Quantum backreaction and stability of topological wormholes

Haris Mehulić and Tomislav Prokopec
ITP, EMMEPH and Spinoza Institute, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands

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We investigate the quantum stability of a timelike topological wormhole with a simple geometry $M_2 \times S^2$, supported classically by anisotropic fluid. We compute the one-loop quantum backreaction generated by the vacuum fluctuations of a minimally coupled, massive scalar field propagating on the wormhole background. Using dimensional regularization we renormalize the one-loop energy-momentum tensor and identify the necessary gravitational counterterms. We then solve the semiclassical Einstein equations to linear order in $\hbar$ for both time-dependent and static metric *Ansätze*. Depending on the choice of finite counterterms, the quantum effects can induce either negative or positive angular pressure, which will tend to destabilize or stablize the wormhole, respectively. We also show that a classically traversable wormhole will remain traversable when the quantum backreaction is taken into account. It would be of interest to investigate whether these conclusions remain true when the equations of semiclassical gravity are self-consistently solved.

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I. INTRODUCTION

Wormholes [1–3] are among the most intriguing solutions of general relativity, providing hypothetical shortcuts between distant regions of spacetime. Classical analyses [4] have long shown that traversable wormholes typically require stress-energy tensors violating null energy condition, motivating the study of exotic matter sources, modified gravity theories, and semiclassical effects as possible mechanisms for realizing such geometries. Nevertheless, it is of interest to explore configurations that evade such requirements at the classical level.

A particularly simple example is a wormhole with spacetime geometry $M_2 \times S^2$, where M_2 denotes two-dimensional Minkowski space and the curvature is entirely localized on the spherical section S^2. The spacetime is otherwise flat, and the Einstein equations admit a solution supported by a homogeneous anisotropic stress-energy tensor, effectively corresponding to a two-dimensional cosmological constant along M_2. Related constructions have appeared in scalar-tensor theories, where spherically symmetric solutions supported by scalar stresses exhibit nontrivial global structure and wormholelike properties without requiring localized material sources [5–8].

Classical consistency, however, does not guarantee quantum stability. Even in classically well-behaved spacetimes, quantum vacuum fluctuations of matter fields generically produce nontrivial backreaction effects [9,10]. In semiclassical gravity [11] these effects are encoded in the renormalized expectation value of the energy-momentum tensor, which acts as a source in Einstein's equations. Assessing whether such quantum corrections preserve or destabilize a given classical geometry is therefore essential, particularly for wormholes whose viability depends sensitively on their stress-energy content.

Early investigations of this question demonstrated that Casimir-type vacuum energies can influence wormhole geometries. In a seminal work, Khabibullin, Khusnutdinov, and Sushkov [9,10] studied the Casimir effect in wormhole spacetimes and showed that vacuum polarization can generate negative energy densities, although typically at microscopic scales and often requiring additional structure to sustain traversability. Related studies of Casimir energies in curved and compact backgrounds, including analyses of the thermal Casimir effect in the Einstein universe with spherical boundaries, further illustrate how global geometry and boundary conditions shape quantum stresses when the background spacetime is treated as fixed [12–15]. Casimir effects have been revisited in the context of semiclassical wormhole constructions, where vacuum energy is combined with additional classical or semiclassical sources to obtain self-consistent solutions of the semiclassical Einstein equations [16] and to explore conditions for sustained traversability [17,18].

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In this work, we address the closely related question of whether quantum vacuum fluctuations stabilize or destabilize a classically admissible wormhole geometry. We also study how traversability is affected by the inclusion of the quantum backreaction. For analytical tractability, we consider a wormhole described by the product spacetime $M_2 \times S^2$, with the two-sphere S^2 of constant radius a. At the classical level, the geometry is supported by a homogeneous anisotropic stress-energy tensor and by localized negative energy shells at both mouths rather than by localized matter at a throat. The wormhole length L_0 is assumed to be much larger than its radius, allowing for a simplified treatment. The classical stress-energy tensor exhibits a thin shell at the interface between the wormhole region and the surrounding Minkowski space, which would be smoothed out in more realistic models.

We quantize a minimally coupled massive scalar field on this background and compute the renormalized one-loop expectation value of its energy-momentum tensor using dimensional regularization and appropriate renormalization conditions. We choose the finite cosmological constant counterterm such that the observed the cosmological constant vanishes both in the Minkowski spaces outside the wormhole and inside the wormhole. The finite Newton constant counterterm is chosen such that the observed Newton constant has identical value inside and outside the wormhole.

Treating gravity semiclassically, we then study the backreaction of the one-loop quantum stress-energy on the classical geometry. We find that, depending on the choice of finite higher derivative counterterms, quantum effects can induce either a negative or a positive angular pressure. When the angular pressure is negative (positive), the quantum backreaction will tend to destablize (stabilize) the wormhole. In both cases a traversable wormholes will remain traversable.

II. CLASSICAL WORMHOLE

In this work we consider a topological wormhole, whose geometry is given by $\mathbb{R} \times \mathbb{R} \times S^2$,[1] and four-dimensional metric of the wormhole is

$$ds^2 = -dt^2 + dz^2 + a^2 d\Omega_2^2 \qquad (d\Omega_2^2 = d\vartheta^2 + \sin^2(\vartheta) d\varphi^2), \tag{1}$$

where a is the wormhole radius, $\vartheta \in [0, \pi)$ and $\varphi \in [0, 2\pi)$ are the usual spherical coordinates on S^2, $z \in (-\infty, \infty)$ is the longitudinal coordinate along the wormhole,and $t \in (-\infty, \infty)$ is time. In this paper we use units in which the speed of light $c = 1$. The wormhole in Eq. (1) is illustrated in Fig. 1.

[1]Note that the wormhole in Eq. (1) has a flat five-dimensional Minkowski space, $M_5 = M_2 \times \mathbb{R}^3$, as the embedding space.

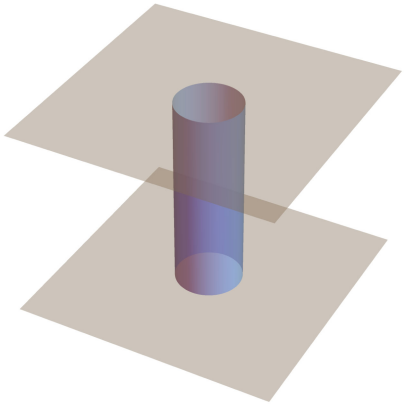


FIG. 1. The wormhole geometry described by the metric in Eq. (1) shown as $\mathbb{R} \times S^1$ for illustrative purpose. The cylinder length is L_0 and its radius is a. The points of contact between the flat Minkowski spaces and the wormhole is the boundary of the cylinder, shown as the two circles of radius a at $z = \pm L_0/2$.

The spherical section of the wormhole is curved, so nonvanishing Chrisoffel symbols are

$$\Gamma^{\vartheta}_{\varphi\varphi} = -\sin(\vartheta)\cos(\vartheta), \qquad \Gamma^{\varphi}_{\vartheta\varphi} = \Gamma^{\varphi}_{\varphi\vartheta} = \frac{\cos(\vartheta)}{\sin(\vartheta)}. \tag{2}$$

The wormhole in Eq. (1) is attached to flat Minkowski spaces above and below as shown in Fig. 1. It is instructive to analyze how the attachment works in some detail. The corresponding flat metrics are

$$ds^2 = -dt^2 + dr_\pm^2 + r_\pm^2(d\vartheta^2 + \sin^2(\vartheta) d\varphi^2) \quad (r_\pm \geq a), \tag{3}$$

where at the points of contact between the wormhole and the flat spaces $r_\pm = a \pm z - L_0/2$, where L_0 is the wormhole length, and $z = 0$ at the center of the wormhole. Nonvanishing Christoffel symbols for the metrics in Eqs. (1) and (3) are

$$\begin{aligned}
&\Gamma^{r_\pm}_{\vartheta\vartheta} = -\sum_{\pm} r_\pm \times \Theta(r_\pm - a), \\
&\Gamma^{r_\pm}_{\varphi\varphi} = -\sum_{\pm} r_\pm \sin^2(\vartheta) \times \Theta(r_\pm - a), \\
&\Gamma^{\vartheta}_{r_\pm\vartheta} = \sum_{\pm} \frac{1}{r_\pm} \times \Theta(r_\pm - a), \qquad \Gamma^{\vartheta}_{\varphi\varphi} = -\sin(\vartheta)\cos(\vartheta), \\
&\Gamma^{\varphi}_{\vartheta\varphi} = \Gamma^{\varphi}_{\varphi\vartheta} = \frac{\cos(\vartheta)}{\sin(\vartheta)},
\end{aligned} \tag{4}$$

where the Heaviside theta functions $\Theta(x)$ indicate that the Christoffel symbols in Eq. (4) exhibit a jump at $r_\pm = a$ ($z = \pm L_0/2$).

Nonvanishing components of the Riemann curvature tensor are then

$$R_{r_\pm\vartheta r_\pm\vartheta}=R_{\vartheta r_\pm\vartheta r_\pm}=-R_{\vartheta r_\pm r_\pm\vartheta}=-R_{r_\pm\vartheta\vartheta r_\pm}=-a\sum_\pm\delta(r_\pm-a), \tag{5}$$

$$\begin{aligned}R_{r_\pm\varphi r_\pm\varphi}&=R_{\varphi r_\pm\varphi r_\pm}=-R_{\varphi r_\pm r_\pm\varphi}\\&=-R_{r_\pm\varphi\varphi r_\pm}=-a\sin^2(\vartheta)\sum_\pm\delta(r_\pm-a),\end{aligned} \tag{6}$$

$$R_{\vartheta\varphi\vartheta\varphi}=R_{\varphi\vartheta\varphi\vartheta}=-R_{\vartheta\varphi\varphi\vartheta}=-R_{\varphi\vartheta\vartheta\varphi}=a^2\sin^2(\vartheta)\Theta\left(\frac{L_0}{2}-|z|\right). \tag{7}$$

These relations imply for nonvanishing components of the Ricci tensor and Ricci scalar

$$\begin{aligned}R_{r_\pm r_\pm}&=-\frac{2}{a}\sum_\pm\delta(r_\pm-a),\\R_{\vartheta\vartheta}&=-a\sum_\pm\delta(r_\pm-a)+\Theta\left(\frac{L_0}{2}-|z|\right),\\R_{\varphi\varphi}&=-a\sin^2(\vartheta)\sum_\pm\delta(r_\pm-a)+\sin^2(\vartheta)\Theta\left(\frac{L_0}{2}-|z|\right),\\R&=-\frac{4}{a}\sum_\pm\delta(r_\pm-a)+\frac{2}{a^2}\Theta\left(\frac{L_0}{2}-|z|\right),\end{aligned} \tag{8}$$

such that nonvanishing components of the Einstein curvature tensor, $G^{\mu\nu}=R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R$, are

$$\begin{aligned}G_{tt}&=-\frac{2}{a}\sum_\pm\delta(r_\pm-a)+\frac{1}{a^2}\Theta\left(\frac{L_0}{2}-|z|\right),\\G_{zz}&=-\frac{1}{a^2}\Theta\left(\frac{L_0}{2}-|z|\right),\qquad G_{\vartheta\vartheta}=a\sum_\pm\delta(r_\pm-a),\\G_{\varphi\varphi}&=a\sin^2(\vartheta)\sum_\pm\delta(r_\pm-a).\end{aligned} \tag{9}$$

The Einstein equation,

$$G_{\mu\nu}=8\pi GT_{\mu\nu}, \tag{10}$$

then implies that the classical energy-momentum tensor that supports our topological wormhole is given by

$$\begin{aligned}T_t^{\ t}&=\frac{1}{4\pi Ga}\sum_\pm\delta(r_\pm-a)-\frac{1}{8\pi Ga^2}\times\Theta\left(\frac{L_0}{2}-|z|\right),\\T_z^{\ z}&=-\frac{1}{8\pi Ga^2}\times\Theta\left(\frac{L_0}{2}-|z|\right)\\T_\vartheta^{\ \vartheta}&=T_\varphi^{\ \varphi}=\frac{1}{8\pi Ga}\sum_\pm\delta(r_\pm-a),\end{aligned} \tag{11}$$

which contains *regular* contributions inside the wormhole and *singular* contributions at the boundary between the wormhole and the flat spacetimes. Inside the wormhole the energy-momentum tensor in Eq. (11) corresponds to an anisotropic fluid with the classical energy-momentum tensor of an anisotropic fluid,

$$T_\mu^{\ \nu}=\mathrm{diag}(-\rho,p_z,p_\perp,p_\perp), \tag{12}$$

with $p_z=-\rho=-1/(8\pi Ga^2)$ and $p_\perp=0$, which is large when converted to SI units, $\rho\approx5\times10^{42}\,(1\ \mathrm{m}/a)^2\,[\mathrm{kg}/(\mathrm{ms}^2)]$, which corresponds to a mass density $\rho/c^2\approx5\times10^{25}\,(1\ \mathrm{m}/a)^2\,[\mathrm{kg}/\mathrm{m}^3]$, where a is here expressed in meters. This means that for a wormhole with $a=10$ km, mass density is about $5\times10^{17}\ \mathrm{kg}/\mathrm{m}^3$, which is comparable to that in a neutron star. This means that the amount of energy needed to build a wormhole of radius a (in meters) is $E\approx2\times10^{43}\,(a/1\ \mathrm{m})\,[\mathrm{J}]$, which is comparable with Earth's rest energy, $E_\oplus=m_\oplus c^2\simeq5.36\times10^{41}$ J. As expected, the energy needed to build a wormhole is very large [4], simply because it takes a large amount of energy to curve space.[2] Therefore, the wormhole is supported by a fluid with a negative pressure along the z direction, and a vanishing pressure in the angular directions, i.e., it behaves as a two-dimensional cosmological constant.

An interesting question is what kind of fluid can support such a wormhole. One possibility is a classical massless scalar field χ, whose energy density reads $T_{\mu\nu}=\partial_\mu\chi\partial_\nu\chi+g_{\mu\nu}[-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\chi\partial_\beta\chi]$. For a χ field with linear z dependence along the wormhole, $\chi=X_0z$ for $|z|\le L_0/2$ and a continuously matched constant outside the wormhole we have

$$\begin{aligned}T_{00}&=\frac{1}{2}(\partial_z\chi)^2=\frac{X_0^2}{2}\Theta\left(\frac{L_0}{2}-|z|\right)\equiv\rho_\chi,\\T_{zz}&=-\frac{1}{2}(\partial_z\chi)^2=-\frac{X_0^2}{2}\Theta\left(\frac{L_0}{2}-|z|\right)\equiv p_z^\chi=-\rho_\chi.\end{aligned} \tag{13}$$

Comparing this with Eq. (11) we see that

$$X_0=\pm\frac{1}{\sqrt{4\pi Ga}} \tag{14}$$

reproduces the nonsingular part of the energy-momentum in Eq. (11). The change of the scalar field across the wormhole is then $|\Delta\chi|=|X_0|L_0=L_0/(\sqrt{4\pi Ga})=(\sqrt{2}L_0/a)M_\mathrm{P}$, where $M_\mathrm{P}=1/\sqrt{8\pi G}\simeq2.4\times10^{18}$ GeV denotes the reduced Planck mass. This means that $|\Delta\chi|$ becomes super-Planckian when $L_0>a$, which

[2]The Universe is known to be spatially flat to a high precision [19]. If the observed universe were positively or negatively curved, one would need an equally large amount of energy to curve it. Even though this energy is large, one does not use it as an argument against a spatially curved universe.

some may consider problematic. Alternatively, one can use [20] the following classical electromagnetic potential $A_\mu = (0, E_z t, 0, 0)$ inside the wormhole ($|z| \leq L_0/2$), which produces constant electric field along the wormhole, $F_{01} = -F_{10} = E_z$, which yields $(T_E)^\nu_\mu = \mathrm{diag}(-E_z^2, -E_z^2, 0, 0) + \delta^\nu_\mu E_z^2/2$. It now follows that the choice $E_z = \pm 1/\sqrt{8\pi G}a$ together with the cosmological constant $\Lambda_E = 4\pi G E_z^2 = 1/(2a^2)$, reproduces the regular part of (11). Such electric fields can be supported by placing a positive electric charge on one side of the wormhole and the compensating negative charge on the other side. The singular shells at $r_\pm = a$ in Eq. (11) are harder to get from a classical field theory.

Note that, for null vectors $\ell_1^\mu = (1, 1, 0, 0)$ and $\ell_2^\mu = (1, 0, a^{-1}, 0)$ ($\ell_1^2 = 0$, $\ell_2^2 = 0$), we have

$$T_{\mu\nu}\ell_1^\mu \ell_1^\nu = T_{00} + T_{zz} = \rho + p_z = 0, \tag{15}$$

$$T_{\mu\nu}\ell_2^\mu \ell_2^\nu = T_{00} + \frac{1}{a^2} T_{\vartheta\vartheta} = \rho + p_\perp = \frac{1}{8\pi G a^2} > 0, \tag{16}$$

implying that null energy condition (NEC) is *marginally satisfied* inside the wormhole for the regular contribution in Eq. (12). On the other hand, the singular contribution in Eq. (11) corresponds to a surface energy density tensor $\sigma_{\mu\nu}$ of matter with

$$\sigma^\nu_\mu = \mathrm{diag}(-\sigma_t, \sigma_z, \sigma_\perp, \sigma_\perp), \qquad \sigma_t = -\frac{1}{4\pi G a},$$
$$\sigma_z = 0, \qquad \sigma_\perp = \frac{1}{8\pi G a}, \tag{17}$$

which is also needed to support the wormhole.[3] The negative value of σ_t makes this wormhole somewhat problematic. In realistic wormholes the singular shells of energy and pressure in Eq. (11) would be smoothed out. Here we keep the geometry singular in order to make our analytic calculations tractable. One way of incorporating the singular shells into an energy condition is by introducing an averaged energy-momentum tensor of the wormhole,

$$\langle T_{\mu\nu} \rangle_V \equiv \frac{1}{L_0} \int_{-\frac{L_0}{2}-\delta}^{\frac{L_0}{2}+\delta} \mathrm{d}z T_{\mu\nu}, \tag{18}$$

with $\delta > 0$ an infinitesimal quantity, resulting in

$$\langle T_t^{\ t} \rangle_V = -\langle \rho \rangle_V = -\frac{1}{8\pi G a^2}\left(1 - \frac{4a}{L_0}\right),$$
$$\langle T_z^{\ z} \rangle_V = \langle p_z \rangle_V = -\frac{1}{8\pi G a^2},$$
$$\langle T_\vartheta^{\ \vartheta} \rangle_V = \langle T_\varphi^{\ \varphi} \rangle_V = \langle p_\perp \rangle_V = \frac{1}{4\pi G a L_0}. \tag{19}$$

Using these results in Eqs. (15)–(16) one obtains for average null energy condition (ANEC),

$$\langle T_{\mu\nu} \rangle_V \ell_1^\mu \ell_1^\nu = \langle T_{00} \rangle_V + \langle T_{zz} \rangle_V$$
$$= \langle \rho \rangle_V + \langle p_z \rangle_V = -\frac{1}{2\pi G a L_0} < 0, \tag{20}$$

$$\langle T_{\mu\nu} \rangle_V \ell_2^\mu \ell_2^\nu = \langle T_{00} \rangle_V + \frac{1}{a^2}\langle T_{\vartheta\vartheta} \rangle_V$$
$$= \langle \rho \rangle_V + \langle p_\perp \rangle_V = \frac{1}{8\pi G a^2}\left(1 - \frac{2a}{L_0}\right), \tag{21}$$

implying that ANEC is *violated* inside the wormhole, which is in agreement with the literature [4,21–25]. This means that the singular shells in Eq. (11) contribute towards violation of ANEC. In order to make our calculations analytically tractable, in this work we assume that the wormhole is supported by the classical energy-momentum tensor given in Eq. (11). However, recent literature [26–32] has established that even standard model matter can be used to form traversable wormholes which may, but need not, be stable.

Curiously, the wormhole in Eq. (1) neither violates the weak energy condition (WEC) nor the average weak energy condition (AWEC),

$$T_{\mu\nu} t^\mu t^\nu = -T_0^{\ 0} = \rho = \frac{1}{8\pi G a^2} > 0, \tag{22}$$

$$\langle T_{\mu\nu} \rangle_V t^\mu t^\nu = -\langle T_0^{\ 0} \rangle_V = \langle \rho \rangle_V = \frac{1}{8\pi G a^2}\left(1 - \frac{4a}{L_0}\right) > 0,$$
$$(L_0 > 4a), \tag{23}$$

where $t^\mu = (1, 0, 0, 0)$ ($t^2 = -1$) is a timelike vector. To get the last inequality in Eq. (23) we assumed that the wormhole is sufficiently long, $L_0 > 4a$.

A note on topology. The wormhole spacetime in Eq. (1) considered in this work is topological in the sense that the second homotopy group of the wormhole manifold is nontrivial, $\pi_2(M_2 \times S^2) = \mathbb{Z}$, and in this sense it accords with the definition promoted in Ref. [22]. Note that higher homotopy groups are also nontrivial. For example, $\pi_3(M_2 \times S^2) = \mathbb{Z}$ and $\pi_4(M_2 \times S^2) = \pi_5(M_2 \times S^2) = \mathbb{Z}_2$. The fact that the wormhole in Eq. (1) is continuously connected to two flat Minkowski spaces as illustrated in Fig. 1 does not change the wormhole topology. On the other hand, when the two wormhole ends are attached to different spatial sections of

[3] Including the negative surface energy in Eq. (17) will violate NEC.

the same Minkowski space, then spacetime topology changes to $\mathbb{R}\times S^1\times S^2$. In this case, in addition to π_2, also fundamental homotopy group π_1 of the manifold becomes nontrivial, $\pi_1(\mathbb{R}\times S^1\times S^2)=\mathbb{Z}$. It is reasonable to assume that topology is preserved by the classical evolution. However, topology can be changed by the quantum evolution [4,31]. Since here we are primarily interested in the classical and quantum dynamics of the wormhole geometry, and topological differences between $M_2\times S^2$ and $\mathbb{R}\times S^1\times S^2$ do not affect the wormhole dynamics, the fate of topology will not be our primary concern in this work.

III. QUANTUM BACKREACTION

In this section we compute the one-loop quantum backreaction induced by a massive scalar field on the classical geometry in Eq. (1). We begin by introducing the theory, and then proceed toward computation of the quantum backreaction.

A. Bare theory

We consider a noninteracting, massive real scalar field, whose bare matter action is given by

$$S_{\mathsf{b}}[g_{\mu\nu},\phi]=\int \mathrm{d}^Dx\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi_{\mathsf{b}})(\partial_\nu\phi_{\mathsf{b}})-\frac{1}{2}m_{\mathsf{b}}^2\phi_{\mathsf{b}}^2\right], \tag{24}$$

where $g_{\mu\nu}$ and $g^{\mu\nu}$ is the metric tensor and its inverse, $g=\det[g_{\mu\nu}]$, ϕ_{b} is the bare field, and m_{b} is the bare mass. Upon defining

$$\phi_{\mathsf{b}}=\sqrt{Z_{\mathsf{b}}}\phi,\quad Z_{\mathsf{b}}=1+\delta Z,\quad Z_{\mathsf{b}}m_{\mathsf{b}}^2=m^2+\delta m^2, \tag{25}$$

where ϕ is classical (tree-level) scalar field, Z_{b} is field strength renormalization, m^2 is tree-level mass, and δm^2 and δZ are used to renormalize loop divergences of the theory. With these remarks in mind, the bare action in Eq. (24) splits into two parts, the classical action S_ϕ and the counterterm action S_ϕ^{ct},

$$\begin{aligned}S_{\mathsf{b}}[g_{\mu\nu},\phi]&=S_\phi[g_{\mu\nu},\phi]+S_\phi^{\mathsf{ct}}[g_{\mu\nu},\phi],\\ S_\phi[g_{\mu\nu},\phi]&=\int \mathrm{d}^Dx\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)-\frac{1}{2}m^2\phi^2\right],\end{aligned} \tag{26}$$

$$S_\phi^{\mathsf{ct}}[g_{\mu\nu},\phi]=\int \mathrm{d}^Dx\sqrt{-g}\left[-\frac{1}{2}\delta Z g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)-\frac{1}{2}\delta m^2\phi^2\right]. \tag{27}$$

In what follows we canonically quantize ϕ to study how its quantum fluctuations backreact onto the spacetime. We shall use the semiclassical gravity approximation, according to which gravity is classical, and matter is quantum. For the purposes of this work, we shall use Einstein's gravity, whose bare action is given by

$$S_{\mathsf{g}}[g_{\mu\nu}]=\frac{1}{\kappa_{\mathsf{b}}^2}\int \mathrm{d}^Dx\sqrt{-g}[R-2\Lambda_{\mathsf{b}}], \tag{28}$$

where $R=R(g_{\mu\nu})$ is the Ricci curvature scalar and $\kappa_{\mathsf{b}}^2=16\pi G_{\mathsf{b}}$ and Λ_{b} are the bare gravitational coupling and the cosmological constant, respectively. The bare gravitational couplings can be split into their classical parts and quantum parts (needed for renormalization),

$$\frac{1}{\kappa_{\mathsf{b}}^2}=\frac{1}{\kappa^2}+\delta\left(\frac{1}{\kappa^2}\right),\qquad \frac{\Lambda_{\mathsf{b}}}{\kappa_{\mathsf{b}}^2}=\frac{\Lambda}{\kappa^2}+\delta\left(\frac{\Lambda}{\kappa^2}\right), \tag{29}$$

where $\kappa^2=16\pi G$ and Λ are the tree-level gravitational coupling constant and cosmological constant (G is the tree-level Newton constant), respectively, and $\delta(\frac{1}{\kappa^2})$ and $\delta(\frac{\Lambda}{\kappa^2})$ are the corresponding Ricci scalar and cosmological constant counterterm couplings. When Eq. (29) is inserted into Eq. (28), the bare gravitational action $S_{\mathsf{g}}[g_{\mu\nu}]$ splits into the tree-level and counterterm action, $S_{\mathsf{g}}[g_{\mu\nu}]=S_{\mathsf{HE}}[g_{\mu\nu}]+S_{\mathsf{HE}}^{\mathsf{ct}}[g_{\mu\nu}]$, where

$$S_{\mathsf{HE}}[g_{\mu\nu}]=\frac{1}{\kappa^2}\int \mathrm{d}^Dx\sqrt{-g}[R-2\Lambda], \tag{30}$$

$$S_{\mathsf{HE}}^{\mathsf{ct}}[g_{\mu\nu}]=\int \mathrm{d}^Dx\sqrt{-g}\left[\delta\left(\frac{1}{\kappa^2}\right)R-2\delta\left(\frac{\Lambda}{\kappa^2}\right)\right]. \tag{31}$$

In the renormalization procedure we shall also use the following higher dimensional counterterm gravitational action,

$$S_{\mathsf{R}^2}[g_{\mu\nu}]=\delta\alpha_{\mathsf{R}^2}\int \mathrm{d}^Dx\sqrt{-g}R^2, \tag{32}$$

where R denotes the Ricci curvature scalar and $\delta\alpha_{\mathsf{R}^2}$ is the corresponding coupling constant. For simplicity, we shall assume that the observed cosmological constant vanishes in the Minkowski spaces that surround the wormhole and that the tree-level Newton constant $G=\kappa^2/(16\pi)$ corresponds to the observed one.

B. Canonical quantization

We are interested in canonical quantization in the background of a classical topological wormhole, whose D-dimensional geometry[4] is given by $\mathbb{R}_t\times\mathbb{R}^{D-3}\times S^2$, and whose classical metric is

[4]We consider a classical topological wormhole in general D spacetime dimensions, needed for dimensional regularization which we shall use to renormalize the divergences of the one-loop energy momentum tensor. After renormalization one can set $D=4$.

$$ds^2 = -dt^2 + d\mathbf{z}\cdot d\mathbf{z} + a^2 d\Omega_2^2, \tag{33}$$

where a is the radius of the wormhole, and $d\Omega_2^2 = d\vartheta^2 + \sin^2(\vartheta)d\varphi^2$ is the metric on the two-dimensional sphere.

The classical matter action (26) implies a canonical momentum, $\pi(x) = \delta S_\phi/\delta\partial_0\phi(x) = -\sqrt{-g}g^{0\nu}(\partial_\nu\phi)$, which in a flat space background simplifies to $\pi(x) = \dot{\phi}(x)$, where $\dot{X} \equiv \frac{d}{dt}X$. Canonical quantization is exacted by

$$[\hat{\phi}(t,\mathbf{x}), \hat{\pi}(t,\mathbf{x}')] = i\hbar\delta^{D-1}(\mathbf{x}-\mathbf{x}'). \tag{34}$$

Varying Eq. (26) with respect to $\phi(x)$ yields a Klein-Gordon equation for the quantum field $\hat{\phi}$,

$$(\Box_x - m^2)\hat{\phi}(x) = 0, \tag{35}$$

where $\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ is the d'Alembertian (wave) operator for curved spacetimes, which for the metric in Eq. (33) simplifies to

$$\Box_x = -\partial_t^2 + \Delta_{D-3} + \frac{1}{a^2}\Delta_{S^2}, \tag{36}$$

where here

$$\Delta_{S^2} = \frac{1}{\sin(\vartheta)}\frac{\partial}{\partial\vartheta}\sin(\vartheta)\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2(\vartheta)}\partial_\varphi^2 \tag{37}$$

is the wave operator on the sphere S^2 and Δ_{D-3} is Laplace operator on the $(D-3)$-dimensional flat Euclidean space, which in Cartesian coordinates reads

$$\Delta_{D-3} = \delta^{ab}_{D-3}\frac{\partial}{\partial x^a}\frac{\partial}{\partial x^b}, \tag{38}$$

where δ^{ab}_{D-3} is a flat metric on the $(D-3)$-dimensional subspace.

Taking account of the background symmetries of the metric (33), the field operator can be decomposed in terms of mode functions $u_{\ell m}(t,\mathbf{k})$ as

$$\hat{\phi}(t,\mathbf{z},\vartheta,\varphi) = \sum_{\ell m}\int\frac{d^{D-3}k}{(2\pi)^{D-3}}(e^{i\mathbf{k}\cdot\mathbf{z}}u_{\ell m}(t,\mathbf{k})\hat{a}_{\ell m}(\mathbf{k})Y_{\ell m}(\vartheta,\varphi) + e^{-i\mathbf{k}\cdot\mathbf{z}}u^*_{\ell m}(t,\mathbf{k})\hat{a}^\dagger_{\ell m}(\mathbf{k})Y^*_{\ell m}(\vartheta,\varphi)), \tag{39}$$

where $\hat{a}^\dagger_{\ell m}(\mathbf{k})$ and $\hat{a}_{\ell m}(\mathbf{k})$ and the creation and annihilation operators which satisfy

$$[\hat{a}_{\ell m}(\mathbf{k}), \hat{a}^\dagger_{\ell' m'}(\mathbf{k}')] = (2\pi)^{D-3}\delta^{D-3}(\mathbf{k}-\mathbf{k}')\delta_{\ell\ell'}\delta_{mm'},$$
$$[\hat{a}_{\ell m}(\mathbf{k}), \hat{a}_{\ell' m'}(\mathbf{k}')] = 0 = [\hat{a}^\dagger_{\ell m}(\mathbf{k}), \hat{a}^\dagger_{\ell' m'}(\mathbf{k}')], \tag{40}$$

such that $\hat{a}_{\ell m}(\mathbf{k})|\Omega\rangle = 0$ destroys the vacuum $|\Omega\rangle$ and $\hat{a}^\dagger_{\ell m}(\mathbf{k})$ creates a particle with quantum numbers $\{\mathbf{k}, \ell, m\}$, where $\ell \in \{0,1,2,\ldots\}$, $-\ell \le m \le \ell$ and $\mathbf{k} \in \mathbb{R}^{D-3}$. Furthermore, $Y_{\ell m}(\vartheta,\varphi)$ are the usual spherical harmonics, and $u_{\ell m}(t,\mathbf{k})$ and $u^*_{\ell m}(t,\mathbf{k})$ are the mode functions of the problem that obey

$$\left(\frac{d^2}{dt^2} + \omega_\ell^2\right)u_{\ell m}(t,\mathbf{k}) = 0, \quad \left(\omega_\ell^2 = \|\mathbf{k}\|^2 + m^2 + \frac{\ell(\ell+1)}{a^2}\right). \tag{41}$$

One can then show that canonically normalized mode functions are

$$u_{\ell m}(t,\mathbf{k}) = \frac{1}{a}\sqrt{\frac{\hbar}{2\omega_\ell}}e^{-i\omega_\ell t}, \qquad u^*_{\ell m}(t,\mathbf{k}) = \frac{1}{a}\sqrt{\frac{\hbar}{2\omega_\ell}}e^{i\omega_\ell t}. \tag{42}$$

It is useful to recall that spherical harmonics are eigenfunctions of the Laplace operator Δ_{S^2},

$$\Delta_{S^2}Y_{\ell m}(\vartheta,\varphi) = -\ell(\ell+1)Y_{\ell m}(\vartheta,\varphi), \tag{43}$$

are orthonormal polynomials on the sphere,

$$\int_0^\pi d\vartheta\sin(\vartheta)\int_0^{2\pi}d\varphi Y_{\ell m}(\vartheta,\varphi)Y^*_{\ell' m'}(\vartheta,\varphi) = \delta_{\ell\ell'}\delta_{mm'}, \tag{44}$$

and they satisfy a completeness relation,

$$\sum_{\ell,m}Y_{\ell m}(\vartheta,\varphi)Y^*_{\ell m}(\vartheta',\varphi') = \delta(\cos(\vartheta)-\cos(\vartheta'))\delta(\varphi-\varphi') = \frac{\delta(\vartheta-\vartheta')}{\sin(\vartheta)}\delta(\varphi-\varphi'), \tag{45}$$

the coincidence limit of which is

$$\sum_m Y_{\ell m}(\vartheta,\varphi)Y^*_{\ell m}(\vartheta,\varphi) = \frac{2\ell+1}{4\pi} \Rightarrow \sum_{\ell,m}Y_{\ell m}(\vartheta,\varphi)Y^*_{\ell m}(\vartheta,\varphi) = \sum_\ell\frac{2\ell+1}{4\pi} = \frac{2\zeta(-1)+\zeta(0)+1}{4\pi}, \tag{46}$$

such that the sum over ℓ diverges. The last equality expresses the divergent sums in terms of the Riemann zeta functions, defined as $\zeta(z) = \sum_{n=1}^\infty\frac{1}{n^z}$.

C. One-loop energy-momentum tensor

The classical energy-momentum tensor associated with the scalar field ϕ is defined as

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g_{\mu\nu}(x)} = (\nabla^\mu\phi)(\nabla^\nu\phi) + g^{\mu\nu}\left(-\frac{1}{2}g^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - \frac{1}{2}m^2\phi^2\right), \tag{47}$$

where S_ϕ is the classical action in Eq. (26), $\nabla^\mu\phi = g^{\mu\nu}\partial_\nu\phi$, and ϕ satisfies the classical equation of motion [cf. Eq. (35)],

$$(\Box_x - m^2)\phi(x) = 0, \tag{48}$$

where $\phi(x) = \langle\hat{\phi}(x)\rangle$, $\langle\hat{\phi}(x)\rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{\phi}(x)]$ and $\hat{\rho}(t)$ is the density operator. For the purpose of the paper, the density operator corresponds to the pure vacuum state $|\Omega\rangle$, and therefore it can be written as

$$\hat{\rho}(t) = |\Omega\rangle\langle\Omega|. \tag{49}$$

In this paper we are not interested in studying classical gravitational backreaction generated by the energy-momentum tensor in Eq. (47), as that depends on the classical field $\phi(x)$, which satisfies Eq. (48), but otherwise it is not constrained. Instead, we consider here the quantum backreaction induced by the one-loop contribution to the energy-momentum tensor, which is defined by

$$\langle\Delta\hat{T}^{\mu\nu}(x)\rangle = \langle T^*[\nabla^\mu\hat{\phi}(x)\nabla^\nu\hat{\phi}(x)]\rangle + g_{\mu\nu}\left(-\frac{1}{2}g^{\alpha\beta}\langle T^*[\partial_\alpha\hat{\phi}(x)\partial_\beta\hat{\phi}(x)]\rangle - \frac{1}{2}m^2\langle T^*[\hat{\phi}^2(x)]\rangle\right) \tag{50}$$

$$= [\nabla^\mu\nabla'^\nu i\Delta_\phi(x;x')]_{x'\to x} + g^{\mu\nu}\left(-\frac{1}{2}g^{\alpha\beta}[\partial_\alpha\partial'_\beta i\Delta_\phi(x;x')]_{x'\to x} - \frac{1}{2}m^2 i\Delta_\phi(x;x)\right), \tag{51}$$

where $i\Delta_\phi(x;x')$ is the Feynman (time-ordered) propagator, defined by

$$i\Delta_\phi(x;x') = \Theta(t-t')\langle\hat{\phi}(x)\hat{\phi}(x')\rangle + \Theta(t'-t)\langle\hat{\phi}(x')\hat{\phi}(x)\rangle, \tag{52}$$

which obeys

$$\sqrt{-g}(\Box - m^2)i\Delta_\phi(x;x') = i\hbar\delta^D(x-x'). \tag{53}$$

To get the second equality (51) we used a T^* ordering, according to which the Heaviside Θ functions in the time-ordering operator T^* commute with the vertex derivatives. One can show that the difference between the more-usual T-ordering prescription and the T^* ordering used here are ultralocal terms containing Dirac δ functions evaluated at spacetime coincidence and/or derivatives acting on δ functions evaluated at spacetime coincidence, which all vanish in dimensional regularization used here. Our considerations in the Appendix imply that, for the purposes of this work, the difference between the T^* and T ordering is immaterial.

Here we are interested in studying the backreaction generated by the one-loop energy-momentum tensor of the quantum scalar field $\hat{\phi}(x)$, which are universal in the sense that they are completely specified once the background metric is fixed.[5] The self-consistent semiclassical metric can then be obtained from the semiclassical Einstein equation,

$$G^{\mu\nu} = \frac{\kappa^2}{2}(T^{\mu\nu} + \langle\Delta\hat{T}^{\mu\nu}(x)\rangle + T^{\mu\nu}_{\mathrm{ct}}(x)), \tag{54}$$

where $G^{\mu\nu}$ denotes the Einstein curvature tensor, $T^{\mu\nu}$ is the classical energy-momentum tensor in Eq. (11) needed to support the wormhole, its quantum contribution is given in Eq. (51), and the corresponding counterterm contribution is defined as

$$T^{\mu\nu}_{\mathrm{ct}}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{ct}}}{\delta g_{\mu\nu}(x)}, \tag{55}$$

where S_{ct} is a sum of the counterterm actions in Eqs. (27), (31), and (32). In this work we assume that classical scalar field $\phi(x)$ vanishes, and therefore we shall not need the matter counterterm action Eq. (27) to renormalize the one-loop energy-momentum tensor in Eq. (51).

Solving self-consistently the system of equations of semiclassical gravity in Eqs. (53) and (54) is a hard problem and it cannot be solved in general. Instead, we shall resort to perturbative methods and construct a perturbative solution, according to which the right-hand side of Eq. (54) can be viewed as the source for the left-hand side. In this scheme, the metric, the Einstein tensor, the propagator, and the energy-momentum tensors can all be expanded in powers of $\hbar$ as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu} + \mathcal{O}(\hbar^3), \tag{56}$$

$$G^{\mu\nu} = G^{\mu\nu(0)} + G^{\mu\nu(1)} + G^{\mu\nu(2)} + \mathcal{O}(\hbar^3), \tag{57}$$

[5]This is only true if the quantum fluctuations are evaluated in a vacuum which respects the symmetries of the background metric, which is what we assume here. This is however not always possible. For example, in de Sitter space quantum fluctuations of a massless scalar field violate de Sitter symmetry even at the tree level [33,34], which is then manifested in secular contributions to the propagator which grow in time. Quite unexpectedly, an analogous violation of de Sitter symmetry occurs in the gauge sector of the photon field in de Sitter space in general covariant gauges [35,36].

$$i\Delta_\phi(x;x') = i\Delta_\phi{}^{(1)}(x;x') + i\Delta_\phi{}^{(2)}(x;x') + \mathcal{O}(\hbar^3), \quad (58)$$

$$T^{\mu\nu}_{\rm ct} = T^{\mu\nu\,(1)}_{\rm ct} + T^{\mu\nu\,(2)}_{\rm ct} + \mathcal{O}(\hbar^3), \quad (59)$$

$$\langle \Delta\hat{T}^{\mu\nu}\rangle = \langle \Delta\hat{T}^{\mu\nu}\rangle^{(1)} + \langle \Delta\hat{T}^{\mu\nu}\rangle^{(2)} + \mathcal{O}(\hbar^3), \quad (60)$$

where $g^{(0)}_{\mu\nu}$ is the classical wormhole metric in Eq. (33). Since the source $\langle \Delta\hat{T}^{\mu\nu}(x)\rangle$ in Eq. (54) is the one-loop energy-momentum tensor, which is $\mathcal{O}(\hbar)$, the expansion in Eq. (56) is indeed an expansion in powers of $\hbar$. The equations of motion (53)–(54) can be expanded as

$$G^{\mu\nu\,(0)} = \frac{\kappa^2}{2}T^{\mu\nu}, \quad G^{\mu\nu\,(1)} = \frac{\kappa^2}{2}(\langle \Delta\hat{T}^{\mu\nu}\rangle^{(1)} + T^{\mu\nu\,(1)}_{\rm ct}), \quad (61)$$

and [cf. Eq. (53)],

$$\sqrt{-g_{(0)}}(\Box_{(0)} - m^2)i\Delta_\phi{}^{(1)}(x;x') = i\hbar\delta^D(x-x'),$$
$$\Box_{(0)} = \frac{1}{\sqrt{-g_{(0)}}}\partial_\mu\sqrt{-g_{(0)}}g^{\mu\nu}_{(0)}\partial_\nu, \quad (62)$$

and to leading order in $\hbar$ Eq. (51) reads

$$\langle \Delta\hat{T}^{\mu\nu}(x)\rangle^{(1)} = [\partial^\mu_{(0)}\partial'^\nu_{(0)}i\Delta^{(1)}_\phi(x;x')]_{x'\to x} + g^{\mu\nu}_{(0)}\left(-\frac{1}{2}g^{\alpha\beta}_{(0)}[\partial_\alpha\partial'_\beta i\Delta^{(1)}_\phi(x;x')]_{x'\to x} - \frac{1}{2}m^2 i\Delta^{(1)}_\phi(x;x)\right). \quad (63)$$

The interested reader can easily write down the relevant equations at $\mathcal{O}(\hbar^2)$. In this work, however, we limit ourselves to studying the leading order solution $\mathcal{O}(\hbar)$ of the quantum backreaction problem, for which Eqs. (61)–(63) suffice.

In the Appendix we compute the one-loop energy-momentum tensor in Eq. (51), and determine the counterterm action $S_{\rm ct}$ needed to renormalize it. It turns out that the Hilbert-Einstein counterterm action in Eq. (31) is sufficient to remove the divergences in the one-loop energy-momentum tensor and to renormalize it. However, our analysis shows that one can also use a finite higher derivative counterterm action in Eq. (32) to further simplify the quantum backreaction.

With this in mind, we can write down nonvanishing components of Eq. (61) as [see Eqs. (9), (11) and (A92)–(A93)],

$$G^{t\,(1)}_t = G^{z\,(1)}_z = \frac{\kappa^2}{2}[T_t{}^t + \langle \Delta\hat{T}_t{}^t\rangle^{(1)}_{\rm ren}] = -\frac{1}{a^2} - \frac{\hbar\kappa^2}{128\pi^2}\left\{\left(m^4 - \frac{2m^2}{3a^2} + \frac{2}{15a^4}\right)\left[\ln\left(\frac{m^2-\frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right] - \frac{1}{12a^2}\left(m^2 - \frac{31}{40a^2}\right) - \frac{31}{4032a^6}\frac{1}{m^2-\frac{1}{4a^2}} + \mathcal{O}(a^{-8}, {\rm e}^{-2\pi am})\right\} + \frac{2\kappa^2}{a^4}\alpha_{\rm R^2} + \frac{\kappa^2}{\bar{\kappa}^2}\frac{1}{a^2} - \Lambda, \quad (64)$$

$$G^{\vartheta\,(1)}_\vartheta = G^{\varphi\,(1)}_\varphi = \frac{\kappa^2}{2}\langle \Delta\hat{T}^\vartheta_\vartheta\rangle^{(1)}_{\rm ren} = -\frac{\hbar\kappa^2}{128\pi^2}\left\{\left(m^4 - \frac{2}{15a^4}\right)\left[\ln\left(\frac{m^2-\frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right] + \frac{1}{4a^2}\left(m^2 - \frac{27}{40a^2}\right) + \frac{457}{20160a^6}\frac{1}{m^2-\frac{1}{4a^2}} + \mathcal{O}(a^{-8}, {\rm e}^{-2\pi am})\right\} - \frac{2\kappa^2}{a^4}\alpha_{\rm R^2} - \Lambda, \quad (65)$$

where $\bar{\kappa}^{-2}$, Λ and $\alpha_{\rm R^2}$ are finite counterterm coefficients, and we dropped the $\mathcal{O}(1/a^8)$ terms. The μ dependence in Eqs. (64)–(65) was introduced by the counterterms needed to renormalize the theory. Indeed, making a different choice of the scale μ, the quantum backreaction changes as

$$\frac{\kappa^2}{2}\langle \Delta\hat{T}^t_t\rangle^{(1)}_{\rm ren}(\mu') = \frac{\kappa^2}{2}\langle \Delta\hat{T}^t_t\rangle^{(1)}_{\rm ren}(\mu) + \frac{\hbar\kappa^2}{64\pi^2}\left(m^4 - \frac{2m^2}{3a^2} + \frac{2}{15a^4}\right)\ln\left(\frac{\mu'}{\mu}\right), \quad (66)$$

$$\frac{\kappa^2}{2}\langle \Delta\hat{T}^\vartheta_\vartheta\rangle^{(1)}_{\rm ren}(\mu') = \frac{\kappa^2}{2}\langle \Delta\hat{T}^\vartheta_\vartheta\rangle^{(1)}_{\rm ren}(\mu) + \frac{\hbar\kappa^2}{64\pi^2}\left(m^4 - \frac{2}{15a^4}\right)\ln\left(\frac{\mu'}{\mu}\right). \quad (67)$$

This scale dependence of the quantum backreaction is also present in Minkowski space, when the cosmological constant backreaction is considered [37].

A natural choice of the finite cosmological constant in Eqs. (64)–(65) is that for which the cosmological constant in Minkowski space vanishes,

$$\Lambda = -\frac{\hbar\kappa^2 m^4}{128\pi^2}\left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right], \tag{68}$$

with the price of introducing a fine-tuning. Equations (66)–(67) suggest that the fine-tuning needs to be repeated for every choice of scale μ. This is however an artifact of the perturbative approximation [38]. Indeed, when the geometric cosmological constant is tuned to cancel the energy associated with the minimum of the RG-improved effective potential of the theory, then fine-tuning ought to be done once, thus resolving the problem of dependence of the quantum backreaction on the choice of the renormalization scale μ, see Ref. [38] for more details.

The considerations of Ref. [38] can be extended to the problem at hand as follows. The μ-dependent fine-tuning of Λ in Eq. (68), which removes the perturbative one-loop quantum backreaction at some scale μ, can be generalized to the RG-improved effective action of the theory such that the total cosmological constant remains zero in the surrounding Minkowski space. However, even when this is done, the wormhole will harbor a nonvanishing quantum energy-momentum tensor,

$$G_t^{t(1)} = G_z^{z(1)} = \frac{\kappa^2}{2}[T_t{}^t + \langle\Delta\hat{T}_t{}^t\rangle_{\text{ren}}^{(1)}] = \frac{1}{a^2}\left(\frac{\kappa^2}{\bar{\kappa}^2} - 1\right) + \frac{\hbar\kappa^2 m^2}{192\pi^2 a^2}\left\{\left(1 - \frac{1}{5(am)^2}\right)\left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E\right] - \left(1 - \frac{4}{105(am)^4}\right) + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am})\right\} + \frac{2\kappa^2}{a^4}\alpha_{\mathsf{R}^2}, \tag{69}$$

$$G_\vartheta^{\vartheta(1)} = G_\varphi^{\varphi(1)} = \frac{\kappa^2}{2}\langle\Delta\hat{T}_\vartheta^\vartheta\rangle_{\text{ren}}^{(1)} = \frac{\hbar\kappa^2}{960\pi^2 a^4}\left\{\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E - \frac{8}{21(am)^2}\right) + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am})\right\} - \frac{2\kappa^2}{a^4}\alpha_{\mathsf{R}^2}. \tag{70}$$

There are two more counterterms in Eqs. (69) and (70) that remain unfixed, namely those of the Ricci scalar and Ricci scalar squared. One way of fixing the Ricci scalar counterterm is to fix the finite part of it to zero. In this case the term $\kappa^2/(a^2\bar{\kappa}^2)$ in Eq. (69) vanishes, leaving us with the classical contribution $-1/a^2$ plus the quantum contribution $\propto \hbar$. The more natural choice is to fix $\bar{\kappa}^2$ to remove as much of the quantum source in Eq. (69) as possible, but such that $\bar{\kappa}^2$ is independent on the wormhole size a,

$$\frac{1}{\bar{\kappa}^2} = -\frac{\hbar m^2}{192\pi^2}\left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E - 1\right]. \tag{71}$$

This choice represents a second tuning. Just like for the cosmological constant tuning in Eq. (68), the choice of the Newton constant counterterm in Eq. (71) guarantees continuity of the Newton constant at the boundary between the wormhole and the asymptotic Minkowski spaces shown in Fig. 1, and from that point of view is not a tuning, but a good way of fixing the counterterms. In this case the tree-level Newton constant in Eq. (30) corresponds to the observed one on large scales both in the Minkowski spaces and inside the wormhole. An analogous choice of the Newton constant counterterm was used in Ref. [39] to prove a *gravitational decoupling theorem* between the ultraviolet and infrared massive scalar fluctuations in Minkowski space, constituting the gravitational version of the Appelquist-Carazzone decoupling theorem, originally proved for gauge fields in Ref. [40].

Finally, one can also fix the finite Ricci scalar squared counterterm in Eqs. (69)–(70) to remove as much as possible of the quantum energy-momen-tum tensor in Eq. (70),

$$\alpha_{\mathsf{R}^2} = \frac{\hbar}{1920\pi^2}\left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E\right], \tag{72}$$

representing yet another fine-tuning. Upon inspecting the dependence on the radius a of the quantum backreaction in Eqs. (69)–(70) one sees that the choice in Eq. (72) corresponds to choosing a vanishing Ricci scalar squared coupling constant in the contribution to the effective action of the form given in Eq. (32). Adding a finite contribution in Eq. (72), $\alpha_{\mathsf{R}^2} \to [\hbar/(1920\pi^2)] \times [\ln(m^2/(4\pi\mu^2)) + \gamma_E] + \alpha$ would imply that the measured coupling constant in the higher derivative action in Eq. (32) is α both inside and outside the wormhole. Since observable modifications of gravity induced by higher derivative terms such as the one in Eq. (72) are tiny in the infrared, our constraints on higher derivative couplings are very weak, see e.g., Ref. [39].[6] Sensitivity to higher derivative terms inside the

[6]An exception is Starobinsky's inflation [41], in which the value of the R^2 coupling is constrained by the normalization of the scalar cosmological perturbations to be $\alpha \sim 10^9$. This means that, by measuring the quantum beckreaction inside a wormhole, one could in principle measure α, which would in turn provide a direct test of Starobinsky inflation.

wormhole is much higher, however, and could be, in principle, used to measure the higher derivative coupling α. With these choices of the counterterms the quantum sources in Eq. (69) simplify to

$$G_t^{t(1)} = G_z^{z(1)} = \frac{\kappa^2}{2}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\rm ren}^{(1)}] = -\frac{1}{a^2} + \frac{\hbar\kappa^2}{5040\pi^2 a^6 m^2} + \mathcal{O}(a^{-8}, {\rm e}^{-2\pi am}), \quad (73)$$

$$G_\vartheta^{\vartheta(1)} = G_\varphi^{\varphi(1)} = \frac{\kappa^2}{2}\langle \Delta\hat{T}_\vartheta^\vartheta \rangle_{\rm ren}^{(1)} = -\frac{\hbar\kappa^2}{2520\pi^2 a^6 m^2} + \mathcal{O}(a^{-8}, {\rm e}^{-2\pi am}). \quad (74)$$

Note that the remaining contributions are tiny in the limit of large wormholes ($am \gg 1$). Note also that our choice of the finite counterterms removed all of the logarithmic contributions. This is so because the ratio of the coefficients in front of the logarithms and the divergent terms $\propto 1/(D-4)$ are fixed by the requirement that the primitive energy-momentum tensor (before renormalization) is μ independent. In what follows we shall investigate perturbative quantum backreaction induced by the general quantum one-loop energy-momentum tensor in Eqs. (69) and (70). We shall also investigate the special case when the additional fine-tuning in Eqs. (71)–(72) is exacted, such that the quantum energy-momentum tensor reduces to Eqs. (73) and (74).

IV. SOLVING SEMICLASSICAL EINSTEIN'S EQUATIONS

Here we investigate stability of the topological wormhole by solving the corresponding semiclassical Einstein's equation (61) to the leading order in the Planck constant $\hbar$. Based on this analysis we arrive at a quantitative stability criterion for the semiclassical wormhole. We also study how traversability of the wormhole is affected by the quantum backreaction.

A. Time dependent *Ansatz*

In what follows we consider the wormhole spacetime in the presence of the one-loop energy-momentum tensor in Eqs. (69)–(70) generated by the quantum fluctuations of a massive scalar inside the wormhole. We assume that the cosmological constant counterterm is chosen as in Eq. (68), which subtracts the vacuum energy of the Minkowski vacuum fluctuations. This choice ensures that the vacuum fluctuations in the Minkowski space surrounding the wormhole do not contribute to the quantum energy-momentum tensor, which is supported by observations. For our considerations the presumed cosmological constant driving the observed accelerated expansion of the Universe can be neglected as long as $aH_0 \ll 1$ (here $H_0 \simeq 78$ km/s/Mpc is the present expansion rate of the Universe), which is amply satisfied. If, in addition, the Newton constant and the Ricci scalar squared counterterms are chosen as in Eqs. (71) and (72), the source in the semiclassical Einstein equation $\frac{\kappa^2}{2}[T_\mu^\nu + \langle \Delta\hat{T}_\mu^\nu \rangle_{\rm ren}^{(1)}]$ simplifies to that in Eqs. (73)–(74). With these remarks in mind, we are now ready to consider the perturbative quantum backreaction induced by Eqs. (69)–(70) or by Eqs. (73)–(74).

The source in Eqs. (69)–(70) in the semiclassical Einstein's equations (61) suggests the following *time-dependent Ansatz* for the metric,

$$ds^2 = -dt^2 + a_z^2(t)dz^2 + a_\perp^2(t)d\Omega_2^2, \quad (75)$$

where $a_z^2(t)$ and $a_\perp^2(t)$ are the scale factors associated with the longitudinal (z) and angular directions of the wormhole, respectively. The corresponding nonvanishing Christoffel symbols implied by Eq. (75) are then

$$\Gamma_{\mu\nu}^t = \frac{1}{2}\partial_t g_{\mu\nu} \Rightarrow \Gamma_{zz}^t = a_z^2 H_z, \quad \Gamma_{\vartheta\vartheta}^t = a_\perp^2 H_\perp, \quad \Gamma_{\varphi\varphi}^t = a_\perp^2 H_\perp \sin^2(\vartheta), \quad \Gamma_{\mu\nu}^z = \frac{1}{2a_z^2}(\partial_\mu g_{\nu z} + \partial_\nu g_{\mu z}) \Rightarrow \Gamma_{tz}^z = \Gamma_{zt}^z = H_z,$$
$$\Gamma_{\mu\nu}^\vartheta = \frac{1}{2a_\perp^2}(2\partial_{(\mu} g_{\nu)\vartheta} - \partial_\vartheta g_{\mu\nu}) \Rightarrow \Gamma_{t\vartheta}^\vartheta = \Gamma_{\vartheta t}^\vartheta = H_\perp, \quad \Gamma_{\varphi\varphi}^\vartheta = -\sin(\vartheta)\cos(\vartheta), \quad \Gamma_{\mu\nu}^\varphi = \frac{1}{a_\perp^2 \sin^2(\vartheta)}\partial_{(\mu} g_{\nu)\varphi} \Rightarrow \Gamma_{t\varphi}^\varphi = \Gamma_{\varphi t}^\varphi = H_\perp,$$
$$\Gamma_{\vartheta\varphi}^\varphi = \Gamma_{\varphi\vartheta}^\varphi = \frac{\cos(\vartheta)}{\sin(\vartheta)}, \quad (76)$$

from which it follows that

$$\Gamma_{\mu\alpha}^\mu = \frac{1}{\sqrt{-g}}\partial_\alpha\sqrt{-g} \Rightarrow \Gamma_{\mu t}^\mu = H_z + 2H_\perp, \quad \Gamma_{\mu\vartheta}^\mu = \frac{\cos(\vartheta)}{\sin(\vartheta)}, \quad (77)$$

where $H_z = \dot{a}_z/a_z$ and $H_\perp = \dot{a}_\perp/a_\perp$. We can now calculate nonvanishing elements of the Ricci tensor and Ricci scalar. Notice first the Ricci tensor is diagonal, so its nonvanishing components are

$$R_t^{t(1)} = (\dot{H}_z + H_z^2) + 2(\dot{H}_\perp + H_\perp^2), \qquad R_z^{z(1)} = (\dot{H}_z + H_z^2) + 2H_z H_\perp, \qquad R_\vartheta^{\vartheta(1)} = R_\varphi^\varphi = H_z H_\perp + (\dot{H}_\perp + 2H_\perp^2) + \frac{1}{a_\perp^2},$$
$$R^{(1)} = g^{\mu\nu} R_{\mu\nu}^{(1)} = 2[(\dot{H}_z + H_z^2) + 2H_z H_\perp + (2\dot{H}_\perp + 3H_\perp^2)] + \frac{2}{a_\perp^2}, \tag{78}$$

such that the Einstein tensor and the corresponding semiclassical Einstein equations read

$$G_t^{t(1)} = -[2H_z H_\perp + H_\perp^2] - \frac{1}{a_\perp^2} = \frac{\kappa^2}{2}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\text{ren}}^{(1)}], \tag{79}$$

$$G_z^{z(1)} = -[2\dot{H}_\perp + 3H_\perp^2] - \frac{1}{a_\perp^2} = \frac{\kappa^2}{2}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\text{ren}}^{(1)}], \tag{80}$$

$$G_\vartheta^{\vartheta(1)} = G_\varphi^{\varphi(1)} = -[H_z H_\perp + (\dot{H}_z + H_z^2) + (\dot{H}_\perp + H_\perp^2)] = \frac{\kappa^2}{2}\langle \Delta\hat{T}_\vartheta^\vartheta \rangle_{\text{ren}}^{(1)}, \tag{81}$$

such that the $G_\varphi^{\varphi(1)}$ equation contains no new information. The structure of these equations suggests to subtract the second equation from the first to obtain

$$(\dot{H}_\perp + H_\perp^2) - H_z H_\perp = 0, \tag{82}$$

from where we conclude

$$H_z = \frac{\dot{H}_\perp}{H_\perp} + H_\perp. \tag{83}$$

This can be used in Eq. (79) to obtain

$$(2\dot{H}_\perp + 3H_\perp^2) + \frac{1}{a_\perp^2} + \frac{\kappa^2}{2}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\text{ren}}^{(1)}] = 0. \tag{84}$$

This is a closed equation for $a_\perp(t)$ and it can be solved as follows. First notice that, for the quantum source in Eqs. (79) and (69), there exists a time independent solution,

$$\begin{aligned}\frac{1}{a_\perp^2} &= \frac{1}{a^2}\left(1 - \frac{\kappa^2}{\bar{\kappa}^2}\right) - \frac{\hbar\kappa^2 m^2}{192\pi^2 a^2}\left\{\left(1 - \frac{1}{5(am)^2}\right) \times \left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E\right] - \left(1 - \frac{4}{105(am)^4}\right) + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am})\right\} - \frac{2\kappa^2}{a^4}\alpha_{\text{Riem}^2}, \\ &= \frac{1}{a^2} - \frac{\hbar\kappa^2}{5040\pi^2 a^6 m^2} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am}),\end{aligned} \tag{85}$$

where the second equality holds when both $\bar{\kappa}$ and α_{Riem^2} are fine-tuned to the values in Eqs. (71) and (72), see Eq. (73). A simple consideration shows that iterating Eq. (86) would result in a self-consistent solution for semiclassical gravity,

$$\begin{aligned}\frac{1}{a_\perp^2} &= \frac{1}{a^2}\left(1 - \frac{\kappa^2}{\bar{\kappa}^2}\right) - \frac{\hbar\kappa^2 m^2}{192\pi^2 a_\perp^2}\left\{\left(1 - \frac{1}{5(a_\perp m)^2}\right) \times \left[\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E\right] - \left(1 - \frac{4}{105(a_\perp m)^4}\right)\right\} \\ &\quad + \mathcal{O}(a_\perp^{-8}, \mathrm{e}^{-2\pi a_\perp m}) - 2\frac{\kappa^2}{a_\perp^4}\alpha_{\text{R}^2} \\ &= \frac{1}{a^2} - \frac{\hbar\kappa^2}{5040\pi^2 a_\perp^6 m^2} + \mathcal{O}\left(\frac{1}{a_\perp^8}, \mathrm{e}^{-2\pi a_\perp m}\right),\end{aligned} \tag{86}$$

which, when solved, gives $a_\perp = a_\perp(a)$. As before, the second equality holds for the fine-tunings in Eqs. (71) and (72). The conjecture in Eq. (86) ought to be checked by constructing a self-consistent solution of semiclassical gravity.

Notice that Eq. (84) allows for a time dependent solutions for $a_\perp(t)$. To better understand the form of these solutions, it is useful to introduce a time variable, $\sqrt{a_\perp(t(\tau))}\mathrm{d}\tau = \mathrm{d}t$, in terms of which Eq. (84) becomes

$$a_\perp''(\tau) + \frac{1}{2} + a_\perp^2\frac{\kappa^2}{4}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\text{ren}}^{(1)}] = 0, \tag{87}$$

which is solved by the Weierstrass elliptic function $\mathcal{P}$ (related to the Jacobi elliptic function),

$$\begin{aligned}a_\perp(\tau) &= \frac{1}{\alpha^{1/3}} \times \mathcal{P}\left(\alpha^{1/3}\tau, \left\{\frac{1}{\alpha^{1/3}}, C\right\}\right), \\ \alpha &= -\frac{3\kappa^2}{2}[T_t^t + \langle \Delta\hat{T}_t^t \rangle_{\text{ren}}^{(1)}],\end{aligned} \tag{88}$$

where C is an integration constant (the other integration constant is absorbed in the definition of time).

Next, Eq. (81) can be recast as

$$(\dot{H}_z + H_z^2) + 2(\dot{H}_\perp + H_\perp^2) + \frac{\kappa^2}{2}\langle \Delta\hat{T}_\vartheta^\vartheta \rangle_{\text{ren}}^{(1)} = 0, \tag{89}$$

which for $a_\perp = \text{const.}$ simplifies to

$$(\dot{H}_z + H_z^2) = \frac{\ddot{a}_z}{a_z} = -\frac{\kappa^2}{2}\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}}, \tag{90}$$

which is solved by

$$a_z(t) = a_{z(0)}\cosh(H_z t),$$
$$H_z^2 = -\frac{\kappa^2}{2}\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}} \simeq \frac{2\hbar G}{315\pi a^6 m^2} > 0, \tag{91}$$

where $a_{z(0)} = a_z(0)$, where the value chosen here is that of the fine-tuned solution in Eq. (74).[7] The solution in Eq. (91) is chosen such that it is symmetric in $t \to -t$. At late times, the exponentially decaying solution in Eq. (91) can be neglected, and the scale factor can be approximated by $a_z(t) = [a_{z(0)}/2]\mathrm{e}^{H_z t}$. This means that, due to the negative angular pressure $p_\perp \simeq -\hbar/(1250\pi^2 a^6 m^2) < 0$ generated by the one-loop vacuum fluctuations inside the wormhole, the wormhole's length (along the z direction) will tend to expand exponentially. The topological wormhole harbors a cosmological horizon along the z direction, $R_z = 1/H_z$, such that for distances larger than R_z the wormhole will not be be traversable, i.e., it will expand faster than the speed of light, as its physical size will grow as

$$L(t) = a_z(t)L_0 = L_0\mathrm{e}^{H_z t}, \tag{93}$$

which means that, even a relatively short wormhole, whose initial length $L_0 \ll R_z$, will grow after some time $\Delta t \simeq R_z \ln(R_z/L_0)$ into a Hubble-sized wormhole, thus becoming nontraversable. One ought to keep in mind that this conclusion is reached based on a perturbative solution of the semiclassical equations. Furthermore, as we argue below, the exponentially expanding solution is credible only if the initial length of the wormhole is sufficiently large, i.e., if $L_0 > 1/H_z$, which will typically not be satisfied for the tiny quantum backreaction in Eq. (74).

To complete the analysis we need to show that the solutions given in Eqs. (84) and (91) are the physical solutions. Recall that the equation for $a_\perp$ also harbours a time-dependent solution given in Eq. (88). We will now argue that that is not a physical solution. Note first that Eq. (81) can be combined with Eq. (83) (and its derivative) to yield

$$\frac{\ddot{H}_\perp}{H_\perp} + 5\dot{H}_\perp + 3H_\perp^2 + \frac{\kappa^2}{2}\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}} = 0. \tag{94}$$

One can subtract from this a time derivative of Eq. (84) (divided by $2H_\perp$) to obtain

$$2\dot{H}_\perp + 3H_\perp^2 + \frac{1}{a_\perp^2} + \frac{\kappa^2}{2}\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}} = 0, \tag{95}$$

which is consistent with Eq. (84) only in the isotropic case, in which $\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}} = T_t{}^t + \langle \Delta\hat{T}^{t}_{t}\rangle^{(1)}_{\text{ren}}$, which is not the case for the topological wormhole. This therefore completes the proof that the unique solution satisfying all three Einstein's equations (79)–(81) is given in Eqs. (84) and (91). One should keep in mind that these solutions are perturbative, and therefore they are not reliable at late times. To get solutions that are reliable at late times, one would have to solve the self-consistent semiclassical Einstein equations, in which the renormalized energy-momentum tensor is computed with the metric in Eq. (75), which is beyond the scope of the present work.

B. Static *Ansatz*

Let us now consider the following metric *Ansatz* for the semiclassical wormhole,

$$\mathrm{d}s^2 = -\mathrm{e}^{2\alpha(z)}\mathrm{d}t^2 + \mathrm{e}^{2\beta(z)}\mathrm{d}z^2 + a_\perp^2\mathrm{d}\Omega_2^2. \tag{96}$$

Nonvanishing Christoffel symbols are

$$\Gamma^t_{zt} = \Gamma^t_{tz} = \alpha'(z), \qquad \Gamma^z_{tt} = \mathrm{e}^{2(\alpha-\beta)}\alpha', \qquad \Gamma^z_{zz} = \beta',$$
$$\Gamma^{\vartheta}_{\varphi\varphi} = -\sin(\vartheta)\cos(\vartheta), \qquad \Gamma^{\varphi}_{\vartheta\varphi} = \Gamma^{\varphi}_{\varphi\vartheta} = \frac{\cos(\vartheta)}{\sin(\vartheta)}, \tag{97}$$

and those of the Ricci tensor and Ricci scalar are

$$R_t{}^t = R_z{}^z = -\mathrm{e}^{-2\beta}[\alpha'' + \alpha'^2 - \alpha'\beta'], \qquad R_{\vartheta}{}^{\vartheta} = R_{\varphi}{}^{\varphi} = \frac{1}{a_\perp^2},$$
$$R = -2\mathrm{e}^{-2\beta}[\alpha'' + \alpha'^2 - \alpha'\beta'] + \frac{2}{a_\perp^2}, \tag{98}$$

such that the Einstein tensor and the corresponding semiclassical Einstein equations read

$$G_t{}^t = G_z{}^z = -\frac{1}{a_\perp^2} = \frac{\kappa^2}{2}[T_t{}^t + \langle \Delta\hat{T}^{t}_{t}\rangle^{(1)}_{\text{ren}}], \tag{99}$$

$$G_{\vartheta}{}^{\vartheta} = G_{\varphi}{}^{\varphi} = \mathrm{e}^{-2\beta}[\alpha'' + \alpha'^2 - \alpha'\beta'] = \frac{\kappa^2}{2}\langle \Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\text{ren}}. \tag{100}$$

Just as in the former *Ansatz* in Eq. (69), Eq. (99) is solved by Eq. (84),

[7] Had we not included the higher dimensional Ricci scalar squared counterterm (corresponding to the choice $\alpha_{\mathrm{R}^2} = 0$), then the pressure in Eq. (91) would be that implied by Eq. (70), which is positive,

$$p_\perp \approx \frac{\hbar}{480\pi^2 a^4}\left\{\ln\left(\frac{m^2}{4\pi\mu^2}\right) + \gamma_E - \frac{8}{21(am)^2}\right\}. \tag{92}$$

In this case the metric would be static, and the corresponding quantum backreaction could be studied in the context of the static metric *Ansatz* considered in the following subsection.

$$\frac{1}{a_\perp^2}=\frac{1}{a^2}\left(1-\frac{\kappa^2}{\bar{\kappa}^2}\right)-\frac{\hbar\kappa^2 m^2}{192\pi^2 a^2}\left\{\left(1-\frac{1}{5(am)^2}\right)\times\left[\ln\left(\frac{m^2}{4\pi\mu^2}\right)+\gamma_E\right]-\left(1-\frac{4}{105(am)^4}\right)+\mathcal{O}(a_\perp^{-6},\mathrm{e}^{-2\pi am})\right\}-2\frac{\kappa^2}{a^4}\alpha_{\mathsf{R}^2}. \tag{101}$$

Notice that the second equation (100) can be solved by $\beta=-\alpha$, in which case Eq. (100) becomes [cf. Eq. (91)],

$$\mathrm{e}^{2\alpha}[\alpha''+2\alpha'^2]=\frac{1}{2}(\mathrm{e}^{2\alpha})''=-H_z^2,$$
$$H_z^2\equiv-\frac{\kappa^2}{2}\langle\Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\mathsf{ren}}=\text{const.}, \tag{102}$$

whose general solution is

$$\mathrm{e}^{2\alpha(z)}=-H_z^2z^2+\omega_1 z+\omega_0. \tag{103}$$

The two integration constants, ω_1 and ω_0, can be fixed by matching the wormhole metric onto the Minkowski metric outside the wormhole. Assuming that the wormhole length is L_0 and that the wormhole center is located at $z=0$, we have

$$\mathrm{e}^{2\alpha(L_0/2)}=1=\mathrm{e}^{2\alpha(-L_0/2)}, \tag{104}$$

from which it follows, $\omega_1=0$, $\omega_0=1+(H_zL_0/2)^2$, such that the metric in Eq. (96) inside the wormhole becomes

$$\mathrm{d}s^2=-\left[1+H_z^2\left(\frac{L_0^2}{4}-z^2\right)\right]\mathrm{d}t^2+\frac{\mathrm{d}z^2}{1+H_z^2(\frac{L_0^2}{4}-z^2)}+a_\perp^2\mathrm{d}\Omega_2^2,\qquad\left(|z|\le\frac{L_0}{2}\right), \tag{105}$$

which at $r=|z|=L_0/2$ continuously matches onto the outside Minkowski metrics in Eq. (3). However, first derivatives of the metric are discontinuous. Notice that, had we attached only one wormholelike spacetime onto the Minkowski space, both the metric and its first derivative could have been matched continuously. Notice also that with a different renormalization choice, namely Λ and $\bar{\kappa}^2$ as in Eqs. (68) and (71), but $\alpha_{\mathsf{R}^2}=0$ instead of that in Eq. (72), one would have obtained [cf. Eq. (70)],

$$H_z^2=-\frac{\kappa^2}{2}\langle\Delta\hat{T}^{\vartheta}_{\vartheta}\rangle^{(1)}_{\mathsf{ren}}\approx-\frac{\hbar\kappa^2}{960\pi^2a^4}\left\{\ln\left(\frac{m^2}{4\pi\mu^2}\right)+\gamma_E-\frac{8}{21(am)^2}\right\}\equiv-\chi_z^2<0, \tag{106}$$

and the metric in Eq. (106) would still give a valid solution,

$$\mathrm{d}s^2=-\left[1+\chi_z^2\left(z^2-\frac{L_0^2}{4}\right)\right]\mathrm{d}t^2+\frac{\mathrm{d}z^2}{1+\chi_z^2(z^2-\frac{L_0^2}{4})}+a_\perp^2\mathrm{d}\Omega_2^2,\qquad\left(|z|\le\frac{L_0}{2}\right). \tag{107}$$

Since this metric has no horizon along the z direction, an initially traversable metric will remain traversable after the quantum backreaction is taken into account. The geometry of the metric in Eq. (107) is that of $\mathrm{AdS}_2\times S^2$. A curious fact is that the effect of the quantum backreaction on the geometry depends on the choice of finite gravitational counterterms.

The metric (105) can be compared with the metric obtained by using the time-dependent *Ansatz* in Eq. (75),

$$\mathrm{d}s^2=-\mathrm{d}t^2+a_{z(0)}^2\cosh(H_zt)\mathrm{d}z^2+a_\perp^2(t)\mathrm{d}\Omega_2^2, \tag{108}$$

where, by a suitable rescaling of the coordinate z, one can set $a_{z(0)}=1$. Below we show that these two solutions are physically equivalent.

C. Quantum energy conditions

Let us now consider nonvanishing Christoffel symbols for the metrics in Eqs. (105) and (3), whose coordinates are continuously connected at $r_\pm-a_\perp=\pm z-L_0/2$,

$$\Gamma^t_{zt}=\Gamma^t_{tz}=-\frac{H_z^2z}{1+H_z^2(\frac{L_0^2}{4}-z^2)}\times\Theta\left(\frac{L_0}{2}-|z|\right),\qquad\Gamma^z_{zz}=\frac{H_z^2z}{1+H_z^2(\frac{L_0^2}{4}-z^2)}\times\Theta\left(\frac{L_0}{2}-|z|\right),$$
$$\Gamma^{r_\pm}_{\vartheta\vartheta}=-\sum_\pm r_\pm\times\Theta(r_\pm-a_\perp),\qquad\Gamma^{r_\pm}_{\varphi\varphi}=-\sum_\pm r_\pm\sin^2(\vartheta)\times\Theta(r_\pm-a_\perp),$$
$$\Gamma^{\vartheta}_{r_\pm\vartheta}=\sum_\pm\frac{1}{r_\pm}\times\Theta(r_\pm-a_\perp),\qquad\Gamma^{\vartheta}_{\varphi\varphi}=-\sin(\vartheta)\cos(\vartheta),\qquad\Gamma^{\varphi}_{\vartheta\varphi}=\Gamma^{\varphi}_{\varphi\vartheta}=\frac{\cos(\vartheta)}{\sin(\vartheta)}. \tag{109}$$

A comparison of these Christoffel symbols with those in Eq. (4) reveals that, apart from the singular components listed in Eqs. (5)–(6), additional singular Riemann tensor components arise,

$$R_{tztz}=R_{ztzt}=-R_{tzzt}=-R_{ztzt}=-\frac{H_z^2L_0}{2}\sum_{\pm}\delta\left(z\mp\frac{L_0}{2}\right)=-\frac{H_z^2L_0}{2}\sum_{\pm}\delta(r_\pm-a_\perp). \tag{110}$$

When these singular contributions are added to the regular contributions one obtains for nonvanishing components of the Ricci tensor and Ricci scalar [cf. Eq. (8)],

$$\begin{aligned}
&R_t^t=\frac{H_z^2L_0}{2}\sum_{\pm}\delta(r_\pm-a_\perp)+H_z^2\Theta\left(\frac{L_0}{2}-|z|\right),\quad R_z^z=\left(\frac{H_z^2L_0}{2}-\frac{2}{a_\perp}\right)\sum_{\pm}\delta(r_\pm-a_\perp)+H_z^2\Theta\left(\frac{L_0}{2}-|z|\right),\\
&R_\vartheta^\vartheta=R_\varphi^\varphi=-\frac{1}{a_\perp}\sum_{\pm}\delta(r_\pm-a_\perp)+\frac{1}{a_\perp^2}\Theta\left(\frac{L_0}{2}-|z|\right),\quad R=\left(H_z^2L_0-\frac{4}{a_\perp}\right)\sum_{\pm}\delta(r_\pm-a_\perp)+\left(H_z^2+\frac{2}{a_\perp^2}\right)\Theta\left(\frac{L_0}{2}-|z|\right),
\end{aligned} \tag{111}$$

from where one arrives at nontrivial components of the Einstein tensor [cf. Eq. (9)],

$$\begin{aligned}
&G_t^{\ t}=\frac{2}{a_\perp}\sum_{\pm}\delta(r_\pm-a_\perp)-\frac{1}{a_\perp^2}\Theta\left(\frac{L_0}{2}-|z|\right),\qquad G_z^{\ z}=-\frac{1}{a_\perp^2}\Theta\left(\frac{L_0}{2}-|z|\right),\\
&G_\vartheta^{\ \vartheta}=G_\varphi^{\ \varphi}=\left(-\frac{H_z^2L_0}{2}+\frac{1}{a_\perp}\right)\sum_{\pm}\delta(r_\pm-a_\perp)-H_z^2\Theta\left(\frac{L_0}{2}-|z|\right).
\end{aligned} \tag{112}$$

The semiclassical Einstein equation (61) then implies that nonvanishing components of the energy-momentum tensor are [cf. Eq. (11)],

$$\begin{aligned}
&T_t^{\ t}+\langle\Delta\hat{T}_t^{\ t}\rangle^{(1)}=\frac{1}{4\pi Ga_\perp}\times\sum_{\pm}\delta(r_\pm-a_\perp)-\frac{1}{8\pi Ga_\perp^2}\times\Theta\left(\frac{L_0}{2}-|z|\right),\qquad T_z^{\ z}+\langle\Delta\hat{T}_z^{\ z}\rangle^{(1)}=-\frac{1}{8\pi Ga_\perp^2}\times\Theta\left(\frac{L_0}{2}-|z|\right),\\
&T_\vartheta^{\ \vartheta}+\langle\Delta\hat{T}_\vartheta^{\ \vartheta}\rangle^{(1)}=T_\varphi^{\ \varphi}+\langle\Delta\hat{T}_\varphi^{\ \varphi}\rangle^{(1)}=\frac{1}{8\pi G}\left(-\frac{H_z^2L_0}{2}+\frac{1}{a_\perp}\right)\times\sum_{\pm}\delta(r_\pm-a_\perp)-\frac{H_z^2}{8\pi G}\times\Theta\left(\frac{L_0}{2}-|z|\right).
\end{aligned} \tag{113}$$

The corresponding results for the metric in Eq. (107) are obtained by exacting the change $H_z^2\to-\chi_z^2$ in Eqs. (109)–(113), as well as in the energy conditions discussed in Eqs. (114)–(120) below. Comparing this with the classical energy-momentum tensor in Eq. (11) we see that inside the wormhole null energy condition reads [cf. Eqs. (15)–(16)],

$$T_{\mu\nu}\ell_1^\mu\ell_1^\nu=-T_t^{\ t}-\langle\Delta\hat{T}_t^{\ t}\rangle^{(1)}+T_z^{\ z}+\langle\Delta\hat{T}_z^{\ z}\rangle^{(1)}=0, \tag{114}$$

$$\begin{aligned}
T_{\mu\nu}\ell_2^\mu\ell_2^\nu&=-T_t^{\ t}-\langle\Delta\hat{T}_t^{\ t}\rangle^{(1)}+T_\vartheta^{\ \vartheta}+\langle\Delta\hat{T}_\vartheta^{\ \vartheta}\rangle^{(1)}\\
&=\frac{1}{8\pi G}\left(\frac{1}{a_\perp^2}-H_z^2\right)=\frac{1}{8\pi G}\left(\frac{1}{a_\perp^2}-\frac{2\hbar G}{315\pi a^6m^2}\right)>0,
\end{aligned} \tag{115}$$

where we made use of Eq. (91). The last inequality in Eq. (115) follows from $a_\perp\simeq a$, $G\ll a^2$ and $am\gg1$. Note that the null vectors in Eqs. (114)–(115) are constructed for the semiclassical metric in Eq. (105),

$$\begin{aligned}
\ell_1^\mu&=\left(\left[1+H_z^2\left(\frac{L_0^2}{4}-z^2\right)\right]^{-1/2},\right.\\
&\quad\left.\times\left[1+H_z^2\left(\frac{L_0^2}{4}-z^2\right)\right]^{1/2},0,0\right),\\
\ell_2^\mu&=\left(\left[1+H_z^2\left(\frac{L_0^2}{4}-z^2\right)\right]^{-1/2},0,a_\perp^{-1},0\right),
\end{aligned} \tag{116}$$

and that they indeed satisfy $\ell_1^2=0$, $\ell_2^2=0$ and they are normalized as usual, $(\ell_1)^t(\ell_1)_t=-1=(\ell_2)^t(\ell_2)_t$. A simple consideration shows that quantum average null energy condition (ANEC) for the metric in Eq. (105) reads

$$\begin{aligned}
\langle T_{\mu\nu}+\langle\Delta\hat{T}_{\mu\nu}\rangle\rangle_V\ell_1^\mu\ell_1^\nu&=-\langle T_t^{\ t}+\langle\Delta\hat{T}_t^{\ t}\rangle\rangle_V+\langle T_z^{\ z}+\langle\Delta\hat{T}_z^{\ z}\rangle\rangle_V\\
&=-\frac{1}{2\pi Ga_\perp L_0}<0,
\end{aligned} \tag{117}$$

$$\begin{aligned}
\langle T_{\mu\nu}+\langle\Delta\hat{T}_{\mu\nu}\rangle\rangle_V\ell_2^\mu\ell_2^\nu&=-\langle T_t^{\ t}+\langle\Delta\hat{T}_t^{\ t}\rangle\rangle_V+\langle T_\vartheta^{\ \vartheta}+\langle\Delta\hat{T}_\vartheta^{\ \vartheta}\rangle\rangle_V\\
&=\frac{1}{8\pi Ga_\perp^2}\left(1-\frac{2a_\perp}{L_0}-2a_\perp^2H_z^2\right)>0\\
&\quad\times(L_0\gg a_\perp),
\end{aligned} \tag{118}$$

which is to be compared with classical ANEC in Eqs. (20)–(21). As explained above, the last contribution in Eq. (118) is expected to be small, and therefore, for sufficiently long wormholes, $L_0 \gg a_\perp$, Eq. (118) is satisfied. However, as in the classical case, quantum ANEC is violated by Eq. (117), for the null vector ℓ_1^μ pointing in the z direction along the wormhole. The only difference between the classical and quantum cases is in a slight change in the radius of the wormhole, $a \to a_\perp$, which is quantitatively insignificant. An interesting question worth investigating is whether this conclusion holds true if one solves the semiclassical gravity equations (54) self-consistently.

Finally we give a couple of remarks on weak energy condition. For a timelike vector t^μ ($t^2 = -1$) we have for quantum WEC and quantum average weak energy condition,

$$T_{\mu\nu}t^\mu t^\nu = -T_t^{\ t} - \langle \Delta \hat{T}_t^{\ t} \rangle^{(1)} = \frac{1}{8\pi a_\perp^2} > 0, \quad (119)$$

$$\begin{aligned}\langle T_{\mu\nu} + \langle \Delta \hat{T}_{\mu\nu} \rangle\!\rangle_V t^\mu t^\nu &= -\langle T_t^{\ t} + \langle \Delta \hat{T}_t^{\ t} \rangle\!\rangle_V \\ &= \frac{1}{8\pi a_\perp^2}\left(1 - \frac{4a_\perp}{L_0}\right) > 0, \quad (L_0 > 4a_\perp),\end{aligned} \quad (120)$$

which is to be compared with Eqs. (22)–(23). The only difference between the classical and semiclassical weak energy conditions is in that the classical radius a ought to be replaced by the semiclassical one, $a \to a_\perp$, so that—provided $L_0 > 4a_\perp$ is satisfied—the semiclassical wormhole satisfies both WEC and AWEC.

D. Coordinates on de Sitter

To shed light on the question whether the metric is expanding or static, let us consider de Sitter space in two spacetime dimensions (t, z), which can be represented by the following three-dimensional flat Minkowski space embedding,

$$\mathrm{d}S^2 = -\mathrm{d}T^2 + \mathrm{d}X^2 + \mathrm{d}Y^2, \qquad -T^2 + (X^2 + Y^2) = \frac{1}{H^2}, \quad (121)$$

where T is a time coordinate, X and Y are spatial coordinates, and $1/H$ is the throat radius of the de Sitter space hyperboloid, which in de Sitter plays the role of the constant expansion rate. When the following coordinate transformations,

$$\begin{aligned}T &= \frac{1}{H}\sinh(Ht), \qquad X = \frac{1}{H}\cosh(Ht)\cos(Hz), \\ Y &= \frac{1}{H}\cosh(Ht)\sin(Hz),\end{aligned} \quad (122)$$

are inserted into Eq. (121) one obtains the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \cosh^2(Ht)\mathrm{d}z^2. \quad (123)$$

This is the global metric of de Sitter space, in which a contracting phase is followed by an expanding phase. On the other hand, the coordinate transformations,

$$\begin{aligned}T &= \frac{1}{H}\sqrt{1-(Hz)^2}\sinh(Ht), \\ X &= \frac{1}{H}\sqrt{1-(Hz)^2}\cosh(Ht), \ Y = z,\end{aligned} \quad (124)$$

yield the metric

$$\mathrm{d}s^2 = -(1 - H^2z^2)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{1 - H^2z^2}, \quad (125)$$

such that the two metrics in Eqs. (123) and (125) describe the same two-dimensional de Sitter space. With this insight in mind, we see that the two semiclassical metrics in Eqs. (108) and (105) describe the same four-dimensional spacetime: $dS_2 \times S^2$, and therefore they are physically equivalent. The metric in Eq. (125) can be transformed to the (t, z) part of the metric in Eq. (105) by a linear coordinate transformation $z \to z/\sqrt{1 + (H_z L_0/2)^2}$ and $t \to t\sqrt{1 + (H_z L_0/2)^2}$, and therefore these two-dimensional metric sections are physically equivalent. We choose the form in Eq. (105) to match it continuously to the external Minkowski metrics in Eq. (3).

The metric in static coordinates in Eq. (105) possesses a cosmological horizon where the vector orthogonal to $z = \text{const.}$ becomes null, $\|\partial_\mu z\|^2 = g^{zz} = 0$. When this is solved it gives

$$z_H = \pm\sqrt{\frac{1}{H_z^2} + \frac{L_0^2}{4}}, \quad (126)$$

which is always outside the wormhole ($|z| \le L_0/2 < |z_H|$) and therefore it is not part of the spacetime manifold.[8] Below we show that consequently a traversable wormhole will remain traversable after the quantum backreaction is taken into account. Note that typically $1/H_z = 6\sqrt{35}\pi a^3 m M_P \gg L_0/2$, where we used Eq. (91) and we defined a reduced Planck mass, $M_P = 1/\sqrt{8\pi G\hbar} \simeq 2.5 \times 10^{18}$ GeV. To see that let us compare $R_z \equiv 1/H_z$ with the cosmological Hubble radius today, $R_{H_0} \simeq 1.3 \times 10^{26}$ m. From

$$\frac{R_z}{R_{H_0}} \simeq 6\sqrt{35}\pi\frac{a}{R_{H_0}}(a^2 m M_P) \simeq 1 \times 10^{25}\left(\frac{m}{\mathrm{MeV}}\right)(a[\mathrm{m}])^3, \quad (127)$$

[8]We thank Farshid Jafarpour for pointing this out to us.

we see that R_z is indeed very large. For example, for a typical particle whose mass is equal to that of the electron ($m_e \simeq 0.5$ MeV), one finds $R_z \sim 10^{25} R_{H_0}$, which is huge. This means that the quantum backreaction on the wormhole generated by the quantum vacuum fluctuations of a massive scalar field is tiny. We expect analogous results to hold for massive particles with spin.

We now know that the expanding and static wormhole metrics in Eqs. (123) and (125) are physically equivalent. Nevertheless, a legitimate question to ask is which of the two metrics gives the correct physical picture. A subsystem of test particles, whose energy is too small to escape from the wormhole, will move on virialized trajectories whose average distance from the wormhole center will be less than $L_0/2$ and independent on time. On the other hand, energetic test particles, whose energy is larger than the escape energy, will move on trajectories whose average distance from the wormhole center will grow in time. These particles will perceive a wormhole that is expanding in the z direction.

E. Traversing the wormhole

Here we shall estimate the time it takes to traverse the wormhole. Firstly, a traversable wormhole satisfies Eq. (127), and its metric is given by Eq. (105). Since the wormhole possesses spherical symmetry, it suffices to consider motion in the equatorial plane of the sphere ($\vartheta = \pi/2$). The two Killing vectors we need is $K = \partial_t$ and $R = \partial_\varphi$, which imply a conserved energy $E = -mK_\mu \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$ and a conserved momentum $L = mR_\mu \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$, which for the metric in Eq. (105) give

$$E = m\left[1 + H_z^2\left(\frac{L_0^2}{4} - z^2\right)\right]\frac{\mathrm{d}t}{\mathrm{d}\tau}, \qquad L = ma_\perp^2 \frac{\mathrm{d}\varphi}{\mathrm{d}\tau}, \tag{128}$$

where τ is proper time ($\mathrm{d}\tau^2 = -\mathrm{d}s^2$). When these are used in Eq. (105) one obtains

$$\frac{\frac{E^2}{m^2} - (\frac{\mathrm{d}z}{\mathrm{d}\tau})^2}{1 + H_z^2(\frac{L_0^2}{4} - z^2)} = 1 + \frac{L^2}{(a_\perp m)^2}. \tag{129}$$

Next, one can perform a separation of variables to obtain

$$\int \frac{\mathrm{d}z}{\sqrt{\frac{E^2}{m^2} - \left[1 + \frac{L^2}{(a_\perp m)^2}\right]\left[1 + H_z^2(\frac{L_0^2}{4} - z^2)\right]}} = \tau. \tag{130}$$

This integral is solved by $\sim \ln[H_z z + \sqrt{(H_z z)^2 + \alpha^2}]$ which, when inverted, results in

$$z(\tau) = \frac{1}{2H_z}\left\{\mathrm{e}^{\omega\tau} - \left[\frac{(E/m)^2}{1 + (\frac{L}{a_\perp m})^2} - \left[1 + \left(\frac{H_z L_0}{2}\right)^2\right]\mathrm{e}^{-\omega\tau}\right\}, \tag{131}$$

with $\omega = \sqrt{1 + (\frac{L}{a_\perp m})^2} H_z$. From this it follows that the proper time interval $\Delta\tau$ needed to traverse the wormhole is given by

$$\Delta\tau = \frac{1}{H_z\sqrt{1 + (\frac{L}{a_\perp m})^2}} \times \ln\left\{\sqrt{\frac{(E/m)^2}{1 + (\frac{L}{a_\perp m})^2} - 1} - \frac{H_z L_0}{2}\right\}, \tag{132}$$

which is well defined only when the following condition is satisfied,

$$\frac{(E/m)^2}{1 + (\frac{L}{a_\perp m})^2} > 1 + \left(\frac{H_z L_0}{2}\right)^2. \tag{133}$$

The condition in Eq. (133) implies a minimum energy E_{min} (in units of c^2) a particle must have in order to be able to traverse the wormhole,

$$E > E_{\mathrm{min}} = m\sqrt{1 + \left(\frac{L}{a_\perp m}\right)^2}\sqrt{1 + \left(\frac{H_z L_0}{2}\right)^2}. \tag{134}$$

As expected, the condition in Eq. (134) becomes less restrictive for a particle whose angular momentum vanishes (for $L = 0$, $E_{\mathrm{min}} = m\sqrt{1 + (H_z L_0/2)^2}$), as in this case the particle does not lose time circling around the internal sphere. This means that, as long as a particle enters the wormhole with a large enough energy satisfying Eq. (134), it will traverse the wormhole in a finite proper time given in Eq. (132).

In what follows we show that the finite proper time interval in Eq. (132) will remain finite when measured in the coordinate time of outside Minkowski space observers, which is the last step needed to establish traversability of the wormhole. We can get the coordinate time interval Δt by solving Eq. (128).

First note that the solution for $t(\tau)$ can be written as

$$t = \frac{E/(2m)}{\sqrt{1 + (\frac{H_z L_0}{2})^2}}\left[\int \frac{\mathrm{d}\tau}{\sqrt{1 + (\frac{H_z L_0}{2})^2} + H_z z(\tau)} + \int \frac{\mathrm{d}\tau}{\sqrt{1 + (\frac{H_z L_0}{2})^2} - H_z z(\tau)}\right]. \tag{135}$$

Upon inserting Eq. (131) into Eq. (135) and integrating over τ one obtains

$$t(\tau)=\frac{\sqrt{1+(\frac{L}{a_\perp m})^2}}{\sqrt{1+(\frac{H_zL_0}{2})^2}}\ln\left\{\frac{e^{2\omega\tau}-[\sqrt{1+(H_zL_0/2)^2}-\frac{E/m}{\sqrt{1+(L/(a_\perp m))^2}}]^2}{e^{2\omega\tau}-[\sqrt{1+(H_zL_0/2)^2}+\frac{E/m}{\sqrt{1+(L/(a_\perp m))^2}}]^2}\right\}, \tag{136}$$

where $t_0\equiv t(0)$ is not zero. The desired time interval Δt is obtained by subtracting $t(0)$ from $t(\tau)$ in Eq. (136) and inserting $\Delta\tau$ from Eq. (132),

$$\Delta t=\frac{\ell}{\ell_0}\ln\left[\frac{(\sqrt{\mathcal{E}^2-1}-\frac{H_zL_0}{2})^2-(\mathcal{E}-\ell_0)^2}{(\sqrt{\mathcal{E}^2-1}-\frac{H_zL_0}{2})^2-(\mathcal{E}+\ell_0)^2}\times\frac{1-(\mathcal{E}+\ell_0)^2}{1-(\mathcal{E}-\ell_0)^2}\right], \tag{137}$$

where we introduced a shorthand notation,

$$\ell_0=\sqrt{1+\left(\frac{H_zL_0}{2}\right)^2},\qquad \ell=\sqrt{1+\left(\frac{L}{a_\perp m}\right)^2},$$
$$\mathcal{E}=\frac{E/m}{\sqrt{1+(L/(a_\perp m))^2}}. \tag{138}$$

These calculations give traversal times for the case when the quantum backreaction is given by Eq. (74), and the effective angular pressure is negative, $p_\perp<0$. In this case $H_z^2>0$, such that the quantum backreaction tends to stretch the wormhole in the z direction, thereby destabilizing it. On the other hand, when the Ricci scalar squared coupling in Eq. (70) is chosen to vanish, $\alpha_{R^2}\to 0$, the corresponding metric is shown in Eq. (107), which is a direct product of the two-dimensional anti–de Sitter space and the sphere, $AdS_2\times S^2$. In that case the quantum backreaction tends to stabilize the wormhole, and a classically traversable wormhole will remain traversable. We leave to the reader to repeat the above calculation of the minimum energy and traversal times in this case.

V. DISCUSSION AND OUTLOOK

In this work we consider the quantum backreaction in a timelike wormhole of topology $M_2\times S^2$, where M_2 denotes two-dimensional Minkowski space. The wormhole metric in Eq. (1) is supported by the classical energy-momentum tensor of an anisotropic fluid in Eq. (11), which marginally satisfies null energy condition (15)–(16); however, it breaks average null energy condition (20)–(21).

In a comprehensive Appendix we calculate the quantum one-loop backreaction of a minimally coupled, massive scalar field induced by the corresponding quantum vacuum fluctuations, in which we use a dimensional regularization scheme to renormalize the energy-momentum tensor. The two counterterms needed are the cosmological constant counterterm and the Ricci scalar counterterm. The finite parts of these counterterms in Eqs. (68) and (71) are chosen such to yield a vanishing observed cosmological constant and a continuous Newton constant in the entire spacetime. To simplify the results even further we have used a finite Ricci scalar squared counterterm in Eq. (72). We neglected the tiny cosmological constant giving rise to dark energy, which is justified as long as the size of the wormhole is much smaller than the cosmological horizon today H_0, i.e., $aH_0\ll 1$, which is amply satisfied. It would be easy to include this contribution, but that would not change our results in any significant way. The renormalized quantum energy-momentum tensor in Eqs. (69)–(70), and its simpler cousin in Eqs. (73)–(74), are then used as the source in the equations of motion of semiclassical gravity (54). We solve these equations perturbatively and find a static metric solution in Eq. (105), with an inverse expansion rate $1/H_z$ in the z direction [see Eqs. (91) and (127)],

$$R_{H_z}\equiv\frac{1}{H_z}\approx 6\sqrt{35}\pi a^3 m M_P, \tag{139}$$

where, to get Eq. (139), we used the fine-tuned energy-momentum tensor in Eq. (74). The length scale in Eq. (139) is typically very large, much larger than the wormhole length, $R_{H_z}\gg L_0$, which means that the quantum backreaction is tiny and does not significantly affect the wormhole geometry.

We also study traversability [42] of the semiclassically corrected wormhole by sending a test particle through the wormhole. We find that large wormholes remain generally traversable, provided the particle has a sufficient energy (133) at the entrance to the wormhole. The proper and external observer time intervals needed to traverse the wormhole are given in Eqs. (132) and (137), respectively. From these results one sees that quantum fluctuations do not significantly affect traversability of the wormhole:

> If a classical wormhole is traversable, it will remain traversable when the quantum backreaction is taken into account.

Even though we solve here the semiclassical Einstein equations perturbatively (to linear order in $\hbar$), based on the fact that the quantum backreaction is very small for large wormholes, we expect that the self-consistent solution of the semiclassical gravity will maintain the principal features of

the perturbative solution. This statement becomes more quantitatively accurate for larger wormholes.

In this paper we do not address the question how to create wormholes. Classical wormholes are characterized by a nontrivial topology [22], which is protected by the classical evolution. Therefore, to create a wormhole, one probably needs quantum fluctuations [4].[9] The Appendix of this paper contains exact expressions (A47)–(A48) for the one-loop quantum backreaction of a massive scalar field. Here we are primarily interested in the quantum backreaction of large wormholes, and these are obtained from asymptotic expansions of the exact results. On the other hand, small argument expansions of Eqs. (A47)–(A48) contain information on the quantum backreaction of small wormholes, and therefore they would be used to get an insight into the formation of wormholes *via* quantum fluctuations.

A natural extension of this work would be to consider a self-consistent quantum backreaction. That can be achieved by firstly constructing the scalar field propagator in a presumed wormhole spacetime, and then use that propagator to compute the quantum backreaction and solve the corresponding semiclassical equations for the metric tensor. Furthermore, one could study the quantum backreaction in the wormhole background induced by massive fermions and gauge fields, which constitute particle content of the standard model (we do not expect gravitons or photons to yield any significant backreaction, since they are both massless particles). Finally, one can modify the shape and topology of the classical wormhole studied in this work from $M_2 \times S^2$ to some other shape/topology and investigate how that would change the quantum backreaction.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: ONE-LOOP ENERGY-MOMENTUM TENSOR

The goal of this Appendix is to compute the leading-order quantum energy-momentum tensor in Eq. (51), where the propagator obeys the equation of motion in flat space given in Eq. (62). In this work we adopt a T^* ordering, as mentioned in the main text. However, in our calculations we shall use the more standard T ordering as the difference in the ordering is immaterial for the calculation of the one-loop energy-momentum tensor.

From Eq. (39) we have

$$\partial_t\hat{\phi}(t,\mathbf{z},\vartheta,\varphi)=\sum_{\ell m}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}(-i\omega_\ell \mathrm{e}^{i\mathbf{k}\cdot\mathbf{z}}u_{\ell m}(t,\mathbf{k})\hat{a}_{\ell m}(\mathbf{k})Y_{\ell m}(\vartheta,\varphi)+i\omega_\ell \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{z}}u^*_{\ell m}(t,\mathbf{k})\hat{a}^\dagger_{\ell m}(\mathbf{k})Y^*_{\ell m}(\vartheta,\varphi)), \tag{A1}$$

$$\partial_{\mathbf{z}}\hat{\phi}(t,\mathbf{z},\vartheta,\varphi)=\sum_{\ell m}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}(i\mathbf{k}\mathrm{e}^{i\mathbf{k}\cdot\mathbf{z}}u_{\ell m}(t,\mathbf{k})\hat{a}_{\ell m}(\mathbf{k})Y_{\ell m}(\vartheta,\varphi)-i\mathbf{k}\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{z}}u^*_{\ell m}(t,\mathbf{k})\hat{a}^\dagger_{\ell m}(\mathbf{k})Y^*_{\ell m}(\vartheta,\varphi)), \tag{A2}$$

$$\partial_\vartheta\hat{\phi}(t,\mathbf{z},\vartheta,\varphi)=\sum_{\ell m}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}(\mathrm{e}^{i\mathbf{k}\cdot\mathbf{z}}u_{\ell m}(t,\mathbf{k})\hat{a}_{\ell m}(\mathbf{k})\partial_\vartheta Y_{\ell m}(\vartheta,\varphi)+\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{z}}u^*_{\ell m}(t,\mathbf{k})\hat{a}^\dagger_{\ell m}(\mathbf{k})\partial_\vartheta Y^*_{\ell m}(\vartheta,\varphi)), \tag{A3}$$

$$\partial_\varphi\hat{\phi}(t,\mathbf{z},\vartheta,\varphi)=\sum_{\ell m}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}(\mathrm{e}^{i\mathbf{k}\cdot\mathbf{z}}u_{\ell m}(t,\mathbf{k})\hat{a}_{\ell m}(\mathbf{k})\partial_\varphi Y_{\ell m}(\vartheta,\varphi)+\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{z}}u^*_{\ell m}(t,\mathbf{k})\hat{a}^\dagger_{\ell m}(\mathbf{k})\partial_\varphi Y^*_{\ell m}(\vartheta,\varphi)), \tag{A4}$$

[9]See, however, Ref. [43] for an example of how the topology of spatial slices of an expanding Universe can be changed by global monopoles.

from which we conclude that only the diagonal terms $\mu = \nu$ contribute to $\langle \partial_\mu \hat{\phi}(x) \partial_\nu \hat{\phi}(x) \rangle$. We therefore have

$$\langle (\partial_t \hat{\phi}(x))^2 \rangle = \sum_{\ell m} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} (\omega_\ell^2 |u_{\ell m}(t,\mathbf{k})|^2 \times Y_{\ell m}(\vartheta,\varphi) Y^*_{\ell m}(\vartheta,\varphi)), \tag{A5}$$

$$\langle (\partial_{\mathbf{z}} \hat{\phi}(x))^2 \rangle = \sum_{\ell m} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} (\|\mathbf{k}\|^2 |u_{\ell m}(t,\mathbf{k})|^2 \times Y_{\ell m}(\vartheta,\varphi) Y^*_{\ell m}(\vartheta,\varphi)), \tag{A6}$$

$$\langle (\partial_\vartheta \hat{\phi}(x))^2 \rangle = \sum_{\ell m} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} (|u_{\ell m}(t,\mathbf{k})|^2 \times \partial_\vartheta Y_{\ell m}(\vartheta,\varphi) \partial_\vartheta Y^*_{\ell m}(\vartheta,\varphi)), \tag{A7}$$

$$\langle (\partial_\varphi \hat{\phi}(x))^2 \rangle = \sum_{\ell m} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} (|u_{\ell m}(t,\mathbf{k})|^2 \times \partial_\varphi Y_{\ell m}(\vartheta,\varphi) \partial_\varphi Y^*_{\ell m}(\vartheta,\varphi)), \tag{A8}$$

where we used the commutation relations in Eq. (40) and $\hat{a}_{\ell m}(\mathbf{k})|\Omega\rangle = 0$, and no time ordering is yet imposed. The next question we address is how the T ordering would affect the results in Eqs. (A5)–(A8). Comparing the definition of time ordering, $T[\hat{A}\,\hat{B}] = \Theta(\Delta t)[\hat{A}\,\hat{B}] + \Theta(-\Delta t)[\hat{B}\,\hat{A}]$, with the operator structure of Eqs. (A5)–(A8), we see that in these equations $\hat{A} = \hat{B}$. Since $T[\hat{A}\,\hat{A}] = (\hat{A})^2$, operator ordering in Eqs. (A5)–(A8) is immaterial. In other words, calculating Eqs. (A5)–(A8) is equivalent to calculating the time-ordered product of the same quantities. With this insight in mind we shall drop time ordering in the rest of the Appendix. Next, making use of Eqs. (46) and (42), Eqs. (A5) and (A6) can be simplified to

$$\langle (\partial_t \hat{\phi}(x))^2 \rangle = \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1)\omega_\ell, \tag{A9}$$

$$\langle (\partial_{\mathbf{z}} \hat{\phi}(x))^2 \rangle = \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1) \frac{\|\mathbf{k}\|^2}{\omega_\ell}. \tag{A10}$$

To evaluate the angular contributions (A7)–(A8), the following spherical harmonics addition theorem is useful,

$$\sum_{m=-\ell}^{\ell} Y_{\ell m}(\vartheta,\varphi) Y^*_{\ell m}(\vartheta',\varphi') = \frac{2\ell+1}{4\pi} P_\ell(\mu), \tag{A11}$$

where

$$\mu = \hat{r}\cdot\hat{r}' = \cos(\vartheta)\cos(\vartheta') + \sin(\vartheta)\sin(\vartheta')\cos(\varphi-\varphi'), \tag{A12}$$

from where it is straightforward to obtain

$$\sum_m \partial_\varphi Y_{\ell m}(\vartheta,\varphi) \partial_\varphi Y^*_{\ell m}(\vartheta,\varphi) = \sin^2(\vartheta) \frac{2\ell+1}{4\pi} [P'_\ell(\mu)]_{\mu=1} = \sin^2(\vartheta) \frac{2\ell+1}{4\pi} \frac{\ell(\ell+1)}{2}, \tag{A13}$$

$$\sum_m \partial_\vartheta Y_{\ell m}(\vartheta,\varphi) \partial_\vartheta Y^*_{\ell m}(\vartheta,\varphi) = \frac{2\ell+1}{4\pi} [P'_\ell(\mu)]_{\mu=1} = \frac{2\ell+1}{4\pi} \frac{\ell(\ell+1)}{2}, \tag{A14}$$

where we used $[P'_\ell(\mu)]_{\mu=1} = \ell(\ell+1)/2$, which can be derived from the Rodrigues formula. With these results in mind Eqs. (A7)–(A8) can be simplified to

$$\langle (\partial_\vartheta \hat{\phi}(x))^2 \rangle = \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1) \frac{\ell(\ell+1)}{2\omega_\ell}, \tag{A15}$$

$$\langle (\partial_\varphi \hat{\phi}(x))^2 \rangle = \sin^2(\vartheta) \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1) \frac{\ell(\ell+1)}{2\omega_\ell}. \tag{A16}$$

Finally, from Eq. (51) we see that we also need $\langle \hat{\phi}^2(x) \rangle$. Repeating the same steps as above, one obtains

$$\langle \hat{\phi}^2(x) \rangle = \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1) \frac{1}{\omega_\ell}. \tag{A17}$$

Making use of the wormhole metric (33) one obtains

$$\begin{aligned} \langle \hat{\mathcal{L}} \rangle &\equiv -\frac{1}{2} g^{\alpha\beta}_{(0)} \langle \partial_\alpha \hat{\phi}(x) \partial_\beta \hat{\phi}(x) \rangle - \frac{1}{2} m^2 \langle \hat{\phi}^2(x) \rangle, \\ &= -\frac{1}{2} \frac{\hbar}{8\pi a^2} \int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} \sum_\ell (2\ell+1) \left[-\omega_\ell + \frac{\|\mathbf{k}\|^2}{\omega_\ell} + \frac{\ell(\ell+1)}{a^2\omega_\ell} + \frac{m^2}{\omega_\ell} \right] = 0, \end{aligned} \tag{A18}$$

such that the energy-momentum tensor in Eq. (51) simplifies to

$$\langle \Delta \hat{T}_\mu{}^\nu(x) \rangle^{(1)} = g^{\nu\rho}_{(0)} \langle \partial_\mu \hat{\phi}(x) \partial_\rho \hat{\phi}(x) \rangle, \tag{A19}$$

and, therefore, its nonvanishing components are

$$\langle \Delta\hat{T}_t^{\ t}(x)\rangle^{(1)} = -\langle(\partial_t\hat{\phi}(x))^2\rangle = -\frac{\hbar}{8\pi a^2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\sum_\ell(2\ell+1)\omega_\ell, \quad \text{(A20)}$$

$$\langle \Delta\hat{T}_{z^i}^{\ z^i}(x)\rangle^{(1)} = \langle(\partial_{z^i}\hat{\phi}(x))^2\rangle = \frac{\hbar}{8\pi a^2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\sum_\ell(2\ell+1)\frac{\|\mathbf{k}\|^2}{(D-3)\omega_\ell}, \quad \text{(A21)}$$

$$\langle \Delta\hat{T}_\vartheta^{\ \vartheta}(x)\rangle^{(1)} = \langle \Delta\hat{T}_\varphi^{\ \varphi}(x)\rangle^{(1)} = \frac{\hbar}{8\pi a^2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\sum_\ell(2\ell+1)\frac{\ell(\ell+1)}{2a^2\omega_\ell}. \quad \text{(A22)}$$

1. Renormalization

The primitive one-loop energy-momentum tensor in Eqs. (A20)–(A22) is expressed in terms of composite field operators, and therefore it is divergent.

Let us now first consider vacuum fluctuations in an infinite wormhole, $a\to\infty$ (i.e., the flat Minkowski space limit), for which the propagator obeys the equation

$$(\partial^2 - m^2)i\Delta_M(x;x') = i\hbar\delta^D(x-x') \qquad (\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu). \quad \text{(A23)}$$

The Minkowski space propagator is given by

$$i\Delta_M(x;x') = \frac{\hbar}{2}\int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\frac{\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}}{\omega(\mathbf{k})}[\Theta(t-t')\mathrm{e}^{-i\omega(\mathbf{k})(t-t')} + \Theta(t'-t)\mathrm{e}^{i\omega(\mathbf{k})(t-t')}] = \frac{\hbar m^{D-2}}{(2\pi)^{D/2}}\frac{K_{\frac{D-2}{2}}(m\sqrt{\Delta x^2})}{(m\sqrt{\Delta x^2})^{\frac{D-2}{2}}}, \quad \text{(A24)}$$

where $K_\nu(z)$ is the Macdonald function (the Bessel function of the second kind),

$$\Delta x^2(x;x') = -(|t-t'|-i\varepsilon)^2 + \|\mathbf{x}-\mathbf{x}'\|^2 \quad \text{(A25)}$$

is the (Lorentz) invariant distance squared between x and x', and the $i\varepsilon$ piece ($\varepsilon>0$ and $\varepsilon\to 0$) generates the imaginary part of the propagator that corresponds that of the vacuum Feynman (time-ordered) propagator. The coincident limit of the Feynman propagator in Eq. (A24) reads

$$i\Delta_M(x;x) = \langle T[\hat{\phi}^2(x)]\rangle = \frac{\hbar}{2}\int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\frac{1}{\omega(\mathbf{k})} = \frac{\hbar m^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\left(1-\frac{D}{2}\right), \quad \text{(A26)}$$

where to get this result we made use of the following master integral [44],

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+M^2)^\beta} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(\beta-\frac{d}{2})}{\Gamma(\beta)}(M^2)^{\frac{d}{2}-\beta}, \quad \text{(A27)}$$

where $\omega(\mathbf{k}) = \sqrt{\|\mathbf{k}\|^2+m^2}$. It is now not difficult to show that

$$\langle(\partial_t\hat{\phi}(x))(\partial^t\hat{\phi}(x))\rangle_M^{(1)} = -\frac{\hbar}{2}\int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\omega(\mathbf{k}) = \frac{\hbar}{2}\frac{m^D}{(4\pi)^{\frac{D}{2}}}\Gamma\left(-\frac{D}{2}\right), \quad \text{(A28)}$$

$$\langle(\partial_i\hat{\phi}(x))(\partial^j\hat{\phi})\rangle_M^{(1)} = \frac{\hbar}{2}\frac{\delta_i^j}{D-1}\int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\frac{\|\mathbf{k}\|^2}{\omega(\mathbf{k})} = \frac{\hbar}{2}\frac{\delta_i^j}{D-1}\int\frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}}\left(\omega(\mathbf{k})-\frac{m^2}{\omega(\mathbf{k})}\right) = \frac{\hbar}{2}\delta_i^j\frac{m^D}{(4\pi)^{\frac{D}{2}}}\Gamma\left(-\frac{D}{2}\right), \quad \text{(A29)}$$

such that

$$\langle\hat{\mathcal{L}}\rangle_M^{(1)} \equiv -\frac{1}{2}\eta^{\mu\nu}\langle(\partial_\mu\hat{\phi}(x))(\partial_\nu\hat{\phi}(x))\rangle_M^{(1)} - \frac{m^2}{2}\langle(\hat{\phi}(x))^2\rangle_M^{(1)} = 0, \quad \text{(A30)}$$

where we made use of the fact that operator ordering in the above calculations is immaterial. From here it then follows that the nonvanishing elements of the one-loop energy-momentum tensor in Minkowski vacuum are

$$\langle\Delta\hat{T}_t^{\ t}\rangle_M^{(1)} = \langle(\partial_t\hat{\phi}(x))(\partial^t\hat{\phi}(x))\rangle_M^{(1)} = \frac{\hbar}{2}\frac{m^D}{(4\pi)^{\frac{D}{2}}}\Gamma\left(-\frac{D}{2}\right), \quad \text{(A31)}$$

$$\langle\Delta\hat{T}_i^{\ j}\rangle_M^{(1)} = \langle(\partial_i\hat{\phi}(x))(\partial^j\hat{\phi}(x))\rangle_M^{(1)} = \frac{\hbar}{2}\delta_i^{\ j}\frac{m^D}{(4\pi)^{\frac{D}{2}}}\Gamma\left(-\frac{D}{2}\right), \quad \text{(A32)}$$

such that

$$\langle \Delta\hat{T}_{\mu}^{\ \nu}\rangle_{M}^{(1)} = \frac{\hbar}{2}\times\frac{m^{D}}{(4\pi)^{D/2}}\Gamma\left(-\frac{D}{2}\right)\delta_{\mu}^{\nu} = -\frac{\hbar}{2}\times\frac{m^{4}}{(4\pi)^{2}}\frac{\mu^{D-4}}{D-4}\left[1+\frac{D-4}{2}\left(\ln\left(\frac{m^{2}}{4\pi\mu^{2}}\right)+\gamma_{E}-\frac{3}{2}\right)\right]\delta_{\mu}^{\nu}, \tag{A33}$$

where to get the last equality we used

$$\Gamma\left(-\frac{D}{2}\right) = -\frac{1}{D-4}\left[1+\frac{D-4}{2}\left(\gamma_{E}-\frac{3}{2}\right)\right], \tag{A34}$$

where $\gamma_E = -\psi(1) = 0.577\cdots$ is the Euler-Mascheroni constant. In Eq. (A33) we have introduced a fiducial scale μ, which will later turn out to be useful for renormalization.

In what follows we use the method of Minkowski vacuum subtraction, which is often used in Casimir effect studies [45]. It is convenient to subtract the one-loop contribution from the Minkowski space vacuum fluctuations, in which fluctuations on the sphere S^2 are replaced by fluctuations on the plane $\mathbb{R}^2$.

Our Minkowski space coordinates are denoted as $t\in\mathbb{R}$, $\mathbf{z}\in\mathbb{R}^{D-3}$ and $\mathbf{x}\in\mathbb{R}^2$, and the corresponding momenta are denoted as $\mathbf{k}\in\mathbb{R}^{D-3}$ and $\mathbf{p}\in\mathbb{R}^2$. With these remarks in mind, subtracting the Minkowski vacuum contributions in Eqs. (A31)–(A32) from the energy-momentum tensors in Eqs. (A20)–(A22) results in

$$\langle \Delta\hat{T}_{t}^{\ t}(x)\rangle^{(1)} - \langle \Delta\hat{T}_{t}^{\ t}(x)\rangle_{M}^{(1)} = -\frac{\hbar}{2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\left[\frac{1}{4\pi a^{2}}\sum_{\ell}(2\ell+1)\omega_{\ell} - \int\frac{\mathrm{d}^{2}p}{(2\pi)^{2}}\omega(\mathbf{k},\mathbf{p})\right], \tag{A35}$$

$$\langle \Delta\hat{T}_{z^i}^{\ z^i}(x)\rangle^{(1)} - \langle \Delta\hat{T}_{z^i}^{\ z^i}(x)\rangle_{M}^{(1)} = \frac{\hbar}{2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\frac{\|\mathbf{k}\|^{2}}{(D-3)}\left[\frac{1}{4\pi a^{2}}\sum_{\ell}(2\ell+1)\frac{1}{\omega_{\ell}} - \int\frac{\mathrm{d}^{2}p}{(2\pi)^{2}}\frac{1}{\omega(\mathbf{k},\mathbf{p})}\right], \tag{A36}$$

$$\langle \Delta\hat{T}_{\vartheta}^{\ \vartheta}(x)\rangle^{(1)} - \langle \Delta\hat{T}_{\vartheta}^{\ \vartheta}(x)\rangle_{M}^{(1)} = \langle \Delta\hat{T}_{\varphi}^{\ \varphi}(x)\rangle^{(1)} - \langle \Delta\hat{T}_{\varphi}^{\ \varphi}(x)\rangle_{M}^{(1)} = \frac{\hbar}{2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\left[\frac{1}{4\pi a^{2}}\sum_{\ell}(2\ell+1)\frac{\ell(\ell+1)}{2a^{2}\omega_{\ell}} - \int\frac{\mathrm{d}^{2}p}{(2\pi)^{2}}\frac{\|\mathbf{p}\|^{2}}{2\omega(\mathbf{k},\mathbf{p})}\right], \tag{A37}$$

where $\omega(\mathbf{k},\mathbf{p}) = \sqrt{\|\mathbf{k}\|^2+\|\mathbf{p}\|^2+m^2}$, with no summation over z^i in Eq. (A36).

The expressions in square brackets can be evaluated by using Abel-Plana formulas [46]. Let us begin our analysis by introducing the following substitutions,

$$\nu \to \ell+\frac{1}{2}, \qquad \ell(\ell+1)\to\nu^{2}-\frac{1}{4}, \qquad \|\mathbf{p}\|^{2}\to\frac{1}{a^{2}}\left(\nu^{2}-\frac{1}{4}\right), \tag{A38}$$

after which the expressions in square brackets in Eqs. (A35) and (A36) become

$$B_{\omega} \equiv \frac{1}{4\pi a^{2}}\sum_{\ell}(2\ell+1)\omega_{\ell} - \int\frac{\mathrm{d}^{2}p}{(2\pi)^{2}}\omega(\mathbf{k},\mathbf{p}), \qquad = \frac{1}{2\pi a^{3}}\left[\sum_{\nu=\frac{1}{2},\frac{3}{2},\cdots}\nu\sqrt{u^{2}-\frac{1}{4}+\nu^{2}} - \int_{\frac{1}{2}}^{\infty}\mathrm{d}\nu\,\nu\sqrt{u^{2}-\frac{1}{4}+\nu^{2}}\right], \tag{A39}$$

$$B_{\frac{1}{\omega}} = \frac{1}{2\pi a}\left[\sum_{\nu=\frac{1}{2},\frac{3}{2},\cdots}\frac{\nu}{\sqrt{u^{2}-\frac{1}{4}+\nu^{2}}} - \int_{\frac{1}{2}}^{\infty}\mathrm{d}\nu\,\frac{\nu}{\sqrt{u^{2}-\frac{1}{4}+\nu^{2}}}\right], \tag{A40}$$

where $u^2 = a^2(m^2+\|\mathbf{k}\|^2)$. Noting that $\int_{\frac{1}{2}}^{\infty}\mathrm{d}\nu = \int_{0}^{\infty}\mathrm{d}\nu - \int_{0}^{\frac{1}{2}}\mathrm{d}\nu$, we can use the half-integer Abel-Plana formula [46],

$$\sum_{n=0}^{\infty}f\left(n+\frac{1}{2}\right) - \int_{0}^{\infty}f(x)\mathrm{d}x = -i\int_{0}^{\infty}\frac{f(it)-f(-it)}{e^{2\pi t}+1}\mathrm{d}t, \tag{A41}$$

which for

$$f(\nu)\to\nu\sqrt{u^2+\nu^2-\frac{1}{4}}\quad\text{and}\quad f(\nu)\to\frac{\nu}{\sqrt{u^2+\nu^2-\frac{1}{4}}} \tag{A42}$$

gives

$$B_\omega=\frac{1}{\pi a^3}\left[\int_0^{\sqrt{u^2-\frac{1}{4}}}\frac{t\sqrt{u^2-\frac{1}{4}-t^2}}{e^{2\pi t}+1}\mathrm{d}t+\frac{1}{2}\int_0^{\frac{1}{2}}\mathrm{d}\nu\nu\sqrt{u^2-\frac{1}{4}+\nu^2}\right], \tag{A43}$$

$$B_{\frac{1}{\omega}}=\frac{1}{\pi a}\left[\int_0^{\sqrt{u^2-\frac{1}{4}}}\frac{t\mathrm{d}t}{\sqrt{u^2-\frac{1}{4}-t^2}(e^{2\pi t}+1)}+\frac{1}{2}\int_0^{\frac{1}{2}}\frac{\nu\mathrm{d}\nu}{\sqrt{u^2-\frac{1}{4}+\nu^2}}\right], \tag{A44}$$

where we assumed that $t<\sqrt{u^2-\frac{1}{4}}$, which is a reasonable assumption for large wormholes, for which $u\geq am\gg 1$. For light particles, for which $am\leq 1/2$, there is no cut contribution from the Abel-Plana formula (A41) to B_ω and $B_{\frac{1}{\omega}}$ in Eqs. (A43)–(A44), and we have

$$B_\omega=0=B_{\frac{1}{\omega}},\qquad\left(am\leq\frac{1}{2}\right), \tag{A45}$$

implying that light and massless scalar particles satisfying $am\leq 1/2$ do not produce any quantum backreaction at the one-loop level inside the wormhole. It would be of interest to investigate whether analogous results hold for particles with spin.

The latter integrals in Eqs. (A43)–(A44) are easily evaluated,

$$\frac{1}{2}\int_0^{\frac{1}{2}}\mathrm{d}\nu\nu\sqrt{u^2-\frac{1}{4}+\nu^2}=\frac{1}{6}\left[u^3-\left(u^2-\frac{1}{4}\right)^{\frac{3}{2}}\right],$$
$$\frac{1}{2}\int_0^{\frac{1}{2}}\mathrm{d}\nu\frac{\nu}{\sqrt{u^2-\frac{1}{4}+\nu^2}}=\frac{1}{2}\left[u-\sqrt{u^2-\frac{1}{4}}\right]. \tag{A46}$$

The former integrals in Eqs. (A43)–(A44) are hard, and one has to resort to approximations to evaluate them. Expanding the integrand in powers of $\mathrm{e}^{-2\pi t}$ and changing the integration variable, $t\to u_0\tau$, with $u_0\equiv\sqrt{u^2-\frac{1}{4}}$, one obtains

$$B_\omega=\frac{1}{\pi a^3}\left[-u_0^3\sum_{n=1}^{\infty}(-1)^n\int_0^1\tau\sqrt{1-\tau^2}\mathrm{e}^{-2\pi n\tau}\mathrm{d}\tau+\frac{1}{6}(u^3-u_0^3)\right], \tag{A47}$$

$$B_{\frac{1}{\omega}}=\frac{1}{\pi a}\left[-u_0\sum_{n=1}^{\infty}(-1)^n\int_0^1\frac{\tau\mathrm{d}\tau}{\sqrt{1-\tau^2}}\mathrm{e}^{-2\pi u_0\tau}+\frac{1}{2}(u-u_0)\right]. \tag{A48}$$

These integrals can be evaluated,

$$B_\omega=\frac{1}{\pi a^3}\left\{-u_0^3\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{3}+\frac{(2\pi nu_0)^2}{15}\times{}_1F_2\left(1;\frac{1}{2},\frac{3}{2};(\pi nu_0)^2\right)-\frac{1}{4nu_0}\times I_2(2\pi nu_0)\right]+\frac{1}{6}(u^3-u_0^3)\right\}, \tag{A49}$$

$$B_{\frac{1}{\omega}}=\frac{1}{\pi a}\left\{-u_0\sum_{n=1}^{\infty}(-1)^n\left[{}_1F_2\left(1;\frac{1}{2},\frac{3}{2};(\pi nu_0)^2\right)-\frac{\pi}{2}I_1(2\pi nu_0)\right]\right]+\frac{1}{2}(u-u_0)\right]\right\}, \tag{A50}$$

where ${}_1F_2$ and $I_\nu(z)$ denote a generalized hypergeometric function and a modified Bessel function of the first kind, respectively. We are primarily interested in large wormholes, which corresponds to the large argument limit (asymptotic expansion) of special functions in Eqs. (A49)–(A50), which can be found, for example, in Sec. 16.11 of Ref. [47]. Since both functions have branch cuts at large argument values, to get the correct expression for the integrals, one should take the average value above and below the cuts, to obtain

$$B_\omega=\frac{1}{\pi a^3}\left\{-u_0^3\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{(2\pi nu_0)^2}-\frac{3}{(2\pi nu_0)^4}-\frac{15}{(2\pi nu_0)^6}-\frac{315}{(2\pi nu_0)^8}-\frac{14175}{(2\pi nu_0)^{10}}+\mathcal{O}(u_0^{-12})\right]+\frac{1}{6}(u^3-u_0^3)\right\}, \tag{A51}$$

$$B_{\frac{1}{\omega}}=\frac{1}{\pi a}\left\{-u_0\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{(2\pi nu_0)^2}+\frac{3}{(2\pi nu_0)^4}+\frac{45}{(2\pi nu_0)^6}+\frac{1575}{(2\pi nu_0)^8}+\frac{99225}{(2\pi nu_0)^{10}}+\mathcal{O}(u_0^{-12})\right]+\frac{1}{2}(u-u_0)\right]\right\}. \tag{A52}$$

Next, one can perform the sums to obtain

$$B_\omega = \frac{1}{\pi a^3}\left\{u_0^3\left[\frac{1}{48u_0^2}-\frac{7}{3840u_0^4}-\frac{31}{129024u_0^6}-\frac{127}{983040u_0^8} -\frac{2555}{17301504u_0^{10}}+\mathcal{O}(u_0^{-12})\right]+\frac{1}{6}(u^3-u_0^3)\right\}, \tag{A53}$$

$$B_{\frac{1}{\omega}} = \frac{1}{\pi a}\left\{u_0\left[\frac{1}{48u_0^2}+\frac{7}{3840u_0^4}+\frac{31}{43008u_0^6}+\frac{127}{196608u_0^8} +\frac{17885}{17301504u_0^{10}}+\mathcal{O}(u_0^{-12})\right]+\frac{1}{2}(u-u_0)\right]\right\}, \tag{A54}$$

where to get these results we made use of

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^z} = -\left(1-\frac{1}{2^{z-1}}\right)\zeta(z). \tag{A55}$$

Here $\zeta(z)=\sum_{n=1}^{\infty}\frac{1}{n^z}$ is the Riemann zeta function, which when evaluated at even integers takes simple values: $\zeta(2)=\pi^2/6$, $\zeta(4)=\pi^4/90$, $\zeta(6)=\pi^6/945$, $\zeta(8)=\pi^8/9450$, $\zeta(10)=\pi^{10}/93555$, etc. In general, one can obtain $\zeta(2n)(n\in\mathbb{N})$ from its generating function,

$$\sum_{n=0}^{\infty}\zeta(2n)x^{2n} = -\frac{\pi x}{2}\coth(\pi x), \tag{A56}$$

and from here one can show that $\zeta(2n)$ can be expressed in terms of Bernoulli numbers B_{2n} as $\zeta(2n)=(-1)^{n+1}B_{2n}(2\pi)^{2n}/[2(2n)!]$.

An inspection of the coefficients of the asymptotic series in the square brackets in Eqs. (A49)–(A50) shows that they can be written with the help of the Pochhammer symbols, $(a)_{n'}\equiv\Gamma(a+n')/\Gamma(a)$, as $2^{2(n'-1)}(-\frac{1}{2})_{n'-1}(\frac{3}{2})_{n'-1}/(2\pi n u_0)^{2n'}$ and $-2^{2n'}(-\frac{1}{2})_{n'}(\frac{1}{2})_{n'}/(2\pi n u_0)^{2n'}$ $(n'=1,2,\ldots)$, respectively. This is however hard to prove from the complicated form of the asymptotic series as given in Ref. [47]. In what follows, we prove that by construction, i.e., by evaluating the integrals in Eqs. (A43)–(A44) by a different approximate method. We begin by expanding the square roots in Eqs. (A43)–(A44),

$$\sqrt{u^2-\frac{1}{4}-t^2} = u_0\sum_{n=0}^{\infty}\frac{(-\frac{1}{2})_n}{n!}\left(\frac{t}{u_0}\right)^{2n}, \qquad \frac{1}{\sqrt{u^2-\frac{1}{4}-t^2}} = \frac{1}{u_0}\sum_{n=0}^{\infty}\frac{(\frac{1}{2})_n}{n!}\left(\frac{t}{u_0}\right)^{2n}, \tag{A57}$$

where $(a)_n=\Gamma(a+n)/\Gamma(a)$ denotes the Pochammer symbol and

$$u_0 = \sqrt{u^2-\frac{1}{4}} = \sqrt{(am)^2+(a\|\mathbf{k}\|)^2-\frac{1}{4}}. \tag{A58}$$

Both series in Eq. (A57) converge for $t<u_0$, which suffices for our purposes. Inserting Eqs. (A46) and (A57) into Eqs. (A43)–(A44) gives

$$B_\omega = \frac{1}{\pi a^3}\left\{\sum_{n=0}^{\infty}\frac{(-\frac{1}{2})_n}{n!}u_0^{1-2n}\int_0^{u_0}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1}+\frac{1}{6}(u^3-u_0^3)\right\}, \tag{A59}$$

$$B_{\frac{1}{\omega}} = \frac{1}{\pi a}\left\{\sum_{n=0}^{\infty}\frac{(\frac{1}{2})_n}{n!}\frac{1}{u_0^{1+2n}}\int_0^{u_0}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1}+\frac{1}{2}(u-u_0)\right\}. \tag{A60}$$

The integral can be evaluated by firstly rewriting it as

$$\int_0^{u_0}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1} = \int_0^{\infty}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1} - \int_{u_0}^{\infty}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1}. \tag{A61}$$

The second integral is exponentially suppressed in u_0 as

$$\int_{u_0}^{\infty}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1} = \frac{{u_0}^{1+2n}}{2\pi}\mathrm{e}^{-2\pi u_0}\left[1+\mathcal{O}\left(\frac{1}{u_0},\mathrm{e}^{-2\pi u_0}\right)\right], \tag{A62}$$

and the first integral in Eq. (A61) can be evaluated exactly,

$$\int_0^{\infty}\frac{t^{1+2n}\mathrm{d}t}{e^{2\pi t}+1} = -\sum_{n'=1}^{\infty}\frac{(-1)^{n'}}{(2\pi n')^{2+2n}}\int_0^{\infty}v^{1+2n}\mathrm{e}^{-v}\mathrm{d}v = \frac{1}{(2\pi)^{2+2n}}\left(1-\frac{1}{2^{1+2n}}\right)\Gamma(2+2n)\zeta(2+2n), \tag{A63}$$

where we made use of Eq. (A55). Inserting Eqs. (A62) and (A63) into Eqs. (A59)–(A60) results in

$$B_\omega = \frac{1}{\pi a^3}\left\{\sum_{n=0}^{\infty}\frac{(-\frac{1}{2})_n}{n!}\left[\frac{{u_0}^{1-2n}}{(2\pi)^{2+2n}}\left(1-\frac{1}{2^{1+2n}}\right)\times\Gamma(2+2n)\zeta(2+2n)+\mathcal{O}(\mathrm{e}^{-2\pi u_0})\right]+\frac{1}{6}(u^3-u_0^3)\right\}, \tag{A64}$$

$$B_{\frac{1}{\omega}} = \frac{1}{\pi a}\left\{\sum_{n=0}^{\infty}\frac{(\frac{1}{2})_n}{n!}\left[\frac{1}{(2\pi)^{2+2n}}\frac{1}{{u_0}^{1+2n}}\left(1-\frac{1}{2^{1+2n}}\right)\times\Gamma(2+2n)\zeta(2+2n)+\mathcal{O}(\mathrm{e}^{-2\pi u_0})\right]+\frac{1}{2}(u-u_0)\right\}, \tag{A65}$$

where, instead of inserting the results from Eq. (A62), we have inserted just an estimate $\mathcal{O}(\mathrm{e}^{-2\pi u_0})$, as we

are not sure of the precise form of the asymptotic expansion that includes exponential suppression. This is because the results in Eq. (A62) multiply a series that, when summed, either gives zero or diverges. The reason for that ambiguous result can be traced back to the fact that the original integral in Eq. (A61) contains a cut at $t \geq u_0$.

With these results we have achieved the intended goal, namely we have obtained an explicit form for the asymptotic series coefficients in Eqs. (A53)–(A54), which in light of Eq. (A55) can be used to reconstruct the coefficients of the asymptotic series in Eqs. (A51)–(A52).

Indeed, upon inserting Eqs. (A64)–(A65) into Eqs. (A35)–(A37) one obtains

$$\langle \Delta\hat{T}_t^{\ t}(x)\rangle^{(1)} - \langle \Delta\hat{T}_t^{\ t}(x)\rangle_M^{(1)} = -\frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} B_\omega \approx \frac{\hbar}{2\pi a}\left\{\frac{16M^{D-2}}{(4\pi)^{\frac{D+3}{2}}a}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{3}{2})\Gamma(n+1-\frac{D}{2})}{(\pi aM)^{2n}}\left(1-\frac{1}{2^{1+2n}}\right)\zeta(2+2n)\right.$$
$$\left. -\frac{a}{4(4\pi)^{\frac{D-2}{2}}}(m^D - M^D)\Gamma\left(-\frac{D}{2}\right) + \mathcal{O}(\mathrm{e}^{-2\pi aM})\right\}, \tag{A66}$$

$$\langle \Delta\hat{T}_{z^i}^{\ z^i}(x)\rangle^{(1)} - \langle \Delta\hat{T}_{z^i}^{\ z^i}(x)\rangle_M^{(1)} = \frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\frac{\|\mathbf{k}\|^2}{(D-3)}B_{\frac{1}{\omega}} \approx \frac{\hbar}{2\pi a}\left\{\frac{16M^{D-2}}{(4\pi)^{\frac{D+3}{2}}a}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{3}{2})\Gamma(n+1-\frac{D}{2})}{(\pi aM)^{2n}}\left(1-\frac{1}{2^{1+2n}}\right)\zeta(2+2n)\right.$$
$$\left. -\frac{a}{4(4\pi)^{\frac{D-2}{2}}}\Gamma\left(-\frac{D}{2}\right)(m^D - M^D) + \mathcal{O}(\mathrm{e}^{-2\pi aM})\right\}, \tag{A67}$$

$$\langle \Delta\hat{T}_\vartheta^{\ \vartheta}(x)\rangle^{(1)} - \langle \Delta\hat{T}_\vartheta^{\ \vartheta}(x)\rangle_M^{(1)} = \langle \Delta\hat{T}_\varphi^{\ \varphi}(x)\rangle^{(1)} - \langle \Delta\hat{T}_\varphi^{\ \varphi}(x)\rangle_M^{(1)} = \frac{\hbar}{4}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}[B_\omega - (\|\mathbf{k}\|^2 + m^2)B_{\frac{1}{\omega}}],$$
$$\approx \frac{\hbar}{4\pi a}\left\{-\frac{16M^{D-2}}{(4\pi)^{\frac{D+3}{2}}a}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{3}{2})\Gamma(n+1-\frac{D}{2})}{(\pi aM)^{2n}}\left(1-\frac{1}{2^{1+2n}}\right)\times\left(2n+\frac{n+1-\frac{D}{2}}{2a^2M^2}\right)\zeta(2+2n)\right.$$
$$\left. -\frac{a}{2(4\pi)^{\frac{D-2}{2}}}\left(m^D - M^D - \frac{D}{8a^2}M^{D-2}\right)\Gamma\left(-\frac{D}{2}\right) + \mathcal{O}(\mathrm{e}^{-2\pi aM})\right. \tag{A68}$$

Summing up all of the contributions in Eqs. (A66)–(A68) results in

$$\sum_{\mu=0}^{D-1}(\langle \Delta\hat{T}_\mu^{\ \mu}(x)\rangle^{(1)} - \langle \Delta\hat{T}_\mu^{\ \mu}(x)\rangle_M^{(1)}) = -\frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}[m^2 B_{\frac{1}{\omega}}] \approx -\frac{\hbar}{2\pi a}\left\{\frac{32M^{D-2}}{(4\pi)^{\frac{D+3}{2}}a}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{3}{2})\Gamma(n+2-\frac{D}{2})}{(\pi aM)^{2n}}\left(1-\frac{1}{2^{1+2n}}\right)\right.$$
$$\times\left(1+\frac{1}{4a^2M^2}\right)\zeta(2+2n) - \frac{a}{2(4\pi)^{\frac{D-2}{2}}}\left(m^D - M^D - \frac{1}{4a^2}M^{D-2}\right)$$
$$\left.\times\Gamma\left(1-\frac{D}{2}\right) + \mathcal{O}(\mathrm{e}^{-2\pi aM})\right\} = -m^2(\langle\hat{\phi}^2(x)\rangle^{(1)} - \langle\hat{\phi}^2(x)\rangle_M^{(1)}), \tag{A69}$$

which is consistent with Eq. (A18), representing a nontrivial check of our results.

For large wormholes, $u \geq am \gg 1$, in most applications one can neglect the exponentially small correction $\propto \mathrm{e}^{-2\pi u_0}$, such that the series expressions in Eqs. (A64)–(A65) are rapidly converging toward the desired answer if truncated at some reasonably high value of n. With this in mind, one can insert Eqs. (A64)–(A65) into Eqs. (A35)–(A37) to obtain

$$\langle \Delta\hat{T}_t^{\ t}(x)\rangle^{(1)} - \langle \Delta\hat{T}_t^{\ t}(x)\rangle_M^{(1)} = -\frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}B_\omega = -\frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\frac{1}{\pi a^3}\left[\left(\frac{u_0}{48} - \frac{7}{3840u_0} - \frac{31}{129024{u_0}^3}\right.\right.$$
$$\left.\left. + \mathcal{O}({u_0}^{-5}, \mathrm{e}^{-2\pi u_0})\right) + \frac{1}{6}(u^3 - u_0^3)\right], \tag{A70}$$

$$\langle\Delta\hat{T}^{\,z^i}_{z^i}(x)\rangle^{(1)}-\langle\Delta\hat{T}^{\,z^i}_{z^i}(x)\rangle^{(1)}_{M}=\frac{\hbar}{2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\frac{\|\mathbf{k}\|^2}{(D-3)}B_{\frac{1}{\omega}}=\frac{\hbar}{2}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}\frac{\|\mathbf{k}\|^2}{(D-3)}\frac{1}{\pi a}\left[\left(\frac{1}{48u_0}+\frac{7}{3840{u_0}^3}+\frac{31}{43008{u_0}^5}+\mathcal{O}({u_0}^{-7},\mathrm{e}^{-2\pi u_0})\right)+\frac{1}{2}(u-u_0)\right],\tag{A71}$$

$$\langle\Delta\hat{T}^{\,\vartheta}_{\vartheta}(x)\rangle^{(1)}-\langle\Delta\hat{T}^{\,\vartheta}_{\vartheta}(x)\rangle^{(1)}_{M}=\langle\Delta\hat{T}^{\,\varphi}_{\varphi}(x)\rangle^{(1)}-\langle\Delta\hat{T}^{\,\varphi}_{\varphi}(x)\rangle^{(1)}_{M}=\frac{\hbar}{4}\int\frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}}[B_{\omega}-(\|\mathbf{k}\|^2+m^2)B_{\frac{1}{\omega}}],\tag{A72}$$

where $u=a\sqrt{\|\mathbf{k}\|^2+m^2}$, $u_0=a\sqrt{\|\mathbf{k}\|^2+m^2-\frac{1}{4}}$.

The next step is to evaluate the integrals in Eqs. (A70)–(A72). For this we use the master integral (A27) with $\beta=-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{5}{2}$ and $M^2=m^2,m^2-\frac{1}{4a^2}$ to obtain

$$\langle\Delta\hat{T}^{\,t}_{t}(x)\rangle^{(1)}-\langle\Delta\hat{T}^{\,t}_{t}(x)\rangle^{(1)}_{M}=\frac{\hbar}{(4\pi)^{\frac{D}{2}}}\left[\frac{\Gamma(-\frac{D-2}{2})}{24a^2}\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}}+\frac{7}{960a^4}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}}+\frac{31}{16128a^6}\Gamma\left(3-\frac{D}{2}\right)\times\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}-\frac{1}{2}\Gamma\left(-\frac{D}{2}\right)m^D+\frac{1}{2}\Gamma\left(-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}+\mathcal{O}(a^{-8},\mathrm{e}^{-2\pi am})\right],\tag{A73}$$

$$\begin{aligned}\langle\Delta\hat{T}^{\,z^i}_{z^i}(x)\rangle^{(1)}-\langle\Delta\hat{T}^{\,z^i}_{z^i}(x)\rangle^{(1)}_{M}=&\frac{\hbar}{(D-3)}\frac{1}{(4\pi)^{\frac{D}{2}}}\left[-\frac{\Gamma(1-\frac{D}{2})}{24a^2}\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}}+\frac{7}{960a^4}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}}+\frac{31}{5376a^6}\right.\\&\left.\times\Gamma\left(3-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}+\frac{3}{2}\Gamma\left(-\frac{D}{2}\right)m^D-\frac{3}{2}\Gamma\left(-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}+\mathcal{O}(a^{-8},\mathrm{e}^{-2\pi am})\right]\\&-\frac{\hbar}{(D-3)}\frac{1}{(4\pi)^{\frac{D}{2}}}\left[\frac{1}{12a^2}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}}+\frac{7}{480a^4}\Gamma\left(3-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}}+\frac{31}{8064a^6}\right.\\&\left.\times\Gamma\left(4-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}-\Gamma\left(1-\frac{D}{2}\right)m^D+\Gamma\left(1-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}+\mathcal{O}(a^{-8},\mathrm{e}^{-2\pi am})\right],\end{aligned}\tag{A74}$$

which can be simplified to

$$\langle\Delta\hat{T}^{\,t}_{t}(x)\rangle^{(1)}=\frac{\hbar}{(4\pi)^{\frac{D}{2}}}\left[\frac{1}{2}\Gamma\left(-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}+\frac{1}{24a^2}\Gamma\left(1-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}}+\frac{7}{960a^4}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}}+\frac{31}{16128a^6}\Gamma\left(3-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}+\frac{1}{2}\Gamma\left(-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}+\mathcal{O}(a^{-8},\mathrm{e}^{-2\pi am})\right],\tag{A75}$$

$$\langle\Delta\hat{T}^{\,z^i}_{z^i}(x)\rangle^{(1)}=\langle\Delta\hat{T}^{\,t}_{t}(x)\rangle^{(1)},\tag{A76}$$

where we added the Minkowski vacuum contributions given in Eqs. (A31)–(A32). Following the analogous procedure, from Eq. (A72) we obtain the angular contributions to the one-loop energy-momentum tensor,

$$\begin{aligned}\langle\Delta\hat{T}^{\,\vartheta}_{\vartheta}(x)\rangle^{(1)}=\langle\Delta\hat{T}^{\,\varphi}_{\varphi}(x)\rangle^{(1)}=&\frac{\hbar}{(4\pi)^{\frac{D}{2}}}\left[\frac{1}{2}\Gamma\left(-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D}{2}}-\frac{1}{8a^2}\Gamma\left(1-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}}\right.\\&\left.-\frac{17}{960a^4}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}}-\frac{457}{80640a^6}\Gamma\left(3-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}+\mathcal{O}(a^{-8},\mathrm{e}^{-2\pi am})\right].\end{aligned}\tag{A77}$$

To complete the calculation, we also evaluate the expectation value of the field operator squared,

$$\langle(\hat{\phi}(x))^2\rangle^{(1)} = \frac{\hbar}{2}\int \frac{\mathrm{d}^{D-3}k}{(2\pi)^{D-3}} B_{\frac{1}{\omega}} + \frac{\hbar}{2}\int \frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}} \frac{1}{\sqrt{m^2+\|\vec{k}\|^2}}$$
$$= \frac{\hbar}{(4\pi)^{\frac{D}{2}}}\left[\Gamma\left(1-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-2}{2}} + \frac{1}{12a^2}\Gamma\left(2-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-4}{2}} + \frac{7}{480a^4}\Gamma\left(3-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-6}{2}}\right.$$
$$\left. + \frac{31}{8064a^6}\Gamma\left(4-\frac{D}{2}\right)\left(m^2-\frac{1}{4a^2}\right)^{\frac{D-8}{2}} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi a m})\right]. \tag{A78}$$

As an accuracy check of our calculations, we now evaluate the expectation value of the quantized Lagrangian,

$$\langle\hat{\mathcal{L}}\rangle^{(1)} = -\frac{1}{2}[\delta^\mu_\nu\langle\Delta\hat{T}^\nu_\mu\rangle^{(1)} + m^2\langle(\hat{\phi}(x))^2\rangle^{(1)}] = -\frac{1}{2}\left\{(D-2)\langle\Delta\hat{T}^t_t\rangle^{(1)} + 2\langle\Delta\hat{T}^\vartheta_\vartheta\rangle^{(1)} + \left[\left(m^2-\frac{1}{4a^2}\right)+\frac{1}{4a^2}\right]\langle(\hat{\phi}(x))^2\rangle^{(1)}\right\} = 0, \tag{A79}$$

which is the expected result.

We are now ready to renormalize the result by using dimensional regularization. To prepare for renormalization, we expand Eqs. (A75)–(A77) around $D = 4$ to obtain

$$\langle\Delta\hat{T}^t_t(x)\rangle^{(1)} = \langle\Delta\hat{T}^{z^i}_{z^i}(x)\rangle^{(1)} \approx -\frac{\hbar}{64\pi^2}\left\{\left(m^4 - \frac{2m^2}{3a^2} + \frac{2}{15a^4}\right)\left[\frac{2\mu^{D-4}}{D-4} + \ln\left(\frac{m^2-\frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right]\right.$$
$$\left. - \frac{1}{12a^2}\left(m^2 - \frac{31}{40a^2}\right) - \frac{31}{4032a^6}\frac{1}{m^2-\frac{1}{4a^2}} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi a m})\right\}, \tag{A80}$$

$$\langle\Delta\hat{T}^\vartheta_\vartheta(x)\rangle^{(1)} = \langle\Delta\hat{T}^\varphi_\varphi(x)\rangle^{(1)} \approx -\frac{\hbar}{64\pi^2}\left\{\left(m^4 - \frac{2}{15a^4}\right)\left[\frac{2\mu^{D-4}}{D-4} + \ln\left(\frac{m^2-\frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right]\right.$$
$$\left. + \frac{1}{4a^2}\left(m^2 - \frac{27}{40a^2}\right) + \frac{457}{20160a^6}\frac{1}{m^2-\frac{1}{4a^2}} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi a m})\right\}, \tag{A81}$$

where we dropped the terms $\mathcal{O}(D-4)$ and we made use of [cf. Eq. (A34)],

$$\frac{M^{D-2n}}{(4\pi)^{\frac{D}{2}}} = \frac{M^{4-2n}\mu^{D-4}}{16\pi^2}\left[1 + \frac{D-4}{2}\ln\left(\frac{M^2}{4\pi\mu^2}\right)\right] + \mathcal{O}((D-4)^2), \tag{A82}$$

$$\Gamma\left(-\frac{D}{2}\right) = -\frac{1}{D-4}\left[1 + \frac{D-4}{2}\left(\gamma_E - \frac{3}{2}\right)\right] + \mathcal{O}(D-4), \tag{A83}$$

$$\Gamma\left(1-\frac{D}{2}\right) = \frac{2}{D-4}\left[1 + \frac{D-4}{2}(\gamma_E - 1)\right] + \mathcal{O}(D-4), \tag{A84}$$

$$\Gamma\left(2-\frac{D}{2}\right) = -\frac{2}{D-4}\left[1 + \frac{D-4}{2}\gamma_E\right] + \mathcal{O}(D-4). \tag{A85}$$

Observe now that the divergent terms $\propto 1/(D-4)$ in Eqs. (A80)–(A81) are not identical, such that vacuum fluctuations in the topological wormhole background are not isotropic. That means that, apart from the cosmological constant counterterm, we also need the Hilbert-Einstein and R^2 counterterms,

$$S_{\mathrm{ct}}[g_{\mu\nu}] = \int \mathrm{d}^D x\sqrt{-g}\left[\delta\left(\frac{1}{\kappa^2}\right)R - 2\delta\left(\frac{\Lambda}{\kappa^2}\right) + \delta\alpha_{\mathsf{R}^2}R^2\right], \tag{A86}$$

which contribute to the energy-momentum tensor $T^{\mu\nu}_{\mathrm{ct}}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{ct}}}{\delta g_{\mu\nu}(x)}$ as [48]

$$(T_{\mathrm{ct}}(x))^\nu_\mu = -2\delta\left(\frac{1}{\kappa^2}\right)G^\nu_\mu - 2\delta\left(\frac{\Lambda}{\kappa^2}\right)\delta^\nu_\mu$$
$$+ \delta\alpha_{\mathsf{R}^2}(-\delta^\nu_\mu R^2 - 4(G^{\mu\nu} - \nabla^\mu\nabla^\nu + g^{\mu\nu}\Box)R). \tag{A87}$$

Upon using Eqs. (9) and (5)–(7) in Eq. (A87) one obtains

$$(T_{\rm ct})_\mu^{\ \nu} = \delta\left(\frac{1}{\kappa^2}\right)\frac{2}{a^2}(\delta_\mu^{\ \nu} - \tilde{\delta}_\mu^{\ \nu}) - 2\delta\left(\frac{\Lambda}{\kappa^2}\right)\delta_\mu^{\ \nu} + \delta\alpha_{\mathsf{R}^2}\frac{4}{a^4}(\delta_\mu^{\ \nu} - 2\tilde{\delta}_\mu^{\ \nu}), \tag{A88}$$

where $\tilde{\delta}_\mu^{\ \nu} \equiv \mathrm{diag}(0,0,\ldots,0,1,1)$ is the diagonal matrix with zeros along the t and z^i directions and one along the angular (ϑ and φ) directions. Note that the result in Eq. (A88) holds close to $D = 4$, and to work in dimensions that differ by an integer from $D = 4$ one would have to add other counterterms. The counterterms in Eq. (A88) have the same structure as the primitive result in Eqs. (A80)–(A81) ($(T_{\rm ct})_t^{\ t} = (T_{\rm ct})_{z^i}^{\ z^i}$, $(T_{\rm ct})_\vartheta^{\ \vartheta} = (T_{\rm ct})_\varphi^{\ \varphi}$), so we need to consider only two linearly independent equations. A simple algebra shows the following choice of the counterterm coefficients,

$$\delta\left(\frac{\Lambda}{\kappa^2}\right) = -\frac{\hbar m^4}{64\pi^2}\times\frac{\mu^{D-4}}{D-4} + \frac{\Lambda}{\kappa^2}, \tag{A89}$$

$$\delta\left(\frac{1}{\kappa^2}\right) = -\frac{\hbar m^2}{96\pi^2}\times\frac{\mu^{D-4}}{D-4} + \frac{1}{\bar{\kappa}^2}, \tag{A90}$$

$$\delta\alpha_{\mathsf{R}^2} = \frac{\hbar m^2}{960\pi^2}\times\frac{\mu^{D-4}}{D-4} + \alpha_{\mathsf{R}^2}, \tag{A91}$$

where Λ/κ^2, $\bar{G} = \bar{\kappa}^2/(16\pi)$ and α_{R^2} are the finite cosmological constant, Newton constant, and R^2 coupling constant associated with the counterterms, respectively, and which in general are not equal to their observed values. Adding Eq. (A88) to Eqs. (A80)–(A81) results in the renormalized energy-momentum tensor, $\langle\Delta\hat{T}_\mu^{\ \nu}\rangle_{\rm ren}^{(1)} \equiv \langle\Delta\hat{T}_\mu^{\ \nu}\rangle^{(1)} + (T_{\rm ct})_\mu^{\ \nu}$,

$$\langle\Delta\hat{T}_t^{\ t}(x)\rangle_{\rm ren}^{(1)} = \langle\Delta\hat{T}_z^{\ z}(x)\rangle_{\rm ren}^{(1)} = -\frac{\hbar}{64\pi^2}\left\{\left(m^4 - \frac{2m^2}{3a^2} + \frac{2}{15a^4}\right)\left[\ln\left(\frac{m^2 - \frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right] - \frac{1}{12a^2}\left(m^2 - \frac{31}{40a^2}\right) - \frac{31}{4032a^6}\frac{1}{m^2 - \frac{1}{4a^2}} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am})\right\} - \frac{2\Lambda}{\kappa^2} + \frac{2}{a^2}\frac{1}{\bar{\kappa}^2} + \frac{4}{a^4}\alpha_{\mathsf{R}^2}, \tag{A92}$$

$$\langle\Delta\hat{T}_\vartheta^{\ \vartheta}(x)\rangle_{\rm ren}^{(1)} = \langle\Delta\hat{T}_\varphi^{\ \varphi}(x)\rangle_{\rm ren}^{(1)} = -\frac{\hbar}{64\pi^2}\left\{\left(m^4 - \frac{2}{15a^4}\right)\left[\ln\left(\frac{m^2 - \frac{1}{4a^2}}{4\pi\mu^2}\right) + \gamma_E - \frac{3}{2}\right] + \frac{1}{4a^2}\left(m^2 - \frac{27}{40a^2}\right) + \frac{457}{20160a^6}\frac{1}{m^2 - \frac{1}{4a^2}} + \mathcal{O}(a^{-8}, \mathrm{e}^{-2\pi am})\right\} - \frac{2\Lambda}{\kappa^2} - \frac{4}{a^4}\alpha_{\mathsf{R}^2}, \tag{A93}$$

where we dropped index i on z^i as there is only one z coordinate in $D = 4$. These results are used in Eqs. (64)–(65) in the main text.

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