

Dynamics of the simplest chiral gauge theories

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Arguably, the simplest chiral gauge theories are $SO(10)$ with N_f fermion fields in the spinor representation **16**. We study their dynamics using their supersymmetric limits perturbed by an infinitesimal anomaly mediated supersymmetry breaking as a guide. We predict the theory is gapped for $N_f = 1, 2$, while the $SU(N_f)$ global symmetry is broken to $SO(N_f)$ for moderately large $N_f \geq 3$.

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Introduction. The fact that the weak interactions distinguish left from right came as a big surprise to physicists, as established by the ^{60}Co β -decay experiment led by Chien-Shiung Wu [1]. This idea of parity violation became the essential foundation of the Standard Model of particle physics, namely that it is based on *chiral* gauge theories. Left-handed and right-handed particles have different quantum numbers, and actually they are fundamentally different particles unless the Higgs boson condensate connects them.

Despite the essential importance of chiral gauge theories in describing nature, its definition has not been clear. Even though perturbation theory can be developed with somewhat *ad hoc* ways of regularizing divergences, a non-perturbative definition on the lattice has been difficult due to the fermion doubling problem [2–5]. There has been a lot of progress in this area (see, e.g., [6,7] for reviews and references therein), yet numerical simulation has not been possible for four-dimensional spacetime due to the sign problem. Therefore, it is of paramount importance to come up with other approaches in defining and working out chiral gauge theories.

Recently, one of the authors (H. M.) [8] proposed the methodology to use a supersymmetric (SUSY) version of gauge theories and perturb them by an infinitesimal supersymmetry breaking in the form called anomaly

mediation [9,10]. In many cases, supersymmetry allows for exact understanding of nonperturbative effects in gauge theories, because the near-SUSY limit and non-SUSY limit appear to be continuously connected with no apparent sign of phase transition, and hence a crossover [11]. In addition, anomaly mediation has the property called “UV insensitivity” such that its impact can be worked out whether the fields are elementary or composites. Therefore, the non-perturbative dynamics can be understood even in the presence of supersymmetry breaking. This methodology has already been applied to some chiral gauge theories [12–15] with rather surprising results that did not agree with previous suggestions.

In the end, we will need computer simulations to understand which suggestions correctly describe nonperturbative dynamics of chiral gauge theories. Kikukawa suggested that $SO(10)$ gauge theories with fermions in spinor (**16**) representations are likely the first of such theories to be simulated on computers [16]. In fact, $SO(10)$ is the smallest simple group that admits chiral fermions (i.e., complex representations) and is anomaly free. Among them, the **16**-dimensional representation is the smallest complex representation. Kikukawa proposed a formulation for its lattice simulations [16] building on prior works [17–21].

In this Letter, we study dynamics of $SO(10)$ gauge theories with N_f Weyl fermions in the **16** representation. Using the methodology of supersymmetry broken by anomaly mediation, we work out exact analytic solutions when the supersymmetry breaking is small. We also discuss what we might expect in the nonsupersymmetric limit and find they are plausibly connected continuously.

Supersymmetric $SO(10)$ with **16’s.** We can obtain non-perturbative superpotentials for the $SO(10)$ theories with N_f **16**s by first introducing additional $2(4 - N_f) + 1$ fields

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in the vector representation. Then the theories “s-confine” and the superpotentials are given already in [22]. By adding mass terms to the extra fields and integrating them out, we obtain the desired superpotential of the original theories. Yet, the expressions in [22] are somewhat sketchy without precise numerical coefficients and thus inadequate for our purpose. Below, we rederive the same results from the microscopic dynamics with precise expressions.

We consider the chiral superfields ψ_α ($\alpha = 1, \dots, N_f$) that are in the **16** representation of $\text{SO}(10)$ [or $\text{Spin}(10)$ to be more precise] and the fundamental representation of the $\text{SU}(N_f)$ global symmetry. They have gauge-invariant polynomials $S_{\alpha\beta;\gamma\delta}$ constructed as

$$S_{\alpha\beta;\gamma\delta} = \frac{1}{3!} A_{\alpha\beta}^{IJK} A_{\gamma\delta}^{IJK} = S_{\gamma\delta;\alpha\beta},$$

$$A_{\alpha\beta}^{IJK} = \psi_\alpha^T C_{10} \Gamma^{IJK} \psi_\beta = -A_{\beta\alpha}^{IJK}. \quad (1)$$

Here, $I, J, K = 1, \dots, 10$ are $\text{SO}(10)$ vector indices, and C_{10} is the “charge conjugation” matrix of $\text{SO}(10)$ that satisfies $C_{10} \Gamma^I C_{10}^{-1} = -(\Gamma^I)^T$. For our convention of $\text{SO}(10)$ gamma matrices see Supplemental Material [23].

We discuss cases $N_f = 1, 2, 3, 4$. For higher N_f , the theory has a magnetic description [24]. Unfortunately these do not have the $\text{SU}(N_f)$ global symmetry manifest in the description. Here we do not attempt to study higher $N_f \geq 5$.

$N_f = 1$ case. This theory was conjectured to dynamically break supersymmetry based on the difficulty in matching $U(1)_R$ anomalies [25] and later supported by instanton analyses [26]. In either case, the theory is strongly coupled. It was proven later that the Witten index of this theory in fact vanishes [27]. To place the theory under quantitative control, we introduce a chiral superfield in the vector representation of $\text{SO}(10)$ with mass $M \ll \Lambda$. Without the Anomaly Mediated Supersymmetry Breaking (AMSB) effects, the theory spontaneously breaks both supersymmetry, leading to a massless goldstino, and $U(1)_R$, a massless Nambu-Goldstone boson. Once AMSB is included, both supersymmetry and $U(1)_R$ are explicitly broken and both of them acquire finite mass, making the theory gapped. See Supplemental Material [23] for details of the calculation.

Both supersymmetry and $U(1)_R$ are explicitly broken with no global symmetry. Correspondingly, we confirmed that both the goldstino and the $U(1)_R$ Nambu Goldstone Boson (NGB) acquire mass in the limit $m \ll M \ll \Lambda$ where a weakly coupled analysis is justified, and hence the spectrum is indeed gapped.

$N_f = 2$ case. The D -flat direction can be understood using the hierarchical symmetry breaking. Under the $\text{SO}(8)$ subgroup, we have two pairs of $(\mathbf{8}_s \oplus \mathbf{8}_a)$. Using the triality of $\text{SO}(8)$, we can regard them as two $(\mathbf{8}_s \oplus \mathbf{8}_v)$. While two $\mathbf{8}_v$ breaks $\text{SO}(10)$ to $\text{SO}(8)$, one $\mathbf{8}_s$ breaks it to

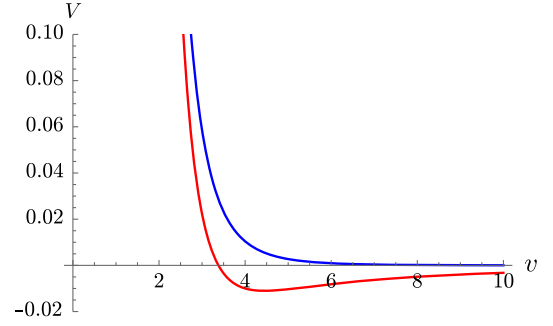


FIG. 1. $N_f = 2$ case, plot of the potential as a function of v with $\Lambda = 1$, without the anomaly mediation $m = 0$ in blue, and with the anomaly mediation $m = 0.01$ in red.

$\text{SO}(7)$ and then the other to G_2 . Recall that a spinor Vacuum Expectation Value (VEV) of $\text{SO}(7)$ breaks it to G_2 .

Explicit D -flat direction for the classical moduli space is described by $S_{\alpha\beta;\gamma\delta} = S \epsilon_{\alpha\beta} \epsilon_{\gamma\delta}$ which is $\text{SU}(2)$ singlet,

$$\psi_1 = \frac{v}{2}(\zeta_4 + \zeta_7), \quad \psi_2 = \frac{v}{2}(\zeta_1 + \zeta_6), \quad (2)$$

giving $S = 3v^4/2$. See Supplemental Material [23] for the definitions of ζ_i . In fact, $45 - 14 = 16 \times 2 - 1$ and hence all $\psi_{1,2}$ are eaten except for the D -flat direction S .

The gaugino condensate of G_2 generates the superpotential

$$W_{\text{dyn}} = 4 \left(\frac{\Lambda^{20}}{S^2} \right)^{1/4}. \quad (3)$$

With AMSB, the potential shown in Fig. 1 using the canonical Kähler potential along v is

$$V = \frac{128\Lambda^{10}}{3v^6} - m \frac{80\Lambda^5}{\sqrt{6}v^2}, \quad (4)$$

which settles to a minimum with no $\text{SU}(2)$ breaking,

$$v = \left(\frac{384 \Lambda^{10}}{25 m^2} \right)^{1/8}. \quad (5)$$

When $m \ll \Lambda$, the minimum is $v \gg \Lambda$ and hence the gauge coupling is weak, justifying the analysis using the canonical Kähler potential. The light spectrum consists of scalar of mass squared $100m^2/3$, pseudoscalar $50m^2/3$, and Majorana fermion $25m^2$, satisfying the vanishing supertrace. The spectrum is gapped.

$N_f = 3$ case. The general D -flat direction breaks $\text{SO}(10)$ to $\text{SU}(2)$. It was nontrivial to come up with explicit parametrization of the general D -flat direction as is often the case for chiral gauge theories. It is helpful to observe that the unbroken $\text{SU}(2)$ can be identified with $\text{SU}(2)_R$ of the maximal Pati-Salam subgroup $\text{SO}(10) \supset \text{SO}(6) \times \text{SO}(4) \simeq \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R$ where the decomposition is

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2}).$$

In order to keep $SU(2)_R$ unbroken, the VEVs reside only in $(\mathbf{4}, \mathbf{2}, \mathbf{1})$. Out of 45 generators of $SO(10)$, $15 + 3 + 3 = 21$ generators belong to the Pati-Salam group, while the remaining 24 generators are $(\mathbf{6}, \mathbf{2}, \mathbf{2})$ that map $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ to $(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ and vice versa. As long as $(\mathbf{4}^*, \mathbf{1}, \mathbf{2})$ all vanish, D flatness under $SU(4) \times SU(2)_L$ is sufficient to guarantee the D flatness under the whole $SO(10)$. Explicit D -flat direction for the classical moduli space is described by

$$S_{\alpha\beta;\gamma\delta} = \epsilon_{\alpha\beta\eta}\epsilon_{\gamma\delta\xi}\bar{S}^{\eta\xi} \quad (6)$$

in the $\mathbf{6}^*$ representation of the global $SU(3)$ symmetry. We find the D -flat direction

$$\begin{aligned} \psi_1 &= v \begin{pmatrix} \cos\theta & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, & \psi_2 &= v \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \cos\theta \end{pmatrix}, \\ \psi_3 &= v \begin{pmatrix} \sin\theta \sin\phi & -\sin\theta \cos\phi \\ 0 & 0 \\ 0 & 0 \\ \sin\theta \cos\phi & \sin\theta \sin\phi \end{pmatrix}, \end{aligned} \quad (7)$$

which gives

$$\bar{S} = v^4 \begin{pmatrix} 0 & -12 \sin^2\theta & 6 \sin 2\theta \sin\phi \\ -12 \sin^2\theta & 0 & 6 \sin 2\theta \sin\phi \\ 6 \sin 2\theta \sin\phi & 6 \sin 2\theta \sin\phi & -24 \cos^2\theta \end{pmatrix}. \quad (8)$$

We verified that all possible eigenvalues can be generated by appropriate choice of v, θ, ϕ .

Along the D -flat directions, $SO(10)$ is generically broken to $SU(2)_R$. Out of $16 \times 3 = 48$ chiral superfields, $45 - 3 = 42$ are eaten, leaving $48 - 42 = 6$ components described by $\bar{S}^{\eta\xi}$. The remaining $SU(2)_R$ gauge group develops a gaugino condensate, which leads to the runaway behavior of $\det \bar{S}$,

$$W_{dyn} = 2 \left(\frac{\Lambda^{18}}{\det \bar{S}} \right)^{1/2}. \quad (9)$$

To work out the potential, it is important that all chiral superfields participate in its derivation even if they do not have an expectation value. We find

$$\begin{aligned} V_{SUSY} &= \frac{\csc^6(\theta) \sec^2(\theta) \sec^2(\phi) \Lambda^{18}}{13824 v^{14}} \\ &\times ((\cos(2\theta) + 3) \sec^2(\theta) \sec^2(\phi) \\ &- 9 \cos(2\theta) + 13), \end{aligned} \quad (10)$$

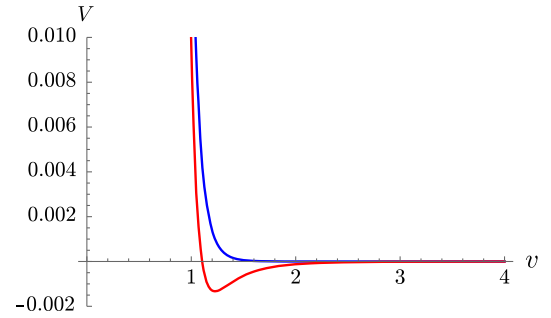
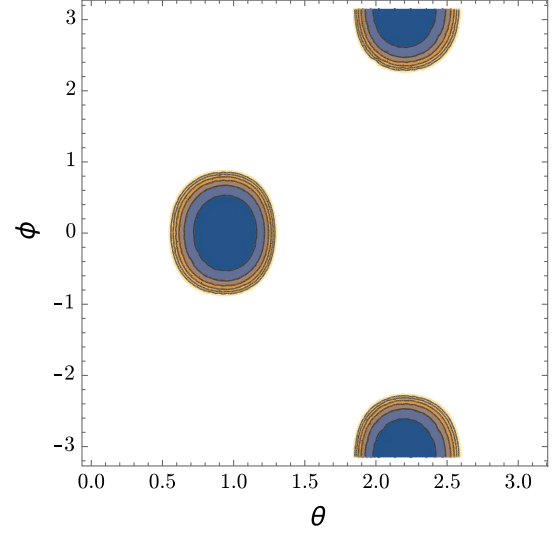


FIG. 2. $N_f = 3$ case, Top: Contour plot of the potential on the (θ, ϕ) plane with $\Lambda = 1$, $m = 0.01$, and v at its minimum Eq. (12). It shows the minimum at $\phi = 0$ and $\theta = \arctan \sqrt{2} \approx 0.955$. The other minimum is equivalent under $(\theta, \phi) \simeq (-\theta, -\phi)$. Bottom: The behavior of the potential along the v direction with (red) and without (blue) the anomaly mediation.

and

$$V_{AMSB} = -\sqrt{\frac{3}{2}} \frac{m \Lambda^9}{4 v^6} \csc^2(\theta) \sec(\theta) \sec(\phi). \quad (11)$$

As shown in Fig. 2, with AMSB, it settles to a minimum with $\theta = \arctan \sqrt{2}$ and $\phi = 0$, where three eigenvalues of $\bar{S}$ are the same up to a unitary transformation. Therefore, the theory breaks $SU(3)$ to $SO(3)$. The minimum is at

$$v = \left(\frac{49 \Lambda^{18}}{18432 m^2} \right)^{1/16} \quad (12)$$

and hence $v \gg \Lambda$ as long as $m \ll \Lambda$, which justifies the weakly coupled analysis using the canonical Kähler potential. The light spectrum consists of multiplets $\mathbf{6}^* = \mathbf{1} + \mathbf{5}$ under $SO(3)$ with mass-squared

$$\text{scalars: } \frac{648}{7} m^2, \frac{162}{49} m^2 \times 5, \quad (13)$$

$$\text{pseudoscalars: } \frac{486}{7}m^2, 0 \times 5, \quad (14)$$

$$\text{fermions: } 81m^2, \frac{81}{49}m^2 \times 5. \quad (15)$$

Five massless pseudoscalars are the NGBs of $SU(3)/SO(3)$, and they together satisfy the vanishing supertrace.

$N_f = 4$ case. In this case, the $SO(10)$ gauge group is generically completely broken on the moduli space. Out of $16 \times 4 = 64$ chiral superfields, 45 are eaten, leaving $64 - 45 = 19$ moduli fields. They are described by **20** of $SU(4)$, or equivalently traceless symmetric tensor $S = S^T$, $\text{Tr} S = 0$ of $SO(6)$ with one constraint. Unfortunately, the expression given in [22] is too sketchy for our purpose. We find that the classical constraint is $4\text{Tr} S^4 - (\text{Tr} S^2)^2 = 0$ by explicitly looking at many D -flat solutions. This factor of four is crucial because it tells us which symmetry breaking pattern is possible within the constraint. Quantum mechanically, the constraint is modified and we have the superpotential

$$W_{\text{dyn}} = X(4\text{Tr} S^4 - (\text{Tr} S^2)^2 - \Lambda^{16}). \quad (16)$$

We studied the possible ground state. We list the candidates in Supplemental Material [23].

As a result, we consider $SO(6)/(SO(3) \times SO(3)) = SU(4)/SO(4)$ as the main candidate for the ground state. However this analysis is not rigorous, given that the theory is in the strongly coupled regime and we do not have control over the Kähler potential. Therefore we leave the possibility of $SO(6)/SO(5) = SU(4)/Sp(4)$ open to be conservative.

Nonsupersymmetric limits. As m is increased and approaches Λ , we cannot solve the theory any more. There may be a phase transition around $m \sim \Lambda$. If there is not, the theory remains in the same universality class, namely that it has the same massless particle content and symmetry breaking pattern. We hope to learn non-SUSY limits this way. At the same time, it is important to discuss what we may expect in the non-SUSY limit to see if it is plausible that they are connected continuously. In this discussion, we abuse our notation to refer to Weyl fermions using the same symbol ψ_α as the chiral superfields above.

There is no fermionic composite operator, given that it would require an odd power of ψ , while each ψ is odd under the $\mathbb{Z}_2$ center of $\text{Spin}(10)$, and hence an odd power of ψ cannot be gauge invariant. Therefore, there cannot be a massless composite fermion for the anomaly matching. The $SU(N_f)$ global symmetry must be dynamically broken to an anomaly-free subgroup. The candidate subgroups are $SO(N_f)$ or $Sp(2[N_f/2])$. Using the Hilbert series techniques developed for the Standard Model effective field theory in [28], one finds the number of gauge-invariant four-fermion operators to be 1, 6, 21, 56 for $N_f = 1, 2, 3, 4$,

respectively [29]. They can be written down as

$$Z_{\alpha\beta;\gamma\delta} = X_{\alpha\beta}^I X_{\gamma\delta}^I, \quad (17)$$

$$X_{\alpha\beta}^I = X_{\beta\alpha}^I \equiv \psi_\alpha^T C C_{10} \Gamma^I \psi_\beta, \quad (18)$$

where $C = i\gamma_0\gamma_2$ is the usual charge conjugation matrix of the $SO(3, 1)$ Lorentz group.

In general, the quadrilinear condensate may be

$$Z_{\alpha\beta;\gamma\delta} \propto \delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} \quad (19)$$

to leave $SO(N_f) \subset SU(N_f)$ unbroken, or

$$Z_{\alpha\beta;\gamma\delta} \propto \delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma} \quad \text{or} \quad J_{\alpha\gamma} J_{\beta\delta} + J_{\alpha\delta} J_{\beta\gamma} \quad (20)$$

to leave $SO(N_f)$ or $Sp(2[N_f/2]) \subset SU(N_f)$ unbroken. For the $N_f = 4$ case, there is another singlet operator,

$$\epsilon^{\alpha\beta\gamma\delta} (\psi_\alpha^T C C_{10} \Gamma^{IJK} \psi_\beta) (\psi_\gamma^T C C_{10} \Gamma^{IJK} \psi_\delta), \quad (21)$$

which is not relevant for symmetry breaking.

One of the few methods proposed to analyze dynamics of chiral gauge theory is called the tumbling [30] (originally suggested in [31]). The tumbling hypothesis assumes a fermion bilinear condensate in the most attractive channel, which suggests $SU(N_f) \rightarrow SO(N_f)$ for all N_f . See discussions in Supplemental Material [23].

On the other hand, the analysis using supersymmetry above shows the preference for $SO(N_f)$ for $N_f = 3$ (19), while for $Sp(2) = SU(2)$ for $N_f = 2$ (20). For $N_f = 4$ our result prefers $SO(4)$ while it is inconclusive and may allow for $Sp(4)$.

In particular, $N_f = 2$ is special where the supersymmetric analysis suggests that the global $SU(2)$ symmetry is unbroken. Namely the tumbling hypothesis and supersymmetric analysis differ in the predictions, which would be very interesting to be settled by future lattice simulations. It may be that $SU(N_f)/Sp(N_f)$ is the preferred ground state when N_f is even.

It is worthwhile recalling the results for other chiral gauge theories. [12–15] It would be useful to study more examples to draw general lessons on chiral gauge theories.

Conclusion. We studied dynamics of $SO(10)$ gauge theory with N_f fermions in the **16** representation. This is arguably the simplest chiral gauge theory to be simulated on lattice in the near future. We solved the theory exactly with supersymmetry broken infinitesimally by anomaly mediation. The theory is gapped for $N_f = 1, 2$, while exhibiting $SU(3)/SO(3)$ symmetry breaking for $N_f = 3$. The case $N_f = 4$ prefers $SU(4)/SO(4)$ while the $SU(4)/Sp(4)$ possibility remains. Even though we cannot exclude a phase transition when the supersymmetry breaking scale is increased, we find it quite plausible that the general pattern

of symmetry breaking is $SU(N_f)/SO(N_f)$ for the non-supersymmetric limit for $N_f > 2$.

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