

## Derivative expansion in a two-scalar field theory

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The derivative expansion of the effective action is considered in the model with two interacting real scalar fields in curved spacetime. Using the functional approach and local momentum representation, the coefficient of the derivative term is calculated up to the first order in curvature in the one-scalar theory. The two-scalar problem is solved by extracting normal modes and consequent reduction to the single-scalar case. The method can be applied to a larger number of scalars. In the theory with strong hierarchy of masses, the renormalized effective potential and the coefficients of the second-order derivative terms demonstrate the quantum decoupling in the low-energy limit.

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### I. INTRODUCTION

Certain cosmological models, such as assisted inflation, are based on two scalar fields with a strong hierarchy of masses (see, e.g., Refs. [1–3] and the more recent Ref. [4]). Since the typical energy scale in the inflationary epoch is high, it may be important to evaluate quantum effects modifying the scalar action. The relevant quantum corrections may include not only the effective potential of scalars but also the lowest-order derivative terms. In this respect, one can note that there are several insufficiently studied aspects of the quantum theory. The first one of these important details is that cosmological inflation is characterized by large values of curvatures, which cannot be neglected in the effective potential and in the quantum contributions to the derivative terms. The curvature-dependent terms are pretty well-known for the effective potential but almost not explored for the derivative expansion. The last unexplored (at least to our knowledge) issue is that the quantum effects may be quite different in the high-energy (UV) and low-energy (IR) limits owing to the quantum decoupling of the large-mass scalar in the latest stage of inflation, where the energy scale gets lower. As a result, at the later phase of inflation, the quantum effects of only one scalar remain relevant. The described features and possibilities make it interesting to explore the curvature-dependent effective potential and the lowest-order

derivative expansion of the effective action in the multiscalar models, especially with different masses.

The derivative expansion of the effective action is a traditional subject of study in quantum field theory [5,6], which has been extensively applied in various branches of effective field theory (see, e.g., [7,8]). The technique was also developed in curved spacetime using Riemann normal coordinates [9] and zeta regularization [10]. It is also worth noting Refs. [11,12] with the development of the derivative expansion for  $O(N)$  scalar model in flat spacetime, i.e.,  $N$  copies of a scalar field with equal masses.

In the present work, we explore the one-loop quantum contributions to the effective action of two scalar fields with different masses in zero and second orders of derivative expansion, in curved spacetime. We follow a simple approach similar to extracting normal modes of the oscillator in classical mechanics. The scheme can be applied to the multiple scalars and arbitrary self-interactions, at least without derivatives. Following the previous literature [9], we use functional methods in curved spacetime, combined with local momentum representation [13] (see also [14] for further references), based on normal coordinates [15] (see also [16] for a more pedagogical introduction) to get the result in an arbitrary curved metric background.

One of the motivations for our work is to elaborate the formalism for taking into account quantum effects in the cosmological models with multiple scalar fields. Such quantum corrections may influence inflationary dynamics and therefore may be relevant. An interesting aspect in these scenarios is the potential (IR) decoupling of heavy fields, which, if present, could effectively reduce the number of relevant degrees of freedom at late times. This phenomenon is particularly important for understanding whether in the later cosmological epoch only the

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quantum effects of a lighter scalar field remain significant, reinforcing the role of effective quantum approach in such kind of models. Beyond the applications to inflation, our results may be also useful for a broader understanding of quantum corrections in multifield theories, where functional techniques provide a powerful framework for systematically extracting both potential and lowest-order derivative terms.

The paper is organized as follows. In Sec. II, we follow the work [6] on the derivative expansion for a single scalar field. After this, the result of this consideration is extended to an arbitrary curved spacetime up to the first order in curvature, by using normal coordinates. Section III reports on the derivative expansion in the model with two scalars with different masses. We perform the calculations in a direct way and also follow the new “double diagonalization” approach to reduce the problem to a single scalar case. It is remarkable that this method is more economical and, in principle, permits a generalization to an arbitrary number of scalars. Section IV is devoted to the decoupling of massive degree of freedom in the theory with a strong hierarchy of masses. Finally, in Sec. V, we draw our conclusions. Throughout this work, we adopt the metric signature  $(+, -, -, -)$  and assume the Wick rotation where necessary, in some cases, without additional comments.

## II. DERIVATIVE EXPANSION FOR A SINGLE SCALAR FIELD

Derivative expansion is a useful extension of the functional methods to calculate the effective potential. In general, the effective action is a nonlocal functional, but one can start from the constant part, i.e., the effective potential, and extract the lowest-order terms corresponding to the weakly varying fields. The scheme may be applied to different kinds of fields, but the most common application is to scalars. In the case of a single mean field  $\phi$ , the first terms in the expansion have the form

$$\Gamma[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left\{ -V_{\text{eff}}(\phi) + \frac{1}{2} Z(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \dots \right\}. \quad (1)$$

The calculation of  $V_{\text{eff}}(\phi)$  can be performed for a constant  $\phi = \phi_0$  to any order of the loop expansion and for the theories with different contents of quantum fields. In this work, we restrict our attention to the self-interacting scalar field in curved spacetime. The one-loop flat-space result was derived in the renowned work [17]. The general investigation based on the functional methods was done in [18]. Let us reproduce the expression for the renormalized one-loop contribution to the effective potential derived in [19] by using the mass-dependent scheme of renormalization and normal coordinates,

$$V_{\text{eff}}^{\text{ren}}(g_{\mu\nu}, \phi) = \rho_\Lambda + \frac{1}{2}(m^2 - \xi R)\phi^2 + V + \frac{1}{2(4\pi)^2} \left[ \frac{1}{2}(m^2 + V'')^2 - \left( \xi - \frac{1}{6} \right) R(m^2 + V'') \right] \ln \left( \frac{m^2 + V''}{\mu^2} \right), \quad (2)$$

where the density of the cosmological constant term  $\rho_\Lambda$  is included for completeness, and  $\mu^2$  is the dimensional parameter of renormalization.

The evaluation of  $Z(\phi)$  requires variable fields. The idea of the standard procedure [5,6] is to split the scalar field into a background constant  $\phi_0$  and the variable part  $\varphi$  as  $\phi(x) = \phi_0 + \varphi(x)$  and evaluate the effective action up to the second order in derivatives. Since  $\partial\phi(x) = \partial\varphi(x)$ , this approach gives a complete result for  $Z$ . Although one can apply this method to calculate terms of the higher orders in derivatives [6], for small oscillations near the minimum, these terms are less relevant, and we do not consider them here.

### A. Single scalar field in flat space

To illustrate the procedure, we first perform the flat-space calculation in the classical theory of a real scalar field with the self-interaction  $\lambda\phi^4$ . The Lagrangian density is

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4. \quad (3)$$

After Wick rotation, the one-loop contribution to the effective action is given by

$$\Gamma^{(1)}(\phi) = \frac{1}{2} \text{Tr} \ln(\square + M^2), \quad (4)$$

where

$$M^2 = m^2 + \frac{\lambda}{2} \phi^2(x). \quad (5)$$

In the next subsection, we consider the generalization to curved spacetime, where

$$\begin{aligned} \Gamma^{(1)}(\phi) &= \int d^4x \sqrt{-g} \left\{ -V^{(1)}(\phi) + \frac{1}{2} Z^{(1)}(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \dots \right\}. \end{aligned} \quad (6)$$

For now, consider the one-loop version of (1) in flat space [6]. To find the one-loop effective potential  $V^{(1)}$ , it is sufficient to consider a constant scalar  $\phi_0$  in Eq. (4). On the other hand, the derivation of  $Z^{(1)}(\phi)$  requires setting  $\phi(x) = \phi_0 + \varphi(x)$ , with a subsequent expansion to the second order in the coordinate-dependent part  $\varphi(x)$ .

The coefficients of this expansion depend only on  $\phi_0$ . In this way, we obtain

$$\Gamma^{(1)}(\phi_0 + \varphi) = \int d^4x \left\{ -V^{(1)}(\phi_0) - \frac{\partial V^{(1)}}{\partial \phi_0} \varphi - \frac{1}{2} \frac{\partial^2 V^{(1)}}{\partial \phi_0^2} \varphi^2 + \frac{1}{2} Z^{(1)}(\phi_0) \partial_\mu \varphi \partial^\mu \varphi + \dots \right\}. \quad (7)$$

To elaborate the last term, one has to expand the logarithm in Eq. (4),

$$\begin{aligned} \Gamma^{(1)}(\phi_0 + \varphi) - \Gamma^{(1)}(\phi_0) &= \frac{\lambda}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} (2\phi_0 \varphi + \varphi^2) \right] \\ &\quad - \frac{\lambda^2 \phi_0^2}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi \frac{1}{p^2 + M_0^2} \varphi \right] \\ &\quad + \dots, \end{aligned} \quad (8)$$

and commute all the momentum operators to the left using the identity [5,6]

$$\begin{aligned} \Phi \frac{1}{p^2 + M^2} &= \frac{1}{p^2 + M^2} \Phi + \frac{1}{(p^2 + M^2)^2} [p^2, \Phi] \\ &\quad + \frac{1}{(p^2 + M^2)^3} [p^2, [p^2, \Phi]] \\ &\quad + \frac{1}{(p^2 + M^2)^4} [p^2, [p^2, [p^2, \Phi]]] + \dots. \end{aligned} \quad (9)$$

The commutators of the momentum operator with an arbitrary function of coordinates  $\Phi$  follows the rules (the four-dimensional momentum operator is identified as  $p_\mu = -i\partial_\mu$ )

$$\begin{aligned} [p^2, \Phi] &= \square \Phi + 2ip^\mu \partial_\mu \Phi, \\ [p^2, [p^2, \Phi]] &= \square^2 \Phi + 4ip^\mu \partial_\mu \square \Phi - 4p^\mu p^\nu \partial_\mu \partial_\nu \Phi. \end{aligned} \quad (10)$$

After the described procedure, we arrive at

$$\begin{aligned} \Gamma^{(1)}(\phi_0 + \varphi) - \Gamma^{(1)}(\phi_0) &= \frac{\lambda \phi_0}{2} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi \right] + \frac{\lambda}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi^2 \right] \\ &\quad - \frac{\lambda^2 \phi_0^2}{4} \text{Tr} \left[ \frac{1}{(p^2 + M_0^2)^2} \varphi^2 \right] - \frac{\lambda^2 \phi_0^2}{4} \text{Tr} \left[ M_0^2 \frac{1}{(p^2 + M_0^2)^4} \varphi \square \varphi \right]. \end{aligned} \quad (11)$$

Compared to Eq. (7), the first three terms in the rhs are the derivatives of  $V^{(1)}$ . Concerning the kinetic part, we need the last term with the d'Alembertian operator. The prescription for the functional trace in momentum space can be found in [6]. In brief,

$$\begin{aligned} \text{Tr} \frac{1}{(p^2 + M^2)^n} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M^2)^n} \\ &= \frac{1}{(4\pi)^2} \int_0^\infty \frac{2k^3 dk}{(k^2 + M^2)^n}. \end{aligned} \quad (12)$$

Deriving the last integral for  $n \geq 2$  is a simple exercise, and we skip the details. After integrating by parts in the last term of (11), we arrive at

$$Z^{(1)}(\phi_0) = \frac{1}{12(4\pi)^2} \frac{\lambda^2 \phi_0^2}{M^2}. \quad (13)$$

As expected from the power counting, this expression is finite and hence is independent of the renormalization procedure.

### B. Effective action of a single scalar field in curved space

Consider now the generalization of the previous result to an arbitrary curved spacetime. In this case, we have several

options for deriving the effective potential of the scalar field and derivative expansion. The pioneer calculation of [20] used a technique specific for the potential of a scalar field in de Sitter spacetime. Technically, more simple is the integration of the renormalization group equation [21]. It can be generalized for other sectors of effective action [22], but since this approach is essentially tied to the Minimal Subtraction scheme of renormalization, it is not appropriate to explore the low-energy decoupling. The same concerns the recently proposed approach of deriving effective potential from the integration of trace anomaly [23]. The most complete result for the massive scalar field can be achieved by evaluating the form factors in the effective action [24]. However, the results for the heat-kernel solution [25] are elaborated explicitly only for the second-order terms [26,27]. This technique enables one to evaluate the  $(\partial\phi)^2$ -type and  $R\phi^2$ -type terms, but the  $R(\partial\phi)^2$ -type corrections, especially in the theory with two or more different masses, would require elaborating on the heat-kernel solution in the third order in “curvatures” [28]. Thus, we need an appropriate method for the effective potential and the derivative expansion in curved spacetime. The mass-dependent scheme for the curved-space effective potential has been developed using Riemann normal coordinates and local momentum representation [13]. The corresponding calculation can be found in [19] for a

general interaction term, so we only need to work out the derivative expansion.

Let us derive the kinetic term  $Z$  for the one-component scalar field theory with a  $\lambda\phi^4$  interaction term, following the procedure which is a direct generalization of the one outlined in Sec. II A. The action under consideration, including the nonminimal term of the first order in curvature, has the form

$$S(\phi, g_{\mu\nu}) = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \phi^2 m^2 + \frac{1}{2} \xi R \phi^2 - \frac{\lambda}{4!} \phi^4 \right\}. \quad (14)$$

For the sake of simplicity, we choose the most common classical potential  $V(\phi) = \lambda\phi^4/4!$ , different from the case of effective potential in (2).

The propagator expansion in the local momentum representation can be done in different forms [13], and

the one which is the most appropriate for deriving the effective potential is, in the Euclidean signature,<sup>1</sup>

$$\tilde{G}(y) = \int \frac{d^4k}{(2\pi)^4} e^{iky} \left[ \frac{1}{p^2 + M_0^2} - \frac{\tilde{\xi} R}{(p^2 + M_0^2)^2} \right], \quad (15)$$

where

$$M_0^2 = m^2 + \frac{\lambda}{2} \phi_0^2 \quad \text{and} \quad \tilde{\xi} = \xi - \frac{1}{6}. \quad (16)$$

Let us note that since we restrict our attention to the  $\mathcal{O}(R)$  terms, hence  $R$  can be treated as a constant. The method can be applied beyond this approximation, but this would require more calculations.

Following the same steps as in the flat case, but with the propagator (15), we obtain

$$\Gamma(\phi_0 + \varphi, g_{\mu\nu}) - \Gamma(\phi_0, g_{\mu\nu}) = \frac{1}{2} \text{Tr} \ln \left\{ 1 + \frac{\lambda}{2} \left( \frac{1}{k^2 + M_0^2} - \frac{\tilde{\xi} R}{(k^2 + M_0^2)^2} \right) (2\phi_0 \varphi + \varphi^2) \right\}. \quad (17)$$

Expanding the logarithm into series, discarding terms of the second order in curvatures, and also higher orders in  $\phi$ , we arrive at the expression

$$\begin{aligned} \Gamma(\phi_0 + \varphi, g_{\mu\nu}) - \Gamma(\phi_0, g_{\mu\nu}) = & \frac{\lambda}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} (2\phi_0 \varphi + \varphi^2) \right] - \frac{\lambda}{4} \tilde{\xi} R \text{Tr} \left[ \frac{1}{(p^2 + M_0^2)^2} (2\phi_0 \varphi + \varphi^2) \right] \\ & - \frac{\lambda^2 \phi_0^2}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi \frac{1}{p^2 + M_0^2} \varphi \right] + \frac{\lambda^2}{2} \tilde{\xi} R \phi_0^2 \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi \frac{1}{(p^2 + M_0^2)^2} \varphi \right]. \end{aligned} \quad (18)$$

As we already know, the kinetic term arises because of the commutation of  $\varphi$  with the momentum operator. Therefore, we can focus only on the last two terms of Eq. (18).

It proves useful to express the coefficient  $Z^{(1)}(\phi)$  as

$$Z^{(1)}(\phi) = Z_0^{(1)} + Z_1^{(1)} R + \mathcal{O}(R^2). \quad (19)$$

The first term,  $Z_0^{(1)}$ , corresponds to the flat space and has already been calculated in (13). One can rewrite it by replacing  $M_0^2 \rightarrow M^2$ ,

$$Z_0^{(1)} = \frac{1}{12} \frac{\lambda^2 \phi_0^2}{16\pi^2 M^2}. \quad (20)$$

The last term in (18), after commutations, becomes

$$\begin{aligned} \frac{\lambda^2}{2} \tilde{\xi} \phi_0^2 R \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi \frac{1}{(p^2 + M_0^2)^2} \varphi \right] = & \frac{\lambda^2}{2} \tilde{\xi} \phi_0^2 R \text{Tr} \left\{ \frac{1}{(p^2 + M_0^2)^3} \varphi^2 + \frac{1}{(p^2 + M_0^2)^4} [p^2, \varphi] \varphi \right. \\ & \left. + \frac{1}{(p^2 + M_0^2)^5} [p^2, [p^2, \varphi]] \varphi + \dots \right\}. \end{aligned} \quad (21)$$

The first term of the last expression contributes to the second derivative of the effective potential because

$$\frac{\lambda}{4} \text{Tr} \left[ \frac{1}{p^2 + M_0^2} \varphi^2 \right] - \frac{\lambda}{4} \tilde{\xi} R \text{Tr} \left[ \frac{1}{(p^2 + M_0^2)^2} \varphi^2 \right] - \frac{\lambda^2 \phi_0^2}{4} \text{Tr} \left[ \frac{1}{(p^2 + M_0^2)^2} \varphi^2 \right] + \frac{\lambda^2 \tilde{\xi} \phi_0^2}{4} \text{Tr} \left[ \frac{1}{(p^2 + M_0^2)^3} \varphi^2 \right] = \frac{1}{2} \int d^4x \frac{\partial^2 V^{(1)}}{\partial \phi_0^2} \varphi^2. \quad (22)$$

<sup>1</sup>This expression includes the normal coordinate expansion of  $\sqrt{-g}$ , such that no corrections in the vertex part are required.

After integrating by parts, we obtain the first-order correction to (13) in the form

$$Z_1^{(1)} = \frac{\lambda^2}{24(4\pi)^2 M_0^4} \tilde{\xi} R \phi_0^2. \quad (23)$$

Replacing  $\phi_0^2$  by  $\phi^2$ , the coefficient  $Z^{(1)}(\phi)$ , in the  $\mathcal{O}(R)$  approximation, is given by

$$Z^{(1)} = \frac{1}{12(4\pi)^2} \frac{\lambda^2 \phi^2}{M^2} \left( 1 + \frac{\tilde{\xi} R}{2M^2} \right). \quad (24)$$

Remarkably, this expression does not depend on the renormalization parameter  $\mu$ . This is explained by the fact that, in the pure scalar theory, at the one-loop level, there are no divergences in the kinetic term.

At this point, it is worthwhile to make a clarification about the approximations used in the formula (24). In flat spacetime, the derivative expansion (1) assumes that the derivatives of the background scalar are subleading compared to the potential part, i.e.,  $(\partial\phi) \ll M^4$  or, in the detailed form,  $(\partial\phi) \ll \lambda\phi^4 + \phi^2 m^2$ . It is important to note that  $M^2$  depends on the background scalar, such that the definition (1) and all consequent formulas hold even in the massless limit [17,18]. The mentioned approximation remains necessary in the next orders of expansion [6] and also in the models with fermions and/or vectors, where the kinetic term is divergent and gains logarithmic corrections. This is the limitation behind Eq. (13), but the curvature correction in (24) implies also the expansion in the Riemann normal coordinates, which means an approximation of weak gravitational field. In our setting, this means  $|R_{\mu\nu\alpha\beta}| \ll M^2$ , that have to hold simultaneous with the aforementioned flat-space restriction  $(\partial\phi) \ll M^4$ .

### III. THE CASE OF A TWO-COMPONENT SCALAR FIELD

Consider a model with two real interacting scalar fields  $\phi$  and  $\chi$ , described by the covariant Lagrangian density,

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} M^2 \chi^2 \\ & + \frac{1}{2} \xi_1 R \phi^2 + \frac{1}{2} \xi_2 R \chi^2 - \frac{\lambda_1}{4!} \phi^4 - \frac{\lambda_2 \chi^4}{4!} - \frac{\lambda_{12}}{8} \phi^2 \chi^2. \end{aligned} \quad (25)$$

Here  $m, M$  are the masses of the two fields, and  $\lambda_1, \lambda_2, \lambda_{12}$  are the coupling constants.

In what follows, we show how to reduce the quantum calculations in the theory with two masses to the single-mass case. Our approach is based on the background field method and the change of variables (diagonalization) which was previously used in [29] (see also [16] for a general discussion of diagonalization and [15] for the references to original papers and more profound discussion, including

the nonpositively defined oscillators). The rest of the procedure is analogous to using normal modes in the multidimensional oscillator.

#### A. Diagonalization of the bilinear form of the action

Following the background field method, we decompose scalar fields into background  $\phi, \chi$  and quantum  $\sigma_1, \sigma_2$  counterparts as

$$\phi \rightarrow \phi + \sigma_1, \quad \chi \rightarrow \chi + \sigma_2. \quad (26)$$

The one-loop contribution depends on the bilinear in quantum fields action

$$S^{(2)} = \frac{1}{2} \int d^4 x \sqrt{-g} \begin{pmatrix} \sigma_1 & \sigma_2 \end{pmatrix} \hat{H} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}. \quad (27)$$

Operator  $\hat{H}$  has a standard matrix form

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \hat{1} \square + \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix}, \quad (28)$$

where

$$\hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{cases} M_{11}^2 = m^2 - \xi_1 R + \frac{\lambda_1}{2} \phi^2 + \frac{\lambda_{12}}{4} \chi^2, \\ M_{12}^2 = M_{21}^2 = \lambda_{12} \phi \chi, \\ M_{22}^2 = M^2 - \xi_2 R + \frac{\lambda_2}{2} \chi^2 + \frac{\lambda_{12}}{4} \phi^2. \end{cases} \quad (29)$$

To diagonalize matrix  $\hat{H}$ , we perform rotation to the angle  $\alpha$  in the space of the fields,

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}. \quad (30)$$

This transformation preserves the diagonal form of the derivative term in (28), and on the other hand, its Jacobian equals one. After rotation, the bilinear action becomes

$$\begin{aligned} S^{(2)} = & \frac{1}{2} \int d^4 x \sqrt{-g} \{ \rho_1 \square \rho_1 + \rho_2 \square \rho_2 \\ & + \rho_1 (M_{11}^2 \cos^2 \alpha + 2M_{12}^2 \sin \alpha \cos \alpha + M_{22}^2 \sin^2 \alpha) \rho_1 \\ & + \rho_2 (M_{11}^2 \sin^2 \alpha - 2M_{12}^2 \sin \alpha \cos \alpha + M_{22}^2 \cos^2 \alpha) \rho_2 \\ & + \rho_1 [(M_{22}^2 - M_{11}^2) \sin 2\alpha + 2M_{12}^2 \cos 2\alpha] \rho_2 \}. \end{aligned} \quad (31)$$

Choosing

$$\theta = \cot 2\alpha = \frac{M_{22}^2 - M_{11}^2}{2M_{12}^2}, \quad (32)$$

we eliminate the nondiagonal components. It is useful to introduce new notations,

$$a = \cos^2 \alpha = \frac{1}{2} + \frac{\theta}{2\sqrt{1+\theta^2}}, \quad b = \sin^2 \alpha = \frac{1}{2} - \frac{\theta}{2\sqrt{1+\theta^2}},$$

$$c = \sin 2\alpha = \frac{1}{\sqrt{1+\theta^2}}. \quad (33)$$

In the new variables, the bilinear form of the action becomes

$$\hat{\mathcal{H}} = \begin{pmatrix} \square + \Pi_1 & 0 \\ 0 & \square + \Pi_2 \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1 & 0 \\ 0 & \mathcal{H}_2 \end{pmatrix}, \quad (34)$$

where

$$\Pi_1 = aM_{11}^2 + bM_{22}^2 - cM_{12}^2,$$

$$\Pi_2 = bM_{11}^2 + aM_{22}^2 + cM_{12}^2. \quad (35)$$

At this point, the diagonalization is concluded. The main difference with the case of two independent scalar fields is that each of the two expressions  $\Pi_1$  and  $\Pi_2$  depend on both background scalars  $\phi$  and  $\chi$ . In what follows, we show how this difference can be eliminated.

### B. Setting the framework

The one-loop effective action is proportional to the expression that can be factorized according to Eq. (34), using the unity of the Jacobian of (30),

$$\ln \text{Det } \hat{H} = \ln \text{Det } \hat{\mathcal{H}} = \text{Tr} \ln \hat{\mathcal{H}} = \text{Tr} \ln \mathcal{H}_1 + \text{Tr} \ln \mathcal{H}_2. \quad (36)$$

Thus, the one-loop contribution to the effective action is

$$\Gamma^{(1)}(\phi, \chi) = \frac{1}{2} \text{Tr} \ln(\square + \Pi_1) + \frac{1}{2} \text{Tr} \ln(\square + \Pi_2). \quad (37)$$

To employ the method described in the previous section, let us first perform the splitting of the scalar fields into constant and weakly variable parts,

$$\phi \rightarrow \phi_0 + \varphi(x) \quad \text{and} \quad \chi \rightarrow \chi_0 + \varepsilon(x). \quad (38)$$

The desired derivative expansion up to the second order in the scalar fields  $\varphi$  and  $\chi$  is

$$\Gamma^{(1)}(\phi, \chi) = \int d^4x \sqrt{-g} \left\{ -V^{(1)}(\phi, \chi) + \frac{1}{2} Z_\phi^{(1)} (\partial\varphi)^2 + \frac{1}{2} Z_\chi^{(1)} (\partial\varepsilon)^2 + Z_{\phi\chi}^{(1)} (\partial\varphi)(\partial\varepsilon) + \dots \right\}, \quad (39)$$

where we used compact notations,

$$(\partial\varphi)^2 = g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi, \quad (\partial\varepsilon)^2 = g^{\mu\nu} \partial_\mu \varepsilon \partial_\nu \varepsilon,$$

$$(\partial\varphi)(\partial\varepsilon) = g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varepsilon. \quad (40)$$

In the rest of this section, we first present direct calculation of the coefficients  $Z^{(1)}$  and then report about the method of reduction to the single scalar case.

### C. Direct calculation of the derivative expansion

Without the loss of generality, one can assume that the coefficients  $Z_\phi^{(1)}$ ,  $Z_\chi^{(1)}$ , and  $Z_{\phi\chi}^{(1)}$  do not depend on the variable scalar fields, but only on the constant components of the scalars, couplings, masses, and nonminimal parameters  $\xi_{1,2}$ .

Since we need only the terms quadratic in the variable components, let us expand the effective action into the power series up to the second order in  $\varphi$  and  $\varepsilon$ ,

$$\Gamma^{(1)}(\phi_0 + \varphi, \chi_0 + \varepsilon) - \Gamma^{(1)}(\phi_0, \chi_0) = \int d^4x \left\{ -\frac{\partial V^{(1)}(\phi, \chi)}{\partial \phi_0} \varphi - \frac{\partial V^{(1)}(\phi, \chi)}{\partial \chi_0} \varepsilon \right. \\ \left. - \frac{1}{2} \frac{\partial^2 V^{(1)}(\phi, \chi)}{\partial \phi_0^2} \varphi^2 - \frac{1}{2} \frac{\partial^2 V^{(1)}(\phi, \chi)}{\partial \chi_0^2} \varepsilon^2 - \frac{\partial^2 V^{(1)}(\phi, \chi)}{\partial \phi \partial \chi} \varphi \varepsilon \right. \\ \left. + \frac{1}{2} Z_\phi^{(1)} (\partial\varphi)^2 + \frac{1}{2} Z_\chi^{(1)} (\partial\varepsilon)^2 + Z_{\phi\chi}^{(1)} (\partial\varphi)(\partial\varepsilon) + \dots \right\}. \quad (41)$$

On the other hand, we have to expand  $\Pi_j$  in Eq. (37) as follows:

$$\Pi_j(\phi_0 + \varphi; \chi_0 + \varepsilon) = \Pi_{j0}(\phi_0, \chi_0) + \frac{\partial \Pi_j}{\partial \phi_0} \varphi + \frac{\partial \Pi_j}{\partial \chi_0} \varepsilon \\ + \frac{1}{2} \frac{\partial^2 \Pi_j}{\partial \phi_0^2} \varphi^2 + \frac{1}{2} \frac{\partial^2 \Pi_j}{\partial \chi_0^2} \varepsilon^2 + \frac{\partial^2 \Pi_j}{\partial \phi_0 \partial \chi_0} \varphi \varepsilon \\ + \dots, \quad (42)$$

As in the one-scalar case, the terms involving second derivatives do not contribute to the kinetic term.

Thus, we need to keep only the terms of the first order in derivatives. Furthermore, in the two first-derivative terms, the coefficients  $\partial \Pi_j / \partial \phi_0$  and  $\partial \Pi_j / \partial \chi_0$  depend solely on the constants  $\phi_0$  and  $\chi_0$ . That being said, we can consider that

$$\frac{\partial \Pi_j}{\partial \chi_0} = \Pi_\chi(\chi_0) = \Pi_\chi \quad \text{and} \quad \frac{\partial \Pi_j}{\partial \phi_0} = \Pi_\phi(\phi_0) = \Pi_\phi \quad (43)$$

are constant coefficients of the linear expansion,

$$\Pi_j(\phi_0 + \varphi; \chi_0 + \varepsilon) = \Pi_{0j}(\phi_0, \chi_0) + \Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon. \quad (44)$$

For the effective potential (2), it is sufficient to set  $\phi = \phi_0$  and  $\chi = \chi_0$  in (37),

$$V^{(1)} = -\frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} [\ln(k^2 + \Pi_{01}) + \ln(k^2 + \Pi_{02})]. \quad (45)$$

The lowest-order derivative terms follow from the relevant part of (42), so we get

$$\Gamma^{(1)} + \int d^4 x V^{(1)} = \frac{1}{2} \sum_{j=1,2} \text{Tr} \ln \left[ 1 + \frac{1}{p^2 + \Pi_{0j}} (\Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon) \right]. \quad (46)$$

From this point onward, the arguments are suppressed, assuming

$$\Gamma^{(1)}(\phi_0 + \varphi, \chi_0 + \varepsilon) = \Gamma^{(1)} \quad \text{and} \quad \Pi_{0j} = \Pi_0(\phi = \phi_0, \chi = \chi_0), \quad (47)$$

such that, for each  $j = 1, 2$ , in the first order of the power series, we get (here  $\Pi_{\phi j} \rightarrow \Pi_\phi$ ,  $\Pi_{\chi j} \rightarrow \Pi_\chi$ , and  $\Pi_{0j} \rightarrow \Pi_0$  for brevity)

$$\begin{aligned} \bar{\Gamma}_j &= \frac{1}{2} \text{Tr} \ln \left[ 1 + \frac{1}{p^2 + \Pi_0} (\Pi_\phi \varphi + \Pi_\chi \varepsilon) \right] = \frac{1}{2} \text{Tr} \left[ \frac{1}{p^2 + \Pi_0} (\Pi_\phi \varphi + \Pi_\chi \varepsilon) \right. \\ &\quad \left. - \frac{1}{2} \frac{1}{p^2 + \Pi_0} (\Pi_\phi \varphi + \Pi_\chi \varepsilon) \frac{1}{p^2 + \Pi_0} (\Pi_\phi \varphi + \Pi_\chi \varepsilon) + \dots \right]. \end{aligned} \quad (48)$$

In the second term of the rhs of the last relation, we commute all of the momentum operators to the left of the field functions, in the way analogous to Eq. (9). Then,

$$\begin{aligned} \bar{\Gamma}_j &= \frac{1}{2} \text{Tr} \left\{ \frac{1}{p^2 + \Pi_0} (\Pi_\phi \varphi + \Pi_\chi \varepsilon) - \frac{1}{2} \frac{1}{(p^2 + \Pi_0)^2} (\Pi_\phi \varphi + \Pi_\chi \varepsilon)^2 \right. \\ &\quad - \frac{\Pi_\phi^2}{2} \frac{1}{(p^2 + \Pi_0)^3} [p^2, \varphi] \varphi - \frac{\Pi_\phi^2}{2} \frac{1}{(p^2 + \Pi_0)^3} [p^2, \varphi] \varepsilon \\ &\quad - \Pi_\chi \Pi_\phi \frac{1}{(p^2 + \Pi_0)^3} [p^2, \varepsilon] \varphi - \frac{\Pi_\phi^2}{2} \frac{1}{(p^2 + \Pi_0)^4} [p^2, [p^2, \varphi]] \varphi \\ &\quad \left. - \frac{\Pi_\phi^2}{2} \frac{1}{(p^2 + \Pi_0)^4} [p^2, [p^2, \varphi]] \varepsilon - \Pi_\chi \Pi_\phi \frac{1}{(p^2 + \Pi_0)^4} [p^2, [p^2, \varepsilon]] \varphi + \dots \right\}. \end{aligned} \quad (49)$$

The part of effective action, which is linear in  $\phi$  and  $\chi$ , corresponds to the first term of last expression, i.e.,

$$\begin{aligned} \int d^4 x \left( \frac{\partial V^{(1)}}{\partial \phi_0} \varphi + \frac{\partial V^{(1)}}{\partial \chi_0} \varepsilon \right) &= \frac{1}{2} \sum_{j=1,2} \text{Tr} \frac{1}{p^2 + \Pi_{0j}} (\Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon) \\ &= \frac{1}{2} \sum_{j=1,2} \int d^4 x \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + \Pi_{0j}} (\Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon). \end{aligned} \quad (50)$$

Similarly, the second term of Eq. (49) produces the quadratic terms

$$\int d^4 x \left( \frac{1}{2} \frac{\partial^2 V^{(1)}}{\partial \phi_0^2} \varphi^2 + \frac{1}{2} \frac{\partial^2 V^{(1)}}{\partial \chi_0^2} \varepsilon^2 + \frac{\partial^2 V^{(1)}}{\partial \phi_0 \partial \chi_0} \varphi \varepsilon \right) = \frac{1}{4} \sum_{j=1,2} \int d^4 x \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p^2 + \Pi_{0j})^2} (\Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon)^2. \quad (51)$$

Now we are in the position to derive the coefficients  $Z_\varphi^{(1)}$ ,  $Z_\chi^{(1)}$ , and  $Z_{\varphi\chi}^{(1)}$  of derivative expansion from Eq. (39). The respective terms with two derivatives appear as a result of the commutation of the momentum operators with the functions of the coordinates in Eq. (49). In the framework of local momentum representation, the commutators are the same (10) as in the flat spacetime. Commuting, taking the traces, and integrating over the momenta, we find, for each component  $\Pi_j$ ,

$$-\frac{1}{24(4\pi)^2} \frac{1}{\Pi_0} \int d^4x (\Pi_\phi^2 \varphi \square \varphi + \Pi_\chi^2 \varepsilon \square \varepsilon + 2\Pi_\phi \Pi_\chi \varepsilon \square \varphi). \quad (52)$$

As explained above, when integrating by parts, one can regard the coefficients  $\Pi_\phi$  and  $\Pi_\chi$  constants and get

$$\begin{aligned} Z_1^{(1)}(\phi, \chi) &= \frac{1}{12(4\pi)^2} \left( \frac{\Pi_{\phi 1}^2}{\Pi_{01}} + \frac{\Pi_{\phi 2}^2}{\Pi_{02}} \right), \\ Z_2^{(1)}(\phi, \chi) &= \frac{1}{12(4\pi)^2} \left( \frac{\Pi_{\chi 1}^2}{\Pi_{01}} + \frac{\Pi_{\chi 2}^2}{\Pi_{02}} \right), \\ Z_{12}^{(1)}(\phi, \chi) &= \frac{1}{12(4\pi)^2} \left( \frac{\Pi_{\phi 1} \Pi_{\chi 1}}{\Pi_{01}} + \frac{\Pi_{\phi 2} \Pi_{\chi 2}}{\Pi_{02}} \right). \end{aligned} \quad (53)$$

Since the variable parts in the expressions for  $Z_\varphi^{(1)}$ ,  $Z_\chi^{(1)}$ , and  $Z_{\varphi\chi}^{(1)}$  are irrelevant in the given approximation, one can simply replace  $\phi_0$  by  $\phi(x)$  and  $\chi_0$  by  $\chi(x)$  in the last formulas to arrive at the final answer. The explicit expressions for the coefficients (53) are rather bulky and cannot be presented here. The coefficients are finite as predicted by the power counting and hence independent of the renormalization condition or parameter  $\mu$ .

It is worth noting that the method of derivative expansion that we used here can be applied for calculating

higher-order derivative terms, as it was done for the single scalar field in flat spacetime in the work [6]. On the other hand, the part which we derived here is certainly dominating in all possible applications.

#### D. Double diagonalization and reduction to the single scalar model

An alternative approach to derive the coefficients  $Z^{(1)}$  involves introducing an auxiliary field  $\rho$  which is a linear combination of  $\varphi$  and  $\varepsilon$ , and expanding  $\Pi_j$  in a single field  $\rho$ ,

$$\Pi_j(\phi_0 + \varphi, \chi_0 + \varepsilon) = \Pi_{0j} + \Pi_{\phi j} \varphi + \Pi_{\chi j} \varepsilon = \Pi_{0j} + \Pi_{\rho j} \rho, \quad (54)$$

where (for both  $j = 1, 2$ )

$$\Pi_\rho = \sqrt{\Pi_\phi^2 + \Pi_\chi^2} \quad \text{and} \quad \rho = \frac{\Pi_\phi}{\Pi_\rho} \varphi + \frac{\Pi_\chi}{\Pi_\rho} \varepsilon. \quad (55)$$

Introducing  $\rho$  is analogous to what is called derivative along the given direction in the analysis of many variables. Together with the rotation (30), the procedure can be called double diagonalization because it includes rotation (30) with the angle (32) in the space of quantum fields and the change of “coordinates” (54) in the space of background fields. Together, these two operations reduce the calculation in the two (and more, if necessary) scalar fields theory to the single-scalar calculation. Let us now see how it works.

In the expansion  $\Pi(\phi_0 + \varphi, \chi_0 + \varepsilon) = \Pi_0 + \Pi_\rho \rho$  in Eq. (54), the coefficients  $\Pi_0$  and  $\Pi_\rho$  are constants. Thus, for each  $j$ , in the relevant order of the expansion in derivatives,

$$\text{Tr} \ln \left[ 1 + \frac{1}{p^2 + \Pi_0} (\Pi_0 + \Pi_\rho \rho) \right] = \frac{1}{2} \text{Tr} \left[ \Pi_\rho \frac{1}{p^2 + \Pi_0} \rho(x) - \frac{\Pi_\rho^2}{2} \frac{1}{p^2 + \Pi_0} \rho(x) \frac{1}{p^2 + \Pi_0} \rho(x) \right]. \quad (56)$$

Taking traces and integrating by parts are essentially the same as in Sec. II, so we do not need to repeat the details. After that, one has to make a replacement according to (54),

$$\frac{1}{24(4\pi)^2} \frac{\Pi_\rho^2}{\Pi_0} \int d^4x (\partial_\mu \rho)(\partial^\mu \rho) = \frac{1}{24(4\pi)^2 \Pi_0} \int d^4x [\Pi_\phi^2 (\partial_\mu \varphi)(\partial^\mu \varphi) + \Pi_\chi^2 (\partial_\mu \varepsilon)(\partial^\mu \varepsilon) + 2\Pi_\phi \Pi_\chi (\partial_\mu \varphi)(\partial^\mu \varepsilon)]. \quad (57)$$

Now, summing up the contribution of  $\Pi_1$  e  $\Pi_2$ , we arrive at the same result (53). So, we can conclude that the described double diagonalization is operational.

Using the described approach, we can easily obtain  $Z^{(1)}$  in the first order in curvature, simply reformulating the result (24) from Sec. II, i.e.,

$$\frac{\xi R}{24(4\pi)^2} \frac{\Pi_\rho^2}{\Pi_0^2} \int d^4x (\partial_\mu \rho)(\partial^\mu \rho). \quad (58)$$

After the same replacement as in (57), we get

$$\begin{aligned} Z_\phi &= \frac{1}{12(4\pi)^2} \left[ \left( 1 + \frac{\tilde{\xi}_1 R}{2\Pi_{01}} \right) \frac{\Pi_{\phi_1}^2}{\Pi_{01}} + \left( 1 + \frac{\tilde{\xi}_2 R}{2\Pi_{02}} \right) \frac{\Pi_{\phi_2}^2}{\Pi_{02}} \right], \\ Z_\chi &= \frac{1}{12(4\pi)^2} \left[ \left( 1 + \frac{\tilde{\xi}_1 R}{2\Pi_{01}} \right) \frac{\Pi_{\chi_1}^2}{\Pi_{01}} + \left( 1 + \frac{\tilde{\xi}_2 R}{2\Pi_{02}} \right) \frac{\Pi_{\chi_2}^2}{\Pi_{02}} \right], \\ Z_{\chi\phi} &= \frac{1}{12(4\pi)^2} \left[ \left( 1 + \frac{\tilde{\xi}_1 R}{2\Pi_{01}} \right) \frac{\Pi_{\phi_1} \Pi_{\chi_1}}{\Pi_{01}} + \left( 1 + \frac{\tilde{\xi}_2 R}{2\Pi_{02}} \right) \frac{\Pi_{\phi_2} \Pi_{\chi_2}}{\Pi_{02}} \right]. \end{aligned} \quad (59)$$

The described reduction of the multiscalar calculation to the single scalar case can be directly applied for a greater number of real scalars. The number of directional derivatives (54) is always equal to the number of original scalars, and we do not expect a degeneracy in this expansion, at least for most of the possible models with real scalars. The application to the system of complex and real scalars (e.g., Higgs field plus an inflaton) is also possible, but would require writing a complex field as two real scalars and also, in the case of the Standard Model Higgs, solving the problem of group indices.

#### IV. LOW-ENERGY LIMIT IN THE THEORY WITH HIERARCHY OF THE MASSES

The notion of quantum decoupling is an important element in the effective approach to quantum field theory and quantum gravity. According to the Appelquist and Carazzone theorem in QED [30], the quantum contribution of the field with a large mass  $M$  vanishes as  $k^2/M^2$  in the IR (low-energy limit), where the Euclidean momentum is small, i.e.,  $k^2 \ll M^2$ . We can observe this effect in many other situations, including for the polarization operator of gravitons in the semiclassical gravity [26] (see more references, e.g., in [14]). The derivative expansion makes sense in the situation when the scalar field performs small oscillations near the point of the minima of the potential. This situation fits pretty well with the decoupling theorem setting, so one can ask how the decoupling can be observed in the effective potential<sup>2</sup> and in the first coefficients of the derivative expansion. Indeed the oscillations may occur not in a simple central minimum of the potential, but also in the broken phase of the “Mexican hat” potential of the spontaneous symmetry breaking. In curved spacetime, this consideration is possible but meets certain complications [32,33]. In the considerations presented below, we consider only local motions in the vicinity of the minimum of the potential, such that the origin of this minimum does not matter, and we may consider only the basic situation. For this reason, we assume that the magnitude of the scalar field

is small when this field oscillates in the vicinity of the point of the minimum.

The decoupling in the case of a single-field effective potential is quite obvious and intuitive. The logarithms in expression (2) for the theory with mass  $M$  comes together with the divergences, in the same way as  $\ln[(k^2 + M^2)/\mu^2]$ -type formula emerges<sup>3</sup> in the nonlocal form factors. It is clear that those are parts of the same logarithm because the source of both is in  $\text{Tr} \ln[\square + M^2 + \lambda\phi^2/2]$ . In the IR, where the  $M^2$  term dominates, in Euclidean signature, we meet the expansion of the type

$$\ln\left(\frac{M^2 + k^2 + \lambda\phi^2/2}{\mu^2}\right) \approx \ln\left(\frac{M^2}{\mu^2}\right) + \frac{k^2 + \lambda\phi^2/2}{M^2} + \dots, \quad (60)$$

which means there are no physically relevant logarithms in the IR, in both mentioned cases, i.e., for the nonlocal form factors (or polarization operator) and for the effective potential.

The situation for the two-scalar case with a strong hierarchy of masses  $m \ll M$  is technically more complicated, but we can apply the method described above to reduce the calculation to the two single-scalar models.

After double diagonalization and the standard calculations for a single scalar (described in detail in [14,19]), we arrive at the renormalized effective potential,

$$V_{\text{ren}}^{(1)} = \frac{\Pi_1^2(\phi, \chi)}{4(4\pi)^2} \ln\left[\frac{\Pi_1(\phi, \chi)}{\mu^2}\right] + \frac{\Pi_2^2(\phi, \chi)}{4(4\pi)^2} \ln\left[\frac{\Pi_2(\phi, \chi)}{\mu^2}\right]. \quad (61)$$

Let us evaluate the regime where the conditions

$$M^2 \gg \phi^2, \quad M^2 \gg m^2 \quad (62)$$

hold and also assume  $m^2 \rightarrow 0$ . According to what was discussed above, this situation can be regarded as an IR limit for the effective potential. Our experience with the

<sup>2</sup>It was known from the epoch of Heisenberg and Euler [31] that in the interaction sector quantum corrections are qualitatively different in the UV and in the IR.

<sup>3</sup>The real expression is more complicated [24], but the difference with this simplified formula is sublogarithmic in the UV, so it is a reasonable approximation.

decoupling implies that the contribution of the large-mass field should vanish at least quadratically, and in the first place, there should not be relevant logarithms with  $M$ . To see whether this is the case, one has to evaluate how the expressions for both  $\Pi_j$  behave in this regime.

It is easy to verify that the coefficient defined in (32) has the limit  $\theta \rightarrow \infty$ . Then, the coefficients  $a$ ,  $b$ , and  $c$  in (33) become

$$\begin{aligned} a &= \left( \frac{1}{2} + \frac{\theta}{2\sqrt{1+\theta^2}} \right) \approx 1, \\ b &= \left( \frac{1}{2} - \frac{\theta}{2\sqrt{1+\theta^2}} \right) \approx 0, \\ c &= \frac{1}{\sqrt{1+\theta^2}} \approx \frac{2\lambda_{12}}{M^2} \phi \chi \approx 0. \end{aligned} \quad (63)$$

These relations mean that the mixing of two *quantum* scalars disappears, exactly like in the  $\lambda_{12} \rightarrow 0$  case with unrestricted masses  $m$  and  $M$ . In this approximation, the expressions for  $\Pi_{1,2}$  reduce to (we intend to take  $m \rightarrow 0$  limit at the end)

$$\begin{aligned} \Pi_1 &\approx M_{11}^2 \approx -m^2 - \frac{\lambda_1}{2} \phi^2 - \frac{\lambda_{12}}{4} \chi^2, \\ \Pi_2 &\approx M_{22}^2 - \frac{2\lambda_{12}}{M^2} \phi \chi M_{12}^2 \approx -M^2. \end{aligned} \quad (64)$$

Let us skip the routine calculations and just go to the final result. In the effective potential, the dominant contribution with “surviving” logarithm has the form

$$V^{(1)} = \frac{1}{4(4\pi)^2} \left( m^2 + \frac{\lambda_1}{2} \phi^2 + \frac{\lambda_{12}}{4} \chi^2 \right)^2 \ln \left( \frac{m^2 + \frac{\lambda_1}{2} \phi^2 + \frac{\lambda_{12}}{4} \chi^2}{\mu^2} \right) + \frac{M^4}{4(4\pi)^2} \ln \left( \frac{M^2}{\mu^2} \right). \quad (65)$$

The last term is the usual contribution to the cosmological constant, which is of no interest to us. Omitting this term and taking the limit  $m \rightarrow 0$ , expression (65) becomes

$$V^{(1)} = \frac{\lambda^2 \rho^4}{16(4\pi)^2} \ln \left( \frac{\lambda \rho^2}{2\mu^2} \right), \quad \text{where } \frac{\lambda}{2} \rho^2 = \frac{\lambda_1}{2} \phi^2 + \frac{\lambda_{12}}{4} \chi^2. \quad (66)$$

Thus, the effective potential boils down to the leading logarithmic contribution of a single scalar field  $\rho$  that is a mixing of the original scalars. When the original theory has no mixing, i.e., when  $\lambda_{12} = 0$ , this scalar coincides with the original light scalar  $\phi$ . In any case, in the IR limit, as we defined it, only the light scalar retains a logarithmic contribution, while for the large-mass scalar, there is a quadratic decoupling.

It remains to consider the IR decoupling in terms of one-loop derivative expansions, that is  $Z^{(1)}$ 's. First of all, the arguments about vanishing mixing (63) in the IR limit (62) remain valid, and hence, it is sufficient to consider only the single-scalar case (24). Taking the corresponding limit  $M \gg \phi$  for the field with the mass  $M$ , we get

$$Z_{\text{IR}}^{(1)} = \frac{\lambda^2}{12(4\pi)^2} \left( 1 + \frac{\xi R}{2M^2} \right) \frac{\phi^2}{M^2}. \quad (67)$$

This formula shows that there is a quadratic decoupling, and for  $|R| \ll M^2$ , the  $R$ -dependent term is vanishing with the double (quartic) speed.

## V. CONCLUSIONS

In this work, we developed a systematic approach to calculate the effective potential and the first-order terms in the derivative expansion in the theory with several scalar fields with different masses. The *double diagonalization* method is based upon the functional approach to effective action [5,6,18] and consists from (i) the diagonalization of the operator  $\hat{H}$  (bilinear form of the action) and (ii) changing the basis of expansion to the linear independent directional derivatives in the background scalar fields. The two-scalars calculation illustrating this method can be easily generalized to a larger number of scalar fields. The use of this method enables a simple derivative expansion not only in Minkowski spacetime, but also in curved space, at least in the first order in scalar curvature.

The analysis of the two-scalar system with a strong hierarchy of masses, when one mass dominates over the magnitudes of scalars while the second mass remains small or even vanishes, has shown the quadratic decoupling of quantum corrections in such an IR limit. The contribution of the large-mass scalar reduces to the one to cosmological constant term. This output is natural if we remember the uniqueness of effective action and the known results for the IR decoupling in the kinetic sector of a scalar field [24].

From the viewpoint of applications, our results mean that the quantum effects of two scalar fields in the assisted models of inflation (see, e.g., [1–3]) boil down to the ones of the single lightest scalar in the course of a fast expansion of the Universe, when the typical energy becomes smaller, and the large-mass field decouples.

Let us discuss the possible extensions of the results of this work. It is technically possible to construct the derivative extension in the effective action of scalars owing to the fermion loops [34]. Using the method presented above, one can extend the existing expansions by including the curvature terms, similar to Eq. (24). The same concerns also the contributions of gauge fields. In both cases, the kinetic scalar term is UV divergent, and the coefficient  $Z$  in (1) is subject of renormalization procedure. However, the power counting tells us that this does not concern curvature-dependent part. We hope to report on this calculation in future work. In view of the possible applications to inflation, it looks interesting to include also graviton and mixed graviton-scalar loops. However, this is a more complicated problem, because quantum gravity based on general relativity is not renormalizable and because such graviton contributions have high degree of ambiguity (see

the discussion and further references, e.g., in [14]). Anyway, this is an attractive idea which may be put into practice in the effective quantum gravity framework.

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## DATA AVAILABILITY

No data were created or analyzed in this study.

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