

Investigating the exclusive toponium production at the LHC and FCC

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An exploratory study of the exclusive toponium production in pp , pPb , and $PbPb$ collisions at the center-of-mass energies of the Large Hadron Collider (LHC) and Future Circular Collider (FCC) is performed. Assuming that the toponium is a pseudoscalar $t\bar{t}$ state, we consider its exclusive production by photon and gluon-induced interactions. Results for the total cross sections and associated rapidity distributions are presented, and the number of events at the LHC and FCC are estimated.

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I. INTRODUCTION

The description of bound states of heavy quarks, generically denoted as quarkonium, is one of the main challenges of quantum chromodynamics. Over the last decades, a vast literature dedicated to the theoretical treatment and experimental observation of $c\bar{c}$ and $b\bar{b}$ bound states has been produced (for a review see, e.g. Ref. [1]). In contrast, until recently, the discussion about the production in hadronic collisions of a color singlet bound state formed by a top and an antitop quark—the toponium, was restricted to some few studies (see, e.g. Refs. [2–6]). Although the formation of toponium in e^-e^+ collisions had been predicted by Fadin and Khoze many years ago [7], the discovery of the top quark with a large mass and rapid decay have reduced the interest about the associated bound state, since it has been widely accepted that there is an insufficient time for the bound state formation before top decay. However, the observation of a 3σ excess of dileptonic top-antitop events in the smallest relative angle bin by the ATLAS collaboration [8] and additional features in the high statistic $t\bar{t}$ events observed by the ATLAS and CMS collaborations [9,10], have largely motivated several groups to revisit the toponium production in hadronic collisions [11–13] as well

as the derivation of the different toponium states [14–16]. In particular, the CMS collaboration has recently released the results associated with the search for resonances in $t\bar{t}$ production [17], where a significant excess of events is observed near the kinematical threshold. Although these results are consistent with the formation of a pseudoscalar $t\bar{t}$ state, denoted η_t state, the observation (or not) of a toponium at the Large Hadron Collider (LHC) is still a theme of debate.

The existing literature for the toponium production in hadronic collisions is focused on the inclusive case, characterized by the fragmentation of the incident protons (see, e.g. Refs. [11–13]). However, in recent years, the study of particle production in exclusive processes, where the incident hadrons remain intact in the final state, became a reality (for recent reviews see, e.g., Refs. [18,19]). Motivated by the results obtained in Refs. [20,21], where the exclusive $t\bar{t}$ production was investigated, and by the fact that for high energies the pseudoscalar mesons are dominantly produced by gluon-gluon and photon-photon interactions, we will calculate in this paper, for the first time, the exclusive toponium production by photon and gluon-induced interactions in hadronic collisions at the LHC and Future Circular Collider (FCC) energies using the formalism developed in Refs. [22–24]. The dominant diagrams are represented in Fig. 1, and can be written in the form

$$h_1 + h_2 \rightarrow h_1 \otimes \eta_t \otimes h_2, \quad (1)$$

where h_i is a proton or a nucleus and η_t represents the pseudoscalar $t\bar{t}$ state. The basic characteristic of these processes is the presence of two rapidity gaps ($\otimes$) in the final state, which are regions devoid of hadronic activity, separating the toponium from the intact outgoing hadrons. Experimentally, these processes have a very clear signal in

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the absence of pile-up, with the presence of the decaying products of the η_t state and no other hadronic activity seen in the central detector. Moreover, these events can be separated at the LHC by measuring the outgoing hadrons using the forward proton detectors such as the ATLAS forward proton detector (AFP) [25,26] and precision proton spectrometer [27] that are installed symmetrically around the interaction point at a distance of roughly 210 m from the interaction point. Similar forward detectors are expected to also be installed at the FCC [28]. In what follows, we will present a brief review of the formalism needed to estimate both contributions for the exclusive η_t production in hadronic collisions. For detailed reviews about the production of other final states in exclusive processes, we refer the interested reader to the Refs. [18,19]. In addition, we indicate Refs. [20,21] for the related discussion about the exclusive open $t\bar{t}$ production in pp collisions.

This paper is organized as follows. In the next section, we will present a brief review of the formalism used to describe the η_t production by $\gamma\gamma$ and gg interactions in exclusive hadronic processes. Moreover, we will discuss the main ingredients used in our calculations. We will focus on the lowest S -wave state ($1S$ state), but the formalism can be directly applied for excited states (e.g. $2S$ and $3S$ states) if the corresponding masses and width decays are used in the calculations. In Sec. III we will present our predictions for the total cross sections considering pp , pPb and $PbPb$ collisions at the LHC and FCC energies, which demonstrate that the η_t production is dominated by $\gamma\gamma$ interactions. In addition, the corresponding invariant mass and rapidity distributions will also be presented. Finally, in Sec. IV, we will summarize our main results and conclusions.

II. FORMALISM

The exclusive toponium production by $\gamma\gamma$ interactions in hadronic collisions at a center-of-mass energy $\sqrt{s}$, represented in Fig. 1 (left panel), is given by [29]

$$\begin{aligned} \sigma(h_1 h_2 \xrightarrow{\gamma\gamma} h_1 \otimes \eta_t \otimes h_2) \\ = \int_0^\infty \frac{d\omega_1}{\omega_1} \int_0^\infty \frac{d\omega_2}{\omega_2} F(\omega_1, \omega_2) \hat{\sigma}_{\gamma\gamma \rightarrow \eta_t}(\omega_1, \omega_2) \end{aligned} \quad (2)$$

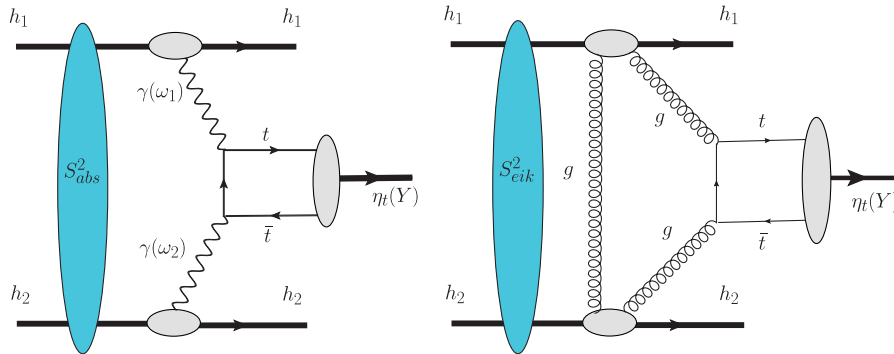


FIG. 1. Exclusive η_t production by photon (left panel) and gluon-induced (right panel) interactions in hadronic collisions.

where $\hat{\sigma}_{\gamma\gamma \rightarrow \eta_t}$ is the cross section for the subprocess $\gamma\gamma \rightarrow \eta_t$ and ω_1 and ω_2 are the energies of the photons which participate of the hard process and that can be expressed in terms of photon-photon center-of-mass energy $W = 4\omega_1\omega_2$ and rapidity Y of the η_t state as follows:

$$\omega_1 = \frac{W}{2} e^Y \quad \text{and} \quad \omega_2 = \frac{W}{2} e^{-Y}. \quad (3)$$

Moreover, F is the folded spectra of the incoming particles (which corresponds to an “effective luminosity” of photons) given by [30]

$$\begin{aligned} F(\omega_1, \omega_2) = 2\pi \int_0^\infty db_1 b_1 \int_0^\infty db_2 b_2 \\ \times \int_0^{2\pi} d\phi N_1(\omega_1, b_1) N_2(\omega_2, b_2) \langle S_{abs}^2 \rangle, \end{aligned} \quad (4)$$

where $N(\omega_i, \mathbf{b}_i)$ is the equivalent spectrum of photons with energy ω_i at a transverse distance $\mathbf{b}_i$ from the center of hadron, defined in the plane transverse to the trajectory, and ϕ is the angle between $\mathbf{b}_1$ and $\mathbf{b}_2$. The impact parameter of the collision $\mathbf{b}$ is related to $\mathbf{b}_1$ and $\mathbf{b}_2$ by $b^2 = b_1^2 + b_2^2 - 2b_1b_2 \cos \phi$ (for a detailed discussion, see e.g. Ref. [31]). The quantity $\langle S_{abs}^2 \rangle$ in Eq. (4) ensures that the incoming particles do not overlap, which we assume to be given by $\langle S_{abs}^2 \rangle = \Theta(b - R_A - R_B)$ [30], where R_i represents the proton or Lead radius. The Weizsäcker-Williams photon spectrum for a given impact parameter is given in terms of the nuclear charge form factor $F(k_\perp^2)$, where $k_\perp$ is the four-momentum of the quasireal photon, as follows [18]

$$N(\omega, b) = \frac{\alpha Z^2}{\pi^2 \omega} \left| \int_0^{+\infty} dk_\perp k_\perp^2 \frac{F\left(\left(\frac{\omega}{\gamma}\right)^2 + \vec{b}^2\right)}{\left(\frac{\omega}{\gamma}\right)^2 + \vec{b}^2} \cdot J_1(bk_\perp) \right|^2, \quad (5)$$

where α is the electromagnetic coupling constant, γ is the Lorentz factor and J_1 is the Bessel function of the first kind. For the nucleus, we will assume the realistic form factor [32], which corresponds to the Woods-Saxon distribution [33] and

is the Fourier transform of the charge density of the nucleus, constrained by the experimental data. Such a model provides a quite good description of the form factors from heavy nuclei. In contrast, for a proton, we will derive the equivalent photon spectrum from its elastic form factor in the dipole approximation (see e.g. [34]). Finally, the cross section for the subprocess $\gamma\gamma \rightarrow \eta_t$ is given by

$$\hat{\sigma}_{\gamma\gamma \rightarrow \eta_t}(\omega_1, \omega_2) = \int dW \delta(4\omega_1\omega_2 - W) \frac{8\pi^2}{M_{\eta_t}} \Gamma_{\eta_t \rightarrow \gamma\gamma}(M_{\eta_t}) \delta(W - M_{\eta_t}^2), \quad (6)$$

where $\Gamma_{\eta_t \rightarrow \gamma\gamma}$ is the partial decay width of the toponium into two photons. In our analysis, we will consider the recent values derived in Refs. [14,16].

On the other hand, in order to estimate the exclusive toponium production by gluon-induced interactions in hadronic collisions, represented in the right panel of Fig. 1, we consider the model proposed two decades ago by Khoze *et al.* [22–24], denoted the KMR model hereafter, which has been used to estimate different final states and have predictions in reasonable agreement with the observed rates for exclusive processes at the Tevatron and LHC (for a review see Ref. [19]). In this model, an additional t channel gluon (denoted spectator gluon hereafter) is needed to screen the color and the cross section for the $gg \rightarrow \eta_t$ subprocess is calculated considering a color singlet production. As a consequence, the incoming gg state satisfies special selection rules, namely it has $J_z = 0$, and positive C and P parity, which implies that only a subset of resonant $t\bar{t}$ states can be produced. As demonstrated in Refs. [22–24], the total cross section for the central exclusive production of a toponium in hadronic collisions can be expressed in a factorized way, as follows

$$\sigma(h_1 h_2 \xrightarrow{gg} h_1 \otimes \eta_t \otimes h_2) = \int dY \langle \mathcal{S}_{eik}^2 \rangle \mathcal{L}_{excl} \frac{2\pi^2}{M_{\eta_t}^3} \Gamma(\eta_t \rightarrow gg), \quad (7)$$

where $\langle \mathcal{S}_{eik}^2 \rangle$ is the gap survival probability (see below), Γ stands for the partial decay width of the toponium η_t in a pair of gluons and $\mathcal{L}_{excl}$ is the effective gluon-gluon luminosity, given by

$$\mathcal{L}_{excl} = \left[C \int \frac{dQ_t^2}{Q_t^4} f_g(x_1, x'_1, Q_t^2, \mu^2) f_g(x_2, x'_2, Q_t^2, \mu^2) \right]^2. \quad (8)$$

In this expression, $C = \pi/[(N_c^2 - 1)b]$, with b the t slope ($b = 4 \text{ GeV}^{-2}$ in what follows [23]), Q_t^2 is the virtuality of the soft gluon needed for color screening (the spectator gluon), x_1 and x_2 are the longitudinal momentum fractions

of the gluons which participate in the hard subprocess and x'_1 and x'_2 the longitudinal momentum fractions of the spectator gluon. Moreover, the quantities f_g are the skewed unintegrated gluon densities. At leading logarithmic approximation, it is possible to express $f_g(x, x', Q_t^2, \mu^2)$ in terms of the conventional integral gluon density and a Sudakov factor, which ensures that the active gluons that participate in the hard process do not radiate in the evolution from Q_t up to the hard scale $\mu = m_\perp \equiv$

$\sqrt{M_{\eta_t}^2 + p_{\eta_t, \perp}^2}$ (For details see Refs. [22–24]). As the KMR model is implemented in the publicly available SuperChic Monte Carlo (MC) [35], we have modified this generator in order to include the possibility of estimating the η_t production. Such a modification allows us to perform a full MC simulation of η_t production in central exclusive processes. In this paper, we will calculate f_g using the CT18NNLO parametrization [36], but we have verified that the impact of a different set on our results is small. Such a result is expected since in the case of toponium production we are probing large values of x ($\approx m_\perp/\sqrt{s} \gtrsim 10^{-2}$ for central rapidities), where the existing data provide important constraints in the behavior of the parton distribution functions (PDFs), which imply that the results obtained by different groups are similar. In addition, a similar result is obtained if other eigenvectors of the CT18 PDF are considered and the corresponding uncertainty is estimated. In particular, the difference between the predictions is smaller than 5%. Such a result is also expected, since for $x \approx 10^{-2}$ the PDF uncertainty is constrained by already existing experimental data [see Fig. 2 (upper right panel) in Ref. [36]].

We will estimate $\langle \mathcal{S}_{eik}^2 \rangle$ using the model proposed in Ref. [37], where the absorptive corrections associated to the additional soft hadron-hadron interactions are estimated considering distinct approaches for the description of the diffractive data. In particular, we will use the model 4 from Ref. [37]. We have verified that if a different model for $\langle \mathcal{S}_{eik}^2 \rangle$ is assumed, the predictions are reduced by a factor ≤ 1.5 . As a consequence, the KMR predictions for the toponium production presented in this paper can be considered an upper limit. Finally, we will assume the values for the partial decay width of the toponium into two gluons estimated in Refs. [14,16].

III. RESULTS

In what follows, we will present our predictions for the exclusive toponium production in hadronic collisions. One of the main inputs in our calculations is the values of the decay widths of the toponium into two photons or two gluons, which can be expressed in terms of the toponium wave function. In recent years, several groups have estimated such a quantity considering different frameworks

TABLE I. Cross sections, in attobarn, for the exclusive toponium production by photon and gluon induced interactions in pp collisions at the LHC and FCC. The lower and higher values of the predictions are associated with the results for $\Gamma_{\eta_t \rightarrow \gamma\gamma}$ and $\Gamma_{\eta_t \rightarrow gg}$ derived in Refs. [14] and [16], respectively.

Channel	LHC	FCC
$\gamma\gamma$	1.5–13.2	11.6–89.9
gg (KMR)	0.0028–0.021	0.024–0.19

and approximations [4,5,11,12,14–16], and derived distinct values for the mass of the η_t state, as well as for the corresponding $\Gamma_{\eta_t \rightarrow \gamma\gamma}$ and $\Gamma_{\eta_t \rightarrow gg}$ decay widths. Here, we will assume two different set of values for the η_t mass and decay widths: (a) $M_{\eta_t} = 343.62$ GeV, $\Gamma_{\eta_t \rightarrow \gamma\gamma} = 7.56$ keV, and $\Gamma_{\eta_t \rightarrow gg} = 1.69$ MeV [14], and (b) $M_{\eta_t} = 341.267$ GeV, $\Gamma_{\eta_t \rightarrow \gamma\gamma} = 57.18$ keV, and $\Gamma_{\eta_t \rightarrow gg} = 12.96$ MeV [16]. These two sets provide, respectively, the lower and upper bounds for our results, and the large difference between them is mainly associated with the distinct treatment of the long-range part of the potential and the different assumptions for the fixing of the coefficient for the Coulomb part. It is important to emphasize that the description of the decay widths is the main source of uncertainty in our calculations. In particular, Ref. [14] also provides results for the decay widths considering a Coulomb potential, but its values are smaller than those derived in Ref. [16], implying predictions that are between our lower and upper bounds.

Initially, we will consider pp collisions at the LHC ($\sqrt{s} = 14$ TeV) and FCC ($\sqrt{s} = 100$ TeV) energies. The predictions for the total cross-sections are presented in Table I. We have that the photon-induced interactions dominate the exclusive toponium production, with the associated predictions being two orders of magnitude larger than the KMR one. Such a result is, in principle, expected, since in pp collisions the effective photon-photon luminosity for producing a system of mass M becomes larger than the gluon-gluon luminosity at large M , as demonstrated in Ref. [23]. In addition, our results indicate that the cross sections increase by one order of magnitude at the FCC. However, the values are still of the order of few dozens of attobarn. Considering the expected integrated luminosities per year for the LHC ($\mathcal{L}_{\text{int}}^{pp} \approx 160 \text{ fb}^{-1}$) [38], we predict a maximum of two events. In contrast, this number of events becomes of the order of 90 at the FCC, where we expect to have $\mathcal{L}_{\text{int}}^{pp} \approx 1 \text{ ab}^{-1}$ [28]. Motivated by this result, we present in Fig. 2 the invariant mass distribution for the exclusive toponium production by $\gamma\gamma$ interactions in pp collisions at the FCC energy ($\sqrt{s} = 100$ TeV), obtained using the partial decay width of the toponium into two photons derived in Refs. [14] and [16]. For comparison, the prediction for the exclusive open $t\bar{t}$ production, computed for a stable top quark, is also presented. As expected for a resonance, the η_t production

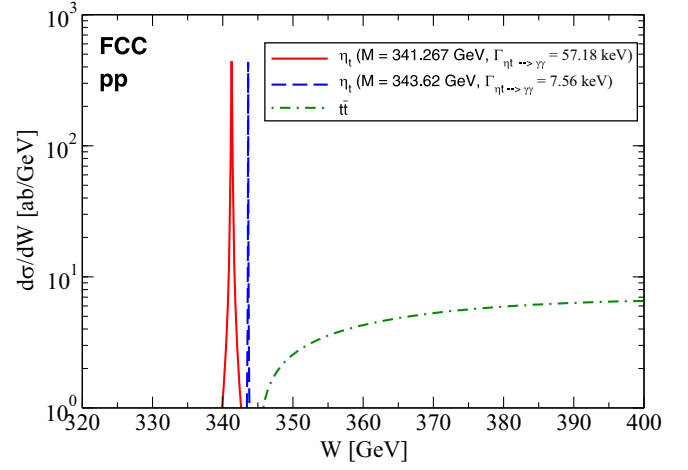


FIG. 2. Invariant mass distribution for the exclusive toponium production by $\gamma\gamma$ interactions in pp collisions at the FCC energy ($\sqrt{s} = 100$ TeV), obtained using the partial decay width of the toponium into two photons derived in Refs. [14] and [16]. For comparison, the prediction for the exclusive open $t\bar{t}$ production is also presented.

implies a peak in the invariant mass distribution at the corresponding mass, while the open $t\bar{t}$ production has a continuum distribution. Another important aspect is that the peak is predicted to occur below the $2m_t$ threshold. Such aspect can be used, in principle, to separate the events associated with the resonance production from the continuum. The studies of exclusive processes in pp collisions, performed by the ATLAS and CMS Collaborations using their forward detectors, have demonstrated that the mass of the central system can be reconstructed with a high precision if the forward protons are tagged, which allow separating the events associated with a small range of invariant masses (see, e.g., Refs. [39,40]). However, it is important to emphasize that the top decay can provide a contribution below threshold already in case of the continuum, and such possibility must be taken into account in order to separate the toponium from the continuum. Such aspects will be investigated in a future publication.

For the nuclear case, previous studies for exclusive gluon-induced processes have pointed out that they are strongly suppressed by the additional soft hadron-hadron interactions [41–43], that imply a very small value for $\langle \mathcal{S}_{eik}^2 \rangle$, with the production of large mass states predicted by the KMR model being much smaller than the results derived considering $\gamma\gamma$ interactions, which are enhanced by a factor Z^2 (Z^4) in pPb ($PbPb$) collisions (see, e.g. Refs. [44,45]).

Based on these previous results and the fact that $\gamma\gamma$ channel already is dominant in pp collisions, in what follows we only will present the corresponding predictions for the exclusive toponium production by photon-induced interactions. For pPb ($PbPb$) collisions, we will estimate the cross sections for $\sqrt{s} = 8.16$ (5.5) TeV at the LHC and $\sqrt{s} = 63$ (39) TeV at the FCC. The results for the rapidity

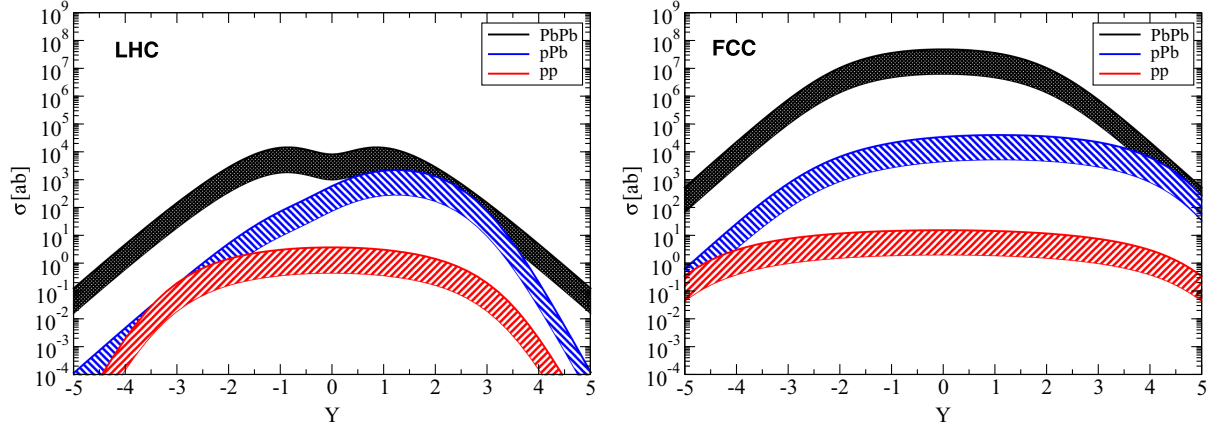


FIG. 3. Rapidity distributions for the exclusive toponium production by $\gamma\gamma$ interactions in pp , pPb , and $PbPb$ collisions at the LHC (left panel) and FCC (right panel). The lower and upper values of the bands are associated with the results for the partial decay width of the toponium into two photons derived in Refs. [14] and [16], respectively.

distributions of the toponium in pp , pPb and $PbPb$ collisions are presented in Fig. 3, where the bands represent the current uncertainty in the value of $\Gamma_{\eta_t \rightarrow \gamma\gamma}$. Our results clearly demonstrate the increasing of the cross section with the center-of-mass energy and that the distribution is asymmetric for pPb collisions, since the photon fluxes associated with the proton and nucleus are distinct. As expected, the predictions are largely enhanced in comparison to the pp one. The corresponding results for the total cross sections in pPb and $PbPb$ collisions are presented in Table II. In particular, for $PbPb$ collisions, the cross sections for the LHC (FCC) energy are four (six) orders of magnitude larger than for pp collisions, with the enhancement being smaller by a factor $\approx 10^2$ in the case of pPb collisions. Unfortunately, the increasing in the cross sections due to the photon-flux enhancement does not imply a larger number of events in comparison to the results presented above, since the expected integrated luminosities for nuclear collisions are orders of magnitude smaller than the value for pp collisions. For example, at the FCC, the expected values are $\mathcal{L}_{\text{int}}^{pPb} \approx 8 \text{ pb}^{-1}$ and $\mathcal{L}_{\text{int}}^{PbPb} \approx 33 \text{ nb}^{-1}$ [46]. As a consequence, we predict a maximum of five events in these collisions at the FCC. Such results indicate that, given the current expected luminosities, an experimental analysis of the exclusive toponium production in these collisions is not feasible.

TABLE II. Cross sections, in attobarn, for the toponium production by photon induced interactions in pPb and $PbPb$ collisions at the LHC and FCC. The lower and higher values of the predictions are associated with the results for $\Gamma_{\eta_t \rightarrow \gamma\gamma}$ derived in Refs. [14] and [16], respectively.

	LHC	FCC
pPb	518.2–4083.5	21000–165000
$PbPb$	5071.1–40697.6	1.8×10^7 – 13.7×10^7

IV. SUMMARY

In this paper, we have performed an exploratory study of the exclusive toponium production in hadronic collisions, motivated by the recent results performed in inclusive processes that indicate a possible formation of a pseudo-scalar $t\bar{t}$ state. One has considered photon and gluon-induced processes and demonstrated that the exclusive η_t production is dominated by $\gamma\gamma$ interactions. We have estimated the rapidity distributions and total cross sections for pp , pPb and $PbPb$ collisions at the LHC and FCC energies. Moreover, the corresponding number of events was calculated. Our results indicate that the study of the exclusive η_t production in pPb and $PbPb$ collisions is not feasible considering the current expected integrated luminosities. However, the predictions for pp collisions at the FCC motivate a more detailed analysis, including the η_t decay and realistic experimental cuts, as well as the treatment of potential backgrounds associated. In particular, the analysis of a toponium decaying into the constituent top and antitop quarks deserves special attention due to the interplay between the continuous $t\bar{t}$ and the toponium production mechanisms, which makes the experimental separation a hard task. An alternative is to consider other toponium decay channels as e.g. into two gluons, which will generate two jets in the final state, or into two photons. If the experimental analysis focus is on the $\eta_t \rightarrow \gamma\gamma$ decay channel, the final state will be characterized by two rapidity gaps and the diphoton system, with the main background being the exclusive diphoton production through the light-by-light scattering [45,47]. In principle, such background can be strongly suppressed by considering the same strategy used to search for axionlike particles in exclusive processes [48–50]. In these studies, the events are selected assuming a cut on the invariant mass of the final state in a range around the mass of the resonance, since the presence of the resonance implies a peak in the invariant mass distribution that dominates over the continuum associated

with the diphoton background. Such aspect will be explored in a forthcoming study, where we intend to determine the observability (or not) of the toponium in exclusive processes at the FCC.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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