

Enhanced quark-loop contribution to pure annihilation nonleptonic B -meson decays in the PQCD approach

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In this study, we conduct an enhanced investigation of pure annihilation-type charmless hadronic B decays within the framework of the perturbative QCD approach. Specifically, we account for the quark-loop enhanced contribution to various decay processes at the next-to-leading order (NLO) in the strong coupling constant α_s . The NLO amplitudes possess large imaginary parts that have signs opposite to those of the leading-order amplitudes. This cancellation effect implies that the NLO contribution does not significantly influence the branching ratios of the processes under consideration. However, the NLO effect explored in this work constitutes an important source of the strong phases in the weak-annihilation nonleptonic $\bar{B}_q$ -meson decay amplitudes. As a result, it can have a substantial impact on CP -violating observables such as the direct CP asymmetry $\mathcal{A}_{CP}^{\text{dir}}$ and mixing-induced CP asymmetry $\mathcal{A}_{CP}^{\text{mix}}$. The numerical result evidently revealed that NLO QCD correction significantly enhances both $\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$ of the pure annihilation-type B decay processes.

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I. INTRODUCTION

Exclusive two-body charmless bottom-meson decays hold fundamental significance. They are crucial for determining the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and exploring CP violation in electroweak interactions. Moreover, these decays offer an ideal context for enhancing our understanding of the diverse aspects of strong interaction dynamics that govern flavor-changing heavy-quark decay processes. Approximately 40 years ago, the factorization approach based on the color transparency assumption was employed to study hadronic B -decay processes. However, it failed to accurately predict experimental observables such as CP violations. In the past two decades, the development of QCD-inspired effective theories has remarkably enhanced the accuracy of theoretical predictions for the partial widths of these decays [1,2]. The QCD factorization (QCDF) [3,4] approach, founded on collinear factorization, has been extensively utilized to study various two-body nonleptonic B -decay channels,

and most of its predictions for decay rates are consistent with experimental measurements. The QCD factorization framework can be more gracefully reformulated using the language of the soft-collinear effective theory [5–8]. The perturbative QCD (PQCD) [9–20] approach is established on the basis of transverse-momentum-dependent factorization. Significantly, within this framework, the CP -violation observables of most nonleptonic B decay processes have been successfully predicted.

The QCDF approach performs effectively at leading power in the heavy-quark expansion. However, power suppressed contributions, like those from the annihilation topology, cannot be accurately computed. This is because of the troublesome end point divergences present in the convolution integrals of perturbation function and the light-cone distribution amplitudes of initial and final states. Although efforts have been made to address the end point singularity within the framework of collinear factorization, a universally accepted solution remains elusive [21–25]. In the QCDF approach, the annihilation contribution is estimated through process-independent parametrizations for logarithmically and linearly divergent integrals [26–29] (see also [30–38] and the references therein); unfortunately, this method leads to substantial theoretical uncertainties. In contrast, within the framework of the PQCD approach, the end point singularity is regularized by taking into account the transverse momentum of the partons within the

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initial- and final-state mesons. As a result, the annihilation diagrams are considered factorizable in the PQCD approach. It has been discovered that this contribution can supply the strong phases necessary for direct CP asymmetries in nonleptonic B -meson decays.

Pure annihilation-type nonleptonic B -meson decays can only take place via annihilation diagrams within the Standard Model. This is because none of the quarks (or antiquarks) in the final states match those of the initial B meson. Studying such processes offers a unique perspective on understanding the QCD dynamics in heavy hadron decays. Within the PQCD framework, various pure annihilation-type B decays have been explored at the α_s order [11,39–50]. The predicted branching ratios are in agreement with experimental measurements. With the continuous accumulation of experimental data, more observables, such as CP asymmetries in the pure annihilation-type B decays, are expected to be measured in the near future. For certain decay modes like $B_s \rightarrow \pi\pi$, the predicted CP violation was very small in the previous studies. This is because the direct CP violation is proportional to the ratio of the magnitudes of the contributions from the tree operators and the penguin operators, as well as the sine of the strong phase difference between them. Moreover, the contribution from the tree operators is suppressed by the CKM mechanism compared with that of the penguin operators, and the strong phase difference between them is also very small. A recent study [51] has revealed that the charm-quark-loop contribution can effectively modify the ratio of the magnitudes and strong phase difference of the contributions from the tree operators and the penguin operators, despite an additional α_s suppression. It has been found that the quark-loop contribution has a significant impact on CP asymmetries, while only slightly affecting the branching ratios. In [51], to ensure the convergence of the convolution integral, only the contributions from twist-2 light-cone distribution amplitudes (LCDAs) of light mesons were considered. As a result, the predicted branching ratios for pure annihilation charmless hadronic B decays are considerably smaller than experimental measurements.

In this paper, within the framework of the PQCD approach, we will conduct a more in-depth study of the pure annihilation-type charmless hadronic B -meson decays, which include processes such as $B^0 \rightarrow K^+K^-$, $K^{*+}K^{*-}$, $\phi\phi$, and $B_s \rightarrow \pi\pi, \rho\rho, \rho\omega, \omega\omega$. The main improvement of this paper lies in taking into account the quark-loop contributions in various decay channels, and this contribution is expected to have a significant impact on the CP -violation observables. Since the PQCD approach can effectively eliminate the end point singularities, in our calculations, we will incorporate the contributions from the twist-2 and two-particle twist-3 light-cone distribution amplitudes of the final-state light mesons. It is expected that, compared with the leading-order contributions in previous studies, the introduction of the quark loop does not significantly change the branching ratios of the pure annihilation decay processes, while the observables of the

direct CP violation and the mixing-induced CP violation have both changed significantly. In this paper, we do not consider the $B \rightarrow PV$ (P denotes a pseudoscalar meson and V stands for a vector meson) decays, because the quark-loop effect does not contribute to these processes. The structure of this paper is arranged as follows. In the next section, we will briefly review the theoretical framework of the PQCD factorization approach and the calculation of the leading-order decay amplitude. Then, in Sec. III, we will perform the perturbative calculations for the enhanced quark-loop contributions, while the numerical results and phenomenological analysis will be presented in Sec. IV. In the last section, we will provide a brief summary and discussion.

II. THE DECAY AMPLITUDES AT LEADING ORDER

We start from the effective weak Hamiltonian of the nonleptonic $\Delta B = 1$ transitions in the Standard Model,

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pq}^* \left[C_1(\nu) Q_1^p(\nu) + C_2(\nu) Q_2^p(\nu) + \sum_{i=3}^{10} C_i(\nu) Q_i(\nu) + C_{8g}(\nu) Q_{8g}(\nu) \right], \quad (1)$$

where we adopt the effective operator basis presented in [52]. The corresponding Wilson coefficients $C_i(\nu)$ with $\nu \sim \mathcal{O}(m_b)$ can be derived through renormalization group evolution. The decay amplitude of $B \rightarrow M_2 M_3$ decays is, in fact, the matrix elements of the effective Hamiltonian, i.e.,

$$\bar{\mathcal{A}}(\bar{B}_q \rightarrow M_2 M_3) = \langle M_2(p_2) M_3(p_3) | \mathcal{H}_{\text{eff}} | \bar{B}_q(p_B) \rangle. \quad (2)$$

In the PQCD approach, the decay amplitudes are free from end point singularities. This is because the transverse momenta of the partons within the initial- and final-state hadrons are retained, and these amplitudes are considered to be completely factorizable. The typical factorization formula in the PQCD approach is given by

$$\begin{aligned} \mathcal{M} \propto & \int_0^1 \prod_{i=1}^3 dx_i \int \prod_{j=1}^3 d^2 \vec{b}_j \phi_B(x_1, \vec{b}_1, t) \phi_{M_2}(x_2, \vec{b}_2, t) \\ & \times \phi_{M_3}(x_3, \vec{b}_3, t) T_H(x_1, x_2, x_3, \vec{b}_1, \vec{b}_2, \vec{b}_3, t) \\ & \times \exp[-S_B(t) - S_2(t) - S_3(t)], \end{aligned} \quad (3)$$

where $\vec{b}_j$ is the impact parameter conjugated to the parton's transverse momentum $\vec{k}_{\perp j}$. The wave functions $\phi_M(x_i, \vec{b}_i, t)$ are related to the transverse-momentum-dependent (TMD) wave functions $\phi_M(x_i, \vec{k}_{\perp i}, t)$ via Fourier transformations. The TMD wave functions defined on the light cone possess light-cone singularity, which needs to be eliminated in a more precise definition. It is convenient to sum up the large

logarithms $\ln^2(k_\perp^2/Q^2)$ and $\ln(k_\perp^2/Q^2)$ (Q^2 denotes a hard scale) arising from the radiative corrections to the wave functions in the impact parameter space. Resummation of the large logarithms leads to the Sudakov form factors $S_B(t)$, $S_2(t)$, and $S_3(t)$, which are given in Appendix A. The perturbative function $T_H(x_1, x_2, x_3, \vec{b}_1, \vec{b}_2, \vec{b}_3, t)$ is calculated in the momentum space, and it is also required to be transformed into the impact space.

The leading-order Feynman diagrams for the pure annihilation-type nonleptonic B decays are depicted in Fig. 1. As a tree-level calculation, the decay amplitudes can be computed using the trace formalism. For final-state light mesons, we disregard the parton's transverse momentum in the wave functions since it has negligible impact on the phenomenological results. The factorization formulas of the $B^0 \rightarrow K^+ K^-$, $K^{*+} K^{*-}$, $\phi\phi$, and $B_s \rightarrow \pi\pi, \rho\rho, \rho\omega, \omega\omega$ decays have already been given in [12,53]. In the following, we present the specific expressions. For the $B \rightarrow PP$ decays, we have

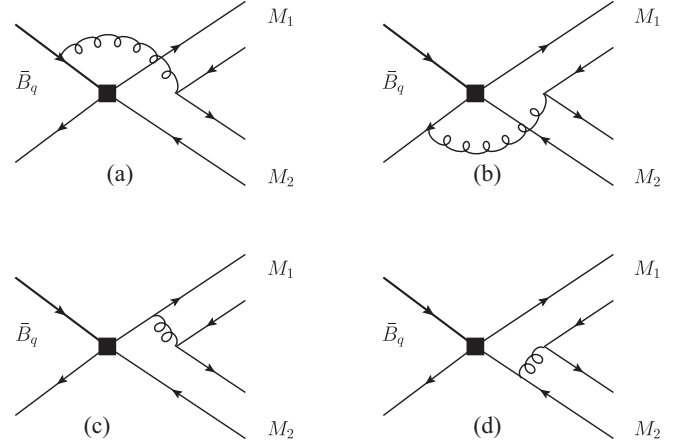


FIG. 1. The leading-order Feynman diagrams for annihilation contribution, with possible four-quark operator insertions. Figures (a) and (b) denote the diagram with a gluon emission from the initial state quarks, and Figs (c) and (d) the diagram with a gluon emission from the final state quarks

$$\begin{aligned} \mathcal{A}_{\text{LO}}(\bar{B}_s^0 \rightarrow \pi^+ \pi^-) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ \mathcal{M}_{\text{LO}}^{LL} \left[2C_4 + \frac{1}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP} \left[2C_6 + \frac{1}{2} C_8 \right] \right\} + \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \mathcal{M}_{\text{LO}}^{SP} [C_2]; \\ \sqrt{2} \mathcal{A}_{\text{LO}}(\bar{B}_s^0 \rightarrow \pi^0 \pi^0) &= \mathcal{A}_{\text{LO}}(\bar{B}_s^0 \rightarrow \pi^+ \pi^-); \\ \mathcal{A}_{\text{LO}}(\bar{B}_d^0 \rightarrow K^+ K^-) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \mathcal{M}_{\text{LO}}^{LL} \left[2C_4 + \frac{1}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP} \left[2C_6 + \frac{1}{2} C_8 \right] \right\} + \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \mathcal{M}_{\text{LO}}^{LL} [C_2], \end{aligned} \quad (4)$$

where $\mathcal{M}_{\text{LO}}^{LL(R)}$ and $\mathcal{M}_{\text{LO}}^{SP}$ are derived through calculating the Feynman diagrams in Figs. 1(c) and 1(d) by inserting four-quark operators $\bar{q}_1 \gamma^\mu (1 - \gamma_5) b \bar{q}_2 \gamma_\mu (1 - (+)\gamma_5) q_1$ and $\bar{q}_1 (1 - \gamma_5) b \bar{q}_2 (1 + \gamma_5) q_1$ (after Fierz transformation), respectively, and they read

$$\begin{aligned} \mathcal{M}_{\text{LO}}^{LL} [C_i] &= 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\ &\times \{ [-x_2 \phi_2^A \phi_3^A - 4r_2 r_3 \phi_2^P \phi_3^P + r_2 r_3 (1 - x_2) (\phi_2^P + \phi_2^T) (\phi_3^P - \phi_3^T) \\ &+ r_2 r_3 x_3 (\phi_2^P - \phi_2^T) (\phi_3^P + \phi_3^T)] a_i(t_d) E'_a(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2) \\ &+ [(1 - x_3) \phi_2^A \phi_3^A + r_2 r_3 (1 - x_3) (\phi_2^P + \phi_2^T) (\phi_3^P - \phi_3^T) \\ &+ r_2 r_3 x_2 (\phi_2^P - \phi_2^T) (\phi_3^P + \phi_3^T)] C_i(t'_d) E'_a(t'_d) h'_{na}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (5)$$

$$\begin{aligned} \mathcal{M}_{\text{ann}}^{SP} [C_i] &= 32\pi C_F M_{B_s}^4 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \\ &\times \{ [(x_3 - 1) \phi_2^A \phi_3^A - 4r_2 r_3 \phi_2^P \phi_3^P + r_2 r_3 x_3 (\phi_2^P + \phi_2^T) (\phi_3^P - \phi_3^T) \\ &+ r_2 r_3 (1 - x_2) (\phi_2^P - \phi_2^T) (\phi_3^P + \phi_3^T)] a_i(t_d) E'_a(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2) \\ &+ [x_2 \phi_2^A \phi_3^A + r_2 r_3 x_2 (\phi_2^P + \phi_2^T) (\phi_3^P - \phi_3^T) \\ &+ r_2 r_3 (1 - x_3) (\phi_2^P - \phi_2^T) (\phi_3^P + \phi_3^T)] C_i(t'_d) E'_a(t'_d) h'_{na}(x_1, x_2, x_3, b_1, b_2) \}. \end{aligned} \quad (6)$$

In the above equations, we have employed the shorthand notation $\phi_i^{A,P,T} \equiv \phi^{A,P,T}(x_i)$. Here ϕ^A represents twist-2 LCDA of a pseudoscalar meson, and $\phi^{P,T}$ stand for two-particle twist-3 LCDAs of a pseudoscalar meson. The ratios $r_i = m_{0i}/m_{B_q}$, where m_{0i} is the chiral scale. The contributions from Figs. 1(a) and 1(b) cancel each other out. This cancellation occurs due to the conservation of the vector current and axial-vector current, which are constructed from massless quark fields.

The $B \rightarrow VV$ decays are more complicated than $B \rightarrow PP$ modes since they contain three kinds of polarization states of final-state vector mesons, namely, longitudinal (L), normal (N), and transverse (T) polarization. The amplitudes that are corresponding to the longitudinal, normal, and transverse polarization amplitudes, respectively, $\mathcal{A}^{(\lambda)}$ can be decomposed as follows:

$$\mathcal{A}(B_q \rightarrow VV) = \mathcal{A}_L + \mathcal{A}_N \epsilon_{2T}^* \cdot \epsilon_{3T}^* + i \frac{\mathcal{A}_T}{m_{B_q}^2} \epsilon^{\alpha\beta\rho\sigma} \epsilon_{2\alpha}^* \epsilon_{3\beta}^* p_{2\rho} p_{3\sigma}. \quad (7)$$

Similar to the $B \rightarrow PP$ decays, the amplitudes $\mathcal{A}_{L,N,T}$ can also be obtained by calculating the Feynman diagrams depicted in Fig. 1. The specific expressions for the longitudinal, normal, and transverse polarization amplitudes at leading order are collected in Appendix B.

III. THE QUARK-LOOP ENHANCED CONTRIBUTION

In this section, our objective is to compute the next-to-leading-order (NLO) corrections to the decay amplitudes of pure annihilation nonleptonic B decays. We will focus on the contributions from quark-loop diagrams and the chromomagnetic penguin operator. The corresponding Feynman diagrams are shown in Figs. 2 and 3. It is true that there are other NLO corrections, such as QCD corrections to the diagrams in Fig. 1. However, these are less significant compared to the contributions considered in this work. This is because the quark-loop diagrams may be enhanced by the CKM matrix elements or Wilson

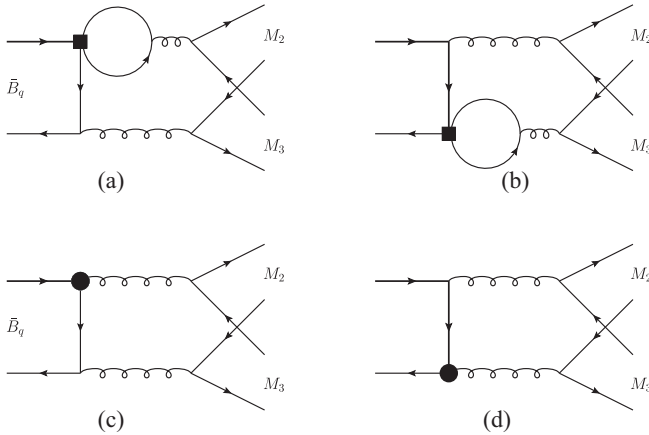


FIG. 2. The Feynman diagrams for next-to-leading-order corrections to annihilation contribution. (a) and (d) describe the quark-loop contribution, where the black square represents four-quark operators; (c) and (d) illustrate the contribution from the chromomagnetic operator, where the black dot denotes O_{8g} . There also exist four diagrams where the gluon propagators attached to M_2 and M_3 exchange with each other, and they are not explicitly shown

coefficients. We take $\bar{B}_s \rightarrow \pi^+ \pi^-$ decay as an example. At leading order, this process is penguin dominated since the tree diagram contribution is suppressed by $|V_{ub} V_{us}^* / V_{tb} V_{ts}^*| \sim 0.02$; meanwhile, the charm-loop diagrams have both large Wilson coefficient C_2 and large CKM matrix element $V_{cb} V_{cs}^*$, albeit with an additional factor $\alpha_s / 4\pi$, and it is expected to be comparable to the leading-order penguin contribution. Upon careful examination [51], it has been determined that the contribution from the triangle diagrams shown in Fig. 3 is significantly more suppressed compared to that of the diagrams in Fig. 2. As a result, these triangle-diagram contributions have been omitted from the current study for the sake of computational efficiency and theoretical focus.

The final states of the pure annihilation-type charmless nonleptonic B decays are particle-antiparticle pairs, and it is obvious from Fig. 2 that NLO amplitude of $B \rightarrow M_1 M_2$ decays can be expressed as

$$\mathcal{A}_{\text{NLO}} = \langle M_2(p_2) M_3(p_3) | g^*(p_g, \alpha) g^*(\tilde{p}_g, \beta) \rangle \times \langle g^*(p_g, \alpha) g^*(\tilde{p}_g, \beta) | \mathcal{H}_{\text{eff}} | \bar{B}_q \rangle. \quad (8)$$

At leading power, the internal gluons must be transversely polarized since it attaches a collinear quark field and an anticollinear quark field; then the $\bar{B}_q \rightarrow g^*(p_g, \alpha) g^*(\tilde{p}_g, \beta)$ transition matrix element can be parametrized as

$$\langle g^*(p_g, \alpha) g^*(\tilde{p}_g, \beta) | \mathcal{H}_{\text{eff}} | \bar{B}_q \rangle = i \epsilon_{\alpha\beta p_g \tilde{p}_g} F_V(p_g^2, \tilde{p}_g^2) + g_{\alpha\beta}^\perp F_A(p_g^2, \tilde{p}_g^2). \quad (9)$$

According to the analysis presented in [51], the form factors $F_{V,A}$ satisfy the relations $F_V(p_g^2, \tilde{p}_g^2) = -F_V(\tilde{p}_g^2, p_g^2)$ and $F_A(p_g^2, \tilde{p}_g^2) = F_A(\tilde{p}_g^2, p_g^2)$. The antisymmetric property of $F_V(p_g^2, \tilde{p}_g^2)$ implies that the NLO contribution considered in this paper will not contribute to $B \rightarrow PV$ decays at leading power. However, if twist-3 LCDAs are taken into account, the internal gluons may not be transverse; this is because the Lorentz structure of a power suppressed collinear quark field differs from that of a leading power one, and the previous conclusion would no longer be valid. Meanwhile, the twist-3 contribution to

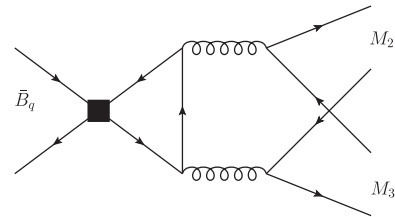


FIG. 3. The triangle diagrams for next-to-leading-order corrections to annihilation contribution. The black square denotes four-quark operators. The gluon propagator exchanged diagram is also not explicitly shown.

$\langle P(p_2)V(p_3)|g^*(p_g,\alpha)g^*(\tilde{p}_g,\beta)\rangle$ indeed vanishes, because the number of γ matrices of the LCDA pseudoscalar and vector mesons is odd.

In the calculation of quark-loop diagrams in Fig. 2, we neglect the transverse momenta of the partons in initial- and final-state mesons since no additional end point divergence will be introduced, and a similar approach

has been adopted in [53]. To facilitate comparison between the NLO results and the leading-order results, we first integrate out the fields within the quark loop, the gluon field connecting the quark loops in Figs. 2(a) and 2(b), and the gluon field coupling to the O_{8g} operator in Figs. 2(b) and 2(c). This process yields two effective operators

$$C_{D,1}^{\text{eff}}\bar{D}\gamma^\mu(1-\gamma_5)T^ab\bar{q}\gamma_\mu T^aq + C_{D,2}^{\text{eff}}\bar{D}\not{q}\gamma^\mu(1+\gamma_5)T^ab\bar{q}\gamma_\mu T^aq, \quad (10)$$

where $D = d, s$ denoting a down-type quark. The effective Wilson coefficients are expressed as

$$\begin{aligned} C_{D,1}^{\text{eff}} &= [V_{ub}V_{uD}^*G(0) + V_{cb}V_{cD}^*G(m_c)]C_2 - V_{tb}V_{tD}^*[G(0) + G(m_b)]C_3 \\ &\quad - (C_4 + C_6)V_{tb}V_{tD}^*[3G(0) + G(m_c) + G(m_b) - 20/3], \\ C_{D,2}^{\text{eff}} &= -\frac{4m_b}{q^2}V_{tb}V_{tD}^*(C_5 + C_{8g}). \end{aligned} \quad (11)$$

The function $G(m)$ arising from the loop diagrams can be decomposed as

$$G(m) = G_q(m) + G_m(m), \quad (12)$$

with

$$\begin{aligned} G_q(m) &= -\frac{4}{3}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2} + \frac{1}{6}\right) + \frac{8}{3}\sqrt{\frac{4m^2}{q^2} - 1} \arctan\frac{1}{\sqrt{\frac{4m^2}{q^2} - 1}}, \\ G_m(m) &= \frac{16m^2}{3q^2}\sqrt{\frac{4m^2}{q^2} - 1} \arctan\frac{1}{\sqrt{\frac{4m^2}{q^2} - 1}} - \frac{28m^2}{3q^2}. \end{aligned} \quad (13)$$

If $m \rightarrow 0$, we have

$$G_0 = G_q(0) = -\frac{4}{3}\left[\frac{1}{\epsilon} + \ln\left(-\frac{\mu^2}{q^2}\right) + \frac{1}{6}\right]. \quad (14)$$

Accordingly, the effective Wilson coefficient $C_{D,1}^{\text{eff}}$ needs to be written as $C_{D,1}^{\text{eff}} = C_{D,1q}^{\text{eff}} + C_{D,1m}^{\text{eff}}$, where $C_{D,1q}^{\text{eff}}$ and $C_{D,1m}^{\text{eff}}$ have the following form:

$$\begin{aligned} C_{D,1q}^{\text{eff}} &= [V_{ub}V_{uD}^*G_0 + V_{cb}V_{cD}^*G_q(m_c)]C_2 - V_{tb}V_{tD}^*[G_0 + G_q(m_b)]C_3 \\ &\quad - (C_4 + C_6)V_{tb}V_{tD}^*[3G_0 + G_q(m_c) + G_q(m_b) - 20/3], \\ C_{D,1m}^{\text{eff}} &= V_{cb}V_{cD}^*G_m(m_c)C_2 - V_{tb}V_{tD}^*G_m(m_b)C_3 - (C_4 + C_6)V_{tb}V_{tD}^*[G_m(m_c) + G_m(m_b)]. \end{aligned} \quad (15)$$

Taking advantage of the effective operators in Eq. (10), we obtain the factorization formulas for NLO amplitudes of $\bar{B}_q \rightarrow PP$ decays as follows:

$$\begin{aligned} \mathcal{M}_{\text{NLO}}^{LV} &= -3C_F^2M^4/\sqrt{6} \int dx_1 dx_2 dx_3 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \{ \mathbf{H}_L(\mathbf{x}, \mathbf{b}) \mathbf{C}_{D,1}^{\text{eff}}(t_n) \\ &\quad \times [(x_2 - x_3 + 1)\Phi_{23}^{AA} + 2r_2 r_3(x_2 - x_3 + 3)\Phi_{23}^{PP} + 2r_2 r_3(-x_2 + x_3 + 1)\Phi_{23}^{TT}] E'_{a'}(t_n) \\ &\quad + \mathbf{H}'_L(\mathbf{x}, \mathbf{b}) \mathbf{C}_{D,1}^{\text{eff}}(t_n)(x_2 - x_3 + 1)[2r_2 r_3(\Phi_{23}^{TT} - \Phi_{23}^{PP}) - \Phi_{23}^{AA}] E'_{a'}(t'_n) \}, \end{aligned} \quad (16)$$

$$\begin{aligned}
\mathcal{M}_{\text{NLO}}^{\text{RV}} = & -3C_F^2 M^5 / \sqrt{6} \int dx_1 dx_2 dx_3 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \{ h_{\text{nan}}(x_1, x_2, x_3, b_1, b_2) C_{D,2}^{\text{eff}}(t_n), \\
& \times [(2x_2 x_3 - x_2 - x_3 + 1) \Phi_{23}^{AA} + 4r_2 r_3 (x_2 x_3 - x_2 + 1) \Phi_{23}^{PP} + 4r_2 r_3 (-x_2 x_3 + x_3) \Phi_{23}^{TT} \\
& + 2r_2 r_3 (x_2 + x_3 - 1) (\Phi_{23}^{TP} + \Phi_{23}^{PT})] E'_a(t_n) + h'_{\text{nan}}(x_1, x_2, x_3, b_1, b_2) C_{D,2}^{\text{eff}}(t'_n) \\
& \times [(2x_2 x_3 - x_2 - x_3 + 1) (\Phi_{23}^{AA} + 4r_2 r_3 \Phi_{23}^{PP}) + 2r_2 r_3 (x_2 + x_3 - 1) (\Phi_{23}^{TP} + \Phi_{23}^{PT})] E'_a(t'_n) \}. \quad (17)
\end{aligned}$$

Here $\Phi_{23}^{ab} \equiv \phi^a(x_2) \phi^b(x_3) + \phi^a(1-x_2) \phi^b(1-x_3)$, where the second term stems from the crossing diagram which is not depicted Fig. 2. The NLO Feynman diagrams involve three propagators. In comparison with the leading-order scenario, a new type of hard function emerges. We write $\mathbf{H}_L^{(i)}(\mathbf{x}, \mathbf{b}) = (h_{na}^{(i)}(x_1, x_2, x_3, b_1, b_2), h'_{\text{nan}}(x_1, x_2, x_3, b_1, b_2))$, and the effective Wilson coefficients $\mathbf{C}_{D,1}^{\text{eff}}(t_n) = (C_{D,1q}^{\text{eff}}(t_n), C_{D,1m}^{\text{eff}}(t_n))^T$. The specific expression for $h_{na}^{(i)}(x_1, x_2, x_3, b_1, b_2)$ is shown in Appendix A. The NLO contribution to the $\bar{B}_s \rightarrow \pi\pi$ decay amplitude is then expressed by

$$\mathcal{A}_{\text{NLO}}(\bar{B}_s^0 \rightarrow \pi^+ \pi^-) = \frac{G_F}{\sqrt{2}} \mathcal{M}_{\text{NLO}}^{LV}[\mathbf{C}_{s,1}^{\text{eff}}] + \frac{G_F}{\sqrt{2}} \mathcal{M}_{\text{NLO}}^{RV}[\mathbf{C}_{s,2}^{\text{eff}}]. \quad (18)$$

The relation $\sqrt{2} \mathcal{A}_{\text{NLO}}(\bar{B}_s^0 \rightarrow \pi^0 \pi^0) = \mathcal{A}_{\text{NLO}}(\bar{B}_s^0 \rightarrow \pi^+ \pi^-)$ still holds. Similarly, the $\bar{B}_d \rightarrow K^+ K^-$ decay amplitude reads

$$\mathcal{A}_{\text{NLO}}(\bar{B}_d^0 \rightarrow K^+ K^-) = \frac{G_F}{\sqrt{2}} \mathcal{M}_{\text{NLO}}^{LV}[\mathbf{C}_{d,1}^{\text{eff}}] + \frac{G_F}{\sqrt{2}} \mathcal{M}_{\text{NLO}}^{RV}[\mathbf{C}_{d,2}^{\text{eff}}]. \quad (19)$$

For the $B \rightarrow VV$ decays, the NLO contribution also depends significantly on the polarization states. If the final states are longitudinally polarized, the decay amplitude is very similar to the case where the final states are pseudoscalar mesons. For the transversely polarized amplitude, however, the matrix element $\langle V_2(p_2, \epsilon_{2\perp}^*) V_3(p_3, \epsilon_{3\perp}^*) | g^*(p_g, \alpha) g^*(\bar{p}_g, \beta) \rangle$ should be proportional to $g_{\perp}^{\beta\nu} \epsilon_{2\perp}^* \cdot \epsilon_{3\perp}^* - \epsilon_{2\perp}^{*\beta} \epsilon_{3\perp}^{\nu} - \epsilon_{2\perp}^{*\nu} \epsilon_{3\perp}^{\beta}$. Contracting this matrix element with the $\bar{B} \rightarrow g^* g^*$ transition matrix element results in a zero contribution. Consequently, only the longitudinal polarization amplitude is affected by the NLO contribution. Similar to the leading-order situation, the longitudinally polarized factorization function $\tilde{\mathcal{M}}_{\text{NLO}}^{LV}$ can be obtained from those in the $B_q \rightarrow PP$ decays with the replacement rule in Eq. (B4) and $r_i \rightarrow m_i/m_{B_q}$. The NLO contribution to the longitudinal $\bar{B}_q \rightarrow VV$ decay amplitude is then expressed as

$$\begin{aligned}
\mathcal{A}_{L,\text{NLO}}(\bar{B}_s^0 \rightarrow \rho^+ \rho^-) &= \frac{G_F}{\sqrt{2}} \tilde{\mathcal{M}}_{\text{NLO}}^{LV}[\mathbf{C}_{s,1}^{\text{eff}}] + \frac{G_F}{\sqrt{2}} \tilde{\mathcal{M}}_{\text{NLO}}^{RV}[\mathbf{C}_{s,2}^{\text{eff}}], \\
\mathcal{A}_{L,\text{NLO}}(\bar{B}_s^0 \rightarrow K^{*+} K^{*-}) &= \frac{G_F}{\sqrt{2}} \tilde{\mathcal{M}}_{\text{NLO}}^{LV}[\mathbf{C}_{d,1}^{\text{eff}}] + \frac{G_F}{\sqrt{2}} \tilde{\mathcal{M}}_{\text{NLO}}^{RV}[\mathbf{C}_{d,2}^{\text{eff}}]. \quad (20)
\end{aligned}$$

The decay amplitude of $B \rightarrow \phi\phi$ has the same form as $\mathcal{A}_{L,\text{NLO}}(\bar{B}_s^0 \rightarrow K^{*+} K^{*-})$. For $B_s \rightarrow \rho^0 \rho^0, \omega\omega$ decays, the second equation in Eq. (B1) still holds.

IV. NUMERICAL ANALYSIS

LCDAs are essential nonperturbative entities within the collinear factorization framework. They are characterized by the fact that they contain only a single variable, namely, the momentum fraction. TMD wave functions, in contrast, are far more complex than a straightforward extension of LCDAs. This complexity arises from the appearance of light-cone singularities. These light-cone divergences can be regularized through the rapidity of non-light-cone Wilson lines. However, this regularization comes at the cost of the emergence of enhanced large logarithms of rapidity, which must be resummed to minimize the scheme

dependence. From a phenomenological perspective, one approach is to utilize a wave function that has been fitted to experimental data. In this study, we employ the model

$$\phi_{B_q}(x, b) = N_{B_q} x^2 (1-x)^2 \exp \left[-\frac{M_{B_q}^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \quad (21)$$

with N_{B_q} the normalization factor. The parameter ω_b is chosen as $\omega_b = 0.40 \pm 0.5$ GeV for B meson and $\omega_b = 0.50 \pm 0.5$ GeV for B_s meson, which has been fixed using the rich experimental data. For the wave function of the light meson, the previous studies have indicated that the transverse-momentum-dependent part is not important numerically; therefore, only the longitudinal momentum-fraction-dependent part is employed. For the pseudoscalar meson, we have

TABLE I. The inputs of parameters in the light meson LCDAs and CKM matrix elements.

f_π (GeV)	f_K (GeV)	$f_{\rho^\pm}^\parallel (f_{\rho^0}^\parallel)$ (GeV)	$f_\omega^\parallel$ (GeV)	$f_{K^*}^\parallel$ (GeV)	$f_\phi^\parallel$ (GeV)
0.130	0.156	0.210(0.212)	0.197	0.204	0.233
$m_{0\pi}$ (GeV)	m_{0K} (GeV)	$f_{\rho^\pm}^\perp (f_{\rho^0}^\perp)$ (GeV)	$f_\omega^\perp$ (GeV)	$f_{K^*}^\perp$ (GeV)	$f_\phi^\perp$ (GeV)
1.400	1.892	0.144(0.146)	0.162	0.159	0.191
a_2^π	$a_2^{\rho,\parallel}$	$a_2^{\rho,\perp}$	$a_2^{\omega,\parallel}$	$a_1^{\omega,\perp}$	$a_2^{\phi,\parallel}$
0.250 ± 0.150	0.180 ± 0.037	0.137 ± 0.030	0.150 ± 0.120	0.140 ± 0.120	0.230 ± 0.080
a_1^K	a_2^K	$a_1^{K^*,\parallel}$	$a_1^{K^*,\perp}$	$a_2^{K^*,\parallel}$	$a_2^{K^*,\perp}$
0.076 ± 0.004	0.221 ± 0.082	0.060 ± 0.040	0.040 ± 0.030	0.160 ± 0.090	0.100 ± 0.080
$a_2^{\phi,\perp}$	m_s (GeV)	$ V_{ud} $	$ V_{us} $	$10^3 V_{td} $	$10^3 V_{ts} $
0.140 ± 0.070	0.130 ± 0.015	0.974	0.224	8.6 ± 0.2	41.5 ± 0.9
$10^3 V_{ub} $	$ V_{tb} $	α	γ	$\eta_{3\pi}$	η_{3K}
3.82 ± 0.20	1.010 ± 0.027	$(84.1^{+4.5}_{-3.8})^\circ$	$(65.7 \pm 3.0)^\circ$	0.025 ± 0.008	0.015 ± 0.005

$$\begin{aligned}
 \phi_P^A(x) &= \frac{3f_P}{\sqrt{6}} x(1-x) [1 + a_1^P(\mu) C_1^{3/2}(t) + a_2^P(\mu) C_2^{3/2}(t)], \\
 \phi_P^P(x) &= \frac{f_P}{2\sqrt{6}} \left[1 + 3\rho^P(1 - 3a_1^P + 6a_2^P)(1 + \ln x) - \frac{3\rho^P}{2}(1 - 9a_1^P + 18a_2^P) C_1^{1/2}(t) \right. \\
 &\quad \left. - 3\rho^P(a_1^P - 5a_2^P) C_2^{1/2}(t) + 30\eta_{3P} C_2^{1/2}(t) - \frac{9\rho^P}{2} a_2^P C_3^{1/2}(t) \right], \\
 \phi_P^T(x) &= \frac{f_P}{2\sqrt{6}} \left\{ -t \left[1 + \frac{\rho^P}{2}(2 - 15a_1^P + 30a_2^P) + 3\rho^P(1 - 3a_1^P + 6a_2^P) \ln x \right. \right. \\
 &\quad \left. + \rho^P \left(3a_1^P - \frac{15}{2} a_2^P \right) C_1^{3/2}(t) + \frac{1}{2} (10\eta_{3P} + 3\rho a_2^P) C_2^{3/2}(t) \right] \\
 &\quad + x(1-x) \left[6\rho^P \left(3a_1^P - \frac{15}{2} a_2^P \right) + 5(10\eta_{3P} + 3\rho^P a_2^P) C_1^{3/2}(t) \right] \\
 &\quad \left. + 3(1-x)\rho^P(1 - 3a_1^P + 6a_2^P) \right\}, \tag{22}
 \end{aligned}$$

where $t = 2x - 1$, $\rho^\pi = 0$, and $\rho^K = m_s^2/m_K^2$. For the longitudinally polarized vector meson, the corresponding LCDAs read

$$\begin{aligned}
 \phi_V(x) &= \frac{3f_V^\parallel}{\sqrt{6}} x(1-x) [1 + a_1^{V,\parallel}(\mu) C_1^{3/2}(t) + a_2^{V,\parallel}(\mu) C_2^{3/2}(t)], \\
 \phi_V'(x) &= \frac{3f_V^\perp}{2\sqrt{6}} t \left[C_1^{1/2}(t) + a_1^{V,\perp} C_2^{1/2}(t) + a_2^{V,\perp} C_3^{1/2}(t) \right], \\
 \phi_V^s(x) &= \frac{3f_V^\perp}{2\sqrt{6}} x(1-x) \left[1 + \frac{a_1^{V,\perp}}{3} C_2^{1/2}(t) + \frac{a_2^{V,\perp}}{6} C_3^{1/2}(t) \right]. \tag{23}
 \end{aligned}$$

We do not list the LCDAs of the transversely polarized vector meson here, since it does not participate in the NLO amplitudes. In the numerical calculation of the leading-order contribution, we take advantage of the same LCDAs with [53]. The decay constants and Gegenbauer moments of the light mesons, as well as the CKM matrix elements and other parameters, are listed in Table I. The CKM matrix elements are taken from the Particle Data Group.

In order to develop a transparent understanding of the numerical feature for the identified enhancement mechanism, we present in Table II the obtained results of $\mathcal{A}_{\text{LO}}(\bar{B}_s^0 \rightarrow \pi^+\pi^-, \rho^+\rho^-)$ and $\mathcal{A}_{\text{NLO}}(\bar{B}_s^0 \rightarrow \pi^+\pi^-, \rho^+\rho^-)$ for the weak-annihilation decays $\bar{B}_s \rightarrow \pi^+\pi^-, \rho^+\rho^-$. It is obvious that the NLO amplitudes contain large imaginary parts, which makes the NLO contribution comparable to the leading-order one. The ratio of their moduli can reach

TABLE II. Numerical predictions of the decay amplitudes $\mathcal{A}_{\text{LO}}$ and $\mathcal{A}_{\text{NLO}}$ and strong phase difference between tree amplitude and penguin amplitude for $\bar{B}_s \rightarrow \pi^+ \pi^-, \rho^+ \rho^-$ decays.

	$\mathcal{A}_{\text{LO}}$	$\mathcal{A}_{\text{NLO}}$	$\frac{ \mathcal{A}_{\text{NLO}} }{ \mathcal{A}_{\text{LO}} }$	$\frac{ \mathcal{A}_{\text{LO+NLO}} }{ \mathcal{A}_{\text{LO}} }$
$\bar{B}_s^0 \rightarrow \pi^+ \pi^-$	$4.97 + 3.97i$	$1.26 - 2.07i$	0.38	1.02
$\bar{B}_s^0 \rightarrow \rho_L^+ \rho_L^-$	$7.06 + 11.5i$	$3.83 - 2.90i$	0.36	1.02
	$\Delta\delta _{\text{twist-2}}$	$\Delta\delta _{\text{twist-3}}$	$\Delta\delta _{\text{total}}$	$\Delta\delta _{\text{LO}}$
$\bar{B}_s^0 \rightarrow \pi^+ \pi^-$	35.8°	33.6°	38.0°	7.2°
$\bar{B}_s^0 \rightarrow \rho_L^+ \rho_L^-$	-166.0°	-112.7°	30.3°	-8.9°

35% $\sim$ 40%, which is in line with our expectation. The imaginary part of the NLO amplitudes have opposite sign to that of the leading-order amplitude, thus there exists a cancellation between them. As the result, the NLO contribution does not cause a significant change in the modulus of the total amplitudes much. Such as the $\bar{B}_s \rightarrow \pi^+ \pi^-$ process, the modulus of the total amplitude remains almost unchanged after including the NLO contribution. Therefore, the prediction of the branching ratio for this process will not change significantly either, which is consistent with the conclusion in Ref. [51], where only the contributions of the leading-twist LCDAs of light mesons were considered. In Table III, the branching ratios of the pure annihilation-type B -decay modes including NLO corrections are presented. For sake of comparison, the leading-order predictions are given in the parentheses. As expected, in almost all processes, the new results are very close to the previous predictions. In Table II, we also list the experimental results of the processes considered in this paper. It can be seen that our predictions are in good agreement with the experimental data. For the $B_q \rightarrow VV$ processes, we also give the predicted values of the longitudinal fraction. It is found that the pure annihilation-type B -meson decays are all dominated by longitudinal polarization. This is because the transverse polarization

amplitudes are severely suppressed. Although the NLO corrections can only contribute to the longitudinal polarization amplitude, they do not cause a significant change in the longitudinal polarization fraction.

As mentioned before, the dominant NLO contribution from the charm-loop diagrams generates the considerable cancellation of the imaginary part of leading power contribution; in consequence, the NLO effect considered in this work provides an important source of the strong phases of the weak-annihilation nonleptonic $\bar{B}_q$ -meson decay amplitudes, which can lead to considerable impact on the CP -violating observables. We first demonstrate the influence of the high-order corrections on the strong phase. The decay amplitudes can be decomposed as $\mathcal{A} \sim V_{ub}V_{ud}^*A_u + V_{tb}V_{td}^*A_t$ ($D = d, s$), then the strong phase of $A_{u,t}$ is written by $\delta_{u,t}$, respectively, and $\Delta\delta \equiv \delta_u - \delta_t$. The direct CP violation is proportional to the sine of the difference in the strong phases and the sine of the difference in the weak phases of the two amplitudes, i.e., $\mathcal{A}_{CP}^{\text{dir}} \sim \sin \Delta\delta \sin \Delta\phi$; it is obviously very sensitive to $\Delta\delta$. We present the difference in the strong phases of $\bar{B}_s^0 \rightarrow \pi^+ \pi^-, \rho^+ \rho^-$ decays in Table II, where $\Delta\delta$ from twist-2 LCDAs and twist-3 LCDAs of the light meson are listed for a comparison. It is found that both of them can generate sizable strong phase difference, which significantly modifies the leading-order result. As a result, the predicted $\sin \Delta\delta$ is over 0.5 for both $\bar{B}_s^0 \rightarrow \pi^+ \pi^-$ and $\bar{B}_s^0 \rightarrow \rho^+ \rho^-$ decays. Therefore, in these processes, objective direct CP violation can be anticipated.

The time-dependent CP asymmetries for the neutral $\bar{B}_q$ meson decaying into CP eigenstates are defined by

$$\begin{aligned} \mathcal{A}_{CP}(t) &= \frac{\Gamma(\bar{B}_q \rightarrow M_1 M_2) - \Gamma(B_q \rightarrow M_1 M_2)}{\Gamma(\bar{B}_q \rightarrow M_1 M_2) + \Gamma(B_q \rightarrow M_1 M_2)} \\ &= -\frac{\mathcal{A}_{CP}^{\text{dir}} \cos(\Delta m_q t) + \mathcal{A}_{CP}^{\text{mix}} \sin(\Delta m_q t)}{\cosh(\Delta\Gamma_q t/2) + \mathcal{A}_{\Delta\Gamma} \sinh(\Delta\Gamma_q t/2)}, \end{aligned} \quad (24)$$

TABLE III. Theory predictions of the branching ratio $\mathcal{B}$ (in units of 10^{-6}) and the percentage of the longitudinal polarizations f_L for the weak-annihilation $\bar{B}_q \rightarrow PP, V_L V_L$ decay processes, where we have included in parentheses the corresponding leading-order (LO) QCD results for a comparison.

	$10^6 \mathcal{B} _{\text{Theory}}$	$10^6 \mathcal{B} _{\text{Exp.}}$	f_L
$\bar{B}_s \rightarrow \pi^+ \pi^-$,	$0.39^{+0.19}_{-0.18}$ (0.36 $^{+0.21}_{-0.18}$)	0.72 ± 0.10	...
$\bar{B}_s \rightarrow \pi^0 \pi^0$	$0.19^{+0.10}_{-0.09}$ (0.18 $^{+0.11}_{-0.10}$)	< 7.7	1.0 (1.0)
$\bar{B}_s \rightarrow \rho^0 \rho^0$	$0.89^{+0.19}_{-0.17}$ (0.82 $^{+0.19}_{-0.16}$)	< 320	~ 1.0 (~ 1.0)
$\bar{B}_s \rightarrow \rho^+ \rho^-$,	$1.71^{+0.36}_{-0.32}$ (1.58 $^{+0.36}_{-0.30}$)	...	~ 1.0 (~ 1.0)
$\bar{B}_s \rightarrow \omega\omega$	$0.62^{+0.27}_{-0.25}$ (0.55 $^{+0.31}_{-0.25}$)	...	~ 1.0 (~ 1.0)
$\bar{B}_s \rightarrow \rho\omega$	~ 0 (~ 0)	...	~ 1.0 (~ 1.0)
$\bar{B}_d \rightarrow K^+ K^-$	$0.12^{+0.05}_{-0.03}$ (0.11 $^{+0.04}_{-0.03}$)	0.078 ± 0.015	...
$\bar{B}_d \rightarrow K^{*+} K^{*-}$	$0.14^{+0.06}_{-0.05}$ (0.12 $^{+0.06}_{-0.04}$)	< 0.4	~ 1.0 (~ 1.0)
$\bar{B}_d \rightarrow \phi\phi$	$0.029^{+0.010}_{-0.010}$ (0.015 $^{+0.007}_{-0.005}$)	< 0.027	0.99 (0.97)

TABLE IV. Theory predictions of the CP asymmetries $\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$ (in units of 10^{-2}) for the weak-annihilation $\bar{B}_q \rightarrow PP, V_L V_L$ decay processes, where we have included in parentheses the corresponding LO QCD results for a comparison.

	$\mathcal{A}_{CP}^{\text{dir}}$	$\mathcal{A}_{CP}^{\text{mix}}$
$\bar{B}_s \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$	$-6.0^{+1.1}_{-2.5} (-3.6^{+1.8}_{-3.1})$	$-4.2^{+21.4}_{-9.0} (35.9^{+15.6}_{-11.2})$
$\bar{B}_s \rightarrow \rho_L^+ \rho_L^-, \rho_L^0 \rho_L^0$	$-4.2^{+0.7}_{-0.5} (-1.9^{+0.7}_{-0.7})$	$-4.3^{+21.5}_{-9.0} (35.9^{+15.6}_{-11.2})$
$\bar{B}_s \rightarrow \omega_L \omega_L$	$-4.6^{+1.2}_{-2.0} (-2.6^{+1.6}_{-2.4})$	$-3.8^{+21.8}_{-9.7} (35.9^{+15.6}_{-11.2})$
$\bar{B}_s \rightarrow \rho_L \omega_L$	$0.0 \pm 0.0 (0.0 \pm 0.0)$	$-71.0^{+6.3}_{-5.4} (-71.0^{+6.3}_{-5.4})$
$\bar{B}_d \rightarrow K^+ K^-$	$41.6^{+12.5}_{-12.3} (38.7^{+13.2}_{-12.2})$	$-2.2^{+19.1}_{-26.4} (-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \rightarrow K_L^{*+} K_L^{*-}$	$36.7^{+16.0}_{-9.5} (25.4^{+17.4}_{-11.1})$	$-1.4^{+19.7}_{-26.9} (-47.0^{+15.7}_{-18.8})$
$\bar{B}_d \rightarrow \phi_L \phi_L$	$-39.7^{+6.1}_{-8.4} (0.0)$	$27.8^{+5.7}_{-25.9} (0.0 \pm 0.0)$

where the observables $\mathcal{A}_{CP}^{\text{dir}}$, $\mathcal{A}_{CP}^{\text{mix}}$, and $\mathcal{A}_{\Delta\Gamma}$ can be found in [54–56], satisfying the exact algebra relation $|\mathcal{A}_{CP}^{\text{dir}}|^2 + |\mathcal{A}_{CP}^{\text{mix}}|^2 + |\mathcal{A}_{\Delta\Gamma}|^2 = 1$. The predictions for the two independent CP asymmetries ($\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$) are displayed in Table IV. It is evidently revealed that NLO QCD correction significantly enhances both $\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$ of the decay processes studied in this paper. The direct CP asymmetry of $B_s \rightarrow \pi\pi, \rho\rho, \omega\omega$ is predicted to about -5% , which needs to be tested in the future experiment. The absolute value of our prediction is smaller than that in [51], which shows the differences in the sources of strong phases between the PQCD and QCDF approaches. Notably, the predicted direct CP asymmetry in $B \rightarrow \phi_L \phi_L$ approaches -40% , and strong phase difference is totally from the NLO contribution. Compared to direct CP violation $\mathcal{A}_{CP}^{\text{dir}}$, the observable $\mathcal{A}_{CP}^{\text{mix}}$ receives more significant corrections from NLO contribution, since it is proportional to the imaginary part of the decay amplitudes. An exception is the $\bar{B}_s \rightarrow \rho_L \omega_L$ decays, where $\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$ are kept unchanged. This is because, in this process, the final-state isospin is 1, and the production through the two-gluon process violates the conservation of isospin; i.e., the matrix element $\langle \rho_L(p) \omega_L(q) | g^*(p_g, \alpha) g^*(\tilde{p}_g, \beta) \rangle$ vanishes if the tiny $\rho - \omega$ mixing effect is neglected. The uncertainties of the branching ratios and CP asymmetries are also present in Tables III and IV. The main sources of the uncertainty is the Gegenbauer moments of the light mesons and the parameter ω in the B -meson wave functions. The uncertainties from difference sources are added in quadrature.

V. CONCLUSIONS

Nonleptonic B -meson decays provide an ideal place for unraveling the fundamental nature of CP violation. In this context, weak-annihilation contributions are not merely significant but absolutely crucial, and they play an indispensable role in driving the CP -violation processes observed in nonleptonic B -meson decays. In this paper, we endeavored to perform an improved study on the pure

annihilation-type charmless hadronic B decays, including $B^0 \rightarrow K^+ K^-, K^{*+} K^{*-}, \phi\phi$, and $B_s \rightarrow \pi\pi, \rho\rho, \rho\omega, \omega\omega$, within the framework of the PQCD approach. Compared to the previous studies, we included a type of enhanced contribution to the various decay processes at next-to-leading order in the strong coupling constant α_s . The enhanced contribution arises from the quark-loop contractions of the four-quark operators, which might be accompanied by large Wilson coefficients and/or the multiplication CKM factors. We only considered the $B \rightarrow PP, VV$ decay processes and do not pay attention to $B \rightarrow PV$ modes since the quark-loop diagram will not contribute to these processes. In the PQCD approach, there is no end point singularity in the convolution of the hard kernel and the wave functions of initial- and final-state mesons, and we have employed both twist-2 and two-particle twist-3 distribution amplitudes in our calculation. The NLO amplitudes contain large imaginary parts with opposite sign to that of leading-order amplitude, and the cancellation effect leads to the fact that the NLO contribution cannot have a significant impact on the branching ratios as well as longitudinal polarization fractions (for $B \rightarrow VV$ modes) of the considered processes. Meanwhile, the NLO effect considered in this work provides an important source of the strong phases of the weak-annihilation nonleptonic $\bar{B}_q$ -meson decay amplitudes, which can lead to considerable impact on the CP -violating observables such as the CP asymmetry $\mathcal{A}_{CP}^{\text{dir}}$ and mixing-induced CP asymmetry $\mathcal{A}_{CP}^{\text{mix}}$. The numerical result evidently revealed that NLO QCD correction significantly enhances both $\mathcal{A}_{CP}^{\text{dir}}$ and $\mathcal{A}_{CP}^{\text{mix}}$ of the decay processes studied in this paper, and the observable $\mathcal{A}_{CP}^{\text{mix}}$ receives more significant corrections from NLO contribution, since it is proportional to the imaginary part of the decay amplitudes.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX A: PQCD FUNCTIONS

The evolution factors $E_a^{(l)}$ and $E_{a'}^{(l)}$ entering in the expressions for the factorization formulas are given by

$$\begin{aligned} E'_a(t) &= \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)], \\ E'_{a'}(t) &= \alpha_s^2(t) \exp[-S_B(t) - S_2(t) - S_3(t)], \end{aligned} \quad (\text{A1})$$

where we have not included the threshold resummation factor since it cannot bring about significant impact on the contribution from annihilation diagrams. The hard scales are chosen as

$$t_c = \max\{\sqrt{\bar{x}_3}M_{B_q}, 1/b_2, 1/b_3\}, \quad (\text{A2})$$

$$t'_c = \max\{\sqrt{x_2}M_{B_q}, 1/b_2, 1/b_3\}, \quad (\text{A3})$$

$$t_d = \max\{\sqrt{x_2\bar{x}_3}M_{B_q}, \sqrt{1 - (\bar{x}_2 - x_1)x_3}M_{B_q}, 1/b_1, 1/b_2\}, \quad (\text{A4})$$

$$t'_d = \max\{\sqrt{x_2\bar{x}_3}M_{B_q}, \sqrt{x_3\bar{x}_2}M_{B_q}, \sqrt{|x_1 - x_2|\bar{x}_3}M_{B_q}, 1/b_1, 1/b_2\}, \quad (\text{A5})$$

$$t_n = \max\{\sqrt{x_2\bar{x}_3}M_{B_q}, \sqrt{x_2\bar{x}_3}M_{B_q}, \sqrt{1 - (\bar{x}_2 - x_1)x_3}M_{B_q}, 1/b_1, 1/b_2\}, \quad (\text{A6})$$

$$t'_n = \max\{\sqrt{x_2\bar{x}_3}M_{B_q}, \sqrt{x_3\bar{x}_2}M_{B_q}, \sqrt{|x_1 - x_2|(1 - x_3)}M_{B_q}, 1/b_1, 1/b_2\}. \quad (\text{A7})$$

The Sudakov factor associated in B_q and light meson wave functions read, respectively, as

$$S_B(x_1, b_1, \mu) = s\left(x_1 \frac{m_B}{\sqrt{2}}, b_1\right) + s_q(b_1, \mu), \quad (\text{A8})$$

$$S_i(x_i, \bar{x}_i, b_i, \mu) = s\left(x_i \frac{m_B}{\sqrt{2}}, b_i\right) + s\left(\bar{x}_i \frac{m_B}{\sqrt{2}}, b_i\right) + s_q(b_i, \mu), \quad (\text{A9})$$

where $\bar{x}_i = 1 - x_i$. The factor $s(Q, b)$ collects the resummation of the double logarithms containing the transverse momentum of partons inside the initial- and final-state mesons,

$$\begin{aligned} s(\xi Q, b) = & \frac{A^{(1)}}{2\beta_1} \hat{q} \ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1\right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b}) - \frac{A^{(1)}\beta_2\hat{q}}{4\beta_1^3} \left[\frac{\ln(2\hat{b}) + 1}{\hat{b}} - \frac{\ln(2\hat{q}) + 1}{\hat{q}}\right] \\ & - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left(\frac{e^{2\gamma_E - 1}}{2}\right)\right] \ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}\beta_2}{8\beta_1^3} [\ln^2(2\hat{b}) - \ln^2(2\hat{q})] - \frac{A^{(1)}\beta_2}{8\beta_1^3} \ln\left(\frac{e^{2\gamma_E - 1}}{2}\right) \left[\frac{\ln(2\hat{b}) + 1}{\hat{b}} - \frac{\ln(2\hat{q}) + 1}{\hat{q}}\right] \\ & - \frac{A^{(2)}\beta_2}{16\beta_1^4} \left[\frac{2\ln(2\hat{q}) + 3}{\hat{q}} - \frac{2\ln(2\hat{b}) + 3}{\hat{b}}\right] - \frac{A^{(2)}\beta_2}{16\beta_1^4} \frac{\hat{q} - \hat{b}}{\hat{b}^2} [2\ln(2\hat{b}) + 1] - \frac{A^{(2)}\beta_2^2}{432\beta_1^6} \frac{\hat{b} - \hat{q}}{\hat{b}^3} [9\ln^2(2\hat{b}) + 6\ln(2\hat{b}) + 2] \\ & - \frac{A^{(2)}\beta_2^2}{1728\beta_1^6} \left[\frac{18\ln^2(2\hat{b}) + 30\ln(2\hat{b}) + 19}{\hat{b}^2} - \frac{18\ln^2(2\hat{q}) + 30\ln(2\hat{q}) + 19}{\hat{q}^2}\right]. \end{aligned} \quad (\text{A10})$$

The abbreviated variables are

$$\hat{q} \equiv \ln\left(\frac{\xi Q}{\sqrt{2}\Lambda}\right), \quad \hat{b} \equiv \ln\left(\frac{1}{b\Lambda}\right), \quad (\text{A11})$$

and the coefficients $A^{(i)}$ and β_i are

$$\begin{aligned} A^{(1)} &= \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{e^{\gamma_E}}{2}\right), \\ \beta_1 &= \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{152 - 19n_f}{24}. \end{aligned} \quad (\text{A12})$$

The factor $s_q(b, \mu)$ arises from the resummation of the single logarithms in the quark self-energy correction

$$s_q(b_1, \mu) = \frac{5}{3} \int_{1/b_1}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})), \quad s_q(b_i, \mu) = 2 \int_{1/b_i}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})), \quad (\text{A13})$$

with the quark anomaly dimension $\gamma_q = -\alpha_s(\mu)/\pi$.

The hard functions h_i in decay amplitudes are given by

$$h_{na}(x_1, x_2, x_3, b_1, b_2) = \frac{\pi i}{2} [\theta(b_2 - b_1) J_0(\sqrt{x_2 \bar{x}_3} M b_1) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_2) + \theta(b_1 - b_2) J_0(\sqrt{x_2 \bar{x}_3} M b_2) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_1)] K_0(\sqrt{1 - (\bar{x}_2 - x_1) x_3} M b_1), \quad (\text{A14})$$

$$h'_{na}(x_1, x_2, x_3, b_1, b_2) = \frac{\pi i}{2} [\theta(b_2 - b_1) J_0(\sqrt{x_2 \bar{x}_3} M b_1) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_2) + \theta(b_1 - b_2) J_0(\sqrt{x_2 \bar{x}_3} M b_2) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_1)] \times \begin{cases} \frac{\pi i}{2} H_0^{(1)}[\sqrt{(x_2 - x_1) \bar{x}_3} M b_1] & x_1 - x_2 < 0 \\ K_0[\sqrt{(x_1 - x_2) \bar{x}_3} M b_1] & x_1 - x_2 > 0, \end{cases} \quad (\text{A15})$$

$$h_{nan}(x_1, x_2, x_3, b_1, b_2) = \frac{\pi i}{2} \frac{1}{(x_2 - x_3) M^2} K_0(\sqrt{1 - (\bar{x}_2 - x_1) x_3} M b_1) \{ [\theta(b_2 - b_1) J_0(\sqrt{x_2 \bar{x}_3} M b_1) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_2) + \theta(b_1 - b_2) J_0(\sqrt{x_2 \bar{x}_3} M b_2) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_1)] - (x_2 \leftrightarrow x_3) \}, \quad (\text{A16})$$

$$h'_{nan}(x_1, x_2, x_3, b_1, b_2) = \frac{\pi i}{2} \frac{1}{(x_2 - x_3) M^2} \{ [\theta(b_2 - b_1) J_0(\sqrt{x_2 \bar{x}_3} M b_1) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_2) + \theta(b_1 - b_2) J_0(\sqrt{x_2 \bar{x}_3} M b_2) H_0^{(1)}(\sqrt{x_2 \bar{x}_3} M b_1)] - (x_2 \leftrightarrow x_3) \} \times \begin{cases} \frac{\pi i}{2} H_0^{(1)}[\sqrt{(x_2 - x_1) \bar{x}_3} M b_1] & x_1 - x_2 < 0 \\ K_0[\sqrt{(x_1 - x_2) \bar{x}_3} M b_1] & x_1 - x_2 > 0. \end{cases} \quad (\text{A17})$$

APPENDIX B: PURE ANNIHILATION-TYPE $B \rightarrow VV$ DECAY AMPLITUDES

The specific expression for the LO amplitudes of each pure annihilation decay process can be written by

$$\begin{aligned} \mathcal{A}_{i,\text{LO}}(\bar{B}_s^0 \rightarrow \rho^+ \rho^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \{ f_{B_s} F_{\text{LO}}^{LL,i} [a_2] + \mathcal{M}_{\text{LO}}^{LL,i} [C_2] \} \\ &\quad - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ f_{B_s} F_{\text{LO}}^{LL,i} \left[2a_3 + \frac{1}{2} a_9 \right] + f_{B_s} F_{\text{LO}}^{LR,i} \left[2a_5 + \frac{1}{2} a_7 \right] \right. \\ &\quad \left. + \mathcal{M}_{\text{LO}}^{LL,i} \left[2C_4 + \frac{1}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP,i} \left[2C_6 + \frac{1}{2} C_8 \right] \right\}, \\ \sqrt{2} \mathcal{A}_{i,\text{LO}}(\bar{B}_s^0 \rightarrow \rho^0 \rho^0) &= \sqrt{2} \mathcal{A}_{i,\text{LO}}(\bar{B}_s^0 \rightarrow \omega \omega) = \mathcal{A}_{i,\text{LO}}(\bar{B}_s^0 \rightarrow \rho^+ \rho^-) \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} 2 \mathcal{A}_{i,\text{LO}}(\bar{B}_s^0 \rightarrow \rho^0 \omega) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \left\{ f_{B_s} F_{\text{LO}}^{LL,i} [a_2] + \mathcal{M}_{\text{LO}}^{LL,i} [C_2] \right\} - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ f_{B_s} F_{\text{LO}}^{LL,i} \left[\frac{3}{2} a_9 \right] \right. \\ &\quad \left. + f_{B_s} F_{\text{LO}}^{LR,i} \left[\frac{3}{2} a_7 \right] + \mathcal{M}_{\text{LO}}^{LL,i} \left[\frac{3}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP,i} \left[\frac{3}{2} C_8 \right] \right\} + [\rho^0 \leftrightarrow \omega], \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \mathcal{A}_{i,\text{LO}}(\bar{B}_d^0 \rightarrow K^{*+} K^{*-}) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left\{ f_{B_d} F_{\text{LO}}^{LL,i} [a_2] + \mathcal{M}_{\text{LO}}^{LL,i} [C_2] \right\} - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ f_{B_d} F_{\text{LO}}^{LL,i} \left[2a_3 + \frac{1}{2} a_9 \right] \right. \\ &\quad \left. + f_{B_d} F_{\text{LO}}^{LR,i} \left[2a_5 + \frac{1}{2} a_7 \right] + \mathcal{M}_{\text{LO}}^{LL,i} \left[2C_4 + \frac{1}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP,i} a \left[2C_6 + \frac{1}{2} C_8 \right] \right\}, \\ \mathcal{A}_{i,\text{LO}}(\bar{B}_d^0 \rightarrow \phi \phi) &= - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ f_{B_d} F_{\text{LO}}^{LL,i} \left[2a_3 + \frac{1}{2} a_9 \right] + f_{B_d} F_{\text{LO}}^{LR,i} \left[2a_5 + \frac{1}{2} a_7 \right] \right. \\ &\quad \left. + \mathcal{M}_{\text{LO}}^{LL,i} \left[2C_4 + \frac{1}{2} C_{10} \right] + \mathcal{M}_{\text{LO}}^{SP,i} \left[2C_6 + \frac{1}{2} C_8 \right] \right\}, \end{aligned} \quad (\text{B3})$$

where $i = L, N, T$. The longitudinal polarization amplitudes $\mathcal{A}_L$ can be obtained from those in the $B_q \rightarrow PP$ decays with the following replacement in the distribution amplitudes:

$$\phi_{2(3)}^A(x) \rightarrow \phi_{2(3)}(x), \phi_{2(3)}^P(x) \rightarrow (-)\phi_{2(3)}^s(x), \phi_{2(3)}^T(x) \rightarrow (-)\phi_{2(3)}^t(x). \quad (\text{B4})$$

In addition, the ratios $r_i \rightarrow m_i/m_{B_q}$ with m_i being vector meson mass. On the contrary, to longitudinal polarization, Figs. 1(a) and 1(b) can contribute to normal polarization and transverse polarization amplitudes, and the results are collected as follows:

$$\begin{aligned} F_{\text{LO}}^{LL(R),N}(a_i) &= -8\pi C_F m_{B_q}^4 r_2 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ E_a(t_c) a_i(t_c) h_a(x_2, 1-x_3, b_2, b_3) \\ &\quad \times [(2-x_3)(\phi_2^v \phi_3^v + \phi_2^a \phi_3^a) + x_3(\phi_2^v \phi_3^a + \phi_2^a \phi_3^v)] - E_a(t'_c) a_i(t'_c) h_a(1-x_3, x_2, b_3, b_2) \\ &\quad \times [(1+x_2)(\phi_2^v \phi_3^v + \phi_2^a \phi_3^a) - (1-x_2)(\phi_2^v \phi_3^a + \phi_2^a \phi_3^v)] \}, \\ F_{\text{LO}}^{LL(R),T}(a_i) &= \pm 16\pi C_F m_{B_q}^4 r_2 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ E_a(t_c) a_i(t_c) h_a(x_2, 1-x_3, b_2, b_3) \\ &\quad \times [x_3(\phi_2^v \phi_3^v + \phi_2^a \phi_3^a) + (2-x_3)(\phi_2^v \phi_3^a + \phi_2^a \phi_3^v)] + E_a(t'_c) a_i(t'_c) h_a(1-x_3, x_2, b_3, b_2) \\ &\quad \times [(1-x_2)(\phi_2^v \phi_3^v + \phi_2^a \phi_3^a) - (1+x_2)(\phi_2^v \phi_3^a + \phi_2^a \phi_3^v)] \}. \end{aligned} \quad (\text{B5})$$

For the nonfactorizable annihilation diagrams shown in Figs. 1(c) and 1(d), we have

$$\begin{aligned} \mathcal{M}_{\text{LO}}^{LL(SP),N}(a_i) &= -64\pi C_F m_{B_q}^4 r_2 r_3 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_q}(x_1, b_1) \\ &\quad \times [\phi_2^v \phi_3^v + \phi_2^a \phi_3^a] E'_a(t_d) a_i(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2), \\ \mathcal{M}_{\text{LO}}^{LL(SP),T}(a_i) &= \mp 128\pi C_F m_{B_q}^4 r_2 r_3 / \sqrt{6} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_q}(x_1, b_1) \\ &\quad \times [\phi_2^v \phi_3^a + \phi_2^a \phi_3^v] E'_a(t_d) a_i(t_d) h_{na}(x_1, x_2, x_3, b_1, b_2). \end{aligned} \quad (\text{B6})$$

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