

Light tetraquark states with $J^{PC} = 1^{--}$ from QCD sum rules

Yi-Wei Jiang,^{1,*} Hua-Xing Chen,^{1,†} Er-Liang Cui,^{2,‡} Ding-Kun Lian,^{1,§} Wen-Ying Liu,^{3,4,5,6,||} and Niu Su^{7,¶}

¹*School of Physics, Southeast University, Nanjing 210094, China*

²*College of Science, Northwest A&F University, Yangling 712100, China*

³*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

⁴*Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, Key Laboratory of Quantum Theory and Applications of MoE, Gansu Provincial Research Center for Basic Disciplines of Quantum Physics, Lanzhou University, Lanzhou 730000, China*

⁵*MoE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China*

⁶*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

⁷*College of Physics and Optoelectronic Engineering, Taiyuan University of Technology, Taiyuan 030024, China*



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We perform a systematic QCD sum rule study of light tetraquark states with $J^{PC} = 1^{--}$ in the diquark-antidiquark picture. A complete set of local interpolating currents is constructed and projected onto six flavor-isospin configurations ($q = u/d$): the isoscalar $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ sectors, the isovector $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ sectors, and the isotensor $qq\bar{q}\bar{q}$ sector. From the nonderivative currents studied in the present work, the extracted masses of the $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ configurations lie in the ranges 1.53–2.37 GeV, 1.86–2.51 GeV, and 2.46–2.60 GeV, respectively. For the $ss\bar{s}\bar{s}$ vector sector, we also quote the previous derivative-current result $2.34^{+0.23}_{-0.30}$ GeV as a complementary benchmark. We further compare the present 1^{--} tetraquark spectrum with previous QCD sum rule results for the 1^{++} tetraquark and hybrid states [N. Su, E.-L. Cui, Y.-W. Jiang, and H.-X. Chen, *Eur. Phys. J. C* **86**, 724 (2026).], aiming to provide useful information for distinguishing tetraquark and hybrid configurations in the light hadron spectrum. As an additional improvement, we complete the previously missing isotensor 1^{++} tetraquark entry and obtain its lowest mass to be $M = 2.19^{+0.26}_{-0.24}$ GeV, which is included in the spectral comparison.

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I. INTRODUCTION

A hadron is a composite subatomic particle made of quarks and gluons and bound together by the strong interaction. In the traditional quark model, mesons are described as quark-antiquark bound states, while baryons are composed of three quarks. This simple classification scheme has achieved remarkable success in describing the observed hadron spectrum and in organizing the properties

of conventional hadrons [1]. Nevertheless, QCD also allows more complicated color-singlet configurations, such as tetraquark states, hadronic molecules, glueballs, and hybrid states [2–24].

Among various quantum-number channels, the vector channel with $J^{PC} = 1^{--}$ plays a special role. Although these quantum numbers are not exotic, vector states are experimentally important because they can be directly produced in electron-positron annihilation. The light vector meson spectrum is already rather rich in the conventional quark-antiquark picture, while additional multi-quark components may also contribute in the same energy region. Therefore, a systematic study of light tetraquark states with $J^{PC} = 1^{--}$ is useful for understanding the possible role of compact four-quark configurations in the light vector spectrum.

The present work is also motivated by the distinction between tetraquark and hybrid configurations. In particular, the $J^{PC} = 1^{++}$ channel has attracted considerable interest because these quantum numbers are forbidden

*Contact author: yiwei Jiang@seu.edu.cn

†Contact author: hxchen@seu.edu.cn

‡Contact author: erliang.cui@nwfafu.edu.cn

§Contact author: liandk@seu.edu.cn

||Contact author: liuwenying@lzu.edu.cn

¶Contact author: suniu@tyut.edu.cn

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for conventional quark-antiquark mesons. Experimentally, the isovector $\pi_1(1600)$ and the isoscalar $\eta_1(1855)$ are important candidates with exotic quantum numbers [25–27]. Their possible internal structures have been discussed in various pictures, including hybrid states [28–32], tightly bound tetraquarks [33], and hadronic molecules [34–36]. However, distinguishing a hybrid state from a compact tetraquark state remains a challenging problem, since the two configurations may appear in similar mass regions and may couple to similar hadronic channels.

For this reason, it is useful to investigate tetraquark and hybrid spectra in different quantum-number channels within a common theoretical framework. The 1^{++} channel provides a direct probe of exotic quantum numbers, while the 1^{--} channel is experimentally accessible through electron-positron annihilation. A combined study of these two channels may therefore offer a broader perspective on the similarities and differences between tetraquark and hybrid configurations. In our previous work we studied the 1^{++} tetraquark and hybrid spectra using QCD sum rules [37–43]. In the present work we extend this line of investigation to the light 1^{--} tetraquark sector, while the corresponding 1^{--} hybrid states will be investigated in future studies.

We systematically construct local diquark-antidiquark interpolating currents for light tetraquark states with $J^{PC} = 1^{--}$. Their flavor structures are classified into the symmetric $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$, antisymmetric $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$, and mixed $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ representations. We then project these currents onto the isoscalar, isovector, and isotensor channels, and perform QCD sum rule analyses for the corresponding $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ configurations. We also compare the resulting mass spectrum with our previous results for the 1^{++} tetraquark and hybrid states [37], thereby providing a useful reference for future studies of light tetraquark and hybrid systems.

This paper is organized as follows. In Sec. II we construct the $J^{PC} = 1^{--}$ tetraquark interpolating currents and classify them according to their flavor and isospin structures. In Sec. III we formulate the QCD sum rules and present the relevant operator product expansion. In Sec. IV we perform the numerical analysis and extract the masses and decay constants. Finally, Sec. V summarizes the main results.

II. INTERPOLATING CURRENTS

In this section we construct the local interpolating currents for the $J^{PC} = 1^{--}$ tetraquark states in the diquark-antidiquark picture. Compared with conventional quark-antiquark mesons, tetraquark systems possess much richer internal structures. It is therefore necessary to organize the corresponding interpolating currents systematically in terms of their flavor, color, and Lorentz

structures, rather than introducing them in an *ad hoc* manner. Such a classification is particularly useful for the present analysis, since the isospin sectors $I = 0, 1$, and 2 will be discussed separately below.

We first summarize the notation used throughout this section. The symbol q denotes a light u or d quark, while s denotes a strange quark. The fields ℓ_{1a} , ℓ_{2b} , $\bar{\ell}_{3c}$, and $\bar{\ell}_{4d}$ denote the quark and antiquark fields entering the formal construction of the currents; the corresponding flavor assignments will be specified explicitly once the isospin channels are projected out. The indices a, b, c, d are color indices, the superscript T denotes transpose in Dirac space, and C is the charge-conjugation matrix.

A. General $(\ell\ell)(\bar{\ell}\bar{\ell})$ currents and symmetry decomposition

The general local derivative-free diquark-antidiquark current can be written as

$$J_\mu \sim (\ell_{1a}^T C \Gamma_1 \ell_{2b})(\bar{\ell}_{3c} \Gamma_2 \bar{\ell}_{4d}^T), \quad (1)$$

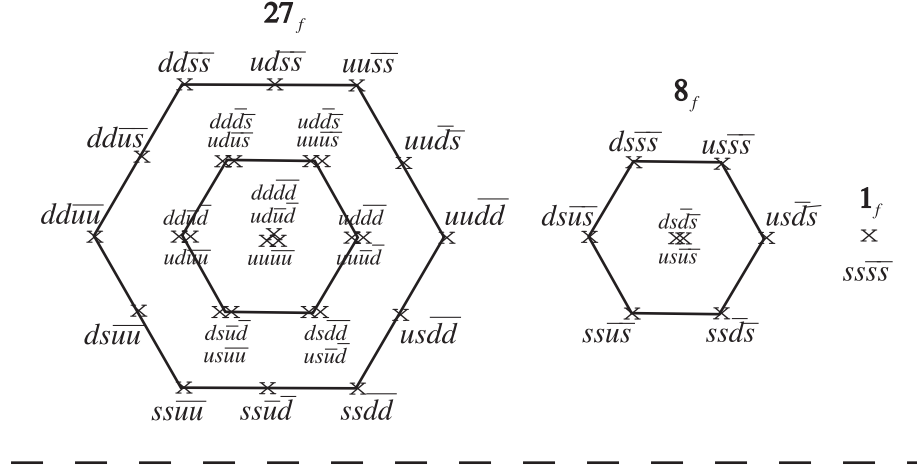
where $\ell_i = u, d, s$, and a, b, c, d are color indices. The explicit form is determined by requiring the current to be a color singlet carrying the quantum numbers $J^{PC} = 1^{--}$. There are two independent ways to couple the diquark and antidiquark colors into a singlet,

$$\bar{\mathbf{3}}_c \otimes \mathbf{3}_c, \quad \mathbf{6}_c \otimes \bar{\mathbf{6}}_c. \quad (2)$$

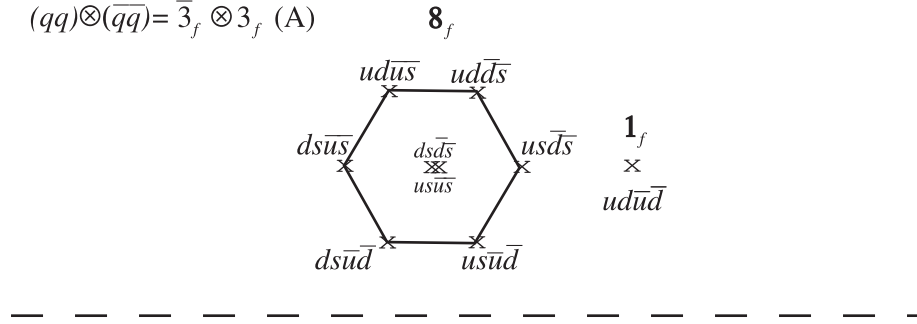
In the explicit currents below, these two color structures are realized through the antisymmetric and symmetric combinations under the interchange of the color indices $a \leftrightarrow b$, respectively. In other words, the relative minus signs in the color part project onto the $\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$ configuration, while the relative plus signs project onto the $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$ configuration.

To construct currents with definite charge-conjugation parity, one should first classify the corresponding flavor structures. Under charge conjugation, a diquark is transformed into an antidiquark while its flavor symmetry is preserved. Therefore, the diquark and antidiquark parts must be combined in compatible flavor representations. These flavor structures are illustrated in Fig. 1 through the corresponding $SU(3)$ weight diagrams. The **S** and **A** flavor structures lead directly to currents with definite charge-conjugation parity. By contrast, neither $\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f$ nor $\mathbf{6}_f \otimes \mathbf{3}_f$ alone has a definite C parity. A definite C parity can only be obtained from suitable linear combinations of these two mixed representations. The tetraquark currents are thus organized into the three classes **S**, **A**, and **M**. Their allowed isospin assignments can be summarized as

$$(qq) \otimes (\bar{q}\bar{q}) = 6_f \otimes \bar{6}_f \text{ (S)}$$



$$(qq) \otimes (\bar{q}\bar{q}) = \bar{3}_f \otimes 3_f \text{ (A)}$$



$$(qq) \otimes (\bar{q}\bar{q}) = \bar{3}_f \otimes \bar{6}_f \text{ (M)}$$

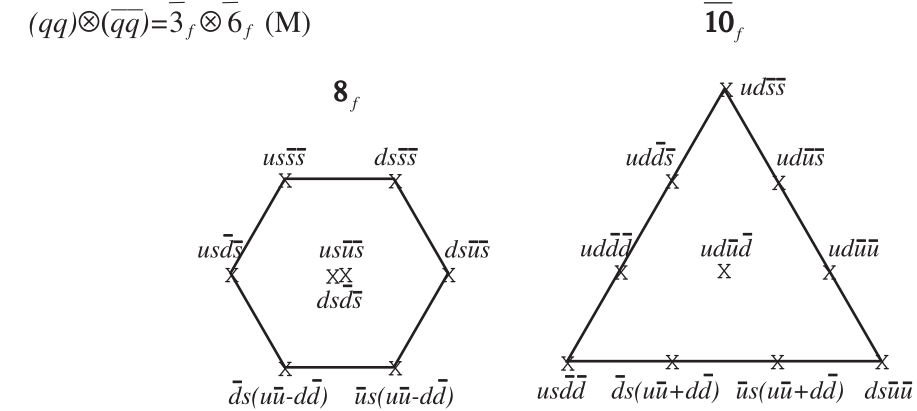


FIG. 1. Weight diagrams for $6_f \otimes \bar{6}_f$ (S) (top panel), $\bar{3}_f \otimes 3_f$ (A) (middle panel), and $\bar{3}_f \otimes \bar{6}_f$ (M) (bottom panel). The weight diagram for $6_f \otimes 3_f$ (M) is obtained from the bottom panel by charge conjugation.

$$\begin{aligned} \mathbf{S} \equiv 6_f \otimes \bar{6}_f: & \begin{cases} qq\bar{q}\bar{q}(\mathbf{S}), & I = 0, 1, 2, \\ qs\bar{q}\bar{s}(\mathbf{S}), & I = 0, 1, \\ ss\bar{s}\bar{s}(\mathbf{S}), & I = 0, \end{cases} & \mathbf{A} \equiv \bar{3}_f \otimes 3_f: & \begin{cases} qq\bar{q}\bar{q}(\mathbf{A}), & I = 0, \\ qs\bar{q}\bar{s}(\mathbf{A}), & I = 0, 1, \end{cases} \\ \mathbf{M} \equiv (\bar{3}_f \otimes \bar{6}_f) \oplus (6_f \otimes 3_f): & \begin{cases} qq\bar{q}\bar{q}(\mathbf{M}), & I = 1, \\ qs\bar{q}\bar{s}(\mathbf{M}), & I = 0, 1. \end{cases} \end{aligned} \quad (3)$$

We now list the $J^{PC} = 1^{--}$ diquark-antidiquark currents explicitly. For the symmetric flavor structure $6_f \otimes \bar{6}_f$ (S), there are two independent $(\ell\ell)(\bar{\ell}\bar{\ell})$ currents:

$$\begin{aligned}\psi_{1\mu}^S &= \ell_{1a}^T C \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma_\mu \gamma_5 C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \gamma_\mu \gamma_5 C \bar{\ell}_{4a}^T) \\ &\quad - \ell_{1a}^T C \gamma_\mu \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma_5 C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \gamma_5 C \bar{\ell}_{4a}^T),\end{aligned}\quad (4)$$

$$\begin{aligned}\psi_{2\mu}^S &= \ell_{1a}^T C \gamma^\nu \ell_{2b} (\bar{\ell}_{3a} \sigma_{\mu\nu} C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \sigma_{\mu\nu} C \bar{\ell}_{4a}^T) \\ &\quad - \ell_{1a}^T C \sigma_{\mu\nu} \ell_{2b} (\bar{\ell}_{3a} \gamma^\nu C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \gamma^\nu C \bar{\ell}_{4a}^T).\end{aligned}\quad (5)$$

For the antisymmetric flavor structure $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f (\mathbf{A})$, there are likewise two independent currents:

$$\begin{aligned}\psi_{1\mu}^A &= \ell_{1a}^T C \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma_\mu \gamma_5 C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \gamma_\mu \gamma_5 C \bar{\ell}_{4a}^T) \\ &\quad - \ell_{1a}^T C \gamma_\mu \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma_5 C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \gamma_5 C \bar{\ell}_{4a}^T),\end{aligned}\quad (6)$$

$$\begin{aligned}\psi_{2\mu}^A &= \ell_{1a}^T C \gamma^\nu \ell_{2b} (\bar{\ell}_{3a} \sigma_{\mu\nu} C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \sigma_{\mu\nu} C \bar{\ell}_{4a}^T) \\ &\quad - \ell_{1a}^T C \sigma_{\mu\nu} \ell_{2b} (\bar{\ell}_{3a} \gamma^\nu C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \gamma^\nu C \bar{\ell}_{4a}^T).\end{aligned}\quad (7)$$

For the mixed flavor structure $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$, neither component alone has a definite charge-conjugation parity. We therefore introduce the currents $\psi_{i\mu}^{ML}$ in $\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f$ and the currents $\psi_{i\mu}^{MR}$ in $\mathbf{6}_f \otimes \mathbf{3}_f$. The four independent $\psi_{i\mu}^{ML}$ currents are

$$\begin{aligned}\psi_{1\mu}^{ML} &= \ell_{1a}^T C \gamma_\mu \ell_{2b} (\bar{\ell}_{3a} C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} C \bar{\ell}_{4a}^T), \\ \psi_{2\mu}^{ML} &= \ell_{1a}^T C \sigma_{\mu\nu} \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma^\nu \gamma_5 C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \gamma^\nu \gamma_5 C \bar{\ell}_{4a}^T), \\ \psi_{3\mu}^{ML} &= \ell_{1a}^T C \ell_{2b} (\bar{\ell}_{3a} \gamma_\mu C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \gamma_\mu C \bar{\ell}_{4a}^T), \\ \psi_{4\mu}^{ML} &= \ell_{1a}^T C \gamma^\nu \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \sigma_{\mu\nu} \gamma_5 C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \sigma_{\mu\nu} \gamma_5 C \bar{\ell}_{4a}^T).\end{aligned}\quad (8)$$

Similarly, the four independent $\psi_{i\mu}^{MR}$ currents are

$$\begin{aligned}\psi_{1\mu}^{MR} &= \ell_{1a}^T C \ell_{2b} (\bar{\ell}_{3a} \gamma_\mu C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \gamma_\mu C \bar{\ell}_{4a}^T), \\ \psi_{2\mu}^{MR} &= \ell_{1a}^T C \gamma^\nu \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \sigma_{\mu\nu} \gamma_5 C \bar{\ell}_{4b}^T + \bar{\ell}_{3b} \sigma_{\mu\nu} \gamma_5 C \bar{\ell}_{4a}^T), \\ \psi_{3\mu}^{MR} &= \ell_{1a}^T C \gamma_\mu \ell_{2b} (\bar{\ell}_{3a} C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} C \bar{\ell}_{4a}^T), \\ \psi_{4\mu}^{MR} &= \ell_{1a}^T C \sigma_{\mu\nu} \gamma_5 \ell_{2b} (\bar{\ell}_{3a} \gamma^\nu \gamma_5 C \bar{\ell}_{4b}^T - \bar{\ell}_{3b} \gamma^\nu \gamma_5 C \bar{\ell}_{4a}^T).\end{aligned}\quad (9)$$

The mixed currents $\psi_{i\mu}^{ML}$ and $\psi_{i\mu}^{MR}$ all have $J^P = 1^-$, but they do not individually have definite charge-conjugation parity. Currents with definite C parity can be formed as

$$\psi_{i\mu}^{M,\pm} = \psi_{i\mu}^{ML} \pm \psi_{i\mu}^{MR}, \quad (10)$$

where the ‘+’ and ‘−’ combinations correspond to positive and negative charge-conjugation parity, respectively. Since the present work is concerned with the $J^{PC} = 1^{--}$ tetraquark states, only the negative- C combinations are used in the following analysis. For simplicity, we omit the superscript ‘−’ below and denote $\psi_{i\mu}^{M,-}$ simply as $\psi_{i\mu}^M$.

The above construction provides a complete and linearly independent $(\ell\ell)(\bar{\ell}\bar{\ell})$ basis of local derivative-free diquark-antidiquark currents with $J^{PC} = 1^{--}$ within the

present framework. These currents are obtained by combining the local Dirac structures of the diquark and antidiquark fields and imposing the required Lorentz structure, parity, charge-conjugation parity, flavor symmetry, color-singlet condition, and the Pauli principle. Operators related by gamma-matrix identities, index relabeling, or symmetry constraints have been removed. We also note that local meson-meson type operators, $(\bar{\ell}\ell)(\bar{\ell}\ell)$, are related to the diquark-antidiquark operators by Fierz transformations in Dirac, color, and flavor spaces. Therefore, they do not constitute an additional independent local derivative-free operator basis, although they are useful for identifying possible two-meson components coupled to the same currents.

In the following subsections, these currents are projected onto the isospin channels $I = 0, 1$, and 2.

B. Isoscalar ($I = 0$) currents

According to Eq. (3), the isoscalar sector receives contributions from the symmetric (**S**), antisymmetric (**A**), and mixed (**M**) flavor representations. More specifically, the $I = 0$ currents arise from $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ in the **S** representation, from $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ in the **A** representation, and from $qs\bar{q}\bar{s}$ only in the **M** representation. We denote the explicit isoscalar currents by $\psi_{i\mu}^{R(f),I=0}$, where $R = S, A, M$ labels the flavor representation, $f = qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, or $ss\bar{s}\bar{s}$ specifies the flavor content, and i enumerates the independent currents in each class.

For the symmetric flavor structure $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f (\mathbf{S})$, there are six independent isoscalar currents:

$$\begin{aligned}\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=0} &\equiv \psi_{1\mu}^S(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma_5 d_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{d}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 d_b (\bar{u}_a \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_5 C \bar{d}_a^T),\end{aligned}\quad (11a)$$

$$\begin{aligned}\psi_{2\mu}^{S(qq\bar{q}\bar{q}),I=0} &\equiv \psi_{2\mu}^S(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma^\nu d_b (\bar{u}_a \sigma_{\mu\nu} C \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{d}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} d_b (\bar{u}_a \gamma^\nu C \bar{d}_b^T - \bar{u}_b \gamma^\nu C \bar{d}_a^T),\end{aligned}\quad (11b)$$

$$\begin{aligned}\psi_{1\mu}^{S(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{1\mu}^S(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_5 s_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 s_b (\bar{u}_a \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_5 C \bar{s}_a^T),\end{aligned}\quad (11c)$$

$$\begin{aligned}\psi_{2\mu}^{S(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{2\mu}^S(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu s_b (\bar{u}_a \sigma_{\mu\nu} C \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} s_b (\bar{u}_a \gamma^\nu C \bar{s}_b^T - \bar{u}_b \gamma^\nu C \bar{s}_a^T),\end{aligned}\quad (11d)$$

$$\begin{aligned}\psi_{1\mu}^{S(ss\bar{s}\bar{s}),I=0} &\equiv \psi_{1\mu}^S(ss\bar{s}\bar{s}) \\ &= s_a^T C \gamma_5 s_b (\bar{s}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{s}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - s_a^T C \gamma_\mu \gamma_5 s_b (\bar{s}_a \gamma_5 C \bar{s}_b^T + \bar{s}_b \gamma_5 C \bar{s}_a^T),\end{aligned}\quad (11e)$$

$$\begin{aligned}\psi_{2\mu}^{S(ss\bar{s}\bar{s}),I=0} &\equiv \psi_{2\mu}^S(ss\bar{s}\bar{s}) \\ &= s_a^T C \gamma^\nu s_b (\bar{s}_a \sigma_{\mu\nu} C \bar{s}_b^T - \bar{s}_b \sigma_{\mu\nu} C \bar{s}_a^T) \\ &\quad - s_a^T C \sigma_{\mu\nu} s_b (\bar{s}_a \gamma^\nu C \bar{s}_b^T - \bar{s}_b \gamma^\nu C \bar{s}_a^T).\end{aligned}\quad (11f)$$

We use the symbol “ $\sim$ ” to indicate that the quark contents written here are only schematic. For example, in the current $\psi_{1\mu}^{S(qs\bar{q}\bar{s}),I=0}$, the component $us\bar{u}\bar{s}$ alone does not have definite isospin $I = 0$; the proper isoscalar flavor structure is the combination $(us\bar{u}\bar{s} + ds\bar{d}\bar{s})$, i.e.,

$$\begin{aligned}\psi_{1\mu}^{S(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{1\mu}^S(qs\bar{q}\bar{s}) \\ &= u_a^T C \gamma_5 s_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 s_b (\bar{u}_a \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_5 C \bar{s}_a^T) \\ &\quad + d_a^T C \gamma_5 s_b (\bar{d}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{d}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - d_a^T C \gamma_\mu \gamma_5 s_b (\bar{d}_a \gamma_5 C \bar{s}_b^T + \bar{d}_b \gamma_5 C \bar{s}_a^T).\end{aligned}\quad (12)$$

In deriving the QCD sum rules, we work in the isospin limit by neglecting the up- and down-quark masses and taking $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$. We also retain only connected contractions of the quark fields. With these approximations, the u and d components in the above current have identical connected operator product expansion (OPE) expressions after the replacement $u \leftrightarrow d$, while the cross contractions between the two flavor components are absent. Therefore, the schematic component $us\bar{u}\bar{s}$ and the full isospin combination lead to the same mass sum rule in the present analysis, up to an overall normalization of the current. However, this equality should not be interpreted as an exact physical degeneracy. Disconnected annihilation-type diagrams are not included in the present calculation. Such contributions may occur in isoscalar channels and can split the $I = 0$ spectrum from the nonisoscalar ones. They may also induce mixing with glue-rich configurations, conventional $\bar{q}q$ mesons, and two-meson components. A quantitative treatment of these effects would require additional disconnected correlators and is not included in the present analysis.

For the antisymmetric flavor structure $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$ (**A**), there are four independent isoscalar currents:

$$\begin{aligned}\psi_{1\mu}^{A(qq\bar{q}\bar{q}),I=0} &\equiv \psi_{1\mu}^A(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma_5 d_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{d}_b^T - \bar{u}_b \gamma_\mu \gamma_5 C \bar{d}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 d_b (\bar{u}_a \gamma_5 C \bar{d}_b^T - \bar{u}_b \gamma_5 C \bar{d}_a^T),\end{aligned}\quad (13a)$$

$$\begin{aligned}\psi_{2\mu}^{A(qq\bar{q}\bar{q}),I=0} &\equiv \psi_{2\mu}^A(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma^\nu d_b (\bar{u}_a \sigma_{\mu\nu} C \bar{d}_b^T + \bar{u}_b \sigma_{\mu\nu} C \bar{d}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} d_b (\bar{u}_a \gamma^\nu C \bar{d}_b^T + \bar{u}_b \gamma^\nu C \bar{d}_a^T),\end{aligned}\quad (13b)$$

$$\begin{aligned}\psi_{1\mu}^{A(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{1\mu}^A(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_5 s_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 s_b (\bar{u}_a \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma_5 C \bar{s}_a^T),\end{aligned}\quad (13c)$$

$$\begin{aligned}\psi_{2\mu}^{A(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{2\mu}^A(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu s_b (\bar{u}_a \sigma_{\mu\nu} C \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} s_b (\bar{u}_a \gamma^\nu C \bar{s}_b^T + \bar{u}_b \gamma^\nu C \bar{s}_a^T).\end{aligned}\quad (13d)$$

For the mixed flavor structure $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ (**M**), the isoscalar sector is contributed by the $qs\bar{q}\bar{s}$ channel only. Following Eq. (10), the currents denoted by $\psi_{i\mu}^M$ below are the negative- C combinations $\psi_{i\mu}^{ML} - \psi_{i\mu}^{MR}$. The corresponding four independent currents are

$$\begin{aligned}\psi_{1\mu}^{M(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{1\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_\mu s_b (\bar{u}_a C \bar{s}_b^T + \bar{u}_b C \bar{s}_a^T) \\ &\quad - u_a^T C s_b (\bar{u}_a \gamma_\mu C \bar{s}_b^T + \bar{u}_b \gamma_\mu C \bar{s}_a^T),\end{aligned}\quad (14a)$$

$$\begin{aligned}\psi_{2\mu}^{M(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{2\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \sigma_{\mu\nu} \gamma_5 s_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma^\nu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma^\nu \gamma_5 s_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T),\end{aligned}\quad (14b)$$

$$\begin{aligned}\psi_{3\mu}^{M(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{3\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C s_b (\bar{u}_a \gamma_\mu C \bar{s}_b^T - \bar{u}_b \gamma_\mu C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu s_b (\bar{u}_a C \bar{s}_b^T - \bar{u}_b C \bar{s}_a^T),\end{aligned}\quad (14c)$$

$$\begin{aligned}\psi_{4\mu}^{M(qs\bar{q}\bar{s}),I=0} &\equiv \psi_{4\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu \gamma_5 s_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} \gamma_5 s_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma^\nu \gamma_5 C \bar{s}_a^T).\end{aligned}\quad (14d)$$

C. Isovector ($I = 1$) currents

According to Eq. (3), the isovector sector receives contributions from the symmetric (**S**), antisymmetric (**A**), and mixed (**M**) flavor representations. More specifically, the $I = 1$ currents arise from $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ in the **S** representation, from $qs\bar{q}\bar{s}$ in the **A** representation, and from $qq\bar{q}\bar{q}$ as well as $qs\bar{q}\bar{s}$ in the **M** representation. We denote the explicit isovector currents by $\psi_{i\mu}^{R(f),I=1}$.

For the symmetric flavor structure $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$ (**S**), there are four independent isovector currents:

$$\begin{aligned} \psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{1\mu}^S(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma_5 d_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{d}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 d_b (\bar{u}_a \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_5 C \bar{d}_a^T), \end{aligned} \quad (15a)$$

$$\begin{aligned} \psi_{2\mu}^{S(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{2\mu}^S(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma^\nu d_b (\bar{u}_a \sigma_{\mu\nu} C \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{d}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} d_b (\bar{u}_a \gamma^\nu C \bar{d}_b^T - \bar{u}_b \gamma^\nu C \bar{d}_a^T), \end{aligned} \quad (15b)$$

$$\begin{aligned} \psi_{1\mu}^{S(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{1\mu}^S(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_5 s_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 s_b (\bar{u}_a \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma_5 C \bar{s}_a^T), \end{aligned} \quad (15c)$$

$$\begin{aligned} \psi_{2\mu}^{S(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{2\mu}^S(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu s_b (\bar{u}_a \sigma_{\mu\nu} C \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} s_b (\bar{u}_a \gamma^\nu C \bar{s}_b^T - \bar{u}_b \gamma^\nu C \bar{s}_a^T). \end{aligned} \quad (15d)$$

Although the explicit flavor wave functions of the current $\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=1}$ in Eq. (15) and the current $\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=0}$ in Eq. (11) are different, the two currents have the same Dirac and color operator structures. As explained in Sec. II.B, under the isospin-symmetric and connected-contraction approximations, their QCD sum rules coincide up to overall flavor-normalization factors. Therefore, they lead to identical numerical entries in Table I within the present calculation.

For the antisymmetric flavor structure $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$ (**A**), there are two independent isovector currents:

$$\begin{aligned} \psi_{1\mu}^{A(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{1\mu}^A(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_5 s_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu \gamma_5 s_b (\bar{u}_a \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma_5 C \bar{s}_a^T), \end{aligned} \quad (16a)$$

$$\begin{aligned} \psi_{2\mu}^{A(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{2\mu}^A(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu s_b (\bar{u}_a \sigma_{\mu\nu} C \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} s_b (\bar{u}_a \gamma^\nu C \bar{s}_b^T + \bar{u}_b \gamma^\nu C \bar{s}_a^T). \end{aligned} \quad (16b)$$

For the mixed flavor structure $(\bar{\mathbf{3}}_f \otimes \bar{\mathbf{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ (**M**), the isovector sector receives contributions from both the $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ channels. Again, the currents $\psi_{i\mu}^M$ displayed below are understood as the negative- C combinations $\psi_{i\mu}^{ML} - \psi_{i\mu}^{MR}$. The corresponding independent currents are

$$\begin{aligned} \psi_{1\mu}^{M(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{1\mu}^M(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma_\mu d_b (\bar{u}_a C \bar{d}_b^T + \bar{u}_b C \bar{d}_a^T) \\ &\quad - u_a^T C d_b (\bar{u}_a \gamma_\mu C \bar{d}_b^T + \bar{u}_b \gamma_\mu C \bar{d}_a^T), \end{aligned} \quad (17a)$$

$$\begin{aligned} \psi_{2\mu}^{M(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{2\mu}^M(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \sigma_{\mu\nu} \gamma_5 d_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma^\nu \gamma_5 C \bar{d}_a^T) \\ &\quad - u_a^T C \gamma^\nu \gamma_5 d_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{d}_b^T + \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{d}_a^T), \end{aligned} \quad (17b)$$

$$\begin{aligned} \psi_{3\mu}^{M(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{3\mu}^M(qq\bar{q}\bar{q}) \\ &\sim u_a^T C d_b (\bar{u}_a \gamma_\mu C \bar{d}_b^T - \bar{u}_b \gamma_\mu C \bar{d}_a^T) \\ &\quad - u_a^T C \gamma_\mu d_b (\bar{u}_a C \bar{d}_b^T - \bar{u}_b C \bar{d}_a^T), \end{aligned} \quad (17c)$$

$$\begin{aligned} \psi_{4\mu}^{M(qq\bar{q}\bar{q}),I=1} &\equiv \psi_{4\mu}^M(qq\bar{q}\bar{q}) \\ &\sim u_a^T C \gamma^\nu \gamma_5 d_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{d}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} \gamma_5 d_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{d}_b^T - \bar{u}_b \gamma^\nu \gamma_5 C \bar{d}_a^T), \end{aligned} \quad (17d)$$

$$\begin{aligned} \psi_{1\mu}^{M(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{1\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma_\mu s_b (\bar{u}_a C \bar{s}_b^T + \bar{u}_b C \bar{s}_a^T) \\ &\quad - u_a^T C s_b (\bar{u}_a \gamma_\mu C \bar{s}_b^T + \bar{u}_b \gamma_\mu C \bar{s}_a^T), \end{aligned} \quad (17e)$$

$$\begin{aligned} \psi_{2\mu}^{M(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{2\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \sigma_{\mu\nu} \gamma_5 s_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{s}_b^T + \bar{u}_b \gamma^\nu \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma^\nu \gamma_5 s_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T + \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T), \end{aligned} \quad (17f)$$

$$\begin{aligned} \psi_{3\mu}^{M(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{3\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C s_b (\bar{u}_a \gamma_\mu C \bar{s}_b^T - \bar{u}_b \gamma_\mu C \bar{s}_a^T) \\ &\quad - u_a^T C \gamma_\mu s_b (\bar{u}_a C \bar{s}_b^T - \bar{u}_b C \bar{s}_a^T), \end{aligned} \quad (17g)$$

$$\begin{aligned} \psi_{4\mu}^{M(qs\bar{q}\bar{s}),I=1} &\equiv \psi_{4\mu}^M(qs\bar{q}\bar{s}) \\ &\sim u_a^T C \gamma^\nu \gamma_5 s_b (\bar{u}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T - \bar{u}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T) \\ &\quad - u_a^T C \sigma_{\mu\nu} \gamma_5 s_b (\bar{u}_a \gamma^\nu \gamma_5 C \bar{s}_b^T - \bar{u}_b \gamma^\nu \gamma_5 C \bar{s}_a^T). \end{aligned} \quad (17h)$$

D. Isotensor ($I=2$) currents

According to Eq. (3), the isotensor sector receives a contribution only from the symmetric flavor representation $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$ (**S**), and only through the $qq\bar{q}\bar{q}$ channel. There are two independent currents, which we denote by $\psi_{i\mu}^{S(qq\bar{q}\bar{q}),I=2}$:

TABLE I. QCD sum rule results for the light tetraquark states with $J^{PC} = 1^{--}$. The quoted uncertainties are obtained by varying M_B^2 , s_0 , and the QCD input parameters within the ranges specified in the numerical analysis. The identical numerical entries for the $qs\bar{q}\bar{s}$ $I = 0$ and $I = 1$ rows, as well as for the S-type $qq\bar{q}\bar{q}$ rows with $I = 0, 1$, and 2 , arise from the isospin limit and the connected-contraction approximation used in the present calculation; they should not be interpreted as exact physical degeneracies once possible disconnected annihilation-type contributions are included.

Current	State [$I^G J^{PC}$]	$s_0^{\min}$	Working regions		Pole	Mass	Decay constant
		[GeV ²]	M_B^2 [GeV ²]	s_0 [GeV ²]	[%]	[GeV]	[GeV ⁵]
$\psi_{1\mu}^{S(qq\bar{q}\bar{q}), I=0}$	$ qq\bar{q}\bar{q}; 0^- 1^{--}\rangle$	6.8	1.86–2.03	$7.4^{+1.0}_{-1.0}$	30–36	$2.37^{+0.12}_{-0.12}$	$0.0150^{+0.0047}_{-0.0041}$
$\psi_{2\mu}^{S(qq\bar{q}\bar{q}), I=0}$	$ qq\bar{q}\bar{q}; 0^- 1^{--}\rangle$	6.3	1.77–1.92	$6.8^{+1.0}_{-1.0}$	30–36	$2.24^{+0.12}_{-0.14}$	$0.0152^{+0.0051}_{-0.0044}$
$\psi_{1\mu}^{S(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	7.2	1.86–2.10	$8.0^{+1.0}_{-1.0}$	30–39	$2.51^{+0.11}_{-0.11}$	$0.0175^{+0.0052}_{-0.0046}$
$\psi_{2\mu}^{S(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	6.7	1.76–1.98	$7.4^{+1.0}_{-1.0}$	30–39	$2.39^{+0.11}_{-0.12}$	$0.0177^{+0.0056}_{-0.0049}$
$\psi_{1\mu}^{S(ss\bar{s}\bar{s}), I=0}$	$ ss\bar{s}\bar{s}; 0^- 1^{--}\rangle$	7.6	1.83–1.86	$7.7^{+1.0}_{-1.0}$	30–31	$2.60^{+0.13}_{-0.10}$	$0.0163^{+0.0052}_{-0.0044}$
$\psi_{2\mu}^{S(ss\bar{s}\bar{s}), I=0}$	$ ss\bar{s}\bar{s}; 0^- 1^{--}\rangle$	7.1	1.74–1.78	$7.2^{+1.0}_{-1.0}$	30–32	$2.46^{+0.13}_{-0.11}$	$0.0165^{+0.0056}_{-0.0048}$
$\psi_{1\mu}^{A(qq\bar{q}\bar{q}), I=0}$	$ qq\bar{q}\bar{q}; 0^- 1^{--}\rangle$	3.3	1.18–1.28	$3.7^{+1.0}_{-1.0}$	30–36	$1.64^{+0.15}_{-0.14}$	$0.0028^{+0.0013}_{-0.0010}$
$\psi_{2\mu}^{A(qq\bar{q}\bar{q}), I=0}$	$ qq\bar{q}\bar{q}; 0^- 1^{--}\rangle$	4.7	1.47–1.60	$5.2^{+1.0}_{-1.0}$	30–37	$1.88^{+0.15}_{-0.18}$	$0.0119^{+0.0048}_{-0.0041}$
$\psi_{1\mu}^{A(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	4.7	1.39–1.52	$5.2^{+1.0}_{-1.0}$	30–37	$1.98^{+0.13}_{-0.12}$	$0.0048^{+0.0019}_{-0.0016}$
$\psi_{2\mu}^{A(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	5.2	1.50–1.64	$5.7^{+1.0}_{-1.0}$	30–37	$2.03^{+0.14}_{-0.15}$	$0.0137^{+0.0054}_{-0.0046}$
$\psi_{1\mu}^{M(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	6.8	1.77–1.99	$7.5^{+1.0}_{-1.0}$	30–38	$2.42^{+0.11}_{-0.11}$	$0.0150^{+0.0047}_{-0.0041}$
$\psi_{2\mu}^{M(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	4.3	1.32–1.46	$4.8^{+1.0}_{-1.0}$	30–38	$1.86^{+0.14}_{-0.14}$	$0.0099^{+0.0042}_{-0.0034}$
$\psi_{3\mu}^{M(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	5.9	1.63–1.79	$6.5^{+1.0}_{-1.0}$	30–37	$2.21^{+0.12}_{-0.13}$	$0.0075^{+0.0027}_{-0.0023}$
$\psi_{4\mu}^{M(qs\bar{q}\bar{s}), I=0}$	$ qs\bar{q}\bar{s}; 0^- 1^{--}\rangle$	6.0	1.62–1.80	$6.6^{+1.0}_{-1.0}$	30–38	$2.24^{+0.12}_{-0.12}$	$0.0136^{+0.0047}_{-0.0040}$
$\psi_{1\mu}^{S(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	6.8	1.86–2.03	$7.4^{+1.0}_{-1.0}$	30–36	$2.37^{+0.12}_{-0.12}$	$0.0150^{+0.0047}_{-0.0041}$
$\psi_{2\mu}^{S(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	6.3	1.77–1.92	$6.8^{+1.0}_{-1.0}$	30–36	$2.24^{+0.12}_{-0.14}$	$0.0152^{+0.0051}_{-0.0044}$
$\psi_{1\mu}^{S(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	7.2	1.86–2.10	$8.0^{+1.0}_{-1.0}$	30–39	$2.51^{+0.11}_{-0.11}$	$0.0175^{+0.0052}_{-0.0046}$
$\psi_{2\mu}^{S(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	6.7	1.76–1.98	$7.4^{+1.0}_{-1.0}$	30–39	$2.39^{+0.11}_{-0.12}$	$0.0177^{+0.0056}_{-0.0049}$
$\psi_{1\mu}^{A(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	4.7	1.39–1.52	$5.2^{+1.0}_{-1.0}$	30–37	$1.98^{+0.13}_{-0.12}$	$0.0048^{+0.0019}_{-0.0016}$
$\psi_{2\mu}^{A(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	5.2	1.50–1.64	$5.7^{+1.0}_{-1.0}$	30–37	$2.03^{+0.14}_{-0.15}$	$0.0137^{+0.0054}_{-0.0046}$
$\psi_{1\mu}^{M(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	6.1	1.71–1.88	$6.7^{+1.0}_{-1.0}$	30–37	$2.24^{+0.12}_{-0.13}$	$0.0119^{+0.0040}_{-0.0035}$
$\psi_{2\mu}^{M(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	3.0	1.13–1.26	$3.5^{+1.0}_{-1.0}$	30–39	$1.53^{+0.17}_{-0.19}$	$0.0060^{+0.0029}_{-0.0024}$
$\psi_{3\mu}^{M(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	5.4	1.61–1.76	$6.0^{+1.0}_{-1.0}$	30–37	$2.07^{+0.14}_{-0.15}$	$0.0066^{+0.0024}_{-0.0021}$
$\psi_{4\mu}^{M(qq\bar{q}\bar{q}), I=1}$	$ qq\bar{q}\bar{q}; 1^+ 1^{--}\rangle$	5.2	1.54–1.68	$5.7^{+1.0}_{-1.0}$	30–36	$2.03^{+0.13}_{-0.15}$	$0.0102^{+0.0039}_{-0.0033}$
$\psi_{1\mu}^{M(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	6.8	1.77–1.99	$7.5^{+1.0}_{-1.0}$	30–38	$2.42^{+0.11}_{-0.11}$	$0.0150^{+0.0047}_{-0.0041}$
$\psi_{2\mu}^{M(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	4.3	1.32–1.46	$4.8^{+1.0}_{-1.0}$	30–38	$1.86^{+0.14}_{-0.14}$	$0.0099^{+0.0042}_{-0.0034}$
$\psi_{3\mu}^{M(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	5.9	1.63–1.79	$6.5^{+1.0}_{-1.0}$	30–37	$2.21^{+0.12}_{-0.13}$	$0.0075^{+0.0027}_{-0.0023}$
$\psi_{4\mu}^{M(qs\bar{q}\bar{s}), I=1}$	$ qs\bar{q}\bar{s}; 1^+ 1^{--}\rangle$	6.0	1.62–1.80	$6.6^{+1.0}_{-1.0}$	30–38	$2.24^{+0.12}_{-0.12}$	$0.0136^{+0.0047}_{-0.0040}$
$\psi_{1\mu}^{S(qq\bar{q}\bar{q}), I=2}$	$ qq\bar{q}\bar{q}; 2^- 1^{--}\rangle$	6.8	1.86–2.03	$7.4^{+1.0}_{-1.0}$	30–36	$2.37^{+0.12}_{-0.12}$	$0.0150^{+0.0047}_{-0.0041}$
$\psi_{2\mu}^{S(qq\bar{q}\bar{q}), I=2}$	$ qq\bar{q}\bar{q}; 2^- 1^{--}\rangle$	6.3	1.77–1.92	$6.8^{+1.0}_{-1.0}$	30–36	$2.24^{+0.12}_{-0.14}$	$0.0152^{+0.0051}_{-0.0044}$

$$\begin{aligned}
\psi_{1\mu}^{S(qq\bar{q}\bar{q}), I=2} &\equiv \psi_{1\mu}^S(qq\bar{q}\bar{q}) \\
&\sim u_a^T C \gamma_5 d_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{d}_a^T) \\
&\quad - u_a^T C \gamma_\mu \gamma_5 d_b (\bar{u}_a \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_5 C \bar{d}_a^T), \quad (18a)
\end{aligned}$$

$$\begin{aligned}
\psi_{2\mu}^{S(qq\bar{q}\bar{q}), I=2} &\equiv \psi_{2\mu}^S(qq\bar{q}\bar{q}) \\
&\sim u_a^T C \gamma^\nu d_b (\bar{u}_a \sigma_{\mu\nu} C \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{d}_a^T) \\
&\quad - u_a^T C \sigma_{\mu\nu} d_b (\bar{u}_a \gamma^\nu C \bar{d}_b^T - \bar{u}_b \gamma^\nu C \bar{d}_a^T). \quad (18b)
\end{aligned}$$

Although the explicit flavor wave function of the current $\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=2}$ in Eq. (18) is different from those of the currents $\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=1}$ in Eq. (15) and $\psi_{1\mu}^{S(qq\bar{q}\bar{q}),I=0}$ in Eq. (11), these currents have the same Dirac and color operator structures. Under the isospin-symmetric and connected-contraction approximations discussed in Sec. II.B, their QCD sum rules coincide up to overall flavor-normalization factors, leading to identical numerical entries in Table I within the present calculation.

III. QCD SUM RULES

Over the past several decades, the QCD sum rule method has proven itself to be a powerful and effective nonperturbative technique [44–48]. In sum rule analyses, we focus on two-point correlation functions, which are defined as

$$\Pi_{\mu\nu}(p^2) \equiv i \int d^4x e^{ipx} \langle 0 | T(\eta_\mu(x) \eta_\nu^\dagger(0)) | 0 \rangle, \quad (19)$$

where η_μ represents the interpolating current associated with the tetraquark state, denoted as Y . The Lorentz structure of the correlation function can be simplified to

$$\Pi_{\mu\nu}(p^2) = \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right) \Pi^{(1)}(p^2) + \frac{p_\mu p_\nu}{p^2} \Pi^{(0)}(p^2). \quad (20)$$

We compute $\Pi^{(1)}(p^2)$ within the framework of the OPE in QCD, truncating the expansion at a certain order to extract the OPE spectral density $\rho_{\text{OPE}}(s)$. This result is then matched with a hadronic parametrization to extract information about hadron properties.

At the hadronic level, the correlation function is represented as a dispersion relation involving a phenomenological spectral function:

$$\Pi^{(1)}(p^2) = \int_{s_<}^{\infty} \frac{\rho_{\text{phen}}(s)}{s - p^2 - i\epsilon} ds, \quad (21)$$

where $s_<$ is the physical threshold. The spectral density $\rho_{\text{phen}}(s)$ is defined as

$$\begin{aligned} & \rho_{\text{phen}}(s) \times \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right) \\ & \equiv \sum_n \delta(s - M_n^2) \langle 0 | \eta_\mu | n \rangle \langle n | \eta_\nu^\dagger | 0 \rangle \\ & = f_Y^2 \delta(s - M_Y^2) \epsilon_\mu \epsilon_\nu^* + \text{continuum}, \end{aligned} \quad (22)$$

where the coupling constant f_Y is defined via the matrix element

$$\langle 0 | \eta_\mu | Y \rangle = f_Y \epsilon_\mu, \quad (23)$$

with ϵ_μ being the polarization vector.

In the second equation, we adopt the standard parametrization, assuming one-pole dominance for the ground state Y and including a continuum contribution. The sum rule analysis is then performed after applying the Borel transformation to Eq. (21):

$$\Pi(\infty, M_B^2) \equiv \mathcal{B}_{M_B^2} \Pi^{(1)}(p^2) = \int_{s_<}^{\infty} e^{-s/M_B^2} \rho_{\text{phen}}(s) ds. \quad (24)$$

Assuming that the contribution from the continuum states can be effectively approximated by the OPE spectral density above a threshold value s_0 , we obtain the following sum rule equation:

$$\Pi(s_0, M_B^2) = f_Y^2 e^{-M_Y^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds. \quad (25)$$

By differentiating this equation with respect to $1/M_B^2$, we obtain the expression for the mass squared of the ground state Y :

$$M_Y^2 = \frac{\frac{\partial}{\partial(-1/M_B^2)} \Pi(s_0, M_B^2)}{\Pi(s_0, M_B^2)} = \frac{\int_{s_<}^{s_0} e^{-s/M_B^2} s \rho_{\text{OPE}}(s) ds}{\int_{s_<}^{s_0} e^{-s/M_B^2} \rho_{\text{OPE}}(s) ds}. \quad (26)$$

Additionally, we arrive at the corollary:

$$f_Y^2 = \Pi(s_0, M_B^2) \times e^{M_Y^2/M_B^2}. \quad (27)$$

In the next section, we will examine both Eqs. (26) and (27) as functions of the Borel mass M_B and the continuum threshold s_0 , for various tetraquark currents.

We have performed the OPE calculation up to dimension ten. Here, we present the OPE expression for the representative current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$, whose isoscalar and isovector forms are given in Eqs. (14b) and (17f), respectively, while the results for other currents are provided in Appendix:

$$\begin{aligned}
\Pi_{2\mu}^{M(qs\bar{q}\bar{s})}(s_0, M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{6144\pi^6} s^4 - \frac{3m_s^2}{512\pi^6} s^3 + \left(\frac{11\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{256\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{32\pi^4} \right) s^2 \right. \\
& + \left(\frac{7m_s^2 \langle g_s^2 GG \rangle}{6144\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{4\pi^4} + \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^4} - \frac{m_s \langle g_s \bar{s}\sigma Gs \rangle}{64\pi^4} + \frac{\langle \bar{q}q \rangle^2}{12\pi^2} + \frac{\langle \bar{s}s \rangle^2}{12\pi^2} \right) s \\
& + \left(\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{64\pi^4} - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{256\pi^4} - \frac{3m_s^2 \langle \bar{q}q \rangle^2}{2\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{8\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{16\pi^2} \right. \\
& - \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{24\pi^2} - \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{24\pi^2} + \left. \frac{\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{16\pi^2} \right) \Big] e^{-s/M_B^2} ds \\
& + \left(\frac{5m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1152\pi^2} + \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{1152\pi^4} - \frac{25m_s \langle g_s^2 GG \rangle \langle g_s \bar{s}\sigma Gs \rangle}{4608\pi^4} + \frac{25 \langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{1728\pi^2} \right. \\
& - \frac{5 \langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{216\pi^2} + \frac{25 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1728\pi^2} + \frac{m_s^4 \langle \bar{q}q \rangle^2}{2\pi^2} - \frac{7m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{8\pi^2} + \frac{m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{12\pi^2} \\
& \left. + \frac{10m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{3} - \frac{\langle g_s \bar{q}\sigma Gq \rangle \langle g_s \bar{s}\sigma Gs \rangle}{24\pi^2} \right),
\end{aligned}$$

In the above equation, $\langle \bar{q}q \rangle$ represents the $D = 3$ up/down quark condensate; $\langle g_s^2 GG \rangle$ refers to the $D = 4$ gluon condensate; and $\langle g_s \bar{q}\sigma Gq \rangle$ is the $D = 5$ mixed condensate. As is customary, the factorized approximation is adopted for higher-dimensional condensates, such as $\langle 0|\bar{q}q\bar{q}q|0 \rangle \sim \langle 0|\bar{q}q|0 \rangle \langle 0|\bar{q}q|0 \rangle$. The possible violation of this approximation is further examined by introducing a factorization parameter κ in the numerical analysis.

IV. NUMERICAL ANALYSIS

To reliably extract the mass using Eq. (26), it is essential to determine the appropriate regions for the two free parameters; the continuum threshold s_0 and the Borel mass M_B . We present the result for the representative current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$, whose isoscalar and isovector forms are given in Eqs. (14b) and (17f), respectively. In the isospin limit and within the connected-contraction approximation adopted in this work, these two currents lead to identical mass sum rules, differing only by their flavor combinations and overall normalization.

In the numerical analysis, we adopt the following QCD input parameters at the renormalization scale $\mu = 2$ GeV [1,49–54]:

$$\begin{aligned}
\langle \bar{q}q \rangle &= -[(0.240 \pm 0.010) \text{ GeV}]^3, \\
\langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\
m_s(2 \text{ GeV}) &= 93_{-3}^{+9} \text{ MeV}, \\
\langle \alpha_s GG \rangle &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\
M_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2, \\
\langle g_s \bar{q}\sigma Gq \rangle &= -M_0^2 \times \langle \bar{q}q \rangle, \\
\langle g_s \bar{s}\sigma Gs \rangle &= -M_0^2 \times \langle \bar{s}s \rangle.
\end{aligned} \tag{28}$$

Here the gluon condensate is quoted in the convention $\langle \alpha_s GG \rangle$. In the OPE expressions, where the convention $\langle g_s^2 GG \rangle$ is used, we make the conversion $\langle g_s^2 GG \rangle = 4\pi \langle \alpha_s GG \rangle$. All scale-dependent input parameters are taken at $\mu = 2$ GeV, and the strange-quark mass is given in the $\overline{\text{MS}}$ scheme.

Before applying the standard QCD sum rule criteria, we first examine the behavior of the Borel-transformed correlation function as a function of the continuum threshold s_0 . This check is particularly useful in the $J^{PC} = 1^{--}$ channel, since these quantum numbers are not exotic and the tetraquark currents may in principle couple to ordinary vector mesons, their radial excitations, and two-meson continua. In the present sum rule framework, a physical pole contribution below the continuum threshold s_0 is parametrized as $f_Y^2 \exp(-M_Y^2/M_B^2)$ and should give a positive contribution to $\Pi(s_0, M_B^2)$.

As a representative example, Fig. 2 shows the behavior of $\Pi(s_0, M_B^2)$ for the current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$ at several fixed values of M_B^2 . One finds that the correlation function is negative in the very low- s_0 region, and hence this region cannot be matched to a physical single-pole contribution. This observation provides a useful consistency check for identifying an unphysical low-threshold region before determining the working window. Nevertheless, it is current-dependent and does not by itself separate compact tetraquark configurations from conventional $\bar{q}q$ excitations or two-meson continuum effects. The pole extracted below should therefore be understood as an effective lowest resonance coupled to the chosen tetraquark current, rather than as direct evidence for a pure compact tetraquark state.

After this preliminary check, we proceed to impose the standard QCD sum rule criteria. We first ensure good OPE convergence by requiring that the combined contribution from the dimension-8 and dimension-10 condensates be

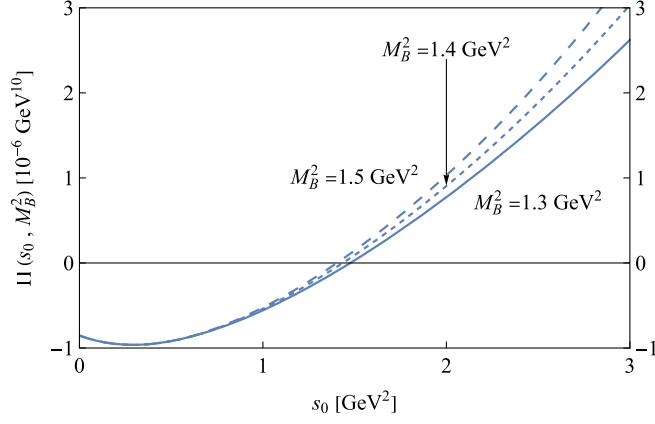


FIG. 2. The Borel-transformed correlation function $\Pi(s_0, M_B^2)$ as a function of the continuum threshold s_0 for the current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$. The curves are obtained by taking $M_B^2 = 1.3, 1.4, 1.5 \text{ GeV}^2$. The negative behavior in the very low- s_0 region indicates that this region cannot be matched to a physical single-pole contribution.

less than 10% of the total:

$$\text{CVG}_{10+8} \equiv \left| \frac{\Pi^{D=10+8}(\infty, M_B^2)}{\Pi(\infty, M_B^2)} \right| \leq 10\%. \quad (29)$$

This condition imposes a lower bound on the Borel mass: $M_B^2 \geq 1.32 \text{ GeV}^2$.

Second, to ensure a sufficient pole contribution, we require that the pole contribution (PC) exceed 30%:

$$\text{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 30\%. \quad (30)$$

For multiquark currents, the pole contribution is usually smaller than that in conventional meson sum rules because of the high dimension of the interpolating currents and the rapid growth of the spectral density. We therefore use $\text{PC} \geq 30\%$ as a minimal working criterion, together with the requirements of OPE convergence and Borel stability. We emphasize that this criterion is relatively loose, and the resulting masses should be regarded as QCD sum rule estimates rather than high-precision determinations. In particular, for currents with very narrow Borel windows and pole contributions close to this lower bound, the extracted masses are less stable and should be treated with caution. For the representative current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$, we find that this condition is satisfied only when $s_0 \geq s_0^{\min} = 4.3 \text{ GeV}^2$. Accordingly, we adopt a slightly larger value of $s_0 = 4.8 \text{ GeV}^2$, resulting in a valid Borel window of $1.32 \text{ GeV}^2 \leq M_B^2 \leq 1.46 \text{ GeV}^2$, as shown in Fig. 3.

Finally, we require that the extracted mass exhibit only mild sensitivity to variations in both the continuum threshold s_0 and the Borel mass M_B^2 . As shown in Fig. 4, the mass dependence on these parameters remains moderate within

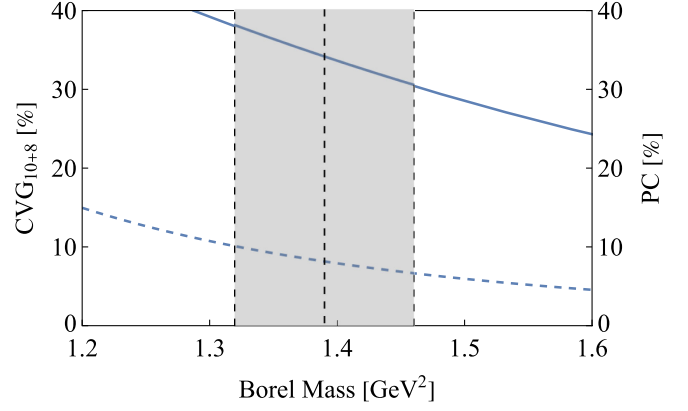


FIG. 3. OPE convergence and pole contribution for the current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$ as functions of the Borel mass M_B at $s_0 = 4.8 \text{ GeV}^2$. The dashed curve denotes CVG_{10+8} , defined as the contribution from the dimension-8 and dimension-10 condensates, while the solid curve denotes the pole contribution.

the working regions $3.8 \text{ GeV}^2 \leq s_0 \leq 5.8 \text{ GeV}^2$ and $1.32 \text{ GeV}^2 \leq M_B^2 \leq 1.46 \text{ GeV}^2$. The resulting mass is determined to be

$$M = 1.86_{-0.14}^{+0.14} \text{ GeV}, \quad (31)$$

where the uncertainty reflects the combined variations in the Borel mass M_B^2 , the continuum threshold s_0 , and the QCD input parameters listed in Eq. (28). In the isospin limit and within the connected-contraction approximation adopted in this work, the isoscalar and isovector currents $\psi_{2\mu}^{M(qs\bar{q}\bar{s}), I=0}$ and $\psi_{2\mu}^{M(qs\bar{q}\bar{s}), I=1}$ have identical connected OPE expressions up to overall flavor-normalization factors. Consequently, they lead to the same extracted mass,

$$M_{\psi_{2\mu}^M(qs\bar{q}\bar{s})}^{I=0} = M_{\psi_{2\mu}^M(qs\bar{q}\bar{s})}^{I=1} = 1.86_{-0.14}^{+0.14} \text{ GeV}. \quad (32)$$

In the above analysis, the factorized four-quark condensates are evaluated under the vacuum saturation approximation. To estimate the possible impact of violations of this approximation for the dimension-six four-quark condensates, we introduce a factorization parameter κ through

$$\begin{aligned} \langle \bar{q}q \rangle^2 &\rightarrow \kappa \langle \bar{q}q \rangle^2, \\ \langle \bar{q}q \rangle \langle \bar{s}s \rangle &\rightarrow \kappa \langle \bar{q}q \rangle \langle \bar{s}s \rangle, \\ \langle \bar{s}s \rangle^2 &\rightarrow \kappa \langle \bar{s}s \rangle^2. \end{aligned} \quad (33)$$

The vacuum saturation limit corresponds to $\kappa = 1$. As a representative sensitivity check, we vary κ in the range $1 \leq \kappa \leq 4$ for the current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$. The extracted mass decreases from

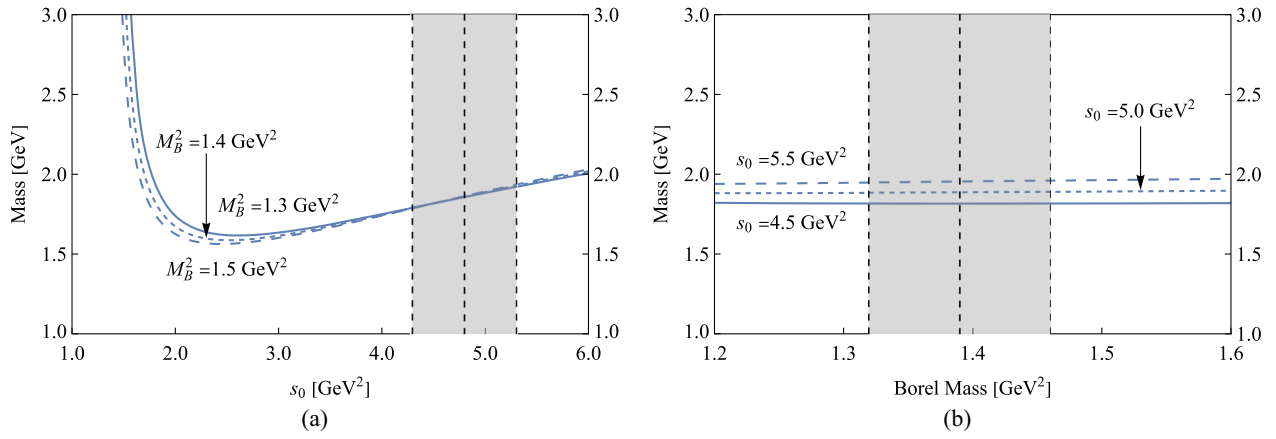


FIG. 4. Dependence of the extracted mass on (a) the continuum threshold s_0 and (b) the Borel mass M_B^2 . In the left panel, the solid, short-dashed, and long-dashed curves correspond to $M_B^2 = 1.3, 1.4, 1.5$ GeV^2 , respectively. In the right panel, the curves correspond to $s_0 = 4.5, 5.0, 5.5$ GeV^2 . Here the current $\psi_{2\mu}^M(qs\bar{q}\bar{s})$ is used as a representative example.

$$M(\kappa = 1) = 1.86 \text{ GeV} \quad (34)$$

to

$$M(\kappa = 4) = 1.63 \text{ GeV}, \quad (35)$$

with intermediate values of 1.74 GeV and 1.67 GeV for $\kappa = 2$ and 3, respectively. Although the variation induced by κ is not negligible, it does not change the qualitative conclusion of the present analysis. In particular, the mass remains in the same low-energy region considered for light vector tetraquark configurations. This dependence should therefore be regarded as an additional systematic effect associated with the vacuum saturation approximation. The uncertainties quoted in Table I are obtained within the standard vacuum-saturation approximation by varying M_B^2 , s_0 , and the QCD input parameters. A complete κ scan for all currents would require repeating the full numerical analysis and is not attempted here.

We apply the above procedures to study the other currents in a similar manner, and the obtained results are summarized in Table I. QCD sum rule analyses for the $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ configurations yield masses of approximately 1.53–2.37 GeV, 1.86–2.51 GeV, and 2.46–2.60 GeV, respectively. The extracted masses should be compared with known light vector resonances with caution, since the $J^{PC} = 1^{--}$ tetraquark currents may also couple to conventional $\bar{q}q$ vector mesons and two-meson continua. The lowest isovector $qq\bar{q}\bar{q}$ mass around 1.5 GeV lies close to the $\rho(1450)$ region and may also be relevant to the $\rho(1700)$ and $\rho(1900)$ [1]. The isoscalar $qq\bar{q}\bar{q}$ results can be compared with the $\omega(1420)$ and $\omega(1650)$, while the hidden-strangeness $qs\bar{q}\bar{s}$ results may be relevant to the $\phi(1680)$ or higher ρ/ω -like vector structures, depending on the isospin assignment. For the $ss\bar{s}\bar{s}$ sector, the mass region around 2.3–2.6 GeV is naturally compared with the $\phi(2170)$ and other higher ϕ -like states. These comparisons

suggest that some observed light vector resonances may contain tetraquark components, but they should not be regarded as unique assignments. Further information from decay properties, production mechanisms, and coupled-channel effects is needed to distinguish compact tetraquark configurations from conventional $\bar{q}q$ excitations and two-meson continuum contributions.

As a further check of the stability of the diagonal-current analysis, we perform a representative two-current mixing analysis in the isovector $qs\bar{q}\bar{s}$ sector. We consider the two lowest isovector currents, $\psi_{2\mu}^{M(qs\bar{q}\bar{s}),I=1}$ and $\psi_{1\mu}^{A(qs\bar{q}\bar{s}),I=1}$. From the diagonal-current sum rules, their masses are

$$\begin{aligned} M_{\psi_{2\mu}^{M(qs\bar{q}\bar{s})}}^{I=1} &= 1.86_{-0.14}^{+0.14} \text{ GeV}, \\ M_{\psi_{1\mu}^{A(qs\bar{q}\bar{s})}}^{I=1} &= 1.98_{-0.12}^{+0.13} \text{ GeV}. \end{aligned} \quad (36)$$

By including the corresponding off-diagonal correlation functions and diagonalizing the resulting two-current correlation matrix, we obtain a small mixing effect,

$$\theta = (1.8 \pm 5.0)^\circ, \quad (37)$$

where the uncertainty represents a conservative estimate of the possible variation of the mixing angle in this analysis. The extracted masses after diagonalization are, within the numerical accuracy,

$$\begin{aligned} M_{\text{lower}}^{I=1} &= 1.86_{-0.14}^{+0.14} \text{ GeV}, \\ M_{\text{higher}}^{I=1} &= 1.98_{-0.12}^{+0.13} \text{ GeV}. \end{aligned} \quad (38)$$

These values are essentially unchanged compared with the masses obtained from the corresponding diagonal-current sum rules. This indicates that, at least for this representative isovector channel, the off-diagonal mixing effect is small and does not lead to an additional lower stable solution.

A complete correlation-matrix analysis involving all currents with the same quantum numbers would require a separate dedicated study.

Note that the $ss\bar{s}\bar{s}$ tetraquark states with $I^G J^{PC} = 0^- 1^{--}$ have also been systematically investigated in Refs. [55–57] within the QCD sum rule approach. In particular, Ref. [57] considered currents containing one derivative operator, for example

$$J_{7,\alpha}^{1^{--}} = [s_a^T C \gamma_\mu s_b] \overleftrightarrow{D}_\alpha [\bar{s}_a \gamma^\mu C \bar{s}_b^T], \quad (39)$$

and obtained

$$M_{ss\bar{s}\bar{s}}^{I=0} = 2.34_{-0.30}^{+0.23} \text{ GeV}. \quad (40)$$

This value is compatible, within uncertainties, with the $ss\bar{s}\bar{s}$ mass region 2.46–2.60 GeV obtained from the nonderivative currents in the present work. Such theoretical studies are also motivated by experimental observations of higher ϕ -like vector structures, including the $Y(2175)$, commonly referred to as $\phi(2170)$ [58–62]. More recently, the GlueX Collaboration reported a structure in the $\phi\pi^+\pi^-$ invariant mass spectrum around 2.24 GeV/ c^2 [63], providing a new experimental benchmark for studies of the $ss\bar{s}\bar{s}$ vector sector.

The nonderivative currents studied in the present work and the derivative current in Ref. [57] correspond to different interpolating structures. In the present work, the negative parity of the $J^{PC} = 1^{--}$ tetraquark currents is realized through local diquark-antidiquark operators containing at least one P -wave diquark or antidiquark component. By contrast, the derivative current given in Eq. (39) consists of an S -wave axial-vector ss diquark and an S -wave axial-vector $\bar{s}\bar{s}$ antidiquark, with the derivative operator introducing a relative orbital excitation between the two clusters. We therefore quote this previous result as a useful complementary benchmark for the $ss\bar{s}\bar{s}$ vector sector. It should not be regarded as a mass newly derived in the present analysis. A systematic study of derivative currents is more involved and will be pursued in future work.

V. SUMMARY

In this paper we have performed a systematic QCD sum rule study of light tetraquark states with $J^{PC} = 1^{--}$ in the diquark-antidiquark picture. We first constructed a complete set of local interpolating currents by classifying their flavor structures into the symmetric $\mathbf{6}_f \otimes \mathbf{\bar{6}}_f$, antisymmetric $\mathbf{\bar{3}}_f \otimes \mathbf{3}_f$, and mixed $(\mathbf{\bar{3}}_f \otimes \mathbf{\bar{6}}_f) \oplus (\mathbf{6}_f \otimes \mathbf{3}_f)$ representations. We then projected these currents onto the isoscalar, isovector, and isotensor channels, and considered the flavor configurations $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ ($q = u/d$).

We performed QCD sum rule analyses using these nonderivative tetraquark currents. The resulting masses

and decay constants are summarized in Table I. In particular, the lowest masses obtained from the nonderivative currents for the six flavor-isospin sectors are summarized as

$$\begin{aligned} M_{qq\bar{q}\bar{q}}^{I=0} &= 1.64_{-0.14}^{+0.15} \text{ GeV}, \\ M_{qs\bar{q}\bar{s}}^{I=0} &= 1.86_{-0.14}^{+0.14} \text{ GeV}, \\ M_{ss\bar{s}\bar{s}}^{I=0} &= 2.46_{-0.11}^{+0.13} \text{ GeV}, \\ M_{qq\bar{q}\bar{q}}^{I=1} &= 1.53_{-0.19}^{+0.17} \text{ GeV}, \\ M_{qs\bar{q}\bar{s}}^{I=1} &= 1.86_{-0.14}^{+0.14} \text{ GeV}, \\ M_{qq\bar{q}\bar{q}}^{I=2} &= 2.24_{-0.14}^{+0.12} \text{ GeV}. \end{aligned} \quad (41)$$

For the $I = 0$ $ss\bar{s}\bar{s}$ vector sector, we also quote the previous derivative-current result $2.34_{-0.30}^{+0.23}$ GeV [57] as a complementary benchmark. This value should not be regarded as a mass newly derived in the present analysis.

The above results are further summarized in Fig. 5, where we also compare the present 1^{--} tetraquark results with our previous results for the 1^{++} tetraquark and hybrid states [37]. Both the 1^{--} and 1^{++} tetraquark spectra contain six flavor-isospin configurations: the isoscalar $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ sectors, the isovector $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ sectors, and the isotensor $qq\bar{q}\bar{q}$ sector. By contrast, the 1^{++} hybrid spectrum contains three flavor-isospin configurations: the isoscalar $\bar{q}qg$ and $\bar{s}s g$ sectors, and the isovector $\bar{q}qg$ sector. Such a comparison provides a useful reference for understanding how tetraquark and hybrid configurations may populate the light hadron spectrum in different quantum-number channels.

It should be noted that the mass of the isotensor tetraquark state with $J^{PC} = 1^{++}$ was not evaluated in our previous study. In analogy with Sec. II D, there are

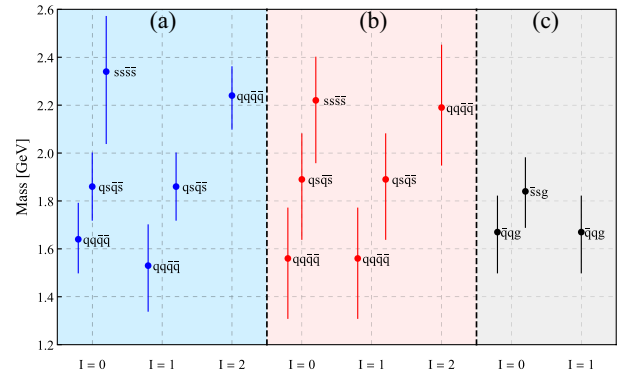


FIG. 5. Comparison of the mass spectra of light tetraquark and hybrid states. Panel (a) shows the $J^{PC} = 1^{--}$ tetraquark masses obtained in the present work, while panels (b) and (c) show our previous results for the $J^{PC} = 1^{++}$ tetraquark and hybrid masses, respectively [37]. The tetraquark spectra include six flavor-isospin configurations: the isoscalar $qq\bar{q}\bar{q}$, $qs\bar{q}\bar{s}$, and $ss\bar{s}\bar{s}$ sectors, the isovector $qq\bar{q}\bar{q}$ and $qs\bar{q}\bar{s}$ sectors, and the isotensor $qq\bar{q}\bar{q}$ sector. The hybrid spectrum includes the isoscalar $\bar{q}qg$ and $\bar{s}s g$ sectors and the isovector $\bar{q}qg$ sector.

two isoscalar $qq\bar{q}\bar{q}$ currents with $J^{PC} = 1^{--}$:

$$\begin{aligned}\psi_{1\mu}^{S^*(qq\bar{q}\bar{q}),I=2} &\equiv \psi_{1\mu}^{S^*(qq\bar{q}\bar{q})} \\ &\sim u_a^T C \gamma_5 d_b (\bar{u}_a \gamma_\mu \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_\mu \gamma_5 C \bar{d}_a^T) \\ &\quad + u_a^T C \gamma_\mu \gamma_5 d_b (\bar{u}_a \gamma_5 C \bar{d}_b^T + \bar{u}_b \gamma_5 C \bar{d}_a^T),\end{aligned}\quad (42a)$$

$$\begin{aligned}\psi_{2\mu}^{S^*(qq\bar{q}\bar{q}),I=2} &\equiv \psi_{2\mu}^{S^*(qq\bar{q}\bar{q})} \\ &\sim u_a^T C \gamma^\nu d_b (\bar{u}_a \sigma_{\mu\nu} C \bar{d}_b^T - \bar{u}_b \sigma_{\mu\nu} C \bar{d}_a^T) \\ &\quad + u_a^T C \sigma_{\mu\nu} d_b (\bar{u}_a \gamma^\nu C \bar{d}_b^T - \bar{u}_b \gamma^\nu C \bar{d}_a^T).\end{aligned}\quad (42b)$$

In the present study we apply the same QCD sum rule procedures to these two currents and derive their masses as $2.19_{-0.24}^{+0.26}$ GeV and $2.40_{-0.22}^{+0.30}$ GeV. The lower of these two mass predictions is included in Fig. 5, thereby completing the comparison with the 1^{--} tetraquark spectrum.

Since $J^{PC} = 1^{--}$ carries exotic quantum numbers, the corresponding hybrid states have attracted considerable interest. However, it remains difficult to distinguish a 1^{--} hybrid state from a 1^{--} tetraquark state, because both configurations may appear in a similar mass region and may couple to similar hadronic channels. Although the 1^{--} channel does not carry exotic quantum numbers, it is experimentally important since vector states can be directly produced in electron-positron annihilation. A combined investigation of the 1^{--} and 1^{--} channels may therefore

provide a broader framework for understanding the similarities and differences between tetraquark and hybrid configurations, and may offer useful clues for distinguishing these two types of exotic hadrons. In the present work we have focused on the 1^{--} tetraquark states, while a further study of the corresponding 1^{--} hybrid states and the decay properties of these states will be carried out in future work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: TWO-POINT CORRELATION FUNCTIONS

In this appendix we present the Borel-transformed correlation functions for all the interpolating currents defined in Sec. II.

$$\begin{aligned}\Pi_{1\mu}^{S(qq\bar{q}\bar{q})}(M_B^2) &= \int_0^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q}q \rangle^2}{18\pi^2} s + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{8\pi^2} \right] e^{-s/M_B^2} ds + \left(\frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{864\pi^2} + \frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{24\pi^2} \right), \\ \Pi_{2\mu}^{S(qq\bar{q}\bar{q})}(M_B^2) &= \int_0^{s_0} \left[\frac{1}{12288\pi^6} s^4 + \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q}q \rangle^2}{12\pi^2} s + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{6\pi^2} \right] e^{-s/M_B^2} ds + \left(\frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{288\pi^2} + \frac{5\langle g_s \bar{q}\sigma Gq \rangle^2}{96\pi^2} \right), \\ \Pi_{1\mu}^{S(qs\bar{q}\bar{s})}(M_B^2) &= \int_{4m_s^2}^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{m_s^2}{512\pi^6} s^3 + \left(-\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{3m_s^4}{256\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{96\pi^4} \right) s^2 \right. \\ &\quad + \left(\frac{m_s^2 \langle g_s^2 GG \rangle}{4608\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{12\pi^4} - \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{384\pi^4} - \frac{5m_s \langle g_s \bar{s}\sigma Gs \rangle}{384\pi^4} + \frac{\langle \bar{q}q \rangle^2}{36\pi^2} + \frac{\langle \bar{s}s \rangle^2}{36\pi^2} \right) s \\ &\quad + \left(-\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{256\pi^4} - \frac{m_s^2 \langle \bar{q}q \rangle^2}{2\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{24\pi^2} + \frac{5\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} \right. \\ &\quad + \left. \frac{5\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(-\frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{4608\pi^4} + \frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{864\pi^2} + \frac{m_s^4 \langle \bar{q}q \rangle^2}{6\pi^2} \right. \\ &\quad - \frac{17m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{48\pi^2} - \frac{m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{48\pi^2} + \frac{10m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} + \frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{64\pi^2} \\ &\quad + \left. \frac{\langle g_s \bar{q}\sigma Gq \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} + \frac{\langle g_s \bar{s}\sigma Gs \rangle^2}{64\pi^2} \right),\end{aligned}$$

$$\begin{aligned}
\Pi_{2\mu}^{S(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{12288\pi^6} s^4 - \frac{3m_s^2}{1024\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{512\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{64\pi^4} \right) s^2 \right. \\
& + \left(\frac{m_s^2 \langle g_s^2 GG \rangle}{2048\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{8\pi^4} - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{192\pi^4} - \frac{m_s \langle g_s \bar{s} \sigma G s \rangle}{64\pi^4} + \frac{\langle \bar{q} q \rangle^2}{24\pi^2} + \frac{\langle \bar{s}s \rangle^2}{24\pi^2} \right) s \\
& + \left(-\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q} q \rangle}{128\pi^4} - \frac{m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{256\pi^4} - \frac{3m_s^2 \langle \bar{q} q \rangle^2}{4\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{16\pi^2} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{16\pi^2} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48\pi^2} \right. \\
& + \frac{\langle \bar{s}s \rangle \langle g_s \bar{q} \sigma G q \rangle}{48\pi^2} + \left. \frac{\langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{16\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(\frac{m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1152\pi^2} - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q} \sigma G q \rangle}{2304\pi^4} \right. \\
& - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{s} \sigma G s \rangle}{4608\pi^4} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{1728\pi^2} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle \langle \bar{s}s \rangle}{432\pi^2} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1728\pi^2} + \frac{m_s^4 \langle \bar{q} q \rangle^2}{4\pi^2} \\
& - \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{2\pi^2} - \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{24\pi^2} + \frac{5m_s \langle \bar{q} q \rangle^2 \langle \bar{s}s \rangle}{3} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{64\pi^2} \\
& \left. + \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48\pi^2} + \frac{\langle g_s \bar{s} \sigma G s \rangle^2}{64\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}
\Pi_{1\mu}^{S(ss\bar{s}\bar{s})}(M_B^2) = & \int_{16m_s^2}^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{m_s^2}{256\pi^6} s^3 + \left(-\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{3m_s^4}{64\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{48\pi^4} \right) s^2 \right. \\
& + \left(\frac{17m_s^2 \langle g_s^2 GG \rangle}{9216\pi^6} - \frac{7m_s^6}{32\pi^6} + \frac{5m_s^3 \langle \bar{s}s \rangle}{24\pi^4} - \frac{m_s \langle g_s \bar{s} \sigma G s \rangle}{32\pi^4} + \frac{\langle \bar{s}s \rangle^2}{18\pi^2} \right) s \\
& + \left(\frac{3m_s^8}{32\pi^6} - \frac{m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{128\pi^4} + \frac{3m_s^5 \langle \bar{s}s \rangle}{8\pi^4} - \frac{29m_s^2 \langle \bar{s}s \rangle^2}{12\pi^2} + \frac{13m_s^3 \langle g_s \bar{s} \sigma G s \rangle}{32\pi^4} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{8\pi^2} \right) \Big] e^{-s/M_B^2} ds \\
& + \left(-\frac{m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{576\pi^4} - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{s} \sigma G s \rangle}{2304\pi^4} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{864\pi^2} - \frac{m_s^5 \langle g_s \bar{s} \sigma G s \rangle}{8\pi^4} + \frac{13m_s^4 \langle \bar{s}s \rangle^2}{12\pi^2} \right. \\
& \left. - \frac{13m_s^2 \langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{8\pi^2} + \frac{20m_s \langle \bar{s}s \rangle^3}{9} + \frac{\langle g_s \bar{s} \sigma G s \rangle^2}{24\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}
\Pi_{2\mu}^{S(ss\bar{s}\bar{s})}(M_B^2) = & \int_{16m_s^2}^{s_0} \left[\frac{1}{12288\pi^6} s^4 - \frac{3m_s^2}{512\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{128\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{32\pi^4} \right) s^2 \right. \\
& + \left(\frac{35m_s^2 \langle g_s^2 GG \rangle}{9216\pi^6} - \frac{21m_s^6}{64\pi^6} + \frac{5m_s^3 \langle \bar{s}s \rangle}{16\pi^4} - \frac{m_s \langle g_s \bar{s} \sigma G s \rangle}{24\pi^4} + \frac{\langle \bar{s}s \rangle^2}{12\pi^2} \right) s \\
& + \left(-\frac{3m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{128\pi^4} + \frac{9m_s^8}{64\pi^6} + \frac{9m_s^5 \langle \bar{s}s \rangle}{16\pi^4} + \frac{19m_s^3 \langle g_s \bar{s} \sigma G s \rangle}{32\pi^4} - \frac{29m_s^2 \langle \bar{s}s \rangle^2}{8\pi^2} \right. \\
& \left. + \frac{\langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{6\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(\frac{m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{576\pi^4} - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{s} \sigma G s \rangle}{768\pi^4} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{288\pi^2} \right. \\
& \left. - \frac{3m_s^5 \langle g_s \bar{s} \sigma G s \rangle}{16\pi^4} + \frac{13m_s^4 \langle \bar{s}s \rangle^2}{8\pi^2} - \frac{19m_s^2 \langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{8\pi^2} + \frac{10m_s \langle \bar{s}s \rangle^3}{3} + \frac{5 \langle g_s \bar{s} \sigma G s \rangle^2}{96\pi^2} \right),
\end{aligned}$$

$$\Pi_{1\mu}^{A(qq\bar{q}\bar{q})}(M_B^2) = \int_0^{s_0} \left[\frac{1}{36864\pi^6} s^4 + \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{36\pi^2} s \right] e^{-s/M_B^2} ds + \left(-\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{864\pi^2} - \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{96\pi^2} \right),$$

$$\begin{aligned}
\Pi_{2\mu}^{A(qq\bar{q}\bar{q})}(M_B^2) = & \int_0^{s_0} \left[\frac{1}{6144\pi^6} s^4 + \frac{11 \langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{6\pi^2} s + \frac{5 \langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{24\pi^2} \right] e^{-s/M_B^2} ds \\
& + \left(\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{96\pi^2} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{24\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}\Pi_{1\mu}^{A(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{36864\pi^6} s^4 - \frac{m_s^2}{1024\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{3m_s^4}{512\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{192\pi^4} \right) s^2 \right. \\ & + \left(-\frac{m_s^2 \langle g_s^2 GG \rangle}{4608\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{24\pi^4} + \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{384\pi^4} - \frac{m_s \langle g_s \bar{s} \sigma G s \rangle}{384\pi^4} + \frac{\langle \bar{q} q \rangle^2}{72\pi^2} + \frac{\langle \bar{s}s \rangle^2}{72\pi^2} \right) s \\ & + \left(\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q} q \rangle}{256\pi^4} - \frac{m_s^2 \langle \bar{q} q \rangle^2}{4\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{48\pi^2} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{96\pi^2} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96\pi^2} - \frac{\langle \bar{s}s \rangle \langle g_s \bar{q} \sigma G q \rangle}{96\pi^2} \right. \\ & \left. \left. + \frac{\langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{96\pi^2} \right) \right] e^{-s/M_B^2} ds + \left(\frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q} \sigma G q \rangle}{4608\pi^4} - \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle \langle \bar{s}s \rangle}{864\pi^2} + \frac{m_s^4 \langle \bar{q} q \rangle^2}{12\pi^2} \right. \\ & \left. - \frac{7m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{48\pi^2} + \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48\pi^2} + \frac{5m_s \langle \bar{q} q \rangle^2 \langle \bar{s}s \rangle}{9} - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{s} \sigma G s \rangle}{96\pi^2} \right),\end{aligned}$$

$$\begin{aligned}\Pi_{2\mu}^{A(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{6144\pi^6} s^4 - \frac{3m_s^2}{512\pi^6} s^3 + \left(\frac{11 \langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{256\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{32\pi^4} \right) s^2 \right. \\ & + \left(\frac{7m_s^2 \langle g_s^2 GG \rangle}{6144\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{4\pi^4} - \frac{m_s \langle g_s \bar{q} \sigma G q \rangle}{96\pi^4} - \frac{m_s \langle g_s \bar{s} \sigma G s \rangle}{64\pi^4} + \frac{\langle \bar{q} q \rangle^2}{12\pi^2} + \frac{\langle \bar{s}s \rangle^2}{12\pi^2} \right) s \\ & + \left(-\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q} q \rangle}{64\pi^4} - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{256\pi^4} - \frac{3m_s^2 \langle \bar{q} q \rangle^2}{2\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{8\pi^2} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{16\pi^2} + \frac{\langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{24\pi^2} \right. \\ & \left. + \frac{\langle \bar{s}s \rangle \langle g_s \bar{q} \sigma G q \rangle}{24\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{s} \sigma G s \rangle}{16\pi^2} \right) \right] e^{-s/M_B^2} ds + \left(\frac{5m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1152\pi^2} - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q} \sigma G q \rangle}{1152\pi^4} \right. \\ & - \frac{25m_s \langle g_s^2 GG \rangle \langle g_s \bar{s} \sigma G s \rangle}{4608\pi^4} + \frac{25 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{1728\pi^2} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle \langle \bar{s}s \rangle}{216\pi^2} + \frac{25 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1728\pi^2} + \frac{m_s^4 \langle \bar{q} q \rangle^2}{2\pi^2} \\ & \left. - \frac{7m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{8\pi^2} - \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{12\pi^2} + \frac{10m_s \langle \bar{q} q \rangle^2 \langle \bar{s}s \rangle}{3} + \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{s} \sigma G s \rangle}{24\pi^2} \right),\end{aligned}$$

$$\begin{aligned}\Pi_{1\mu}^{M(qq\bar{q}\bar{q})}(M_B^2) = & \int_0^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{18\pi^2} s + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{12\pi^2} \right] e^{-s/M_B^2} ds \\ & + \left(-\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{864\pi^2} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{48\pi^2} \right),\end{aligned}$$

$$\begin{aligned}\Pi_{2\mu}^{M(qq\bar{q}\bar{q})}(M_B^2) = & \int_0^{s_0} \left[\frac{1}{6144\pi^6} s^4 + \frac{11 \langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{6\pi^2} s + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{24\pi^2} \right] e^{-s/M_B^2} ds \\ & + \left(\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{864\pi^2} - \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{24\pi^2} \right),\end{aligned}$$

$$\begin{aligned}\Pi_{3\mu}^{M(qq\bar{q}\bar{q})}(M_B^2) = & \int_0^{s_0} \left[\frac{1}{36864\pi^6} s^4 + \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{36\pi^2} s + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{24\pi^2} \right] e^{-s/M_B^2} ds \\ & + \left(\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{864\pi^2} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{96\pi^2} \right),\end{aligned}$$

$$\begin{aligned}\Pi_{4\mu}^{M(qq\bar{q}\bar{q})}(M_B^2) = & \int_0^{s_0} \left[\frac{1}{12288\pi^6} s^4 + \frac{\langle g_s^2 GG \rangle}{18432\pi^6} s^2 + \frac{\langle \bar{q} q \rangle^2}{12\pi^2} s + \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{12\pi^2} \right] e^{-s/M_B^2} ds \\ & + \left(-\frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{864\pi^2} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{96\pi^2} \right),\end{aligned}$$

$$\begin{aligned}
\Pi_{1\mu}^{M(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{18432\pi^6} s^4 - \frac{m_s^2}{512\pi^6} s^3 + \left(-\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{3m_s^4}{256\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{96\pi^4} \right) s^2 \right. \\
& + \left(\frac{m_s^2 \langle g_s^2 GG \rangle}{4608\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{12\pi^4} + \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{384\pi^4} - \frac{5m_s \langle g_s \bar{s}\sigma Gs \rangle}{384\pi^4} + \frac{\langle \bar{q}q \rangle^2}{36\pi^2} + \frac{\langle \bar{s}s \rangle^2}{36\pi^2} \right) s \\
& + \left(\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{256\pi^4} - \frac{m_s^2 \langle \bar{q}q \rangle^2}{2\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{24\pi^2} + \frac{5\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} - \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} - \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} \right. \\
& + \left. \frac{5\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(\frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{4608\pi^4} - \frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{864\pi^2} + \frac{m_s^4 \langle \bar{q}q \rangle^2}{6\pi^2} \right. \\
& - \frac{17m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{48\pi^2} + \frac{m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{48\pi^2} + \frac{10m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} + \frac{\langle g_s \bar{q}\sigma Gq \rangle^2}{64\pi^2} \\
& \left. - \frac{\langle g_s \bar{q}\sigma Gq \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} + \frac{\langle g_s \bar{s}\sigma Gs \rangle^2}{64\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}
\Pi_{2\mu}^{M(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{6144\pi^6} s^4 - \frac{3m_s^2}{512\pi^6} s^3 + \left(\frac{11\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{256\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{32\pi^4} \right) s^2 \right. \\
& + \left(\frac{7m_s^2 \langle g_s^2 GG \rangle}{6144\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{4\pi^4} + \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^4} - \frac{m_s \langle g_s \bar{s}\sigma Gs \rangle}{64\pi^4} + \frac{\langle \bar{q}q \rangle^2}{12\pi^2} + \frac{\langle \bar{s}s \rangle^2}{12\pi^2} \right) s \\
& + \left(\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{64\pi^4} - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{256\pi^4} - \frac{3m_s^2 \langle \bar{q}q \rangle^2}{2\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{8\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{16\pi^2} - \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{24\pi^2} \right. \\
& - \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{24\pi^2} + \left. \frac{\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{16\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(\frac{5m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1152\pi^2} + \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{1152\pi^4} \right. \\
& - \frac{25m_s \langle g_s^2 GG \rangle \langle g_s \bar{s}\sigma Gs \rangle}{4608\pi^4} + \frac{25\langle g_s^2 GG \rangle \langle \bar{q}q \rangle^2}{1728\pi^2} - \frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{216\pi^2} + \frac{25\langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1728\pi^2} + \frac{m_s^4 \langle \bar{q}q \rangle^2}{2\pi^2} \\
& \left. - \frac{7m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{8\pi^2} + \frac{m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{12\pi^2} + \frac{10m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{3} - \frac{\langle g_s \bar{q}\sigma Gq \rangle \langle g_s \bar{s}\sigma Gs \rangle}{24\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}
\Pi_{3\mu}^{M(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{36864\pi^6} s^4 - \frac{m_s^2}{1024\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{3m_s^4}{512\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{192\pi^4} \right) s^2 \right. \\
& + \left(-\frac{m_s^2 \langle g_s^2 GG \rangle}{4608\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{24\pi^4} - \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{384\pi^4} - \frac{m_s \langle g_s \bar{s}\sigma Gs \rangle}{384\pi^4} + \frac{\langle \bar{q}q \rangle^2}{72\pi^2} + \frac{\langle \bar{s}s \rangle^2}{72\pi^2} \right) s \\
& + \left(-\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{256\pi^4} - \frac{m_s^2 \langle \bar{q}q \rangle^2}{4\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{48\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} \right. \\
& + \left. \frac{\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} \right) \Big] e^{-s/M_B^2} ds + \left(-\frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{4608\pi^4} + \frac{5\langle g_s^2 GG \rangle \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{864\pi^2} + \frac{m_s^4 \langle \bar{q}q \rangle^2}{12\pi^2} \right. \\
& \left. - \frac{7m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{48\pi^2} - \frac{m_s^2 \langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{48\pi^2} + \frac{5m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} + \frac{\langle g_s \bar{q}\sigma Gq \rangle \langle g_s \bar{s}\sigma Gs \rangle}{96\pi^2} \right),
\end{aligned}$$

$$\begin{aligned}
\Pi_{4\mu}^{M(qs\bar{q}\bar{s})}(M_B^2) = & \int_{4m_s^2}^{s_0} \left[\frac{1}{12288\pi^6} s^4 - \frac{3m_s^2}{1024\pi^6} s^3 + \left(\frac{\langle g_s^2 GG \rangle}{18432\pi^6} + \frac{9m_s^4}{512\pi^6} + \frac{m_s \langle \bar{s}s \rangle}{64\pi^4} \right) s^2 \right. \\
& + \left(\frac{m_s^2 \langle g_s^2 GG \rangle}{2048\pi^6} - \frac{m_s^3 \langle \bar{s}s \rangle}{8\pi^4} + \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{192\pi^4} - \frac{m_s \langle g_s \bar{s}\sigma Gs \rangle}{64\pi^4} + \frac{\langle \bar{q}q \rangle^2}{24\pi^2} + \frac{\langle \bar{s}s \rangle^2}{24\pi^2} \right) s \\
& + \left(\frac{m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{128\pi^4} - \frac{m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{256\pi^4} - \frac{3m_s^2 \langle \bar{q}q \rangle^2}{4\pi^2} + \frac{m_s^2 \langle \bar{s}s \rangle^2}{16\pi^2} + \frac{\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle}{16\pi^2} \right. \\
& \left. - \frac{\langle \bar{q}q \rangle \langle g_s \bar{s}\sigma Gs \rangle}{48\pi^2} - \frac{\langle \bar{s}s \rangle \langle g_s \bar{q}\sigma Gq \rangle}{48\pi^2} + \frac{\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{16\pi^2} \right) \Big] e^{-s/M_B^2} ds
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{m_s^3 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{1152\pi^2} + \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{q} \sigma G q \rangle}{2304\pi^4} \right. \\
& - \frac{5m_s \langle g_s^2 GG \rangle \langle g_s \bar{s} \sigma G s \rangle}{4608\pi^4} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle^2}{1728\pi^2} - \frac{5 \langle g_s^2 GG \rangle \langle \bar{q} q \rangle \langle \bar{s}s \rangle}{432\pi^2} + \frac{5 \langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{1728\pi^2} + \frac{m_s^4 \langle \bar{q} q \rangle^2}{4\pi^2} \\
& - \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle}{2\pi^2} + \frac{m_s^2 \langle \bar{q} q \rangle \langle g_s \bar{s} \sigma G s \rangle}{24\pi^2} + \frac{5m_s \langle \bar{q} q \rangle^2 \langle \bar{s}s \rangle}{3} + \frac{\langle g_s \bar{q} \sigma G q \rangle^2}{64\pi^2} \\
& \left. - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{s} \sigma G s \rangle}{48\pi^2} + \frac{\langle g_s \bar{s} \sigma G s \rangle^2}{64\pi^2} \right),
\end{aligned}$$

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