

Quantal theory of restoration of strong CP symmetry

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We propose a mechanism for relaxing a gauge theory CP violating phase in discrete steps to very small values. The idea is that the CP violating phase includes the magnetic dual of a 4-form flux that can discharge by the nucleation of membranes. Inside the bubbles surrounded by the membranes, the total CP violating phase is reduced. When the bubbles are produced rapidly during radiation domination in the early Universe, near the chiral symmetry breaking scale, they will collide and percolate, melting away into gauge theory radiation and dramatically relaxing CP violation.

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I. INTRODUCTION

What if there is no QCD axion? Or, what if some UV physics disrupts the axion too much to solve the strong CP problem of QCD? Is a smoothly varying axion field even necessary to relax the strong CP -violating phase? Here we put forward an alternative to axions for dynamically relaxing the strong CP violation, based on discrete reduction of the CP -breaking phase $\theta_{Q\bar{P}}$ in the early Universe, which minimizes the $\theta_{Q\bar{P}}$ -dependent contribution to the total energy of the QCD vacuum below chiral symmetry breaking scale. This mechanism of cosmological relaxation by the production of percolating bubbles of the CP -conserving phase around the time of chiral symmetry breaking may be an example of how $\theta_{Q\bar{P}}$ could be relaxed cosmologically [1].

Our proposal is inspired by an insight provided in the paper [2]. In 1980, Aurilia, Nicolai, and Townsend noted that both the strong CP violating phase $\theta_{Q\bar{P}}$ and the cosmological constant¹ are degenerate with a magnetic dual of a nonvanishing 4-form flux. This follows from the dynamics of a strongly coupled $SU(N)$ gauge theory in the large N limit, below chiral symmetry breaking, where it can be described by a nonlinear σ model with CP violation, with new extended degrees of freedom [5–10]. These degrees of freedom are membranes charged under a 4-form field strength (also known as a “top” form, the term designating that its rank is the same as the number of dimensions).

Lüscher teased the top form Lagrangian out from the non-perturbative contributions to the correlation functions of the duals of gauge theory anomaly terms [11] even earlier. These results were studied and confirmed in two-dimensional CP^{n-1} σ models at low energies in more detail [12,13].

Including explicitly the 4-form sector of the theory in the action provides a very direct and lucid tool for understanding both the CP violation and the possible solutions for restoring CP . Dvali has advocated this approach forcefully in several papers [14–18], explaining that the axion solution of the CP problem corresponds to Higgsing the top form gauge theory [16]. A similar viewpoint was also presented long ago by Aurilia, Takahashi, and Townsend [19]. Moreover, Dvali has also noted that a membrane discharge dynamics could address the problem even when the axion is absent [17]. While it has been noted that nonsupersymmetric large- N gauge theories have a landscape of infinitely many nondegenerate vacua [20,21], concerns were raised that the decay rates are slow, both in the large N limit [22] and for smaller gauge group ranks like in QCD [23]. To enhance the membrane nucleation rates, [17] proposed that the membrane charge depends on the top form flux, and that $\theta_{Q\bar{P}} = 0$ is a very strong attractor for the terminal flux. Further, explicit examples [24,25] exist where the barrier to tunneling diminishes as N grows large.

Here we take a more mundane approach, which allows us to circumvent this subtle issue altogether, as outlined in [26]. We will show that when the strong CP -violating phase is degenerate with a magnetic dual of the flux of a new top form, which is a replica of Lüscher’s top form that emerges below chiral symmetry breaking in QCD, the discharges that relax $\theta_{Q\bar{P}}$ occur. Adopting a bottom-up approach, we treat the charge and tension of the membranes that source the new top form as input parameters, whose properties are not directly determined by QCD dynamics. We select their values so that the nucleations of membranes allow the fluxes in question to relax fast. Since the new top form flux

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¹The same conclusion about the cosmological constant was also noted separately in [3,4].

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is degenerate with the flux of Lüscher's top form, the discharge cascade simultaneously reduces both, and so it relaxes the strong CP violation. This occurs near the QCD chiral symmetry breaking scale, when the Universe is radiation dominated, expanding slowly. Therefore the bubbles of true vacuum inside the membranes will expand at the speed of light, colliding with each other. Upon collisions, the membranes separating them will decay into the strongly coupled QCD excitations. This conflagration of membranes yields the percolation of the true vacuum regions, which have a tiny θ_{QCD} , meaning that the CP violation would be reduced over a huge swath of the Universe.

The mechanism is completely analogous to the discharge of a Maxwell $U(1)$ field by pair production. We will work out the discharge rates in the decoupling limit of gravity when $M_{\text{Pl}} \rightarrow \infty$, treating the background spacetime as locally Minkowski when membranes nucleate, which works since the dynamics of nucleations is covariant, and the membrane radius at nucleation is very small. This will suffice for our purposes. Hence we will ignore any implications or interplay of the strong CP problem and the cosmological constant conundrum, fine-tuning the latter at will to be as small as required.

Since the relaxation of θ_{QCD} toward small values occurs in discrete steps, controlled by the membrane charges, generically the typical terminal value of the CP -violating phase θ_{QCD} will not be exactly zero. Therefore, the neutron dipole moment could naturally be nonvanishing. Further, the relaxation dynamics is similar to first order phase transitions near the QCD scale, and so bubble collisions might also produce a signal in the stochastic gravity wave background [27,28]. Since the nucleations are very prolific, determining precise abundance of those gravity waves requires a more careful study. There could be additional effects, which we outline in the discussion below.

II. EMERGENT TOP FORM SECTOR

Nontrivial topology of the gauge sector in QCD forces an extension of the perturbative Lagrangian, which must include an extra topological term

$$\mathcal{L}_{\text{QCD}} \ni \frac{g^2}{64\pi^2} \theta \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a, \quad (1)$$

with a canonically normalized gauge field strength G^a . Here $\sum_a$ is the sum over colors, and θ is a constant “vacuum angle,” $\theta \in [0, 2\pi]$. Note, that aside from the $1/g^2$ factor in the gauge kinetic term, we follow the normalizations used in [11].

This term is the chiral anomaly, and is required to ensure that the theory is invariant under large gauge transformations. It arises even in the pure glue limit, when the quarks are absent. This term leads to the resolution of the $U(1)$

problem and the explanation of the missing Goldstone meson [29,30]. The θ parameter is *a priori* arbitrary; if there were no other contributions to (1), a nonzero θ would yield CP violation. However, in the regime where the theory has chiral symmetry, unbroken in the fermion sector, all the values of θ are degenerate, implying that the theory has many different, topologically distinct superselection sectors.

One way to see this is to note that this term is purely topological, being a total derivative. Indeed, using the gauge potentials B_μ^a and structure constants f_{abc} , this readily follows since

$$\begin{aligned} & \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a \\ &= \frac{g^2}{16\pi^2} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \left(\sum_a B_\nu^a \partial_\lambda B_\sigma^a + \frac{g}{3} \sum_{a,b,c} f_{abc} B_\nu^a B_\lambda^b B_\sigma^c \right). \end{aligned} \quad (2)$$

Therefore, in perturbation theory the anomaly term drops out from the stress energy tensor. Hence in perturbation theory before chiral symmetry breaking, the parameter θ is a fixed constant, but by (2), in this regime all the states parametrized by a fixed value of $\theta \in [0, 2\pi]$ with all other quantum numbers equal are energetically degenerate. Specifically the Poincaré-invariant states with an arbitrary fixed θ are all admissible as the vacuum states of QCD, and, as noted, give rise to the infinitely many degenerate superselection sectors of perturbation theory before chiral symmetry breaking.

Since the anomaly term is a total derivative, we can rewrite the right-hand side of (2) as

$$\frac{g^2}{32} \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a G_{\lambda\sigma}^a = \partial_\mu K^\mu, \quad (3)$$

where K_μ is the Chern-Simons current. We can trade this auxiliary composite vector field for its Hodge dual 3-form, $K_\mu = \epsilon_{\mu\nu\lambda\sigma} A^{\nu\lambda\sigma}/6$, which, since

$$\begin{aligned} \partial_\mu K^\mu &= \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu A_{\nu\lambda\sigma} = \frac{1}{24} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma}, \quad \text{with} \\ F_{\mu\nu\lambda\sigma} &= 4\partial_{[\mu} A_{\nu\lambda\sigma]}, \end{aligned} \quad (4)$$

yields

$$F_{\mu\nu\lambda\sigma} = \frac{3}{4} g^2 \sum_a G_{[\mu\nu}^a G_{\lambda\sigma]}^a, \quad (5)$$

with [...] denoting antisymmetrization over the enclosed indices. Thus the θ term in the perturbative Lagrangian $\mathcal{L}_{\text{QCD}}$ can be rewritten as

$$\frac{1}{48\pi^2} \theta \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}. \quad (6)$$

In the presence of quarks with a mass matrix $\mathcal{M}$, the anomaly term (1) is interpreted as receiving contributions from both the *a priori* “vacuum angle” θ and the overall phase of the quark mass matrix $\text{Arg det } \mathcal{M}$. This contribution is extracted from $\mathcal{M}$ via a change of fermion basis implemented by chiral gauge transformations. Hence in the basis where the determinant of the quark mass matrix is real, the total CP -violating phase is

$$\theta \rightarrow \theta_{\mathcal{CP}} = \hat{\theta} = \theta + \text{Arg det } \mathcal{M}. \quad (7)$$

We will work in this basis in what follows.

The formula (7) illuminates the circumstances which lead to CP violation very clearly. But it also points to one possible solution. If even one of the quarks were exactly massless, the determinant of $\mathcal{M}$ would have been zero, and the phase $\text{Arg det } \mathcal{M}$ would have been completely arbitrary. That would have implied that the total CP -violating phase $\hat{\theta}$ is completely ambiguous, and that it could have always been chosen to be zero by freely picking an appropriate value of $\text{Arg det } \mathcal{M}$ to cancel θ . Thus there would have been no physical manifestation of CP violation after all.

A physical example of this case is QCD before chiral symmetry breaking in the UV where all the quarks are massless, and so $\hat{\theta} \neq 0$ is unphysical. The problem arises after complete chiral symmetry breaking, in the IR, when the quarks are massive (see the still-relevant discussion in [1]). We will formulate the problem and its diagnostic using the effective theory of the top form, which extends (6) below full chiral symmetry breaking.

To establish the dictionary we review the formalism of the theory relevant for the top form sector. We will use the vacuum expectation values of operators $\mathcal{O}$, which only depend on the gauge field variables as used by [11] after decoupling the fermions for simplicity: we assume that they are massive, which absorbs the mass matrix $U(1)$ phase into $\hat{\theta}$, and leaves us with a path integral over the gauge fields $B_\mu^a = \{\mathcal{B}\}$. Although the quark mass scale is comparable to the chiral symmetry breaking scale in real QCD, this simplification captures the key features of the dynamics well. In this limit,

$$\langle \mathcal{O} \rangle_{\hat{\theta}} = \frac{1}{\mathcal{Z}[\hat{\theta}]} \int [\mathcal{DB}] \mathcal{O} e^{iS + i\hat{\theta} \int q(x)}, \quad (8)$$

where

$$\mathcal{Z}[\hat{\theta}] = \int [\mathcal{DB}] e^{iS + i\hat{\theta} \int q(x)}, \quad (9)$$

is the standard partition function with the “source” term $\hat{\theta} \int q(x)$. Here the operator $q(x)$ is defined by

$q(x) = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\lambda\sigma} \sum_a G_{\mu\nu}^a(x) G_{\lambda\sigma}^a(x)$. The notation $\langle \dots \rangle_{\hat{\theta}}$ represents averaging with the Euclidean path integral of the gauge theory including the anomaly term $\propto \hat{\theta}$. Note that by Eqs. (3) and (4),

$$q(x) = \frac{1}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}. \quad (10)$$

The Euclidean continuation of the partition function (9) has very specific properties that were elucidated by Vafa and Witten [31]. The function $\mathcal{Z}[\hat{\theta}]$ has minima at $\hat{\theta} = 0 + 2n\pi$, where CP is restored. This implies that $\mathcal{Z}[\hat{\theta}]$ is an even function of $\hat{\theta}$. In turn, the vacuum expectation value of $\mathcal{O} = q(x)$ itself is [11]

$$\langle q \rangle_{\hat{\theta}} = iH(\hat{\theta}), \quad (11)$$

where by translational invariance, $\langle q(x) \rangle_{\hat{\theta}} = \langle q(0) \rangle_{\hat{\theta}} \propto i\partial_{\hat{\theta}} \mathcal{Z}[\hat{\theta}] / \mathcal{Z}[\hat{\theta}]$. Therefore, $H(\hat{\theta})$ is an odd function, $H(-\hat{\theta}) = -H(\hat{\theta})$ [1,11].

For the gauge group $SU(2)$ with a particular value of $\hat{\theta}$ state, Lüscher found that $H(\hat{\theta}) = 0.078(\mu_0 \frac{8\pi^2}{g^2})^4 \times (\mu_0 \rho_c)^{10/3} e^{-8\pi^2/g^2(\mu_0)} \sin(\hat{\theta})$, where μ_0 is a dimensional normalization parameter, $g(\mu_0)$ the running coupling and ρ_c the IR cutoff for the instanton size. A qualitatively similar result also holds for CP^{n-1} nonlinear σ models in $1+1$ dimensions. Similar features are also found in more realistic setups in [22,32,33]. Because the vacuum angle $\hat{\theta}$ is removable by chiral transformations above complete breaking of chiral symmetry, where at least one fermion remains massless, this expectation value must be zero at scales above Λ_{QCD} . Since $q(x) = \frac{1}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}$, below chiral symmetry breaking the “secret” top form field strength is not zero when $\hat{\theta}$ is not zero since $q \neq 0$: inverting the Hodge dual, and transforming to Lorentzian signature variables, we find

$$F_{\mu\nu\lambda\sigma} = -2\pi^2 i \epsilon_{\mu\nu\lambda\sigma} q = 2\pi^2 \epsilon_{\mu\nu\lambda\sigma} H(\hat{\theta}). \quad (12)$$

Hence $F_{\mu\nu\lambda\sigma} \neq 0$ is a faithful diagnostic of CP violation parametrized by $\hat{\theta}$. Hence restoring CP is equivalent to selecting a state in which the top form field strength is zero.

Further, although $F^{\mu\nu\lambda\sigma}$ starts out looking like an auxiliary term, it turns out that it has nontrivial dynamics, albeit without² local fluctuating degrees of freedom. To see it, we start with the calculation by Lüscher [11], who showed how the 3-form potential $A_{\mu\nu\lambda}$ gains a propagator, generalizing the work [34,35] in the CP^{n-1} σ model in two dimensions. In Lorenz gauge $\partial_\mu A^{\mu\nu\lambda} = 0$, the propagator is

²This follows from the unitarity of the theory.

given by the connected Green's function computed with the $\hat{\theta}$ term in the action,

$$\frac{1}{(3!)^2} \epsilon_{\mu\alpha\beta\gamma} \epsilon_{\nu\sigma\rho\delta} \langle A^{\alpha\beta\gamma}(x) A^{\sigma\rho\delta}(y) \rangle_{\hat{\theta}} = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \frac{p_\mu p_\nu}{p^4} \mathcal{X}. \quad (13)$$

The residue of the momentum space Green's function at $p^2 \rightarrow 0$ is the topological susceptibility $\mathcal{X} \simeq (\Lambda_{\text{QCD}})^4$ of the theory, defined by the integral of the time-ordered correlator $T(q(x)q(0))$,

$$\mathcal{X} = -i \int d^4 x \langle T(q(x)q(0)) \rangle_{\hat{\theta}}. \quad (14)$$

Its precise form is model dependent. Some explicit examples are discussed by Lüscher in [11]. Since the momentum space Green's function is $\mathcal{X} = -4\pi^4 i \partial_{\hat{\theta}} \langle q(0) \rangle_{\hat{\theta}} = 4\pi^4 \partial_{\hat{\theta}} \mathbf{H}(\hat{\theta})$, the function $\mathbf{H}$ is

$$\mathbf{H}(\hat{\theta}) = \mathcal{X} \hat{\theta} + \dots \quad (15)$$

The full quantum effective Lagrangian includes even power terms $\propto (F_{\mu\nu\lambda\sigma}^2)^n$, induced by nonperturbative physics, starting with the quadratic [7–9]. They are duals to the higher-order correlators $\langle q(x_1)q(x_2)\dots q(x_n) \rangle_{\hat{\theta}}$, which are nonzero below chiral symmetry breaking/confinement scale (when $\mathcal{X} \neq 0$) while vanishing when chiral symmetry is restored (and so $\mathcal{X} = 0$). We can reverse engineer the action to restore the quadratic 4-form field strength $F_{\mu\nu\lambda\sigma}$ by reproducing the expression for the Green's function (13), by recalling the canonical 3-form field theory, which in Lorenz gauge is $\mathcal{L}_{\mathcal{F}^2} = -\frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2$ with $\partial_\mu \mathcal{A}^{\mu\nu\lambda} = 0$. Its momentum space Green's function is [see e.g., [36], Eq. (34) with $\xi = 0$]

$$\langle \mathcal{A}_{\mu\nu\lambda}(p) \mathcal{A}_{\mu'\nu'\lambda'}(-p) \rangle = \epsilon_{\mu\nu\lambda\rho} \epsilon_{\mu'\nu'\lambda'\rho'} \frac{p^\rho p^{\rho'}}{p^4}. \quad (16)$$

Comparing this expression with the Fourier transform of Eq. (13),

$$\langle \mathcal{A}_{\mu\nu\lambda}(p) \mathcal{A}_{\mu'\nu'\lambda'}(-p) \rangle_{\theta} = \epsilon_{\mu\nu\lambda\rho} \epsilon_{\mu'\nu'\lambda'\rho'} \frac{p^\rho p^{\rho'}}{p^4} 4\pi^4 \mathcal{X}, \quad (17)$$

shows that the momentum space tensor structure is the same, and that the Green's function residue at $p^2 \rightarrow 0$ leads to the wave function renormalization

$$\mathcal{A}_{\mu\nu\lambda} = \frac{1}{2\pi^2 \sqrt{\mathcal{X}}} \mathcal{A}_{\mu\nu\lambda}. \quad (18)$$

Therefore, switching to the canonically normalized 4-forms, the theory to quadratic order in $\mathcal{F}_{\mu\nu\lambda\sigma}$ includes

$$\mathcal{L}_{\text{QCD}} \ni \frac{\sqrt{\mathcal{X}}}{24} \hat{\theta} \epsilon_{\mu\nu\lambda\sigma} \mathcal{F}^{\mu\nu\lambda\sigma} - \frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2, \quad (19)$$

up to gauge-fixing terms. We must highlight a similar analysis given in [32,33] who point out that the top form is an emergent degree of freedom below the chiral symmetry breaking scale, which encapsulates faithfully the long range physics induced by the nonperturbative effects. Here, for our purposes it will suffice to keep terms only up to quadratic, ignoring higher order terms discussed in [7–9].

As is well known, classically the 4-form field strength does not include new local degrees of freedom. Briefly, by total antisymmetry the 4-form field strength $\mathcal{F}_{\mu\nu\lambda\sigma}$ is proportional to the action measure, $\mathcal{F}_{\mu\nu\lambda\sigma} dx^\mu dx^\nu dx^\lambda dx^\sigma = 4! \mathcal{F} d^4 x$, and then the field equation, for a purely quadratic theory,³ is $d^* \mathcal{F} = 0$, which yields $\partial_\mu \mathcal{F} = 0$, and so $\mathcal{F} = \text{const}$. This constant of integration is the hidden constant elucidated in [2,6–9].

It is convenient and illuminating to canonically transform the action (19) by trading the electric field strength $\mathcal{F}_{\mu\nu\lambda\sigma}$ for its magnetic dual $\mathcal{F}_{\mu\nu\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} / 4!$. The dualization procedure is discussed at length in [2,16,36–39]. To carry this out, we add a Lagrange multiplier term to the 4-form action, recasting it in the first order form,

$$S_{\mathcal{F}} = \int d^4 x \left(-\frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2 + \frac{\sqrt{\mathcal{X}}}{24} \hat{\theta} \epsilon_{\mu\nu\lambda\sigma} \mathcal{F}^{\mu\nu\lambda\sigma} + \frac{\mathcal{F}}{24} \epsilon^{\mu\nu\lambda\sigma} (\mathcal{F}_{\mu\nu\lambda\sigma} - 4\partial_\mu \mathcal{A}_{\nu\lambda\sigma}) \right). \quad (20)$$

Next, we treat $\mathcal{F}$, $\mathcal{F}_{\mu\nu\lambda\sigma}$, $\mathcal{A}_{\nu\lambda\sigma}$ as independent variables, complete the squares, shift the electric 4-form variable $\tilde{\mathcal{F}}_{\mu\nu\lambda\sigma} = \mathcal{F}_{\mu\nu\lambda\sigma} - (\sqrt{\mathcal{X}} \hat{\theta} + \mathcal{F}) \epsilon^{\mu\nu\lambda\sigma}$ and integrate it out, using the path integral formulation of the theory. That leaves us with the dual action

$$S_{\mathcal{F}} = \int d^4 x \left(-\frac{\mathcal{X}}{2} \left(\hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}} \right)^2 - \frac{\mathcal{F}}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma} \right). \quad (21)$$

Since $\mathcal{F}_{\mu\nu\lambda\sigma}$ is a rank-4 tensor, $\mathcal{F}$ is a pseudoscalar. Because the action (21) is a first-order formulation of the theory, both $\mathcal{F}$ and $\mathcal{A}_{\mu\nu\lambda}$ are independent variables to be varied over. On the other hand $\mathcal{X}$ and $\hat{\theta}$ are fixed parameters—i.e., integrals of motion—when we ignore additional terms that can extend (21). The field equations that follow are

$$\begin{aligned} \delta \mathcal{A}_{\nu\lambda\sigma}: \quad & \partial_\mu \mathcal{F} = 0, \\ \delta \mathcal{F}: \quad & \epsilon^{\mu\nu\lambda\sigma} \mathcal{F}_{\mu\nu\lambda\sigma} = 4 \epsilon^{\mu\nu\lambda\sigma} \partial_{[\mu} \mathcal{A}_{\nu\lambda\sigma]} = -4! \left(\sqrt{\mathcal{X}} \hat{\theta} + \mathcal{F} \right). \end{aligned} \quad (22)$$

³For a bilinear theory, the equation is corrected by a total derivative, as shown below.

Note also that the quadratic term

$$V = \frac{\mathcal{X}}{2} \left(\hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}} \right)^2, \quad (23)$$

is precisely the contribution to the vacuum energy when CP is violated (see, e.g., [32,33]). Further note that now the total CP -violating phase is

$$\theta_{\mathcal{CP}} = \hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}}. \quad (24)$$

The expression for $\theta_{\mathcal{CP}}$ becomes more intricate with the addition of new phases that arise with the inclusion of subleading nonperturbative corrections, which need to be carefully collected together.

At first glance the potential term (23) seems puzzling when one recalls the familiar dilute instanton gas approximation for it,

$$V(\theta_{\mathcal{CP}}) = \sum_{\ell} V_{\ell} (1 - \cos(\ell \theta_{\mathcal{CP}})), \quad (25)$$

which is manifestly periodic in $\theta_{\mathcal{CP}}$. However, the magnetic dual $\mathcal{F}$ is a *variable*, which is fixed to a constant only on shell, by field equations (22), which have infinitely many solutions. Hence, (23) classically has a continuous shift symmetry $\hat{\theta} \rightarrow \hat{\theta}(1 + \vartheta)$, $\mathcal{F} \rightarrow \mathcal{F} - \sqrt{\mathcal{X}}\vartheta\hat{\theta}$, which is broken to discrete shifts $\vartheta = 2\pi n$ by the quantization of $\mathcal{F}$, which follows from the quantization of charges in the units of $\hat{\theta}$. Thus, the potential (23) should be understood as a combination of individual branches generated by $\mathcal{F} \rightarrow \mathcal{F} - 2\pi\sqrt{\mathcal{X}}$, which is the foundation of the flux monodromy models. Of course, the quadratic (23) will be modified when higher order top form corrections are included, but the branch structure will remain.

In retrospect, these are the exact same equations that arise if we vary the theory based on the Lagrangian (19) with respect to $\mathcal{A}_{\nu\lambda\sigma}$. Recalling that since $\mathcal{F}_{\mu\nu\lambda\sigma} = 4\partial_{[\mu}\mathcal{A}_{\nu\lambda\sigma]}$, we get the single field equation

$$\partial_{\mu}(\mathcal{F}_{\mu\nu\lambda\sigma} - \sqrt{\mathcal{X}}\hat{\theta}\epsilon^{\mu\nu\lambda\sigma}) = 0, \quad (26)$$

which after integrating once and contracting both sides of the integral with $\epsilon_{\mu\nu\lambda\sigma}$ yields precisely the two Eqs. (22), with $\mathcal{F}$ as the integration constant. In what follows, without any loss of generality, we will mainly resort to the magnetic pseudoscalar dual because the ensuing picture is simpler.

III. CP VIOLATION AND CP RESTORATION

A. The QCD top form as a diagnostic of CP violation

The emergent top form is a direct probe of CP violation in the theory. This is obvious even before the inclusion of the top form dynamics *à la* Lüscher [11], as displayed by

Eqs. (10), (11), (15), and (18). Combining these, to the leading order of the nonperturbatively corrected effective theory we find, in the Lorentzian signature, and taking into account our redefinition of Eq. (18)

$$\frac{\sqrt{\mathcal{X}}}{24} \epsilon_{\mu\nu\lambda\sigma} \mathcal{F}^{\mu\nu\lambda\sigma} = -\mathcal{X}\hat{\theta}. \quad (27)$$

Inverting the Hodge dual yields

$$\mathcal{F}_{\mu\nu\lambda\sigma} = \sqrt{\mathcal{X}}\hat{\theta}\epsilon_{\mu\nu\lambda\sigma}. \quad (28)$$

As we explained explicitly in the previous Section, this shows that the top form $\mathcal{F}_{\mu\nu\lambda\sigma}$ is a faithful probe of CP violation below chiral symmetry breaking: if CP is unbroken, $\hat{\theta} = 0$ and so $\mathcal{F}_{\mu\nu\lambda\sigma} = 0$. Conversely, if $\mathcal{F}_{\mu\nu\lambda\sigma} \neq 0$, $\hat{\theta} \neq 0$ as well and CP is violated.

The next order of the nonperturbatively corrected theory becomes more interesting. The quadratic $\mathcal{F}_{\mu\nu\lambda\sigma}$ terms extend the dynamics of the top form, which now is controlled by the equation (26) for the electric field strength, whose solution is

$$\mathcal{F}_{\mu\nu\lambda\sigma} = (\sqrt{\mathcal{X}}\hat{\theta} + \mathcal{F})\epsilon_{\mu\nu\lambda\sigma} = \sqrt{\mathcal{X}}\theta_{\mathcal{CP}}\epsilon_{\mu\nu\lambda\sigma}, \quad (29)$$

as per Eq. (24). This includes the totally arbitrary magnetic dual of top form, $\mathcal{F}$, which is an as-yet unspecified integration constant, and which arises due to the presence of the quadratic $\mathcal{F}^2$ in the Lagrangian (19); i.e., such a term only appears after full chiral symmetry breaking, in the IR regime. One can see that this term is precisely the integration constant noted in [2,6,7], which we reviewed above.

Now, first off, we see that (29) is still a solution of (26), which means that $\mathcal{F}_{\mu\nu\lambda\sigma}$ remains a faithful probe of CP violation. Second, from (7), it is clear that the integration constant $\mathcal{F}$, the “pure glue” vacuum angle θ and the overall phase of the quark mass matrix are completely degenerate in the full low energy theory. The only difference between θ and $\mathcal{F}$ is that $\mathcal{F}$ could be attributed to boundary conditions that select the solutions of (26).

This degeneracy between $\mathcal{F}$ and θ points to a possible way of addressing the observational fact that strong CP symmetry is approximately valid. If the boundary conditions for (26) could be chosen so that they select $\mathcal{F} = -\sqrt{\mathcal{X}}\hat{\theta}$, screening $\hat{\theta}$ with the top form flux, the strong CP would be completely restored. Alternatively, if additional degrees of freedom are introduced in the theory, which can dynamically adjust the flux to yield $\mathcal{F} = -\sqrt{\mathcal{X}}\hat{\theta}$, then the CP symmetry would also get restored.

B. The axion solution

Let us review here an explicit example of dynamical restoration [40] of strong CP symmetry, which relies on the axion, introduced in [41,42]. The axion could be any

dynamical phase with anomalous QCD couplings, which appears in the effective theory via the modification of the vacuum angle θ in (1) by

$$\theta \rightarrow \theta + \frac{\phi}{f_\phi} \quad (30)$$

where f_ϕ is the axion decay constant. In the case of a Peccei-Quinn (PQ) axion, it is controlled by the PQ symmetry breaking scale [40], and realistic models were considered in [43–46].

Once a dynamical field ϕ is added, the theory must be Lorentz covariantized by adding kinetic terms for ϕ . This means that the theory of the top form extended with the axion in terms of the magnetic dual of the top form is given by the (pseudo)scalar field action

$$S_{\mathcal{F}+\phi} = \int d^4x \left(\frac{1}{2} (\partial\phi)^2 - \frac{\mathcal{X}}{2} \left(\frac{\phi}{f_\phi} + \hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}} \right)^2 - \frac{\mathcal{F}}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \mathcal{A}_{\nu\lambda\sigma} \right). \quad (31)$$

Dualizing this action back to the electric top form yields, after integrating out $\mathcal{F}$, the theory in the mixed top form/scalar picture as given by [16,19,36]

$$S_{\mathcal{F}+\phi} = \int d^4x \left(\frac{1}{2} (\partial\phi)^2 - \frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2 + \frac{\sqrt{\mathcal{X}}}{24} \left(\frac{\phi}{f_\phi} + \hat{\theta} \right) \epsilon_{\mu\nu\lambda\sigma} \mathcal{F}^{\mu\nu\lambda\sigma} \right). \quad (32)$$

Finally, redefining the axion to $\varphi = \phi + f_\phi \hat{\theta}$ and defining the axion mass $m = \sqrt{\mathcal{X}}/f_\phi$, we can eliminate the scalar φ from the theory and rewrite the action purely in terms of the top form and its gauge potential. After integrating by parts the last term, completing the square for $\partial\varphi$, and then integrating out $\partial\varphi$ by trading it for the Stückelberg field $h_{\nu\lambda\sigma}$, we find [36]

$$S_{\mathcal{F}+\phi} = \int d^4x \left(-\frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2 + \frac{m^2}{12} (\mathcal{A}_{\nu\lambda\sigma} - h_{\nu\lambda\sigma})^2 + \frac{m\varphi}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu h_{\nu\lambda\sigma} \right). \quad (33)$$

Now φ is an auxiliary field; integrating it out reduces (33) to the massive 4-form gauge theory in an axial gauge. On the other hand, integrating out $h_{\mu\nu\lambda}$ brings the theory back to the mixed top form/axion picture (32).

The actions (31)–(33) all represent the same theory, as has been discussed⁴ at some length before [16,19,36].

⁴In particular, these actions were used in the formulation of high scale monodromy inflation [47–49] due to the large degree of symmetry that they exhibit, which, once mass scale m is fixed, protects the theory from both perturbative and nonperturbative corrections in quantum field theory and in quantum gravity.

In this work, our main interest is how the vacuum selection occurs in these descriptions, and what are its implications for the CP problem. The specifics were discussed in [16,19], which we follow here. The CP restoration occurs via the familiar axion rolling in the scalar axion picture (31), where the field equations and Poincaré invariance of the vacuum of the theory yield $\mathcal{F} = \text{const.}$, $\partial\phi = 0$. Therefore the vacuum value of ϕ is determined precisely by

$$\frac{\phi}{f_\phi} + \hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}} = 0. \quad (34)$$

Varying the action (31) with respect to $\mathcal{F}$ yields $\epsilon^{\mu\nu\lambda\sigma} \mathcal{F}_{\mu\nu\lambda\sigma} = -4! \sqrt{\mathcal{X}} (\phi/f_\phi + \hat{\theta} + \mathcal{F}/\sqrt{\mathcal{X}})$. Hence $\mathcal{F}_{\mu\nu\lambda\sigma}$ vanishes in this particular state, and so the state is CP invariant.

In the picture based on (32), CP restoration works similarly. Although the constant $\mathcal{F}$ does not appear in the action, it reemerges as the integration constant after solving the field equation for $\mathcal{F}_{\mu\nu\lambda\sigma}$, which yields $\mathcal{F}_{\mu\nu\lambda\sigma} = \sqrt{\mathcal{X}} \epsilon_{\mu\nu\lambda\sigma} (\phi/f_\phi + \hat{\theta} + \mathcal{F}/\sqrt{\mathcal{X}})$. The field equation for the axion is $\partial^2\phi = \frac{m}{24} \epsilon^{\mu\nu\lambda\sigma} \mathcal{F}_{\mu\nu\lambda\sigma}$, which in the Poincaré-invariant vacuum, by $\partial\phi = 0$, yields $\epsilon^{\mu\nu\lambda\sigma} \mathcal{F}_{\mu\nu\lambda\sigma} = 0$. As a consequence, the axion ϕ again satisfies (34), and CP is again restored in this state.

Finally, in the picture based on (33), we find just a theory of a massive top form. In the Proca gauge, the top form field solves

$$\partial^\mu \mathcal{F}_{\mu\nu\lambda\sigma} + m^2 \mathcal{A}_{\nu\lambda\sigma} = 0. \quad (35)$$

Now Poincaré invariance requires that $\partial^\mu \mathcal{F}_{\mu\nu\lambda\sigma}$ must vanish, and then (35) implies that $\mathcal{A}_{\nu\lambda\sigma}$ must also vanish up to a gauge transformation. In turn, $\mathcal{F}_{\mu\nu\lambda\sigma} = 0$ and this state, again, is CP invariant.

This picture has been recognized as the Higgs phase of the top form gauge theory [16,19] which led Dvali to infer that the axion mechanism for restoring CP , using the top form language, corresponds to screening of the CP violating top form, which is induced by the gauge field mass. Actually the top form theory in the Higgs phase shares many features of the effective theory of superconductivity [36]. We note that the question about effective theory describing condensates of extended objects has been raised by [50], and explored in [51].

While an axion is a very elegant mechanism for addressing the strong CP problem, it could have problems if it couples to some UV physics. The new UV physics could introduce new phases in the theory, which prevent the axion from acquiring the vacuum value (34), which selects $\theta_{\text{QCD}} = 0$. For example, if a new potential contribution $V(\phi)$ were to arise in (34), the axion vacuum value would instead be $m^2 f_\phi (\phi/f_\phi + \hat{\theta} + \mathcal{F}/\sqrt{\mathcal{X}}) + \partial_\phi V(\phi) = 0$. When the new potential is steeper, being controlled by higher scales,

and has minima displaced away from those in the QCD sector, the axion vacuum would be deflected from zero to values where the total strong CP -violating phase is

$$\theta_{\mathcal{CP}} = \frac{\phi}{f_\phi} + \hat{\theta} + \frac{\mathcal{F}}{\sqrt{\mathcal{X}}} = -\frac{1}{m^2 f_\phi} \partial_\phi V(\phi), \quad (36)$$

which would typically be greater than 10^{-10} without fine-tuning or new dynamics in the UV. In such situations one may need a different mechanism for CP restoration.

A clue that a different mechanism may be available is provided by the integration constant $\mathcal{F}$, which so far has been merely a spectator. In what follows, we will set the axion aside and focus on the general reinterpretation of the CP violating phases as integration constants, to seek a different relaxation mechanism to discharge them.

IV. QUANTAL THEORY OF CONSTANTS

A. Top form charges

It has long been argued that the 4-form sector of the effective theory of QCD below chiral symmetry breaking scale must be completed with the inclusion of the sources charged under the top form [2,11,32,33,37,52]. Given the spacetime properties of the gauge field potential $\mathcal{A}_{\mu\nu\lambda}$, the sources must be codimension-1 objects, which must be compact to have finite action. This means that in 4D the top form sources are membranes. In the effective theory we can view them as spherical domain walls separating the vacua with a different value of $\theta_{\mathcal{CP}}$, and this implies that they must have nonzero energy, represented by nonvanishing tension.

The key motivation for introducing these objects, as argued in [11,32,33,37,52], is related to confinement and the existence of light glueball states of QCD. Since QCD confines, the QCD vacuum at short and long distances is believed to have very different properties, which can be parametrized by $\mathcal{F}_{\mu\nu\lambda\sigma}$ variation over the spatial domain of a bound state. In particular, in the interior and exterior of the bound state, the value of $\mathcal{F}_{\mu\nu\lambda\sigma}$ is different: due to confinement, the field ranges of gauge fields are finite, and so all of them vanish at infinity. Inside, the theory is in deconfined phase below chiral symmetry breaking, and so it is characterized by $\mathcal{F}_{\mu\nu\lambda\sigma} \sim \theta_{\mathcal{CP}}$. Thus the value of $\mathcal{F}_{\mu\nu\lambda\sigma}$ must be controlled by boundary conditions far away and in the core.

While these effects originate from strong coupling regime in a quantum field theory (QFT), at low energies we can model the variation of $\mathcal{F}_{\mu\nu\lambda\sigma}$ by introducing a spherical membrane that is required to maintain Gauss's law by absorbing the flux of $\mathcal{F}_{\mu\nu\lambda\sigma}$ at the boundary of the bound state, to confine the interior fields. We can see this explicitly by going back to (19) and interpreting the bilinear term as a boundary term, $\frac{\sqrt{\mathcal{X}}}{24} \partial_\mu \epsilon_{\mu\nu\lambda\sigma} \mathcal{F}^{\mu\nu\lambda\sigma} = \frac{\sqrt{\mathcal{X}}}{6} \hat{\theta} \epsilon_{\mu\nu\lambda\sigma} \partial^\mu \mathcal{A}^{\nu\lambda\sigma}$, which, after integrating over the spherical surface bounding a region with $\mathcal{F}_{\mu\nu\lambda\sigma} \neq 0$ and $\mathcal{F}_{\mu\nu\lambda\sigma} = 0$ yields

$$S_{\mathcal{F}} = - \int d^4x \frac{1}{48} \mathcal{F}_{\mu\nu\lambda\sigma}^2 + \frac{\sqrt{\mathcal{X}} \hat{\theta}}{6} \int d^3\xi \mathcal{A}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}, \quad (37)$$

where the second term involves an integral over the sphere separating a deconfined interior and an exterior vacuum with all fields confined inside a small ball.

The surface term looks precisely like a membrane charge term for an infinitely thin membrane [37,52],

$$\mathcal{Q} = -\sqrt{\mathcal{X}} \hat{\theta}. \quad (38)$$

To complete the membrane Lagrangian, one would also need to add the tension term, which must arise as a measure of the rest energy of a membrane, essentially gapping it from the vacuum [52], which has the form

$$-\mathcal{T} \int d^3\xi \sqrt{\left| \det \left(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|}, \quad (39)$$

where the integration is over the membrane world volume, and $\mathcal{T}$ is the membrane tension. From this perspective, it appears that QCD in the strong coupling regime below chiral symmetry breaking “manufactures” its own system of membranes. This has been used to analyze the spectrum of glueballs in QCD, which were compared to the system of membranes in strong coupling in interesting papers [32,33]. There, and also earlier, in [52], numerical estimates of the scales of charges and tensions of the membranes were obtained, being set by the QCD scale $\Lambda_{\text{QCD}} \simeq 100$ MeV.

In the presence of these membranes, one would expect by comparison with [2,37], but also [38,39,53,54], that the magnetic dual top form flux $\mathcal{F}$ should be quantized in the units of charge $\mathcal{Q}$,

$$\mathcal{F} = \mathcal{N} \mathcal{Q} = -\mathcal{N} \sqrt{\mathcal{X}} \hat{\theta}, \quad (40)$$

where $\mathcal{N}$ is an integer. Further, the quantum nucleation of membranes [38,39,53,54], completely analogous to the Schwinger processes for particle production in a background field [55], would provide a channel for nonperturbative relaxation of $\mathcal{F}$ that could lead to a process for adjusting $\theta_{\mathcal{CP}}$ in discrete steps.

However there are questions concerning this idea, if one wants to use the membranes “emergent” within QCD itself. First of all, it is not clear how reliable is the thin-wall approximation for the membranes [32]. It is conceivable that at least some of these objects are really smeared rather than localized configurations. Specific examples were provided in the AdS/CFT approach to large N gauge theories in [25], where in fact the theory appears to have both thin and thick “membranes.” Second, investigations of the stability of the gauge theory vacua to nucleation of membranes, given in [22] in the large N limit, and in [23]

for smaller gauge group ranks like in QCD, suggest the decay rates may be small; e.g., [22] estimates $\Gamma \sim e^{-\text{const} \times N^4}$, where for small membranes in approximately flat space, appropriate for this case, the constant factor in the exponent already includes a factor of $27\pi^2/2$ [56]. If the remaining factor were $\mathcal{R} \simeq T^4/Q^6 \gtrsim 1$, on dimensional grounds, the rate would be too slow: e.g., using $N_{\text{QCD}} = 3$ suggests that $\Gamma \sim \exp(-\frac{27\pi^2}{2} N_{\text{QCD}}^4 \mathcal{R}) \lesssim \exp(-10^5 \mathcal{R})$. Another estimate [23] for finite N_{QCD} arrives at the similar quantitative conclusion. On the other hand, explicit examples [24,25] exist that suggest that the barrier to tunneling diminishes as N grows large.

This example nevertheless points to a dynamical channel that could yield a viable mechanism for restoring CP . Since the main unknown is the nucleation rate suppression, a possible avenue is to introduce a system of “fundamental” membranes with charges and tensions that are determined by dynamics outside of QCD, and then mix it with the CP -violating operators in QCD. This could happen if the top form of QCD kinetically mixes with some fundamental 4-form field strength that arises in the UV completion of the theory, as in the supergravity examples considered in [2], or with an emergent top form from another hidden sector gauge theory. We take this bottom-up route, and treat the

charge and tension of the membranes as input parameters, whose values are to be selected to ensure that the mechanism succeeds.

B. Adding another top form

We begin by extending the magnetic dual top form action (21) with an inclusion of a copy of the top form sector, which is almost identical to the “emergent” top form of QCD. The difference is that we also add a membrane charged under the new top form, with tension $\mathcal{T}$ and charge $\mathcal{Q}$, which are input parameters of the theory, unrelated to the QCD scale. We ignore the “emergent” QCD defects reviewed above given the uncertainties about their decay rates derived from QCD, and our bottom-up approach. We model the nontrivial dynamics as arising only in the new top form sector, and continue to treat the emergent electric top form of QCD as a spectator, which will discharge since it mixes with the new top form.

Our choice of the couplings of the two top forms is dictated by the axion couplings [41,42], replicating precisely the expression for the potential in the axion-extended action (31) [16]. Using the magnetic dual variables, the new action is

$$S_{F+\mathcal{H}} = \int d^4x \left(-\frac{1}{2} \left(\mathcal{H} + \sqrt{\mathcal{X}} \hat{\theta} + \frac{F}{2\pi^2 \sqrt{\mathcal{X}}} \right)^2 + \frac{1}{24\pi^4 \mathcal{X}} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (F) A_{\nu\lambda\sigma} + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (\mathcal{H}) C_{\nu\lambda\sigma} \right) - \mathcal{T} \int d^3\xi \sqrt{\left| \det \left(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|} - \frac{\mathcal{Q}}{6} \int d^3\xi C_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}, \quad (41)$$

where $\mathcal{H}$ is the new top form. The 3-form $\mathcal{C}$ comes in as a Lagrange multiplier enforcing the field equation $\partial\mathcal{H} = 0$. Here we have reverted to using the emergent QCD top form, which is not canonically normalized, that was directly calculated by Lüscher [11]. The rescaling is given in Eq. (18). This will aid in the phenomenological considerations to follow. The bilinear terms $\propto \epsilon^{\mu\nu\lambda\sigma}$ in the first row were integrated by parts to correctly covariantize the top forms on the membranes, because dynamically they are spherical boundaries of regions with different $\theta_{\mathcal{CP}}$ (see, e.g., [38,39]). Here $\mathcal{T}$ and $\mathcal{Q}$ are related to a mass scale $\mathcal{M}$, which controls the new top form dynamics, be it emergent or fundamental, by formulas like $\mathcal{T} = \zeta \mathcal{M}^3$ and $\mathcal{Q} = \xi \mathcal{M}^2$, where ζ and ξ are dimensionless constants, presumably $\mathcal{O}(1)$. We will confirm this expectation shortly.

Varying the action with respect to the variables $F, A_{\nu\lambda\sigma}$, and $\mathcal{H}, C_{\nu\lambda\sigma}$ yields bulk equations away from the membranes, which look like two copies of Eq. (22), with slightly modified terms reflecting the field contents in (42). Inverting the Hodge dual in the second of (22) and incorporating (42), we obtain, using $F_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} A_{\nu\lambda\sigma]}$, $\mathcal{H}_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} C_{\nu\lambda\sigma]}$,

$$\begin{aligned} \partial_\mu F &= \partial_\mu \mathcal{H} = 0, \\ \frac{1}{2\pi^2 \sqrt{\mathcal{X}}} F_{\mu\nu\lambda\sigma} &= \mathcal{H}_{\mu\nu\lambda\sigma} = \left(\mathcal{H} + \sqrt{\mathcal{X}} \hat{\theta} + \frac{F}{2\pi^2 \sqrt{\mathcal{X}}} \right) \epsilon_{\mu\nu\lambda\sigma}. \end{aligned} \quad (42)$$

As before, away from the membranes both fluxes $\mathcal{F}$ and $\mathcal{H}$ are constant. However, once membranes are included, the flux $\mathcal{H}$ that is sourced by them will change discretely across a membrane, and acquire different values inside and out. Note that from the second of Eq. (42) we immediately see that after the inclusion of the second top form $\mathcal{H}$, the CP -violating phase is

$$\theta_{\mathcal{CP}} = \frac{\mathcal{H}}{\sqrt{\mathcal{X}}} + \hat{\theta} + \frac{F}{2\pi^2 \mathcal{X}}. \quad (43)$$

Note also that the theory (42), as it stands, is well defined after chiral symmetry breaking, when $\mathcal{X} > 0$, with some subtleties to account for before chiral symmetry breaking when $\mathcal{X} = 0$. In this limit the term that depends on the

vacuum angle $\hat{\theta}$ immediately drops out, as it should. The magnetic dual F is decoupled; in the action (42), in the limit $\mathcal{X} \rightarrow 0$ the topological susceptibility $\mathcal{X}$ plays the role analogous to $G_N \sim 1/M_{\text{Pl}}^2$ in gravity, and so its vanishing implies decoupling of F , which is fixed to a constant background value. The only acceptable value is zero, since otherwise the action diverges. This is analogous to how a

flat space is singled out in the weak coupling limit of gravity. Further the QCD top form has not emerged yet; from Eq. (42) we see that $F_{\mu\nu\lambda\sigma} \equiv 0$. The CP symmetry has not broken, since the theory still has massless quarks, and so the quark mass matrix phase is undetermined. Thus in this limit the top form part of the action comes down to only

$$S_{F+\mathcal{H}} \rightarrow \int d^4x \left(-\frac{1}{2} \mathcal{H}^2 + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu(\mathcal{H}) \mathcal{C}_{\nu\lambda\sigma} \right) - \mathcal{T} \int d^3\xi \sqrt{\left| \det \left(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|} - \frac{\mathcal{Q}}{6} \int d^3\xi \mathcal{C}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}. \quad (44)$$

As per the discussion in Sec. II and in [38,39], it is straightforward to see that the first line of this action is just the dual of $-\frac{1}{48} \int d^4x \mathcal{H}_{\mu\nu\lambda\sigma}^2$: the top form $\mathcal{H}$ completely decoupled from QCD. In this limit, there may well be a term in the vacuum angle $\theta \ni \epsilon^{\mu\nu\lambda\sigma} \mathcal{H}_{\mu\nu\lambda\sigma}$ that however is invisible since it always gets compensated by another phase; it only reappears after chiral symmetry breaking. This means, that the quantum channels that can discharge the CP -violating phase θ_{QCD} are completely inaccessible until after chiral symmetry breaking.

With this in mind, we now assume that chiral symmetry breaking has taken place, $\mathcal{X} > 0$, and we consider quantum mechanical membrane production via Schwinger processes [55]. The original, general analysis was presented in [53,54] for both theories without and with gravitational effects included. As we declared before, here we neglect gravity, decoupling it by taking the limit $M_{\text{Pl}} \rightarrow \infty$, and consider the field theory limit in Euclidean space. We will confirm the validity of this approximation in what follows below.

We need the Euclidean action to calculate the membrane nucleation rates [56–58]. The reformulation of (42) to Euclidean space is straightforward; we follow the steps in [38,39], but we ignore the gravitational terms. First, we Wick rotate the action using $t = -ix_E^0$, which gives $-i \int d^4x \sqrt{g} \mathcal{L}_{\text{QFT}} = - \int d^4x_E \sqrt{g} \mathcal{L}_{\text{QFT}}^E$. Next, using $A_{0jk} = A_{0jk}^E$, $A_{jkl} = A_{jkl}^E$, and likewise for $\mathcal{C}$, and $\epsilon_{0ijk} = \epsilon_{0ijk}^E$ and $\epsilon^{0ijk} = -\epsilon^{0ijk}_E$, the tension and charge terms, with notation $\gamma = |\det(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b})|$, transform to $-iT \int d^3\xi \sqrt{\gamma} = -\mathcal{T} \int d^3\xi_E \sqrt{\gamma}$ and $i\mathcal{Q} \int \mathcal{C}_i = -\mathcal{Q} \int \mathcal{C}_i$, and the scalars do not change. The Euclidean action, defined by $iS = -S_E$, is

$$S_E = \int d^4x_E \left(\frac{1}{2} \left(\mathcal{H} + \sqrt{\mathcal{X}} \hat{\theta} + \frac{F}{2\pi^2 \sqrt{\mathcal{X}}} \right)^2 + \frac{1}{24\pi^4 \mathcal{X}} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu(F) A_{\nu\lambda\sigma}^E + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu(\mathcal{H}) \mathcal{C}_{\nu\lambda\sigma}^E \right) + \mathcal{T} \int d^3\xi_E \sqrt{\gamma} - \frac{\mathcal{Q}}{6} \int d^3\xi_E \mathcal{C}_{\mu\nu\lambda}^E \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}. \quad (45)$$

From here on we drop the index E .

We can use the Euclidean action (45) to construct the configurations that are comprised of sections of the 4D sphere S^4 , cojoined along a fixed S^3 latitude sphere, which represents the Euclidean world volume of a spherical membrane bearing the tension $\mathcal{T}$ and charge $\mathcal{Q}$. Detailed constructions of such configurations and an investigation of their properties can be found in [38,39,53,54], where many configurations arise because they are supported by gravitational effects. When gravity is decoupled by taking the limit $M_{\text{Pl}} \rightarrow \infty$, only one of those configurations remains viable. This configuration is the Euclidean bounce relating the backgrounds with non-negative vacuum energy given by

$$V = \frac{1}{2} \left(\mathcal{H} + \sqrt{\mathcal{X}} \hat{\theta} + \frac{F}{2\pi^2 \sqrt{\mathcal{X}}} \right)^2, \quad (46)$$

where the interior and exterior regions have the values of $\mathcal{H}$ that differ from one another by a unit of charge. This follows immediately from (45): varying this action with respect to $\mathcal{C}_{\nu\lambda\sigma}$ and taking the induced metric on the membrane to be S^3 yields [38,39,53,54]

$$n^\mu \partial_\mu \mathcal{H} = \mathcal{Q} \delta(r - r_0), \quad (47)$$

where n^μ is the outward-oriented normal to the membrane, r_0 its radius and r the coordinate along n^μ . Integrating this gives

$$\mathcal{H}_{\text{out}} - \mathcal{H}_{\text{in}} = \mathcal{Q}, \quad (48)$$

as claimed. Since $\mathcal{H}$ can only vary in steps of $\mathcal{Q}$, we can take it to be quantized, $\mathcal{H} = \mathcal{N} \mathcal{Q}$. However, $\mathcal{H}$ is degenerate with $\sqrt{\mathcal{X}} \hat{\theta}$ and $\frac{F}{2\pi^2 \sqrt{\mathcal{X}}}$, and both of these variables can take an arbitrary value, e.g., $\hat{\theta} \ni \text{Arg} \det \mathcal{M}$, which given the arbitrariness of the overall phase of the quark mass matrix, implies that $\hat{\theta}$ is a continuous variable on a circle. Thus we will ignore the question about the quantization of $\mathcal{H}$ since it is moot here.

There is one more constraint governing membrane dynamics, which follows from energy conservation. The energy

difference between the values of V inside and outside the membrane, induced by the jump in $\mathcal{H}$ by a unit of membrane charge $\mathcal{Q}$ in Eq. (47), must be precisely balanced by the membrane tension $\mathcal{T}$. The membrane nucleates with the radius set by total energy conservation, so that the gain in energy incurred by the decay of $\mathcal{H}$ will be compensated by the cost of nucleating a membrane whose total rest energy is given by tension times its proper Euclidean volume [56]. To find it, starting with the action (45), we note that (48) implies that the bilinear terms from the top line of (45) cancel against the charge term from the bottom line of (45). Next we recall that the relevant action governing the nucleation dynamics is the tunneling bounce action, which is a difference of the action (45) on a geometry that includes a single bubble, and the same action on the initial smooth background geometry without any bubbles. Subtracting the two, what remains is the gain minus the cost: the integral of the potential difference over the volume of S^4 , minus the tension integrated over the membrane volume S^3 [56]. Since both integrands are constant over their respective domain of integration, and the relevant volume factors are $V_{S^4} = \pi^2 r_0^4/2$ and $V_{S^3} = 2\pi^2 r_0^3$, formally the membrane radius minimizes the action

$$S_{\text{membrane}} = 2\pi^2 r_0^3 \mathcal{T} - \frac{1}{2} \pi^2 r_0^4 \Delta V, \quad (49)$$

where $\Delta V = V_{\text{out}}(\mathcal{H}_{\text{out}}) - V_{\text{in}}(\mathcal{H}_{\text{in}})$, and $\mathcal{H}_{\text{out,in}}$ are related to each other by (48). Minimizing (49) with respect to r_0 yields the membrane radius at nucleation [56],

$$r_0 = \frac{3\mathcal{T}}{\Delta V}, \quad (50)$$

and, by evaluating (49) at $r = r_0$, we find the actual bounce action [56]

$$B = \frac{27\pi^2}{2} \frac{\mathcal{T}^4}{(\Delta V)^3}. \quad (51)$$

The bubble nucleation rate per unit time per unit volume then is given by $\Gamma = Ae^{-B}$ [56,57], and to compute it we need the prefactor A . The prefactor for membrane production in the flat space limit, where gravity is negligible, has been obtained by Garriga in [58], and using his results, we finally obtain

$$\Gamma \simeq 9 \frac{\mathcal{T}^4}{(\Delta V)^2} \exp\left(-\frac{27\pi^2}{2} \frac{\mathcal{T}^4}{(\Delta V)^3}\right). \quad (52)$$

Note that inside each membrane, the potential V jumps by approximately

$$\Delta V = \mathcal{Q}\left(\mathcal{H} + \sqrt{\mathcal{X}}\hat{\theta} + \frac{F}{2\pi^2\sqrt{\mathcal{X}}}\right). \quad (53)$$

The top form also decreases because it depends on $\mathcal{H}$. Inside each new bubble, the top form decreases by

$$\Delta F_{\mu\nu\lambda\sigma} = 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}\epsilon_{\mu\nu\lambda\sigma}. \quad (54)$$

Since the membrane tension is strictly positive, membrane discharge drives θ_{CP} toward restoring CP , as dictated by [31]. The reverse processes are excluded by (50) in the flat space limit, since transitions increasing the potential and making CP violation worse require $\Delta V < 0$, which with positive membrane tension also requires $r_0 < 0$, which is unphysical.

An extension of this argument shows that discharges must stop before $F_{\mu\nu\lambda\sigma} = 0$, unless the initial value of $F_{\mu\nu\lambda\sigma}$ is fine-tuned to be an integer multiple of $2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$. Once $|F_{\mu\nu\lambda\sigma}| < 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$, the next discharge would have to produce a state with $|F_{\mu\nu\lambda\sigma}| > 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$, which would *increase* the potential inside the membrane. But for such a bubble to get on shell, r_0 would have to be negative by Eq. (50), since $\mathcal{T} > 0$ and $\Delta V < 0$. Again, this is unphysical; therefore the nucleation processes terminate when $|F_{\mu\nu\lambda\sigma}| < 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$. This was also found in [54], and it remains true regardless of whether the discharges are applied to relaxing the CP -violating phase or the cosmological constant.

Further conditions on the dynamics come from phenomenological considerations. The discharge cascade needs to relax θ_{CP} from an initial natural value of $\mathcal{O}(1)$ down to $\theta_{\text{CP}} \lesssim 10^{-10}$. Since $F_{\mu\nu\lambda\sigma} = 2\pi^2\mathcal{X}\theta_{\text{CP}}\epsilon_{\mu\nu\lambda\sigma}$, the bound $|F_{\mu\nu\lambda\sigma}| < 2\pi^2\sqrt{\mathcal{X}}\mathcal{Q}$ and Eq. (54) means that we must take

$$\frac{\mathcal{Q}}{\sqrt{\mathcal{X}}} \lesssim 10^{-10}. \quad (55)$$

Since $\mathcal{X}^{1/4} \simeq \text{few} \times 100 \text{ MeV}$, this implies

$$\mathcal{Q}^{1/2} \lesssim \sqrt{10^{-11} \text{ GeV}^2} \simeq 3 \text{ keV}. \quad (56)$$

Next, the discharges should be fast [59–61]. The nucleation rate (52) should be more efficient than the cosmic dilution at the time of chiral symmetry breaking. The corresponding Hubble scale is roughly $H_{\text{QCD}} \simeq (100 \text{ MeV})^2/M_{\text{Pl}} \simeq 10^{-20} \text{ GeV}$, and imposing $\Gamma/H_{\text{QCD}}^4 \gtrsim 1$ [59–61] gives

$$10^{81} \frac{\mathcal{T}^4}{(\Delta V)^2 (\text{GeV})^4} \gtrsim \exp\left(\frac{27\pi^2}{2} \frac{\mathcal{T}^4}{(\Delta V)^3}\right). \quad (57)$$

The nucleation processes are the slowest close to the CP -restoring state, at which point, for charges satisfying (55), we have $\Delta V = (\mathcal{Q})^2 \lesssim 10^{-22} \text{ GeV}^4$. Thus we need

$$10^{31} \frac{\mathcal{T}}{(\text{GeV})^3} \gtrsim \exp\left(\frac{27\pi^2}{8} \times 10^{66} \times \left(\frac{\mathcal{T}}{\text{GeV}^3}\right)^4\right). \quad (58)$$

The right-hand side term is the controlling term here. The discharges would be extremely slow unless $\frac{27\pi^2}{8} \times 10^{66} \times \left(\frac{\mathcal{T}}{\text{GeV}^3}\right)^4 \lesssim 1$. In turn this requires that $\mathcal{T}^{1/3} \lesssim 10^{-3} \text{ MeV} = \text{keV}$, give or take. Taking into account the factor of 10^{31} on the left-hand side, the numbers are a bit relaxed, yielding

$$\mathcal{T}^{1/3} \lesssim 3 \text{ keV}. \quad (59)$$

Given (55), as long as the membrane tension satisfies this inequality, the nucleation processes would be fast enough to discharge θ_{CP} within $\mathcal{O}(10)$ Hubble times at the QCD phase transition scale, even if the initial state was close to the minimal achievable θ_{CP} . Curiously, both $\mathcal{Q}$ and $\mathcal{T}$ appear to be set by the same scale, $\mathcal{M} \sim 3 \text{ keV}$.

For a generic initial state with $\theta_{\text{CP}} \gg 10^{-10}$, the discharges would be even faster. Indeed, Eq. (58) for a general initial θ_{CP} is modified to

$$10^{31} \frac{\mathcal{T}}{(\text{GeV})^3} \sqrt{\frac{\Delta\theta_{\text{CP}}}{\theta_{\text{CP}}}} \gtrsim \exp\left(\frac{27\pi^2}{8} \times 10^{66} \times \left(\frac{\mathcal{T}}{\text{GeV}^3}\right)^4 \times \left(\frac{\Delta\theta_{\text{CP}}}{\theta_{\text{CP}}}\right)^3\right), \quad (60)$$

and so as θ_{CP} increases from 10^{-10} toward unity, the left-hand side decreases as a square-root of θ_{CP} , but the right-hand side decreases much faster, as its cubic power. When Eq. (59) holds and $\theta_{\text{CP}} \simeq 1$, early on in the discharge cascade, the right-hand side is unity, for all practical intents and purposes, and the tunneling is extremely rapid.

The Eq. (55) implies that there would be many individual steps in the discharge cascade of 10^{10} units of $\mathcal{Q}$ required to reduce θ_{CP} to meet the experimental bounds. Since the nucleation processes are so fast, this appears viable. As a comparison, the axion also needs to traverse—continuously—the same target space distance as it sets the modulus θ_{CP} to zero. In any case as long as the tension and the charge of the membranes satisfy Eqs. (59) and (55), respectively, no part of the Universe would end up being stuck in regions where $\theta_{\text{CP}} > 10^{-10}$ for long enough to raise the question of why we do not see a larger CP violation.

Since at this point we have at least some quantitative ideas about the scales controlling the membrane and $\mathcal{H}$ dynamics, we can verify that our neglecting gravity is justified. Let us compare the radius of a bubble at nucleation (50) to the Hubble length at the QCD chiral symmetry breaking scale. Since $H_{\text{QCD}} \simeq \sqrt{\mathcal{X}}/M_{\text{Pl}}$, we find

$$r_0 H_{\text{QCD}} \simeq \frac{\mathcal{T}}{\Delta V} \frac{\sqrt{\mathcal{X}}}{M_{\text{Pl}}} \simeq \frac{\mathcal{T}}{\sqrt{\mathcal{X}} M_{\text{Pl}}} \frac{1}{\theta_{\text{CP}} \Delta\theta_{\text{CP}}}. \quad (61)$$

Next, using Eq. (59), we find that for $\theta_{\text{CP}} \sim \Delta\theta_{\text{CP}} \sim 10^{-10}$,

$$r_0 H_{\text{QCD}} \lesssim 10^{20} \frac{\mathcal{T}}{\sqrt{\mathcal{X}} M_{\text{Pl}}} \simeq 10^{-12}. \quad (62)$$

Initially, when $\theta_{\text{CP}} \sim 1$, the bubbles are much smaller, with $r_0 H_{\text{QCD}} \lesssim 10^{-22}$. So at the QCD chiral symmetry breaking scale, all the bubbles are very small compared to the background curvature scale $1/H_{\text{QCD}} \simeq M_{\text{Pl}}/\sqrt{\mathcal{X}} \simeq 10^{11} (eV)^{-1} \simeq 100 \text{ km}$. We can always pick a large freely falling local frame, smaller than $1/H_{\text{QCD}}$ and much larger than the size of a nucleating bubble, inside which the nucleation occurs in the same way as in flat space. Further, while in the presence of gravity additional tunneling processes become possible, when the principal channel that remains as $M_{\text{Pl}} \rightarrow \infty$ is so fast, these new channels are inconsequential, since they are very suppressed. Ergo, we can ignore any gravitational corrections in the semiclassical approximation; a quick check is to see that the bubble nucleation equations are precisely the same as those of [38,39,53,54] in the limit $M_{\text{Pl}} \rightarrow \infty$.

We note that even if some domain walls had not completely disappeared, but instead had survived to the present day, or had been recreated at some scale much lower than the QCD chiral symmetry breaking scale, they would have remained very stealthy so far, provided that they do not move too fast. Since the tension that satisfies the bound (58) is well below the limits discussed in [62], any lingering domain wall fragments could not have affected late cosmology (i.e., induced anisotropy in the CMB) in too dramatic a way. A more precise reevaluation of this issue might be warranted. We will set this question aside for the moment, and move on to a more detailed discussion of the cosmology of CP restoration.

V. COSMOLOGICAL CP RESTORATION

We assume that the Universe underwent inflation followed by efficient reheating. After reheating the Universe is evolving as a radiation-dominated, spatially flat, isotropic, and homogeneous Friedmann-Robertson-Walker (FRW) cosmology, with temperature well above the QCD scale $\Lambda_{\text{QCD}} \sim \text{GeV}$. We take this background as given, and ignore the details about where it came from, how the cosmological constant was canceled, and, once we have the FRW metric, even the dynamical effects of gravity.

When the early Universe temperature significantly exceeds Λ_{QCD} , the QCD sector will initially have unbroken chiral symmetry, and the quarks will be massless. The unbroken symmetry implies two important features of the dynamics: 1) initially, all the θ_{CP} states will be degenerate, and 2) due to massless quarks, the mass matrix phase $\text{Arg det } \mathcal{M}$ in Eq. (7) will be completely indeterminate. As a result, the QCD sector will not contribute to CP violation initially.

As the Universe cools, the situation changes dramatically. First, electroweak symmetry will break as the temperature

drops to below TeV. Subsequently chiral symmetry breaking will also occur. The main electroweak breaking consequence will be fixing the contribution of the quark mass matrix to θ_{CP} , although different θ_{CP} phases may still remain degenerate until chiral symmetry breaking when the instanton effects arise (see, e.g., [63] for a review).

Note that from Eq. (57), the discharges of θ_{CP} induced by the discharges of the flux $\mathcal{H}$ do not occur until $H \lesssim H_{\text{QCD}}$ because the vacua with different θ_{CP} are still degenerate. The topological susceptibility is zero early on, and the QCD top form is effectively set to zero, and so it is decoupled from $\mathcal{H}$. The flux of $\mathcal{H}$ may change, but even its discharges not affecting θ_{CP} will be slower early on, because thanks to our choice of the membrane tension and charge, the barrier to membrane nucleation will suppress more efficiently the flux decay for earlier times $t < 1/H_{\text{QCD}}$.

As the Universe cools, the electroweak breaking fixes the quark matrix and its contribution to the CP -breaking phase θ_{CP} . Soon after, the instanton contributions lift degeneracy between different θ_{CP} states and, with the tunneling barrier becoming low, rapid discharges begin. Indeed, even if the initial phase varied randomly from one causal domain at the QCD scale to another,⁵ these regions would develop vacuum energy induced by the condensate, $V = \frac{1}{2} \mathcal{X} \theta_{\text{CP}}^2$, as given in Eq. (46). Consequently, the domains with $\theta_{\text{CP}} = \frac{\mathcal{H}}{\sqrt{\mathcal{X}}} + \hat{\theta} + \frac{F}{2\pi^2 \mathcal{X}} \gg 0$ become unstable to nucleation of bubbles surrounded by membranes with tension $\mathcal{T}$ and charge $\mathcal{Q}$. Since the bubbles are very small compared to the background curvature, $r_0 H_{\text{QCD}} \sim 10^{-22} - 10^{-12} \ll 1$, the flat space analysis suffices⁶ [64].

We can see this manifestly by noting that we can use the strong CP -violating phase θ_{CP} as the discharging variable. In this “axial gauge,” since $\mathcal{H}$ is the only variable flux, we can treat the definition of the total CP -violating phase $\theta_{\text{CP}} = \frac{\mathcal{H}}{\sqrt{\mathcal{X}}} + \hat{\theta} + \frac{F}{2\pi^2 \mathcal{X}}$ as the change of variables from $\mathcal{H}$ to θ_{CP} . Adopting this viewpoint, we can trade $\mathcal{H}$ in Eq. (42) for θ_{CP} , and drop the remaining terms $\propto F$ since they do not change. In those variables, the truncated action (42) becomes, after rescaling $\mathcal{C}_{\nu\lambda\sigma} = \frac{1}{\sqrt{\mathcal{X}}} \mathcal{Y}_{\nu\lambda\sigma}$,

$$S_{\text{CP}} = \int d^4x \left(-\frac{\mathcal{X}}{2} \theta_{\text{CP}}^2 + \frac{1}{6} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu (\theta_{\text{CP}}) \mathcal{Y}_{\nu\lambda\sigma} \right) - \mathcal{T} \int d^3\xi \sqrt{\left| \det \left(\eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right) \right|} - \frac{\mathcal{Q}}{6\sqrt{\mathcal{X}}} \int d^3\xi \mathcal{Y}_{\mu\nu\lambda} \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \frac{\partial x^\lambda}{\partial \xi^c} \epsilon^{abc}. \quad (63)$$

⁵One would not expect this randomness of θ_{CP} in a region that originates from a single causally connected domain before inflation.

⁶As is commonly done, we fine-tune the cosmological constant of the minimal θ_{CP} state to a sufficiently small value.

It is now obvious that the membrane discharges relax θ_{CP} in finite steps of $\frac{\mathcal{Q}}{\sqrt{\mathcal{X}}}$. Further it is also obvious that as $\mathcal{X} \rightarrow 0$, and so chiral symmetry is restored, the discharge processes are inaccessible. The CP -violating contribution to the potential vanishes, and the effective charge $\frac{\mathcal{Q}}{\sqrt{\mathcal{X}}}$ for θ_{CP} diverges in this limit. This makes the barrier to tunneling impassable, as we can readily check by changing variables in the decay rate formula (52).

Since the potential difference between adjacent vacua is $\Delta V \simeq \mathcal{X} \theta_{\text{CP}} \Delta \theta_{\text{CP}} = \sqrt{\mathcal{X}} \mathcal{Q} \theta_{\text{CP}}$,

$$\Gamma \simeq 9 \frac{\mathcal{T}^4}{\mathcal{X} \mathcal{Q}^2 (\theta_{\text{CP}})^2} \exp \left(-\frac{27\pi^2}{2} \frac{\mathcal{T}^4}{\mathcal{X}^{3/2} \mathcal{Q}^3 (\theta_{\text{CP}})^3} \right). \quad (64)$$

Recalling that $\mathcal{T} = \zeta \mathcal{M}^3$ and $\mathcal{Q} = \xi \mathcal{M}^2$, with ζ and ξ being $\mathcal{O}(1)$, and that $\mathcal{X} \simeq \Lambda_{\text{QCD}}^4$, we can rewrite the bounce action in Eq. (64) as

$$B = \frac{27\pi^2 \zeta^4}{2\xi^3} \left(\frac{\mathcal{M}}{\Lambda_{\text{QCD}}} \right)^6 \frac{1}{(\theta_{\text{CP}})^3}, \quad (65)$$

and the prefactor, in the units of H_{QCD} , as

$$\frac{A}{H_{\text{QCD}}^4} = \frac{9\zeta^4}{\xi^2} \left(\frac{\mathcal{M}}{\Lambda_{\text{QCD}}} \right)^8 \left(\frac{M_{\text{Pl}}}{\Lambda_{\text{QCD}}} \right)^4 \frac{1}{(\theta_{\text{CP}})^2}. \quad (66)$$

As we have chosen $\mathcal{M} \sim \text{keV}$, the ratios of scales are $\mathcal{M}/\Lambda_{\text{QCD}} \sim 10^{-5}$ and $M_{\text{Pl}}/\Lambda_{\text{QCD}} \sim 2 \times 10^{19}$. Thus,

$$B \simeq 100 \frac{\zeta^4}{\xi^3} \frac{1}{(10^{10} \theta_{\text{CP}})^3}, \quad \frac{A}{H_{\text{QCD}}^4} \simeq \frac{\zeta^4}{\xi^2} \frac{1}{(\theta_{\text{CP}})^2}, \quad (67)$$

which indeed confirm that the decay rates for the membranes with tension and charge in the keV range are just right to restore CP . Note in particular that as soon as chiral symmetry breaking condensate emerges, the barrier to tunneling that leads to membrane discharges is completely irrelevant when $\theta_{\text{CP}} \sim 1$, and those discharges are very fast. Only as θ_{CP} is discharged down to $\theta_{\text{CP}} \sim 10^{-10}$, the tunneling barrier can suppress discharges. Further, we also see from Eqs. (65) and (66) that in the limit $\Lambda_{\text{QCD}} \rightarrow 0$, of vanishing chiral symmetry-breaking condensate, the membrane discharge stops. The prefactor normalized to the QCD scale does diverge as $1/\Lambda_{\text{QCD}}^{12}$, but the exponent of the bounce action converges to zero faster.

The discharge dynamics has some similarities with the irrational axion proposal of [65]. Since the $\theta_{\text{CP}} \neq 0$ vacua are unstable, they will decay toward the CP -invariant vacuum $\theta_{\text{CP}} = 0$ by discharging the $\mathcal{H}$ flux. This will proceed as long as the emergent top form $F_{\mu\nu\lambda\sigma}$ is nonzero, and is large enough such that discharge reduces the potential (46). These two criteria mean that the discharges will proceed until $|F_{\mu\nu\lambda\sigma}| < 2\pi^2 \sqrt{\mathcal{X}} \mathcal{Q}$, as we explained in

Sec. IV.2. Once this value is reached, the total CP -violating phase is reduced to

$$\theta_{CP} \lesssim \frac{Q}{\sqrt{\chi}}. \quad (68)$$

The nucleations will not stop any sooner than this.

There are however important differences between our proposal and the model presented in [65]. In the irrational axion model, while the transitions between subsequent θ_{CP} in a cascade toward $\theta_{CP} = 0$ are discrete, the step $\Delta\theta_{CP}$ is controlled by the approximation of an integer by an irrational number, and can be arbitrarily small. Hence, the universe in [65] will continue to evolve toward $\theta_{CP} = 0$. However, the adjacent θ_{CP} states are generically separated by huge field space distances, and therefore the potential barrier separating them is typically very broad. Those transitions would behave as membranes with large tension, whose nucleation rate would be extremely slow. Thus while the universe continues to search for the state that restores CP , finding it typically would take a time much larger than the age of the universe. In contrast, in our case we consider membranes with small finite charges $Q/\sqrt{\chi} \lesssim 10^{-10}$ (55) and small tension (59). For these reasons, the discharge stops at $\theta_{CP} \lesssim 10^{-10}$, and this occurs fast.

We should stress that the terminal $\theta_{CP} \sim 10^{-10}$ should not be viewed as a prediction. In our proposal 10^{-10} is really an input, which we adopt since we are interested in a proof of principle. However, if we have a $\sim \text{keV}$ scale top form and membranes, we will get that the decay rate (52) is fast compared to the age of the universe at the chiral symmetry breaking scale, as seen in (57). Thus the nucleations would rapidly minimize CP -breaking effects throughout the universe by the production of bubbles of the smallest attainable θ_{CP} . This argument actually illustrates some flexibility of our mechanism being able to address even smaller terminal values of the strong CP phase θ_{CP} , if they are eventually found. For example, values of θ_{CP} smaller than 10^{-10} could be tested by future experiments [66,67], that might reveal θ_{CP} to be as small as 10^{-13} . Even so, our mechanism would remain operational if the charge and tension were reduced down to about $\sim \mathcal{O}(100)eV$.

After nucleation, the membranes would expand at the speed of light. Upon their collision, they would burn away by the production of strongly interacting particles, i.e., predominantly pions. We can deduce this as follows; from the potential (46) given above, we see that below the chiral symmetry breaking, the theory contains a term

$$\begin{aligned} \int d^4x \mathcal{L}_{F+\mathcal{H}} &\ni \frac{1}{2\pi^2\sqrt{\chi}} \int d^4x \mathcal{H} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma} \\ &\sim \frac{12}{\pi^2\sqrt{\chi}} \int d^4x \mathcal{H} \partial_\mu K^\mu, \end{aligned} \quad (69)$$

where in the last step we used Eqs. (3) and (5). After a chiral transformation the Chern-Simons current will shift, on shell, to $\partial K \rightarrow \partial K + f_\pi \partial^2 \mathcal{P}$, with $\mathcal{P}$ denoting the pion field. Substituting this into (69) yields

$$\frac{12f_\pi}{\pi^2\sqrt{\chi}} \int d^4x \mathcal{H} \partial^2 \mathcal{P}. \quad (70)$$

Integrating this term by parts and recalling that $\partial \mathcal{H} = 0$ away from membranes, and $\Delta \mathcal{H} = Q$ on a membrane, we finally find

$$\begin{aligned} \frac{12f_\pi}{\pi^2\sqrt{\chi}} \int d^4x \mathcal{H} \partial^2 \mathcal{P} &\sim \frac{2f_\pi}{\pi^2} \frac{Q}{\sqrt{\chi}} \int d^3\xi n \cdot \partial \mathcal{P} \det \left| \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \right| \\ &\sim 10^{-11} f_\pi \int_{R \times S^2} d^3\xi n \cdot \partial \mathcal{P}, \end{aligned} \quad (71)$$

where n is the outward normal to the membrane. In this equation we used the bound on Q from Eq. (55). Hence a membrane colliding with another membrane will bremsstrahlung pions, breaking up and transferring its energy to the pion shower.

Although the formula (71) includes a suppression of 10^{-11} , when the membranes are nucleated, they are very small, $r_0 H_{\text{QCD}} \sim 10^{-22} - 10^{-12}$. Since they are moving at the speed of light, by the time they reach the size of the horizon at the chiral symmetry breaking scale, they will have propagated through the background FRW for a time that is at least a factor of 10^{12} larger than their natural size. So even if they only emit a pion with a small probability over a period r_0 , the emission rate will accumulate over the time of order the age of the universe at the QCD scale, and the energy transfer from the membrane walls can be completed.

Additionally, since the relaxation dynamics occurs in discrete steps, and the bubbles are colliding with each other, these processes could produce a contribution to the stochastic gravity wave background similar to the mechanisms of [27,28]. Estimating the qualitative features of their spectrum is complicated because the bubble nucleation processes here are very fast and very prolific. The bubbles nucleated are very small, and as many as 10^{10} could be produced inside each Hubble domain, prompting a careful exploration along the lines of [28] and subsequent detailed work. For this reason we set aside the precise discussion of gravity wave production for a later time.

Let us mention one more possible class of phenomena that could be viewed as the signatures of the model. It has been noted that the bubbles of other phases of QCD could be produced in high energy collisions [17,68–72]. In the interior of these bubbles, one would find the CP -violating vacua, as argued in [68–72]. In our case, the smaller bubbles, which would be more accessible to local scattering processes, and which would have large CP violation inside, would be very short-lived. Their radius, by using Eq. (61), would be as small as $r_0 \sim 10^{-14} \text{ mm}$, and so we expect they

would implode in about $\sim 10^{26}$ sec, since the interior CP -breaking phase is unstable. The large bubbles with very small CP violation could be longer-lived, collapsing within $\sim 10^{16}$ sec. However since already in standard QCD similar $\theta_{Q\bar{P}}$ -inducing effects can appear [68–72], given that the charge of membranes sourcing $\mathcal{H}$ is quite small, the new effects would be a correction on top of the standard QCD phenomena. In any case, understanding better such phenomena might still be interesting, and we hope to return to it in the future.

VI. SUMMARY

Nonabelian gauge theories with chiral symmetry breaking have a rich and complex structure, including nontrivial topological sectors. Those can give rise to the nonperturbatively induced corrections to the effective theory below the chiral symmetry breaking, which, while not propagating local fluctuations, can mediate transitions between different topological sectors.

An example is a top form in QCD. Its flux is sourced by the CP -breaking phase, and so it is a faithful diagnostic of strong CP violation. The top form flux however need not be globally constant because it could be relaxed by tunneling between different topological sectors in the theory. The tunneling dynamics could be modeled by including charged tensional membranes, which can discharge the flux by the Schwinger processes catalyzed by the background field [55]. This mechanism is as ubiquitous in gauge theory as is particle pair production due to the uncertainty principle, where the pair gets on shell due to the work exerted by the background fields.

Hence the discharge is not only possible, but inevitable, as long as the field energy density exceeds the membrane rest energy. Thus a discharge of the QCD top form could be a channel for reducing CP violation, since the top form flux is proportional to the CP -violating phase $\theta_{Q\bar{P}}$, if the discharges are sufficiently fast. A possible channel that facilitates fast discharges may arise if the QCD top form mixes with another top form, either emergent or fundamental. As long as the charges of the membranes sourcing the new top form are comparable to their tensions, and the tensions are not too great, ~ 3 keV the discharge processes could be fast enough, and remove the CP -violating phase rapidly.

In this work we presented a simple model showing how this can proceed, which could approximately restore CP by rapid cosmological membrane production, which looks like very fast phase transitions around the chiral symmetry breaking scale. The discharges should be fast since CP breaking occurs around the chiral symmetry breaking scale, which is within a few orders of magnitude from the Big-Bang Nucleosynthesis (BBN) scale. To play it safe the transitions should complete by BBN, maintaining the

delicate balance between the various ingredients of the universe at that time.

The price to pay is to have membranes with both charges and tensions being rather small. A small charge ensures that the discharge will be refined enough to yield the terminal $\theta_{Q\bar{P}}$ below 10^{-10} , and a small membrane tension then makes the discharges fast, achieving $\theta_{Q\bar{P}} \lesssim 10^{-10}$ before BBN. Curiously, both criteria can be fitted naturally by picking a single scale for the membrane charge and tension, which is about 3 keV. Obvious questions about these requirements is how well are such regimes under control, and how they might be realized.

Regarding the smallness of charges and tensions, the former could be accomplished by, e.g., kinetic mixing, similar to how the millicharge particles arise in field theory [73]. Another way to do this is by UV completing the theory in some framework with compact large extra dimensions, which can make the final $4D$ value of charge of a membrane smaller than the fundamental scale since it involves a factor of compact internal space volume [74]. It is also possible that there are multiple forms of $\mathcal{H}_i$, with larger charges but with small differences between them.

The paper [74] also points out a way to make small tensions, by wrapping p branes on shrinking cycles in internal tensions. However, recall that small tension is merely a simple way to implement fast transitions. An alternative could be to have enhanced decay rates even without making tensions small. This requires different decay channels for false vacua with $\theta_{Q\bar{P}} > 10^{-10}$. Several different approaches were devised over time, including enhancements from thermal background effects [75,76], favorable energetics of transition that enhance decay rates by favoring simultaneous multimembrane emissions [75,77,78], catalysis of fast transitions by classical bubble collisions [79], or rapid discharge cascades by unwinding of higher dimensional objects that appear as membranes after compactification to $4D$ [80]. Yet another possibility might also be using heavier axions, which reside in their potential minima at chiral symmetry breaking, but where potentials have a large number of vacua [81,82]. Some of these vacua might be “close” in field space, with enhanced tunneling rates, allowing for fast transitions. Those transitions could appear as rapid membrane nucleations if the axion multiverse was realized in ways that incorporate flux monodromies [83].

Given the existence of the flux discharge by membrane emissions, and the possibility to use it as an extension of the axion relaxation of $\theta_{Q\bar{P}}$, it seems warranted to examine the flux discharge dynamics as a possible means to resolve the strong CP problem more closely.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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