

Out-of-equilibrium chiral condensate in AdS/QCD

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We study the chiral condensate at finite temperature in AdS/QCD in a time-dependent background, in which the position of the black hole horizon z_h changes with time, producing an increasing or decreasing temperature. Conformal invariance is broken, as in the soft-wall model, by a static quadratic dilaton. Two different scenarios are analyzed; in the first a general power-law time dependence is assumed for z_h , while in the second the energy-momentum tensor at late times reproduces the one found in viscous hydrodynamics. Depending on the rate at which the system evolves, the transition shifts toward lower temperatures. If the chiral condensate is far from equilibrium at low temperatures, oscillations around its equilibrium value are observed before thermalization. A prethermalization stage is found in the chiral limit if the initial condition is set at a temperature close to the critical one. In the hydrodynamic setup the evolution of the medium is slow enough, and the chiral condensate soon matches the equilibrium curve.

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I. INTRODUCTION

Quantum chromodynamics (QCD) has a rich phase diagram in the chemical potential–temperature plane, each phase characterized by different degrees of freedom. In the hadronic phase, realized at small temperature and chemical potential, quarks are confined in colorless states, the hadrons, and chiral symmetry is broken. At higher values of temperature and chemical potential QCD is in the deconfined quark-gluon plasma (QGP) phase in which quarks and gluons are free and chiral symmetry is restored. For small values of the chemical potential a crossover separates the two phases, becoming a second-order phase transition in the case of two massless quarks. Heavy-ion collision experiments at the LHC and RHIC have provided a wealth of data to investigate the fundamental properties of QGP by studying the dynamical evolution of the fireballs generated in the collisions [1,2]. Data are well-described by viscous hydrodynamics, showing that QGP behaves as a nearly perfect fluid with very low viscosity, and has collective flow patterns which indicate strong interactions among quarks and gluons. These flow patterns are characterized by anisotropic flow coefficients, which provide information about the initial geometry and the subsequent evolution of the QGP.

Many theoretical models developed to study QGP involve a static system at thermal equilibrium, with a fixed temperature. Anyway, the strongly interacting matter produced in heavy-ion collisions undergoes rapid expansion and cooling, so out-of-equilibrium effects should be taken into account to provide a realistic theoretical description of the phenomenon, in particular when fast-changing conditions are involved, thus making the equilibrium approximation inappropriate. In this paper we use AdS/QCD to dynamically study the chiral phase transition at zero chemical potential, with out-of-equilibrium effects included by assuming a time-dependent metric in the bulk and either on shell or perturbed initial conditions.

In the past, some theoretical efforts have already been devoted to the analysis of the evolution of the chiral condensate and the chiral transition in out-of-equilibrium dynamical systems in some phenomenological models [3–9]. The chiral phase transition has been also studied in different holographic models, as e.g., Sakai-Sugimoto [10,11], where a particular brane construction is implemented in order to introduce flavors and study chiral symmetry breaking at equilibrium. In [12] the dynamics of a second-order chiral phase transition is studied in the Nambu-Jona-Lasinio model, and in [13] the chiral phase transition in an expanding quark-antiquark system is investigated solving the Vlasov and gap equations. Chiral transition and the chiral condensate have been investigated using a linear σ model in [14], observing strong oscillations of the chiral field during the transition.

Some approaches involving holographic methods to dynamically study the chiral condensate have been developed as well. In [15] the chiral phase transition is studied by placing probe D7 branes in a time-dependent geometry

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describing a growing/shrinking black hole in anti-de Sitter (AdS) with a background magnetic field, which is needed to induce chiral symmetry breaking in this framework. In [16,17] the holographic soft-wall model has been used to study the evolution of the chiral condensate by considering a time-dependent scalar field in a static black hole geometry, perturbing the initial conditions. In this paper we adopt a bottom-up holographic approach and consider different time-dependent geometries, in which the black hole horizon moves either following a generic power-law behavior or according to viscous hydrodynamics, thus changing the effective temperature of the system. A static dilaton field is introduced to break conformal invariance, and the dynamics of the scalar field containing the chiral condensate is studied.

The paper is organized as follows. In Sec. II we introduce the holographic model we use to study a time-dependent chiral condensate. In Sec. III we briefly review the results of the model in the static case (at equilibrium), in which the metric is time-independent. In Sec. IV we introduce two time-dependent metrics we use to study the dynamics of the chiral condensate, one in which the temperature follows a power-law variation with time, described in Sec. IV A, the other describing a hydrodynamiclike regime, investigated in Sec. IV B. In Sec. V we summarize our conclusions and outlook.

II. THE MODEL

In order to study a dynamical chiral phase transition, we consider a time-dependent AdS-like geometry. The starting point is a general five-dimensional metric in Eddington-Finkelstein coordinates:

$$ds^2 = -A(z, v)dv^2 + \Sigma(z, v)^2 e^{B(z, v)} dx_\perp^2 + \Sigma(z, v)^2 e^{-2B(z, v)} dy^2 - \frac{2}{z^2} dv dz, \quad (1)$$

respecting spatial 3d translation invariance along the boundary directions $\{x^1, x^2, y\}$, and $O(2)$ rotation invariance in the plane $x_\perp = \{x^1, x^2\}$ [18]. For a purely AdS metric the metric functions are: $A(z, v) = 1/z^2$, $\Sigma(z, v) = 1/z$ and $B(z, v) = 0$. In an AdS space with a black hole the metric has: $A(z) = 1/z^2(1 - z^4/z_h^4)$, where z_h is the position of the black hole horizon, related to the temperature T of the dual gauge theory by $T = 1/(\pi z_h)$.

In the holographic model considered in this work, chiral $U(n_f)_L \times U(n_f)_R$ symmetry of QCD, where n_f is the number of quark flavors, is implemented via a dual local (gauge) symmetry in the bulk, with gauge fields A_L and A_R . The scalar field dual to the $q\bar{q}$ operator is $X = e^{i\pi}(X_0 + S)e^{i\pi}$, where the field $\pi(z, v, \vec{x})$ describes pseudoscalar mesons and $S(z, v, \vec{x})$ scalar mesons [19,20]. The vacuum expectation value (VEV) $X_0(z, v)$ contains the scalar condensate, responsible for the breaking of

such symmetry in a Higgs-like fashion. For $n_f = 2$ identical quarks, the VEV is given by $X_0(z, v) = \frac{1}{2}X_q(z, v)\text{diag}(1, 1)$. In this framework X_q does not depend on other spatial variables, but such a dependence could be introduced considering an anisotropic metric as e.g., the one considered in [21], which depends on y as well. This effect could be investigated in the future in order to take into account the dynamics of the expanding inhomogeneous fireball formed in heavy-ion collision. This could be relevant to detect new effects in the evolution of the chiral condensate, as e.g., at the kinetic freezeout. The breaking of conformal symmetry is implemented introducing a background dilaton field $\phi(z) = c^2 z^2$ which generates the QCD mass scale Λ_{QCD} [22]. We work in the probe approximation and neglect the backreaction of the flavor fields and dilaton to the metric. In [23] it has been shown that the backreaction of X_0 on the metric should be small or negligible in order to reproduce the crossover transition predicted by lattice QCD. We have checked *a posteriori* that at equilibrium the contribution of the terms in the action containing the field X_0 is indeed small compared to the gravitational part of the action, being at least 2–3 orders of magnitude smaller in the ranges of z and T in which X_0 is sizeable. In the standard soft-wall model the chiral condensate is proportional to the quark mass, so it vanishes in the chiral limit $m_q \rightarrow 0$. Following Refs. [24,25], in order to guarantee the existence of a finite chiral condensate at zero temperature also in the case of vanishing quark mass, a quartic term in the potential and an anomalous dimension (depending on the holographic coordinate z) in the five-dimensional mass of the X field,

$$m_5^2 = -3 - \mu^2 z^2, \quad (2)$$

are introduced, resulting in the full action for $X_q(z, v)$ [26,27],

$$S \propto \int d^5x \sqrt{-g} e^{-\phi(z)} \left[(\partial_M X_q)(\partial^M X_q) + m_5^2 X_q^2 + \frac{\lambda}{4} X_q^4 \right], \quad (3)$$

and equation of motion,

$$\partial^M (\sqrt{-g} e^{-\phi(z)} \partial_M X_q(z, v)) - m_5^2 \sqrt{-g} e^{-\phi(z)} X_q(z, v) - \frac{\lambda}{2} \sqrt{-g} e^{-\phi(z)} X_q(z, v)^3 = 0. \quad (4)$$

The values of the parameters we use are $c = 440$ MeV, $\mu = 1450$ MeV, $\lambda = 80$ [25]. In the following, we will briefly describe the static (equilibrium) case first, and then focus on the dynamic case, in which the position of the black hole horizon changes with time, effectively taking the system out of equilibrium. We will study the behavior of the chiral condensate in both cases.

III. CHIRAL CONDENSATE IN THE STATIC CASE

In the metric with a black hole, having $A(z) = 1/z^2(1 - z^4/z_h^4)$, $\Sigma(z, v) = 1/z$, and $B(z, v) = 0$, the time-independent equation of motion for the scalar field $X_q(z, v) = X_q(z)$ is

$$\left(1 - \frac{z^4}{z_h^4}\right)X_q''(z) + \left(-\frac{3}{z} - \frac{z^3}{z_h^4} - 2c^2z\left(1 - \frac{z^4}{z_h^4}\right)\right)X_q'(z) - \frac{1}{z^2}m_5^2X_q(z) - \frac{\lambda}{2z^2}X_q(z)^3 = 0. \quad (5)$$

It can be solved imposing that the behavior of the field near the boundary $z \rightarrow 0$ is (following the convention of Ref. [16])¹

$$X_q(z) \xrightarrow{z \rightarrow 0} m_q \gamma z + \sigma z^3 + \left(c^2 m_q \gamma + (m_q \gamma)^3 \lambda - \frac{1}{2} m_q \gamma \mu^2\right) z^5 \times \log z + \mathcal{O}(z^5), \quad (6)$$

where $\gamma = \sqrt{N_c}/2\pi$ and σ is related to the chiral condensate at equilibrium: $\langle \bar{q}q \rangle = \gamma \sigma$ [16],² and requiring the solution to have a regular behavior as $z \rightarrow z_h$ via a shooting method on the parameter σ [16,27]. Therefore, in this model the quark mass only enters as a boundary condition. In Fig. 1 the solutions at $z_h = 2 \text{ GeV}^{-1}$ for $m_q = 0$ and $m_q \neq 0$ are shown.

By fitting the numerical solution to (6) for $z \rightarrow 0$ the value of σ can be obtained for any fixed temperature. Such values are shown in Fig. 2 for $m_q = 0$ and $m_q = 3.22 \text{ MeV}$ at different temperatures T , reproducing the result of Ref. [25]. In the chiral limit a second-order transition from $\sigma \neq 0$ at $T < T_c$ to $\sigma = 0$ at $T > T_c$ is observed, with $T_c \sim 163 \text{ MeV}$. For $m_q > 0$ a crossover is obtained instead. In the chiral limit the value of σ at zero temperature is $\sigma_0 = 0.0536 \text{ GeV}^3$, corresponding to a chiral condensate $\langle \bar{q}q \rangle = (245 \text{ MeV})^3$. For $m_q = 3.22 \text{ MeV}$, the value of σ at zero temperature is $\sigma_0 = 0.0545 \text{ GeV}^3$, corresponding to a chiral condensate $\langle \bar{q}q \rangle = (247 \text{ MeV})^3$.

IV. TIME-DEPENDENT CHIRAL CONDENSATE

Let us consider an out-of-equilibrium system in which the position of the black hole horizon changes with time $[z_h(v)]$. We change variable and define the new set (u, v') with

¹With the factor σ/γ of Ref. [16] replaced in this work by σ .

²The numerical factors multiplying m_q and $\langle \bar{q}q \rangle$ depend on the model one is considering and on the convention adopted therein, so they are different in e.g., [16,20]. Nonetheless both models reproduce the QCD high- Q^2 expansion of the two-point correlation function of two scalar operators and adopt the same definition of the chiral condensate.

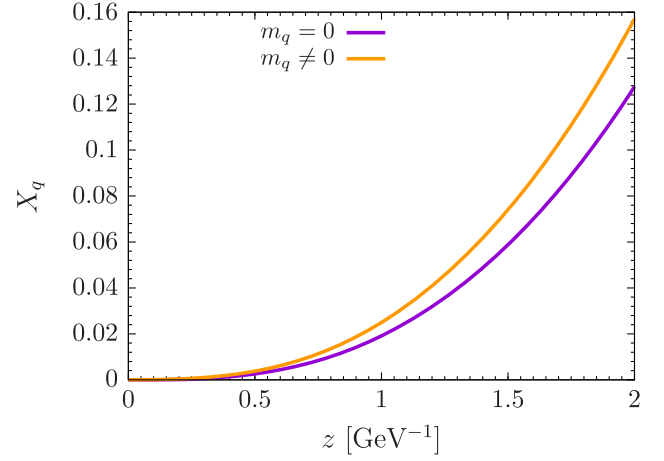


FIG. 1. Profile of $X_q(z)$ for $z_h = 2 \text{ GeV}^{-1}$, and for $m_q = 0$ (purple curve) and $m_q = 3.22 \text{ MeV}$ (orange curve).

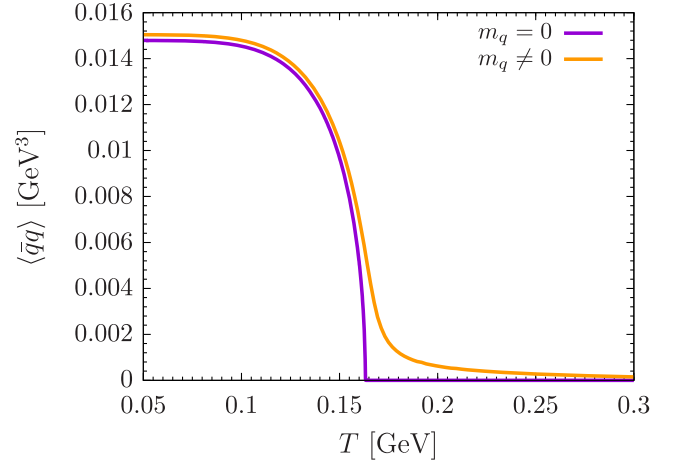


FIG. 2. Chiral condensate vs temperature in the static case for $m_q = 0$ (purple curve) and $m_q = 3.22 \text{ MeV}$ (orange curve).

$$\begin{cases} u = \frac{z}{z_h(v)} \\ v' = v, \end{cases} \quad (7)$$

such that the horizon is at $u = 1$ and $u \in [0, 1]$. For clarity of notation, we will continue calling the new v' variable v . Using the metric in Eq. (1), the equation of motion (4) for the scalar field becomes,

$$\begin{aligned} X_q''(2uz_h + Az_h^2u^2) + z_h \dot{X}_q \left(-3 \frac{\Sigma'}{\Sigma} + \phi' \right) - 2z_h \dot{X}_q' \\ + X_q' \left(6\dot{z}_h \frac{\Sigma'}{\Sigma} u - u\dot{z}_h \phi' + 2\dot{z}_h + 3u^2 z_h^2 A \frac{\Sigma'}{\Sigma} \right. \\ \left. - u^2 z_h^2 A \phi' + u^2 z_h^2 A' + 2uz_h^2 A - 3z_h \frac{\dot{\Sigma}}{\Sigma} \right) \\ - \frac{m_5^2}{u^2} X_q - \frac{\lambda}{2u^2} X_q^3 = 0, \end{aligned} \quad (8)$$

where the prime indicates a derivative with respect to u and the dot with respect to v . We solve this partial differential equation using the Chebyshev pseudospectral method [28,29]. We expand the solution $X_q(u, v)$ in a truncated series of N Chebyshev polynomials in the variable $y = 2u - 1$, with coefficients depending on v . By requiring the equation to be satisfied in N Gauss-Lobatto collocation points,

$$u_j = \frac{1}{2} \left(1 - \cos \left(\frac{j\pi}{N-1} \right) \right) \quad j = 0, 1, \dots, N-1, \quad (9)$$

one is left with a system of N (nonlinear) ordinary differential equations in the variable v , which we then solve using a standard Runge-Kutta algorithm. $N = 30$ is used, and we have checked the final results do not change for higher values of N .

In the following, the behavior of the chiral condensate as a function of time v will be investigated and compared to the static result in two models, which assume two different functional forms for $z_h(v)$. In Sec. IV A a power-law behavior for $z_h(v)$ is assumed, while in Sec. IV B we use the metric discussed in [30–32], which is an analytic solution of Einstein equations in the large-time expansion and holographically reproduces the viscous hydrodynamics energy-momentum tensor of the gauge theory. Results will be mostly shown in parametric plots of $\sigma(v)$ versus $T(v)$ to get a direct comparison with the static curve. In all the considered cases the quark mass m_q is assumed to be constant.

A. Power-law behavior for $z_h(v)$

For simplicity, we consider a toy model in which we assume that the metric functions are the same as in the static case, the only difference being that z_h varies with

time; $\Sigma(z, v) = 1/z$, $A(z, v) = 1/z^2(1 - z^4/z_h(v)^4)$, and $B(z, v) = 0$. Although the system is not at thermodynamical equilibrium, we define an effective temperature $T(v) = 1/(\pi z_h(v))$, coinciding with the temperature of a black hole metric in the static case. Two different profiles are used for $z_h(v)$:

$$z_h(v)/z_h(0) = 1 \pm \Lambda_\ell(v/z_h(0))^a \quad a \geq 1, \quad (10)$$

corresponding to an effective temperature

$$T(v)/T(0) = (1 \pm \Lambda_\ell(v/z_h(0))^a)^{-1}, \quad (11)$$

which is increasing (–) or decreasing (+) with time v .

In the first case, we consider a decreasing $z_h(v)$, starting from $v = 0$ and spanning the phase diagram from a state in which the temperature is initially small to higher values. We set $z_h(0) = 4 \text{ GeV}^{-1}$, corresponding to the low-temperature region of Fig. 2 in which the chiral condensate is approximately constant. We assume the system is initially at equilibrium, so we require as initial condition that $X_q(u, 0)$ is the solution computed at $z_h(0)$ in the static case. As boundary conditions we require that $X_q(0, v) = 0$ and $\partial_u X_q(0, v) = m_q \gamma z_h(v)$. We also assume that the quark mass is constant. Also in this case, as in the static one, the quark mass only enters as a boundary condition. At each fixed time v , $\sigma(v)$ is found, as in the static case, as the coefficient of the z^3 term in the low- z expansion (6) of the solution $X_q(u, v)$ with $u = z/z_h(v)$. The obtained values of $\sigma(v)$ at the corresponding $T(v)$ are shown as parametric plots in Fig. 3 for $m_q = 0$ (left panel) and $m_q = 3.22 \text{ MeV}$ (right panel), and for $a = 1$, $\Lambda_\ell = 0.01, 0.1$, and $a = 2$, $\Lambda_\ell = 0.001, 0.01$, together with the values in the static case. The transition to $\sigma = 0$ is smooth for both $m_q = 0$ and $m_q \neq 0$, as emphasized in Ref. [15], and the curves in the

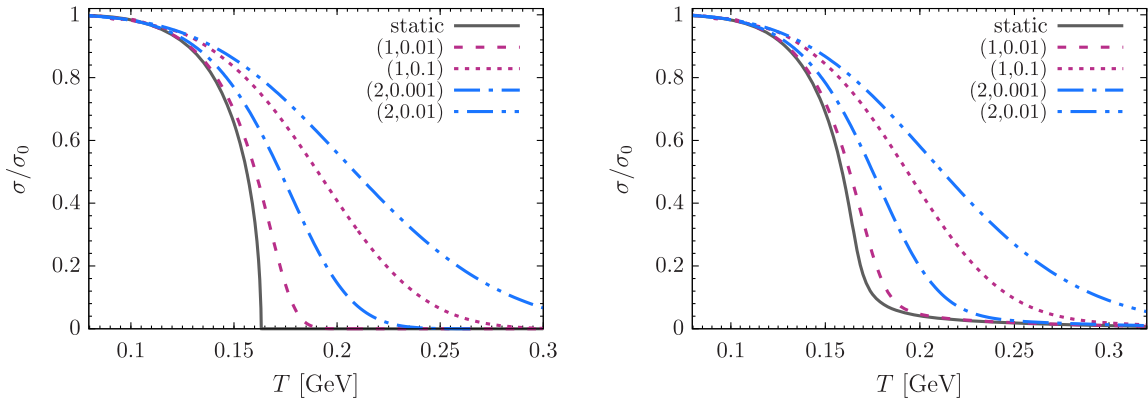


FIG. 3. Time-dependent chiral condensate as a function of the effective temperature $T(v)$ in Eq. (11) (minus sign) for $m_q = 0$ (left panel) and $m_q = 3.22 \text{ MeV}$ (right panel), with the values of the parameters (a, Λ_ℓ) indicated in the legend. Time increases from left to right. The initial condition $X_q(u, 0)$ is set at $z_h(0) = 4 \text{ GeV}^{-1}$, corresponding to $T(0) \sim 80 \text{ MeV}$. $X_q(u, 0)$ is the solution to Eq. (5) at $z_h(0)$. The values of the chiral condensate computed in the static case are shown by the gray plain curve. σ_0 is the value of σ at $T = 0$ at equilibrium.

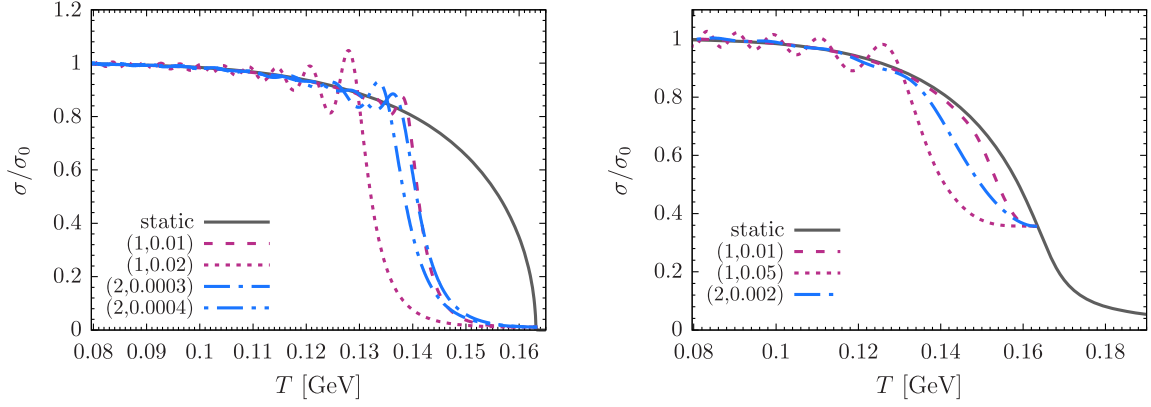


FIG. 4. Time-dependent chiral condensate as a function of $T(v)$ in Eq. (11) (plus sign) for $m_q = 0$ (left panel) and $m_q = 3.22$ MeV (right panel), with the values of the parameters (a, Λ_ℓ) indicated in the legend. Time increases from right to left. The initial condition $X_q(u, 0)$ is set at $z_h(0) = 1.949$ GeV $^{-1}$, corresponding to $T(0) \sim 163$ MeV. $X_q(u, 0)$ is the solution to Eq. (5) at $z_h(0)$. The values of the chiral condensate computed in the static case are shown by the gray plain curve. σ_0 is the value of σ at $T = 0$ at equilibrium.

two cases are similar. The curve with $\Lambda_\ell = 0.01$ and $a = 1$ is the closest to the static one, while they in general deviate more from the equilibrium curve as a and Λ_ℓ increase. The effective temperature corresponding to the inflection point increases with Λ_ℓ and a , meaning that it increases as T varies more rapidly with v .

As a second example, we choose an increasing $z_h(v)$ according to Eq. (10) with the plus sign, so the system is at high temperature at first, and then cools down as time goes by. We set the initial condition at $z_h(0) = 1.949$ GeV $^{-1}$, close to the critical temperature T_c in the static case at $m_q = 0$. We again assume that the system is initially at equilibrium, so the initial condition $X_q(u, 0)$ is the solution computed at $z_h(0)$ in the static case. If $\Lambda_\ell \rightarrow 0$ the static curve is reproduced by the dynamical evolution. Results for higher values of Λ_ℓ are shown in Fig. 4. Since we are considering the case in which temperature decreases with time, higher values of temperature correspond to initial times, while the temperature gets low at the end of the evolution. At the beginning ($T \sim 160$ MeV) the system tends to stay close to the initial condition, so the chiral condensate remains small, and then it starts growing, running after the static curve. Eventually, the dynamical solution approaches the static one. At this stage oscillations can occur, which are damped as temperature further decreases. An analogous oscillatory behavior towards equilibrium has been observed in [16] when studying the evolution of the chiral condensate in a static background when the initial value of σ is different from the on shell one. In [15] oscillatory solutions obtained with an excited initial condition, having an energy excess with respect to the on shell one, are linked to bubble formation. In particular, a large homogeneous bubble forms in the mixed phase when there is an excess of energy due to a thermal fluctuation. Therefore, it is also interesting to investigate what happens in a time-dependent background

if the initial condition is different from the on shell static one. This analysis is described in the rest of the section and in the next one, using a different metric.

We now assume the scalar field is at some time perturbed from the equilibrium profile, getting the expression in Eq. (6) with a given value of σ .

For an increasing temperature, the initial condition given by Eq. (6) with $\sigma = 1/z_h(0)^3$ is set at $z_h(0) = 4$ GeV $^{-1}$. The values of σ obtained letting T increase according to (11) (minus sign) are shown in Fig. 5 for $m_q = 0$ (left panel) and $m_q = 3.22$ MeV (right panel). Damped oscillations are observed in the beginning, which disappear when the system approaches the equilibrium curve. In particular, for smaller Λ_ℓ the oscillations are faster and the system reaches equilibrium at lower temperatures.

In case of a decreasing temperature and in the chiral limit, we start from $z_h(0) = 1.7$ GeV $^{-1}$ [or $T(0) \sim 187$ MeV] and set the initial condition given by Eq. (6) with $\sigma = 0.1/z_h(0)^3$. The values of σ obtained letting z_h increase according to (10) (plus sign) are shown in Fig. 6. In general, in the chiral limit we find that if the initial condition is $X_q(u, 0) = 0$, the solution vanishes for any subsequent v . For $\Lambda_\ell = 0.05$ and $a = 1$, σ is finite during the evolution. In the other considered cases the system undergoes a transition (crossover) from very tiny values of σ ($\sigma \sim 0$) to a phase in which chiral symmetry is broken, taking place at a temperature $T < T_c$. Such temperature increases with decreasing Λ_ℓ , and thermalization occurs at a higher T . However, for even smaller Λ_ℓ , in particular if $\Lambda_\ell < 0.00215$ for $a = 1$, at the beginning of the evolution, when $T > T_c$, the system thermalizes to $X_q = 0$ and $\sigma = 0$ (numerically $\sigma < 10^{-13}$ GeV ^{3}) and then they continue to be zero for subsequent times in the range of temperatures we are interested in. Oscillations are also observed. In the $m_q \neq 0$ case, starting from the initial condition $X_q(u, 0) = 0$ at $z_h(0) = 1$ GeV $^{-1}$, we observe

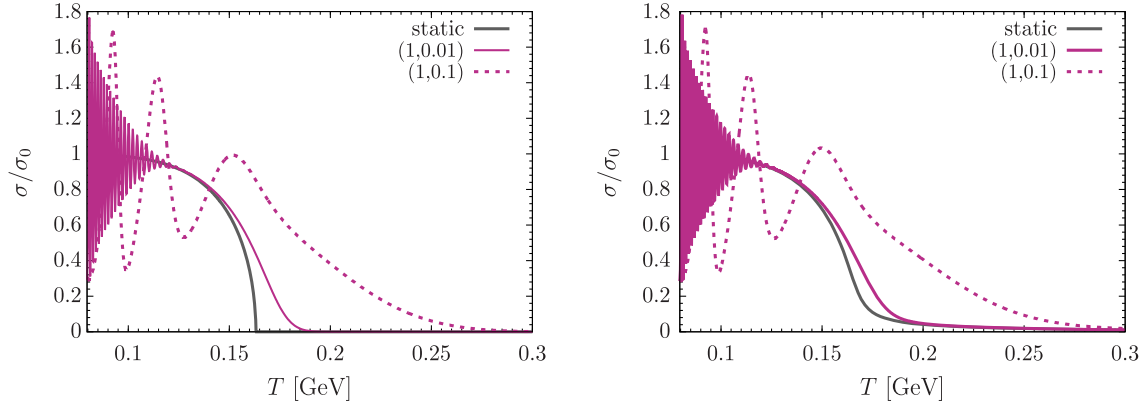


FIG. 5. Time-dependent chiral condensate as a function of $T(v)$ in Eq. (11) (minus sign) for $m_q = 0$ (left panel) and $m_q = 3.22$ MeV (right panel), with the values of the parameters (a, Λ_ℓ) indicated in the legend. Time increases from left to right. The initial condition $X_q(u, 0)$ is set at $z_h(0) = 4 \text{ GeV}^{-1}$ and it is given by Eq. (6) with $\sigma = 1/z_h(0)^3$. The values of the chiral condensate computed in the static case are shown by the gray curve. σ_0 is the value of σ at $T = 0$ at equilibrium.

that the system quickly reaches equilibrium or small oscillations are observed as long as the parameters a and Λ_ℓ are small, i.e., the variation of temperature is slow. This is shown in Fig. 7. For smaller values of Λ_ℓ the dynamic curve approaches the static one at higher temperatures. In the linear case $a = 1$, if $\Lambda_\ell < 0.001$ the process is quasistatic and the dynamic curve coincides with the static one. The effective temperature corresponding to the inflection point is smaller than the one in the static case, and it decreases with increasing Λ_ℓ and a .

In out-of-equilibrium studies a prethermalization phase has been found, in which the ratio of pressure over energy density is almost constant although the system is still not

equilibrated [33]. This has been also observed when investigating collision of shock waves using gauge/gravity duality producing a far-from-equilibrium strongly-coupled plasma which relaxes towards a hydrodynamic regime, see e.g., [30,34]. In general a prethermalization phase is a transient quasistationary state different from the true thermal equilibrium [33] while after a sufficiently long time the system thermalizes. In this phase the memory of the initial state is partially kept, while it is completely lost in the thermal phase [35]. Prethermalization usually occurs when there are relevant separated time scales and relaxation proceeds in several steps [36]. Prethermalization and thermalization phases have been studied in AdS/QCD in [16] in the case of a system at fixed temperature

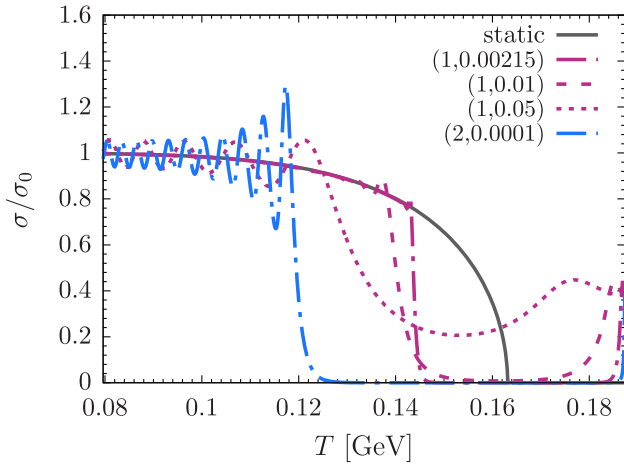


FIG. 6. Time-dependent chiral condensate as a function of $T(v)$ in Eq. (11) (plus sign) for $m_q = 0$, with the values of the parameters (a, Λ_ℓ) indicated in the legend. Time increases from right to left. The initial condition $X_q(u, 0) = 10^{-1}u^3$ is set at $z_h(0) = 1.7 \text{ GeV}^{-1}$, corresponding to $T(0) \sim 187 \text{ MeV}$. The static result is shown by the plain gray curve. σ_0 is the value of σ at $T = 0$ at equilibrium.

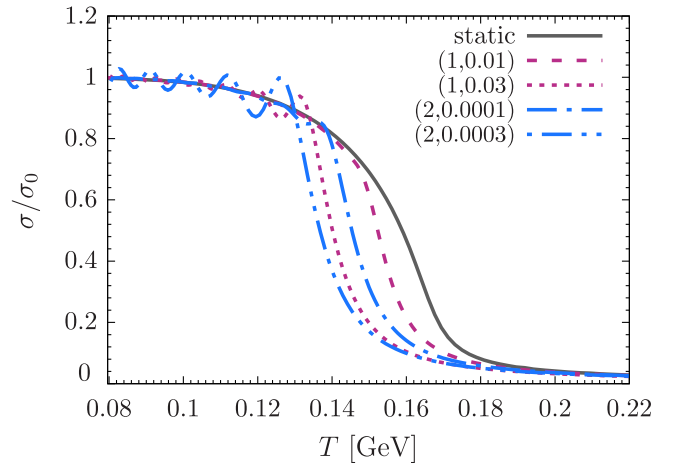


FIG. 7. Time-dependent chiral condensate as a function of $T(v)$ in Eq. (11) (plus sign) for $m_q = 3.22$ MeV, with the values of the parameters (a, Λ_ℓ) indicated in the legend. Time increases from right to left. The initial condition $X_q(u, 0) = 0$ is set at $T(0) = 0.32 \text{ GeV}$ ($z_h(0) = 1 \text{ GeV}^{-1}$). The static result is shown by the gray plain curve. σ_0 is the value of σ at $T = 0$ at equilibrium.

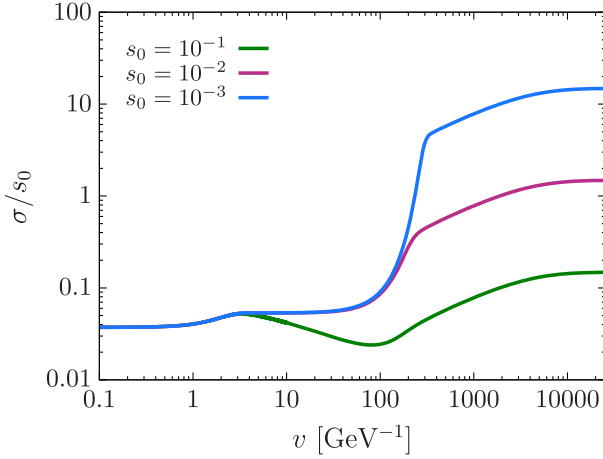


FIG. 8. Time-dependent chiral condensate for $m_q = 0$ in a background with decreasing temperature according to (11) (plus sign) with $a = 1$, $\Lambda_\ell = 10^{-4}$ and $z_h(0) = 1.949 \text{ GeV}^{-1}$, corresponding to $T(0) \sim T_c$. The initial condition is $X_q(u, 0) = s_0 u^3$, with the values of s_0 indicated in the legend. v is in units (GeV^{-1}).

quenched to a perturbed initial state, and the interplay between order parameter and the corresponding Goldstone modes has been investigated in [17]. The system considered here is in contact with an evolving background, in which temperature changes with time. In the chiral limit, if the initial temperature is very close to the critical one T_c and if z_h slowly changes with time, the initial behavior of the system is akin to a prethermalization phase. This is shown in Figs. 8 and 9. In Fig. 8 the time evolution of the chiral condensate starts from $z_h(0) = 1.949 \text{ GeV}^{-1}$,

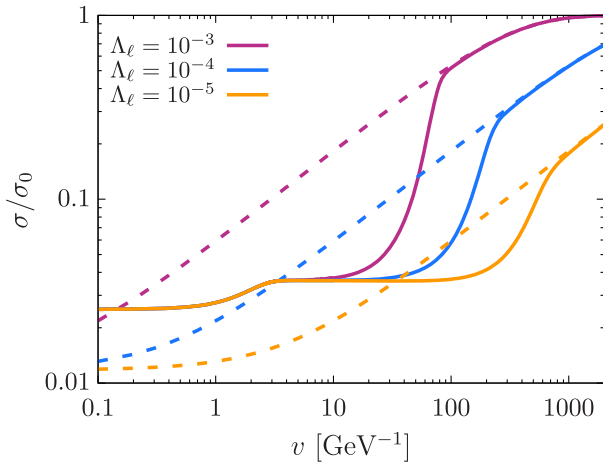


FIG. 9. Time-dependent chiral condensate for $m_q = 0$ in a background with decreasing temperature according to (11) (plus sign) with $a = 1$, $z_h(0) = 1.949 \text{ GeV}^{-1}$ and the values of Λ_ℓ indicated in the legend. The initial condition is $X_q(u) = s_0 u^3$, with $s_0 = 10^{-2}$. The static results corresponding to each value of Λ_ℓ are shown by the dashed curves. σ_0 is the value of σ at $T = 0$ at equilibrium.

corresponding to $T(0) \sim T_c$, with $X_q(u, 0) = s_0 u^3$ and $s_0 = 10^{-1}, 10^{-2}, 10^{-3}$. Temperature and black hole horizon change with time according to Eqs. (11) and (10), respectively, with $a = 1$ and $\Lambda_\ell = 10^{-4}$. A prethermalization stage, in which the system is almost stationary and trapped in a state different from the equilibrium one, can be observed from $v \sim 4 \text{ GeV}^{-1}$, and it lasts more as s_0 gets closer to the equilibrium value. As shown in Ref. [16] during the prethermalization stage the ratio σ/s_0 is almost constant and does not depend on s_0 . In Fig. 9 the dependence on Λ_ℓ is investigated, showing that the system remains in the prethermalization phase longer for smaller values of Λ_ℓ . Hence, the duration of the prethermalization phase depends on how fast the temperature changes with time and how much the initial condition deviates from the equilibrium one. Equilibrium, represented by the dashed lines, is eventually reached. For $m_q \neq 0$ we find no evidence of prethermalization.

B. Hydrodynamicslike behavior for $z_h(v)$

We now consider a different time-dependent metric, which has been obtained in Refs. [30–32] to describe a viscous hydrodynamic system in a holographic model. It has been found (see, e.g., [18]) that if an external source perturbs (quenches) a boost-invariant boundary metric within a definite time interval, a horizon appears in the bulk. The quench, represented by the profile $\gamma(v)$, modifies the line element of the $4d$ boundary:

$$ds_4^2 = -dv^2 + e^{\gamma(v)} dx_\perp^2 + v^2 e^{-2\gamma(v)} dy^2, \quad (12)$$

leaving the spatial three-volume unchanged and preserving the translational and $O(2)$ symmetries in the transverse plane. The $5d$ spacetime on which the gravity dual is defined, having the metric (12) as boundary, is described by the line element in Eq. (1). It has also been observed that, after a very short time from the end of the quench, the system thermalizes reaching a behavior which is well described by viscous hydrodynamics. Indeed, the energy-momentum tensor of the gauge theory, computed from the $5d$ metric using the holographic renormalization procedure developed in [37], approaches soon after the end of the quench the energy-momentum tensor obtained in viscous hydrodynamics through a Bjorken expansion [38–41]:

$$T_\nu^\mu = \frac{N_c^2}{2\pi^2} \text{diag}(-\epsilon, p_\perp, p_\perp, p_\parallel) \quad (13)$$

with

$$\epsilon(v) = \frac{3\pi^4 \Lambda^4}{4(\Lambda v)^{4/3}} \left[1 - \frac{2c_1}{(\Lambda v)^{2/3}} + \frac{c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right) \right] \quad (14)$$

$$p_{\parallel}(v) = \frac{\pi^4 \Lambda^4}{4(\Lambda v)^{4/3}} \left[1 - \frac{6c_1}{(\Lambda v)^{2/3}} + \frac{5c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right) \right] \quad (15)$$

$$p_{\perp}(v) = \frac{\pi^4 \Lambda^4}{4(\Lambda v)^{4/3}} \left[1 - \frac{c_2}{(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right) \right], \quad (16)$$

where $c_1 = \frac{1}{3\pi}$, $c_2 = \frac{1+2\log 2}{18\pi^2}$, and Λ is a parameter. ϵ is the energy density, $p_{\parallel}$ is the pressure in the longitudinal direction along which the plasma expands, $p_{\perp}$ is the pressure along one of the two transverse directions. Hydrodynamic models are able to describe the behavior of the QGP in relativistic heavy-ion collisions [42–45]. The metric that produces such an energy-momentum tensor, solution of Einstein equations in the late time expansion, is (1) with [31]:

$$A(z, v) = \frac{1}{z^2} \left(1 - \frac{4z^4}{3} \epsilon(v) \right), \quad (17)$$

$$\Sigma(z, v) = \frac{1}{z} (v + z)^{1/3}, \quad (18)$$

$$B(z, v) = \frac{z^4}{3} (p_{\perp}(v) - p_{\parallel}(v)) - \frac{2}{3} \log(v + z). \quad (19)$$

The effective temperature, defined as

$$\epsilon(v) = \frac{3}{4} \pi^4 T(v)^4, \quad (20)$$

is given by

$$T(v) = \frac{\Lambda}{(\Lambda v)^{1/3}} \left[1 - \frac{1}{6\pi(\Lambda v)^{2/3}} + \frac{-1 + \log 2}{36\pi^2(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right) \right], \quad (21)$$

and the position of the horizon [i.e., the value of z at which $A(z, v)$ vanishes] by

$$z_h(v) = \frac{(\Lambda v)^{1/3}}{\pi \Lambda} \left[1 + \frac{c_1}{2(\Lambda v)^{2/3}} + \frac{5c_1^2 - 2c_2}{8(\Lambda v)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda v)^2}\right) \right]. \quad (22)$$

Four models have been considered in [30], each characterized by a different form of the quench. The parameter Λ takes the following values; $\Lambda = 2.25$ GeV in model $\mathcal{A}_1$, $\Lambda = 1.73$ GeV in model $\mathcal{A}_2$, $\Lambda = 1.12$ GeV in model $\mathcal{B}$, and $\Lambda = 1.59$ GeV in model $\mathcal{C}$. Notice that in [30] no energy scale was set and no conformal symmetry breaking scheme was introduced, so the values of Λ were dimensionless. In this work we have introduced an energy scale

through a static dilaton, and all quantities get a scale dimension. In the following we shall study the scalar field in the background described by Eqs. (14)–(19), with the above four values of Λ .

In the chiral limit, we have set the initial condition $X_q(u, v_0) = 10^{-5} u^3$ at initial time v_0 corresponding to $z_h(v_0) = 1.9$ GeV⁻¹ [or $T(v_0) \sim 168$ MeV]. Temperature decreases with increasing time v . Solving the partial differential equation and fitting the solution to Eq. (6), we have obtained the values of σ shown in Fig. 10. In general, $z_h(v) = z_0$ has two real solutions and, since we are considering the large-time limit, we take the largest value of v . As found in the previous section, we again find that in all these models, for $m_q = 0$, if $X_q(u, v_0) = 0$ is set for any v_0 or if a finite X_q is set as initial condition at a temperature sufficiently higher than the critical one, the solution vanishes for all times. Therefore, a transition to a chirally broken phase is triggered by fluctuations of the scalar field $X_q \neq 0$ for temperatures close to the critical one or lower. Figure 10 shows that, if a fluctuation of the scalar field occurs at $T \gtrsim T_c$, the system takes some time to reach equilibrium and the transition to a chirally broken phase starts at $T < T_c$. Once σ gets its equilibrium value, it continues to follow the equilibrium curve as the temperature decreases. We have checked that this behavior is also realized when the initial fluctuation is larger than the considered one, with the coefficient of u^3 in $X_q(u, v_0)$ up to $\mathcal{O}(1)$.

If a fluctuation of X_q occurs at a much lower temperature, short oscillations are observed before equilibrium is reached. This is shown in Fig. 11, obtained setting the initial condition $X_q(u, v_0) = 0.001 u^3$

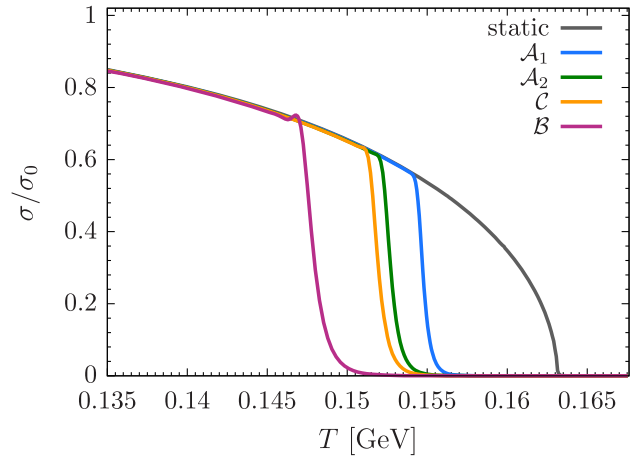


FIG. 10. Time-dependent chiral condensate as a function of the effective temperature $T(v)$ in (21) in the hydrodynamic model described by Eqs. (14)–(19) with $m_q = 0$. The initial condition $X_q(u, v_0) = 10^{-5} u^3$ is set at $z_h(v_0) = 1.9$ GeV⁻¹, corresponding to $T(v_0) \sim 168$ MeV. Time increases from right to left. The static result is shown by the gray curve.

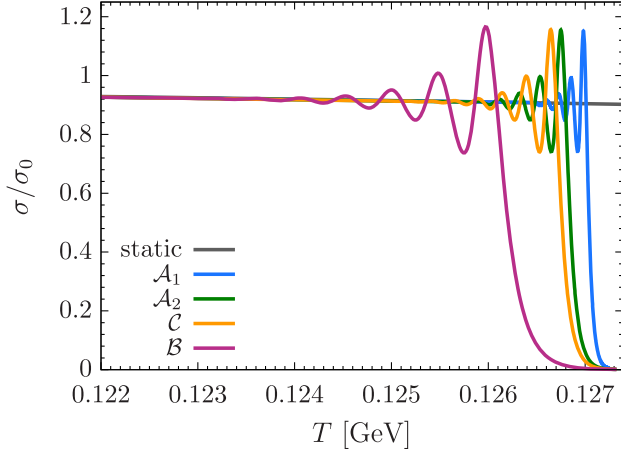


FIG. 11. Time-dependent chiral condensate as a function of the effective temperature $T(v)$ (21) in the hydrodynamic model described by Eqs. (14)–(19) with $m_q = 0$. The initial condition $X_q(u, v_0) = 0.001u^3$ is set at $z_h(v_0) = 2.5 \text{ GeV}^{-1}$, corresponding to $T(v_0) \sim 127.3 \text{ MeV}$. Time increases from right to left. The static result is shown by the gray curve.

at $z_h(v_0) = 2.5 \text{ GeV}^{-1}$, corresponding to an initial temperature $T(v_0) = 127.3 \text{ MeV}$. The height of the first oscillation is the same in the four models, while the amplitude is inversely proportional to Λ , and model B , which has the smallest Λ , reaches equilibrium at lower temperatures. Notice that, according to Eq. (22), $z_h(v)$ varies more slowly with time when Λ is larger.

When $m_q = 3.22 \text{ MeV}$, we again find that any initial condition for the scalar field leads to a chirally broken phase at low temperature, and $\sigma(T)$ rapidly approaches the static value. In particular, if the initial state is $X_q(u, v_0) = 0$ at $z_h(v_0) = 1 \text{ GeV}^{-1}$, the system behaves as at equilibrium during the whole evolution in all the models. This also occurs if $X_q(u, v_0) \neq 0$ since thermalization turns out to be very rapid. For higher $z_h(v_0)$, if the initial $\sigma(v_0)$ is different from the expected value in the static limit, a short oscillation can form, after which the static values are reproduced. In this respect, we have investigated what happens if at some point during the crossover the value of σ is modified by some external effect. To this aim, we have solved the equation of motion for X_q setting an initial condition at $z_h(v_0) = 2.5 \text{ GeV}^{-1}$ such that X_q is equal to the asymptotic expression in Eq. (6) with $\sigma = 0.02 \text{ GeV}^{-3}$, a value different from the equilibrium one. Results reported in Fig. 12 show that, after a very short oscillation, the system returns to equilibrium.

It is possible to observe prethermalization in this system in the chiral limit by choosing an initial temperature close to the critical one T_c . In Fig. 13, σ is plotted as a function of $v - v_0$, where v_0 is the initial time, such that $z_h(v_0) = 1.949 \text{ GeV}^{-1}$. For all the models, the initial condition is $X_q(u, v_0) = 10^{-4}u^3$. Model B , having the

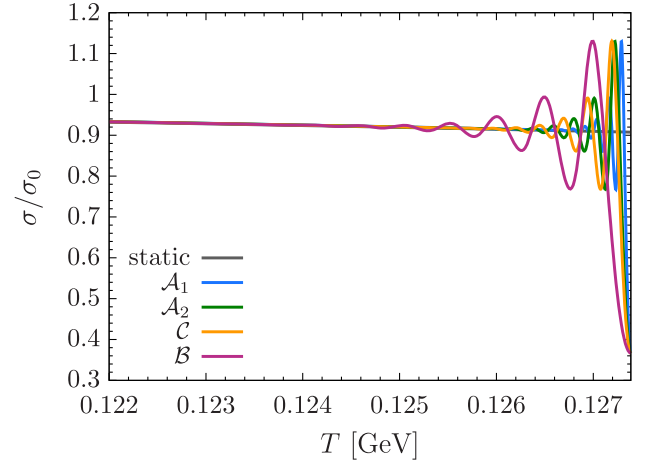


FIG. 12. Time-dependent chiral condensate as a function of $T(v)$ in the hydrodynamic model described by Eqs. (14)–(19) with $m_q = 3.22 \text{ MeV}$. The initial condition $X_q(u, v_0)$ is given by Eq. (6) with $\sigma = 0.02 \text{ GeV}^{-3}$ and is set at $z_h(v_0) = 2.5 \text{ GeV}^{-1}$, corresponding to $T(v_0) = 127.3 \text{ MeV}$. Time increases from right to left. The static result is shown by the gray curve.

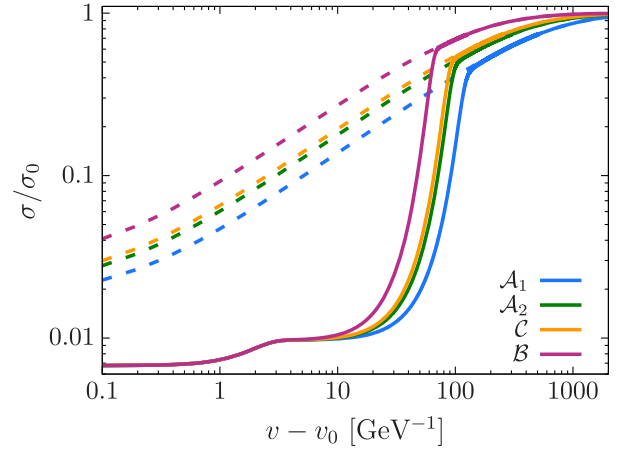


FIG. 13. Time-dependent chiral condensate as a function of $v - v_0$ in the hydrodynamic model described by Eqs. (14)–(19) with $m_q = 0$. The initial condition $X_q(u, v_0) = 10^{-4}u^3$ is set close to T_c at $z_h(v_0) = 1.949 \text{ GeV}^{-1}$. The static results are shown by the dashed curves.

smallest Λ , has the shortest prethermalization stage, in agreement with the results found in the previous section.

V. CONCLUSIONS

We have studied the chiral condensate at finite temperature in AdS/QCD in a time-dependent background, with conformal invariance broken by a static dilaton, as in the soft-wall model. A quartic potential term and the anomalous dimension of the scalar field X_q allow us to study chiral symmetry breaking for massless quarks. We have considered two kinds of initial conditions for the evolution

equation; the equilibrium on shell solution of X_q and a perturbed profile, where the initial chiral condensate is abruptly shifted from its equilibrium value.

A toy model has been analyzed first, assuming a general power-law time dependence for the position of the black hole horizon $z_h(v)$, describing an increasing or decreasing temperature. The parametric curve representing the evolution of the chiral condensate as a function of temperature remains smooth for both $m_q = 0$ and $m_q \neq 0$. Its deviation from the static case increases as the rate of temperature change over time becomes more pronounced. In the chiral limit, if the chiral condensate initially vanishes, the transition to $\sigma \neq 0$ does not occur. For $m_q \neq 0$, if z_h varies slowly with time, X_q evolves through quasistatic transformations (i.e., adiabatically), so σ gets almost the same values as in the static case, and this is roughly independent of the initial condition, since thermalization is very quick. In the chiral limit, the dependence on the initial condition, for a given $z_h(v)$, is subtler. If the system is perturbed at $T > T_c$ with a small initial quench, where T_c is the critical temperature characterizing the second-order transition in the static case, the system soon thermalizes to $\sigma = 0$, and then it needs some time to reach the static behavior at low temperatures, in which $\sigma \neq 0$. The smaller the initial σ , the higher the thermalization time for a given $z_h(v)$. If instead the system is perturbed at $T < T_c$, thermalization time decreases when the initial condition is closer to the on shell one. We have also observed that at low temperatures the chiral condensate oscillates around the equilibrium value when the solution for X_q approaches the static one with an energy excess.

Then, we have considered a specific metric describing a cooling system in which the energy-momentum tensor is the one found in viscous hydrodynamics. Also in this case, $z_h(v)$ follows a power-law behavior, but with a noninteger exponent and with corrections containing viscous effects. In the hydrodynamic setup the evolution of the medium is slow enough, and the chiral condensate computed in a dynamical model, solving partial differential equations, is almost equal to the one computed assuming the system is at equilibrium. Oscillations are observed if the initial condition for the scalar field is different from the equilibrium value at $T < T_c$, which however soon disappear, and then the system behaves as at equilibrium.

A prethermalization stage, i.e., a transient quasistationary state different from the true thermal equilibrium, is found in the chiral limit by choosing an initial temperature close to the critical temperature determined in the static case.

It would be interesting to study a system with dynamical dilaton, coupled to the metric and obtained, together with the metric functions, by solving time-dependent Einstein equations, and check if the general behavior described here is confirmed. Indeed, using dynamical and time-dependent dilaton and metric would produce a more detailed and comprehensive description of the gluodynamics contributing to the time evolution of the system. Moreover, a further step forward would be the inclusion of the backreaction of the field X_0 in the Einstein equations to see how the geometry responds to the presence of the chiral condensate. Going further, it could be interesting to investigate how a spatial dependence of the field X_0 , which can be caused either by inhomogeneous initial conditions or by requiring that the metric functions depend on y ,³ as done e.g., in [46] to study heavy quark energy loss, can affect the dynamics of the chiral condensate. We leave these refinements for future work.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [47].

³In this case the line element contains an additional term $2F(t, y, z)dtdy$.

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