

Revisiting $SU(5)$ Yukawa sectors through quantum corrections

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This article revisits the validity of tree-level statements regarding the Yukawa sector of various minimal-renormalizable $SU(5)$ frameworks at the loop level. It is well known that an $SU(5)$ model with only the 45_H dimensional irreducible representation (irrep) contributing to the Yukawa sector is highly incompatible in yielding the low-energy observables. However, this study shows that when one-loop corrections from heavy degrees of freedom are included in the various Yukawa vertices, the model can reproduce the charged fermion mass spectrum and mixing angles within the assumed experimental uncertainty. The fitted Yukawa couplings remain within the perturbative range. The scenario also necessitates mass splitting among various scalars of 45_H dimensional irrep, with some of the scalars' mass deviating significantly from the matching scale (M_{GUT}), collectively providing substantial threshold corrections. The effect of the lightest scalar on the renormalization group evolution of the standard model (SM) parameters has also been considered. It is shown that this scenario reproduces the SM spectrum at low energy. As an extension, the minimal $SU(5)$ model with only the 45_H irrep is augmented with the 15_H -dimensional irrep, which also successfully reproduces the observed charged and neutral fermion mass spectra. Finally, the study considers an alternative $SU(5)$ model incorporating both 5_H and 15_H irreps, which also yields the desired fermion mass spectra and mixing angles. This work demonstrates the viability of a minimal $SU(5)$ Yukawa sector in different setups when quantum corrections are considered.

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I. PRELUDE

Grand unified theories (GUTs) are well-motivated frameworks in dealing with some of the inconsistencies of the Standard Model (SM) [1,2]. Once the quarks and leptons are unified in a common irreducible representation (irrep) and GUTs offer a plethora of irreps comprising of distinct scalar fields, in addition to SM Higgs [3–5].

A grand unification model with only a single irrep in the Yukawa sector often leads to inconsistent tree-level Yukawa relations. For example, in an $SU(5)$ model with a single scalar irrep 5_H contributing to the Yukawa sector, the relation $Y_d = Y_e^T$ arises, which is observationally inconsistent. To resolve this, multiple scalar irreps are typically introduced, modifying the tree-level Yukawa relations and making them phenomenologically viable [6–11]. However, this approach introduces the hierarchy problem and numerous tunable parameters in the scenario, which are one of the most pronounced problems of the GUTs. Alternative solutions involve incorporating

higher-dimensional operators [12–18] or vectorlike fermions [19–28], both of which alter the tree-level matching conditions. The inconsistency in Yukawa relations for minimal¹ GUT models are not exclusive to $SU(5)$ but also applies to $SO(10)$ GUTs. However, our findings suggest that a minimal $SU(5)$ model with only 5_H in the Yukawa sector, extended with $SU(5)$ -singlet fermion (s), can yield a realistic fermion mass spectrum and mixing angles when quantum corrections are included [29].

This study, on the same line of [29], aims to comprehensively investigate various minimal $SU(5)$ frameworks with different irreps in the Yukawa sector, assessing their potential to yield a viable fermion mass spectrum when quantum corrections are included. Such an investigation is essential for constructing a minimal and calculable GUT model. In $SU(5)$, only two irreps— 5_H and 45_H —can couple to the matter multiplets at the renormalizable level, with each containing a submultiplet transforming like the SM Higgs. Minimal versions that include only one of these irreps, i.e., 5_H and 45_H , in the Yukawa sector are known to produce inconsistent tree-level Yukawa relations, as previously discussed. The present work revisits the validity of this tree-level statement at the loop level.

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¹Minimality here refers to having only a single irrep consisting of SM Higgs contributing to the Yukawa sector.

The central focus of this study is to evaluate the viability of Yukawa sectors where only a single irrep containing the SM Higgs contributes to the Yukawa sector. It examines various renormalisable $SU(5)$ frameworks—with 45_H , $45_H - 15_H$, and $5_H - 15_H$ irreps—regarding their potential to reproduce the observed fermion mass spectra and mixing angles when loop corrections to tree-level Yukawa vertices are incorporated. The quantum corrections arise from the heavy degrees of freedom inherent to the models, which are various scalar fields arising from the same scalar irrep and the heavy gauge boson. To provide sufficient threshold corrections at the matching scale, a mass splitting among the scalar masses is necessary, which extends beyond the extended survival hypothesis (ESH) [30–32]. The ESH suggests that only the scalars necessary for breaking a symmetry at a given energy scale are present at that scale and can remain light. The consequence of 45_H dimensional irrep in contributing to fermion masses [10,33,34], gauge coupling unification [9,35,36] and flavor anomalies [11,37] are already present in the literature. Additionally, 15_H dimensional irrep is long known to contribute to neutrino mass [38,39] via type II seesaw mechanism [40,41].

This article is organized as follows: Section II begins with a discussion on one loop matching condition of the Yukawa sector, followed by subsections II A, II B, and II C, which examine different $SU(5)$ models in various frameworks: 45_H , $45_H - 15_H$, and $5_H - 15_H$, respectively, and calculate the one-loop corrections to various Yukawa vertices. Section III presents the numerical analysis of the frameworks discussed in Sec. II, evaluating the effectiveness of the one-loop-corrected Yukawas in reproducing the largest Yukawas in the charged fermion sectors. Section IV provides the benchmark values of the fitted parameters for the various $SU(5)$ models with different Yukawa sectors considered in the study. Section V summarized the carried out study and the Appendix is included to support the contents of the main text.

II. $SU(5)$ YUKAWA RELATIONS AT ONE LOOP

Consider a generic $SU(5)$ model with three generations of $\bar{\mathbf{5}}$ and $\mathbf{10}$ irreps consisting of SM quarks and leptons. The SM fermionic embedding of the $SU(5)$ matter multiplets, i.e., $\bar{\mathbf{5}}$ and $\mathbf{10}$, is shown below [5].

$$\begin{aligned} \bar{\mathbf{5}}_a &= \epsilon_{ab} l^b, & \bar{\mathbf{5}}_\alpha &= d_\alpha^C, \\ \mathbf{10}^{aa} &= \frac{1}{\sqrt{2}} q^{aa}, & \mathbf{10}^{a\beta} &= \frac{1}{\sqrt{2}} \epsilon^{a\beta\gamma} u_\gamma^C, & \mathbf{10}^{ab} &= \frac{1}{\sqrt{2}} \epsilon^{ab} e^C, \end{aligned} \quad (1)$$

where, Greek letters ($1 \leq \alpha, \beta, \gamma \dots \leq 3$) denotes $SU(3)_C$ indices while $SU(2)_L$ labels are depicted by the lowercase Latin alphabets ($4 \leq a, b, c \dots \leq 5$). The convention of

two-indexed Levi-Civita tensor is as follows: $\epsilon_{45} = 1 = \epsilon^{54} = -\epsilon_{54} = -\epsilon^{45}$. Further, three indexed Levi-Civita follows the convention where $\epsilon_{123} = 1$ and for other cyclic permutations. The generation labels are designated by upper case Latin alphabets, i.e., ($1 \leq A, B, C, \dots \leq 3$).

This section discusses different versions of $SU(5)$ models—namely, the 45_H , $45_H - 15_H$, and $5_H - 15_H$ models—and evaluates the one-loop corrections to the tree-level Yukawa vertices. It is to be noted that the 50_H -dimensional $SU(5)$ irreducible representation also couples to the $SU(5)$ fermion multiplets at the renormalizable level; however, it does not contain the SM Higgs doublet [42]. Following Refs. [19,43–47], the one-loop matching condition for the Yukawa couplings at a given renormalization scale μ was derived in Ref. [29] and is given by

$$Y_f(\mu) = Y_f^0 \left(1 - \frac{K_H(\mu)}{2} \right) + \delta Y_f(\mu) - \frac{1}{2} (K_f^T(\mu) Y_f^0 + Y_f^0 K_f^C(\mu)), \quad (2)$$

where Y_f^0 denotes the tree-level Yukawa coupling of the fermions $f \supset \{q, u^C, d^C, l, e^C\}$ to the SM Higgs. The one-loop corrected Yukawa coupling at the scale μ , namely $Y_f(\mu)$, is fully determined in terms of the finite part of the vertex correction, δY_f , and the wave function renormalization factors K_H and K_f . The relevant one-loop diagrams for the vertex Y_d are shown in Fig. 1, where the first row corresponds to vertex corrections, the second row to wave function corrections to the fermion legs induced by scalar fields and heavy gauge bosons, and the third row to the wave function correction to the Higgs leg.

A. Model I: $SU(5)$ with only 45_H

The scalar sector of this $SU(5)$ model consists only of 45_H -dimensional irrep. The various scalar fields residing in 45_H are listed in Table I. The decomposition of 45_H -plet into various irreps with canonically normalized kinetic terms can be inferred from [48] and is reproduced below for convenience:

$$\begin{aligned} 45_{H\gamma}^{a\beta} &\equiv S_\gamma^{a\beta} + \frac{1}{2\sqrt{2}} (\delta_\gamma^a T^\beta - \delta_\gamma^\beta T^a), & 45_{Ha}^{a\beta} &\equiv \Omega_a^{a\beta}, \\ 45_{H\beta}^{aa} &\equiv \frac{1}{\sqrt{2}} O_\beta^{aa} + \frac{1}{2\sqrt{6}} \delta_\beta^a H^a, & 45_{H\beta}^{ab} &\equiv \frac{1}{\sqrt{2}} \epsilon^{ab} T_\beta^\dagger, \\ 45_{Ha}^{ba} &\equiv \frac{1}{\sqrt{2}} T_a^{ba} - \frac{1}{2\sqrt{2}} \delta_a^b T^a, & 45_{Hc}^{ab} &\equiv -\frac{\sqrt{3}}{2\sqrt{2}} (\delta_c^a H^b - \delta_c^b H^a). \end{aligned} \quad (3)$$

The interaction of $\bar{\mathbf{5}}$ and $\mathbf{10}$ with 45_H -dimensional field at renormalizable level is parametrized as follows:

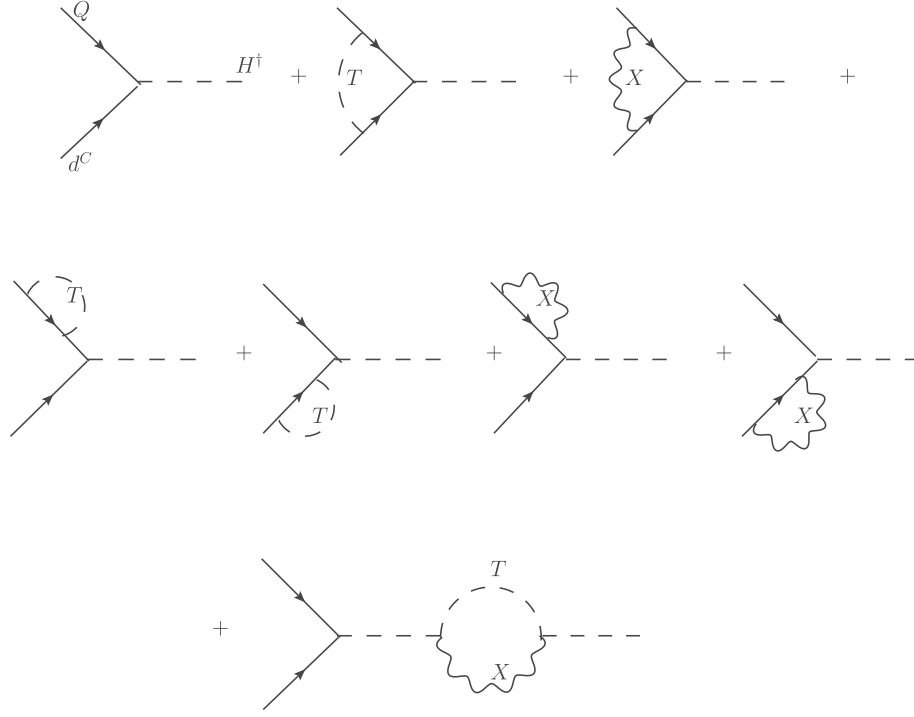


FIG. 1. One-loop correction diagrams for Y_d are shown, with analogous diagrams also applying to Y_e and Y_u . Here, T denotes a scalar field transforming in an irreducible representation that participates in the Yukawa interactions, while X denotes the gauge boson. The first line of diagrams represents the vertex corrections, the second line represents the wave function corrections to the fermion legs arising from different scalar fields and heavy gauge bosons, and the last line represents the wave function correction to the Higgs leg due to scalar fields and gauge bosons.

$$-\mathcal{L}_Y^{45_H} = \frac{1}{2}(Y_1)_{AB} \mathbf{10}_A^T C^{-1} \mathbf{10}_B 45_H + \sqrt{2}(Y_2)_{AB} \mathbf{10}_A^T C^{-1} \bar{\mathbf{5}}_B 45_H^\dagger + \text{H.c.}, \quad (4)$$

where Y_1 is antisymmetric in flavor indices. Using the dictionary provided in Eq. (1) together with the canonically normalized decomposition of 45_H given in Eq. (3), the vertices involving SM-Higgs (H) and fermions can be evaluated as follows:

$$-\mathcal{L}_Y^{45_H} \supset -Y_{1AB} \frac{2}{\sqrt{6}} \varepsilon_{ab} q_A^{a\alpha T} C^{-1} u_{aB}^C H^b - Y_{2AB} \left(\frac{1}{\sqrt{6}} q_A^{a\alpha T} C^{-1} d_{aB}^C H_a^\dagger - \sqrt{\frac{3}{2}} e_A^{CT} C^{-1} l_B^a H_a^\dagger \right) + \text{H.c.} \quad (5)$$

The interaction terms written in Eq. (5) lead to the following tree-level Yukawa relations valid at the GUT scale:

$$(Y_u)_{AB} = \frac{2}{\sqrt{6}} (Y_1)_{AB},$$

$$(Y_d)_{AB} = \frac{1}{\sqrt{6}} (Y_2)_{AB},$$

$$(Y_e)_{AB} = -\sqrt{\frac{3}{2}} (Y_2^T)_{AB}. \quad (6)$$

TABLE I. Scalar multiplets residing inside 45_H dimensional irrep.

Notation	SM charge
H	$(1, 2, \frac{1}{2})$
T	$(3, 1, -\frac{1}{3})$
$\mathcal{T}$	$(\bar{3}, 1, \frac{4}{3})$
Ω	$(\bar{3}, 2, -\frac{7}{6})$
$\mathbb{T}$	$(3, 3, -\frac{1}{3})$
S	$(\bar{6}, 1, -\frac{1}{3})$
O	$(8, 2, \frac{1}{2})$

The tree-level Yukawa relations mentioned in Eq. (6) are inconsistent due to the following reasons:

- (1) Inconsistent Yukawa relations in down type and charged lepton sector. The Yukawa relations mentioned in Eq. (6) lead to $3Y_d = -Y_e^T$ which result in $\frac{y_d}{y_e} = \frac{y_c}{y_\mu} = \frac{y_b}{y_\tau} = \frac{1}{3}$. However, at the traditional GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV), the renormalization

group evolution (RGE) extrapolated SM values requires $\frac{y_d}{y_e} = 2$, $\frac{y_c}{y_\mu} = \frac{1}{5}$, and $\frac{y_b}{y_\tau} = \frac{2}{3}$.

- (2) Inaccurate spectrum in the up-quark sector. The antisymmetric nature of Y_1 in three dimensions makes its determinant zero, implying at least one of the three eigenvalues is zero. Moreover, this scenario also predicts two degenerate eigenvalues at the tree-level.

This subsection aims to investigate whether the one-loop corrections imparted by various scalars residing in the 45_H can modify the tree-level relations mentioned in Eq. (6). The couplings of various scalars stemming from 45_H with SM fermions at renormalizable level are shown below:

$$\begin{aligned} -\mathcal{L}_Y^{45_H} \supset Y_{1AB} & \left(-\frac{1}{\sqrt{2}} \epsilon_{ae} \epsilon_{\beta\gamma\alpha} q_A^{e\beta T} C^{-1} q_B^{b\gamma} \mathbb{T}_b^{aa} + \sqrt{2} u_A^{CT} C^{-1} e_B^C T^\alpha + \frac{1}{\sqrt{2}} \epsilon_{\beta\gamma\alpha} e_A^{CT} C^{-1} q_B^{a\gamma} \Omega_a^{\dagger\alpha\beta} - \frac{1}{\sqrt{2}} \epsilon^{\gamma\lambda\theta} u_A^{CT} C^{-1} u_{\lambda B}^C T_\theta^\dagger \right. \\ & + \sqrt{2} \epsilon_{ab} q_A^{aaT} C^{-1} u_{\beta B}^C O_\alpha^{\beta b} - \frac{1}{2\sqrt{2}} \epsilon_{ab} \epsilon_{\gamma\alpha\beta} q_A^{a\gamma T} C^{-1} q_B^{b\theta} S_\theta^{\alpha\beta} \Big) + Y_{2AB} \left(\frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\rho} u_{\rho A}^{CT} C^{-1} d_{\gamma B}^C S_{\alpha\beta}^{\dagger\gamma} + \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\rho} u_{\rho A}^{CT} C^{-1} d_{\alpha B}^C T_\beta^\dagger \right. \\ & + \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\rho} \epsilon_{ab} u_{\rho A}^{CT} C^{-1} l_B^b \Omega_{\alpha\beta}^a - \sqrt{2} q_A^{aaT} C^{-1} d_{\beta B}^C O_{aa}^{\dagger\beta} + \sqrt{2} \epsilon_{ab} q_A^{paT} C^{-1} l_B^b \mathbb{T}_{pa}^{\dagger a} - \sqrt{2} e_A^{CT} C^{-1} d_{\beta B}^C T_\beta^\dagger - \frac{1}{\sqrt{2}} \epsilon_{ab} q_A^{aaT} C^{-1} l_B^b T_\alpha^\dagger \Big) \\ & + \text{H.c.} \end{aligned} \quad (7)$$

It is to be noted that the coupling of T with quarks, i.e., qqT , is not allowed due to the antisymmetric nature of Y_1 . Further, the scalar fields T , $\mathbb{T}$ exhibits diquark as well as leptoquark couplings, while all the other scalars either have diquark or leptoquark couplings. But, only scalar field T and $\mathbb{T}$ can induce tree-level proton decay and all the other scalars exhibiting leptoquark couplings can induce proton decays at loop level.

The scalars contribute to one-loop Yukawa relations by inducing vertex corrections and correcting the external fermion leg. In addition to the corrections from the heavier scalar fields, the Yukawa vertices also receive contributions from the heavier gauge bosons. The couplings of heavy gauge boson (X) with the SM fermions is shown below [4,49]:

$$-\mathcal{L}_G^{(X)} \supset \frac{g_5}{\sqrt{2}} \bar{X}_\mu (\bar{d}^C_i \bar{\sigma}^\mu l_i - \bar{q}_i \bar{\sigma}^\mu u_i^C - \bar{e}^C_i \bar{\sigma}^\mu q_i) + \text{H.c.}, \quad (8)$$

where X has SM charge of $(3, 2, -\frac{5}{6})$, g_5 is the $SU(5)$ gauge coupling and σ is the Pauli matrix.

The vertex corrections to the various Yukawa vertices are produced by heavier scalars and gauge bosons by propagating inside the loop. These vertex corrections can be computed from Eqs. (7) and (8) and are shown below:

$$\begin{aligned} (\delta Y_u)_{AB} &= \frac{8}{\sqrt{6}} g_5^2 (Y_1)_{AB} f[M_X^2, 0] - \sqrt{\frac{3}{2}} (Y_1^T Y_2^* Y_2^T)_{AB} f[M_\Omega^2, 0] + \sqrt{\frac{3}{2}} (Y_2 Y_2^\dagger Y_1^T)_{AB} f[M_T^2, 0] + \sqrt{6} (Y_2 Y_2^\dagger Y_1)_{AB} f[M_O^2, 0], \\ (\delta Y_d)_{AB} &= \frac{2}{\sqrt{6}} g_5^2 (Y_2)_{AB} f[M_X^2, 0] + 2\sqrt{6} (Y_1 Y_1^\dagger Y_2)_{AB} f[M_O^2, 0], \\ (\delta Y_e)_{AB} &= -6\sqrt{\frac{3}{2}} g_5^2 (Y_2^T)_{AB} f[M_X^2, 0] + 2\sqrt{6} (Y_2^T Y_1^\dagger Y_1^T)_{AB} f[M_\Omega^2, 0] - \sqrt{6} (Y_2^T Y_1^* Y_1)_{AB} f[M_T^2, 0]. \end{aligned} \quad (9)$$

where $(\delta Y)_f$ corresponds to the finite part of the vertex correction to the Yukawa vertex of fermion f with SM Higgs. $f[M_i^2, 0]$ is the loop integration factor, with its form provided in the Appendix, M_i being the masses of scalars listed in Table I or the mass of the gauge boson (X). These vertex corrections are a function of the renormalization scale μ and are completely calculable in terms of the tree-level Yukawa couplings $Y_{1,2}$ and also depend upon the loop function involving the masses of heavy degrees of freedom. All the SM fields have been assumed massless while

computing the vertex corrections to the Yukawa vertices. It can be noticed that δY_d and δY_e receive contributions from different scalars and hence are different.

The wave function renormalization factor of the (scalar or fermion) field f is computed by taking the derivative of the self-energy correction of field f with respect to the outgoing momentum and then setting the momentum to zero. The contribution to the wave function renormalization due to various scalars and gauge bosons to the external leg of SM fermions and Higgs are shown below:

$$\begin{aligned}
(K_q)_{AB} &= 3g_5^2\delta_{AB}h[M_X^2, 0] - (6h[M_O^2, 0] + 4h[M_T^2, 0] + 0.5h[M_T^2, 0])(Y_2Y_2^\dagger)_{AB} \\
&\quad - \left(2h[M_T^2, 0] + 6h[M_O^2, 0] + \frac{3}{4}h[M_S^2, 0]\right)(Y_1Y_1^\dagger)_{AB} - 2h[M_\Omega^2, 0](Y_1^TY_1^*)_{AB}, \\
(K_{u^c})_{AB} &= 4g_5^2\delta_{AB}h[M_X^2, 0] - (3h[M_S^2, 0] + 1.5h[M_T^2, 0] + 2h[M_\Omega^2, 0])(Y_2Y_2^\dagger)_{AB} \\
&\quad - \left(2h[M_T^2, 0] + \frac{1}{2}h[M_T^2, 0] + 6h[M_O^2, 0]\right)(Y_1Y_1^\dagger)_{AB}, \\
(K_{d^c})_{AB} &= 2g_5^2\delta_{AB}h[M_X^2, 0] - (6h[M_S^2, 0] + h[M_T^2, 0] + 12h[M_O^2, 0] + 2h[M_T^2, 0])(Y_2^TY_2^*)_{AB}, \\
(K_l)_{AB} &= 3g_5^2\delta_{AB}h[M_X^2, 0] - (6h[M_\Omega^2, 0] + 6h[M_T^2, 0] + 1.5h[M_T^2, 0])(Y_2^TY_2^*)_{AB}, \\
(K_{e^c})_{AB} &= 6g_5^2\delta_{AB}h[M_X^2, 0] - (6h[M_T^2, 0] + h[M_\Omega^2, 0])(Y_1Y_1^\dagger)_{AB} - 6h[M_T^2, 0](Y_1^TY_1^*)_{AB}, \\
K_H &= \frac{g_5^2}{2}[2(f[M_X^2, M_T^2] + g[M_X^2, M_T^2]) + 4(f[M_X^2, M_T^2] + g[M_X^2, M_T^2]) + 4(f[M_X^2, M_T^2] + g[M_X^2, M_T^2])]. \quad (10)
\end{aligned}$$

Different K_f characterises the finite part of the wave function renormalization factor corresponding to the field f . It receives corrections from heavy scalars and gauge bosons. g and h are different loop integration factors whose functional form is given in the Appendix.

The following one-loop Yukawa relations at the renormalization scale μ are obtained by substituting Eqs. (9) and (10) into Eq. (2):

$$\begin{aligned}
Y_u &= \frac{2Y_1}{\sqrt{6}}(1 - 0.5 \times K_H) + \delta_{Y_u} - 0.5 \times \frac{2}{\sqrt{6}} \times (K_q^TY_1 + Y_1K_{u^c}), \\
Y_d &= \frac{Y_2}{\sqrt{6}}(1 - 0.5 \times K_H) + \delta_{Y_d} - \frac{0.5}{\sqrt{6}} \times (K_q^TY_2 + Y_2K_{d^c}), \\
Y_e &= -\sqrt{\frac{3}{2}}Y_2^T(1 - 0.5 \times K_H) + \delta_{Y_e} - 0.5 \times \left(-\sqrt{\frac{3}{2}}\right) \times (K_l^TY_2^T + Y_2^TK_{e^c}). \quad (11)
\end{aligned}$$

The later part of this study is devoted to discussing the ability of Eq. (11) to yield a valid charged fermion mass spectrum. This scenario, involving only 45_H in the Yukawa sector, is labeled as model I for future reference.

B. Model I-A: $SU(5)$ with 45_H and 15_H

The earlier subsection considered an $SU(5)$ model with only 45_H contributing to the Yukawa sector and examined its impacts on the Yukawa relations of charged fermions at one loop. This case is now extended by including 15_H in the Yukawa sector to assess the model's ability to yield realistic charged and neutral fermion Yukawas and mixing angles and is termed as model I-A. The scalar fields residing in 15_H are listed in Table II. In addition to generating neutrino

masses, the scalars stemming from 15_H can also contribute to the correction of the Yukawa vertices.

The inclusion of 15_H in the Yukawa sector modifies the Lagrangian as shown below:

$$\mathcal{L}_Y = \mathcal{L}_Y^{45_H} + \mathcal{L}_Y^{15_H}, \quad (12)$$

where $\mathcal{L}_Y^{45_H}$ is written in Eq. (4). The decomposition of 15_H in its scalar submultiplets with canonically normalized kinetic term is as follows:

$$15^{ab} = t^{ab}, \quad 15^{a\alpha} = \frac{1}{\sqrt{2}}\Delta^{a\alpha}, \quad \text{and} \quad 15^{\alpha\beta} = \Sigma^{\alpha\beta} \quad (13)$$

The interaction of 15_H with $SU(5)$ fermion plets can be analogously decomposed into various effective vertices using the decomposition of 15_H and the resulting expression is shown below

TABLE II. Scalar multiplets residing in 15_H dimensional irrep.

Notation	SM charge
t	(1, 3, 1)
Δ	(3, 2, $\frac{1}{6}$)
Σ	(6, 1, $-\frac{2}{3}$)

$$\begin{aligned}
-\mathcal{L}_Y^{15_H} &= (Y_3)_{AB} \bar{\mathbf{5}}_A^T C^{-1} \bar{\mathbf{5}}_B 15_H + \text{H.c.} \\
&= Y_{3AB} (\epsilon_{am} \epsilon_{bn} l_A^{mT} C^{-1} l_B^n t^{ab} \\
&\quad + \sqrt{2} \epsilon_{ab} l_A^{bT} C^{-1} d_{aB}^C \Delta^{aa} + d_{aA}^{CT} C^{-1} d_{\beta B}^C \Sigma^{a\beta}) + \text{H.c.},
\end{aligned} \tag{14}$$

where Y_3 is symmetric in family labels. The interaction of SM fermions with the scalars originating from 15_H is evaluated by decomposing 15_H into its constituent scalars with canonically normalized kinetic terms and can be inferred from [48]. The scalars residing in 15_H solely couple to fermions residing in $\bar{\mathbf{5}}$ dimensional irrep; consequently, they can only contribute to their wave function renormalization factor. Additionally, as there are no common scalars in 45_H and 15_H , none of the scalars of 15_H can contribute to the vertex correction. The finite part of the corrections to the external leg of d^C and l due to Δ , Σ and t are as follows:

$$\begin{aligned}
(k_{d^C})_{AB} &= -4h[M_\Sigma^2, 0](Y_3 Y_3^\dagger)_{AB} - 4h[M_\Delta^2, 0](Y_3^T Y_3^*)_{AB} \\
(k_l)_{AB} &= -3h[M_t^2, 0](Y_3 Y_3^\dagger)_{AB} - 6h[M_\Delta^2, 0](Y_3 Y_3^\dagger)_{AB},
\end{aligned} \tag{15}$$

where $h[m_1^2, m_2^2]$ is loop integration factor also used earlier. As Σ and t exhibit only diquark and dilepton couplings, respectively; these fields can specifically contribute to the external leg corrections of d^C and l . In contrast, Δ , which exhibits leptoquark couplings, can correct the external legs of both d^C and l fermions.

The scalar irrep 15_H is conventionally known to contribute to the neutrino masses. The allowed $SU(5)$ invariant terms of the Lagrangian necessary for the generation of neutrino masses are as follows:

$$\begin{aligned}
V_\phi &= M_{45_H}^2 45_H^\dagger 45_H + M_{15_H}^2 15_H^\dagger 15_H \\
&\quad + (\lambda 45_H 45_H 15_H^\dagger + \rho 45_H 45_H 45_H 15_H + \text{H.c.}),
\end{aligned} \tag{16}$$

where, λ is a dimension-full trilinear coupling and is assumed to be close to the GUT scale while ρ is the $\mathcal{O}(1)$ quartic coupling and has been set to unity. The scalar field t (cf. Table II) contributes to tree-level neutrino masses. At the loop level, contributions to neutrino masses arise from the scalar pairs $T - \Delta$, associated with trilinear coupling λ and $\mathbb{T} - \Omega$ with the strength ρ [39,50]. The combined contribution to the neutrino mass from various sources up to one loop is as follows:

$$\begin{aligned}
M_\nu &= -2\lambda \frac{\langle H \rangle^2}{M_t^2} Y_3 + \frac{3}{2} \lambda \langle H \rangle^2 [(Y_2^T Y_d^* Y_3^T) + (Y_2^T Y_d^* Y_3^T)^T] \\
&\quad \times p[M_T^2, M_\Delta^2] + 6\rho \langle t \rangle \langle H \rangle^2 [(Y_2^T Y_u^* Y_2) \\
&\quad + (Y_2^T Y_u^* Y_2)^T] \times p[M_\mathbb{T}^2, M_\Omega^2].
\end{aligned} \tag{17}$$

$\langle t \rangle$, $\langle H \rangle$ are vacuum expectation values of scalar fields t and H respectively. The definition of loop integration factor $p[m_1^2, m_2^2]$ has been relegated to the Appendix. The first term is the tree-level contribution, while the other two terms are the one-loop-level contribution. The loop-level expressions involve the contributions from one-loop generated Y_d and Y_u . It is to be noted that Y_u at tree-level is antisymmetric while the effectively generated Y_u at one loop is not antisymmetric.

C. Model II: $SU(5)$ with 5_H and 15_H

Toward the end of this section, an alternative version of $SU(5)$ with a 5_H -dimensional irrep contributing to the Yukawa sector is considered. This version of $SU(5)$ is known to produce generational mass degeneracy in the down-quark and charged lepton sectors, i.e., $Y_d = Y_e^T$ [4]. This generational mass degeneracy was addressed by extending the minimal $SU(5)$ model with $SU(5)$ fermion singlet(s), as presented in [29]. In the present study, an alternative approach to ameliorate the generational mass degeneracy is considered by extending the minimal $SU(5)$ framework with a 15_H irrep. Analogous to the previous subsections, one-loop corrections to various Yukawa vertices are computed, induced by the scalars in 5_H and 15_H as well as the heavy gauge bosons (X), and is labeled as model II.

The interaction of 5_H and 15_H with the matter multiplets of $SU(5)$ is parametrized as shown below:

$$\begin{aligned}
-\mathcal{L}_Y^{5_H+15_H} &= \frac{1}{4} (Y_1)_{AB} \mathbf{10}_A^T C^{-1} \mathbf{10}_B 5_H \\
&\quad + \sqrt{2} (Y_2)_{AB} \mathbf{10}_A^T C^{-1} \bar{\mathbf{5}}_B 5_H^\dagger \\
&\quad + (Y_3)_{AB} \bar{\mathbf{5}}_A^T C^{-1} \bar{\mathbf{5}}_B 15_H + \text{H.c.},
\end{aligned} \tag{18}$$

where Y_1 and Y_3 are symmetric in family indices. The decomposition of the term written in the last line of the Eq. (18) is already given in Eq. (14) while the decomposition of the remaining terms is shown below:

$$\begin{aligned}
-\mathcal{L}_Y^{5_H} &= -Y_{1AB} \left(u_{\gamma A}^{CT} C^{-1} e_B^C + \frac{1}{2} \epsilon_{\alpha\beta\gamma} \epsilon_{ab} q_A^{a\alpha T} C^{-1} q_B^{b\beta} \right) T^\gamma \\
&\quad - Y_{2AB} (-\epsilon_{ab} q_A^{a\gamma T} C^{-1} l_B^b - \epsilon^{\alpha\beta\gamma} u_{aA}^C C^{-1} d_{\beta B}^C) T_\gamma^* \\
&\quad - Y_{1AB} \epsilon_{ab} q_A^{a\alpha} C^{-1} u_{aB}^C H^b - Y_{2AB} (q_A^{a\alpha T} C^{-1} d_{aB}^C \\
&\quad + e_A^{CT} C^{-1} l_B^a) H_a^* + \text{H.c.}
\end{aligned} \tag{19}$$

It is evident from the above expression that the scalar field T exhibits diquark as well as leptoquark coupling and hence can induce tree level proton decay.

The one-loop corrections to $Y_{u,d,e}$ in the presence of singlet fermions have been computed in [29]. These results are reproduced below for convenience, ignoring the singlet fermions' contributions. The finite part of the vertex

corrections arising from the scalar field T and the gauge bosons X is as follows:

$$\begin{aligned}(\delta Y_u)_{AB} &= 4g_5^2(Y_1)_{AB}f[M_X^2, 0] + (Y_1 Y_2^* Y_2^T + Y_2 Y_2^* Y_1^T)_{AB}f[M_T^2, 0], \\(\delta Y_d)_{AB} &= 2g_5^2(Y_2)_{AB}f[M_X^2, 0] + (Y_1 Y_1^* Y_2)_{AB}f[M_T^2, 0], \\(\delta Y_e)_{AB} &= 6g_5^2(Y_2^T)_{AB}f[M_X^2, 0] + 3(Y_2^T Y_1^* Y_1)_{AB}f[M_T^2, 0],\end{aligned}\quad (20)$$

where f is the loop integration factor given in the Appendix. The first term is the contribution from heavy gauge boson while the other term is the contribution of the scalar field T . The wave function renormalization factor of various fermion fields due to correction from T and X is also shown below:

$$\begin{aligned}(K_q)_{AB} &= 3g_5^2\delta_{AB}h[M_X^2, 0] - \frac{1}{2}(Y_1^* Y_1^T + 2Y_2^* Y_2^T)_{AB}h[M_T^2, 0], \\(K_{uc})_{AB} &= 4g_5^2\delta_{AB}h[M_X^2, 0] - (Y_1^* Y_1^T + 2Y_2^* Y_2^T)_{AB}h[M_T^2, 0], \\(K_{dc})_{AB} &= 2g_5^2\delta_{AB}h[M_X^2, 0] - 2(Y_2^* Y_2)_{AB}h[M_T^2, 0] \\(K_l)_{AB} &= 3g_5^2\delta_{AB}h[M_X^2, 0] - 3(Y_2^* Y_2)_{AB}h[M_T^2, 0], \\(K_{ec})_{AB} &= 6g_5^2\delta_{AB}h[M_X^2, 0] - 3(Y_1^* Y_1)_{AB}h[M_T^2, 0], \\K_H &= \frac{g_5^2}{2}(f[M_X^2, M_T^2] + g[M_X^2, M_T^2]),\end{aligned}\quad (21)$$

where, again, h and g are loop integration factors can be inferred from the Appendix. The first term is the contribution of the gauge boson, while the second term is a contribution from the scalar field T . As there is no common multiplet in 5_H and 15_H , scalars residing in 15_H cannot contribute to the vertex correction of various Yukawa vertices. However, the scalars of 15_H can contribute to external leg correction of fermion living in $\bar{5}$, which are d^C and l , and such corrections have already been computed in Eq. (15) and contribute to the K_{dc} and K_l . The one-loop matching condition, analogous to Eq. (11), in this scenario can be computed by substituting Eqs. (20), (21), and (15) into Eq. (2) at a given renormalization scale μ , yielding the following:

$$\begin{aligned}Y_u &= Y_1(1 - 0.5 \times K_H) + \delta Y_u - 0.5 \times (K_q^T Y_1 + Y_1 K_{uc}), \\Y_d &= Y_2(1 - 0.5 \times K_H) + \delta Y_d - 0.5 \times (K_q^T Y_2 + Y_2 K_{dc}), \\Y_e &= Y_2^T(1 - 0.5 \times K_H) + \delta Y_e - 0.5 \times (K_l^T Y_2^T + Y_2^T K_{ec}).\end{aligned}\quad (22)$$

Turning attention toward the contribution of 15_H toward the neutrino mass. The $SU(5)$ invariant allowed terms of the Lagrangian relevant for neutrino mass are as follows:

$$V_\phi = M_{15_H}^2 15_H^\dagger 15_H + (\lambda 5_H 5_H 15_H^\dagger + \text{H.c.}), \quad (23)$$

where the λ is dimension-full cubic coupling and is assumed to be at the conventional GUT scale. The tree and loop contribution of 15_H in the neutrino mass can be

analogously computed as shown in the Sec. II B and the result is shown below:

$$\begin{aligned}M_\nu &= \langle H \rangle^2 \lambda \left(-\frac{2}{M_T^2} Y_3 - 3\sqrt{2}[(Y_3 Y_d^\dagger Y_2) \right. \\&\quad \left. + (Y_3 Y_d^\dagger Y_2)^T] \times p(M_\Delta^2, M_T^2) \right),\end{aligned}\quad (24)$$

where, the loop function $p[m_1^2, m_2^2]$ is defined in the Appendix. The first term in the above expression represents the tree-level contribution, while the second term is the loop-level contribution. The loop-level contribution is due to the propagation of a pair of scalars $T - \Delta$ inside the loop and includes the effectively generated Y_d at one loop.

III. VIABILITY OF MODELS

In Secs. II A, II B, and II C, one-loop corrections to various Yukawa vertices are computed, with the latter two subsections also includes computations of neutrino mass within the considered framework. This section is dedicated to examining the viability of model I and model II in yielding the largest desired Yukawa values. The computed one-loop Yukawa relations depend on the Yukawa matrices ($Y_{1,2,3}$), masses of the scalars and the heavy gauge boson, matching scale (μ), and the $SU(5)$ gauge coupling (g_5). The adopted strategy for the analysis fixes the masses of all scalars except two, along with the matching scale at the conventional GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV) and a fixed gauge coupling value ($g_5 \sim 0.5$). The largest Yukawa entry of $Y_{1,2,3}$ is then varied within a specified range to reproduce

the desired largest Yukawa entry, i.e., $Y_{u,d,e}$. The two-loop SM RGE extrapolated values require the largest entry of $(Y_u)_{23} \sim 0.4$, while the desired values of Y_d is 6×10^{-3} and for Y_e is 9×10^{-3} at $\mu = 10^{16}$ GeV [29]. The specified range of the elements of the Yukawa matrices $Y_{1,2,3}$ is chosen such that it does not violate perturbative limits, i.e., $(Y_{1,2,3})_{ij} \leq (-3.5, 3.5)$, which is derived from $2 \rightarrow 2$ tree-level scatterings at high energy limit [51].

A. Third generation numerical analysis: Model I

Following form of Y_1 and Y_2 are chosen to check the ability of Eq. (11) to address the inconsistencies mentioned in points (1 and 2) simplifying the analysis:

- (i) Y_1 is assumed to be an antisymmetric matrix with complex entries, while Y_2 is taken as a real-diagonal

matrix, as depicted below:

$$Y_1 = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}, \quad Y_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d \end{pmatrix}, \quad (25)$$

where, a, b, c are complex numbers while d is purely real. The particular form of Y_2 , in the above expression, has been specifically chosen to perform the third generation analysis.

Substituting $Y_{1,2}$ given in Eq. (25) into Eq. (11) results in the following functional form of $(Y_u)_{33}$, where the contribution of K_H has been neglected

$$(Y_u)_{33} = 2ia \operatorname{Im}(cb^*) \left(2f[M_\Omega^2, 0] + 2f[M_T^2, 0] + \frac{1}{2}f[M_T^2, 0] + \frac{3}{4}f[M_S^2, 0] + 2f[M_\mp^2, 0] \right), \quad (26)$$

where Im represents the imaginary part. It is to be noted that if any of a, b or c is 0, then $(Y_u)_{33}$ is also 0. Further, in the limit $\{a, b\} \rightarrow 0$, $(Y_u)_{23}$ is obtained as follows:

$$(Y_u)_{23} = c\sqrt{\frac{2}{3}} + \frac{g_5^2}{\sqrt{6}}(8h[M_X^2, 0] - 7f[M_X^2, 0]) + \sqrt{\frac{2}{3}}cd^2 \left(f[M_\Omega^2, 0] + \frac{3}{2}f[M_S^2, 0] + \frac{3}{4}f[M_T^2, 0] + \frac{3}{2}h[M_\Omega^2, 0] \right) \\ + \sqrt{\frac{2}{3}}|c|^2c \left(f[M_\Omega^2, 0] + 6f[M_O^2, 0] + f[M_T^2, 0] + \frac{1}{4}f[M_T^2, 0] + \frac{3}{8}f[M_S^2, 0] + f[M_\mp^2, 0] \right). \quad (27)$$

Following comments and observations regarding the expressions given in Eqs. (26) and (27) are in order:

- (1) Y_u is not antisymmetric: Since the product of a diagonal matrix Y_2 and an antisymmetric matrix Y_1 contains only off-diagonal elements, δY_u does not contribute to $(Y_u)_{33}$. The only contribution to $(Y_u)_{33}$ arises from a specific term in the wave function renormalization correction, K_q , which involves $Y_1 Y_1^\dagger$. This term is a Hermitian and determinant-less, and but includes diagonal elements. Although each individual contribution to Y_u is determinantless (since Y_1 itself is determinantless), the combined effect of these contributions results in a Y_u that becomes

determinant-full due to the influence of K_q . Therefore, it is the contribution from K_q that renders Y_u a matrix with $\det \neq 0$.

- (2) $(Y_u)_{33} < (Y_u)_{23}$. The element $(Y_u)_{33}$ is generated at the loop level, whereas $(Y_u)_{23}$ is generated at the tree level; thus, $Y_{23} > Y_{33}$ and hence would be the largest element of Y_u provided $c > b$.

Having computed the largest entry in the up-quark sector, now down and charged lepton sectors are considered. In the limit $\{a, b\} \rightarrow 0$, the expression of $\frac{(Y_d)_{33}}{(Y_e)_{33}} \equiv \frac{y_b}{y_\tau}$ turns out to be as follows:

$$\frac{y_b}{y_\tau} \sim -\frac{1}{3} + \frac{4g_5^2}{3}(4f[M_T^2, 0] - 5h[M_X^2, 0]) - cc^* \left[f[M_\Omega^2, 0] + 3f[M_T^2, 0] + \frac{2}{3}(f[M_T^2, 0] + f[M_\mp^2, 0]) \right. \\ \left. + f[M_O^2, 0] + \frac{1}{8}f[M_S^2, 0] + 4(h[M_\Omega^2, 0] + h[M_O^2, 0] - 2h[M_T^2, 0]) \right] \\ - 2d^2 \left[\frac{3}{2}f[M_O^2, 0] + \frac{1}{2}f[M_S^2, 0] + \frac{1}{2}f[M_T^2, 0] + \frac{1}{6}f[M_T^2, 0] + \frac{7}{12}f[M_\mp^2, 0] \right]. \quad (28)$$

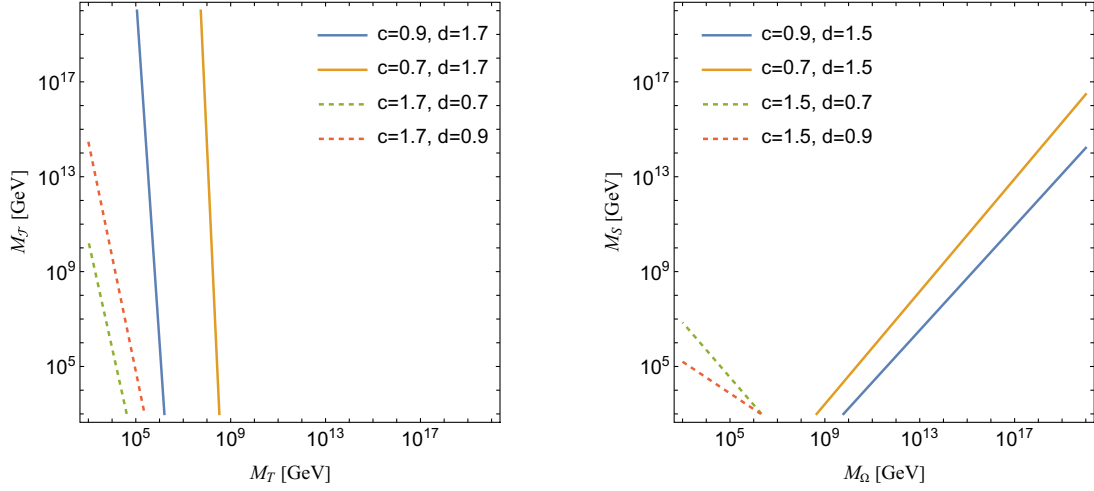


FIG. 2. Contours of $(Y_u)_{23} = 0.4$ for different values of c and d in $M_T - M_T$ (left panel) and $M_O - M_S$ planes (right panel). All the other scalars and gauge bosons are assumed to be at the GUT scale.

As evident from the above expression that at the tree level $\frac{y_b}{y_\tau}$ equals $-\frac{1}{3}$, which it should be. At the one loop level this ratio receives corrections from various heavy degrees of freedom.

Plots presented in Fig. 2 examine the viability of Eq. (27) in achieving specific values for $(Y_u)_{23}$. The elements of Y_1 and Y_2 , as defined in Eq. (25), are taken within the perturbative range $(-3.5, 3.5)$, while the masses of the two relevant scalars are varied from 1 TeV to the Planck scale ($M_p \sim 10^{19}$ GeV), keeping other heavy degrees of freedom at the conventional GUT scale $M_{\text{GUT}} = 10^{16}$ GeV. Similarly, the plots in Fig. 3 examine the feasibility of Eqs. (27) and (28) in achieving specific values of $y_b = 6 \times 10^{-3}$ and $y_\tau = 9 \times 10^{-3}$ in the $M_T - M_O$ and $M_T - M_T$ planes, respectively. All other parameters remain same with those in Fig. 2.

These plots reveal that either c or d must exceed unity to achieve the desired values of $(Y_u)_{23}$, y_b and y_τ . Furthermore, when $c > d$, one of the scalar masses is more closer toward the matching scale (M_{GUT}). Conversely, for $c < d$, the difference between the matching scale and the scalar mass increases. To summarize, in order to counteract suppression from loop factors, larger values of Yukawa couplings or a greater disparity between the matching scale and scalar mass are preferred. Thus far, the case with $b = 0$ and $c \neq 0$ has been considered in Eq. (25). If $b \neq 0$ and $b > c$, then $(Y_u)_{13}$ or $(Y_u)_{31}$ would dominate, resulting in analogous outcomes.

B. Third generation numerical analysis: Model II

The ability of the expressions in Eq. (22) together with (15) to yield specific values of the largest Yukawa couplings is examined in this subsection. For this purpose,

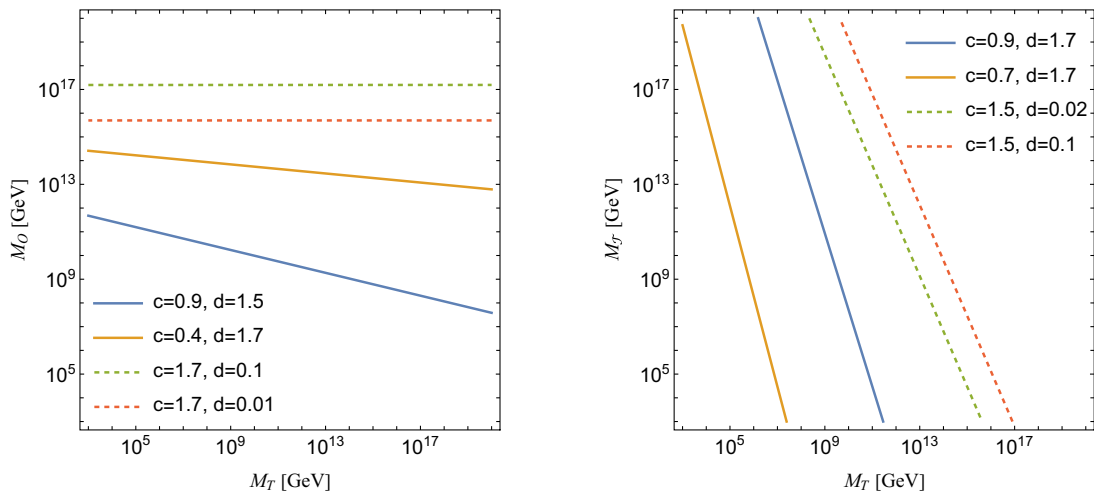


FIG. 3. Contours of $(Y_d)_{33} = 6 \times 10^{-3}$ (left panel) and $(Y_e)_{33} = 9 \times 10^{-3}$ (right panel) for different values of c and d in $M_T - M_O$ and $M_T - M_T$ planes respectively. All the other scalars and gauge bosons are assumed to be at the GUT scale.

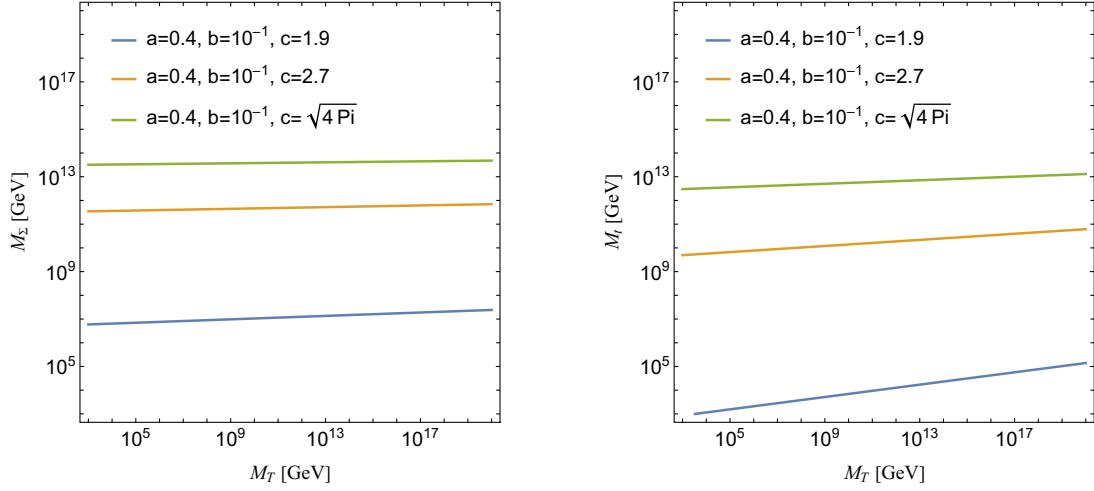


FIG. 4. Contours of $(Y_d)_{33} = 6 \times 10^{-3}$ (left panel) and $(Y_e)_{33} = 9 \times 10^{-3}$ (right panel) for different values of c while holding a and b fixed, as they appear in Eq. (30), in $M_T - M_\Sigma$ and $M_T - M_t$ planes respectively. All the other scalars and gauge bosons are assumed to be at the GUT scale.

the matrices Y_1 , Y_2 , and Y_3 are chosen in the following form:

$$Y_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix}, \quad Y_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b \end{pmatrix},$$

and $Y_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c \end{pmatrix}.$ (29)

where b and c can be purely real while a is a complex number. Substituting the above form of $Y_{1,2,3}$ in Eqs. (22) and (15) leads to the following form of $\frac{y_b}{y_\tau}$:

$$\begin{aligned} \frac{y_b}{y_\tau} \sim & 1 + 2g_5^2(f[M_X^2, 0] - 2h[M_X^2, 0]) \\ & - aa^* \left(\frac{5}{4}f[M_T^2, 0] + 2h[M_T^2, 0] \right) + \frac{b^2}{2}f[M_T^2, 0] \\ & - c^2 \left(f[M_\Delta^2, 0] - f[M_S^2, 0] + \frac{3}{2}f[M_t^2, 0] \right) \end{aligned} \quad (30)$$

The tree-level contribution to $\frac{y_b}{y_\tau}$ is unity, deviating from unity due to contributions from various sectors. In this case, Y_1 at tree level can be freely chosen to produce the desired value of $(Y_u)_{33}$, hence its analysis is not done.

The plots in Fig. 4 show contours of $y_b = 6 \times 10^{-3}$ and $y_\tau = 9 \times 10^{-3}$ for different values of c in the $M_T - M_\Sigma$ and $M_T - M_t$ planes, with a and b held constant at 0.4 and 10^{-1} , respectively. Additionally, the scalar field Δ , and the gauge boson X are also fixed at the conventional

GUT scale, i.e., $M_X = M_\Delta = \mu = M_{\text{GUT}} \sim 10^{16}$ GeV. The contours in Fig. 4 reveal that, as the magnitude of c increases, the difference between the matching scale and the mass of one of the scalars also increases. This arises due to the requirement for ample threshold corrections from the respective scalar. Therefore to achieve desired threshold corrections, either large couplings or a substantial difference between the mass of the scalar and the matching scale is preferred, which has also pointed out in the previous subsection.

IV. QUANTITATIVE ANALYSIS

The discussions in Sec. III establish that the one-loop Yukawa relations computed for the different models in Sec. II can, at least, reproduce the largest Yukawas of Y_u , Y_d , and Y_e . This section presents the three-generation χ^2 analysis for the different models considered. The χ^2 definition and optimization procedure can be inferred from [29,52,53] and is reproduced below for convenience;

$$\chi^2 = \sum_i \left(\frac{O_{i,\text{theo}} - O_{i,\text{exp}}}{\sigma_{i,\text{exp}}} \right)^2, \quad (31)$$

where, the theoretical prediction for each observable is denoted by $O_{i,\text{theo}}$, while $O_{i,\text{exp}}$ represents the corresponding experimentally measured value, where i runs over all observables in the dataset. The experimental uncertainty associated with each measurement is given by σ_i . As, all the $O_{i,\text{theo}}$ are computed at the GUT scale, $O_{i,\text{exp}}$ will also be taken at GUT scale. The GUT scale values of the different O_i 's are done performing two SM RGE from M_Z to the

TABLE III. The benchmark best-fit values obtained for solutions I and II. In addition to the charged fermion sector observable, χ^2 also includes observables of neutral leptons in the case of solution II. The extrapolated values at the scale of $\mu = 10^{16}$ GeV are provided along with the values reproduced through χ^2 minimization and the corresponding pulls. The fitted value of mass of the various scalars, cubic coupling λ , and vev of the scalar field t are also specified at the bottom of the table.

Observable	O_{exp}	Solution I		Solution I-A	
		O_{th}	Pull	O_{th}	Pull
y_u	2.81×10^{-6}	2.77×10^{-6}	0	2.80×10^{-6}	0
y_c	1.42×10^{-3}	1.42×10^{-3}	0	1.42×10^{-3}	0
y_t	4.27×10^{-1}	4.28×10^{-1}	0.1	4.25×10^{-1}	0
y_d	6.14×10^{-6}	3.77×10^{-6}	-1.2	5.45×10^{-6}	-0.4
y_s	1.25×10^{-4}	1.78×10^{-4}	1.4	1.28×10^{-4}	0.1
y_b	5.80×10^{-3}	5.80×10^{-3}	0	5.39×10^{-3}	-0.7
y_e	2.75×10^{-6}	2.81×10^{-6}	0.2	2.74×10^{-6}	0
y_μ	5.72×10^{-4}	5.30×10^{-4}	-0.7	5.93×10^{-4}	0.4
y_τ	9.68×10^{-3}	9.68×10^{-3}	0	1.01×10^{-2}	0.4
$ V_{us} $	0.2286	0.2160	-0.6	0.2286	0
$ V_{cb} $	0.0457	0.0438	-0.4	0.0443	-0.3
$ V_{ub} $	0.0042	0.0044	0.4	0.0043	0
$\sin \delta_{\text{CKM}}$	0.78	0.88	1.3	0.78	0
$\Delta m_{\text{sol}}^2 [\text{eV}^2]$	7.49×10^{-5}	...	...	7.74×10^{-5}	-0.1
$\Delta m_{\text{atm}}^2 [\text{eV}^2]$	2.534×10^{-3}	...	...	2.686×10^{-3}	0.1
$\sin^2 \theta_{12}$	0.307	...	...	0.327	0.3
$\sin^2 \theta_{23}$	0.561	...	...	0.526	0.5
$\sin^2 \theta_{13}$	0.02195	...	...	0.02249	0
χ_{min}^2			6.6		2.0
$M_T [\text{GeV}]$		1.002×10^{14}		5.973×10^{15}	
$M_{\bar{T}} [\text{GeV}]$		1.060×10^{14}		1.201×10^{15}	
$M_\Omega [\text{GeV}]$		8.115×10^9		6.411×10^{15}	
$M_{\bar{T}} [\text{GeV}]$		1.001×10^{14}		1.133×10^{15}	
$M_S [\text{GeV}]$		3.122×10^8		2.034×10^5	
$M_O [\text{GeV}]$		3.204×10^7		5.064×10^{15}	
$M_\Delta [\text{GeV}]$		...		4.899×10^{13}	
$M_\Sigma [\text{GeV}]$		...		1.928×10^{11}	
$M_t [\text{GeV}]$		...		2.565×10^{15}	
$\lambda [\text{GeV}]$		...		1.089×10^{16}	
$\langle t \rangle [\text{GeV}]$		...		1.000	

GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV) [cf. footnote²]. While the experimental uncertainties of various observables differ and are only known at the low-energy scale, a conservative estimate has been adopted for these observables at the GUT scale. For lighter quarks, an experimental uncertainty of $\pm 30\%$ is assumed, while for other observables the uncertainty is set to $\pm 10\%$. Although leptonic parameters are

²The calculation starts with a basis at M_Z where $Y_{u,e} = \text{Diag}(M_{u,e})/v$, $Y_d = V_{\text{CKM}} M_d/v$. Here $M_{u,d,e}$ are the diagonal mass matrices of charged fermions at M_Z and v is the vacuum expectation value of SM Higgs. These values are used as input for the 2-loop SM RGE equations. The running performed using the [54] gives the observables at the GUT scale. After diagonalization, the GUT-scale observables are obtained and results are listed in Table III.

known with high precision at low energies [55], a conservative uncertainty of $\pm 10\%$ is imposed at the GUT scale due to several factors: (1) unaccounted scalar contributions to RGE evolution, (2) uncertainties in scalar masses, and (3) uncertainty in the matching scale (4) next-to-next leading order threshold corrections. The ambiguity in the known physics at the GUT scale is the primary reason of the increased uncertainty (cf. [56,57] for fits having different uncertainty level for the leptonic spectrum).

A. Model I

The viability of Y_u , Y_d , and Y_e given in Eq. (11) to yield realistic low-energy observables: Yukawa couplings of charged fermions and the quark mixing matrix—is assessed through χ^2 optimization. The χ^2 definition includes six

quark Yukawas, three charged lepton Yukawas, and four quark mixing parameters. The two-loop RGE evolved values of the SM Yukawas of the charged fermions at the GUT scale ($M_{\text{GUT}} \sim 10^{16}$ GeV) has been sourced from [29] and is listed as O_{exp} in Table III.

The basis given in Eq. (25) for Y_1 and Y_2 is chosen again, making Y_1 a complex antisymmetric matrix and Y_2 a real diagonal matrix, providing a total of fifteen parameters, which are nine Yukawa couplings and six scalar masses, to be fitted in order to reproduce thirteen aforementioned observables. Additionally, the $SU(5)$ gauge coupling is taken to be $g_5 = 0.524$, and the matching scale (μ) is set at

the conventional GUT scale (M_{GUT}), which is also chosen as the mass of the heavy gauge bosons (M_X): $\mu = M_{\text{GUT}} = 10^{16}$ GeV = M_X . The mass of the heavy gauge boson (M_X) has been chosen to comply with the proton decay bounds. The Yukawa couplings have been restricted to lie within the perturbative regime i.e. $|(Y_{1,2})_{ij}| \leq 4\pi$. The values of various parameters obtained in the minimum χ^2 analysis are provided in Table III and is labeled as solution I.

The fitted values of $Y_{1,2}$ corresponding to entries of solution I in Table III are as follows:

$$Y_1 = \begin{pmatrix} 0 & (0.895 + i1.337) \times 10^{-4} & -0.585455 - i0.834302 \\ (-0.895 - i1.337) \times 10^{-4} & 0 & -0.0298835 - i0.0452801 \\ 0.585455 + i0.834302 & 0.0298835 + i0.0452801 & 0 \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} -2.6 \times 10^{-6} & 0 & 0 \\ 0 & 0.0004304 & 0 \\ 0 & 0 & -1.51563 \end{pmatrix}. \quad (32)$$

It is evident from the Yukawa relations derived in Eq. (11) that $Y_{1,2}$ collectively contribute to $Y_{u,d,e}$ at one loop. Moreover, as seen from the fitted values of $Y_{1,2}$ given in Eq. (32), the largest entry of Y_2 is comparable to the largest entry of Y_1 . However, the largest entry of Y_u must be more than two orders of magnitude greater than the largest entry of $Y_{d,e}$ in order to reproduce the correct order of the Yukawa couplings. Therefore, a cancellation occurs between the tree-level Yukawa coupling of $Y_{d,e}$ and the loop-corrected Yukawa, resulting in smaller Yukawa values, as show below:

$$\left| \frac{1}{\sqrt{6}} Y_2 \right| = \begin{pmatrix} -1.06 \times 10^{-6} & 0 & 0 \\ 0 & 1.75 \times 10^{-4} & 0 \\ 0 & 0 & 0.615 \end{pmatrix} \quad (33)$$

$$|\delta Y_d| = \begin{pmatrix} 3.37 \times 10^{-6} & 2.96 \times 10^{-5} & 1.65 \times 10^{-5} \\ 1.79 \times 10^{-7} & 2.19 \times 10^{-6} & 3.09 \times 10^{-4} \\ 2.82 \times 10^{-11} & 8.78 \times 10^{-8} & 1.98 \end{pmatrix}, \quad (34)$$

$$\left| \frac{0.5}{\sqrt{6}} \times (K_q^T Y_2 + Y_2 K_{dc}) \right| = \begin{pmatrix} 5.93 \times 10^{-7} & 5.24 \times 10^{-6} & 2.91 \times 10^{-6} \\ 3.16 \times 10^{-8} & 8.79 \times 10^{-8} & 5.47 \times 10^{-5} \\ 4.99 \times 10^{-12} & 1.55 \times 10^{-8} & 2.59 \end{pmatrix}.$$

$$\Rightarrow \left| \frac{1}{\sqrt{6}} Y_2 \right|_{33} + |\delta Y_d|_{33} - \left| \frac{0.5}{\sqrt{6}} \times (K_q^T Y_2 + Y_2 K_{dc}) \right|_{33} \approx 6 \times 10^{-3} \quad (35)$$

As shown above, the Yukawa correction involves the product of three Yukawa matrices. Since the largest entry of each matrix, $Y_{1,2}$, is $\mathcal{O}(1)$, and the loop function can contribute at most one order less than the largest entry, the largest entry of the loop-corrected Yukawa is also $\mathcal{O}(1)$, due to which the cancellation happens leading to small Yukawa.

The χ^2 analysis indicates that all the thirteen known parameters can be nicely fitted within the allowed range, resulting in a χ^2 value of 6.6. As evident from the values of $Y_{1,2}$ given in Eq. (32), the Yukawa entries remain well within perturbativity

constraints. The masses of the three scalars M_Ω , M_S , and M_O differ maximum from the matching scale, contributing ample threshold corrections while other scalars are closer toward the matching scale. However, this substantial mass splitting among the scalars challenges the extended survival hypothesis. As evident from this case, the splitting among the masses of different scalars belonging to the same irrep (i.e., 45_H) significantly contributes to altering the tree-level Yukawa relations. But, this requires an alternate mechanism that creates substantial hierarchies among different scalar fields within the same multiplet.

Before discussing the three-generation χ^2 analysis of models I A and II, constraints on the scalar fields T and $\mathbb{T}$ from the proton decays are presented in the following subsection.

I. Constraints on scalar fields T and $\mathbb{T}$ from proton decay

As evident from Eq. (7), the scalar fields T , $\mathcal{T}$, and $\mathbb{T}$ exhibit diquark and leptoquark couplings and can thus induce proton decay. The scalar field $\mathcal{T}$, however, couples to

up quarks with flavor-antisymmetric couplings, resulting in proton decay mediated by $\mathcal{T}$ occurring only at the loop level. In contrast, T and $\mathbb{T}$ can induce tree-level proton decays. Additionally, the scalar field Ω exhibits leptoquark couplings that can mix with the diquark coupling of $\mathbb{T}$, thereby inducing proton decay. Such mixing occurs when the SM Higgs acquires a vev , leading to the mixing of states with identical electric charges, particularly the $2/3$ -charged state in this case. However, this mixing is ruled out if there are only 24_H and 45_H irreps present in the model as none of the combinations $45_H 45_H 45_H$, $45_H 45_H 45_H^\dagger$, $45_H 45_H^\dagger 45_H^\dagger$, $24_H 45_H 45_H 45_H$, $24_H 45_H 45_H 45_H^\dagger$ and $24_H 45_H 45_H^\dagger 45_H^\dagger$ are $SU(5)$ invariant. Such mixing can arise only if the scalar sector also includes the 15_H -dimensional irrep which is $15_H 45_H 45_H 45_H$, as in the case of model I-A.

As a result, only tree-level proton decays mediated by T and $\mathbb{T}$ have been considered. Integrating out the degrees of freedom of T and $\mathbb{T}$ results in the following dimension-six four-fermion operators:

$$\begin{aligned} \mathcal{L}_{\text{eff}}^\beta &= h_1[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (u_A^{\alpha T} C^{-1} d_B^\beta) (u_C^{\gamma T} C^{-1} e_D) + h_2[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (u_A^{\alpha T} C^{-1} d_B^\beta) (d_C^{\gamma T} C^{-1} \nu_D) \\ &+ h_3[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (u_{\alpha A}^C C^{-1} d_{\beta B}^C)^* (u_A^{\gamma T} C^{-1} e_B) + h_4[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (u_{\alpha A}^C C^{-1} d_{\beta B}^C)^* (d_A^{\gamma T} C^{-1} \nu_B) + \text{H.c.} \\ &= h_1[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (\overline{u_L^{\alpha C}} d_{LB}^\beta) (\overline{u_L^{\gamma C}} e_{LD}) + h_2[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (\overline{u_L^{\alpha C}} d_{LB}^\beta) (\overline{d_L^{\gamma C}} \nu_{LD}) \\ &+ h_3[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (\overline{u_R^{\alpha C}} d_{RB}^\beta) (\overline{u_L^{\gamma C}} e_{LD}) + h_4[A, B, C, D] \varepsilon_{\alpha\beta\gamma} (\overline{u_R^{\alpha C}} d_{RB}^\beta) (\overline{d_L^{\gamma C}} \nu_{LD}) + \text{H.c.}, \end{aligned} \quad (36)$$

where $h_{1,2,3,4}$ are effective strengths of the operators and depend upon particular flavor indices. The last line of the above expression represents the baryon number violating operator in the conventional form where L and R denotes the chirality of the respective fermion. These operators violate the baryon number and induces proton decay modes having $B - L = 0$. The functional form of $h_{1,2}$ in the mass basis in terms of coupling $Y_{1,2}$ are shown below:

$$\begin{aligned} h_1[A, B, C, D] &= \frac{1}{M_\mathbb{T}^2} [(U^T Y_1 D)_{BA} (D^T Y_2 E)_{CD} + 2(U^T Y_1 U)_{CB} (D^T Y_2 E)_{AD}] \\ h_2[A, B, C, D] &= \frac{1}{M_\mathbb{T}^2} [(U^T Y_1 D)_{BA} (D^T Y_2 N)_{CD} + 2(D^T Y_1 D)_{AC} (U^T Y_2 N)_{BD}] \\ h_3[A, B, C, D] &= \frac{1}{2M_T^2} [(U^{C\dagger} Y_2^\dagger D^C)_{AB} (U^T Y_2 E)_{CD}] \\ h_4[A, B, C, D] &= \frac{-1}{2M_T^2} [(U^{C\dagger} Y_2^\dagger D^C)_{AB} (D^T Y_2 N)_{CD}], \end{aligned} \quad (37)$$

where U, U^C, D, D^C, E, N are the unitary matrices which diagonalizes the Y_u, Y_d, Y_e , and M_ν , respectively and their evaluation can be conferred from [48]. The effective strengths are suppressed by the quadratic mass of the scalar fields T and $\mathbb{T}$. Two-component Fierz transformation rules for Weyl fermions have been used in order to evaluate the effective strengths of the operators [58].

The scalar field $\mathbb{T}$ couples only to left-chiral fermions, consequently the proton decay operator it induces only involves left-chiral fermions, while the scalar field T couples to right and left chiral fermions. To evaluate their contribution to proton decay, the contribution from either T or $\mathbb{T}$ is considered at a time, assuming the other scalar is significantly heavier than the one under consideration. The expressions of leading two body proton decay modes can be inferred from [5] while the values of various parameters entering in the decay width expression and the procedure of evaluating the decay widths can be referenced from [48]. Substituting the fitted values of Y_1 and Y_2 from Eq. (32) into Eq. (37) and using matrices for $U, D,$

TABLE IV. Proton decay branching fraction estimated for the solution I of Table III induced by the scalar fields T and $\mathbb{T}$. Only the contribution of a single scalar at a time is considered when computing the bounds on the mediators.

Branching ratio [%]	T	$\mathbb{T}$
$\text{BR}[p \rightarrow \bar{\nu}\pi^+]$	99.9	< 1
$\text{BR}[p \rightarrow \bar{\nu}K^+]$	< 1	99.8
$\text{BR}[p \rightarrow e^+\pi^0, \mu^+\pi^0, e^+K^0, \mu^+K^0]$	< 1	< 1

U^C , D^C and E , one can compute the branching ratios of leading two body proton decay modes and are given in Table IV. Additionally, as neutrinos are massless in the setting of solution I, consequently N matrix appearing in Eq. (37) is taken to be an identity matrix.

When the contribution to proton decay arises solely from T , the leading decay mode is $p \rightarrow \bar{\nu}\pi^+$, and is due to the diagonal nature of Y_2 . The lower bound on the mass of the scalar field T , ensuring consistency with the experimental lower limit on proton decay mode $p \rightarrow \bar{\nu}\pi^+$ [59], is given as follows:

$$\tau/\text{BR}[p \rightarrow \bar{\nu}\pi^+] = 3.9 \times 10^{32} \text{ yrs} \times \left(\frac{M_T}{1.7 \times 10^{13} \text{ GeV}} \right)^4. \quad (38)$$

It is also evident from the Table IV that the leading branching mode is $p \rightarrow \bar{\nu}K^+$ when proton decay is only induced with $\mathbb{T}$ and is due to summing over all the flavors of neutrinos and presence of contribution of Y_1 matrix. The lower bound on the mass of $\mathbb{T}$ from the current lower bound on the mode $p \rightarrow \bar{\nu}K^+$ [60] is as follows:

$$\tau/\text{BR}[p \rightarrow \bar{\nu}K^+] = 5.9 \times 10^{33} \text{ yrs} \times \left(\frac{M_{\mathbb{T}}}{1.0 \times 10^{14} \text{ GeV}} \right)^4. \quad (39)$$

The fitted value of M_T and $M_{\mathbb{T}}$ in solution I of Table III is greater than the lower bound on it from the proton decays. Moreover, the fitted value of $\mathcal{T}$, being only two orders of magnitude less than the GUT scale, also adheres to the constraints from loop-induced proton decays.

2. Effect of lightest scalar in model I

It has been noted above that some of the scalars deviate significantly from the matching scale (μ) in order to viably reproduce the SM spectra. The mass of these scalars can, in principle, affect the RGE of the SM parameters and change their GUT scale values. Below, the model I has is considered (i.e., Yukawa sector with only 45_H) as it has least number of free parameters. It is evident from Table III, that the lightest scalar is ($O(8, 2, 1/2)$) and contribution of the lightest scalar to the SM RGE is

considered at its mass M_O , and the process is elaborated below:

Step 1: The SM parameters are extrapolated using two-loop RGE equations from M_t (top pole mass) to the GUT scale and used for fitting the GUT scale parameters. This step (i.e., zeroth iteration) will yield the Yukawa matrices $Y_{1,2}$ and the masses of the various scalars listed in Table III. From here, the mass of the lightest scalar is determined, which is $M_O \sim 10^7 \text{ GeV}$.
Step 2: The interaction of the scalar field O with SM fermions is constructed as shown below;

$$\mathcal{L} \supset Y_{O_1} q^T C^{-1} u^C O + Y_{O_2} q^T C^{-1} d^C O^\dagger + \text{H.c.} \quad (40)$$

Since O ($O^\dagger$) stems from 45_H ($45_H^\dagger$), the largest entry of Y_1 becomes the (1,1) element of Y_{O_1} , and the largest element of Y_2 becomes the (2,2) element of Y_{O_2} , while all other elements of $Y_{O_{1,2}}$ are taken to be zero.³ It is to be noted that coupling of $Y_{O_{1,2}}$ is also complex in nature.

Step 3: Having constructed the $Y_{O_{1,2}}$ Yukawa couplings from $Y_{1,2}$, the parameters are run down from M_{GUT} to M_O (determined in step 1) using two-loop SM + Octet RGE equations. The running parameters include the fitted values of $Y_{u,d,e}$, the formulated values of $Y_{O_{1,2}}$, and the computed values of $g_{1,2,3}$.⁴

Step 4: After obtaining the values of $Y_{O_{1,2}}$ at M_O , these values are inserted as initial conditions for the two-loop SM + Octet RGE equations and solved from M_O to $\mu = M_{\text{GUT}}$. The initial conditions for these RGE equations include the solutions for $Y_{u,d,e}$ and $g_{1,2,3}$ obtained from solving the SM RGE equations from M_t to M_O . Now, $Y_{u,d,e}$ and CKM parameters are obtained at the GUT scale, incorporating the contribution of the octet scalar.

Step 5: Since the octet contribution has been included in the RGE equations, its threshold correction at the matching scale has been set to zero. With this, the obtained SM parameters are fitted at the GUT scale, yielding a new set of $Y_{1,2}$ (first iteration). After this, steps 2–5 are repeated for fifty iterations. While fitting the parameters all the Yukawa couplings are restricted to lie between $(-3.5, 3.5)$. The Yukawa couplings $Y_{u,d,e}$ predicted at the GUT scale, when evolved down to the octet mass scale using SM + octet RGEs, are again run down to M_t to yield SM parameters at M_t . The resulting input values and the predicted SM

³This particular choice of $Y_{O_{1,2}}$ ensures no large contribution of the scalar field's Yukawa coupling to the SM parameters.

⁴The considered scenario does not achieve gauge coupling unification. Therefore, while performing the fit, g is fixed at $g \equiv g_5 = 0.524$, but for the beta function, the values of $g_{1,2,3}$ at the respective scale obtained from running are used.

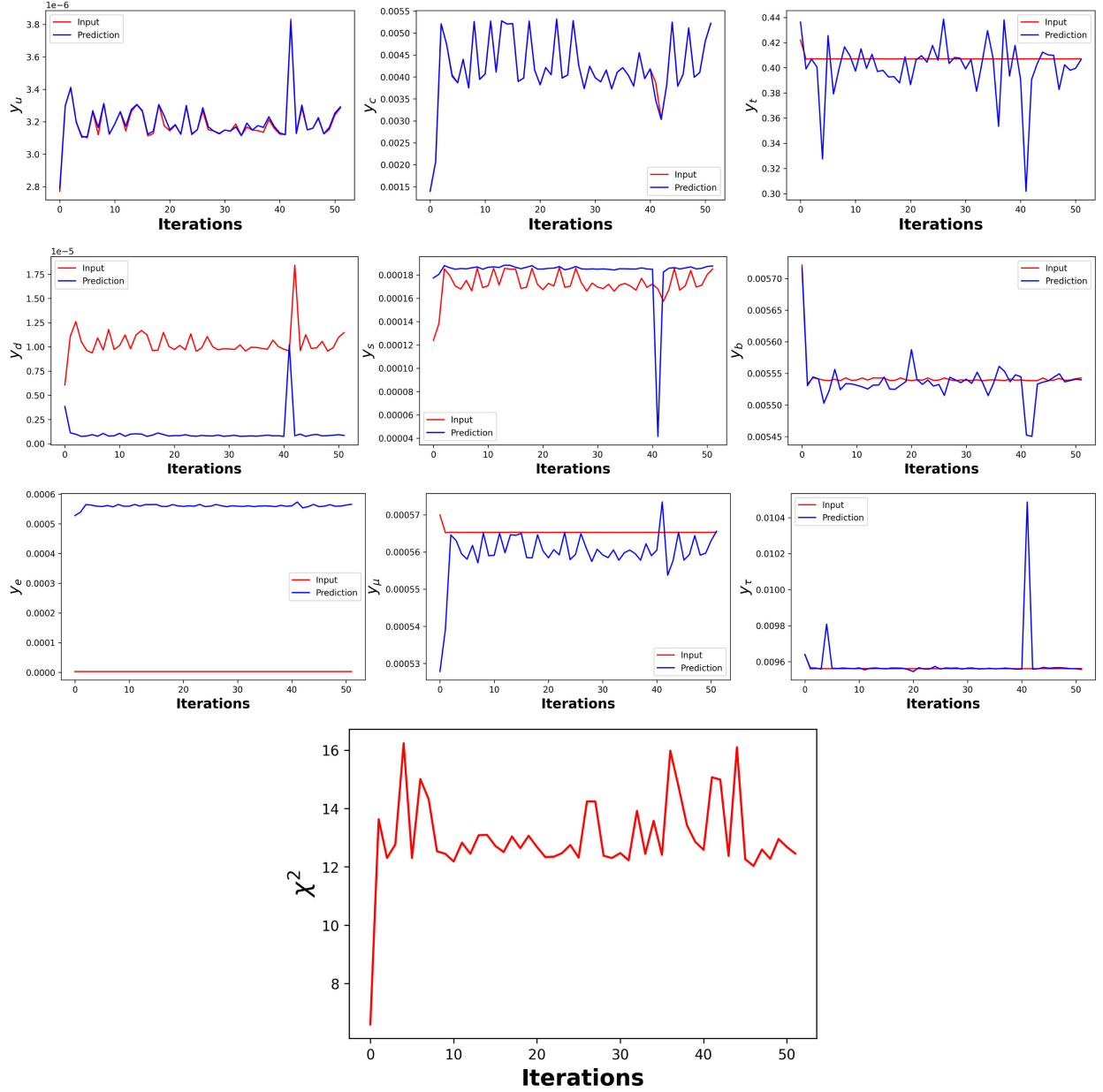


FIG. 5. The figure compares the input Yukawa couplings (red) with the predicted values (blue) at the matching scale across multiple iterations. The figure in the last line shows the obtained χ^2 at different iteration, where the zeroth iteration is without considering the scalar contribution.

parameters, both at the matching scale (M_{GUT}) and M_t , are extracted and presented in Figs. 5 and 6.

The above analysis demonstrates that the obtained solution can approximately reproduce the SM parameters, even after accounting for the contribution of the lightest scalar in the RGE evolution of the SM parameters. The top Yukawa coupling exhibits the largest deviation, both at the GUT scale and at M_t . A specific form of the Yukawa coupling of the scalar field O is chosen to minimize its impact on the SM parameters. Any alternative choice would lead to different SM parameters at the GUT scale, and if these deviations are significant, their low-energy

equivalents would differ substantially from the original values. Moreover, it is observed that the χ^2 value increases when the octet's contribution to the RGEs is included. This occurs because its threshold correction contribution is set to zero at the GUT scale.

B. Model I-A

The viability of Eq. (17) in conjunction with Eq. (11) is assessed in reproducing the observed charged and neutral fermion mass spectrum and mixing angles through χ^2 optimization. For this purpose, the basis for Y_1 and Y_2

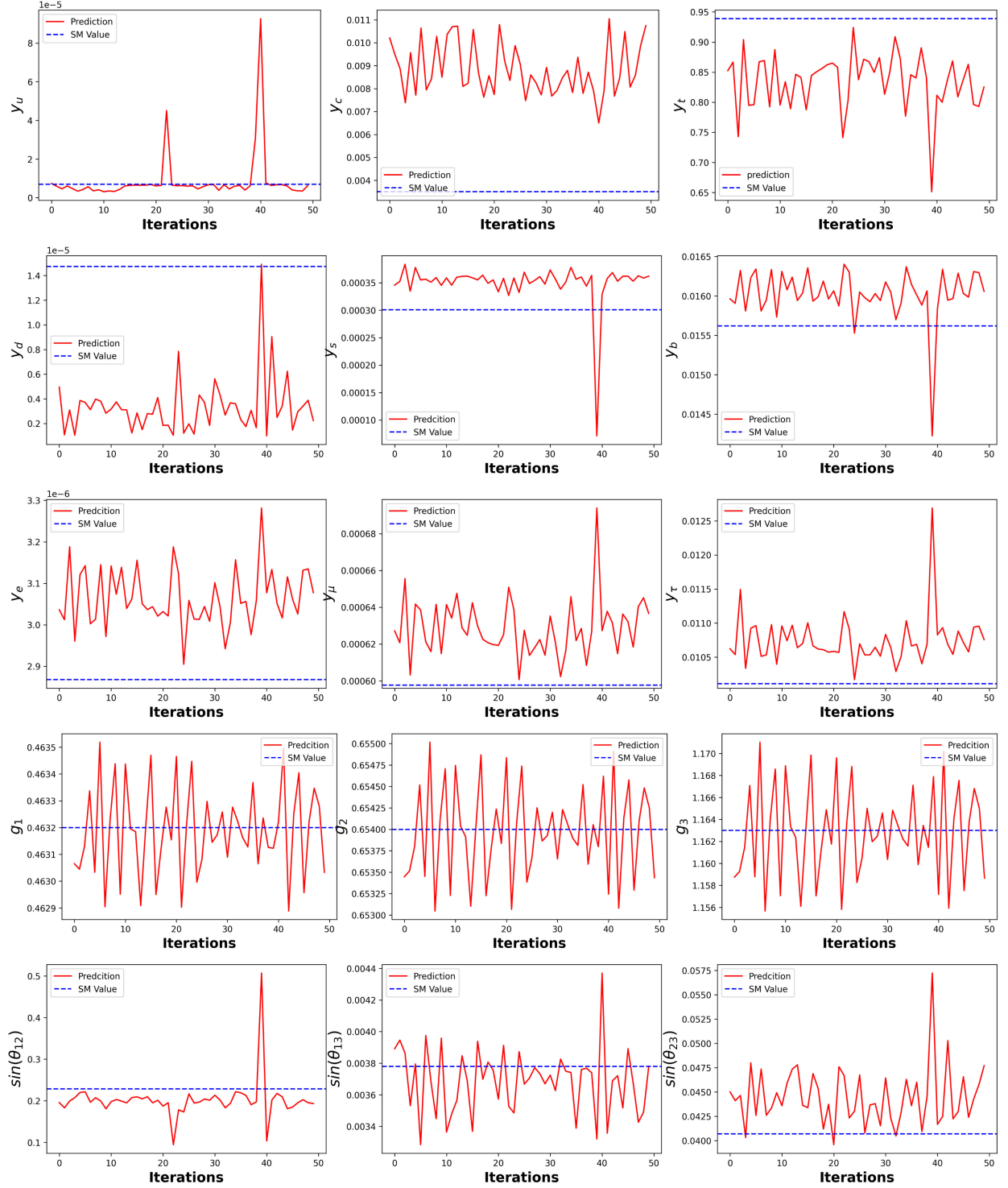


FIG. 6. The figure compares the predicted Standard Model (SM) parameters (solid red lines) at the top quark mass scale (M_t) with the input SM parameter (blue dashed line).

is chosen as before, with Y_3 set as a real-symmetric matrix. The ability of Eqs. (17) and (11) to yield six quark Yukawas, three charged lepton Yukawas, four CKM parameters, two neutral fermion mass-squared differences, and four PMNS parameters is examined. The reference

values of neutral lepton parameters are sourced from a recent update of *NuFIT-6.0* [61].

As done in the previous subsection IV A, two-loop RGE-extrapolated values for charged fermion Yukawas are used for fitting. Although Y_u and Y_d appearing in Eq. (17) are at

the GUT scale, the neutral lepton parameters are fitted at the low scale, assuming negligible running effects [62–65]. Choosing Y_3 as a real symmetric matrix adds six more Yukawa parameters, while 15_H introduces three additional scalars, a cubic coupling (λ), and the vev of t , all of which contribute additional new parameters. This results in a total of 26 parameters to be fitted to reproduce 18 observables. Again all the Yukawa couplings are varied with the perturbative limit specified in the beginning of this section, masses of scalars are varied within 1 TeV and the Planck scale, while mass of the gauge boson is equal to the matching scale ($\mu = M_{\text{GUT}} = M_X = 10^{16}$) GeV. The λ

and $\langle t \rangle$ parameter appearing in Eq. (17) are also varied near the GUT scale and $(10^{-2} - 5)$ GeV respectively. Additionally, the vacuum expectation value of H is set at the electroweak scale (i.e., $\langle H \rangle = 174$ GeV). The discussion of Sec. IV A 1 puts lower bound on the mass of scalars T and $\mathbb{T}$ which also has been taken care while fitting the parameters. The best fitted of various parameters, in this case, are labeled as solution I-A in Table III.

The fitted values of $Y_{1,2,3}$ corresponding to the solution II given in Table III are as follows:

$$\begin{aligned}
 Y_1 &= \begin{pmatrix} 0 & (3.76776 + i0.968243) \times 10^{-5} & (1.95037 + i0.984594) \\ (-3.76776 - i0.968243) \times 10^{-4} & 0 & -1.09659 - i0.553524 \\ (-1.95037 - i0.984594) & 1.09659 + i0.553524 & 0 \end{pmatrix}, \\
 Y_2 &= \begin{pmatrix} -8.03887 \times 10^{-5} & 0 & 0 \\ 0 & -2.51338 \times 10^{-3} & 0 \\ 0 & 0 & -1.01186 \end{pmatrix}, \quad \text{and}, \\
 Y_3 &= \begin{pmatrix} 0.374622 & 0.302098 & -2.04874 \\ 0.302098 & 2.10474 & 0.377314 \\ -2.04874 & 0.377314 & -0.318043 \end{pmatrix}.
 \end{aligned} \tag{41}$$

It is evident from the solution I-A, given in Table III, that the entire charged and neutral fermion mass spectrum, along with the mixing angles, can be adequately fitted by including 15_H in addition to 45_H in the Yukawa sector. The entries of Y_3 are of $\mathcal{O}(1)$ in Eq. (41) and are essential for reproducing the correct neutrino mass scales. The scalar fields S deviates the most from the matching scale and of the TeV order, while the other scalars are closer to the matching scale. The fitted value of the vev of scalar field t is close to 1.00 GeV and the obtained χ^2 corresponding to the fitted parameters is 2.0.

It is noteworthy that even in this case, the required mass of the scalar field $\mathcal{T}$ is close to the GUT scale, complying with the any constraints that may arise from loop-induced proton decays. Furthermore, the scalar field Ω is also near the GUT scale, ensuring that the resulting state from the mixing of $\mathbb{T}$ and Ω remains of the order of the GUT scale and satisfies proton decay constraints.

C. Model II

Finally, model II given in Sec. II C is assessed and the ability of Eqs. (22), (15), and (24) to reproduce the

observed charged and neutral fermion mass spectra, as also performed in previous subsections, through χ^2 analysis. The chosen basis includes Y_1 as a complex-symmetric matrix, Y_2 as a real-diagonal matrix, and Y_3 as a real-symmetric matrix, resulting in a total of 21 unknown Yukawa parameters to be fitted. The Yukawa couplings are varied within the range $(-1.0, 1.0)$, and the matching scale (μ) is fixed at the GUT scale, which is also set equal to the mass of the heavy gauge boson, consistent with previous settings. Additionally, the scalar masses are varied between 1 TeV and the Planck scale. The scalar field T , known to induce tree-level proton decay, has its lower bound set to satisfy proton decay constraints [66]. The λ parameter appearing in M_ν of Eq. (24) is varied again close to the GUT scale. The total number of unknown parameters to be fitted includes the Yukawa couplings, scalar masses, and λ parameter, giving a total of 26 parameters. The results of the χ^2 optimization are shown in Table V as the solution II, and the fitted values of $Y_{1,2,3}$ are provided in corresponding to the solution III is as follows:

$$\begin{aligned}
Y_1 &= \begin{pmatrix} (-4.5031 - i5.5048) \times 10^{-5} & (7.6529 - i3.8840) \times 10^{-4} & (-3.0386 + i3.4412) \times 10^{-3} \\ (7.6529 - i3.8840) \times 10^{-4} & (4.1431 - i1.1378) \times 10^{-3} & (-3.2919 + i3.3610) \times 10^{-2} \\ (-3.0386 + i3.4412) \times 10^{-3} & (-3.2919 + i3.3610) \times 10^{-2} & (1.7848 - i3.7993) \times 10^{-1} \end{pmatrix}, \\
Y_2 &= \begin{pmatrix} -1.3211 \times 10^{-5} & 0 & 0 \\ 0 & 1.2873 \times 10^{-2} & 0 \\ 0 & 0 & 1.1107 \times 10^{-1} \end{pmatrix}, \quad \text{and} \\
Y_3 &= \begin{pmatrix} -0.21650 & -0.99153 & 0.71898 \\ -0.99153 & -0.31605 & -0.71097 \\ 0.71898 & -0.71097 & -0.76757 \end{pmatrix}.
\end{aligned} \tag{42}$$

The Yukawa entries $Y_{1,2,3}$ corresponding to solution II are provided in Eq. (42). As there are 26 unknown parameters, the entries of $Y_{1,2,3}$ are feeble and well within the perturbative range. Among the scalars, M_Δ shows the greatest deviation from the matching scale, while other

scalars are closer to the matching scale. The fitted value of the trilinear coupling is approximately 10^{16} GeV, and the corresponding χ^2 for the fitted values is ~ 0 .

V. SUMMARY

In general, tree-level conclusions in any model—particularly Yukawa relations—can sometimes be unreliable and may not fully represent the actual situation, as also demonstrated by this article. Tree-level statements alone cannot capture the full dynamics of any model and must be augmented with loop corrections to provide a more accurate picture. The Yukawa sector of a renormalizable $SU(5)$ GUT model has been comprehensively analyzed in this study across different minimal settings, with a focus on the inclusion of quantum corrections. The quantum corrections to the various Yukawa vertices are induced by heavy degrees of freedom—scalar fields and heavy gauge bosons. These heavy degrees of freedom impart threshold corrections at a particular matching scale. The noteworthy and revealing features of this study are as follows:

- (i) The Yukawa sector of an $SU(5)$ model containing only the 45_H -dimensional irrep can reproduce the correct charged lepton spectrum when one-loop corrections to various Yukawa vertices are included. Furthermore, adding the 15_H irrep to the Yukawa sector alongside the 45_H yields the observed charged and neutral fermion mass spectra and mixing angles when quantum corrections are switched on, even after considering the RGE effects of lightest scalars.
- (ii) Incorporating loop corrections to the Yukawa relations of an $SU(5)$ model comprising of 5_H and 15_H dimensional irreps also successfully reproduces the correct fermion mass spectrum and mixing angles.

A general feature of this analysis is that, to achieve a desirable Yukawa spectrum, either larger Yukawa couplings are needed or the mass of one or more scalar particles must significantly deviate from the matching scale, as these are the primary sources for achieving the

TABLE V. The benchmark best-fit values of different observables for the charged and neutral fermion mass spectrum and quark and lepton mixing parameters. Other details are same as in Table III.

Observable	O_{exp}	Solution II O_{th}
y_u	2.81×10^{-6}	2.81×10^{-6}
y_c	1.42×10^{-3}	1.42×10^{-3}
y_t	4.27×10^{-1}	4.27×10^{-1}
y_d	6.14×10^{-6}	6.12×10^{-6}
y_s	1.25×10^{-4}	1.23×10^{-4}
y_b	5.80×10^{-3}	5.83×10^{-3}
y_e	2.75×10^{-6}	2.75×10^{-6}
y_μ	5.72×10^{-4}	5.73×10^{-4}
y_τ	9.68×10^{-3}	9.62×10^{-3}
$ V_{us} $	0.2286	0.2288
$ V_{cb} $	0.0457	0.0458
$ V_{ub} $	0.0042	0.0042
$\sin \delta_{\text{CKM}}$	0.78	0.78
$\Delta m_{\text{sol}}^2 [\text{eV}^2]$	7.49×10^{-5}	7.84×10^{-5}
$\Delta m_{\text{atm}}^2 [\text{eV}^2]$	2.534×10^{-3}	2.65×10^{-3}
$\sin^2 \theta_{12}$	0.307	0.306
$\sin^2 \theta_{23}$	0.561	0.559
$\sin^2 \theta_{13}$	0.02195	0.02195
χ^2_{min}		~ 0
$M_T [\text{GeV}]$		2.34×10^{14}
$M_\Delta [\text{GeV}]$		1.01×10^3
$M_S [\text{GeV}]$		3.26×10^8
$M_t [\text{GeV}]$		1.82×10^{15}
$\lambda [\text{GeV}]$		7.98×10^{15}

necessary threshold corrections. An appealing aspect of GUTs is that they possess multiple heavy degrees of freedom, often arising from a single irrep, which can contribute to Yukawa relations at loop level, as harnessed in this study. However, for a viable spectrum, these heavy degrees of freedom—i.e., scalar fields—require mass splitting, with many needing significant deviation from the matching scale. This scenario calls for a mechanism to generate mass splittings in nonsupersymmetric GUTs at all loop levels.

As the focus of this study is to construct a minimal Yukawa sector, other important aspects like gauge coupling unification have not been addressed. To achieve gauge coupling unification, one can include additional (scalar and fermion) irreps that modify the RGE of gauge couplings without affecting the Yukawa sector [67]. It is noteworthy that quantum corrections can significantly alter the tree-level Yukawa relations, making them phenomenologically desirable. This serves as a motivation to explore the Yukawa sectors of other GUT models.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: LOOP INTEGRATION FACTORS

The loop integration factors appearing in Eqs. (9), (10), (15), (17), (20), (21), and (24) are given by

$$f[m_1^2, m_2^2] = -\frac{1}{16\pi^2} \left(\frac{m_1^2 \log \frac{m_1^2}{\mu^2} - m_2^2 \log \frac{m_2^2}{\mu^2}}{m_1^2 - m_2^2} - 1 \right), \quad (\text{A1})$$

$$h[m_1^2, m_2^2] = \frac{1}{16\pi^2} \left(\frac{1}{2} \log \frac{m_1^2}{\mu^2} + \frac{\frac{1}{2} r^2 \log r - \frac{3}{4} r^2 + r - \frac{1}{4}}{(1-r)^2} \right), \quad (\text{A2})$$

$$g[m_1^2, m_2^2] = \frac{1}{16\pi^2} \frac{\frac{r^3}{6} - r^2 + \frac{r}{2} + r \log r + \frac{1}{3}}{(1-r)^3}, \quad (\text{A3})$$

where $r = m_2^2/m_1^2$ in the last two equations, and,

$$p(m_1^2, m_2^2) = \frac{1}{16\pi^2} \frac{1}{m_1^2 - m_2^2} \log \left(\frac{m_1^2}{m_2^2} \right) \quad (\text{A4})$$

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