

Entanglement, trace anomaly, and confinement in QCD

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We formulate *confinement* in quantum chromodynamics (QCD) as an entropic surface phenomenon. Quark and gluon quantum information is localized on a transverse, *entangling two-sphere* of radius R_{EE} ; at this radius the QCD vacuum—partitioned by a hadron into interior and exterior regions—reaches its maximal entanglement entropy. Lattice-QCD determinations of the scalar (trace) gravitational form factors fix both R_{EE} and the transverse trace-anomaly density $\rho_h(R_{EE})$, yielding a *parameter-free* slope $c_h = 8\pi^2 R_{EE}^2 \rho_h(R_{EE})$ and a mechanical entropy $S_{EE}(y) = c_h y$ that grows linearly with rapidity y . The entropy gradient $\partial_R S_{EE}$ changes sign at R_{EE} : it pushes colored degrees of freedom outward for $r < R_{EE}$ and pulls them inward for $r > R_{EE}$, thereby localizing them on the codimension-2 entangling two-sphere $\Sigma_\perp = S_{R_{EE}}^2$ (which, in the infinite-momentum frame (IMF), projects onto the transverse plane)—the “information wall.” This provides a high-energy (large- y) entropic confinement diagnostic that complements—rather than replaces—Wilson’s area-law criterion, which probes long-distance dynamics near the rest frame ($y \rightarrow 0$). Imposing unitarity on an entropic ansatz for the amplitude yields $\sigma(s) \propto y^\delta$. World data *favor* $\delta = 2$ for elastic $pp(p\bar{p})$ scattering and heavy-quark photoproduction, whereas ϕ photoproduction favors a softer $\delta = 0.387$. All extracted cross sections remain well below the Froissart-Martin bound. These results provide a confinement criterion *quantified* directly from nonperturbative QCD inputs, unifying the trace anomaly, entanglement entropy, and high-energy scattering within a single quantitative framework.

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Introduction. Why do colored quarks and gluons never emerge from hadrons? The textbook answer invokes an *area law* for large Wilson loops, which diagnoses a linear confining potential in the infrared (IR) [1]. Yet that criterion carries no information about ultraviolet (UV) dynamics, where asymptotic freedom reigns.

Here we propose a complementary, UV-sensitive diagnostic: *entropic confinement*. Quantum information carried by quarks and gluons localizes on a codimension-2 entangling surface of fixed radius R_{EE} . Geometrically, the entangling surface is the two-sphere $\Sigma_\perp = S_R^2$ equipped with the induced metric h_{ab} and area element $\sqrt{h}d^2x$. The universal contribution to the von Neumann entropy of the reduced state is then controlled by the quantum chromodynamics (QCD) trace anomaly on $\Sigma_\perp$ [2,3]

$$\begin{aligned} S_{EE}^{\text{univ}}(R) &= 2\pi \ln \frac{R}{\varepsilon} \int_{\Sigma_\perp} \sqrt{h} \langle T^\mu{}_\mu \rangle \\ &= 2\pi \ln \frac{R}{\varepsilon} \int_{r=R} R^2 d\Omega \langle T^\mu{}_\mu \rangle, \end{aligned} \quad (1)$$

as established by general arguments in relativistic QFT [4–14]. See the Supplemental Material [15] for a detailed derivation of Eq. (1). In the infinite-momentum frame (IMF), the two-sphere S_R^2 projects onto the transverse (impact-parameter) plane, and the surface measure reduces to the standard light-front impact-parameter measure used to define parton densities [16,17]. We use this mapping solely to express densities; the underlying entangling surface remains S_R^2 .

By “mechanical entropy” we mean the *universal logarithmic* term S_{EE}^{univ} evaluated under two physical renormalization conditions: (i) a maximal-entropy radius $R = R_{EE}$ set by the hadron; and (ii) a Lorentz-contracted cutoff $\varepsilon = R_{EE}e^{-y}$ for boost rapidity y . With these choices, spherical symmetry on $\Sigma_\perp$ gives $\int_{\Sigma_\perp} \sqrt{h} \langle T^\mu{}_\mu \rangle = 4\pi R_{EE}^2 \rho_h(R_{EE})$, so that

$$S_{EE}(y) = c_h y, \quad c_h = 8\pi^2 R_{EE}^2 \rho_h(R_{EE}). \quad (2)$$

At $y = 0$ the *universal logarithmic* contribution is tuned to vanish by construction since $\varepsilon = R_{EE}$ (therefore, our

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renormalization choice is valid only for large- y); this does not imply that the full state is pure.

For complementary 1 + 1D derivations of the linear-in- y growth of entanglement, see [18–21]. In particular, Ref. [21] computes the energy-momentum tensor (EMT) of a “hadron-like” primary of dimension h and finds [their Eq. (3.15)] in pure 1 + 1D CFT $T_{++}(x_+) = C_h/x_+^2$ with $C_h = h$; in the mass-deformed case [their Eq. (3.19)] one reads off the same coefficient from $C_h = x_+^2 T_{++}(x_+)$ at $x_+ \rightarrow 0, \infty$. This gives a universal slope $S_{EE}^{\text{univ}}(y) = 2\pi C_h y$, implying the dictionary $C_h = c_h/(2\pi)$ when compared to Eq. (2). Our result is the 3 + 1D QCD realization: we determine c_h nonperturbatively from the hadron’s trace anomaly localized on the entangling two-sphere, $c_h = 8\pi^2 R_{EE}^2 \rho_h(R_{EE})$, using lattice-QCD gravitational form factor (GFF) inputs in 2D impact-parameter space Eq. (9).

It is convenient to parametrize the slope c_h in terms of an *operational* (nonperturbative) occupation number on the information wall

$$\frac{c_h}{8\pi^2 R_{EE}^2} \stackrel{\text{op.}}{=} \int_0^1 dx \int d^2 k_\perp \int d^2 b_\perp \delta_\Sigma(b_\perp) n_{q+g}(x, \mathbf{k}_\perp, \mathbf{b}_\perp), \quad (3)$$

where $\delta_\Sigma(b_\perp) \equiv \frac{\delta(b_\perp - R_{EE})}{2\pi R_{EE}}$, and $n_{q+g}(x, \mathbf{k}_\perp, \mathbf{b}_\perp)$ denotes the total (sea-quark+gluon) occupation number with longitudinal momentum fraction x and transverse momentum $\mathbf{k}_\perp$ per unit transverse phase-space area $d^2 b_\perp$; this identification serves as a nonperturbative analog of the gluon occupation number in the color glass condensate (CGC)/Balitsky-Fadin-Kuraev-Lipatov (BFKL) framework [22–39]. Also note that c_h per nucleon shows rapid saturation with increasing mass number A in nuclei; see Fig. 1.

Formally, we define the hadron’s entanglement by starting from the vacuum density operator $|0\rangle\langle 0|$ and

tracing out all quark- and gluon-field modes whose support lies *outside* a sphere of radius R , see Fig. S1 in the Supplemental Material [15]. The reduced density matrix admits the Euclidean path-integral representation [2]

$$\hat{\rho}_<(R) = \text{Tr}_{r>R} |0\rangle\langle 0| = \frac{1}{\mathcal{N}} \int_{\phi(C_{<})=\phi_< \atop \phi(C_{>})=\phi_>} \mathcal{D}\phi e^{-I(\phi)}, \quad (4)$$

which describes the quantum state of the interior region ($r < R$). Here $C_{<or>}$ denote the two sides of a three-dimensional cut $\mathcal{C}$ with boundary $\partial\mathcal{C} = \Sigma_\perp$; ϕ collectively denotes all quantum field theory (QFT) fields, and $\phi_{<or>}$ are fixed field configurations on $C_{<or>}$. The eigenvalues $\{\lambda_n(R)\}$ of $\hat{\rho}_<(R)$ form the *entanglement spectrum*. The von Neumann entropy $S_{EE}(R) = -\sum_n \lambda_n \ln \lambda_n$ reduces, in the continuum limit, to Eq. (2), with *spectral density* $\rho_h(b_\perp) = \langle T^\mu_\mu(b_\perp) \rangle$ that appears in the QCD trace anomaly density in impact-parameter space Eq. (9).

Percent-level lattice QCD calculations of the *scalar* (trace) gravitational form factors (GFFs) [40–43] provide a direct, nonperturbative handle on $\langle T^\mu_\mu \rangle$. Independent experimental extractions corroborate these values: (i) the *quark-sector* GFF is obtained from deep-virtual Compton scattering at Jefferson Lab [44], analyzed with the dispersive-factorization approaches of [45–47]; (ii) the *gluon-sector* GFF is deduced from near-threshold J/ψ photo-production [48] using both holographic-QCD [49,50] and an NRQCD–GPD factorization scheme [51].

The sign-changing entropy gradient (see Fig. 2) is an intrinsic property of the QCD vacuum, akin to Casimir-like stresses in non-Abelian gauge theories [52], and *not* a by-product of the $2^{++}-0^{++}$ resonances admixture that governs the Breit-frame three-dimensional EMT pressure inside a hadron [53–58] (see Fig. S2 in the Supplemental Material [15] for the separate mechanical-pressure node).

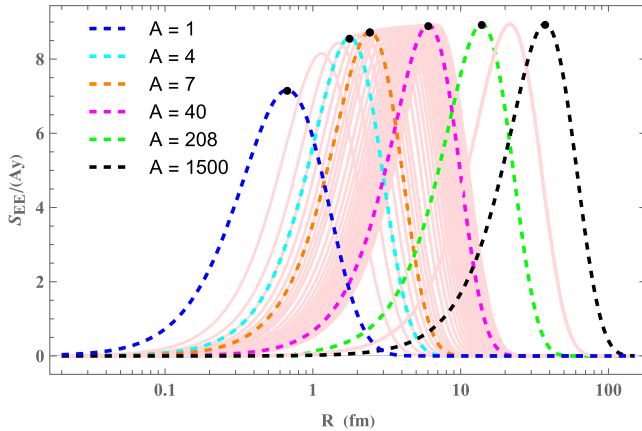


FIG. 1. Mechanical entanglement entropy per nucleon, $S_{EE}/(Ay)$, versus entangling radius R . The light band spans $2 \leq A \leq 500$; dashed curves highlight representative nuclei. All curves level off at $S_{EE}/(Ay) \simeq 8.94$, signaling rapid saturation with A .

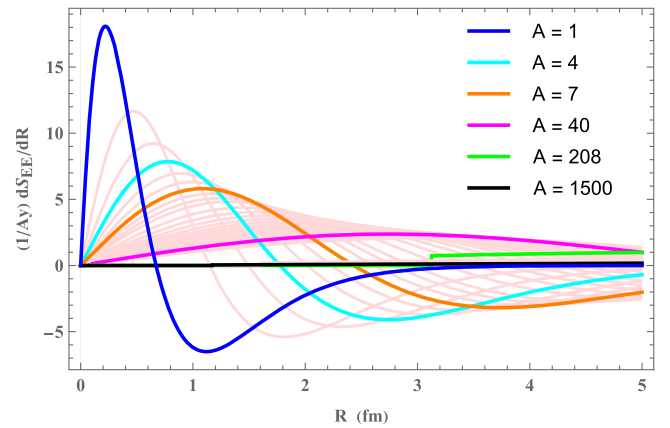


FIG. 2. Radial derivative $\partial_R S_{EE}$ for the systems in Fig. 1. Positive values ($R < R_{EE}$) represent an outward entropic pressure; negative values ($R > R_{EE}$) pull inward. The zero crossing defines the “information wall” at R_{EE} .

An *entropic ansatz* relates an elastic $2 \rightarrow 2$ cross section to the mechanical entropy of the color field produced in the collision [59,60]; see *Cross sections and data* below for details. Reference [61], following the exponential entropic ansatz of Ref. [20] for deep inelastic scattering (DIS) structure functions, models total cross sections as $\sigma \propto e^{\delta y}$. Although this form reproduces vector-meson photoproduction data at high energies, it fails near threshold and overshoots the Froissart-Martin unitarity bound in the $y \rightarrow \infty$ limit. By contrast, our power-law ansatz $\sigma \propto y^\delta$ describes both threshold and high-energy photoproduction data with a single exponent and remains well below the Froissart-Martin limit across the full kinematic range, as illustrated in Figs. 4(a)–4(c). See also [62–74] for other proposals which relate entanglement entropy with multiplicity of final-state hadrons produced in high-energy collisions.

In the remainder of this paper we (i) summarize the lattice-QCD inputs that fix ρ_h and R_{EE} ; (ii) quantify the entropic force associated with $\partial_R S_{EE}$; and (iii) show how unitarity reduces an entropic ansatz for elastic $2 \rightarrow 2$ processes to $\sigma(s) = \mathcal{N}^2 y^\delta$, with δ reflecting the UV sensitivity of the channel.

Trace anomaly and lattice inputs. Entropic confinement depends only on the transverse anomaly density $\rho_h(b_\perp) = \langle T^\mu_\mu(b_\perp) \rangle$ (two-dimensional impact-parameter density). Modern lattice data [40–43,75] and perturbative counting rules [76–81] fix the necessary scalar form factors.

Nucleon ($J = \frac{1}{2}$): With $t = (P' - P)^2$, $\bar{P} = (P' + P)/2$,

$$\langle P' | T_i^{\mu\nu} | P \rangle = \bar{u}(P') \left[A_i(t) \gamma^{(\mu} \bar{P}^{\nu)} + D_i(t) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4m_N} \right] u(P), \quad (5)$$

with $i \equiv$ sea-quark(q) or gluon(g) and the trace is governed by the dipole

$$A_i^S(t) = A_i(t) - \frac{3t}{4m_N^2} D_i(t) = \alpha_{S,i} (1 - t/\Lambda_{S,i}^2)^{-2}. \quad (6)$$

Pion ($J = 0$): With monopole form

$$A_\pi^S(t) = A_\pi(t) - \frac{t}{4m_\pi^2} [2A_\pi(t) + 3D_\pi(t)]. \quad (7)$$

Spin-0 nuclei: With $n_A = 3A - 1$,

$$A_i^{S,A}(t) = A\alpha_{S,i} (1 - t/\Lambda_{S,i}^2)^{-n_A}. \quad (8)$$

After 2D Fourier transform with respect to $t = -\Delta_\perp^2$, any n -pole form yields the Bessel- K density

TABLE I. Charge radii r_c , maximum-entropy radii R_{EE} , and specific entropies $S_{EE}/(Ay)$ for representative hadronic and nuclear systems. All radii are in fm. Proton r_c is from PRad [84]; the pion value is the PDG average [85]; nuclear radii are taken from [86]. Over the mass range shown the maximum-entropy radius is reproduced to better than 0.1% by $R_{EE}(A) = 0.964\sqrt{A} - 0.279/\sqrt{A} - 0.014$. Note that $R_{EE}(A)$ need not match the liquid-drop $r_0 A^{1/3}$ law for *geometric* radii because the underlying weights (2D trace anomaly density vs 3D mass density) and the codimension (surface vs volume) are different.

Species	r_c	R_{EE}	$S_{EE}/(Ay)$
$\pi^\pm$	0.659(4)	0.98(6)	12(4)
N	0.831(14)	0.67(8)	7.17(36)
^4He	1.68(3)	1.77(21)	8.57(43)
^7Li	2.44(4)	2.43(28)	8.74(44)
^{40}Ar	3.43(3)	6.04(71)	8.91(45)
^{208}Pb	5.50(1)	13.9(1.6)	8.94(45)

$$\rho_i(b_\perp) = \alpha_{S,i} \frac{\Lambda_{S,i}^2}{2\pi} \frac{(\Lambda_{S,i} b_\perp)^{n-1}}{2^{n-1} \Gamma(n)} K_{n-1}(\Lambda_{S,i} b_\perp). \quad (9)$$

Summing over $i = q, g$ and evaluating at $b_\perp = R_{EE}$ fixes $c_h = 8\pi^2 R_{EE}^2 \rho_h(R_{EE})$ in Eq. (2). All lattice fit parameters are summarized in Table S1 of the Supplemental Material [15].

Mechanical entanglement and confinement. Equation (2), together with Figs. 1 and 2, shows that the entropy gradient reverses sign at $r = R_{EE}$ (see Table I for values of R_{EE} for pion and nuclei, also see Fig. 3 for a visual comparison of various proton radii inferred from the lattice GFFs). Accordingly, the radial derivative $\partial_R S_{EE}$ acts as an *entropic*

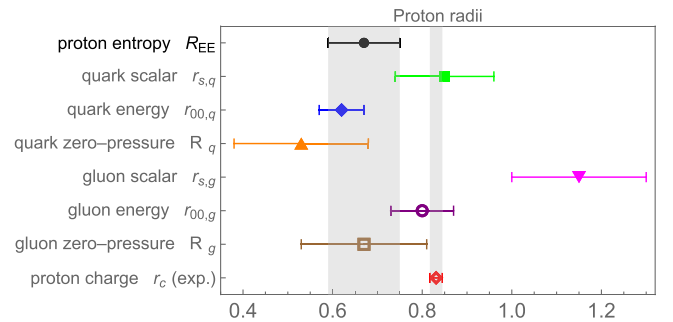


FIG. 3. Visual comparison of proton radii using the lattice gravitational form factor (5) from Ref. [41] and the fit parameters summarized in Table S1 of the Supplemental Material [15]: maximum-entropy radius R_{EE} , mechanical core radii $R_{q,g}$ (from the node of the 3D Breit-frame pressure density; see Fig. S2 in the Supplemental Material [15]), rms scalar radii r_s , rms energy radii r_{00} , and the experimental rms charge radius r_c from PRad [84]. All rms radii shown are computed separately for the quark and for the gluon sectors using 3D Breit-frame densities, as detailed in the Supplemental Material [15]; no $q + g$ sum is taken. However, R_{EE} is computed from $q + g$ 2D (impact-parameter space) trace anomaly density Eq. (9).

pressure or force in the sense of Refs. [82,83]. A parton that strays *inside* the surface ($r < R_{EE}$) is pushed outward, whereas one that moves *outside* ($r > R_{EE}$) is pulled back. Colored degrees of freedom are therefore locked onto the two-dimensional “information wall” at $r = R_{EE}$. This entropic bag complements the Wilson area-law criterion: it provides a high-energy (UV) diagnostic while the area law governs long-distance (IR) dynamics.

Cross sections and data. The entropic ansatz ties an elastic $2 \rightarrow 2$ cross section to the entropy of the intermediate color field [60]. In the standard eikonal representation, unitarity gives

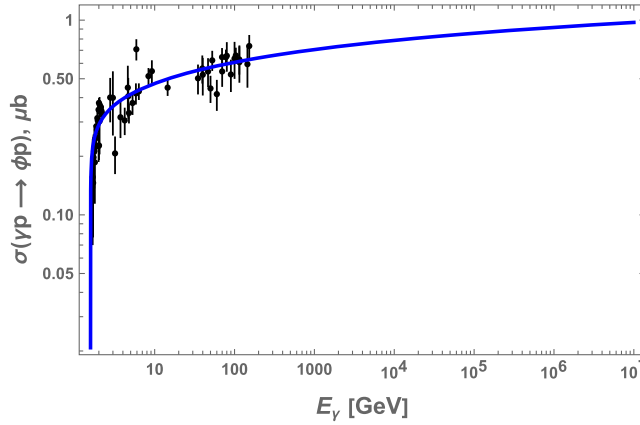
$$\sigma_{\text{el}}(s) = \int d^2 b_{\perp} |1 - e^{i\chi(b_{\perp}, y)}|^2, \quad (10)$$

and, because the phase accumulates additively with rapidity, we take an entropy-controlled eikonal $\chi(b_{\perp}, y) \propto y$. This yields a single-scale power law at high energy. In particular, for small-eikonal (“moderate opacity”) regime, we find $\sigma_{\text{el}}(s) \propto \int d^2 b |\chi|^2 \propto y^2$.

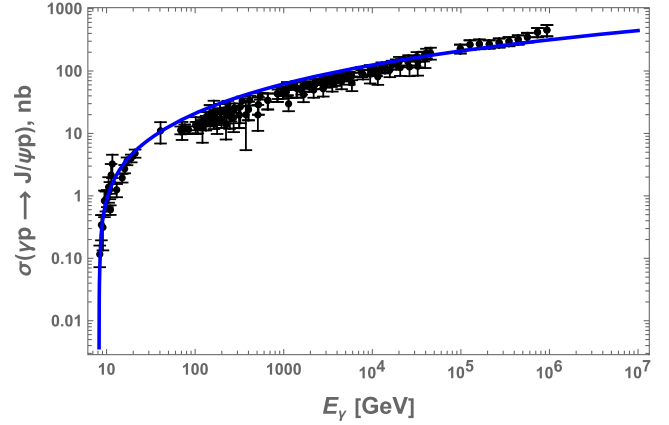
Consequently, for each elastic/exclusive channel, we write the entropic ansatz

$$\sigma_{\mathcal{C}}(s) = \mathcal{N}_{\mathcal{C}}^2 y^{\delta_{\mathcal{C}}}, \quad y = \text{arcosh } \gamma, \quad (11)$$

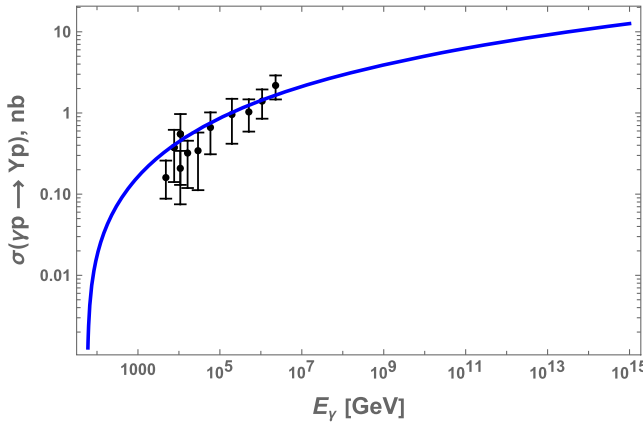
where $\mathcal{N}_{\mathcal{C}}$ is a single normalization constant per elastic/exclusive channel $\mathcal{C}$ and the exponent $\delta_{\mathcal{C}}$ quantifies how efficiently the process probes the ultraviolet anomaly density on the information wall [60].



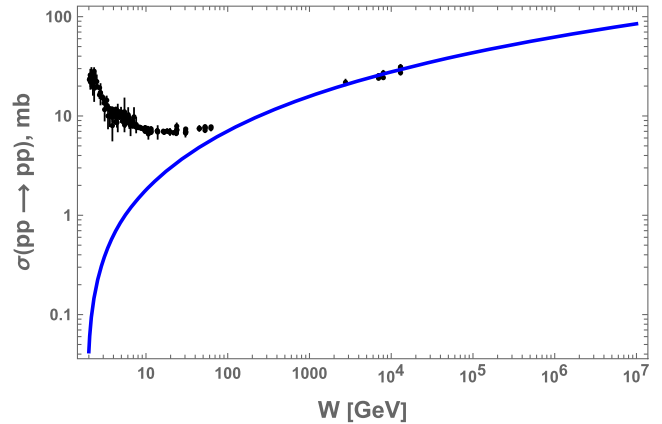
(a) ϕ photoproduction



(b) J/ψ photoproduction



(c) $\Upsilon(1S)$ photoproduction



(d) Elastic pp

FIG. 4. Cross sections σ are plotted versus the rapidity $y = \text{arcosh } \gamma = \ln(\gamma + \sqrt{\gamma^2 - 1})$, with $\gamma_{\text{beam}} \equiv E_{\gamma}/E_{\gamma, \text{thr}}(V)$ for photoproduction and $\gamma_{\text{CM}} \equiv \sqrt{s}/(2m_N)$ for elastic pp scattering. Panels display (a) ϕ photoproduction, (b) J/ψ photoproduction, (c) $\Upsilon(1S)$ photoproduction, and (d) elastic pp scattering. The solid blue curves show the scaling ansatz $\sigma \propto y^{\delta}$ with $\delta_{\phi} = 0.387$ in (a) and $\delta = 2$ in (b)–(d). The shallower $y^{0.39}$ rise for ϕ indicates enhanced infrared sensitivity of light-quark dynamics, whereas heavy quarkonia and elastic pp retain the full y^2 growth expected from the high-energy eikonal picture. Data sources: ϕ from Refs. [87,88]; J/ψ and Υ from the compilation in Ref. [49]; elastic pp from Refs. [89–91].

UV-dominated channels: Rapidity is defined as $y = \text{arcosh } \gamma = \ln(\gamma + \sqrt{\gamma^2 - 1})$. For vector-meson photoproduction with $s = W^2 = (p_\gamma + p_p)^2$, we adopt the dimensionless beam-energy ratio $\gamma_{\text{beam}} \equiv E_\gamma/E_{\gamma,\text{thr}}(V)$, where $E_{\gamma,\text{thr}}(V) = [(m_N + M_V)^2 - m_N^2]/(2m_N)$, so that $\gamma_{\text{beam}} = 1$ at threshold and $y_{\text{beam}} = \text{arcosh } \gamma_{\text{beam}}$. For elastic pp scattering (with $\sqrt{s} = W$) in the c.m. frame the Lorentz factor of either proton is $\gamma_{\text{CM}} \equiv \sqrt{s}/(2m_N)$, yielding $y_{\text{CM}} = \text{arcosh } \gamma_{\text{CM}}$. Reactions characterized by compact transverse sizes (elastic pp , J/ψ , and Υ production) sample the full ultraviolet content of the wall; their exponents saturate at the unitarity limit $\delta = 2$

$$\begin{aligned} \mathcal{N}_{pp}^2 &= 0.324 \text{ mb}, & \mathcal{N}_{J/\psi}^2 &= 2.1 \times 10^{-6} \text{ mb}, \\ \mathcal{N}_\Upsilon^2 &= 1.3 \times 10^{-8} \text{ mb}. \end{aligned}$$

Even at the largest measured rapidities ($y_{\text{CM}} \simeq 10$ at the LHC) the resulting cross sections remain below 32 mb, smaller by a factor $\gtrsim 2$ than the canonical $\pi/m_\pi^2 \simeq 62.8$ mb scale; see Figs. 4(b)–4(d).

IR-sensitive channel: The more diffuse ϕ meson samples the wall less efficiently, yielding a softer rise

$$\delta_\phi = 0.387, \quad \mathcal{N}_\phi^2 = 3.3 \times 10^{-4} \text{ mb},$$

so that $\sigma_{\gamma p \rightarrow \phi p}$ stays far below σ_{FM} across the entire energy range from threshold to multi-TeV photoproduction; see Fig. 4(a).

All extracted cross sections remain well below the canonical Froissart-Martin scale $(\pi/m_\pi^2) \ln^2 s$ for σ_{tot} (with $\pi/m_\pi^2 \simeq 62.8$ mb after unit conversion): elastic pp lies by factors $\gtrsim 2$ below at present energies, whereas heavy-quarkonia photoproduction is suppressed by $\sim 10 - 10^3$.

Conclusions. We have shown that the QCD trace anomaly, quantified with percent-level lattice determinations of the scalar gravitational form factors, fixes a transverse

information wall of radius R_{EE} . Tracing over quark and gluon modes outside that surface yields a finite, *parameter-free* mechanical entropy $S_{EE} = c_h y$ with c_h fixed entirely by lattice inputs—whose sign-changing radial derivative locks colored fields to the wall at *all* momentum scales. This entropic mechanism provides a high-energy diagnostic that *complements* the traditional Wilson-loop area law: it remains consistent with asymptotic freedom in the ultraviolet and with linear confinement in the infrared. Imposing unitarity on an entropic ansatz for the amplitude yields $\sigma(s) \propto y^\delta$. Hard processes (pp , J/ψ , Υ) saturate the unitarity limit $\delta = 2$, exhibiting the familiar $y^2 \sim \ln^2 s$ growth pattern, while the more diffuse ϕ channel selects $\delta_\phi = 0.387$. All extracted cross sections remain well below the canonical Froissart-Martin bound $(\pi/m_\pi^2) \ln^2 s$ for σ_{tot} .

Outlook. The entropic framework developed here delivers a quantitative baseline against which future high-precision data can be benchmarked. Upcoming measurements at the HL-LHC, Jefferson Lab (CLAS12, GlueX, and SoLID), and the Electron-Ion Collider will confront its predictions across an unprecedented kinematic range. Any systematic deviation—for example from Odderon exchange, Regge exchanges, exotic multiquark resonances—will stand out cleanly against the fully quantified entropy background, offering a sharpened path to discovering genuinely new QCD dynamics.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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