

Quantum slow roll

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We consider a scalar field coupled to massless fermions through Yukawa couplings, such as the Higgs field, in a Robertson-Walker spacetime. We compute the nonlocal quantum effective action as a functional of the background scalar field and the scale factor at one loop order by integrating the Weyl anomaly resulting from the fermions in the loop. We show that the mode expansion of the scalar field is modified by the quantum nonlocality and the power spectrum of scalar perturbations exiting the horizon exhibits a slow roll with a red tilt proportional to the square of the Yukawa coupling even though the classical spacetime is exactly de Sitter. We comment on possible implications for cosmology.

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I. INTRODUCTION

Loops of massless particles in curved spacetime result in a quantum effective action that depends nonlocally on the background metric and other background fields. This nonlocality of the effective action is a potential source of interesting quantum effects in cosmology or in black hole physics and holography. Computing the effective action is a well-posed problem in perturbation theory using the background field method, but is forbiddingly difficult to implement even at the one-loop level. One needs to compute the heat kernel for the background-dependent quadratic fluctuation operator for all values of the proper time. This in general is not tractable.

The computation can be greatly simplified for Weyl-flat metrics for conformally coupled matter [1,2] with Weyl-invariant classical action. In this case, under suitable conditions, one can determine the quantum effective action by integrating the Weyl anomaly. Since the anomaly is a consequence of the short-distance regularization of the path integral, it is determined by the short proper-time expansion of the heat kernel which can be computed explicitly in a local expansion. As a result, it is possible to determine [2] the quantum effective action in a closed form¹ as we review in Sec. II.

¹We are interested in the consequences of the β -function anomalies away from a conformal fixed point and not in the conformal anomalies usually considered at the conformal fixed point, for example, the ones used to determine the Polyakov action or the Riegert action.

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In four dimensions this method was implemented for the Yang-Mills field coupled to conformal matter [2]. The resulting action takes a simple form when expressed in terms of the conformal compensator Ω but has a rather complicated form when expressed covariantly in terms of the physical metric. One can check [2] that it reduces² to the nonlocal action to third order obtained using the Barvinsky-Vilkovisky covariant perturbation theory [3–6] in the regime $R^2 \ll \nabla^2 R$ for a typical curvature scale R . The advantage of our method for the class of Weyl-flat metrics is that it has a greater range of validity effectively resumming the covariant perturbation theory to all orders in curvatures. In particular, it is applicable also in the opposite regime $R^2 \gg \nabla^2 R$, which is the relevant regime in a number of interesting situations in cosmology. For example, the quantum effective action for Maxwell gauge theory in Robertson-Walker spacetimes enables a systematic investigation of scenarios for anomaly-induced primordial magnetogenesis [7] during inflation where the curvatures vary much more slowly compared to the Hubble scale.

The general lesson both from Yang-Mills theory in four dimensions is that the effective coupling of different fields to the metric can be different from the classical canonical coupling because of nonlocal quantum effects. The relevant effective metric depends upon the observable of interest and can be different from the classical metric.

²The complicated structure of the curvature expansion simplifies in a Weyl-flat spacetime because a large part of the curvature expansion evaluates to zero [3,4]. This nontrivial cancellation can be understood from the point of view of integrating the Weyl anomaly. The effective action can be decomposed into a Weyl-anomalous part and a Weyl-invariant part. The Weyl invariant part simplifies to the flat space action when the metric is Weyl-flat. One expects that this simplification must be present also in the covariant curvature expansion.

In this paper, we examine another four-dimensional example where it is possible to explicitly determine the quantum effective action. We consider a (pseudo) scalar field in four dimensions coupled to N_f massless fermions through Yukawa coupling, such as the Higgs field. Our considerations can be easily extended to a complex scalar field and to axionic fields of interest in various contexts [8,9]. Since the scalar may have mass and nonconformal coupling to the metric, the classical action is not Weyl invariant. However, in the limit of large N_f , the quantum effective action is dominated by the fermionic contributions and the method of integrating the Weyl anomaly is still applicable. In Sec. III we determine the nonlocal quantum effective action by applying the method of [2] described in Sec. II.

The quantum corrected dynamics of the scalar field is particularly interesting in a cosmological context because the quantum fluctuations of the scalar field can contribute to primordial density perturbations. With this motivation we examine in Sec. IV how the mode expansion of the scalar field is altered because of the quantum nonlocality. Even though the quantum-modified equations of motion for the scalar are rather complicated, we are able to find analytic expressions for the modes at late times in terms of Airy functions. In Sec. V we compute the scalar power spectrum in de Sitter spacetime and the spectral index. We find that, even in exactly de Sitter spacetime, the power spectrum has a nonzero red tilt that depends on the Yukawa coupling and the number of fermions. Thus, the effective metric relevant for the scalar quantum fluctuations in this case is different from the classical de Sitter metric.

We expect that this general mechanism of quantum slow roll can have interesting applications in cosmology. In inflationary models, the inflaton plays a dual role as the field whose dynamics drives the nearly exponential expansion of the Universe as well as the field that generates the primordial density perturbations. These two roles are logically separate. The classically slowly rolling potential required for these purposes is not natural from the perspective of effective field theory unless one can invoke some approximate global symmetry. By contrast, quantum slow roll is naturally slow in a weakly coupled theory without invoking artificially flat potentials. Thus, it may be interesting to consider scenarios where the two aspects of the inflationary cosmology are not necessarily linked. We have not considered here the reentry of the scalar perturbations into the horizon after the end of inflation to produce density perturbations. It would be interesting to explore if this mechanism of quantum slow roll can be embedded in specific cosmological scenarios.

II. QUANTUM EFFECTIVE ACTION FROM WEYL ANOMALIES

Consider a classically Weyl invariant theory with classical Lorentzian action $\mathcal{I}_0[g, \chi_f]$ wick-rotated to

$\mathcal{S}_0[g, \chi_f]$. The local Weyl transformation of the metric $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} \rightarrow e^{2\xi(x)} g_{\mu\nu}, \quad g^{\mu\nu} \rightarrow e^{-2\xi(x)} g^{\mu\nu}, \quad (2.1)$$

Other fields denoted collectively as $\{\chi_f\}$ transform with their respective Weyl weights $\{\Delta_f\}$

$$\chi_f(x) \rightarrow e^{-\Delta_f \xi(x)} \chi_f(x). \quad (2.2)$$

In particular, in four dimensions, a conformally coupled scalar field has Weyl weight 1, a fermion field has weight 3/2, a gauge field has weight 0 so that the kinetic terms are scale invariant.

The local Weyl group $\mathcal{G}$ is an infinite dimensional Abelian group with generators $\{J_x\}$ acting on the space of fields

$$J_x := -2g^{\mu\nu}(x) \frac{\delta}{\delta g^{\mu\nu}(x)} - \Delta_f \chi_f(x) \frac{\delta}{\delta \chi_f(x)}. \quad (2.3)$$

Treating the coordinate x of the local scaling parameter $\xi(x)$ as a continuous index, we can write an element of this group as

$$e^{\xi \cdot J} \quad (2.4)$$

with the summation convention

$$\xi \cdot J := \sum_x \xi_x J_x := \int d^4x \xi(x) J_x. \quad (2.5)$$

Weyl invariance of the classical action implies that

$$J_x(\mathcal{S}_0[g, \chi_f]) = 0. \quad (2.6)$$

Consider now the one-loop effective action $\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1$, obtained by integrating the conformally coupled massless fields. Regularization of the path integral, for example, using short proper time cutoff of the heat kernel, is manifestly diffeomorphism invariant but not Weyl invariant. Consequently, the Weyl-variation of the resulting action is nonzero. Since the lack of Weyl invariance is a result of a short distance cutoff, the anomalous Weyl-variation is determined by the short proper time expansion of the coincident heat kernel given by [2]

$$J_x(\mathcal{S}_1[g, \chi_f]) := -\mathcal{A}(x)\sqrt{g} = -2 \sum_f n_f \text{tr} K_f(x, x; \epsilon) \sqrt{g} \quad (2.7)$$

where n_f is a coefficient that differs for each type of field running in the loops. In particular $n_\psi = -\frac{1}{2}$ for a Weyl fermion ψ and $n_\phi = \frac{1}{2}$ for a real scalar ϕ [2].

The anomaly equation can be viewed as a first order differential equation on field space. If a metric (g, χ_f) is on the Weyl-orbit of $(\bar{g}, \bar{\chi}_f)$ with Weyl factor $\xi(x)$

$$(g, \chi_f) = e^{\xi J}(\bar{g}, \bar{\chi}_f). \quad (2.8)$$

then given the action $\mathcal{S}_1[\bar{g}, \bar{\chi}_f]$ one can compute the quantum effective action $\mathcal{S}[g, \chi_f]$ by integrating the anomaly equation. It is convenient to implement this idea using the lemma discussed in [2]. One starts with the trivial identity

$$e^{\xi J} = \mathbf{1} + \int_0^1 dt e^{t\xi J} \xi \cdot J. \quad (2.9)$$

to write³ $\mathcal{S}_1[g, \chi_f]$ as

$$e^{\xi J}(\mathcal{S}_1[\bar{g}, \bar{\chi}_f]) = \left(\mathbf{1} + \int_0^1 dt e^{t\xi J} \xi \cdot J \right) (\mathcal{S}_1[\bar{g}, \bar{\chi}_f]) \quad (2.10)$$

$$= \mathcal{S}_1[\bar{g}, \bar{\chi}_f] + \int_0^1 dt e^{t\xi J} \xi \cdot J(\mathcal{S}_1[\bar{g}, \bar{\chi}_f]) \quad (2.11)$$

$$= \mathcal{S}_1[\bar{g}, \bar{\chi}_f] - \int_0^1 dt e^{t\xi J} \left(\int d^4x \xi(x) \sqrt{g} \mathcal{A}[\bar{g}, \bar{\chi}_f](x) \right). \quad (2.12)$$

One concludes

$$\mathcal{S}_1[g, \chi_f] = \mathcal{S}_1[\bar{g}, \bar{\chi}_f] + \mathcal{S}_A[\bar{g}, \bar{\chi}_f, \xi], \quad (2.13)$$

where

$$\begin{aligned} \mathcal{S}_A[\bar{g}, \bar{\chi}_f, \xi] \\ := - \int_0^1 dt \int d^4x \sqrt{\bar{g}} e^{2t\xi(x)} \xi(x) \mathcal{A}[\bar{g} e^{2t\xi}, \bar{\chi}_f e^{-\Delta_f t \xi}](x) \end{aligned} \quad (2.14)$$

is the anomalous contribution to the action. This lemma expresses the quantum effective action in terms of its Weyl variation as an integral over the Weyl orbit.

Since $\mathcal{S}_A[\bar{g}, \bar{\chi}_f, \xi]$ is thus determined by the anomaly which is computable from the short-time expansion of the heat kernel, one can determine $\mathcal{S}_1[g, \chi_f]$ completely, if the integration constant $\mathcal{S}_1[\bar{g}, \bar{\chi}_f]$ can be computed. For a Weyl-flat metric of the form

$$g_{\mu\nu}(x) = e^{2\Omega(x)} \eta_{\mu\nu} \quad (2.15)$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric one can choose $\bar{g} = \eta$ and $\xi(x) = \Omega(x)$. In this case our task is reduced to computing the quantum effective action $\mathcal{S}[\eta, \bar{\chi}_f]$ with Minkowski metric which is often much simpler to evaluate.

Lorentzian continuation of (2.13) gives a similar equation

$$\mathcal{I}[g, \chi_f] = \mathcal{I}_0[g, \chi_f] + \mathcal{I}_1[\bar{g}, \bar{\chi}_f] + \mathcal{I}_A[\bar{g}, \bar{\chi}_f, \xi] \quad (2.16)$$

but with $\mathcal{I}_A$ given by

$$\begin{aligned} \mathcal{I}_A[\bar{g}, \bar{\chi}_f, \xi] \\ := \int_0^1 dt \int d^4x \sqrt{-\bar{g}} e^{2t\xi(x)} \xi(x) \mathcal{A}[\bar{g} e^{2t\xi}, \bar{\chi}_f e^{-\Delta_f t \xi}](x) \end{aligned} \quad (2.17)$$

because the anomaly scalar does not change sign under Wick rotation.

By construction, the effective action (2.13) can only depend on the physical background fields $g_{\mu\nu} = e^{2\xi} \bar{g}_{\mu\nu}$ and $\chi_f = e^{-\Delta_f \xi} \bar{\chi}_f$. This implies that the effective action $\mathcal{S}$ should be invariant a fiducial Weyl transformation of the fiducial metric $\bar{g}$ and other fields

$$\xi \rightarrow \xi - \alpha(x), \quad \bar{g}_{\mu\nu} \rightarrow e^{2\alpha(x)} \bar{g}_{\mu\nu}, \quad \bar{\chi}_f \rightarrow e^{2\alpha(x)} \bar{\chi}_f. \quad (2.18)$$

Unlike the physical Weyl transformation corresponding to the Weyl transformation of the physical metric, fiducial Weyl transformation is a gauge transformation. It reflects the arbitrariness in splitting the physical metric g into a fiducial metric $\bar{g}$ and the Weyl factor ξ . As a result, fiducial Weyl invariance must be respected as a gauge symmetry even in the quantum theory and cannot be anomalous. We further discuss the fiducial Weyl invariance of the action in the Appendix.

III. SCALAR COUPLED TO MASSLESS FERMIONS

Consider the Euclidean action for a pseudoscalar coupled to N_f massless fermions

$$\begin{aligned} S_0 = \int_x \sqrt{g} \left[\frac{1}{2} |\nabla \varphi|^2 + \frac{1}{2} m^2 \varphi^2 + \frac{\xi}{2} R \varphi^2 \right. \\ \left. - \sum_{j=1}^{N_f} \bar{\psi}_j (i \gamma^\mu \nabla_\mu + i y \varphi \gamma_5) \psi_j \right]. \end{aligned} \quad (3.1)$$

The action is invariant under parity under which the pseudoscalar is odd.⁴ We choose to work with a pseudo-scalar instead of a scalar so that linear and cubic terms are avoided in the effective action as a consequence of parity. While the fermionic action is invariant under Weyl

³The argument g of the action functional $\mathcal{S}[g, \chi_f]$ here refers to the covariant tensor $g_{\mu\nu}$ and not $g^{\mu\nu}$.

⁴To avoid notational confusion we denote the Yukawa coupling by y .

transformations, the scalar kinetic term is not. Hence, one can apply the lemma above to the fermionic contributions but not to the scalar contributions. This limitation can be circumvented by considering N_f large so that the quantum effective action is dominated by the fermionic contributions and the error from ignoring the scalar contribution is small.

The contribution from the N_f fermions at one-loop order is given by

$$N_f \log \det (\not{D} + V\gamma_5) \quad (3.2)$$

where $V \equiv y\varphi$. In even dimensions γ^μ and $-\gamma^\mu$ are equally good representations of the Clifford algebra, we can conclude that nothing can depend on the sign of $\not{D}$. Therefore we can write

$$\log \det (\not{D} + V\gamma_5) = \frac{1}{2} \log \det (\not{D} + V\gamma_5)(-\not{D} + V\gamma_5). \quad (3.3)$$

The one-loop contribution of N_f fermions to the effective action can then be written as

$$\frac{N_f}{2} \log \det (-\not{D}^2 - \gamma_5 \not{D} V + V^2 \mathbf{1}) := \frac{N_f}{2} \log \det (\mathcal{O}) \quad (3.4)$$

with

$$\mathcal{O} := -D^2 + \left(-3V^2 + \frac{R}{4}\right) \mathbf{1} \quad (3.5)$$

where we have defined $D_\mu := \nabla_\mu + V\gamma_5\gamma_\mu$ and used $-\not{D}^2 = -\nabla^2 + \frac{R}{4}\mathbf{1}$. The contribution to the anomaly from N_f fermions can then be read off from the heat kernel⁵ for the operator $\mathcal{O}$ and is given by [11–17]

$$\mathcal{A} = \frac{N_f}{16\pi^2} \left[\frac{11}{360} E_4 - \frac{1}{36} W^2 + 2V^4 - \frac{1}{3} V^2 R - 2(\nabla V)^2 \right] \quad (3.6)$$

The purely gravitational terms E_4 and W^2 are not presently of interest to us since they do not affect the mode expansion of the scalar field. Each of the remaining terms can be integrated separately to obtain the anomalous part of the effective action.

- (i) The φ^4 term is related to the running of the quartic scalar coupling. When integrated we obtain an expression that is a local functional of $\bar{g}_{\mu\nu}$ and Ω , but a nonlocal functional of $g_{\mu\nu}$. This nonlocal coupling to the metric is necessary because a running coupling implies the trace of momentum tensor is modified as a result of the Weyl anomaly. By integrating the V^4 term one obtains

$$\int_0^1 dt \sqrt{\bar{g}} e^{4\Omega t} \bar{V}^4 e^{-4\Omega t} \Omega = \sqrt{\bar{g}} \bar{V}^4 \Omega; \quad (3.7)$$

- (ii) The remaining two terms are the nonlocal effects corresponding to the kinetic term renormalization. The integration of such terms is slightly more convoluted since they involve derivatives of φ . Applying the lemma first to the $|\nabla V|^2$ term one obtains

$$\int_0^1 dt \sqrt{\bar{g}} e^{4\Omega t} e^{-2\Omega t} [\bar{\nabla}(e^{-\Omega t} \bar{V})]^2 \Omega = \sqrt{|\bar{g}|} \Omega \left[(\bar{\nabla} \bar{V})^2 + \frac{1}{3} \bar{V}^2 (\bar{\nabla} \Omega)^2 - \frac{1}{2} (\bar{\nabla} \Omega \cdot \bar{\nabla} \bar{V}^2) \right]; \quad (3.8)$$

Similarly the $V^2 R$ term gives

$$\int_0^1 dt \sqrt{\bar{g}} e^{4\Omega t} \Omega e^{-2\Omega t} \left[\frac{R}{6} - (\bar{\nabla} \Omega)^2 t^2 - \bar{\nabla}^2 \Omega t \right] = \sqrt{|\bar{g}|} \Omega \bar{V}^2 \left[\frac{R}{6} - \frac{1}{2} \bar{\nabla}^2 \Omega - \frac{1}{3} (\bar{\nabla} \Omega)^2 \right] \quad (3.9)$$

Combining the two and integrating by parts one obtains

$$\sqrt{\bar{g}} \left\{ \Omega \left[(\bar{\nabla} \bar{V})^2 + \frac{R}{6} \bar{V}^2 \right] + \frac{1}{2} (\bar{\nabla} \Omega)^2 \bar{V}^2 \right\}. \quad (3.10)$$

⁵The cohomology of the Weyl group on field space classifies the consistent terms that can appear as a Weyl anomaly and separates the nontrivial cohomological terms from the ones that are simply a variation of a local term in the action [10], such as $\nabla^2 R$ and $\nabla^2 \varphi^2$. In the model we consider, the only cohomologically nontrivial terms are the Euler density E_4 , the square of the Weyl tensor W^2 , the quartic potential φ^4 and the conformally coupled kinetic term $|\nabla \varphi|^2 + R\varphi^2/6$, as we consistently obtain from the heat kernel expansion.

Thus, the one-loop Lorentzian effective action is given by

$$\mathcal{I}[g, \varphi] = \mathcal{I}_0[g, \varphi] + \mathcal{I}_1[\bar{g}, \bar{\varphi}] + \frac{N_f}{16\pi^2} \int_x \sqrt{-\bar{g}} \left\{ 2\Omega \left[\bar{V}^4 - \left((\bar{\nabla} \bar{V})^2 + \frac{\bar{R}}{6} \bar{V}^2 \right) \right] - (\bar{\nabla} \Omega)^2 \bar{V}^2 \right\} \quad (3.11)$$

By substituting $\bar{V} = ye^{\Omega}\varphi$ and $\bar{g}_{\mu\nu} = e^{2\Omega}g_{\mu\nu}$, the last term in (3.11) can be written as

$$\frac{y^2 N_f}{16\pi^2} \int_x \sqrt{-g} \left\{ 2\Omega \left[V^4 - \left((\nabla V)^2 + \frac{R}{6} V^2 \right) \right] + (\nabla \Omega)^2 V^2 \right\}. \quad (3.12)$$

The integration constant is the Weyl invariant part of the effective action. For $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, it is given by

$$\mathcal{I}_1[\eta, \bar{\varphi}] = N_f \text{Tr} \log [\not{\partial} + ye^{\Omega}\varphi\gamma_5] \quad (3.13)$$

which can be computed by Feynman diagrams in flat spacetime for rapidly fluctuating $\bar{\varphi}$. More precisely, extra powers of $\bar{\varphi}^2$ can be ignored compared to extra derivatives of $\bar{\varphi}$ if

$$-\partial^2(y^2 e^{2\Omega}\varphi^2) \gg y^4 e^{4\Omega}\varphi^4. \quad (3.14)$$

This is precisely the condition for the generalized curvature expansion [3,6,18,19], where the trace of the logarithm involving the background field $\bar{\varphi}$ is expanded around the free theory with insertions of the generalized curvature $y^2\bar{\varphi}^2$ up to the desired power. To make sense of the expansion of the operator in the logarithm around the free

(derivative) operator, it is necessary that extra powers of derivatives are more important than extra powers of generalized curvatures. For (nearly) de Sitter metric this condition reads

$$-\partial^2\left(\frac{\varphi^2}{H^2}\right) + 2\tau\frac{\partial}{\partial\tau}\left(\frac{\varphi^2}{H^2}\right) + 2\left(\frac{\varphi^2}{H^2}\right) \gg y^2\left(\frac{\varphi^4}{H^4}\right). \quad (3.15)$$

which can be satisfied either when $y^2\left(\frac{\varphi^2}{H^2}\right) \ll 1$ or when the derivatives of $\left(\frac{\varphi^2}{H^2}\right)$ are large compared to its background value. This condition can be satisfied as long as the occupation number is not too high. When $\varphi^2 \gg H^2$, the backreaction on the background metric be significant and one cannot regard the metric to be evolving independently of the φ dynamics.

Under these conditions, $\mathcal{I}_1[\eta, \bar{\varphi}]$ can be evaluated using Feynman diagrams and it contains the flat space logarithms corresponding to each nonlocal term in $\mathcal{I}_A$

$$\begin{aligned} \mathcal{I}_1[\eta, \bar{\varphi}] &\sim 2\alpha \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{2\pi} \bar{\varphi}_{\vec{k}}(\omega) (k^2 - \omega^2) \log\left(\frac{k^2 - \omega^2}{\mu^2}\right) \bar{\varphi}_{-\vec{k}}(-\omega) + \mathcal{O}(\bar{\varphi}^4) \\ &= 2\alpha \int_{\tau\tau'\vec{k}\omega} a^2(\tau) \left[\varphi_{\vec{k}}''(\tau) + 2\frac{a'}{a}\varphi_{\vec{k}}'(\tau) + \frac{a''}{a}\varphi_{\vec{k}}(\tau) \right] \varphi_{-\vec{k}}(\tau') e^{i\omega(\tau-\tau')} \frac{a(\tau')}{a(\tau)} \log\left(\frac{k^2 - \omega^2}{\mu^2}\right) \end{aligned} \quad (3.16)$$

where we have ignored the $y^2\varphi^4$ terms, since we are only interested in the quadratic piece of the action to compute the mode expansion for φ . Here μ is the scale at which the scalar is canonically normalized, introduced in the process of renormalizing the UV divergences in Z_φ and Z_ξ . The logarithms which depend on this renormalization scale have the same coefficients as their nonlocal counterparts in (3.11) and a constant shift in Ω in (3.11) can be compensated by a shift in $\log\mu$.

To better understand the nonlocal logarithm, we approximate the ω Fourier integral by a delta function

$$\int \frac{d\omega}{2\pi} e^{i\omega(\tau-\tau')} \log\left(\frac{k^2 - \omega^2}{\mu^2}\right) \sim \delta(t - t') \log\left(\frac{k^2}{\mu^2}\right). \quad (3.17)$$

In this approximation

$$\begin{aligned} \mathcal{I}_1[\eta, \bar{\varphi}] &\sim 2\alpha \int_{\tau\vec{k}} a^2(\tau) \left[\varphi_{\vec{k}}''(\tau) + 2\frac{a'}{a}\varphi_{\vec{k}}'(\tau) + \left(k^2 + \frac{a''}{a}\right)\varphi_{\vec{k}}(\tau) \right] \varphi_{-\vec{k}}(\tau) \log\left(\frac{k^2}{\mu^2}\right) \\ &= 2\alpha \int_{\tau\vec{k}} a^4(\tau) \left[-a^{-2}\varphi_{\vec{k}}'\varphi_{-\vec{k}}' + k^2 a^{-2}\varphi_{\vec{k}}\varphi_{-\vec{k}} + \frac{R}{6}\varphi_{\vec{k}}\varphi_{-\vec{k}} \right] \log\left(\frac{k^2}{\mu^2}\right) \end{aligned} \quad (3.18)$$

which can be seen as the logarithmic running of the kinetic and the $\xi R\varphi^2$ term as k^2 is varied.

It would be interesting to obtain this result by directly computing the one-loop Feynman diagrams in nearly de Sitter spacetime. There are a number of subtleties in global de Sitter spacetime [20,21] which are a consequence of the special symmetries that exist only when the metric is exactly de Sitter. For example, all curvatures are constant for the exact de Sitter metric, and the usual expressions for the scale factor in terms of $g_{\mu\nu}$ become IR divergent. One can sidestep these issues by considering a more general

Robertson-Walker metric which does not have all the symmetries of global de Sitter and approach de Sitter as a limit.

IV. MODE EXPANSION

The mode expansion of the field φ is modified by the nonlocal coupling to the scale factor Ω . The relevant quadratic Lorentzian effective action for determining the modes is given by

$$\begin{aligned} \mathcal{I}[g, \varphi] &= -\frac{1}{2} \int d^4x \sqrt{-g} \left[I^2(\Omega) |\nabla \varphi|^2 + \frac{2\alpha}{3} \Omega R \varphi^2 - 2\alpha |\nabla \Omega|^2 \varphi^2 + m^2 \varphi^2 + \xi R \varphi^2 \right] \\ &= \frac{1}{2} \int d^4x a^2 I^2(\Omega) \left[(\varphi')^2 - |\vec{\nabla} \varphi|^2 - \left(\frac{m^2 a^2}{I^2} + \frac{a''}{a} \left(1 + \frac{6\xi - 1}{I^2} \right) + \frac{2\alpha}{I^2} \left(\frac{a'}{a} \right)^2 \right) \varphi^2 \right] \end{aligned} \quad (4.1)$$

where $I^2(\Omega) := 1 + 4\alpha(\Omega - \log(k/\mu))$, with $\alpha := \frac{y^2 N_f}{16\pi^2}$. For $\chi := aI\varphi$, the action takes the form

$$\mathcal{I} = \frac{1}{2} \int d^4x [(\chi')^2 - |\vec{\nabla} \chi|^2 - m_{\text{eff}}^2(\tau) \chi^2] \quad (4.2)$$

where the effective time dependent mass is given by

$$m_{\text{eff}}^2(\tau) \sim m^2 a^2 \left(1 - 4\alpha \log \frac{aH}{k} \right) - \frac{a''}{a} \left[2\alpha + (1 - 6\xi) \left(1 - 4\alpha \log \frac{aH}{k} \right) \right] \quad (4.3)$$

Notice that the mass depends both on τ and the wave number mode k .

One can expand the field χ in terms of Fourier modes using translation invariance

$$\chi(\vec{x}, \tau) = \frac{1}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^3} [a_{\vec{k}} v_{\vec{k}}^*(\tau) + a_{\vec{k}}^\dagger v_{\vec{k}}(\tau)] e^{i\vec{k} \cdot \vec{x}}. \quad (4.4)$$

The mode functions $v_k(\tau)$ satisfy the equation of motion

$$v_k''(\tau) + [k^2 + m_{\text{eff}}^2(\tau)] v_k(\tau) = 0. \quad (4.5)$$

In particular, in de Sitter spacetime with $a(\tau) = \exp[\Omega(\tau)] = -\frac{1}{H\tau}$, one obtains

$$m_{\text{eff}}^2(\tau) = \frac{1}{\tau^2} \left\{ \frac{m^2}{H^2} + 12\xi - 2 + 4\alpha \left[\left(\frac{m^2}{H^2} + 12\xi - 2 \right) \log \left(\frac{H|\tau|k}{\mu} \right) - 1 \right] \right\}. \quad (4.6)$$

To avoid large logarithms from RG running, a natural choice for the renormalization scale is $\mu = H$ so that the scalar is canonically normalized at the scale of the de Sitter horizon.

At early times, k^2 dominates over $m_{\text{eff}}^2(\tau)$ and the two independent solutions are asymptotically proportional to $e^{\pm i k \tau}$. One can choose the Bunch-Davies vacuum by selecting

$$v_k(\tau) = \frac{1}{\sqrt{k}} e^{i k \tau}. \quad (4.7)$$

To analyze the late time solution, it is useful to change variables from τ to $N := \log(aH/k)$, to write the equation of motion as

$$\frac{d^2 v_k}{dN^2} + \frac{dv_k}{dN} + \left\{ e^{-2N} + \frac{m^2}{H^2} + 12\xi - 2 - 4\alpha \left[\left(\frac{m^2}{H^2} + 12\xi - 2 \right) N + 1 \right] \right\} v_k = 0. \quad (4.8)$$

Deep outside of the horizon, one can neglect the e^{-2N} term to obtain

$$\frac{d^2 v_k}{dN^2} + \frac{dv_k}{dN} + \left\{ \frac{m^2}{H^2} + 12\xi - 2 - 4\alpha \left[\left(\frac{m^2}{H^2} + 12\xi - 2 \right) N + 1 \right] \right\} v_k = 0. \quad (4.9)$$

For comparison, recall that when $\alpha = 0$, one has the standard late time solution

$$v_k(N) = e^{-N/2} [\tilde{C}_k e^{-\nu N} + \tilde{D}_k e^{\nu N}] = C_k a^{-(\nu+1/2)} + D_k a^{(\nu-1/2)}, \quad (4.10)$$

where $\nu := \sqrt{\frac{9}{4} - \frac{m^2}{H^2} - 12\xi}$ and C_k and D_k are constants. At late times $D_k a^{(\nu-1/2)}$ dominates. Matching solutions at the time of horizon crossing $|k\tau| = 1$, results in the late time solution

$$v_k(\tau) \sim \frac{1}{\sqrt{k}} \left(\frac{Ha(\tau)}{k} \right)^{\nu-1/2} = \frac{1}{\sqrt{k}} |k\tau|^{\frac{1}{2}-\nu}. \quad (4.11)$$

With $\alpha \neq 0$, the nonlocal effects are turned on. It is nevertheless possible to find an analytic solution at late times in terms of the two independent Airy functions $A_i(z)$ and $B_i(z)$

$$v_k(N) = e^{-N/2} [C_k A_i(z) + D_k B_i(z)]. \quad (4.12)$$

where C_k and D_k are constants and

$$z := 2^{-4/3} \frac{\nu^2 + 16\alpha + 4\alpha \left(\frac{m^2}{H^2} + 12\xi - 2 \right) N}{(\alpha)^{2/3} \left(\frac{m^2}{H^2} + 12\xi - 2 \right)^{2/3}},$$

$$\nu := \sqrt{\frac{9}{4} - \frac{m^2}{H^2} - 12\xi} \quad (4.13)$$

In the $\alpha \rightarrow 0$ limit the combination of Airy functions (4.12) reduces to the polynomial solution (4.10), up to constants that can be reabsorbed in the definition of C_k and D_k . At late times ($N \gg 1$) $B_i(z)$ dominates and has the small α expansion

$$B_i(N) \propto \exp \left[\left(\nu + \frac{8\alpha}{\nu} \right) N + \frac{\alpha b}{\nu} N^2 \right], \quad (4.14)$$

where $b := 2 + \frac{m^2}{H^2} + 12\xi$. Matching the solution at horizon crossing $|k\tau| = 1$ leads to

$$v_k \sim \frac{1}{\sqrt{k}} |k\tau|^{\frac{1}{2}-\nu-\frac{8\alpha}{\nu}} \exp \left[\frac{\alpha b}{\nu} (\log |k\tau|)^2 \right]. \quad (4.15)$$

It is important to clarify explicitly the distinction between our analysis and that of Sivasankaran and Woodard [22]. Their regime considers slowly varying scalar fields with large amplitudes, specifically in the regime $y\varphi \gg H$. In that context, the effective potential takes a negative form, leading to negative mass-squared terms and eventually a big rip singularity. By contrast, our regime assumes rapidly varying scalar fields with relatively small amplitudes, specifically satisfying $y\varphi \ll H$. Hence, our conclusions differ significantly, as we explore distinct, complementary regimes. Similar distinctions appear in flat-space analyses, placing bounds on the number of fermion species.

V. QUANTUM RED TILT IN THE POWER SPECTRUM

The power spectrum $\Delta^2(k)$ for the field $\varphi_k = v_k/(aI)$ is given by

$$\Delta^2(k) = \frac{k^3 |v_k|^2}{(2\pi a I)^2}. \quad (5.1)$$

Before horizon crossing the modes are oscillatory functions as in (4.7) with power spectrum

$$\begin{aligned} \Delta^2(k) &\sim \frac{1}{4\pi^2} \left(\frac{k}{a} \right)^2 \left[1 - 4\alpha \log \frac{aH}{k} \right] \\ &= \frac{H^2}{4\pi^2} |k\tau|^2 (1 + 4\alpha \log |k\tau|). \end{aligned} \quad (5.2)$$

Note that as a function of the physical wavelength $L_{ph} = 2\pi(k/a)^{-1} = 2\pi/(Hk|\tau|)$, the spectrum before horizon crossing grows as

$$\Delta^2(k) \sim L_{ph}^{-2} \left[1 - 4\alpha \log \left(\frac{HL_{ph}}{2\pi} \right) \right] \quad (5.3)$$

which indicates a slightly faster decay compared to $\alpha = 0$.

For the power spectrum after horizon crossing, consider first the unperturbed case $\alpha = 0$. In this case the mode function is given approximately by (4.11). Thus,

$$\Delta^2(k) = \frac{H^2}{4\pi^2} |k\tau|^{3-2\nu} \quad (\alpha = 0) \quad (5.4)$$

In the massless, minimally coupled limit, $m^2 = 0$, $\xi = 0$ and therefore $\nu = 3/2$. The unperturbed spectrum is exactly flat as expected

$$\Delta^2(k) = \frac{H^2}{4\pi^2} \quad (\alpha = 0, m^2 = 0, \xi = 0). \quad (5.5)$$

Let us now include the nonlocal effects by turning $\alpha \neq 0$.

$$\begin{aligned} \Delta^2(k) &= \frac{H^2}{4\pi^2} |k\tau|^{3-2\nu-\frac{16\alpha}{\nu}} (1 + 4\alpha \log |k\tau|) \\ &\times \exp \left[\frac{2\alpha b}{\nu} (\log |k\tau|)^2 \right]. \end{aligned} \quad (5.6)$$

Substituting $\nu = 3/2$ and $\xi = 0$, corresponding to a massless minimal scalar, one obtains

$$\begin{aligned} \Delta^2(k) &= \frac{H^2}{4\pi^2} |k\tau|^{-\frac{32\alpha}{3}} (1 + 4\alpha \log |k\tau|) \exp \left[\frac{4\alpha}{3} (\log |k\tau|)^2 \right] \\ &\sim \frac{H^2}{4\pi^2} |k\tau|^{-\frac{20\alpha}{3}} \exp \left[\frac{4\alpha}{3} (\log |k\tau|)^2 \right]. \end{aligned} \quad (5.7)$$

In terms of the physical wavelength $\Delta^2(k)$ scales as

$$\Delta^2(k) \sim \left(\frac{H}{2\pi} \right)^2 \left(\frac{HL_{ph}}{2\pi} \right)^{\frac{20\alpha}{3}} \exp \left[\frac{4\alpha}{3} \left(\log \frac{HL_{ph}}{2\pi} \right)^2 \right]. \quad (5.8)$$

This is a very slowly decaying function as $\tau \rightarrow 0$ (in future). Perturbation theory breaks down at very late times because of large logarithms and one cannot trust the solution anymore. However, the modes will have exited the horizon way before this happens.

The spectral index n_s is defined as

$$n_s = 1 + \frac{d \log (\Delta^2(k))}{d \log k}. \quad (5.9)$$

It is convenient to cast the power spectrum (5.6) as

$$\begin{aligned} \Delta^2(k) &= \frac{H^2}{4\pi^2} \exp \left[\left(3 - 2\nu - \frac{16\alpha}{\nu} + 4\alpha \right) \log |k\tau| \right. \\ &\quad \left. + \frac{2\alpha b}{\nu} (\log |k\tau|)^2 \right]. \end{aligned} \quad (5.10)$$

It then follows that

$$n_s = 1 + 3 - 2\nu + \alpha \left(4 - \frac{16}{\nu} + \frac{4b}{\nu} \log |k\tau| \right). \quad (5.11)$$

In particular, in the $m^2 = 0$ and $\xi = 0$ case, one obtains

$$n_s = 1 - \alpha \left(\frac{20}{3} - \frac{8}{3} \log |k\tau| \right). \quad (5.12)$$

A rough estimate for the parameter $\alpha = \frac{y^2 N_f}{16\pi^2}$ can be made by considering realistic Yukawa couplings. For instance, taking $y \sim 10^{-2}$ and $N_f \sim 10^2$, one obtains $\alpha \sim 10^{-5}$, indicating small but potentially observable quantum effects.

In summary, the power spectrum at horizon exit for a massless scalar is not flat even in exact de Sitter spacetime. Instead, it exhibits a quantum slow roll induced by the quantum nonlocality which mimics the power spectrum of a classically slowly rolling scalar field such as an inflaton. The first part of the effect mimics the $\alpha = 0$ case with a mass $\frac{m^2}{H^2} = 10\alpha - \frac{100\alpha^2}{9}$. The second part is quite distinct and has a characteristic logarithmic growth.

VI. CONCLUSIONS

The full action of a scalar field in an inflationary geometry is impossible to compute exactly, so one is always working under a certain approximation valid in some regime. In the case of the Coleman-Weinberg potential one assumes the scalar field to be constant (both spatially and temporally). It can be a good approximation if the derivatives of ϕ are negligible compared to the value of ϕ itself (at the appropriate power).

We adopted the strategy of starting from integrating the Weyl anomaly. At that point no assumption has yet been made in regards to ϕ or the metric. However once we compute the Weyl invariant part of the metric we do choose a regime to work on (3.15). Our paper therefore studies a regime in which H dominates over ϕ (low occupation number) and would break down if ϕ gets comparable to H . This is quite a different regime from the previously studied in the literature. For instance in [22] the opposite regime is considered (large field compared to H).

We have shown that the effective action can be studied in this novel regime and the quantum effects arising from fermions coupled through Yukawa interactions induce significant modifications to scalar dynamics even in de Sitter spacetime. Quantum nonlocal effects generate a slow-roll behavior in scalar perturbations, characterized by a red tilt, without relying on classically flat potentials. This “quantum

slow-roll” mechanism contrasts with previously studied regimes, which emphasize different conditions on scalar field variations and amplitudes. Investigating transitions between these distinct quantum regimes presents an intriguing direction for future research, potentially offering insights into the role of quantum effects in cosmological scenarios involving phase transitions.

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APPENDIX: FIDUCIAL WEYL INVARIANCE

The integrated anomaly (2.14) is not manifestly invariant under fiducial Weyl transformations. We now show that it is indeed invariant by using the structure of the Weyl group. Applying the transformation (2.18) to (2.14) one obtains

$$\mathcal{S}'[g, \chi_f] = \mathcal{S}[e^{2\alpha}\bar{g}, e^{-\Delta\alpha}\bar{\chi}_f] + \mathcal{S}_{\mathcal{A}}[e^{2\alpha}\bar{g}, e^{-\Delta\alpha}\bar{\chi}_f, \xi - \alpha]. \quad (\text{A1})$$

The term $\mathcal{S}[e^{2\alpha}\bar{g}, e^{-\Delta\alpha}\bar{\chi}_f]$ is just the Weyl transformation of $\mathcal{S}[\bar{g}, \bar{\chi}_f]$ with parameter α , for which one can apply the same lemma again to obtain

$$\mathcal{S}'[g, \chi_f] = \mathcal{S}[\bar{g}, \bar{\chi}_f] + \mathcal{S}_{\mathcal{A}}[\bar{g}, \bar{\chi}_f, \alpha] + \mathcal{S}_{\mathcal{A}}[e^{2\alpha}\bar{g}, e^{-\Delta\alpha}\bar{\chi}_f, \xi - \alpha] \quad (\text{A2})$$

which in turn should be equal to $\mathcal{S}[g, \chi_f]$. This implies the condition⁶

$$\mathcal{S}[g, \chi_f] - \mathcal{S}'[g, \chi_f] = \mathcal{S}_{\mathcal{A}}[\bar{g}, \bar{\chi}_f, \xi] - \mathcal{S}_{\mathcal{A}}[\bar{g}, \bar{\chi}_f, \alpha] - \mathcal{S}_{\mathcal{A}}[e^{2\alpha}\bar{g}, e^{-\Delta\alpha}\bar{\chi}_f, \xi - \alpha] = 0. \quad (\text{A3})$$

It is useful to change variables in the t integral from t to $u = e^{at+b}$ to obtain

⁶This condition is tightly related to the Wess-Zumino consistency condition [23]. The latter can be seen as an infinitesimal form of the latter.

$$\int_0^1 dt a e^{at+b} f(t) = \frac{1}{d} \int_{e^b}^{e^{a+b}} du f\left(\frac{1}{d} \log u\right). \quad (\text{A4})$$

To simplify the notation let us define the following scaled quantities

$$g_{\xi t} \equiv e^{2\xi t} \bar{g}, \quad \chi_{\xi t} \equiv e^{-\Delta\xi t}, \quad \mathcal{A}_{\xi t} \equiv \mathcal{A}[g_{\xi t}, \chi_{\xi t}]. \quad (\text{A5})$$

Applying (A4) to (A3) one can write

$$\mathcal{S}_{\mathcal{A}}[g_{\alpha}, \chi_{f\alpha}, \xi - \alpha] = - \int d^d x \sqrt{\bar{g}} \int_0^1 dt (\xi - \alpha) \times e^{d(\xi-\alpha)t+d\alpha} \mathcal{A}_{(\xi-\alpha)t+\alpha} \quad (\text{A6})$$

$$= - \frac{1}{d} \int d^d x \sqrt{\bar{g}} \int_{e^{d\alpha}}^{e^{d\xi}} du \mathcal{A}_{\frac{1}{d} \log u}. \quad (\text{A7})$$

On the other hand,

$$\mathcal{S}_{\mathcal{A}}[\bar{g}, \bar{\chi}_f, \xi] - \mathcal{S}_{\mathcal{A}}[\bar{g}, \bar{\chi}_f, \alpha] = - \int d^d x \sqrt{\bar{g}} \int_0^1 dt (e^{d\xi t} \xi \mathcal{A}_{\xi t} - e^{d\alpha t} \xi \mathcal{A}_{\alpha t}) \quad (\text{A8})$$

$$= - \frac{1}{d} \int d^d x \sqrt{\bar{g}} \int_{e^{d\alpha}}^{e^{d\xi}} du \mathcal{A}_{\frac{1}{d} \log u} = \mathcal{S}_{\mathcal{A}}[g_{\alpha}, \chi_{f\alpha}, \xi - \alpha]. \quad (\text{A9})$$

Thus, the condition for fiducial Weyl invariance (A3) is indeed satisfied.

The particular choice of the Minkowski metric as the fiducial metric, $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, is arbitrary and does not affect the final result as we have seen above. With this particular choice, the Weyl parameter is the conformal factor of the metric Ω . An advantage of this choice is that the integration constant in the lemma discussed in Sec. II can be evaluated relatively easily using flat space Feynman diagrams. Another interesting choice for the fiducial metric could be the de Sitter metric especially if one is interested in analyzing the dynamics in quasi de Sitter spacetime as in classically slowly rolling inflation. In this which the Weyl parameter would be the conformal factor of the metric relative to the de Sitter metric, that is $\Omega - \Omega_{dS}$. For quasi-de Sitter space, this difference would be small.

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