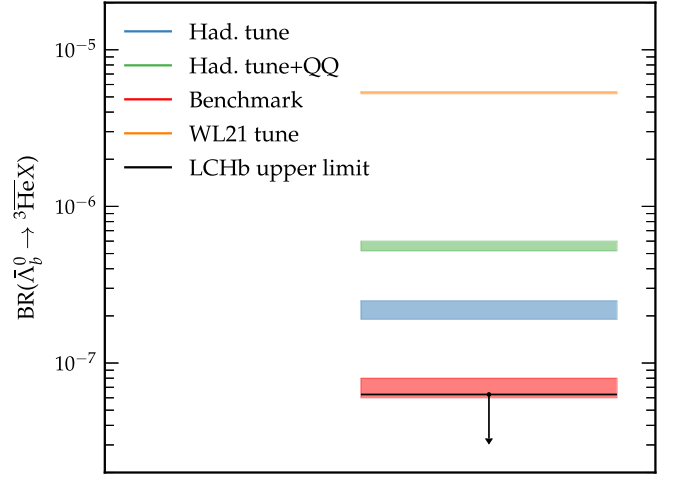


FIG. 4. This figure shows the estimate for $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{3}\text{He}X)$ obtained with the different models tested in the paper along with the upper limit measured by LHCb [37].

Moreover, our predictions are consistent with the LHCb upper limit for $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{3}\text{He}X)$ (see Fig. 4). In the rest of the paper, we label this setup as Benchmark.

VII. RESULTS FOR THE SPECTRA OF $\bar{D}$ AND $\bar{3}\text{He}$

We now present our predictions for the antideuteron and antihelion spectra. We choose a DM mass of 50 GeV with the $b\bar{b}$ annihilation channel we run simulations with 10^{10} annihilation events, which yield, for the Benchmark, a total of approximately 2×10^6 $\bar{D}$ and 500 $\bar{3}\text{He}$ events. Figure 5 displays the resulting spectra: the total spectrum (solid line) along with the separate contributions from b -baryons (dashed line) and B -mesons (dotted line).

Under the Benchmark, B mesons contribute about 20% of the total b -hadron yield in the $\bar{D}$ spectrum, while they do not contribute to the $\bar{3}\text{He}$ spectrum. Overall, antinuclei produced from b -hadron decays are a minor component, contributing only at the high-energy end of the spectrum (i.e., for energies close to the kinematic cutoff of the DM

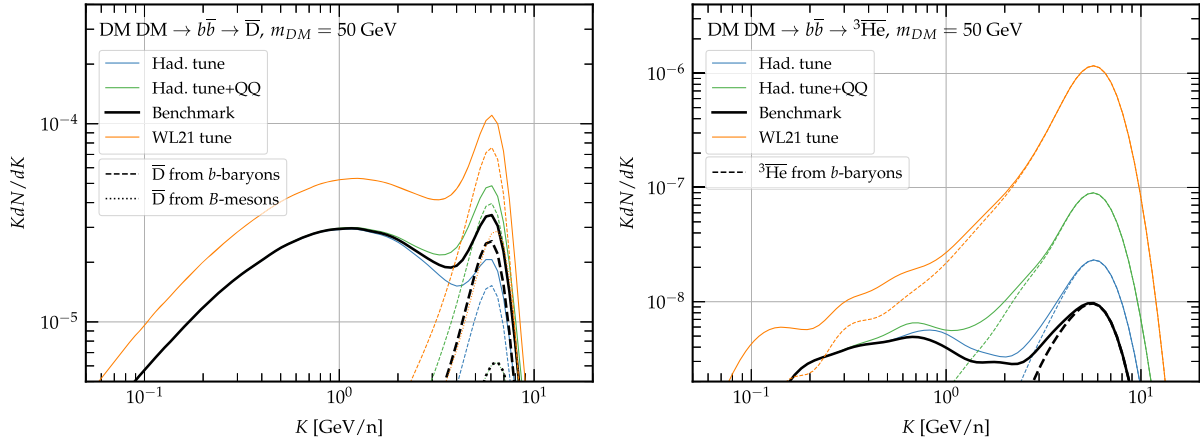


FIG. 5. Source spectra for the production of $\bar{D}$ (left panel) and $\bar{^3\text{He}}$ (right panel) from DM annihilations into $b\bar{b}$ with a mass of 50 GeV. For the prompt production of $\bar{D}/\bar{^3\text{He}}$, we always use Had.tune. For the $\bar{D}/\bar{^3\text{He}}$ produced by weakly decaying b hadrons (and B mesons), represented by dashed (dotted) lines, we show the results for different cases: Had.tune (blue), Had.tune+QQ (green), Benchmark (black), and WL21 tune (orange). Solid lines represent the sum of the prompt production, weakly decaying b baryon, and B -meson contributions.

mass), with contributions of about 8% and 16% of the total for $\bar{D}$ and $\bar{^3\text{He}}$, respectively. Even when focusing solely on the highest energies, b hadrons account for at most about 50% of the total counts.

The use of the default Pythia settings for the Λ_b^0 decay channels, as done in Ref. [26], has important implications. In the case labeled as Had.tune+QQ, where the default Λ_b^0 decay modes are employed, the spectra are approximately 30% larger for $\bar{D}$ and a factor of 10 larger for $\bar{^3\text{He}}$ compared to the Benchmark. Thus, the modifications we introduced to the rare $\bar{\Lambda}_b^0$ decay modes—made to conform with the LHCb upper limits on $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{^3\text{He}}X)$ —have a significant impact on the $\bar{^3\text{He}}$ spectrum. In contrast, employing the `probQQtoQ` value from Had.tune, which is closer to the default Pythia setting, leads to a reduced b -hadron contribution; in particular, the $\bar{^3\text{He}}$ spectrum is reduced by a factor of 3.

We also demonstrate that we recover the significant enhancement reported in Ref. [26] when using a `probQQtoQ` value 3 times larger than the default, the default $\bar{\Lambda}_b^0$ branching ratios and $p_c = 0.124$ GeV. In that scenario, the enhancement in the $\bar{D}$ and $\bar{^3\text{He}}$ spectra due to the weakly decaying b hadrons is approximately a factor of 3 and 50, respectively, relative to the predictions obtained with the default Pythia setting. However, as noted above, this model is severely excluded by the LHCb upper limits.

VIII. CONCLUSIONS

In this paper we have developed a hadronization model for the prompt production of antinucleons that is fully compatible with a comprehensive LEP dataset. In addition, we tuned the antinucleon yield from weakly decaying b -hadrons using multiplicity data and b -hadron

fragmentation function estimates. We constrained the coalescence process by fitting ALICE data for the $\bar{^3\text{He}}$ spectrum and found the model to be compatible at 1σ with the ALEPH $\bar{D}$ data. Finally, we adjusted the Pythia settings for the decay of $\bar{\Lambda}_b^0$ to be consistent with the LHCb upper limit for $\text{BR}(\bar{\Lambda}_b^0 \rightarrow \bar{^3\text{He}}X)$. With these refinements, we provide the most precise estimates to date for the production of $\bar{D}$ and $\bar{^3\text{He}}$ from B -meson and b -baryon weak decays, finding that their contribution is negligible compared to that from prompt production. This implies that if the tentative detections of antihelium events by AMS-02 are confirmed, new production mechanisms must be considered.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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