

Exclusive J/ψ production off a dilute proton within a refined hotspot description

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We revisit the calculation of exclusive J/ψ production in electron-proton scattering within the QCD dipole model, employing a refined description of the proton in terms of gluonic hot spots in the dilute regime. In contrast to earlier studies, we incorporate key missing elements; the event-by-event fluctuations in both the number of hot spots and their local saturation scales, as well as the first relativistic corrections to the J/ψ wave function. We perform a Bayesian analysis to constrain the model parameters using HERA data. These enhancements significantly improve the agreement with HERA data. Saturation scale fluctuations are found to be more important than fluctuations in the number of hot spots. Including the first relativistic correction is found to be important, especially in order to describe the observed power-law behavior in the incoherent cross section at large t .

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I. INTRODUCTION

Recent advances in QCD have driven substantial progress in unraveling the internal structure of hadronic states—particularly protons and nuclei—in terms of their fundamental constituents; quarks and gluons. A central goal of this line of research is to elucidate how hadronic substructure influences various high-energy scattering observables, thereby bridging theoretical QCD calculations with experimental measurements. Experimentally, insights into hadronic substructure are indispensable for interpreting a wide range of data from high-energy scattering experiments. These insights rely on data from the H1 and ZEUS experiments at the DESY HERA or from the active experimental program of ultraperipheral collisions at the LHC and RHIC [1,2]. In the future, the Electron-Ion Collider (EIC) [3] is expected to provide data which shall refine our understanding of the internal structure of protons and nuclei.

The most basic description of a proton is the valence quark picture, in which the proton is treated as a bound state of three constituent quarks. This simplified model provides a reasonable approximation for the proton in low-energy processes (see, e.g., Refs. [4–6]). However, in the context of high-energy scattering—such as deep inelastic scattering (DIS) or proton-proton collisions at current and future collider energies—a more refined description becomes

necessary. At high energies, the proton is better described as a highly evolved quantum state, where additional partons (gluons and sea quarks) are dynamically generated. These partonic cascades emerge from the original valence quarks and are described by perturbative QCD formalisms, such as the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [7–10] or Balitsky-Fadin-Kuraev-Lipatov (BFKL) [11–14] evolutions.

A recent widely adopted and phenomenologically successful approach to modeling this evolved structure is the hot spot model, in which the proton is represented as an ensemble of localized regions of high gluon density, commonly referred to as gluonic hot spots [15–28]. In this picture, the gluon content is not uniformly distributed but is instead concentrated in spatially confined domains with a small transverse radius. This reflects the fluctuating nature of the proton’s internal structure at high energies and plays a critical role in describing observables sensitive to event-by-event fluctuations, such as diffractive cross sections (see, e.g., Refs. [17,20,22]) and particle correlations in proton-proton and proton-lead collisions (see e.g., [16,29–31]). For a review of the proton (and nuclear) shape fluctuations, see Ref. [32].

Many contemporary descriptions of hot spots in the proton structure (see, e.g., Refs. [17,22,26,27]) are grounded in the color glass condensate (CGC) effective field theory [33]. The CGC framework provides a natural theoretical foundation for analyzing various sources of fluctuations within hadronic wave functions. In this formalism, partons carrying large momentum fractions x are treated as static color charges that radiate gluons at small- x . Due to the high occupancy of these small- x gluons, they

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can be effectively described by classical gluon fields. To model the distribution of the color-charge sources, Gaussian correlations are assumed, as in the McLerran-Venugopalan (MV) model [34–36]. In the full nonlinear regime, hot spot models typically require numerical implementation due to the intrinsic complexity of the underlying dynamics. In the dilute limit where observables such as scattering cross sections can be expressed as expansions into the lowest nontrivial order in the gluon field strength, analytical computations become feasible and have been successfully carried out in Refs. [26,27]. However, a numerical implementation of the dilute-limit calculation of exclusive J/ψ photoproduction in electron-proton scattering in Ref. [27] has demonstrated poor agreement with the available HERA data. This discrepancy is unlikely to stem from the use of the dilute-limit approximation. In fact, some previous studies [37,38] have indicated that the linearized framework can yield satisfactory fits for exclusive heavy vector meson production in the scattering off the proton target. Instead, the observed mismatch is more plausibly attributed to the absence of certain components in the model, such as the fluctuations in the number of hot spots and in the local saturation scales.

In the present study, we revisit the above-mentioned proton hot spot model in the dilute limit and its application to exclusive J/ψ production in electron-proton scattering. We work in the small- x kinematics where the process can be described in the dipole picture [39]. Exclusive vector meson production is known to be highly sensitive to fluctuations in the hadronic target wave function. As such, it serves as a powerful probe of the proton’s internal gluonic structure—especially in the incoherent channel. To improve upon previous implementations of the model, we introduce additional sources of fluctuations; namely, event-by-event variations in both the number of gluonic hot spots and in the local saturation scales. These ingredients, which were absent in the earlier analysis presented in Ref. [27], are expected to play a crucial role in shaping the incoherent scattering spectrum.

Moreover, we examine the impact of relativistic corrections to the light cone wave function of the J/ψ . Specifically, we incorporate the first-order relativistic correction to the fully nonrelativistic approximation, with the aim of quantifying its effect on the predicted production cross sections. This correction addresses another missing component in the previous calculation and is anticipated to improve the accuracy of the model.

By construction, the model contains several free parameters that must be constrained in order to make quantitative predictions. In previous works [26,27], these parameters were manually chosen based on physical intuition and prior experience, rather than being systematically determined by comparing with the available experimental data. In the present analysis, we adopt a more rigorous and data-driven approach by employing Bayesian inference to extract the model parameters. Bayesian analysis has recently gained

significant traction in the fields of high-energy QCD and nuclear physics, where it has been successfully applied to various problems (see, e.g., [28,40–42]). In a particularly relevant application, Ref. [28] employed Bayesian inference to constrain the parameters of a hot spot model for exclusive J/ψ production in electron-proton scattering within the full nonlinear CGC framework. Their analysis yielded a set of parameters that provided excellent agreement with H1 photoproduction data at low-momentum transfer [43], demonstrating the power of Bayesian inference in extracting reliable, uncertainty-quantified model inputs from experimental measurements.

The paper is organized as follows. In Sec. II, we provide an overview of the QCD dipole picture for exclusive J/ψ production in electron-proton scattering, which serves as a powerful theoretical framework for both photoproduction and electroproduction regimes. This section also introduces the modeling of the proton wave function in terms of gluonic hot spots, along with a detailed discussion of the projectile structure—specifically, the overlaps between the virtual photon and J/ψ light cone wave functions. In Sec. III, we derive the relevant expressions for the differential cross sections associated with both coherent and incoherent exclusive J/ψ productions. The Bayesian inference framework employed for the parameter estimation is outlined in Sec. IV, and the resulting posterior distributions, along with the discussions of the model’s performance against the HERA data, are presented in Sec. V. Finally, Sec. VI summarizes our main findings and discusses potential relevant perspectives.

II. EXCLUSIVE J/ψ PRODUCTION IN THE DIPOLE PICTURE AND PROTON HOT SPOTS

A. Vector meson production

We consider exclusive J/ψ production in electron-proton scattering. Within the single-photon exchange approximation, the interaction between the electron and the proton is mediated by a virtual (or quasireal) photon γ^* of virtuality Q^2 . In the framework of the QCD dipole model, the virtual photon fluctuates into a $c\bar{c}$ dipole of transverse size $\mathbf{r}$, which then interacts with the proton target. At leading order, this dipole undergoes elastic scattering and subsequently hadronizes into a J/ψ meson in the final state (see Fig. 1). The scattering amplitude depends on the polarization of the incoming photon—either longitudinal (L) or transverse (T)—and is given by [32,44,45]

$$\begin{aligned} \mathcal{A}_{L,T}^{J/\psi}(W, Q^2, t) &= i \int d^2\mathbf{b} \int d^2\mathbf{r} \int \frac{dz}{4\pi} e^{-i\Delta \cdot [\mathbf{b} + (\frac{1}{2}-z)\mathbf{r}]} \\ &\times (\Psi_{\gamma^*}^* \Psi_{J/\psi})_{L,T}(Q^2, \mathbf{r}, z) \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}}(\mathbf{r}, \mathbf{b}, x_{IP}). \end{aligned} \quad (1)$$

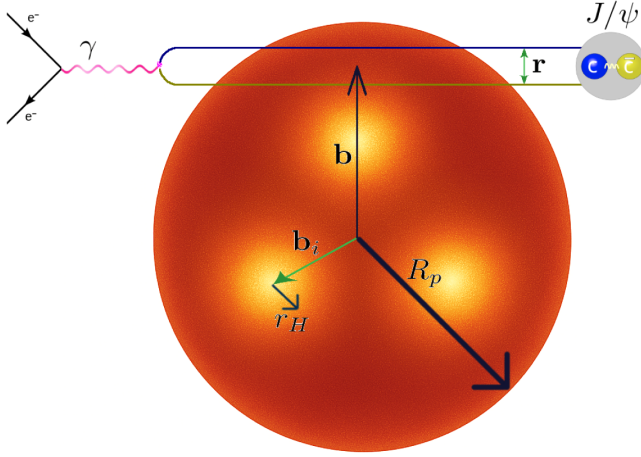


FIG. 1. Illustration of exclusive J/ψ production in electron–proton scattering, modeled within the QCD dipole framework incorporating a gluonic hot spot structure of the target proton. The hot spots are depicted as yellow regions with characteristic size r_H . Additional geometric variables shown in the figure are described in the main text.

Here z is the fraction of the large photon light cone “+” momentum carried by the quark in the dipole, and W is the center-of-mass energy of the $\gamma^* p$ process. The transverse momentum transfer Δ is the Fourier conjugate to $\mathbf{b} + (\frac{1}{2} - z)\mathbf{r}$ which is the center-of-mass of the dipole, when $\mathbf{b}$ is the center of the $q\bar{q}$ system. Furthermore, in a high-energy scattering $t = -\Delta^2$. The target momentum fraction x_{IP} is related to the invariants Q^2 , W , and t as

$$x_{IP} \equiv \frac{Q^2 + M_{J/\psi}^2 - t}{Q^2 + W^2 - m_p^2}, \quad (2)$$

where m_p is the proton mass.

According to the dipole factorization formalism in Eq. (1), the calculation of the scattering amplitude reduces to evaluating two key components; the overlap of the photon and vector meson wave functions, $(\Psi_{\gamma^*}^* \Psi_{J/\psi})$, and the dipole-proton cross section at a fixed impact parameter, $\frac{d^2\sigma_{dip}}{d^2\mathbf{b}}$. The wave function overlap encapsulates the structure of the probing system; namely, the transition of the virtual photon into a quark-antiquark dipole and its subsequent formation into a vector meson such as the J/ψ in the current study. Meanwhile, the dipole-proton cross section contains information about the internal structure of the target proton. In QCD, the proton is described as a highly complex quantum state composed of quarks and gluons, capable of exhibiting an infinite number of possible configurations. As a result, the microscopic configuration of the proton can fluctuate from one scattering event to another, leading to event-by-event variations in the interaction dynamics.

The exclusive production of the J/ψ meson—and more generally, of any meson—in high-energy $e + p$ scattering

can be categorized into two classes according to the final state of the target. If the target remains intact, the process is called coherent production. In this case, the scattering is quasielastic, and the corresponding cross section reflects an average over all possible configurations of the target without resolving its internal structure,

$$\frac{d\sigma_{L,T}^{\gamma^* p \rightarrow J/\psi p}}{dt} = \frac{1}{16\pi} |\langle A_{L,T}^{J/\psi}(W, Q^2, t) \rangle_{\text{target}}|^2. \quad (3)$$

Coherent production is therefore sensitive to the average spatial distribution of gluons within the target. In contrast, if the target breaks up after the interaction while still leaving a rapidity gap in the final state, the process is referred to as incoherent production. The corresponding cross section can be expressed in terms of the variance of the amplitude as

$$\frac{d\sigma_{L,T}^{\gamma^* p \rightarrow J/\psi p^*}}{dt} = \frac{1}{16\pi} [\langle |A_{L,T}^{J/\psi}(W, Q^2, t)|^2 \rangle_{\text{target}} - |\langle A_{L,T}^{J/\psi}(W, Q^2, t) \rangle_{\text{target}}|^2]. \quad (4)$$

This production channel is then sensitive to fluctuations in the target’s substructure and provides valuable insight into event-by-event variations in its internal gluonic configuration. See Refs. [17,32,46–49] for more detailed discussions.

B. A hot spot description of the proton

In the CGC approach, the wave function of an evolved proton involves large- x color sources and small- x gluon fields generated by the former. We can assume that the color charges are located around N_h centers $\mathbf{b}_i$ ($i = 1, \dots, N_h$) measured from the center of mass of the proton, which can be referred to as the hot spots. The transverse-space color charge density $\mu(\mathbf{x})$ within a hot spot is taken to be Gaussian,

$$\mu^2(\mathbf{x}) = \frac{\mu_0^2 Q_s^2 e^{-\sigma_s^2/2}}{2\pi r_H^2} \exp\left(-\frac{\mathbf{x}^2}{2r_H^2}\right), \quad (5)$$

where r_H is the hot spot radius and μ_0^2 is a parameter characterizing the average amount of color charge in the hot spot (average saturation scale $\langle Q_s \rangle$).

The factor $Q_s^2 = Q_s^2 / \langle Q_s^2 \rangle$ describes the saturation scale fluctuations. Beyond the calculation in Refs. [26,27], we include this source of fluctuations in the current analysis. The event-by-event Q_s^2 fluctuations are taken to follow the log-normal distribution of width σ_s as in Refs. [18,50,51]:

$$P[\log(Q_s^2)] = \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left[-\frac{\log^2(Q_s^2)}{2\sigma_s^2}\right]. \quad (6)$$

The expectation value of Q_s^2 computed from the log-normal distribution given in Eq. (6) is $e^{\sigma_s^2/2}$, rather than 1.

To correct for this effect and ensure that the average Q_s^2 remains unchanged, we scale the values of Q_s^2 generated from this distribution by the inverse of this factor. This normalization procedure accounts for the appearance of the compensating factor $e^{-\sigma_s^2/2}$ in Eq. (5).

The color charge density $\rho^a(\mathbf{x})$ at a transverse position $\mathbf{x}$ fluctuates on an event-by-event basis. Based on the MV model [34–36], we assume Gaussian statistics for the distribution of the color charges, according to which the one-point and two-point correlators of color charge density read,

$$\langle \rho^a(\mathbf{x}) \rangle_{\text{CGC}} = 0, \quad (7a)$$

$$\begin{aligned} \langle \rho^a(\mathbf{x}) \rho^b(\mathbf{y}) \rangle_{\text{CGC}} &= \frac{3}{N_h} \sum_{i=1}^{N_h} \mu^2 \left(\frac{\mathbf{x} + \mathbf{y}}{2} - \mathbf{b}_i \right) \\ &\times \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta^{ab}. \end{aligned} \quad (7b)$$

Note that here the longitudinal dependence is implicitly integrated out, and the average over the Q_s fluctuation has not yet been performed in these correlators. The two-point correlator (7b) is normalized to the number of hot spots N_h to avoid the increase of color charge in the proton as the number of hot spots increases. The factor 3 in Eq. (7b) is to recover the unnormalized correlator in Refs. [26,27] in which the number of hot spots was fixed at the value $N_h = 3$. The condition (7a) comes from the requirement of color neutrality. The Dirac delta function in Eq. (7b) reflects a feature of the MV model in which two color charges at two different transverse positions are not correlated. Higher-order correlators can then be calculated from Eq. (7) using Wick's theorem.

The transverse positions $\mathbf{b}_i$ ($i = 1, \dots, N_h$) of the hot spots also fluctuate on an event-by-event basis according to a Gaussian distribution given by

$$T_p(\mathbf{b}_i) = \frac{1}{2\pi R_p^2} \exp\left(-\frac{\mathbf{b}_i^2}{2R_p^2}\right), \quad (8)$$

where R_p is the effective size of the proton. Following [26,27], we require the center of mass of the hot spots to coincide with the center of mass of the proton, $(1/N_h) \times \sum_{i=1}^{N_h} \mathbf{b}_i = 0$.

In addition to the saturation scale fluctuation, we also allow the number of hot spots N_h to vary from event to event around an expected value $\bar{N}_h > 1$. This type of fluctuation is modeled using a zero-truncated Poisson distribution, following the approach described in Ref. [21]; the probability of having N_h hot spots is

$$P_\Lambda(N_h) = \frac{\Lambda^{N_h}}{(e^\Lambda - 1)N_h!}. \quad (9)$$

Here $\Lambda > 0$ solves $\Lambda/(1 - e^{-\Lambda}) = \bar{N}_h$.

With the above description of the proton wave function, the average over target configurations in Eqs. (3) and (4) reads

$$\begin{aligned} \langle \mathcal{O} \rangle_{\text{target}} &= \sum_{N_h=1}^{+\infty} P_\Lambda(N_h) \left(\frac{2\pi R_p^2}{N_h} \right) \\ &\times \int \left[\prod_{k=1}^{N_h} d^2 \mathbf{b}_k T_h(\mathbf{b}_k) \right] \delta^{(2)} \left(\frac{1}{N_h} \sum_{i=1}^{N_h} \mathbf{b}_i \right) \\ &\times \int \left\{ \prod_{j=1}^{N_h} d[\log(Q_{s,j}^2)] P[\log(Q_{s,j}^2)] \right\} \langle \mathcal{O} \rangle_{\text{CGC}}, \end{aligned} \quad (10)$$

where the factor $(2\pi R_p^2)/N_h$ is to ensure the normalization condition $\langle \mathbf{1} \rangle_{\text{target}} = 1$.

To describe the data across various values of Q^2 and W , the model should account for the energy evolution of the system. This can be achieved by incorporating the JIMWLK [52–58] or BK [59,60] evolutions into the framework, or by appropriately modeling the energy dependence of the relevant parameters (see, e.g., [20,21,25]). However, in this work, we focus specifically on the effects of fluctuations in the saturation scale Q_s and the number of hot spots N_h , as well as of the first relativistic correction to the exclusive J/ψ production within the dilute proton hot spot model. To this end, we fit the model to data at fixed values of Q^2 and W , allowing the momentum transfer t to vary within a limited range. In this kinematics, the energy dependence can be considered negligible.

C. Wave function overlaps

The calculation of the wave function overlaps that appear in Eq. (1) requires information on the $c\bar{c}$ wave functions of the virtual photon γ^* and of J/ψ . The branching $\gamma^* \rightarrow c\bar{c}$ is a simple perturbative QED process and the photon wave function can be computed using the light-cone perturbation theory [61–63]. They are quoted in Appendix A.

The J/ψ light cone wave function is a nonperturbative object. As c is a heavy quark, the relative velocity of the $c\bar{c}$ pair is parametrically small. Thus, one can expand the J/ψ wave function in terms of this small parameter [the so-called nonrelativistic QCD (NRQCD) expansion]. In Ref. [64], the authors proposed a parametrization for the heavy vector meson wave functions based on NRQCD long-distance matrix elements. Using this method, they extracted the $q\bar{q}$ component of the wave functions up to the first relativistic correction. This correction was shown to have a significant effect on J/ψ production phenomenology. On the other hand, in a recent analytical hot spot model calculation for exclusive J/ψ production [27], the authors used the fully nonrelativistic limit of the wave function. For highly massive states like $b\bar{b}$, this is a very

good approximation. However, for charmonium states like J/ψ , the relativistic correction may have a significant effect. Therefore, we shall include the first relativistic correction in our calculation to assess its effect on the production cross sections. The corresponding wave functions are quoted in Appendix B. The wave function overlaps can be computed using the wave functions quoted in Appendixes A and B. The results after integrating over the quark's momentum fraction z read [64]

$$\int_0^1 \frac{dz}{4\pi} (\Psi_{J/\psi}^* \Psi_{\gamma^*})_{L}^{c\bar{c}} e^{i(z-\frac{1}{2})\mathbf{r}\cdot\Delta} = -\frac{e_c e Q \sqrt{2N_c}}{8\pi\sqrt{m_c}} \left\{ AK_0(\epsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{9}{2} K_0(\epsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\epsilon|\mathbf{r}|) - \frac{Q^2 |\mathbf{r}|}{4\epsilon} K_1(\epsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\epsilon|\mathbf{r}|) \right] \right\}, \quad (11a)$$

$$\int_0^1 \frac{dz}{4\pi} (\Psi_{J/\psi}^* \Psi_{\gamma^*})_{T}^{c\bar{c}} e^{i(z-\frac{1}{2})\mathbf{r}\cdot\Delta} = -\frac{e_c e \sqrt{2m_c N_c}}{4\pi} \left\{ AK_0(\epsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{7}{2} K_0(\epsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\epsilon|\mathbf{r}|) - \frac{|\mathbf{r}|}{2\epsilon} (Q^2 + 2m_c^2) K_1(\epsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} |\mathbf{r}|^2 \Delta^2 K_0(\epsilon|\mathbf{r}|) \right] \right\}, \quad (11b)$$

where N_c is the number of colors, $e_c = 2/3$ is the fractional charge of the charm quark, θ_r is the angle between $\mathbf{r}$ and Δ , and $\epsilon^2 = \frac{Q^2}{4} + m_c^2$. The coefficients A and B read

$$A \approx 0.213 \text{ GeV}^{3/2}, \quad B \approx -0.0157 \text{ GeV}^{7/2}. \quad (12)$$

In the fully nonrelativistic limit, B vanishes and the corresponding value of A is slightly different from the above value, $A_{NR} \approx 0.211 \text{ GeV}^{3/2}$ [27]. At $|\mathbf{r}| = r_m \sim 0.6 \text{ fm}$, the wave function overlaps (11) become negative when the first relativistic correction is taken into account. We will set the overlaps to zero at $|\mathbf{r}| > r_m$ to avoid the appearance of the node in the J/ψ wave functions. In Ref. [64], this cutoff was checked to have a negligible effect when convoluting with the IPsat dipole cross section. In this study, the effect of this cutoff is more significant (although still not large), as the dipole cross section derived in the dilute limit in the next section does not saturate at large $|\mathbf{r}|$.

III. EXCLUSIVE SCATTERING IN THE DILUTE LIMIT

A. Coherent production

The dipole-proton cross section at a fixed impact parameter can be expressed in terms of Wilson lines $V(\mathbf{x})$ as

$$\frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}}(\mathbf{r}, \mathbf{b}) = 2 \left(1 - \frac{1}{N_c} \text{Tr} V(\mathbf{x}) V^\dagger(\mathbf{y}) \right), \quad (13)$$

where the impact parameter $\mathbf{b}$ and the transverse dipole size $\mathbf{r}$ are related to the transverse coordinates $\mathbf{x}$ and $\mathbf{y}$ of the (anti)quark as

$$\mathbf{b} = \frac{\mathbf{x} + \mathbf{y}}{2}, \quad \mathbf{r} = \mathbf{x} - \mathbf{y}. \quad (14)$$

The lightlike fundamental Wilson line $V(\mathbf{x})$ represents the eikonal propagation of a quark at transverse position $\mathbf{x}$ through the target color field, and is defined as a path-ordered exponential along the plus direction,

$$V(\mathbf{x}) = \mathcal{P}_+ \exp \left[ig \int_{-\infty}^{+\infty} dz^+ A_a^-(z^+, \mathbf{x}) t^a \right] = \mathcal{P}_+ \exp \left[ig \int d^2\mathbf{z} G(\mathbf{x} - \mathbf{z}) \rho_a(\mathbf{z}) t^a \right], \quad (15)$$

where g is the strong coupling constant and the Green's function is given by

$$G(\mathbf{x} - \mathbf{z}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{z})}}{\mathbf{k}^2 + m^2}, \quad (16)$$

with an infrared cutoff m^2 to suppress the long-range Coulomb tail of the color field. Following Ref. [27], the CGC average of the dipole cross section in the dilute limit employing Eq. (7) reads

$$\begin{aligned} \left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}} \right\rangle_{\text{CGC}} &= \frac{3g^2 C_F}{N_h} e^{-\sigma_s^2/2} \int d^2\mathbf{z} \sum_{i=1}^{N_h} \mu^2(\mathbf{z} - \mathbf{b}_i) \\ &\times \left[G\left(\mathbf{b} + \frac{\mathbf{r}}{2} - \mathbf{z}\right) G\left(\mathbf{b} + \frac{\mathbf{r}}{2} - \mathbf{z}\right) \right. \\ &+ G\left(\mathbf{b} - \frac{\mathbf{r}}{2} - \mathbf{z}\right) G\left(\mathbf{b} - \frac{\mathbf{r}}{2} - \mathbf{z}\right) \\ &\left. - 2G\left(\mathbf{b} + \frac{\mathbf{r}}{2} - \mathbf{z}\right) G\left(\mathbf{b} - \frac{\mathbf{r}}{2} - \mathbf{z}\right) \right], \quad (17) \end{aligned}$$

where $C_F = (N_c^2 - 1)/(2N_c)$ is the fundamental Casimir of $su(N_c)$. The additional factor of $e^{-\sigma_s^2/2}$ enforces the same average Q_s^2 independently of the Q_s fluctuations, see the discussion in Sec. III. This factor cancels out exactly when

averaging over Q_s^2 fluctuations using Eq. (6). Therefore, the Q_s^2 fluctuations have no effect on the average dipole cross section in the dilute limit.

Performing the average over the hot spot transverse positions as in Ref. [27], we obtain

$$\left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}} \right\rangle_{\text{target}} = 3g^2 C_F \sum_{N_h=1}^{+\infty} P_\Lambda(N_h) \times \int d^2\mathbf{z} F_1(\mathbf{z}) \Omega\left(\mathbf{b} + \frac{\mathbf{r}}{2}, \mathbf{b} - \frac{\mathbf{r}}{2}, \mathbf{z}, \mathbf{z}\right), \quad (18)$$

where

$$\Omega(\mathbf{x}, \mathbf{y}, \mathbf{z}_1, \mathbf{z}_2) \equiv G(\mathbf{x} - \mathbf{z}_1)G(\mathbf{x} - \mathbf{z}_2) + G(\mathbf{y} - \mathbf{z}_1)G(\mathbf{y} - \mathbf{z}_2) - 2G(\mathbf{x} - \mathbf{z}_1)G(\mathbf{y} - \mathbf{z}_2) \quad (19)$$

and

$$F_1(\mathbf{z}) \equiv \frac{\mu_0^2}{2\pi(r_H^2 + \frac{N_h-1}{N_h}R_p^2)} \exp\left[-\frac{1}{2} \frac{\mathbf{z}^2}{r_H^2 + \frac{N_h-1}{N_h}R_p^2}\right]. \quad (20)$$

The function $F_1(\mathbf{z})$ is the one-point function of the color charge content in the hot spots averaging over their spatial distribution, $F_1(\mathbf{z}) = \sum_{i=1}^{N_h} \langle \mu^2(\mathbf{z} - \mathbf{b}_i) \rangle_{hs}$. After the Fourier transform, we obtain

$$\begin{aligned} & \int d^2\mathbf{b} e^{-i\mathbf{b} \cdot \Delta} \left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}} \right\rangle_{\text{target}} \\ &= \frac{3g^2\mu_0^2 C_F}{\pi} \exp\left[-\frac{1}{2}(r_H^2 + R_p^2)\Delta^2\right] \\ & \times \left[\cos\left(\frac{1}{2}\mathbf{r} \cdot \Delta\right) \psi(0, \Delta) - \psi(\mathbf{r}, \Delta) \right] \\ & \times \sum_{N_h=1}^{+\infty} P_\Lambda(N_h) \exp\left(\frac{R_p^2\Delta^2}{2N_h}\right), \end{aligned} \quad (21)$$

where

$$\begin{aligned} \psi(\mathbf{r}, \Delta) &= \int_0^{1/2} d\eta \cos(\eta \mathbf{r} \cdot \Delta) \\ & \times \frac{|\mathbf{r}| K_1\left(|\mathbf{r}| \sqrt{-\Delta^2 \eta^2 + \frac{\Delta^2}{4} + m^2}\right)}{\sqrt{-\Delta^2 \eta^2 + \frac{\Delta^2}{4} + m^2}}, \end{aligned} \quad (22a)$$

$$\begin{aligned} \psi(0, \Delta) &= \int_0^{1/2} \frac{d\eta}{-\Delta^2 \eta^2 + \frac{\Delta^2}{4} + m^2} \\ &= \frac{2 \operatorname{arctanh} \frac{|\Delta|}{\sqrt{\Delta^2 + 4m^2}}}{|\Delta| \sqrt{\Delta^2 + 4m^2}}. \end{aligned} \quad (22b)$$

The coherent exclusive J/ψ cross section can now be computed from Eqs. (1), (3), (11), and (21). It reads

$$\frac{d\sigma_{T,L}^{J/\psi p}}{dt} = \frac{9C_{T,L}^2 g^4 \mu_0^4 C_F^2}{16\pi \pi^2} e^{-(r_H^2 + R_p^2)|t|} \times \left[\sum_{N_h=1}^{+\infty} P_\Lambda(N_h) e^{\frac{R_p^2}{2N_h}|t|} \right]^2 \mathcal{M}_{T,L}^2, \quad (23)$$

where

$$C_T = \frac{e_c e \sqrt{2m_c N_c}}{4\pi}, \quad C_L = \frac{e_c e Q}{8\pi} \sqrt{\frac{2N_c}{m_c}}, \quad (24)$$

$$\begin{aligned} \mathcal{M}_T &= \int_{|\mathbf{r}| < r_m} d^2\mathbf{r} \left[\cos\left(\frac{1}{2}\mathbf{r} \cdot \Delta\right) \psi(0, \Delta) - \psi(\mathbf{r}, \Delta) \right] \\ & \times \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{7}{2} K_0(\varepsilon|\mathbf{r}|) \right. \right. \\ & \left. \left. + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \left(\frac{Q^2}{4\varepsilon} + \varepsilon \right) |\mathbf{r}| K_1(\varepsilon|\mathbf{r}|) \right. \right. \\ & \left. \left. + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}, \end{aligned} \quad (25)$$

and

$$\begin{aligned} \mathcal{M}_L &= \int_{|\mathbf{r}| < r_m} d^2\mathbf{r} \left[\cos\left(\frac{1}{2}\mathbf{r} \cdot \Delta\right) \psi(0, \Delta) - \psi(\mathbf{r}, \Delta) \right] \\ & \times \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{9}{2} K_0(\varepsilon|\mathbf{r}|) \right. \right. \\ & \left. \left. + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \frac{Q^2 |\mathbf{r}|}{4\varepsilon} K_1(\varepsilon|\mathbf{r}|) \right. \right. \\ & \left. \left. + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}. \end{aligned} \quad (26)$$

As was mentioned in the previous section, the cutoff $|\mathbf{r}| < r_m$ in the above integrals is to avoid the large- $|\mathbf{r}|$ region where the wave-function overlaps change sign (there should be no node in the J/ψ wave function). Comparing to the corresponding expression for the coherent cross sections in Ref. [27], we see that Eq. (23) involves additionally the dependence on the N_h fluctuation and on the relativistic correction to the J/ψ nonrelativistic wave function. The Q_s fluctuations have no influence on the coherent diffractive cross sections (23) in the dilute limit. The dependence on the number of hot spot fluctuations factorizes as a squared summation which modifies the t spectra and as such effectively rescales the proton's size. To see that more explicitly, let us perform a small- $|t|$ estimation. In this limit, we have

$$\begin{aligned} \sum_{N_h=1}^{+\infty} P_\Lambda(N_h) e^{\frac{R_p^2}{2N_h}|t|} &\approx \sum_{N_h=1}^{+\infty} P_\Lambda(N_h) \left(1 + \frac{R_p^2}{2N_h}|t|\right) \\ &= 1 + \frac{R_p^2}{2}|t| \mathcal{F}(\Lambda) \approx e^{\frac{R_p^2}{2}|t| \mathcal{F}(\Lambda)}, \end{aligned} \quad (27)$$

where

$$\mathcal{F}(\Lambda) \equiv \sum_{N_h=1}^{\infty} P_\Lambda(N_h) \frac{1}{N_h} = \frac{Ei(\Lambda) - \log(\Lambda) - \gamma_E}{e^\Lambda - 1}, \quad (28)$$

with $Ei(x)$ being the exponential integral and $\gamma_E \approx 0.5772 \dots$ the Euler-Mascheroni constant. At small $|t|$, the differential cross section behaves as

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow J/\psi p}}{dt} \sim e^{-[r_H^2 + R_p^2(1 - \mathcal{F}(\Lambda))]|t|}. \quad (29)$$

In the case of no N_h fluctuation, we have

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow J/\psi p}}{dt} \propto e^{-[r_H^2 + R_p^2(1 - \frac{1}{N_h})]|t|}. \quad (30)$$

The effect of fluctuations in the number of hot spots on the t -spectrum can be estimated as follows. Taking a typical value $\overline{N_h} = 3$ corresponding to $\Lambda \approx 2.82$, we get $1 - \mathcal{F}(\Lambda) \approx 0.54 < 1 - 1/3 \approx 0.67$. This effectively makes the spectrum with N_h fluctuations less steeply falling corresponding to a smaller proton compared to the case with a fixed number of $N_h = 3$ hot spots. The factors $\mathcal{F}(\Lambda)$ and $1/N_h$ in Eqs. (29) and (30) are a consequence of forcing the center of mass of the hot spots to coincide with the center of mass of the proton. In the limit of a large number of hot spots, or when the mentioned center-of-mass constraint is relaxed, these terms modifying the t slope and the proton size would vanish. In the latter case, the N_h fluctuations would have no effect on the coherent J/ψ production in the dilute limit. From Eq. (23), one can observe that this effect holds for the whole t range.

B. Incoherent production

In order to evaluate the incoherent cross section, Eq. (4), we first need to calculate the following quantity:

$$\left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}}(\mathbf{r}, \mathbf{b}) \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}'}(\mathbf{r}', \mathbf{b}') \right\rangle_{\text{target}}. \quad (31)$$

Expanding the squared amplitude to the order ρ^4 and performing the CGC average as in Ref. [27], we get

$$\begin{aligned} &\left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}}(\mathbf{r}, \mathbf{b}) \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}'}(\mathbf{r}', \mathbf{b}') \right\rangle_{\text{CGC}} \\ &= 9g^4 \int d^2\mathbf{z}_1 d^2\mathbf{z}_2 \left[C_F^2 \Omega(\mathbf{x}, \mathbf{y}, \mathbf{z}_1, \mathbf{z}_1) \Omega(\mathbf{x}', \mathbf{y}', \mathbf{z}_2, \mathbf{z}_2) \right. \\ &\quad + \frac{C_F}{2N_c} \Omega(\mathbf{x}, \mathbf{y}, \mathbf{z}_1, \mathbf{z}_2) \Omega(\mathbf{x}', \mathbf{y}', \mathbf{z}_1, \mathbf{z}_2) \\ &\quad + \left. \frac{C_F}{2N_c} \Omega(\mathbf{x}, \mathbf{y}, \mathbf{z}_1, \mathbf{z}_2) \Omega(\mathbf{x}', \mathbf{y}', \mathbf{z}_2, \mathbf{z}_1) \right] \\ &\quad \times \left[\frac{1}{N_h^2} \sum_{i=1}^{N_h} \mu^2(\mathbf{z}_1 - \mathbf{b}_i) \mu^2(\mathbf{z}_2 - \mathbf{b}_i) \right. \\ &\quad + \left. \frac{1}{N_h^2} \sum_{i \neq j}^{N_h} \mu^2(\mathbf{z}_1 - \mathbf{b}_i) \mu^2(\mathbf{z}_2 - \mathbf{b}_j) \right]. \end{aligned} \quad (32)$$

Here $\mathbf{r}' = \mathbf{x}' - \mathbf{y}'$ and $\mathbf{b}' = (\mathbf{x}' + \mathbf{y}')/2$. The first term proportional to C_F^2 in the first square bracket of Eq. (32) represents the contribution in which the color structures of the amplitude and of the conjugate amplitude are disconnected (DC). Meanwhile, the two remaining terms are connected (C) color structures.

In the second bracket, the first term corresponds to the case where the color fields originate from the same hot spot. Averaging that contribution over the hot spot transverse positions (denoted by $\langle \mathcal{O} \rangle_{\text{hs}}$) and the Q_s fluctuations ($\langle \mathcal{O} \rangle_{Q_s}$) gives

$$\begin{aligned} F_2(\mathbf{z}_1, \mathbf{z}_2) &\equiv \sum_{i=1}^{N_h} \langle \mu^2(\mathbf{z}_1 - \mathbf{b}_i) \mu^2(\mathbf{z}_2 - \mathbf{b}_i) \rangle_{\text{hs}, Q_s} = \sum_{i=1}^{N_h} e^{-\sigma_s^2} \int d[\log(Q_{s,i}^2)] P[\log(Q_{s,i}^2)] Q_{s,i}^4 \\ &\quad \times \left(\frac{\mu_0^4 R_p^2 N_h}{2\pi r_H^4} \right) \int \prod_{k=1}^{N_h} d^2\mathbf{b}_k T_h(\mathbf{b}_k) \delta^{(2)}\left(\sum_{j=1}^{N_h} \mathbf{b}_j\right) \exp\left[-\frac{(\mathbf{z}_1 - \mathbf{b}_i)^2}{2r_H^2} - \frac{(\mathbf{z}_2 - \mathbf{b}_i)^2}{2r_H^2}\right] \\ &= e^{\sigma_s^2} \left(\frac{\mu_0^4 R_p^2 N_h^2}{2\pi r_H^4} \right) \int \prod_{k=1}^{N_h} d^2\mathbf{b}_k T_h(\mathbf{b}_k) \exp\left[-\frac{(\mathbf{z}_1 - \mathbf{b}_1)^2}{2r_H^2} - \frac{(\mathbf{z}_2 - \mathbf{b}_1)^2}{2r_H^2}\right] \delta^{(2)}\left(\sum_{j=1}^{N_h} \mathbf{b}_j\right) \\ &= e^{\sigma_s^2} \left(\frac{\mu_0^2}{2\pi r_H^2} \right)^2 \frac{N_h}{1 + \frac{N_h-1}{N_h} \frac{2R_p^2}{r_H^2}} \exp\left[-\frac{(\mathbf{z}_1 + \mathbf{z}_2)^2}{4r_H^2 \left(1 + 2\frac{N_h-1}{N_h} \frac{R_p^2}{r_H^2}\right)} - \frac{(\mathbf{z}_1 - \mathbf{z}_2)^2}{4r_H^2}\right]. \end{aligned} \quad (33)$$

Similar calculations lead to the following expression for the second summation in the second square bracket in Eq. (32):

$$F_3(\mathbf{z}_1, \mathbf{z}_2) \equiv \sum_{i \neq j}^{N_h} \langle \mu^2(\mathbf{z}_1 - \mathbf{b}_i) \mu^2(\mathbf{z}_2 - \mathbf{b}_j) \rangle_{hs, Q_s} \\ = \left(\frac{\mu_0^4}{(2\pi)^2 (r_H^2 + R_p^2)} \right) \left(\frac{N_h(N_h - 1)}{r_H^2 + \frac{N_h - 2}{N_h} R_p^2} \right) \\ \times \exp \left[-\frac{(\mathbf{z}_1 + \mathbf{z}_2)^2}{4r_H^2 \left(1 + \frac{N_h - 2}{N_h} \frac{R_p^2}{r_H^2} \right)} - \frac{(\mathbf{z}_1 - \mathbf{z}_2)^2}{4(r_H^2 + R_p^2)} \right]. \quad (34)$$

Substituting Eqs. (33) and (34) into Eq. (32) and performing the Fourier transform, one obtains

$$\int d^2\mathbf{b} d^2\mathbf{b}' e^{-i(\mathbf{b}-\mathbf{b}') \cdot \Delta} \left\langle \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}} \frac{d^2\sigma_{\text{dip}}}{d^2\mathbf{b}'} \right\rangle_{\text{CGC}, hs, Q_s} \\ = e^{\sigma_s} \{C1\} + \{C2\} + e^{\sigma_s} \{DC1\} + \{DC2\}, \quad (35)$$

where

$$\{DC1\} \equiv \frac{9g^4 \mu_0^4 C_F^2}{\pi^2} \frac{e^{-r_H^2 \Delta^2}}{N_h} \\ \times \left\{ \psi(\mathbf{r}, \Delta) - \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) \psi(0, \Delta) \right\} \\ \times \left\{ \psi(\mathbf{r}', \Delta) - \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) \psi(0, \Delta) \right\}, \quad (36a)$$

$$\{DC2\} \equiv \frac{9g^4 \mu_0^4 C_F^2}{\pi^2} \frac{N_h - 1}{N_h} e^{-(r_H^2 + R_p^2) \Delta^2} \\ \times \left\{ \psi(\mathbf{r}, \Delta) - \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) \psi(0, \Delta) \right\} \\ \times \left\{ \psi(\mathbf{r}', \Delta) - \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) \psi(0, \Delta) \right\}, \quad (36b)$$

$$\{C1\} \equiv \frac{9g^4 \mu_0^4 C_F}{8\pi^4 N_c} \frac{1}{N_h} \int d^2\mathbf{k} d^2\mathbf{k}' e^{-r_H^2 (\mathbf{k} + \mathbf{k}')^2} \\ \times \frac{1}{[(\mathbf{k} + \frac{\Delta}{2})^2 + m^2] [(\mathbf{k} - \frac{\Delta}{2})^2 + m^2]} \\ \times \frac{1}{[(\mathbf{k}' + \frac{\Delta}{2})^2 + m^2] [(\mathbf{k}' - \frac{\Delta}{2})^2 + m^2]} \\ \times \left\{ 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) \right. \\ \left. - 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) e^{i\mathbf{k}' \cdot \mathbf{r}'} + e^{i\mathbf{k} \cdot \mathbf{r} + i\mathbf{k}' \cdot \mathbf{r}'} \right. \\ \left. - 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) e^{i\mathbf{k} \cdot \mathbf{r}} + e^{i\mathbf{k} \cdot \mathbf{r} - i\mathbf{k}' \cdot \mathbf{r}'} \right\}, \quad (36c)$$

$$\{C2\} \equiv \frac{9g^4 \mu_0^4 C_F}{8\pi^4 N_c} \frac{N_h - 1}{N_h} \int d^2\mathbf{k} d^2\mathbf{k}' e^{-(r_H^2 + R_p^2) (\mathbf{k} + \mathbf{k}')^2} \\ \times \frac{1}{[(\mathbf{k} + \frac{\Delta}{2})^2 + m^2] [(\mathbf{k} - \frac{\Delta}{2})^2 + m^2]} \\ \times \frac{1}{[(\mathbf{k}' + \frac{\Delta}{2})^2 + m^2] [(\mathbf{k}' - \frac{\Delta}{2})^2 + m^2]} \\ \times \left\{ 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) \right. \\ \left. - 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}\right) e^{i\mathbf{k}' \cdot \mathbf{r}'} + e^{i\mathbf{k} \cdot \mathbf{r} + i\mathbf{k}' \cdot \mathbf{r}'} \right. \\ \left. - 2 \cos\left(\frac{1}{2} \Delta \cdot \mathbf{r}'\right) e^{i\mathbf{k} \cdot \mathbf{r}} + e^{i\mathbf{k} \cdot \mathbf{r} - i\mathbf{k}' \cdot \mathbf{r}'} \right\}. \quad (36d)$$

Averaging Eq. (35) over the number of hot spots N_h corresponds to taking the full target average. When this result is convoluted with the wave function overlap, the diffractive cross section involving both coherent ($\gamma^* + p \rightarrow J/\psi + p$) and incoherent ($\gamma^* + p \rightarrow J/\psi + Y$) processes reads

$$\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow J/\psi(p,Y)}}{dt} = \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow J/\psi p}}{dt} + \frac{d\sigma_{T,L}^{\gamma^* p \rightarrow J/\psi Y}}{dt} \\ = e^{\sigma_s} \frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{DC1} + \frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{DC2} \\ + e^{\sigma_s} \frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{C1} + \frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{C2}. \quad (37)$$

The terms originating from the color-disconnected contribution read

$$\frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{DC1} = \frac{9C_{T,L}^2}{16\pi} \frac{g^4 \mu_0^4 C_F^2}{\pi^2} e^{-r_H^2 |t|} \mathcal{M}_{T,L}^2 \mathcal{F}(\Lambda), \quad (38a)$$

$$\frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{DC2} = \frac{9C_{T,L}^2}{16\pi} \frac{g^4 \mu_0^4 C_F^2}{\pi^2} e^{-(r_H^2 + R_p^2) |t|} \mathcal{M}_{T,L}^2 [1 - \mathcal{F}(\Lambda)]. \quad (38b)$$

The remaining terms from the color-connected part are

$$\frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{C1} = \frac{9C_{T,L}^2}{16\pi} \frac{g^4 \mu_0^4 C_F}{\pi^2 N_c} \mathcal{K}_{T,L}(t; r_H^2) \mathcal{F}(\Lambda), \quad (39a)$$

$$\frac{d\sigma_{T,L}^{\text{ci}}}{dt} \Big|_{C2} = \frac{9C_{T,L}^2}{16\pi} \frac{g^4 \mu_0^4 C_F}{\pi^2 N_c} \mathcal{K}_{T,L}(t; r_H^2 + R_p^2) [1 - \mathcal{F}(\Lambda)]. \quad (39b)$$

The auxiliary functions $\mathcal{K}_{T,L}$ are defined as

$$\mathcal{K}_{T,L}(t; \mathfrak{R}) = \int d^2\mathbf{k} d^2\mathbf{k}' \frac{e^{-\mathfrak{R}(\mathbf{k}+\mathbf{k}')^2}}{[(\mathbf{k}+\frac{\Delta}{2})^2 + m^2][(\mathbf{k}-\frac{\Delta}{2})^2 + m^2]} \times \frac{(\mathcal{X}_\Delta - \mathcal{X}_{\mathbf{k}'})_{T,L}}{[(\mathbf{k}'+\frac{\Delta}{2})^2 + m^2][(\mathbf{k}'-\frac{\Delta}{2})^2 + m^2]}, \quad (40)$$

where

$$\mathcal{X}_{\Delta;L} \equiv \frac{1}{2\pi} \int_{|\mathbf{r}| < r_m} d^2\mathbf{r} \cos\left(\frac{1}{2}\Delta \cdot \mathbf{r}\right) \times \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{9}{2} K_0(\varepsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \frac{Q^2 |\mathbf{r}|}{4\varepsilon} K_1(\varepsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}, \quad (41a)$$

$$\mathcal{X}_{\Delta;T} \equiv \int_{|\mathbf{r}| < r_m} \frac{d^2\mathbf{r}}{2\pi} \cos\left(\frac{1}{2}\Delta \cdot \mathbf{r}\right) \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{7}{2} K_0(\varepsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \left(\frac{Q^2}{4\varepsilon} + \varepsilon \right) |\mathbf{r}| K_1(\varepsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}, \quad (41b)$$

$$\mathcal{X}_{\mathbf{k};L} \equiv \frac{1}{2\pi} \int_{|\mathbf{r}| < r_m} d^2\mathbf{r} e^{i\mathbf{k}\mathbf{r}} \times \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{9}{2} K_0(\varepsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \frac{Q^2 |\mathbf{r}|}{4\varepsilon} K_1(\varepsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}, \quad (42a)$$

and

$$\mathcal{X}_{\mathbf{k};T} \equiv \frac{1}{2\pi} \int_{|\mathbf{r}| < r_m} d^2\mathbf{r} e^{i\mathbf{k}\mathbf{r}} \left\{ AK_0(\varepsilon|\mathbf{r}|) + \frac{B}{m_c^2} \left[\frac{7}{2} K_0(\varepsilon|\mathbf{r}|) + m_c^2 |\mathbf{r}|^2 K_0(\varepsilon|\mathbf{r}|) - \left(\frac{Q^2}{4\varepsilon} + \varepsilon \right) |\mathbf{r}| K_1(\varepsilon|\mathbf{r}|) + \frac{1 + \cos 2\theta_r}{8} (|\mathbf{r}|^2 \Delta^2) K_0(\varepsilon|\mathbf{r}|) \right] \right\}. \quad (42b)$$

Different from the corresponding result in Ref. [27], the total exclusive (coherent + incoherent) J/ψ production cross section (37) involves the dependence on both the Q_s and N_h fluctuations, as well as the dependence on the relativistic correction (terms proportional to B/m_c^2). From

Eq. (37), we see that the Q_s fluctuations affect only the single-hot-spot contributions ($DC1$ and $C1$).

To estimate the effect of the Q_s fluctuations on the incoherent cross section, let us ignore the N_h fluctuations and subtract the coherent contribution from Eq. (37) as was done in Ref. [27]. We denote by R_X , $X \in \{DC1, DC2, C1, C2\}$, the ratios of the 4 contributions to the incoherent cross section in the case where the Q_s fluctuations are not included. As such, $\sum_X R_X = 1$. From Ref. [27] we get that $0 < R_{C1} \sim R_{C2} < 1$, $R_{DC2} < 0$ and $R_{DC} > 0$ (and this $DC1$ component can be larger than unity). Note that the R_{DC2} component can be negative as we have subtracted the coherent cross section. As such, the Q_s fluctuations increase the incoherent cross section by a t -dependent factor,

$$\Phi_{\sigma_s} \equiv \frac{(\frac{d\sigma}{dt})_{\sigma_s > 0}^{\text{incoh}}}{(\frac{d\sigma}{dt})_{\sigma_s = 0}^{\text{incoh}}} = 1 + (e^{\sigma_s^2} - 1) \left(R_{DC1} + R_{C1} + \frac{1}{N_h} \underbrace{\frac{(\frac{d\sigma}{dt})^{\gamma^* p \rightarrow J/\psi p}}{(\frac{d\sigma}{dt})_{\sigma_s = 0}^{\text{incoh}}}}_{\mathcal{R}} \right). \quad (43)$$

Here $\mathcal{R}$ is the incoherent-to-coherent cross section ratio in the case where there are no Q_s fluctuations.¹ Note that the dependence on coherent cross section appears in Eq. (43) as we are calculating the modification factor for incoherent cross section, which is calculated by subtracting the coherent contribution from the total diffractive cross section Eq. (37).

Let us next consider the effect of Q_s fluctuations in three different kinematical regimes:

- (1) At large $|t| \gg 1/r_H^2$, color charge fluctuations, as described by the C terms, dominate. Furthermore, the coherent cross section is subleading in this regime, so we can ignore the coherent-over-incoherent ratio $\mathcal{R}$. In this case, $R_{DC1} + R_{C1} \approx R_{C1}$, $1 = \sum R_X \approx R_{C1} + R_{C2}$ and $R_{C1} \sim R_{C2}$. Therefore, $1 < \Phi_{\sigma_s} \approx 1 + (e^{\sigma_s^2} - 1)R_{C1} < e^{\sigma_s^2}$. In other words, in this regime, the incoherent cross section is enhanced by a factor less than $e^{\sigma_s^2}$;
- (2) At intermediate $|t| \sim 1/r_H^2$ where the hot spot fluctuations (DC terms) dominate and the coherent contribution is still significantly smaller than the incoherent one, $R_{DC1} \simeq 1$ and $\Phi_{\sigma_s} \approx 1 + (e^{\sigma_s^2} - 1) \times R_{DC1} \simeq e^{\sigma_s^2}$. This means that the Q_s fluctuations increase the incoherent cross section approximately by a factor of $e^{\sigma_s^2}$;
- (3) Finally, at small $|t| \ll 1/r_H^2$, the color charge fluctuations again dominate over hot spot fluctuations. However, in this range, the coherent contribution

¹Recall that the Q_s fluctuations have no effect on the coherent production.

becomes important, and the coherent-over-incoherent ratio $\mathcal{R} \sim 10$ [43]. In this case, the enhancement Φ_s depends strongly on $\mathcal{R}$ and N_h . For $\mathcal{R}/N_h > 1$, the enhancement can be significant, with $\Phi_{\sigma_s} > e^{\sigma_s^2}$.

To summarize, the Q_s fluctuations enhance the incoherent cross section by a $|t|$ -dependent factor, which is typically of the order of $e^{\sigma_s^2}$.

The dependence on the N_h fluctuations [or on Λ that controls the mean number of hot spots in (8)] also factorizes in both the single-hot-spot [as $\mathcal{F}(\Lambda)$] and the double hot spot [as $1 - \mathcal{F}(\Lambda)$] terms. At large $|t| \gtrsim 1/r_H^2$, when the coherent contribution can be negligible, the ratio of the incoherent cross section involving the N_h fluctuations versus the one where the number of hot spots is fixed to N_h^f reads

$$\begin{aligned} \Phi_\Lambda &\equiv N_h^f \mathcal{F}(\Lambda) (R_{DC1} + R_{C1}) \\ &+ \frac{N_h^f}{N_h^f - 1} [1 - \mathcal{F}(\Lambda)] (R_{DC2} + R_{C2}) \\ &= \frac{N_h^f [1 - \mathcal{F}(\Lambda)]}{N_h^f - 1} \left[1 + \frac{N_h^f \mathcal{F}(\Lambda) - 1}{1 - \mathcal{F}(\Lambda)} (R_{DC1} + R_{C1}) \right]. \end{aligned} \quad (44)$$

In particular, at $|t| \sim 1/r_H^2$ we find $\Phi_\Lambda \simeq N_h^f \mathcal{F}(\Lambda)$, which implies that the modification to the incoherent cross section depends strongly on the value of $N_h^f \mathcal{F}(\Lambda)$.

To estimate the phenomenological impact of the N_h fluctuations, we fix the number of hot spots in the reference to a typical value $N_h^f = 3$. We obtain $\Phi_\Lambda = 1.01 \dots 1.37$, where the band is obtained by varying $\overline{N}_h$ in the range $\overline{N}_h = 3 \dots 4$. For this specific choice, the N_h fluctuations have a modest effect, as Φ_Λ is close to unity.

IV. BAYESIAN PARAMETER ESTIMATION

The hot spot model contains several nonperturbative parameters that must be extracted from data. In this analysis, their values are constrained using Bayesian inference. In particular, the unknown parameters Θ are estimated from their posterior distribution, which is determined using Bayes's theorem as

$$\mathbb{P}(\Theta | \mathbf{Y}_{\text{exp}}) \propto \mathbb{P}(\mathbf{Y}_{\text{exp}} | \Theta) \mathbb{P}(\Theta). \quad (45)$$

The distribution $\mathbb{P}(\Theta)$ in the above equation encodes the prior knowledge on the model parameters. Here we choose the flat multivariate distribution with bounds corresponding to physically motivated parameter ranges.

The function $\mathbb{P}(\mathbf{Y}_{\text{exp}} | \Theta)$ is the likelihood of the model calculations with parameters Θ to match the experimental data. It is chosen to be the multivariate Gaussian distribution, for which the log-likelihood reads

$$\begin{aligned} \log [\mathbb{P}(\mathbf{Y}_{\text{exp}} | \Theta)] &= -\frac{1}{2} \log [(2\pi)^n \det \Sigma] \\ &- \frac{1}{2} [\mathbf{Y}(\Theta) - \mathbf{Y}_{\text{exp}}]^T \Sigma^{-1} [\mathbf{Y}(\Theta) - \mathbf{Y}_{\text{exp}}], \end{aligned} \quad (46)$$

where $\mathbf{Y}(\Theta)$ is the model result calculated using parameters Θ , $\mathbf{Y}_{\text{exp}}$ represents the experimental data, and $n = \dim(\mathbf{Y}_{\text{exp}})$ is the number of experimental data points used in the analysis. The matrix $\Sigma = \Sigma_{\text{exp}} + \Sigma_{\text{model}}$ is the $(n \times n)$ covariance matrix encoding the uncertainties from both experimental and model analyses. In the current study, the experimental part is assumed to be diagonal,

$$\Sigma_{\text{exp}} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2), \quad (47)$$

where σ_i , ($k = 1, \dots, n$) are experimental errors. We use the H1 2006–2007 dataset [43] for exclusive J/ψ photoproduction differential cross sections $d\sigma/dt$ in both coherent and incoherent channels at the kinematics $Q^2 = 0.1 \text{ GeV}^2$ and $W = 78 \text{ GeV}$ for the current analysis.

The model uncertainties are assessed using the covariance matrix obtained from trained Gaussian process emulators (GPEs) [65], which act as computationally efficient surrogates for the full theoretical calculations. These emulators enable rapid evaluations of model predictions across the parameter space, significantly reducing computational cost. The parameter space is explored using the Markov chain Monte Carlo (MCMC) method, implemented with the Python-based EMCEE package [66,67] with multiple random walkers. Training of the GPEs is performed using a set of points selected via the maximin Latin hypercube sampling strategy, which ensures a well-distributed and representative coverage of the parameter space.

The set of free parameters Θ to be constrained by the H1 data on the exclusive photoproduction of the J/ψ involves:

- (i) $g\mu_0$: the average amount of color charge in the hot spot;
- (ii) m^2 : the IR regulator in Eq. (16);
- (iii) R_p : parameter controlling the proton size in Eq. (8);
- (iv) r_H : parameter controlling the hot spot size in Eq. (5);
- (v) $\overline{N}_h$: the average number of hot spots when fluctuations in this quantity are taken into account. In the absence of such fluctuation, the corresponding free parameter reduces to the number of hot spots N_h . In the previous analyses [26,27], N_h is fixed at the value $N_h = 3$. In the present work, however, we treat it as a free parameter. Specifically, we sample N_h within the hypercube and define $[N_h + 0.5]$ as the instantaneous number of hot spots;
- (vi) σ_s : the magnitude of fluctuations in the saturation scale Q_s as parametrized in Eq. (6). When such fluctuations are neglected, the saturation scale is assumed to be fixed at its average value across all events, corresponding to $\sigma_s = 0$.

The charm quark mass is set to $m_c = 1.4$ GeV as in Ref. [64], consistently with the extraction of the applied NRQCD long-distance matrix elements [68].

V. NUMERICAL RESULTS

The first Bayesian fit is performed under the assumption that both N_h and Q_s fluctuations are neglected, i.e., in the nonrelativistic limit it corresponds to the setup studied in Ref. [27]. Posterior distributions obtained in the non-relativistic limit, and when the first relativistic correction is included, are shown in Fig. 2 and the corresponding maximum *a posteriori* (MAP) parameters in Table I. Comparison to the H1 data [43,69] is shown in Fig. 3.

In the fully nonrelativistic (NR or nonrel) limit, the resulting cross sections computed using the maximum *a posteriori* (MAP) values yield poor agreement with the incoherent cross section data at large $|t| \gtrsim 4$ GeV², as indicated by large reduced chi-squared values ($\chi^2/\text{d.o.f.} = 4.14$). On the other hand, the description of the available coherent and incoherent cross section data at smaller $|t|$ is good. Note that the corresponding MAP hot spot size is unphysically small ($r_H \approx 0$ GeV⁻¹), implying that the color charge density collapses into a sharply peaked, delta-functionlike structure. However, the field of an individual point charge should extend to a distance of order $1/m$ [27]. When this parameter is adjusted to a more physically reasonable value, $r_H = 0.2$ GeV⁻¹, the resulting predictions (denoted as mMAP in Fig. 3) still yield a satisfactory description of the data except at high- $|t|$. As the color field obtained from the unphysically small

TABLE I. Parameter estimation from the posterior distribution shown in Fig. 2.

Parameters	Prior range	MAP (R)	MAP (NR)
$g\mu_0$	[2.4, 4.9]	$4.037^{+0.559}_{-0.225}$	$2.857^{+1.130}_{-0.198}$
m^2 [GeV ²]	[0.008, 0.65]	$0.037^{+0.067}_{-0.023}$	$0.076^{+0.242}_{-0.048}$
R_p [GeV ⁻¹]	[1.5, 3.0]	$2.366^{+0.344}_{-0.356}$	$2.310^{+0.69}_{-0.491}$
r_H [GeV ⁻¹]	[0, 0.9]	$0.699^{+0.201}_{-0.261}$	$0.001^{+0.219}_{-0.001}$
N_h	[0.5, 7.5]	$1.760^{+1.907}_{-0.138}$	$1.608^{+4.797}_{-0.213}$
$\chi^2/\text{d.o.f.}$		1.63	4.14

color charge density already extends to distances $\sim 1/m$, adjusting the hot spot size here has only a small effect at the cross-section level.

This finding stands in contrast to the normalization mismatch argument given in Ref. [27], where a multiplicative factor of approximately 2.5 was proposed to reconcile the incoherent cross section with experimental data, attributed to the absence of additional sources of fluctuations. However, a direct comparison between the MAP (or mMAP) parameter set and the manually selected parameters used in Ref. [27] suggests that the claimed mismatch is likely an artifact of parameter selection, rather than a fundamental issue with missing fluctuations. As an example, in this fit, the number of hot spots peaks at $N_h = 2$, while it was chosen to be $N_h = 3$ in Refs. [26,27].

When relativistic corrections are included, a good description of the H1 data with $\chi^2/\text{d.o.f.} = 1.63$ is obtained at all t . Furthermore, parameter values are physically better motivated; in particular, the hot spot size can now be constrained to a finite value. The main effect of the relativistic correction is to significantly suppress the cross section, as reflected by the larger color charge density needed to compensate for this correction. This effect is expected based on Ref. [64], and can be seen from the wave function overlap (11) noticing that the nonrelativistic contribution and the relativistic correction come with opposite signs ($A > 0$, $B < 0$).

The relativistic correction is also found to have a significant effect on the shape of the incoherent t spectrum at large $|t|$, as shown in Fig. 3 (right panel). This correction changes the spectrum from its exponential shape to a power-law behavior at $|t| \gtrsim 3.5$ GeV². However, this effect actually comes from the magnitude of r_H . As discussed in Ref. [27], the hot spot fluctuations are probed at $|t| \sim 1/r_H^2$, and beyond this scale, there is a transition to the power-law (more precisely $\ln |t|/t^2$, see Ref. [27]) behavior originating from the color charge fluctuations. Therefore, there should be a transition to the power-law behavior in the non-relativistic setup also, but this transition takes place at much larger $|t|$.

Before quantifying the impact of the Q_s and number of hot spot fluctuations, let us analyze in more detail the effect

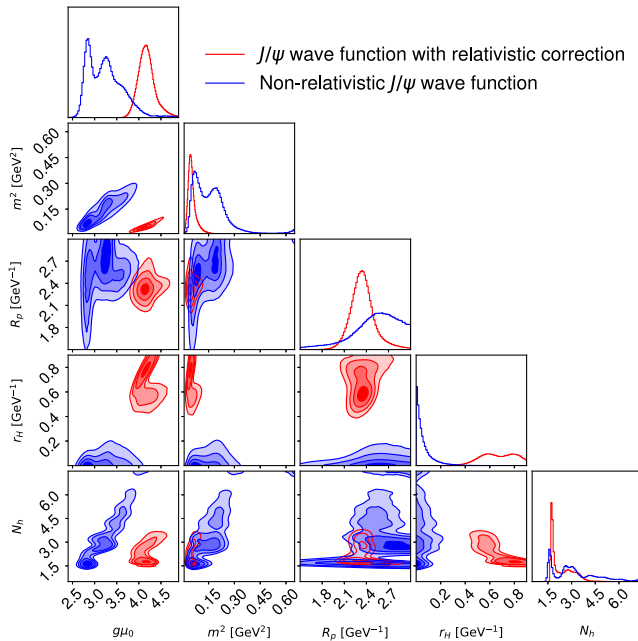


FIG. 2. Posterior distribution of parameters in the scheme without Q_s or N_h fluctuations.

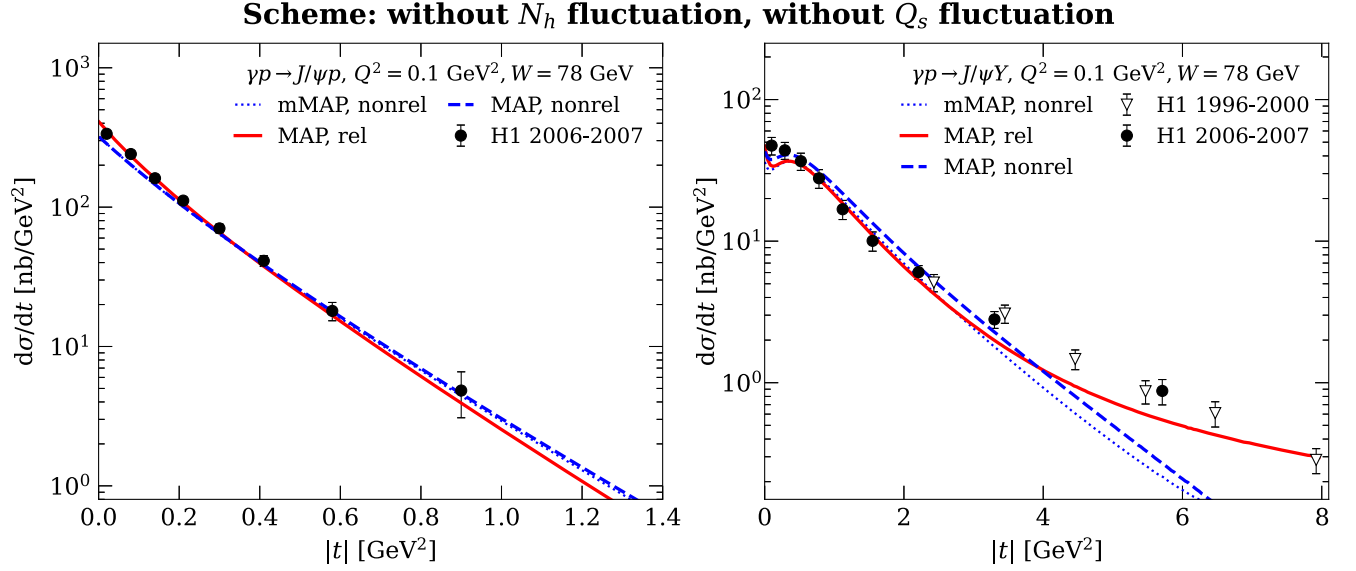


FIG. 3. Fit results compared to the H1 data [43,69] up to $|t| = 8 \text{ GeV}^2$. For the fully nonrelativistic setup, the mMAP parameter set is that of MAP but with $r_H = 0.2 \text{ GeV}^{-1}$.

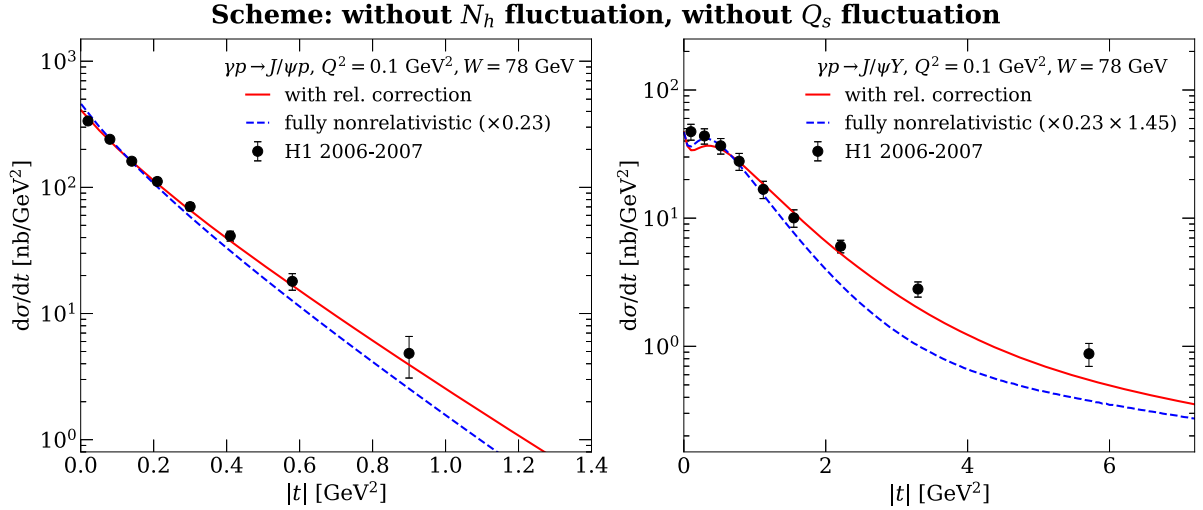


FIG. 4. Coherent and incoherent cross sections calculated in the nonrelativistic limit and with the first relativistic correction included. The scaling factor 0.23 is to scale the corresponding coherent curve to the experimental data.

of the relativistic correction to the J/ψ wave function. As already seen above and in Ref. [64], the relativistic correction significantly suppresses the photoproduction cross section. In order to quantify the effect of the relativistic correction on coherent and incoherent cross sections separately, we calculate these cross sections using the MAP parameters corresponding to the nonrelativistic setup listed in Table I, with and without the relativistic correction included in the model. Then, we separately scale the coherent and incoherent cross sections obtained in the nonrelativistic limit such that they approximately match the calculation with the relativistic correction included. The obtained spectra and the determined scaling factors are shown in Fig. 4.

We find that the scaling factors required for the coherent and incoherent cross sections differ significantly. In particular, the first relativistic correction suppresses the coherent cross section much more than the incoherent one. This is because the relativistic correction puts more weight on smaller dipoles that can more effectively resolve substructure fluctuations enhancing the incoherent cross section. As such, the relativistic correction can be accounted for as a missing normalization factor of about 2.5 for the incoherent cross section, as was discussed in Ref. [27]. Furthermore, as discussed above, including this relativistic correction is essential for describing the exponential-to-power-law transition observed in the experimental data at $t \gtrsim 3.5 \text{ GeV}^2$.

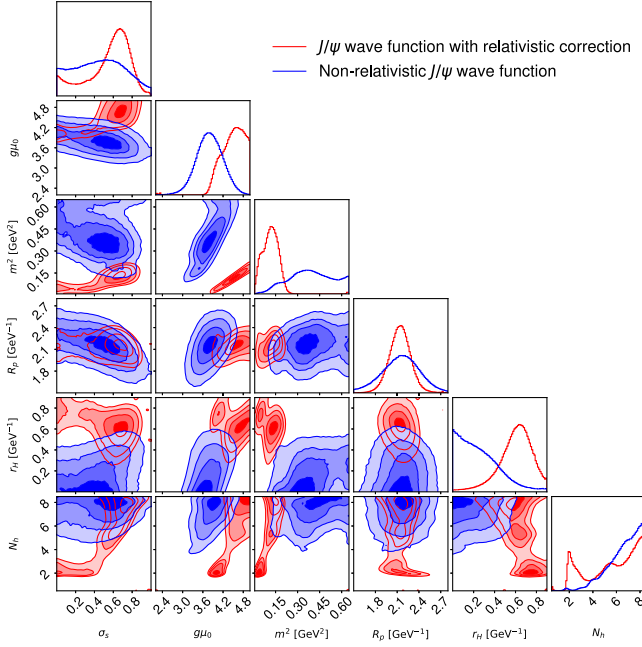


FIG. 5. Posterior distribution of parameters in the scheme with Q_s fluctuations and without N_h fluctuations (variable N_h).

TABLE II. Parameter estimation from the posterior distribution in Fig. 5.

Parameters	Prior range	MAP (R)	MAP (NR)
σ_s	[0.0, 1.0]	$0.595^{+0.252}_{-0.583}$	$0.586^{+0.290}_{-0.586}$
$g\mu_0$	[2.2, 5.0]	$4.517^{+0.483}_{-0.577}$	$3.558^{+1.014}_{-0.501}$
m^2 [GeV ²]	[0.008, 0.65]	$0.107^{+0.095}_{-0.088}$	$0.191^{+0.459}_{-0.046}$
R_p [GeV ⁻¹]	[1.5, 2.8]	$2.108^{+0.323}_{-0.264}$	$2.109^{+0.521}_{-0.479}$
r_H [GeV ⁻¹]	[0.0, 0.9]	$0.627^{+0.273}_{-0.328}$	$0.008^{+0.613}_{-0.008}$
N_h	[0.5, 8.5]	$5.098^{+3.402}_{-3.076}$	$8.186^{+0.314}_{-4.132}$
$\chi^2/\text{d.o.f.}$		1.24	1.73

Next we move beyond the setup used in Ref. [27] and include the Q_s fluctuations into the model. The posterior distribution obtained for the model parameters in this case is shown in Fig. 5, with the MAP parameters summarized in Table II. Comparison to the HERA data is shown in Fig. 6. The resulting MAP parameter sets yield improved fits to the H1 data across both modeling schemes—with and without relativistic corrections—as evidenced by the consistently lower reduced χ^2 values. Notably, for the incoherent cross section, the MAP fit incorporating relativistic corrections provides a more accurate description of the data, particularly in the small- and large- $|t|$ regions. From the MAP fit with the relativistic correction, we can conclude that by including both the relativistic correction and the event-by-event Q_s fluctuations allows a good description of the cross section data in the whole $|t|$ range, provided that $|t|$ is not very large.

Similarly to the first case, the MAP value for the hot spot size r_H is peaked at zero in the nonrelativistic limit. Consequently, the transition to the power-law behavior is again not seen at $|t| \sim 3.5$ GeV² in this setup. However, the corresponding marginal distribution of r_H is considerably flatter than the one from the setup without Q_s fluctuations.

We also consider the number of hot spots N_h to be a free parameter. However, we find that it is not well-constrained by the H1 data. Similar results have been obtained in Ref. [28]. This is because the number of hot spots is correlated with the magnitude of Q_s fluctuations, a correlation which is especially clear in the setup with relativistic corrections. With large Q_s fluctuations, some hot spots will always have a significant reduction in their density, and as such, the effective number of hot spots is reduced.

As an alternative scenario, we consider a model in which only fluctuations in the number of hot spots N_h are included, while the fluctuations in the saturation scale Q_s are absent. The results of this fit are presented in Figs. 7 and 8 and Table III. Again, the hot spot size parameter r_H

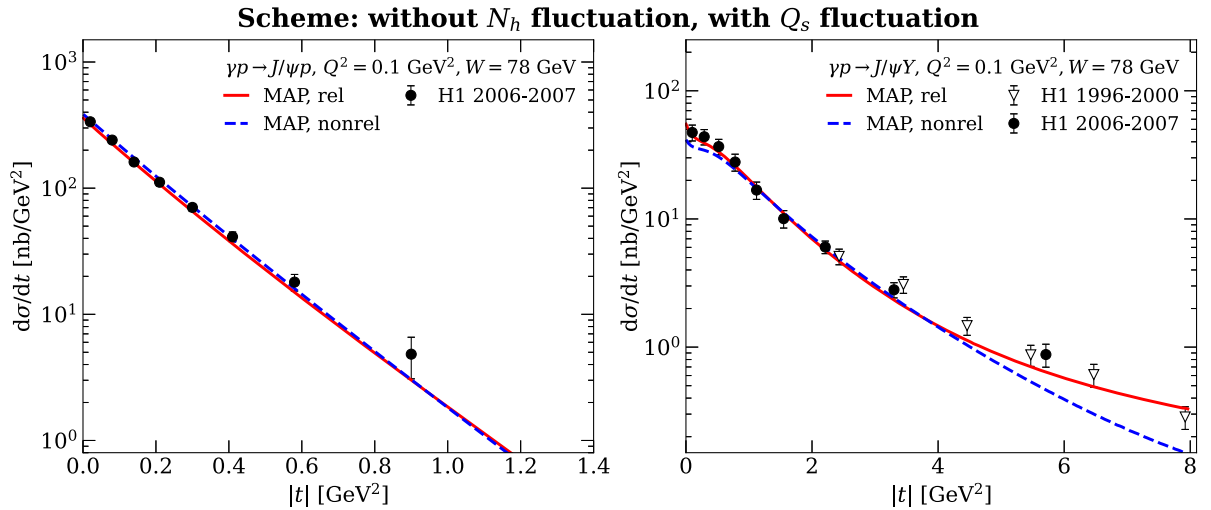


FIG. 6. Fit result compared to the H1 data [43,69].

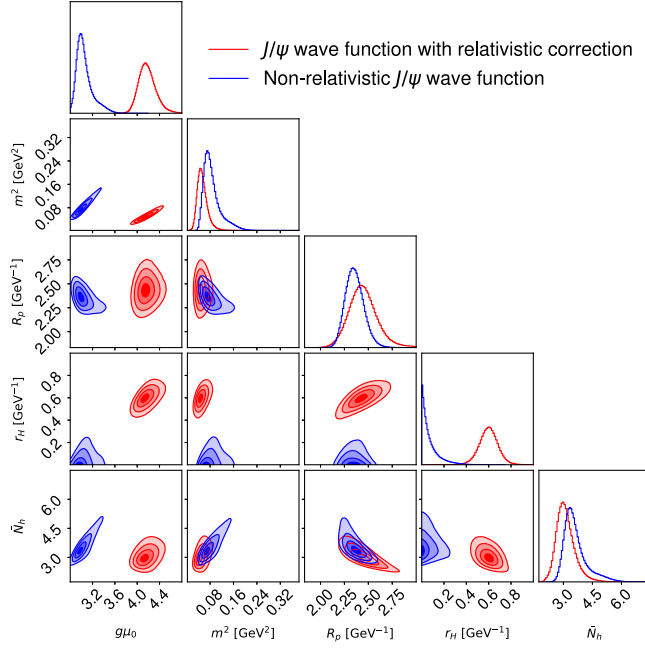


FIG. 7. Posterior distribution of parameters in the scheme without Q_s fluctuation and with N_h fluctuation.

TABLE III. Parameter estimation from the posterior distribution in Fig. 7.

Parameters	Prior range	MAP (R)	MAP (NR)
σ_s	No Q_s fluctuation	0	0
$g\mu_0$	[2.8, 4.8]	$4.116^{+0.38}_{-0.226}$	$2.946^{+0.439}_{-0.125}$
m^2 [GeV ²]	[0.001, 0.86]	$0.045^{+0.044}_{-0.022}$	$0.063^{+0.090}_{-0.021}$
R_p [GeV ⁻¹]	[1.6, 3.0]	$2.458^{+0.289}_{-0.302}$	$2.382^{+0.167}_{-0.205}$
r_H [GeV ⁻¹]	[0.0, 1.0]	$0.595^{+0.173}_{-0.17}$	$0.002^{+0.246}_{-0.001}$
$\bar{N}_h$	[1.2, 7.5]	$2.925^{+1.207}_{-0.691}$	$3.303^{+1.696}_{-0.623}$
$\chi^2/\text{d.o.f.}$		1.04	1.94

peaks at zero in the nonrelativistic limit, but otherwise all model parameters are rather well constrained by the HERA data. In particular, the mean number of hot spots $\bar{N}_h$ is constrained to be close to 3 in both setups. In this case, the MAP parameter sets yield reasonably good agreement with the H1 data. This agreement is particularly notable in the scenario that incorporates the relativistic correction. Nonetheless, we note that the description of the incoherent cross section at very small t is worse than in the setup with Q_s fluctuations described above.

In the most comprehensive scenario, the fluctuations in both N_h and Q_s are incorporated into the analysis. The outcomes of this scenario are presented in Figs. 9 and 10 and Table IV. When the relativistic correction to the J/ψ wave function is included, the analysis reveals that the MAP parameter set achieves a remarkably good agreement with the experimental data. In this setup, we find that the H1 data do not constrain the mean number of hot spots, which is strongly correlated with the magnitude of the Q_s fluctuations, as discussed above in the case with Q_s fluctuations and a fixed number of hot spots. In the fully nonrelativistic limit, the MAP parameter set provides a reasonably good description of the experimental data at $t < 4$ GeV². This outcome suggests that the model retains substantial predictive power even in the absence of relativistic corrections, particularly at small enough $|t|$.

VI. CONCLUSION

In this work, we have investigated exclusive J/ψ production in the dilute limit with an emphasis on a more comprehensive treatment of event-by-event fluctuations in the hot spot substructure of the target proton. Beyond the previously considered geometry and color charge fluctuations, we incorporated additional sources of fluctuations, allowing the number of hot spots N_h and the saturation scale Q_s associated with each hot spot to fluctuate from

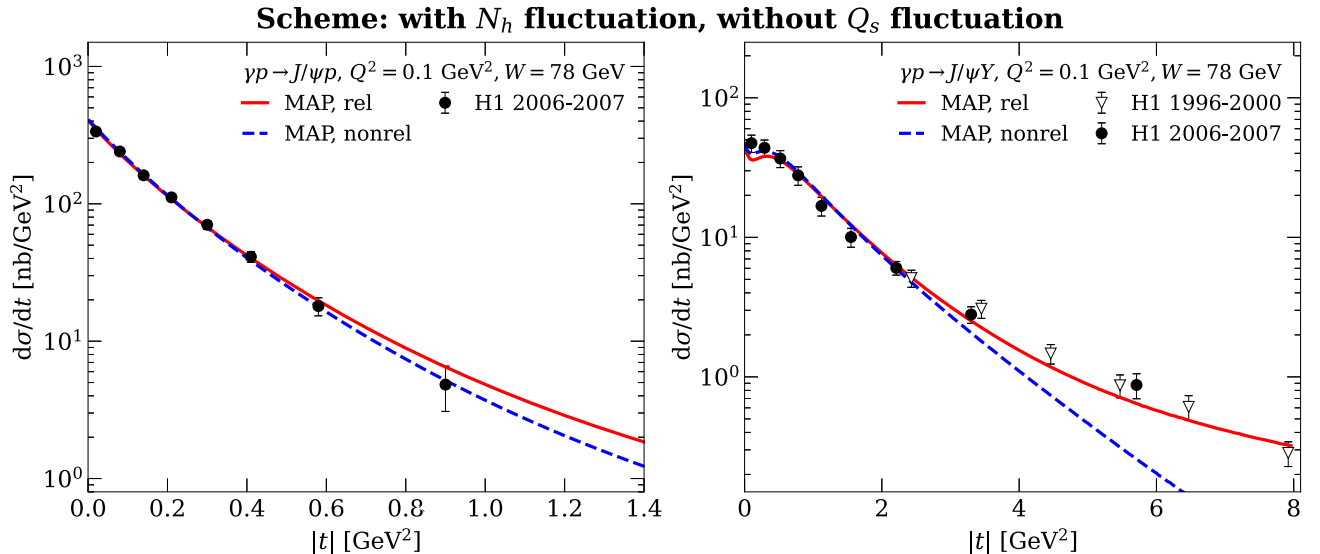


FIG. 8. Fit result compared to the H1 data [43,69].

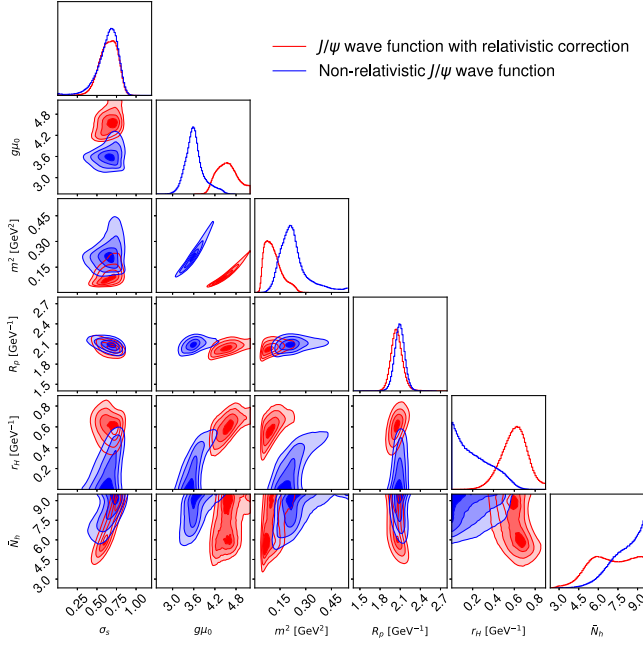


FIG. 9. Posterior distribution of parameters in the scheme with Q_s fluctuation and with N_h fluctuation.

TABLE IV. Parameter estimation from the posterior distribution in Fig. 9.

Parameters	Prior range	MAP (R)	MAP (NR)
σ_s	[0.0, 1.2]	$0.571^{+0.281}_{-0.145}$	$0.685^{+0.184}_{-0.321}$
$g\mu_0$	[2.5, 5.2]	$4.306^{+0.821}_{-0.249}$	$3.467^{+0.783}_{-0.326}$
m^2 [GeV ²]	[0.002, 0.55]	$0.069^{+0.147}_{-0.033}$	$0.175^{+0.258}_{-0.084}$
R_p [GeV ⁻¹]	[1.4, 2.8]	$2.048^{+0.176}_{-0.167}$	$2.057^{+0.183}_{-0.115}$
r_H [GeV ⁻¹]	[0.0, 0.9]	$0.578^{+0.264}_{-0.231}$	$0.0002^{+0.5278}_{-0.0002}$
$\tilde{N}_h$	[0.5, 9.5]	$5.585^{+3.915}_{-1.164}$	$9.012^{+0.488}_{-3.186}$
$\chi^2/\text{d.o.f.}$		0.45	1.08

event to event. Furthermore, we included the leading relativistic correction to the J/ψ wave function, which provides a more accurate representation of the heavy quarkonium structure, and quantified its impact on the production cross section. The formalism developed here enabled us to derive (semi)analytical results for both coherent and incoherent production, offering both theoretical clarity and computational efficiency. The obtained cross sections are Eqs. (23) and (37).

Model parameters were constrained through a Bayesian analysis, ensuring a statistically robust determination of the proton substructure properties and associated uncertainties. The obtained posterior distributions are available in the Supplemental Material [70]. The results demonstrated that including additional sources of fluctuations beyond the hot spot geometry, together with relativistic corrections, significantly improves the agreement with experimental data and provides valuable insights into the fluctuating nature of the proton substructure. In particular, both the N_h and Q_s fluctuations improve the quality of the fit. The latter is found to be more important, especially at small $|t|$. Furthermore, we demonstrated that even if no Q_s or N_h fluctuations are included, it is still possible to get a reasonably good description of the HERA data when relativistic corrections are included. Notably, the leading relativistic correction is also essential for describing the transition of the incoherent cross section to a power-law behavior at $|t| \gtrsim 3.5$ GeV².

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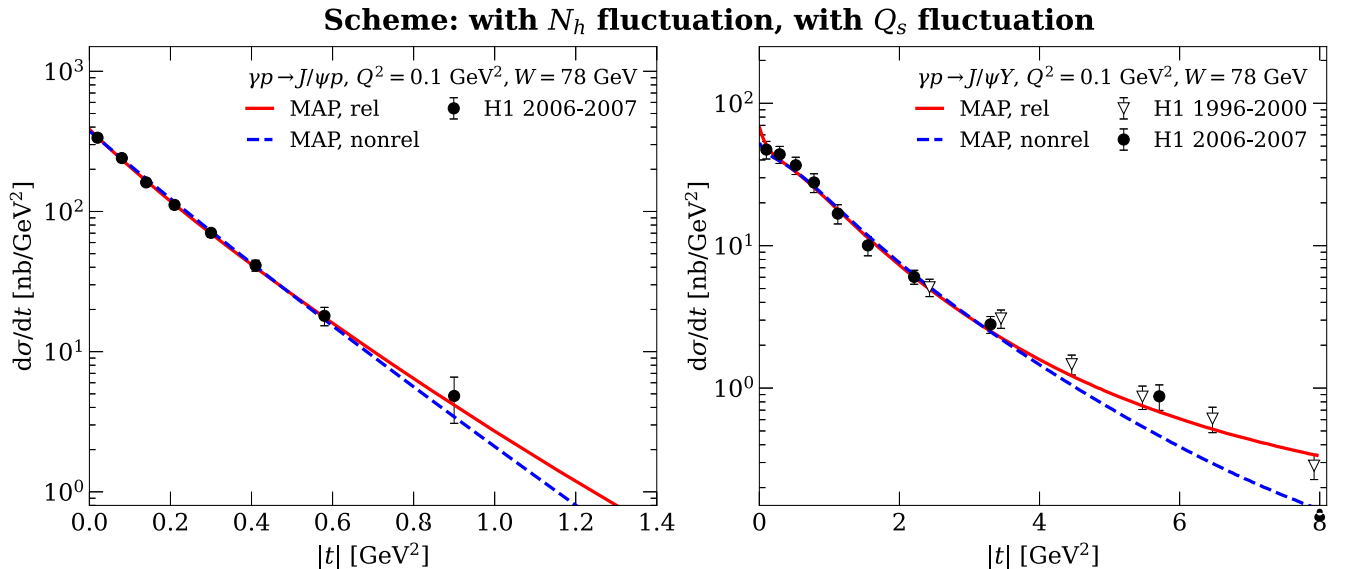


FIG. 10. Fit result compared to the H1 data [43,69].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [70].

APPENDIX A: THE $c\bar{c}$ WAVE FUNCTIONS OF γ^*

The wave functions of γ^* to a charm-anticharm ($c\bar{c}$) dipole can be calculated using light-front perturbation theory. They can be found in the literature, e.g., Refs. [61–64]. For longitudinal polarization of the photon ($\lambda = 0$), the wave function reads

$$\Psi_{\gamma^*;hh'}^{\lambda=0} = -e_q e \sqrt{N_c} \delta_{h,-h'} 2Qz(1-z) \frac{K_0(\varepsilon_z|\mathbf{r}|)}{2\pi}, \quad (\text{A1})$$

while for the transverse polarizations ($\lambda = \pm 1$), the corresponding wave functions are given by

$$\begin{aligned} \Psi_{\gamma^*;hh'}^{\lambda=+1} = & -e_q e \sqrt{2N_c} \left[m_c \frac{K_0(\varepsilon_z|\mathbf{r}|)}{2\pi} \delta_{h+}\delta_{h'+} \right. \\ & \left. + (z\delta_{h+}\delta_{h'-} - (1-z)\delta_{h-}\delta_{h'+}) i e^{i\theta_r} \frac{\varepsilon K_1(\varepsilon_z|\mathbf{r}|)}{2\pi} \right], \end{aligned} \quad (\text{A2a})$$

$$\begin{aligned} \Psi_{\gamma^*;hh'}^{\lambda=-1} = & -e_q e \sqrt{2N_c} \left[m_c \frac{K_0(\varepsilon_z|\mathbf{r}|)}{2\pi} \delta_{h-}\delta_{h'-} \right. \\ & \left. + ((1-z)\delta_{h+}\delta_{h'-} - z\delta_{h-}\delta_{h'+}) i e^{-i\theta_r} \frac{\varepsilon K_1(\varepsilon_z|\mathbf{r}|)}{2\pi} \right]. \end{aligned} \quad (\text{A2b})$$

Here h and h' are helicities of c and c' , respectively, and $\varepsilon_z = z(1-z)Q^2 + m_c^2$.

APPENDIX B: THE $c\bar{c}$ WAVE FUNCTIONS OF J/ψ TO THE FIRST RELATIVISTIC CORRECTION

The leading-order light front wave function of heavy quarkonia with the first relativistic correction can be computed based on the long-distance matrix elements constrained by decay width analyses in the nonrelativistic QCD (see Ref. [64]). In the following, we list the wave functions for different polarizations and helicities for J/ψ obtained in Ref. [64].

$$\begin{aligned} \Psi_{J/\psi;+-}^{\lambda=0} = \Psi_{J/\psi;-+}^{\lambda=0} = & \frac{\pi\sqrt{2}}{\sqrt{m_c}} \left\{ A\delta\left(z - \frac{1}{2}\right) \right. \\ & \left. + \frac{B}{m_c^2} \left[\left(\frac{5}{2} + r^2 m_c^2\right) \delta\left(z - \frac{1}{2}\right) - \frac{1}{4} \partial_z^2 \delta\left(z - \frac{1}{2}\right) \right] \right\}, \end{aligned} \quad (\text{B1a})$$

$$\begin{aligned} \Psi_{J/\psi;++}^{\lambda=1} = \Psi_{J/\psi;--}^{\lambda=-1} = & \frac{2\pi}{\sqrt{m_c}} \left\{ A\delta\left(z - \frac{1}{2}\right) \right. \\ & \left. + \frac{B}{m_c^2} \left[\left(\frac{7}{2} + r^2 m_c^2\right) \delta\left(z - \frac{1}{2}\right) - \frac{1}{4} \partial_z^2 \delta\left(z - \frac{1}{2}\right) \right] \right\}, \end{aligned} \quad (\text{B1b})$$

$$\begin{aligned} \Psi_{J/\psi;+-}^{\lambda=1} = -\Psi_{J/\psi;-+}^{\lambda=1} = & (\Psi_{J/\psi;+-}^{\lambda=-1})^* = (-\Psi_{J/\psi;-+}^{\lambda=-1})^* \\ = & -\frac{2\pi i}{m_c^{3/2}} B \delta\left(z - \frac{1}{2}\right) (r_1 + i r_2), \end{aligned} \quad (\text{B1c})$$

$$\Psi_{J/\psi;-+}^{\lambda=1} = \Psi_{J/\psi;+-}^{\lambda=-1} = \Psi_{J/\psi;--}^{\lambda=0} = \Psi_{J/\psi;++}^{\lambda=0} = 0. \quad (\text{B1d})$$

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