

Energy dependence of exclusive heavy vector meson photoproduction cross sections and NLO BFKL evolution

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We study the energy dependence of the cross section for exclusive photoproduction of vector mesons J/ψ and Υ , using a solution to the next-to-leading order (NLO) Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation. Our goal is to use BFKL evolution as a benchmark to provide evidence for the presence of nonlinear QCD dynamics and signs for the onset of gluon saturation at the highest center-of-mass energies. Our approach determines initial conditions for the proton from the Bartels-Golec Biernat-Kowalski (BGK) dipole model and evolves the resulting unintegrated gluon distribution using NLO BFKL evolution. For the nucleus, initial conditions are generated through both the impact parameter dependent saturation model (IP-Sat), using a Wood-Saxon distribution, and a $A^{1/3}$ scaling of the saturation scale of the original BGK model. We find that NLO BFKL evolution provides a very good description of the nuclear modification factor for J/ψ production if initial conditions are generated through an $A^{1/3}$ scaled BGK model, while the description fails if initial conditions are created using the IP-Sat model.

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I. INTRODUCTION

Exclusive photoproduction of vector mesons in ultra-peripheral collisions at the Large Hadron Collider (LHC) is a very useful process to explore quantum chromodynamics (QCD) at ultrasmall values of x . Here, $x = M_V^2/W^2$, with M_V the mass of the vector meson and W the center-of-mass energy of the photon-proton or photon-nucleus reaction. The process is primarily of interest for the photoproduction of charmonium, where the charm mass provides a perturbative scale at the boundary to nonperturbative dynamics; it is therefore suitable to potentially observe effects related to the saturation of gluon densities at ultralow values of x [1–4]. At low x , perturbative QCD predicts through the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [5–7] a powerlike rise of the gluon distribution. Bounds due to the unitarity state, however, that such a rise cannot continue to arbitrarily small values of x . At some value of x it must start to slow down and eventually come to hold. A theoretical framework which predicts such a slow down and saturation of gluon densities are the JIMWLK-BK (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner-Balitsky, Kovchegov) equations [8–14], which generalize BFKL evolution to the case of high gluon densities.

Photoproduction of charmed vector mesons in exclusive reactions has been extensively explored during the recent years; see, e.g., [15–36] for recent theory and [37–40] for experimental proposals as well as the reviews [41–43]. If datasets collected at both HERA and LHC experiments are combined, such reactions allow us to explore low x dynamics over several orders of magnitude, from $x \simeq 0.1$ down to $x = 10^{-6}$. Exclusive photoproduction on large nuclei is of special interest since gluon densities are in this case further enhanced through the nuclear mass number A . Nevertheless, comparison of recently measured ALICE and CMS data for photonuclear production on lead [44,45] with theory predictions do not yet allow us to draw firm conclusions on the observation of gluon saturation effects. In particular, descriptions based on a leading twist approach, using nuclear modification factors of parton distribution functions (PDFs), provide currently a competitive description, albeit x dependence characteristic for gluon saturation close to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) starting scale might be hidden in this case in the initial conditions.

A question which one therefore would like to answer in such reactions is whether the observed energy dependence can be explained through an approach which assumes low gluon densities or whether inclusion of high density effects is needed. Predictions which include high density effects are usually based on solutions of nonlinear QCD evolution equations, such as the JIMWLK-BK evolution equations. Nevertheless the ability of such schemes to describe data is

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insufficient to provide evidence for high gluon densities and potentially gluon saturation, since the evolution equations themselves are also applicable in the low density regime. One therefore needs, in addition, a framework which assumes low gluon densities and which fails to describe the dataset. At first sight, a natural framework for the low density limit is collinear factorization, which provides predictions not only for the low x region but for all values of x , while nonperturbative input to DGLAP evolution equations can be constrained by global fits. While appealing for the aforementioned reasons, relying on collinear factorization only bears the danger to overlook the presence of high gluon densities at low hard scales. DGLAP evolution evolves PDFs from low to high hard scales and testing its validity requires two reactions with a hierarchy of hard scales, i.e., photoproduction of the J/ψ (low hard scale) and the Υ (high hard scale). Since $x = M_V^2/W^2$ and with the maximal center-of-mass energy W fixed, bottomonium is limited to larger x values than charmonium production, $x_\Upsilon > x_{J/\psi}$. In addition, predictions at the bottom scale probe only initial conditions at the charm scale for $x \geq x_\Upsilon$. Parametrizations at $x < x_\Upsilon$ do not enter the DGLAP evolution. Predictions at low hard scales and low $x < x_\Upsilon$, such as J/ψ photoproduction, therefore either depend strongly on extrapolation of the parametrization used at larger values of x or—if J/ψ data are included in the fit—merely the observed x dependence.

To provide at the very least an alternative framework to test the validity of a low density approach, one may therefore use BFKL evolution to evolve hadronic structure at intermediate $x \simeq 0.01$ down to low x . If the BFKL prediction significantly deviates from observation, one can quantify the degree of deviation and therefore arrive at a conclusion to which degree the low density description breaks down. Note that such an approach has been taken before in the literature; see, in particular, [17,22,33]. Here [17,22] rely on a fit of the proton impact factor to HERA data and therefore are naturally limited to photoproduction on a proton. Reference [33] provides, on the other hand, predictions for J/ψ and Υ photoproduction on both proton and a lead nucleus. While the scope of the present study and [33] are very similar, there exist a few important differences which we would like to highlight in the following. Reference [33] provides predictions for both BK (nonlinear) and BFKL (linear) evolution equations based on the leading order BFKL kernel, supplemented with next-to-leading order (NLO) running coupling corrections; in addition [33] uses a gluon mass to regulate a certain infrared instability associated with impact parameter dependence in the evolution equation and is therefore capable to provide predictions for finite momentum transfer t between a photon and proton/lead nucleus. Our treatment is, on the other hand, restricted to BFKL evolution only, while we will take into account the complete NLO corrections, including a resummation of configurations enhanced by collinear logarithms, which have been found to provide important contributions

for a correct description of data; see, e.g., [46,47]. The solution to the NLO BFKL evolution equation is however restricted to zero momentum transfer.

Apart from potential uncertainties related to the restriction to $t = 0$, this approach is subject to uncertainties due to the transverse momentum distribution of the proton or lead impact factor at the initial x value as well as on the particular choice that one takes to solve the NLO BFKL evolution equations. To constrain the initial transverse momentum distribution for the proton, we will use, unlike [15,17,22], the recently refitted [48] Bartels-Golec Biernat-Kowalski (BGK) model [49] to generate an unintegrated gluon densities at $x = 0.01$. For the lead nucleus, we use the impact parameter dependent saturation model [50], based on the BGK model, as well as the BGK model with the effective saturation scale rescaled by a factor $A^{1/3}$. Since study of these models is of interest in its own right, we will provide in the following also a brief comparison of the energy dependence of these models to data, while our main interest is in using them as a way to model initial conditions for NLO BFKL evolution. Our solution of the NLO BFKL equation follows closely the solution of [46,47], which has been tested both in a fit to HERA data as well as LHC phenomenology [15,17,22,51,52]. The important difference to these studies is that instead of fitting the initial transverse momentum distribution to HERA data, we will use an already existing dipole model, from which we extract the required impact factor.

To test this setup, we will first explore the proton case, including a comparison to both J/ψ and $\Upsilon(1s)$ data. Comparison with the Υ photoproduction cross section provides a test in a region of phase space where the BFKL evolution is expected to perform well. J/ψ photoproduction takes, on the other hand, place at the boundary between perturbative and nonperturbative physics and associated uncertainties are naturally large. Of particular interest is here the study of the nuclear modification factor, since for this quantity uncertainties present in both proton and lead cross sections will cancel to a large extent.

The outline of this paper is as follows: In Sec. II we summarize the setup which allows us to obtain the energy dependence of photoproduction cross sections from the inclusive unintegrated gluon distribution. We also compare the energy dependence predicted by the BGK model and its nuclear generalizations to data. Section III discusses the implementation of the solution of the NLO BFKL equation. Section IV contains our results, including a comparison to data. Section V provides our conclusions.

II. PHOTOPRODUCTION CROSS SECTIONS

We study the process,

$$\begin{aligned} \gamma(q) + N(p) &\rightarrow V(q') + N(p'), \\ V &= J/\psi, \Upsilon(1s), N = p, Pb, \end{aligned} \quad (1)$$

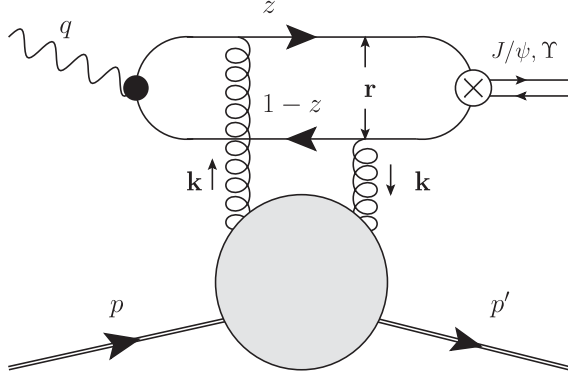


FIG. 1. Exclusive photoproduction of vector mesons J/ψ and $\Upsilon(1s)$. For the quark-anti-quark dipole, we indicate photon momentum fractions z and $1-z$ as well as the transverse separation r . Finally, k denotes the transverse momentum transmitted from the unintegrated gluon distribution of the proton or lead nucleus; the latter is indicated through the gray blob; r denotes the transverse size of the dipole.

where γ denotes a quasireal photon with virtuality $Q^2 \simeq 0$. $W^2 = (q + p)^2$ is the center-of-mass energy squared of the $\gamma(q) + N(p)$ reaction; see also Fig. 1. With $t = (q - q')^2$, the differential cross section for the exclusive photoproduction of a vector meson can be written as

$$\frac{d\sigma}{dt}(\gamma N \rightarrow VN) = \frac{1}{16\pi} |\mathcal{A}_T^{\gamma p \rightarrow Vp}(x, t)|^2, \quad (2)$$

$V = J/\psi, \Upsilon(1s),$

with $\mathcal{A}_T(x, t)$ the scattering amplitude for a transverse polarized real photon with color singlet exchange in the t channel, where an overall factor W^2 has been already extracted. $x = M_V^2/W^2$ with M_V the mass of the vector meson; see also e.g. [15]. To access the inclusive gluon distribution, we follow a two-step procedure, frequently employed in the literature: First one determines the differential cross section at zero momentum transfer $t = 0$, which can be related to the inclusive gluon distribution; in a second step one uses the ansatz $d\sigma/dt \sim \exp[-|t|B_D]$ with the diffractive slope parameter B_D , which is determined from data. The total cross section then reads

$$\sigma^{\gamma p \rightarrow Vp}(W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt}(\gamma p \rightarrow Vp)|_{t=0}. \quad (3)$$

For photon-proton collisions, we use

$$B_{D,p}^{J/\psi}(W) = \left[b_{0,p} + 4\alpha'_p \ln \frac{W}{W_0} \right] \text{GeV}^{-2}, \quad (4)$$

where $W_0 = 90 \text{ GeV}$ and $b_{0,p}$ and α'_p have been determined in [53] from a fit to HERA data with $b_{0,p}^{J/\psi} = 4.62$, while $\alpha_p^{J/\psi} = 0.171$. For the Υ we use

$B_{D,p}^{\Upsilon}(W) = B_{D,p}^{J/\psi}(W) - b_1 \ln(m_\Upsilon^2/m_{J/\psi}^2)$ [53] with $b_1 = 0.45 \text{ GeV}^{-2}$ and $m_\Upsilon = 9.46040 \text{ GeV}$ while $m_{J/\psi} = 3.096916 \text{ GeV}$. For photonuclear J/ψ production the value $B_{D,\text{Pb}} = (4.01 \pm 0.15) \times 10^2/\text{GeV}^2$ has been determined in [54] from a fit to ALICE data [55], which we will also use for Υ photoproduction. The specific form of Eq. (4) indicates a relatively weak energy dependence for the diffractive slope and we therefore do not expect that the above approximations significantly affect our theoretical predictions. Nevertheless our predictions rely on an extrapolation of this parameter at low center-of-mass energies and a drastic change of its energy dependence at higher energies cannot be excluded with certainty.

The subleading real part of the scattering amplitude can be estimated from the imaginary part using

$$\frac{\Re \mathcal{A}(x)}{\Im \mathcal{A}(x)} = \tan \frac{\lambda(x)\pi}{2}, \quad \lambda(x) = \frac{d \ln \Im \mathcal{A}(x)}{d \ln 1/x}. \quad (5)$$

As noted in [15], the dependence of the slope parameter λ on x provides a sizable correction to the W dependence of the complete cross section. We therefore do not assume $\lambda = \text{const}$, but determine the slope λ directly from the x -dependent imaginary part of the scattering amplitude.

A. Wave function overlap

Within high energy factorization, the imaginary part of the scattering amplitude at $t = 0$ is obtained as a convolution of the light-front wave function overlap and the dipole cross section. Using a simple boosted Gaussian model for the vector meson wave function, based on the Brodsky-Huang-Lepage prescription [56–58], we have

$$\Im \mathcal{A}_T^{\gamma p \rightarrow Vp}(x, t=0) = \int d^2r \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi_T)(r, z) \sigma_{q\bar{q}}(x, r), \quad (6)$$

where $r = |\mathbf{r}|$ denotes the transverse separation of the dipole and [59]

$$(\Psi_V^* \Psi_T) = \frac{\hat{e}_f e N_c}{\pi z(1-z)} \{ m_f^2 K_0(m_f r) \phi_T(r, z) - [z^2 + (1-z)^2] m_f K_1(m_f r) \partial_r \phi_T(r, z) \}, \quad (7)$$

with

$$\begin{aligned} \phi_T^{1s}(r, z) &= \mathcal{N}_{T,1s} z(1-z) \\ &\times \exp \left(-\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2} \right). \end{aligned} \quad (8)$$

The free parameters of Eq. (8) have been determined in various studies from the normalization and orthogonality of

TABLE I. Parameters of the boosted Gaussian vector meson wave functions for J/Ψ [60] and $\Upsilon(1s)$ [61].

Meson	m_f/GeV	$\mathcal{N}_T$	$\mathcal{R}^2/\text{GeV}^{-2}$	M_V/GeV
J/Ψ	$m_c = 1.4$	0.57	2.45	3.097
$\Upsilon(1s)$	$m_b = 4.2$	0.481	0.57	9.460

the wave functions as well as the decay width of the vector mesons. In the following we use the values found in [60,61] which we summarize in Table I.

B. Dipole cross sections

A simple model for the dipole cross section is provided by the Bartels, Golec-Biernat, Kowalski (BGK) model [49],

$$\sigma_{q\bar{q}}^{\text{BGK}}(x, r) = \sigma_0^{\text{BGK}} \left[1 - \exp \left(-\frac{r^2 \pi^2 \alpha_s(\mu_r^2) x g(x, \mu_r^2)}{3 \sigma_0^{\text{BGK}}} \right) \right], \quad (9)$$

which is obtained through exponentiating the collinear dipole cross section [49,62]. The model has been recently refitted [48] for dipole scattering on a proton to combined HERA data. The collinear gluon distribution $g(x, \mu^2)$ is subject to a leading order DGLAP evolution equation without quarks,

$$\begin{aligned} \frac{d}{d\mu^2} g(x, \mu^2) &= \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_{gg}(z) g(x/z, \mu^2), \\ xg(x, Q_0^2) &= A_g x^{-\lambda_g} (1-x)^{5.6}, \end{aligned} \quad (10)$$

where $xg(x, Q_0^2)$ denotes the gluon distribution at the initial scale $Q_0 = 1$ GeV. Following [48], we evaluate the gluon distribution and the QCD running coupling at the dipole size dependent scale

$$\mu_r^2 = \frac{\mu_0^2}{1 - \exp(-\mu_0^2 r^2 / C)}. \quad (11)$$

The remaining parameters of the model have been obtained as $\sigma_0^{\text{BGK}} = (22.93 \pm 0.27)$ mb, $A_g = 1.07 \pm 0.13$, $\lambda_g = 0.11 \pm 0.03$, $C = 0.27 \pm 0.04$, $\mu_0^2 = (1.74 \pm 0.16) \text{ GeV}^2$. To arrive at a suitable generalization of the BGK model for dipole scattering on a large nucleus, we consider two options: the impact parameter dependent saturation (IP-Sat) model and a description where the effective saturation scale $Q_s^2(x)$ is scaled by a factor $A^{1/3}$ with A the number of nucleons in the nucleus. The formulation of the IP-Sat model is based on the optical Glauber model. Using a Wood-Saxon distribution, one averages over the position of different nucleons and finds [50,63]

$$\frac{d\sigma_{q\bar{q}}^{\text{IPSat}}}{d^2b} = 2 \left[1 - \left(1 - \frac{1}{2} T_A(b) \sigma_{q\bar{q}}(x, r) \right)^A \right], \quad (12)$$

with nuclear thickness function $T(b)$ and Wood-Saxon distribution $\rho^{\text{WS}}(r)$ given by

$$T(b) = \int_{-\infty}^{\infty} dz \rho^{\text{WS}}(\sqrt{z^2 + b^2}), \quad \rho^{\text{WS}}(r) = \frac{\mathcal{N}^{\text{WS}}}{1 + e^{\frac{r-R_{\text{pb}}}{d}}}; \quad (13)$$

$\mathcal{N}^{\text{WS}}$ is fixed through $\int d^3r \rho^{\text{WS}}(r) = 1$. For the following study we will employ the values obtained in [64], $R_{\text{pb}} = 6.624$ fm and $d = 0.549$ fm, which have been also used in [65]. As an alternative to the IP-Sat model, we consider the scenario where the dipole no longer scatters on individual nucleons, but on the Lorentz contracted nucleus as a whole. Within high energy factorization, the exchanged t -channel gluons have deltalike support and it is therefore natural to assume scattering on a Lorentz contracted large nucleus, which gives rise to $A^{1/3}$ scaling of the saturation scale. To include such a scaling within the BGK model we use

$$\sigma_{q\bar{q}}^{A^{1/3}}(x, r) = \sigma_0^{\text{Pb}} \left[1 - \exp \left(-\frac{r^2 \pi^2 \alpha_s(\mu_r^2) A^{1/3} x g(x, \mu_r^2)}{3 \sigma_0^{\text{BGK}}} \right) \right]. \quad (14)$$

Based on simple geometric considerations, see also [54], one expects the parameter σ_0^{Pb} to scale approximately as $\sim A^{2/3}$; see also the discussion in [66]. In practice, such a simple rescaling yields still a considerable offset in the overall normalization. We therefore determine σ_0^{Pb} from a fit to combined ALICE-CMS dataset as $\sigma_0^{\text{Pb}} = (1.587 \pm 0.008)$ barn. Comparing to simple scaling we find that $\sigma_0^{\text{Pb}}/(\sigma_0 \cdot 208^{2/3}) \simeq 1.97$, while the obtained value is approximately 0.63 times the maximal value of $\sigma_{q\bar{q}}^{\text{IPSat}}(x, r \rightarrow \infty)$.

The energy dependence predicted by these dipoles models, including a comparison to data, is shown in Figs. 2 and 3. Since we are mainly interested in the energy dependence, we allow for adjustments in the overall normalization, through introducing a parameter N through $\Im m \mathcal{A} \rightarrow N \cdot \Im m \mathcal{A}$, which we adjust through a fit to J/ψ data for $W < 500$ GeV. While we find a very good description of the energy dependence of the J/ψ photoproduction cross section on a proton, Fig. 2, for $W < 500$ GeV, the model undershoots proton data for $W > 500$ GeV. This seems to indicate that the size of saturation effects obtained from the fit to HERA data are too strong. Since we will use in the following the model to create initial conditions at $x = 0.01$, this does not provide a problem to us. For the $\Upsilon(1s)$ photoproduction cross section we use the normalization determined from the J/ψ fit. We observe that both energy dependence and normalization are well described in this case. As a side note we mention that our own earlier attempts used instead of the BGK model the GBW model; while the energy dependence is still well described for the Υ , one finds a considerable offset in normalization, if the latter is fixed using J/ψ data. Comparison of the photonuclear cross section with data is shown in Fig. 3. While the IP-Sat model provides a good

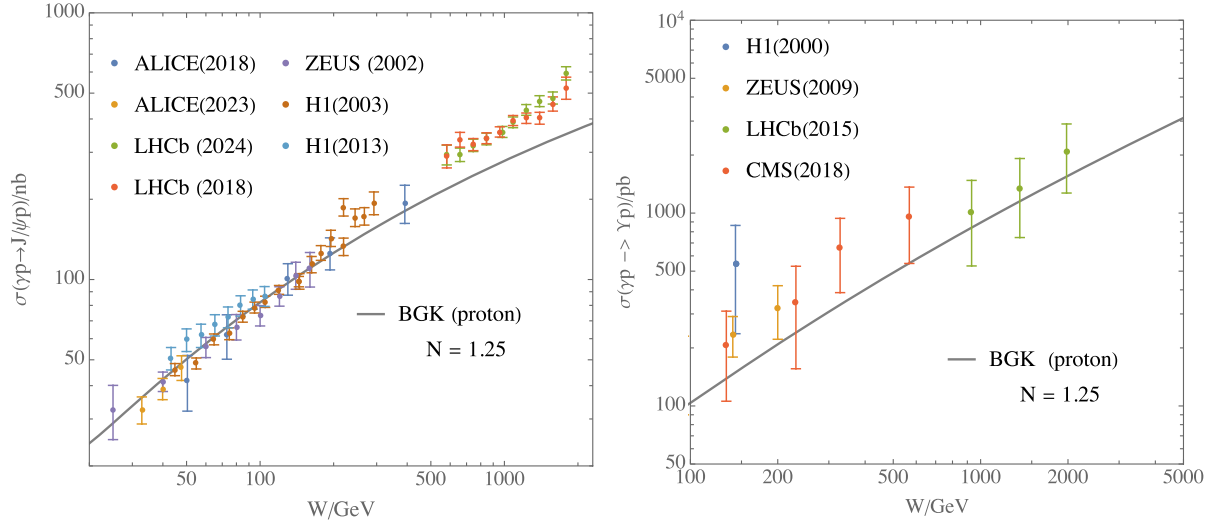


FIG. 2. Energy dependence of the total cross sections for exclusive photoproduction of J/Ψ (left) and $\Upsilon(1s)$ on a proton, using the BGK dipole model. We further display photoproduction data for J/ψ from ZEUS [68,69], H1 [70–72], ALICE [73,74], and LHCb [75,76] as well as $\Upsilon(1s)$ data obtained at H1 [77], ZEUS [78], LHCb [79], and CMS [80].

estimate of the overall normalization of the cross section, which for the $A^{1/3}$ scaled BGK model has been fitted to data, its energy dependence is in general too strong and does not agree with data. Even with the proton dipole cross section undershooting data, the IP-Sat model—which has been obtained from the proton dipole cross section—overshoots data. The $A^{1/3}$ scaled BGK model provides, on the other hand, a good description of the energy dependence, in particular, if one ignores the offset with high energy CMS data. Even though the observation is merely made on the levels of saturation models, our results seems to indicate that assuming scattering of the dipole on individual nucleons is no longer justified. Treating the lead nucleus as a scaled-up proton provides, on the other hand, a more

accurate description of the energy dependence. We further present results for the nuclear suppression factor which is defined as $S_{\text{Pb}} = \sqrt{\sigma_{\gamma\text{Pb}}/\sigma_{\gamma\text{Pb}}^{\text{IA}}}$. Following [44,67] the cross section in the impulse approximation $\sigma_{\gamma\text{Pb}}^{\text{IA}}$ is obtained as

$$\sigma_{\gamma\text{Pb}}^{\text{IA}} = \frac{d\sigma_{\gamma p \rightarrow J/\psi p}}{dt} \Big|_{t=0} \cdot \int_{|t|_{\min}}^{\infty} d|t| |F(t)|^2, \quad (15)$$

$$F(t) = 208 \cdot \int d^3r e^{i\vec{q} \cdot \vec{r}} \rho^{\text{WS}}(r),$$

with $t = -\vec{q}^2$ and $t_{\min} \simeq 0$ for LHC kinematics. As expected, the IP-Sat model does not describe the energy

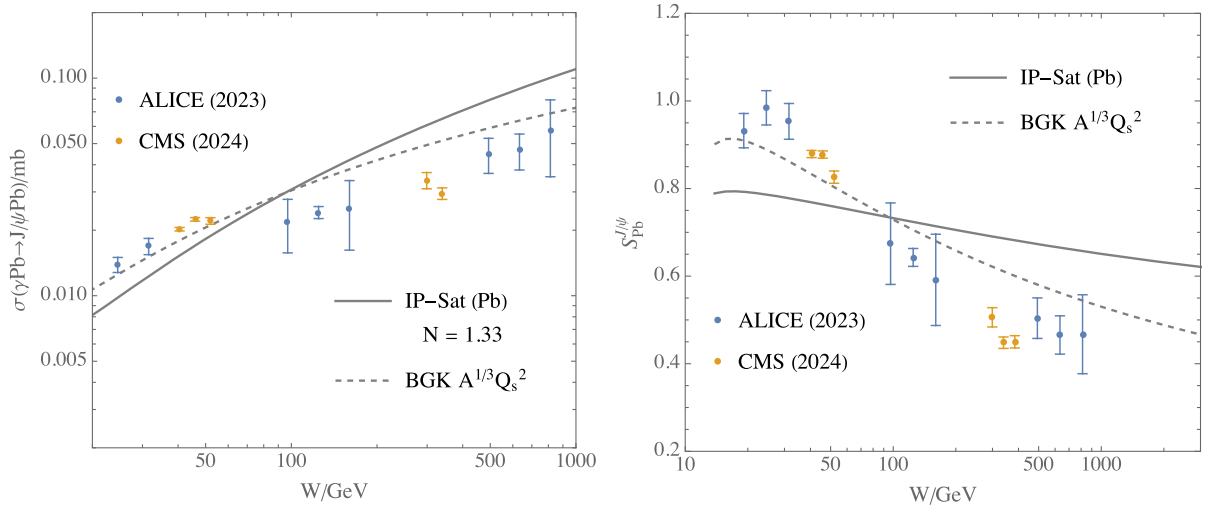


FIG. 3. Energy dependence of the J/ψ photoproduction cross section on lead (left) and the nuclear modification factor (right) obtained from the dipole models Eqs. (12) and (14). We further show ALICE [44] and CMS [45] data.

dependence of the nuclear modification factor. The $A^\frac{1}{3}$ scaled BGK model provides, on the other hand, a good description of data, even though it overshoots high energy CMS data points.

III. ENERGY DEPENDENCE THROUGH BFKL

In the following the above dipole models are used to generate initial conditions for BFKL evolution.

A. BFKL evolution of dipole cross sections

Our starting point is the following factorization of the elastic scattering amplitude in the forward limit, $t = 0$,

$$\Im \mathcal{M}_T^{\gamma p \rightarrow \gamma p}(x, 0) = \int dk^2 \int dq^2 \Phi^{\gamma V}(k) \mathcal{F}_{\text{BFKL}}^{\text{DIS}} \times \left(\frac{x_0}{x}, k, q \right) G(x_0, q), \quad (16)$$

where $\mathcal{F}_{\text{BFKL}}^{\text{DIS}}$ denotes the NLO BFKL Green's function in the deep inelastic scattering (DIS) scheme and

$$\Phi^{\gamma V}(k, m_f) = \frac{4\pi^2}{N_c} \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \frac{1 - e^{ik \cdot \mathbf{r}}}{k^2} (\Psi_V^* \Psi_T)(r, z) \quad (17)$$

provides the impact factor for the photon vector meson transition. $G(x_0, q)$ denotes, on the other hand, the unintegrated gluon distribution at $x_0 = 0.01$. The expression can be constructed as follows: at leading order, one has the following relation between the dipole cross section and unintegrated gluon distribution $G(x, k)$:

$$\sigma_{q\bar{q}}(x, r) = \frac{4\pi^2\alpha_s}{N_c} \int \frac{d^2\mathbf{k}}{\pi} \frac{1 - e^{ik \cdot \mathbf{r}}}{k^2} G(x, k), \quad k = |\mathbf{k}|. \quad (18)$$

To evolve $G(x_0, k)$ to low x , we use a solution of the NLO BFKL Green's function elaborated in [46,47,51]

$$G(x, k) = \int_0^\infty \frac{dq^2}{q^2} \mathcal{F}_{\text{BFKL}}^{\text{DIS}} \left(\frac{x_0}{x}, k, q \right) G(x_0, q). \quad (19)$$

The unintegrated gluon distribution at x_0 can be finally determined from the dipole cross section using¹

$$G(x_0, k) = \frac{N_c k^2}{8\pi^2 \alpha_s} \int_0^\infty dr r J_0(rk) [\sigma_0 - \sigma_{q\bar{q}}(x_0, r)],$$

$$\sigma_0 = \lim_{r \rightarrow \infty} \sigma_{q\bar{q}}(x_0, r). \quad (20)$$

Equation (19) describes correctly the low x evolution of the unintegrated gluon distribution if high density corrections

are suppressed by powers of the strong coupling α_s , i.e., if the gluon distribution does not saturate the bound $G \sim 1/\alpha_s$. This is the central hypothesis of this section: we assume that such corrections are irrelevant and we explore to which extent this leads to tension with data. We further note that both Eqs. (18) and (20) depend on α_s , while the dependence cancels if both expressions are combined. In general, it is not clear if both strong couplings are to be evaluated at the same scale, which gives rise to a normalization ambiguity, see also the discussion in [81].

To solve the NLO BFKL evolution, we follow closely [46,47]. In transverse Mellin space we have

$$\mathcal{F}_{\text{BFKL}}^{\text{DIS}} \left(\frac{x_0}{x}, k, q \right) = \frac{1}{k^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} (k^2)^{\gamma-1} \hat{f} \times \left(\frac{x_0}{x}, \frac{M^2}{Q_s^2(x_0)}, \frac{\bar{M}^2}{M^2}, \gamma \right) (q^2)^{-\gamma}, \quad (21)$$

with $\hat{f}$ an operator in γ space and defined as

$$\hat{f} = \left(\frac{1}{x} \right)^{\chi(\gamma, \mu, M)} \cdot \left\{ 1 - \frac{\bar{\alpha}_s^2 \beta_0}{8N_c} \log \left(\frac{x_0}{x} \right) \left[\overleftarrow{\partial}_\gamma \chi_0(\gamma) - \chi_0(\gamma) \overrightarrow{\partial}_\gamma - 2\chi_0(\gamma) \ln M^2 \right] \right\}, \quad (22)$$

see also the related discussion in [15,51,82,83]. Here $\bar{\alpha}_s = \alpha_s(\mu)N_c/\pi$, while μ denotes the renormalization scale. M is, on the other hand, a parameter which has been introduced to identify the central value of the renormalization scale of the problem; it cancels to NLO accuracy. In the following, we first determine a suitable choice for M and then vary μ in the range $M/2, 2M$ to estimate the uncertainty associated with our prediction. The determination of M will be discussed below. $\chi(\gamma, \mu, M)$ is the next-to-leading logarithmic (NLL) BFKL kernel after collinear improvements; in addition, large terms proportional to the first coefficient of the QCD beta function, $\beta_0 = 11N_c/3 - 2n_f/3$, are resummed through employing a Brodsky-Lepage-Mackenzie (BLM) optimal scale setting scheme [84]. For those details we follow very closely the treatment in [15,46,47]. The NLL kernel with collinear improvements reads

$$\chi(\gamma, \mu, M) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1(\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi'_0(\gamma) \chi_0(\gamma) + \chi_{RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}) - \frac{\bar{\alpha}_s^2 \beta_0}{4N_c} \chi_0(\gamma) \log \frac{\mu^2}{M^2}, \quad (23)$$

with the leading-order BFKL eigenvalue,

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma). \quad (24)$$

Employing BLM optimal scale setting and the momentum space physical renormalization scheme based on a

¹See [81] for a compact summary of the procedure.

symmetric triple gluon vertex with $Y \simeq 2.343907$ and gauge parameter $\xi = 3$ one obtains the following next-to-leading order BFKL eigenvalue:

$$\begin{aligned} \tilde{\chi}_1(\gamma) = & \tilde{\mathcal{S}}\chi_0(\gamma) + \frac{3}{2}\zeta(3) \\ & + \frac{\Psi''(\gamma) + \Psi''(1-\gamma) - \phi(\gamma) - \phi(1-\gamma)}{4} \\ & - \frac{\pi^2 \cos(\pi\gamma)}{4\sin^2(\pi\gamma)(1-2\gamma)} \left[3 + \left(1 + \frac{n_f}{N_c^3}\right) \frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)} \right] \\ & + \frac{1}{8} \left[\frac{3}{2}(Y-1)\xi + \left(1 - \frac{Y}{3}\right)\xi^2 + \frac{17Y}{2} - \frac{\xi^3}{6} \right] \chi_0(\gamma), \quad (25) \end{aligned}$$

where $\tilde{\mathcal{S}} = 1/3 - \pi^2/12$. We note that the specific choice on scheme and parameters follows the one of [46,85]; see also the discussion in [86]. Within this scheme one then replaces $\bar{\alpha}_s \rightarrow \tilde{\alpha}_s(\gamma)$ with

$$\tilde{\alpha}_s(\bar{\alpha}_s, \gamma) = \frac{\bar{\alpha}_s(\mu)}{1 + \frac{\bar{\alpha}_s \beta_0}{4N_c} \left[\frac{1}{2}\chi_0(\gamma) - \frac{5}{3} + 2\left(1 + \frac{2}{3}Y\right) \right]}, \quad (26)$$

which yields the effective running coupling constant to be used for the Green's function. For the numerical study we use the NLO QCD running coupling with $\alpha_s(M_Z) = 0.1181$ and $M_Z = 91.1876$ GeV. The term which achieves a resummation of collinear enhanced terms in the NLO BFKL kernel reads

$$\begin{aligned} \chi_{RG}(\bar{\alpha}_s, \gamma, a, b) = & \bar{\alpha}_s(1 + a\bar{\alpha}_s)(\psi(\gamma) - \psi(\gamma - b\bar{\alpha}_s)) \\ & - \frac{\bar{\alpha}_s^2}{2} \psi''(1-\gamma) - \frac{b\bar{\alpha}_s^2 \cdot \pi^2}{\sin^2(\pi\gamma)} \\ & + \frac{1}{2} \sum_{m=0}^{\infty} \left(\gamma - 1 - m + b\bar{\alpha}_s - \frac{2\bar{\alpha}_s(1 + a\bar{\alpha}_s)}{1 - \gamma + m} \right. \\ & \left. + \sqrt{(\gamma - 1 - m + b\bar{\alpha}_s)^2 + 4\bar{\alpha}_s(1 + a\bar{\alpha}_s)} \right). \quad (27) \end{aligned}$$

For details on the derivation of this term we refer to the discussion in [46]. The coefficients $\tilde{a}, \tilde{b}$ which enter the collinear resummation term equation (27) are obtained as the coefficients of the $1/\gamma$ and $1/\gamma^2$ poles of the NLO eigenvalue. We have

$$\begin{aligned} \tilde{a} = & -\frac{13}{36} \frac{n_f}{N_c^3} - \frac{55}{36} + \frac{3Y-3}{16} \xi + \frac{3-Y}{24} \xi^2 - \frac{1}{48} \xi^3 + \frac{17}{16} Y, \\ \tilde{b} = & -\frac{n_f}{6N_c^3} - \frac{11}{12}. \quad (28) \end{aligned}$$

With the elements of Eq. (22) fixed, we determine as a next step the Mellin transform of Φ^{rV} and $G(x_0, q)$. We have [15]

$$\begin{aligned} \phi^{rV}(\gamma, m_f) = & \int_0^\infty \frac{dk^2}{k^2} \left(\frac{k^2}{m_f^2} \right)^\gamma \Phi^{rV}(k, m_f) = \int_0^1 \frac{dz}{4\pi} \tilde{\phi}_{V,T}(\gamma, z, m_f) \\ \tilde{\phi}_{V,T}(\gamma, z, m_f) = & e\hat{e}_f 8\pi^2 \mathcal{N}_T \frac{\Gamma(\gamma)\Gamma(1-\gamma)}{m_f^2} \left(\frac{m_f^2 \mathcal{R}^2}{8z(1-z)} \right)^{2-\gamma} e^{-\frac{m_f^2 \mathcal{R}^2}{8z(1-z)}} e^{\frac{m_f^2 \mathcal{R}^2}{2}}, \\ & \times \left[U\left(2-\gamma, 1, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)}\right) + [z^2 + (1-z)^2] \frac{\Gamma(3-\gamma)}{\Gamma(2-\gamma)} U\left(3-\gamma, 2, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)}\right) \right], \quad (29) \end{aligned}$$

where $U(a, b, z)$ is a hypergeometric function of the second kind or Kummer's function.² We note that normalization of the above impact factor is inversely proportional to the heavy quark mass; its precise value takes therefore an important role in fixing the normalization of the photo-production cross section. While the heavy quark mass is the natural scale of the impact factor describing the photon-vector meson transition, the choice is less clear for the hadronic impact factor. While one might use as a first guess the saturation scale of the dipole model at initial $x_0 = 0.01$, the NLO BFKL fit to the HERA data of [46,47] suggests

using a smaller value, of the order of a few hundred MeV. Within the current setup, we then define for each unintegrated gluon distribution at $x_0 = 0.01$ a characteristic scale Q_0 through

$$1 - \sigma_{q\bar{q}}(x_0, r = 1/Q_0)/\sigma_0 = e^{-1}; \quad (30)$$

the resulting scale Q_0 is then half the saturation scale of a corresponding GBW type model [48]. Note that the above choice only affects our result in the way we fix the scale M . We then finally arrive at

$$\phi^{\text{ugd}}(\gamma, Q_0) = \int_0^\infty dq^2 \left(\frac{q^2}{Q_0^2} \right)^{-\gamma} G^{\text{ugd}}(x_0, q, Q_0). \quad (31)$$

²With respect to the expression found in [15], we corrected an incorrect power of m_f in the exponent as well as relative factor of 2 between the first and the second term.

To determine this quantity, we first determine numerically $G^{\text{ugd}}(x_0)$ from the dipole cross section and then from the resulting expression $\phi^{\text{ugd}}(\gamma)$. With these ingredients we have

$$\begin{aligned} \Im \mathcal{A}(x, 0) = N \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{m_f^2}{Q_0^2} \right)^\gamma \\ \times \left(\frac{x_0}{x} \right)^{\chi(\gamma, \mu, M)} \phi^{\gamma V}(\gamma, m_f) \phi^{\text{ugd}}(\gamma, Q_0) \\ \cdot \left\{ 1 - \frac{\tilde{\alpha}_s^2(\tilde{\alpha}_s, \gamma) \beta_0 \chi_0(\gamma)}{8N_c} \log \left(\frac{x_0}{x} \right) \right. \\ \left. \cdot \left[\frac{d}{d\gamma} \ln \frac{\phi^{\gamma V}(\gamma, m_f)}{\phi^{\text{ugd}}(\gamma, Q_0)} - 2 \ln \frac{M^2}{m_f Q_0} \right] \right\}, \quad (32) \end{aligned}$$

where N is a factor that collects the various sources of normalization uncertainty. While $N = \mathcal{O}(1)$, its precise value will be fixed by a comparison to data. We finally set $M^2 = k \cdot m_f \cdot Q_0$ with k a parameter which we vary. While $k = 1$ is the natural choice, we will explore also other values in the following; recall that the value of M sets the central value for the renormalization scale μ , which we vary in addition in the range $[M/2, 2M]$.

B. Instability of the NLO solution

The above solution to the NLO BFKL equation possesses an instability for ultralow values of x ; see also the discussion in [17,22]: for sufficiently small values of k , the second term in the second line of Eq. (32) is negative and growing in magnitude with diminishing x . Unlike [17,22], which employed directly the results of the Hentschinski Salas Sabio Vera fit, the effect can be avoided in the current case through choosing a sufficiently large value of k . Such a choice comes however at an undesired cost: the growth with energy of the photoproduction cross section is too strong and does not agree with data. We only find agreement with data for a negative NLO correction. Since we would like to explore the degree to which BFKL evolution can be adapted to still describe photoproduction data, we allow in the following for values of k which yield such a contribution. At the same time, we demand that the BFKL prediction is still well justified within QCD perturbation theory. We define to this end two criteria, which our solution must fulfill. We first define

$$\Im \mathcal{A}(x, 0) = \Im \mathcal{A}^{(0)}(x, 0) + \Im \mathcal{A}^{(1)}(x, 0), \quad (33)$$

where

$$\begin{aligned} \Im \mathcal{A}^{(0)}(x, 0) &= N \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{m_f^2}{Q_0^2} \right)^\gamma \left(\frac{x_0}{x} \right)^{\chi(\gamma, \mu, M)} \phi^{\gamma V}(\gamma, m_f) \phi^{\text{ugd}}(\gamma, Q_0), \\ \Im \mathcal{A}^{(1)}(x, 0) &= N \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{m_f^2}{Q_0^2} \right)^\gamma \left(\frac{x_0}{x} \right)^{\chi(\gamma, \mu, M)} \phi^{\gamma V}(\gamma, m_f) \phi^{\text{ugd}}(\gamma, Q_0) \\ &\cdot \left\{ -\frac{\tilde{\alpha}_s^2(\tilde{\alpha}_s, \gamma) \beta_0 \chi_0(\gamma)}{8N_c} \log \left(\frac{x_0}{x} \right) \cdot \left[\frac{d}{d\gamma} \ln \frac{\phi^{\gamma V}(\gamma, m_f)}{\phi^{\text{ugd}}(\gamma, Q_s^2(x_0))} - 2 \ln k \right] \right\}, \quad (34) \end{aligned}$$

see also the discussion in [17]. Given these expressions we can define now two criteria that indicate that the choice of k is no longer natural and stability of the perturbative description is to be lost:

- (i) First we require that $|\Im \mathcal{A}^{(1)}(x^{\min}, 0)| / \Im \mathcal{A}^{(0)}(x^{\min}, 0) < 0.5$ at the lowest observed value of x ; for the proton $x_p^{\min} \simeq 3.0 \times 10^{-6}$, whereas for lead one currently has $x_{pb}^{\min} \simeq 1.45 \times 10^{-5}$.
- (ii) A second criteria is that we require a positive slope of the scattering amplitude $\lambda > 0$; i.e., $\Im \mathcal{A}(x^{\min}, 0)$ must still grow with energy.

For J/ψ photoproduction, these criteria allow us to push the BFKL prediction to the limits of its applicability. Photoproduction of the Υ provides, on the other hand, an important cross-check since one expects a stable perturbative prediction in this case.

IV. RESULTS AND DISCUSSION

As a first step, we provide predictions for the energy dependence of the photoproduction on the proton, both for

J/ψ and Υ vector mesons. Condition equation (30) yields in this case $Q_0^{\text{prot}} = 0.256$ GeV, which is numerically very close to the scale of the impact factor found in [46,47]. Our results are shown in Fig. 4. We would like to stress that even for the proton case, the ability of the NLO BFKL evolution to describe the energy dependence of photoproduction is a nontrivial observation. The initial conditions for the unintegrated gluon distribution were not fitted to collider data, employing NLO BFKL evolution as the underlying evolution equation. Instead, the initial conditions have been obtained from the BGK dipole model, which itself has been fitted to inclusive DIS data. The fact that NLO BFKL evolution provides a good description of the energy dependence of both the J/ψ and Υ photoproduction cross section is yet another confirmation that NLO BFKL is in general terms a useful framework to predict the energy dependence of perturbative QCD cross sections. We both study the scenario of $k = 1$ as well as of minimal $k = 0.949$. The deviation of the overall normalization constant is very close to one and has been fixed

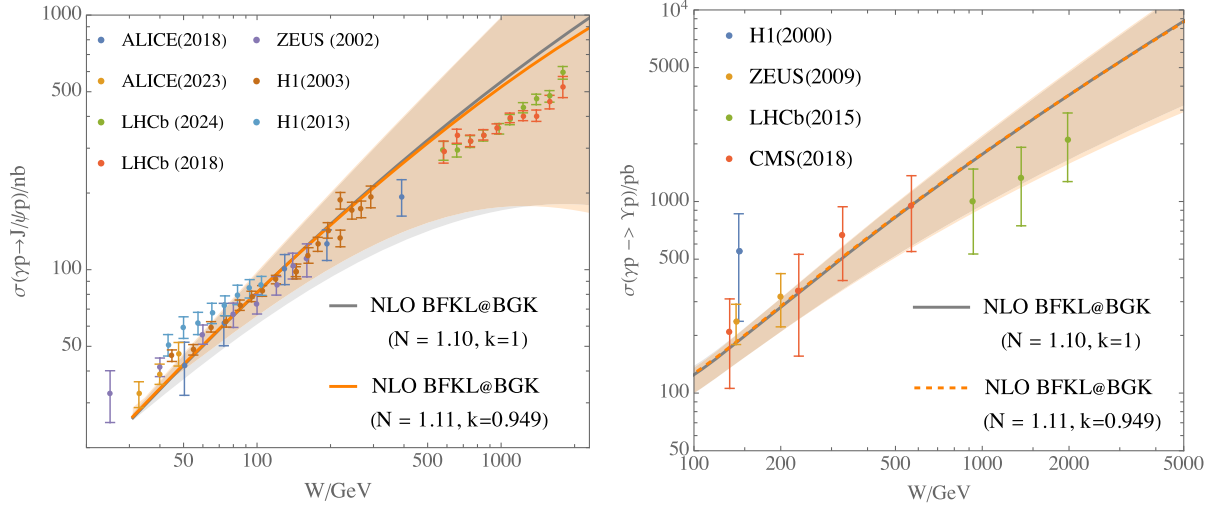


FIG. 4. Energy dependence of the J/ψ (left) and Υ (right) photoproduction cross section as obtained from acting with the NLO BFKL Green's function on the BGK dipole cross section at $x_0 = 0.01$. We displayed the same datasets as in Fig. 2.

through a fit to J/ψ data at $W < 500$ GeV. The central values of the BFKL prediction overshoot in both cases data points at $W > 500$ GeV. An improved description with a smaller value of k , would however violate our stability conditions, which were formulated in Sec. III B. The depicted uncertainty band is obtained due to a variation of the renormalization scale in the range $\mu \in [M/2, 2 \cdot M]$. For $W > 500$ GeV, the uncertainty associated with our prediction of the J/ψ photoproduction cross section is already considerable. While this is unpleasant, such a behavior was to be expected, based on the results of [17,22]. As argued there, the BFKL prediction leaves certain space for nonlinear QCD evolution—which in some cases can provide an improved description—but the uncertainties are too large to provide definite evidence. Uncertainties are considerably reduced for Υ

photoproduction. While our predictions agree within uncertainties with experimental results, our central predictions overshoots data. This is contrast to the result of [33], where BFKL was found to provide a good description of the energy dependence. Since [33] uses a t -dependent solution to the BFKL equation, this might indicate a stronger growth of the diffractive slope parameter B_D than provided by the parametrization of HERA data equation (4). Last but not least we would like to comment that one may also try to obtain the dipole distribution function using the conceptually simpler Golec-Biernat-Wüsthof (GBW) dipole model. While this model has the advantage that analytic expressions can be obtained for the Mellin transform of the unintegrated gluon distribution, it lacks corrections due to DGLAP evolution at the initial x_0 , which gives rise to a certain offset in normalization between the J/ψ and Υ

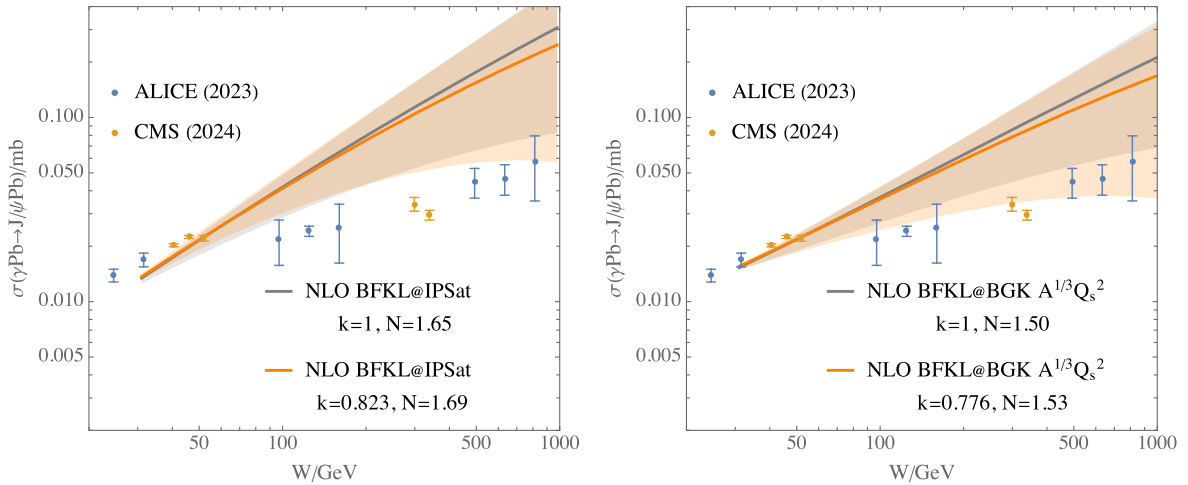


FIG. 5. Energy dependence of the J/ψ photoproduction cross section as obtained from acting with the NLO BFKL Green's function on the IP-Sat (left) and $A^{1/3}$ scaled BGK dipole cross section at $x_0 = 0.01$. We displayed the same datasets as in Fig. 3.

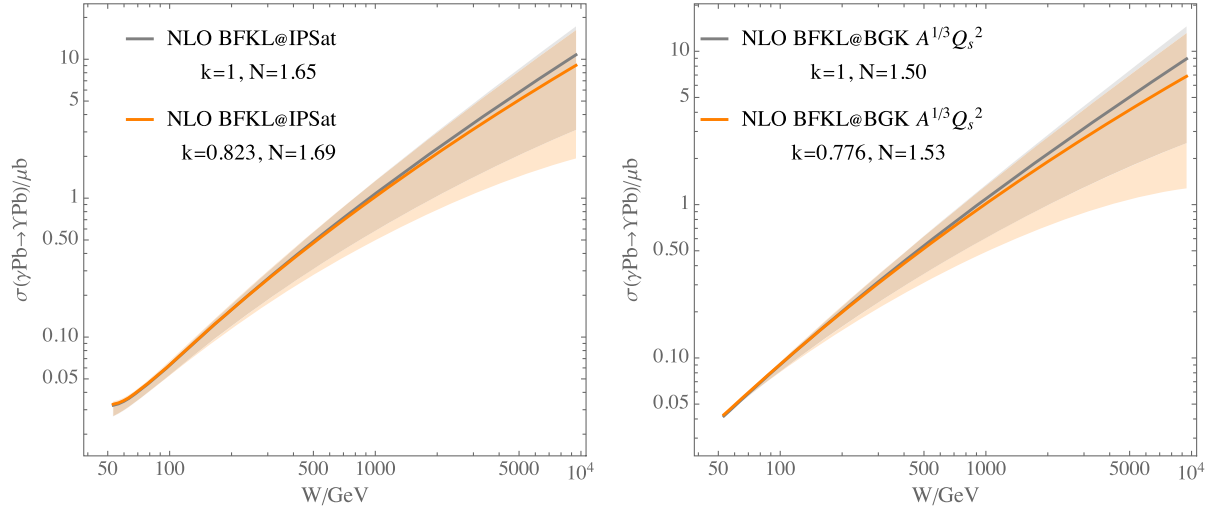


FIG. 6. Energy dependence of the Υ photoproduction cross section as obtained from acting with the NLO BFKL Green's function on the IP-Sat (left) and $A^{1/3}$ scaled BGK dipole cross section at $x_0 = 0.01$.

photoproduction cross section. While not central to the objective of this paper, we believe that this is an interesting observation since it indicates that DGLAP type corrections are also needed for impact factors.

BFKL evolution for J/ψ photoproduction on lead is shown in Fig. 5, including a comparison to data. For the IP-Sat model we find $Q_0^{\text{IPs}} = 0.344$ GeV, which is only marginally larger than the proton results. For the $A^{1/3}$ scaled proton saturation scale, we have instead $Q_0^{A^{1/3}} = 0.622$ GeV. As for the proton, we show predictions both for the case $k = 1$ and for the minimal k value, which is allowed by the stability requirements of Sec. III B. While the higher values of Q_0 allow us to extend the prescription down to lower values of k , the impact on the energy dependence is rather small. For the determination of the normalization, we

exclude data points with $W > 280$ GeV. The resulting adjustments in normalization are larger than in the case of the proton. We note that not fixing the normalization leads to an improved description of values at $W > 280$ GeV, while one clearly misses the low energy data points. Since our main goal is to study predictions for the energy dependence, fixing the low energy normalization appears to be the right choice here. If initial conditions for BFKL evolution are generated through the IP-Sat model, we find strong tension between the obtained energy dependence and data. Assuming that the IP-Sat model provides the correct description of the dipole cross section at values of $x > 0.01$, we can exclude almost with certainty that the energy dependence of photonuclear J/ψ production is described by low density QCD low x evolution. While the $A^{1/3}$ scaled BGK model also provides a BFKL

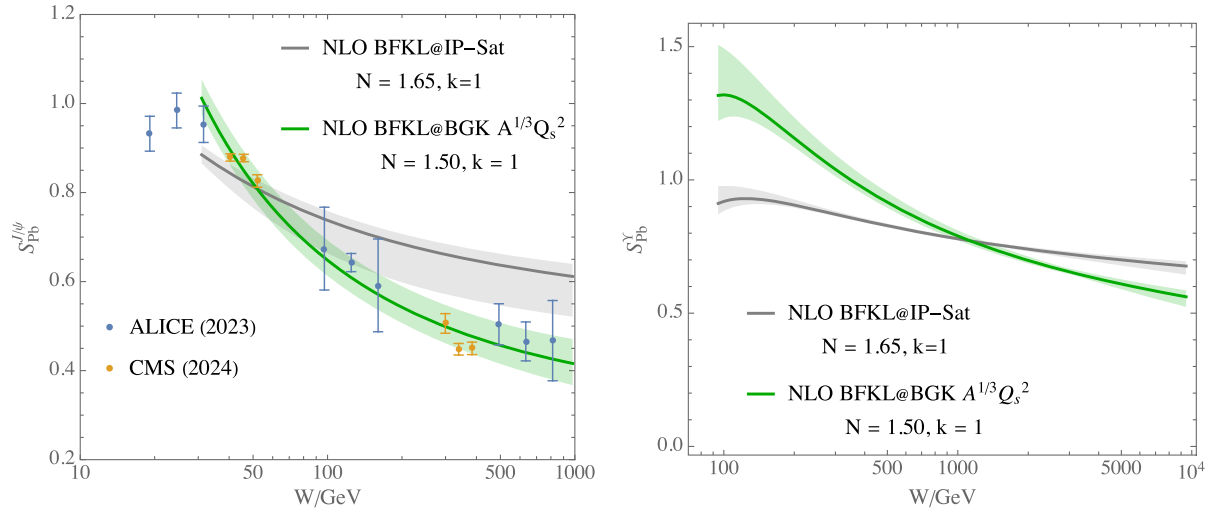


FIG. 7. Nuclear modification factor for J/ψ (left) and Υ (right). We further show ALICE [44] and CMS [45] data.

prediction which clearly overshoots data, the situation is less clear in that case. In particular for $k = 0.776$, the majority of data points is still described within the large uncertainty of the BFKL prediction. At the same time, one needs to keep in mind that the description is based on a significantly increased value of Q_0 , which requires strong correlations between individual nucleons in the lead nucleus.

In Fig. 6 we finally provide predictions for Υ photonuclear production. As in the proton case, uncertainties are reduced for the Υ due to the higher hard scale. Similar to the J/ψ case we find that BFKL based on IP-Sat initial conditions grows stronger with energy than the $A^{1/3}$ scaled BGK saturation model.

In Fig. 7 we finally provide the nuclear modification factor for both J/ψ and Υ photonuclear production. As for saturation models we determine the nuclear modification factor as $S_{\gamma\text{Pb}} = \sqrt{\sigma_{\gamma\text{Pb}}/\sigma_{\gamma\text{Pb}}^{\text{IA}}}$, where $\sigma_{\gamma\text{Pb}}^{\text{IA}}$ is determined from Eq. (15), using now the proton cross section with BFKL evolution. As one might expect, the theoretical uncertainties associated with BFKL evolution reduce significantly for the nuclear modification factor $S_{\gamma\text{Pb}}^{J/\psi}$. While the energy dependence predicted by the NLO BFKL evolution with IP-Sat model initial conditions is within uncertainties in agreement with ALICE data, it clearly overshoots CMS data points at high W . The description based on NLO BFKL evolution based on the $A^{1/3}$ scaled BGK model describes the energy dependence of the nuclear modification factor very well, over the entire range in energies. For photonuclear production of $\Upsilon(1s)$, the nuclear modification factor is within our framework significantly reduced, albeit the deviation from one is still sizable at highest values of W .

V. CONCLUSIONS

We explored to which extent NLO BFKL evolution is able to describe the energy dependence of photoproduction cross sections of vector mesons J/ψ and $\Upsilon(1s)$ on a proton and on a lead nucleus. Unlike previous studies, e.g., [33], which provided a solution of the BFKL equation for nonzero momentum transfer $t \neq 0$, but were restricted to leading order in the perturbative expansion, our solution is restricted to $t = 0$, while the complete NLO corrections have been included, together with a resummation of collinear enhanced terms in the NLO BFKL Green's function. As a first result we find that we need to push the NLO BFKL description to the limits of its applicability to describe the observed energy dependence. To guarantee perturbative stability, we defined two criteria, i.e., a limit on the size of a perturbative correction to the Green's function as well as a positive slope at smallest observed x values. With these criteria we find that for J/ψ photoproduction, central values of the BFKL description overshoot both proton and lead data. While proton data agree with the

BFKL description within the sizable theory uncertainties, lead data only agree with the BFKL prediction within uncertainties, if the $A^{1/3}$ scaled BGK model is used to generate initial conditions for BFKL evolution. The photoproduction cross section of the Υ on the proton is, on the other hand, very well described.

A natural question to ask is whether an improved description would have been obtained, if a solution to the NLO BFKL equation with $t \neq 0$ would have been used, similar to the study presented in [33]. We believe that this is not the case, since the observed energy dependence of the diffractive slope B_D is, in general, weak and should not lead to such a drastic change. Instead it seems more likely that a more suitable choice of initial conditions could provide an improved description of data. On the other hand, [33] finds a strong dependence on the impact parameter in their solution to the BFKL evolution equation in the case of the proton, which is somehow in contradiction to this line of argument. While we are currently not able comment on the details of the solution of [33], we believe that it would be a worthwhile exercise to compare in the future both solutions as well as to explore possibilities to arrive at NLO BFKL predictions with t dependence; see [87] for a framework which might allow for progress in this direction. For the time being, a particularly interesting observable is provided by the nuclear modification factor: we find that the sizable uncertainties of the BFKL description reduces significantly in this case and we find an excellent description of data for initial conditions created by the $A^{1/3}$ scaled BGK model, whereas BFKL evolution based on the IP-Sat model clearly overshoots data.

Our results indicate the following: at first there are strong indications that the nuclear gluon density per transverse area scales as $A^{1/3}$. The $A^{1/3}$ scaled dipole model provides itself a good description of the photonuclear cross section as well as of the nuclear modification factor. At the same time, the $A^{1/3}$ scaled dipole model provides adequate initial conditions for NLO BFKL evolution. This is particularly remarkable since descriptions based on the IP-Sat model fails. Our result for the nuclear modification factor seems furthermore to indicate that a description of the energy dependence does currently not require the inclusion of nonlinear terms into low x evolution. Nevertheless one should keep in mind that the NLO BFKL description is only possible due to the negative perturbative correction to the NLO BFKL Green's function. Choosing the renormalization scale in a way such that this term is very small or positive, a description of the energy dependence would be not possible. In this sense, our result cannot exclude completely the need of a nonlinear terms in the evolution (which arguably would have a similar effect). However our results seem to imply that the size of nonlinear terms should be of similar magnitude for both photoproduction on the proton and a lead nucleus.

For future studies it would be interesting to generate initial conditions for NLO BFKL evolution directly from nuclear PDFs. While this is challenging, it would probably

reduce further the uncertainty in fixing initial conditions. A future measurement of the Υ production at lead targets would be of great use to further verify the current approach. While one expects nonlinear dynamics to further weaken in this case, different initial conditions to NLO BFKL evolution provide significantly different results for the nuclear modification factor in the case of the Υ ; its measurement would therefore clearly contribute to clarify the situation further and to fix the correct initial conditions. Finally, measurement of the dependence of the vector meson production cross section on photon virtuality at the future EIC would allow us to determine the lead impact factor from a fit to EIC data and therefore reduce uncertainties.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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