

Topological stabilization via Higgs and Z-boson mediated repulsions in electroweak monopole-antimonopole pairs

Dan Zhu¹ and Xurong Chen²

Southern Center for Nuclear Science Theory, Institute of Modern Physics, Chinese Academy of Sciences, Huizhou, 516000, China and School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing, 100049, China

Khai-Ming Wong³

School of Physics, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia



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We identify two distinct repulsive mechanisms in the Cho-Maison monopole-antimonopole pair (MAP) configuration. Our results show that the Higgs-mediated repulsion exhibits a nonmonotonic dependence on both topological charge and Higgs self-coupling, confirming its topological origin while revealing a mass-controlled range transition that deviates from the exponential form of a Yukawa potential. Simultaneously, the Z-boson field generates localized repulsive cores of radius $R_c \approx 0.8m_W^{-1}$, consistent with the weak interaction scale. The collaborative effect of these mechanisms—operating in different physics regimes—counteracts the magnetic attraction, establishing a stabilization paradigm for the Cho-Maison MAP that extends naturally to other topological solitons in the Standard Model.

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I. INTRODUCTION

Since the seminal discovery of the 't Hooft-Polyakov monopole [1,2], these topological defects have become essential to the grand unified theories, where their existence is fundamentally required [3]. Beyond particle physics, they provide critical insights into nonperturbative phenomena including cosmological phase transitions [3], color confinement mechanisms [4], monopole-catalyzed baryogenesis [5,6], etc.

Following Taubes's rigorous proof of existence for static monopole-antimonopole pairs (MAPs) in the SU(2) Yang-Mills-Higgs theory [7], their first numerical realization was achieved by Kleihaus and Kunz [8]. Subsequent studies on the scattering, pair creation [9], and interaction potentials [10,11] revealed rich dynamics of such systems. The MAP configuration serves as an ideal platform for probing force equilibria where long-range topological attraction competes against short-range repulsive interactions. However, these studies remained fundamentally limited by their SU(2) symmetry—incapable of describing electroweak-scale physics where the U(1)_Y gauge field critically modifies topological interactions.

Contrary to the expectation that the second homotopy $\pi_2[\text{SU}(2)_L \times \text{U}(1)_Y/\text{U}(1)_{\text{em}}] = 0$ precludes magnetic monopoles in the Weinberg-Salam model, Cho and Maison demonstrated its CP^1 interpretation [12]. The Weinberg-Salam theory is viewed as a gauged CP^1 model in which the normalized Higgs doublet plays the role of the CP^1 field, endowing the SU(2) sector with $\pi_2(S^2) = \mathbb{Z}$ topology identical to the Georgi-Glashow model [13]. This breakthrough legitimized electroweak monopoles theoretically and enabled the construction of Cho-Maison MAP solutions [14].

In this work, we study the monopole-antimonopole interactions within the Cho-Maison MAP configuration by means of a detailed numerical analysis of the stress-energy tensor $T_{\mu\nu}$, validated against the well-established SU(2) MAP solutions [8]. Our results indicate that repulsive interactions mediated by the Higgs and Z-boson fields compete with the topological magnetic attraction, thereby resisting annihilation.

Specifically, the Higgs-mediated repulsion possesses a double nonmonotonic dependence on topological charge and Higgs boson mass, revealing a complexity beyond the exponential decay of a Yukawa potential. Simultaneously, the Z-boson repulsion generates localized cores of radius $R_c \approx 0.8m_W^{-1}$, reflecting the weak interaction scale. Together, these mechanisms establish a paradigm for topological stabilization that is applicable to other nonperturbative configurations in the Standard Model, such as electroweak vortex rings and sphaleron chains.

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II. THEORETICAL FRAMEWORKS

We work in flat spacetime with the mostly positive metric, $(-+++)$. The following Subsections detail the two underlying field theories.

A. The Weinberg-Salam Model

The Cho-Maison MAP numerical solutions are obtained in the bosonic sector of the Weinberg-Salam model, described by the Lagrangian [12,14]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - (\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) - \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{\mu_H^2}{\lambda}\right)^2, \quad (1)$$

with the $SU(2) \times U(1)$ covariant derivative

$$\mathcal{D}_\mu = \partial_\mu - \frac{i}{2}gA_\mu^a\sigma^a - \frac{i}{2}g'B_\mu. \quad (2)$$

Here, $F_{\mu\nu}^a$, A_μ^a , and g are the $SU(2)$ field strength, gauge potential, and coupling, respectively, while their $U(1)$ counterparts are denoted $f_{\mu\nu}$, B_μ , and g' . The spacetime indices μ, ν range from 0 to 3.

Variations of the action for Eq. (1) yield the stress-energy tensor and the equations of motion,

$$T_{\mu\nu} = 2(\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}_\nu\phi) + F_{\mu\beta}^a F_{\nu\beta}^a + f_{\mu\beta}f_{\nu\beta} + g_{\mu\nu}\mathcal{L}, \quad (3)$$

$$D^\mu F_{\mu\nu}^a = \frac{ig}{2}[\phi^\dagger\sigma^a(\mathcal{D}_\nu\phi) - (\mathcal{D}_\nu\phi)^\dagger\sigma^a\phi], \quad (4)$$

$$\partial^\mu f_{\mu\nu} = \frac{ig'}{2}[\phi^\dagger(\mathcal{D}_\nu\phi) - (\mathcal{D}_\nu\phi)^\dagger\phi], \quad (5)$$

$$\mathcal{D}^\mu\mathcal{D}_\mu\phi = \lambda\left(\phi^\dagger\phi - \frac{\mu_H^2}{\lambda}\right)\phi. \quad (6)$$

B. The Yang-Mills-Higgs theory

For comparison, the $SU(2)$ MAP configuration is constructed in the Yang-Mills-Higgs theory, governed by the Lagrangian [15]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{\lambda}{4}\left(\Phi^a\Phi^a - \frac{\mu_H^2}{\lambda}\right)^2, \quad (7)$$

where the $SU(2)$ covariant derivative is

$$D_\mu = \partial_\mu + g\epsilon^{abc}A_\mu^b. \quad (8)$$

The corresponding stress-energy tensor and equations of motion are

$$T_{\mu\nu} = D_\mu\Phi^a D_\nu\Phi^a + F_{\mu\beta}^a F_{\nu\beta}^{a\beta} + g_{\mu\nu}\mathcal{L}, \quad (9)$$

$$D^\mu F_{\mu\nu}^a = g\epsilon^{abc}\Phi^b D_\nu\Phi^c, \quad (10)$$

$$D^\mu D_\mu\Phi^a = \lambda\Phi^a\left(\Phi^b\Phi^b - \frac{\mu_H^2}{\lambda}\right). \quad (11)$$

In this theory, the Higgs field is an isotriplet Φ^a , whereas in the Weinberg-Salam model, it is a complex scalar doublet ϕ as shown in Eq. (1). In both cases, μ_H and λ represent the Higgs field mass and self-coupling, respectively.

III. MAGNETIC ANSATZ

Solutions to the equations of motion corresponding to the MAP configurations are obtained when the fundamental fields adopt the following form [14,15] (spatial index $i = 1, 2, 3$):

$$gA_i^a = -\frac{1}{r}\psi_1(r, \theta)\hat{n}_\phi^a\hat{\theta}_i + \frac{n}{r}\psi_2(r, \theta)\hat{n}_\theta^a\hat{\phi}_i + \frac{1}{r}R_1(r, \theta)\hat{n}_\phi^a\hat{r}_i - \frac{n}{r}R_2(r, \theta)\hat{n}_r^a\hat{\phi}_i, \quad (12)$$

$$A_0 = 0,$$

$$g'B_i = \frac{n}{r\sin\theta}B_s(r, \theta)\hat{\phi}_i, \quad (13)$$

$$B_0 = 0,$$

$$\Phi^a = \Phi_1(r, \theta)\hat{n}_r^a + \Phi_2(r, \theta)\hat{n}_\theta^a = H(r, \theta)\hat{\Phi}^a, \quad (14)$$

where $H(r, \theta) = |\Phi| = \sqrt{\Phi_1^2 + \Phi_2^2}$ is the Higgs modulus and the corresponding unit vector is

$$\hat{\Phi}^a = -\xi^\dagger\sigma^a\xi$$

$$= \cos(\alpha - \theta)\hat{n}_r^a + \sin(\alpha - \theta)\hat{n}_\theta^a = h_1\hat{n}_r^a + h_2\hat{n}_\theta^a,$$

$$\xi = i\begin{pmatrix} \sin\frac{\alpha(r, \theta)}{2}e^{-in\phi} \\ -\cos\frac{\alpha(r, \theta)}{2} \end{pmatrix}. \quad (15)$$

The Higgs field orientation in the $SU(2)$ internal space is parametrized by the angle $\alpha(r, \theta)$, whose asymptotic form $\alpha \rightarrow p\theta$ [16] is set by the integer p . This parameter specifies the number of poles, which is set to $p = 2$ (corresponding to MAPs) in this study.

The implementation of the magnetic ansatz differs between the two theories. The Cho-Maison MAP in the Weinberg-Salam model utilizes the full ansatz, whereas for its $SU(2)$ counterpart, only Eqs. (12) and (14) are required. Additionally, in Eq. (12), the spatial spherical coordinate unit vectors are defined as

$$\begin{aligned}\hat{r}_i &= \sin \theta \cos \phi \delta_{i1} + \sin \theta \sin \phi \delta_{i2} + \cos \theta \delta_{i3}, \\ \hat{\theta}_i &= \cos \theta \cos \phi \delta_{i1} + \cos \theta \sin \phi \delta_{i2} - \sin \theta \delta_{i3}, \\ \hat{\phi}_i &= -\sin \phi \delta_{i1} + \cos \phi \delta_{i2},\end{aligned}\quad (16)$$

and the unit vectors for isospin coordinate system are given by

$$\begin{aligned}\hat{n}_r^a &= \sin \theta \cos n\phi \delta_1^a + \sin \theta \sin n\phi \delta_2^a + \cos \theta \delta_3^a, \\ \hat{n}_\theta^a &= \cos \theta \cos n\phi \delta_1^a + \cos \theta \sin n\phi \delta_2^a - \sin \theta \delta_3^a, \\ \hat{n}_\phi^a &= -\sin n\phi \delta_1^a + \cos n\phi \delta_2^a,\end{aligned}\quad (17)$$

where n denotes the ϕ -winding number that controls the topological charge carried by each pole.

IV. NUMERICAL METHOD

The magnetic ansatz is substituted into the equations of motion, reducing the Weinberg-Salam system to seven coupled nonlinear partial differential equations, while the corresponding SU(2) system yields six. To facilitate the numerical calculations, we introduce the following dimensionless variables:

$$x = m_W r, \quad \tilde{H} = \frac{H}{H_0}, \quad \tan \theta_W = \frac{g'}{g}, \quad \beta^2 = \frac{\lambda}{g^2}, \quad (18)$$

where m_W is the W -boson mass and H_0 the Higgs vacuum expectation value. These quantities are defined differently in the two theories: in the SU(2) theory, $m_W = gH_0$ and $H_0 = \mu_H/\sqrt{\lambda}$, whereas in the Weinberg-Salam model, $m_W = gH_0/2$ and $H_0 = \sqrt{2}\mu_H/\sqrt{\lambda}$. In addition, $1/m_W^{-1}$ is the characteristic range of weak interactions, which is roughly 2.5 attometers (10^{-18} m).

The dimensionless radial coordinate x is compactified via $x_c = x/(x+1)$, mapping the semi-infinite domain $r \in [0, \infty)$ to $x_c \in [0, 1]$. This compactification scheme naturally increases node density near the origin upon discretization, thereby enhancing the resolution in regions with the most field variations. The polar coordinate $\theta \in [0, \pi]$ is kept unchanged.

After the dimensionless transformation, the Weinberg-Salam model is fully characterized by two running parameters: the weak mixing angle $\tan \theta_W$ and the rescaled Higgs self-coupling β . Using the measured values $m_H = 125.10$ GeV, $m_W = 80.379$ GeV, $m_Z = 91.1876$ GeV [17], together with the relations $\beta = m_H/(2m_W)$, $\cos \theta_W = m_W/m_Z$, we obtain $\beta = 0.7782$ and $\tan \theta_W = 0.5356$. In the SU(2) theory, only the parameter β is present after the transformation.

The system of equations is then discretized onto a nonequidistant grid of 70×60 using a second-order finite difference method. At the boundaries, forward/backward

differences are applied, while central differences are used on the interior. The resulting nonlinear algebraic equations are solved iteratively using a trust-region reflective algorithm supplied with a Jacobian sparsity pattern to accelerate convergence.

Boundary conditions for the Cho-Maison MAP configuration are imposed as follows [14]. Along the positive and negative z axis, when $\theta = 0$ and π ,

$$\partial_\theta \psi_A = R_A = \partial_\theta \Phi_1 = \Phi_2 = \partial_\theta B_s = 0, \quad (19)$$

where $A = 1, 2$. Asymptotically,

$$\begin{aligned}\psi_A(\infty, \theta) &= 2, & R_A(\infty, \theta) &= B_s(\infty, \theta) = 0, \\ \Phi_1(\infty, \theta) &= \cos \theta, & \Phi_2(\infty, \theta) &= \sin \theta,\end{aligned}\quad (20)$$

and at the origin,

$$\begin{aligned}\psi_A(0, \theta) &= R_A(0, \theta) = 0, & B_s(0, \theta) &= -2, \\ \Phi_1(0, \theta) \sin \theta + \Phi_2(0, \theta) \cos \theta &= 0, \\ \partial_r(\Phi_1(r, \theta) \cos \theta - \Phi_2(r, \theta) \sin \theta)|_{r=0} &= 0.\end{aligned}\quad (21)$$

For the SU(2) MAP solutions, the same set of boundary conditions applies [15] except that there is no B_s .

The obtained numerical solutions exhibit high accuracy, with the squared Euclidean norm of the residuals ranging from 10^{-18} to 10^{-23} and the first-order optimality conditions satisfied within 10^{-4} to 10^{-7} . The truncation errors scale as $\mathcal{O}(\Delta x_c^2) \sim \mathcal{O}(1/70^2)$ radially and $\mathcal{O}(\Delta \theta^2) \sim \mathcal{O}(\pi^2/60^2)$ polarly.

V. PROPERTIES OF THE MAP CONFIGURATIONS

A critical distinction between the Cho-Maison MAP and its single-monopole counterpart lies in the mathematical nature of their singularities. While the isolated monopole exhibits a linear divergence in the total energy, the singularities in the MAPs are fundamentally different and follow a logarithmic growth.

As in the case of a single Cho-Maison monopole [12], the MAP configuration possesses singularities at the monopole locations. However, these singularities are not confined to the U(1) sector. The energy density, T_{00} , of this system can be decomposed as follows [cf. Eq. (3)]:

$$\begin{aligned}T_{00}^{\text{Higgs}} &= (\mathcal{D}_i \phi)^\dagger (\mathcal{D}^i \phi) + \frac{\lambda}{2} \left(\phi^\dagger \phi - \frac{\mu_H^2}{\lambda} \right)^2, \\ T_{00}^{\text{SU}(2)} &= \frac{1}{4} F_{ij}^a F^{aij}, & T_{00}^{\text{U}(1)} &= \frac{1}{4} f_{ij} f^{ij}, \\ T_{00} &= T_{00}^{\text{Higgs}} + T_{00}^{\text{SU}(2)} + T_{00}^{\text{U}(1)} = -\mathcal{L}.\end{aligned}\quad (22)$$

The term responsible for the singularities in T_{00}^{Higgs} is

$$\frac{n^2 H^2 B_s}{4r^2 \sin \theta} \left[\frac{1}{\sin \theta} + (1 - \psi_2)h_2 + (R_2 - \cot \theta)h_1 \right]. \quad (23)$$

Likewise, the singular term in $T_{00}^{U(1)}$ is $n^2 B_s'^2 / (2r^2 \sin^2 \theta)$. Given that both B_s and its radial derivative B_s' are non-vanishing along the z axis, it is evident that these singularities originate from $1/\sin \theta$, which integrates to

$$\int \frac{1}{\sin \theta} d\theta = \ln \left| \tan \frac{\theta}{2} \right| + C. \quad (24)$$

This logarithmic form indicates a significantly milder divergence compared to the $1/r^2$ behavior of a single Cho-Maison monopole. This qualitative difference is crucial for analyzing the Cho-Maison MAP configuration without regularization, as the slow, logarithmic growth localizes the singular behavior, leaving the interactions at the asymptotic region well defined.

Our analysis of the monopole-antimonopole interactions in both the Cho-Maison and SU(2) MAP solutions focuses on the T_{33} component of the stress-energy tensor. In these configurations, the monopole and antimonopole are positioned symmetrically along the z axis, establishing it as the principal axis of force transmission. This cylindrical symmetry necessitates the examination of T_{33} .

Similar to Eq. (22), we decompose T_{33} into contributions from the fundamental fields. In the Weinberg-Salam model, the decomposition reads,

$$\begin{aligned} T_{33}^{\text{Higgs}} &= 2(\mathcal{D}_3 \phi)^\dagger (\mathcal{D}_3 \phi) - T_{00}^{\text{Higgs}} \\ T_{33}^{\text{SU}(2)} &= F_{3i}^a F_3^{ai} - T_{00}^{\text{SU}(2)}, \\ T_{33}^{\text{U}(1)} &= f_{3i} f_3^i - T_{00}^{\text{U}(1)}, \\ T_{33} &= T_{33}^{\text{Higgs}} + T_{33}^{\text{SU}(2)} + T_{33}^{\text{U}(1)}. \end{aligned} \quad (25)$$

Likewise in the SU(2) Yang-Mills-Higgs theory, starting from Eq. (9),

$$\begin{aligned} T_{33}^{\text{Higgs}} &= D_3 \Phi^a D_3 \Phi^a - \frac{1}{2} D_i \Phi^a D^i \Phi^a - \frac{\lambda}{4} \left(\Phi^a \Phi^a - \frac{\mu_H^2}{\lambda} \right)^2, \\ T_{33}^{\text{SU}(2)} &= F_{3i}^a F_3^{ai} - \frac{1}{4} F_{ij}^a F^{aij}, \\ T_{33} &= T_{33}^{\text{Higgs}} + T_{33}^{\text{SU}(2)}. \end{aligned} \quad (26)$$

While T_{33} is physical, its decomposition into individual field contributions is not. To address this, we reformulate the decomposition using observable fields. In the Weinberg-Salam model, these are defined in terms of the fundamental fields by,

$$\begin{pmatrix} A_i^{\text{em}} \\ Z_i \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_i \\ A_i'^3 \end{pmatrix}, \quad (27)$$

with inverse relations,

$$B_i = \cos \theta_W A_i^{\text{em}} - \sin \theta_W Z_i, \quad (28)$$

$$A_i'^3 = \sin \theta_W A_i^{\text{em}} + \cos \theta_W Z_i, \quad (29)$$

where

$$g A_i'^3 = \frac{n}{r} \left(\psi_2 h_2 - R_2 h_1 - \frac{1 - \cos \alpha}{\sin \theta} \right) \hat{\phi}_i, \quad (30)$$

obtained by applying the unitary gauge,

$$\begin{aligned} G &= -i \begin{pmatrix} \cos \frac{\alpha}{2} & \sin \frac{\alpha}{2} e^{-i\alpha} \\ \sin \frac{\alpha}{2} e^{i\alpha} & -\cos \frac{\alpha}{2} \end{pmatrix} \\ &= \cos \frac{\alpha}{2} + i \hat{u}_r^a \sigma^a \sin \frac{\alpha}{2}, \\ \hat{u}_r^a &= \sin \frac{\alpha}{2} \cos n\phi \delta_1^a + \sin \frac{\alpha}{2} \sin n\phi \delta_2^a + \cos \frac{\alpha}{2} \delta_3^a. \end{aligned} \quad (31)$$

In this study, we interpret constituents of T_{33} through physical field combinations [Eqs. (28) and (29)], rather than fundamental field decompositions.

Finally, to further elucidate the repulsive mechanisms, particularly the role of the Z-boson field, we also compute the neutral charge distribution $\mathcal{N}$ of the configuration, defined as,

$$\begin{aligned} \mathcal{N} &= \partial^i \mathcal{B}_i^{\text{neutral}} = \partial^i \left(-\frac{1}{2} \varepsilon_{ijk} \mathcal{Z}^{jk} \right) \\ &= \partial^i \left[-\frac{1}{2} \varepsilon_{ijk} (\partial^j Z^k - \partial^k Z^j) \right], \end{aligned} \quad (32)$$

where $\mathcal{B}_i^{\text{neutral}}$, $\mathcal{Z}^{jk}$, and Z^j [as in Eq. (27)], are the neutral field, strength tensor, and potential, respectively.

VI. RESULTS AND DISCUSSION

A. The anomalous scaling of pole separation

Our investigation of the Cho-Maison MAP configurations reveals a profound departure from classical expectations of electromagnetic interaction. It is manifested in the anomalous scaling of the pole separation d_z , which is measured as the distance between the minima of the Higgs modulus $|\Phi|$.

Figure 1 compares the 3D surface plots of $|\Phi|$ for an SU(2) MAP and a Cho-Maison MAP in cylindrical coordinate system (z, ρ) , where $\rho = \sqrt{x^2 + y^2}$. The behavior of $|\Phi|$ along the z axis is entirely determined by Φ_1 , since the boundary conditions [Eq. (19)] require $\Phi_2(x_c, 0) = \Phi_2(x_c, \pi) = 0$. The surface plot of profile function Φ_1 and its cross section $\Phi_1(x_c, 0)$ for the Cho-Maison MAP [Fig. 1(b)] are shown in Fig. 2. The magnetic monopole is located where $\Phi_1(x_c, 0) = 0$, labeled as point C

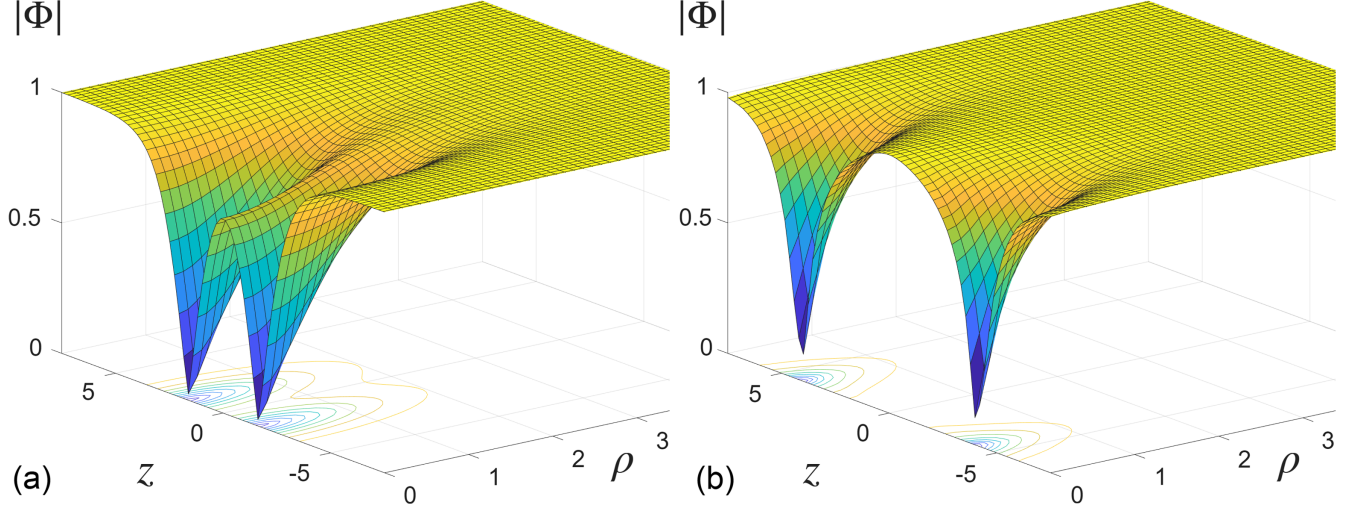
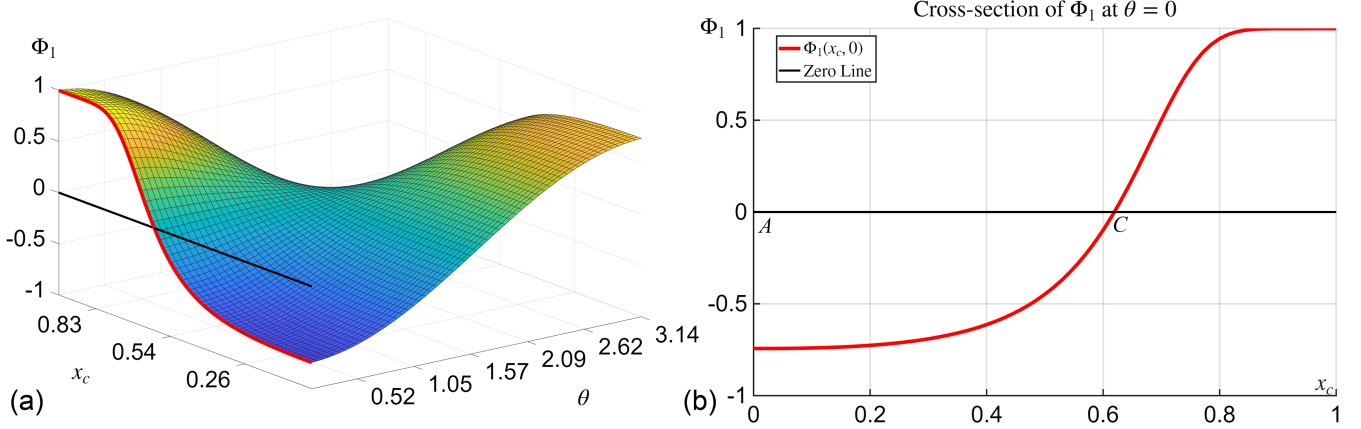


FIG. 1. Higgs modulus comparison between (a) an SU(2) MAP and (b) a Cho-Maison MAP.


 FIG. 2. Determining the pole separation d_z of a MAP solution from (a) Φ_1 surface plot and (b) the cross section $\Phi_2(x_c, 0)$.

in Fig. 2(b). Similarly, the antimonopole is found at $\Phi_1(x_c, \pi) = 0$. Due to the symmetry of the configuration, d_z can be calculated as $2 \times AC$ using the noncompactified radial coordinate x .

Intuitively, an increase in ϕ -winding number, n , would strengthen the topological attraction, thereby reducing the pole separation. For the SU(2) MAPs, this trend is indeed observed, as evidenced by the variation of d_z with Higgs self-coupling β for solutions with different n , shown in Fig. 3. Note that when $n = 2$ (blue curve), numerical stability deteriorates when $\beta > 1.35$, marked by a sharp increase in residual norm and first-order optimality; these points are consequently excluded from the plot. Furthermore, when $n \geq 3$, vortex-ring formation [15] alters the topology and d_z is no longer defined.

In contrast, Fig. 4 reveals the anomalous scaling of d_z versus β for Cho-Maison MAPs with different n . A striking deviation from classical expectations emerges when $\beta < 1$, where $d_{z(n=1)}$ is the unexpectedly the smallest. Specifically, at $\beta \approx 0.91$, the $n = 2$ curve (blue) intersects the $n = 1$

curve (black) under quadrupled magnetic attraction. This implies the presence of a repulsive interaction that scales with the doubled topological charge exactly four times. As β increases, however, the scaling systematically reverts to the expected behavior. This transition necessitates a Higgs-mediated repulsive mechanism that depends on both the topological charge and Higgs boson mass ($m_H \propto \beta$).

While the inverse correlation between pole separation and Higgs self-coupling is well established [8,15], Figs. 3 and 4 reveal a previously overlooked nonmonotonicity: all curves exhibit a “recovery phase” where d_z increases with β after reaching a minimum—most clearly seen in the minima indicated in Fig. 3. This phenomenon cannot be explained by the exponential form of a Yukawa potential. If the Higgs-mediated repulsive interaction obeys such a potential ($\sim e^{-m_H r}/r$), the curves for d_z versus β would decrease monotonically.

To further probe the nature of this Higgs-mediated repulsive interaction, we push the topological complexity of this configuration to $n = 7$, while fixing β , $\tan \theta_W$ at

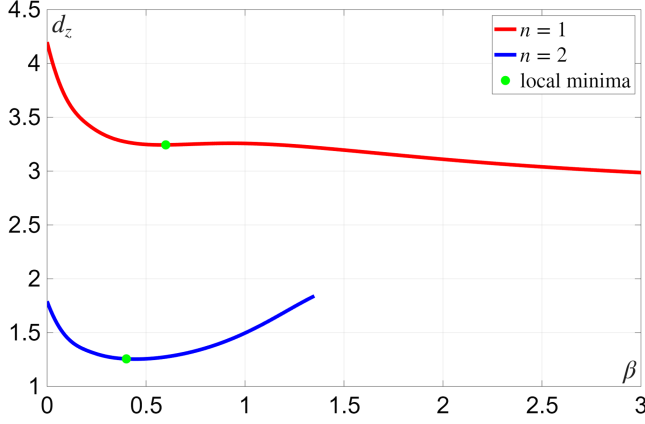


FIG. 3. Pole separation d_z vs Higgs self-coupling β for SU(2) MAP solutions with different ϕ -winding number n .

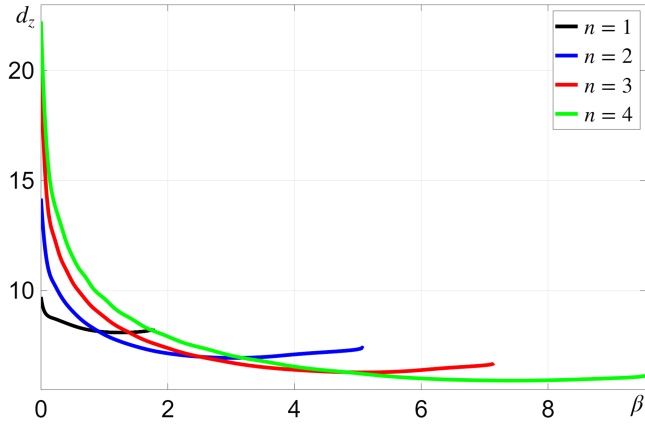


FIG. 4. Pole separation d_z vs Higgs self-coupling β for Cho-Maison MAP solutions with different ϕ -winding number n .

physical values (Table I). Numerical solutions cease to converge for $n \geq 8$, marking the computational stability limit for this configuration. In Table I, the pole separation exhibits a mirrored nonmonotonicity in n , peaking at $n = 4$ —revealing the topological charge threshold where repulsive dominance reverses. This dual nonmonotonicity (with respect to both β and n) reveals complex Higgs-mediated repulsion dynamics and establishes it as a multi-scale phenomenon.

B. The Higgs repulsion

The anomalous scaling and the dual nonmonotonic dependence together signal a complex balance between

TABLE I. Pole separation d_z for Cho-Maison MAP configurations at physical Higgs self-coupling $\beta = 0.7782$ and Weinberg angle $\tan \theta_W = 0.5356$ with different topological charges n .

n	1	2	3	4	5	6	7
d_z	8.20	8.38	9.35	10.30	10.14	9.81	8.73

attractive and repulsive interactions in the Cho-Maison MAP system. To unveil the underlying mechanism of this balance, we analyze the stress-energy tensor component T_{33} , which encodes the pressure distribution along the z axis. Specifically, we investigate the Higgs and SU(2) contributions to T_{33} . For the U(1) contribution, however, the physics is obscured by singularities that cannot be alleviated by a weighting factor.

Crucially, our analysis of the Higgs contribution T_{33}^{Higgs} focuses on the case of $\beta = 0$ (massless Higgs), where the interaction mediated by the Higgs boson is long ranged. In the asymptotic region, the influence of the logarithmic singularities is negligible. Conducting this analysis without an explicit regularization scheme is justified, as well-established regularization procedures are designed to preserve asymptotic field distributions and the long-range behavior of the Cho-Maison monopole [18–20]. For instance, Ref. [20] regularizes the singularity at the origin by introducing a nontrivial U(1) hypercharge permeability in the form of a dimensionless function $\epsilon(\phi) = (H/H_0)^8$. Since $H \rightarrow H_0$ at spatial infinity, the field configuration in the far region remains unchanged. The application of a similar regularization scheme to the Cho-Maison MAP studied here would leave our core findings unaffected.

Figure 5 presents the evolution of $x^2 \sin \theta \cdot T_{33}^{\text{Higgs}}$ with increasing β for both MAP configurations, the Weinberg angle is fixed at $\tan \theta_W = 0.5356$ for the Cho-Maison case. To interpret the plots, we examine the mathematical structure of T_{33}^{Higgs} . Derived from Eq. (2), it comprises,

- (1) Direct Higgs contribution: $(\partial_3 \phi)^2$
- (2) Gauge-Higgs couplings: $(A_i^a \sigma^a \phi)^2, \dots$
- (3) Cross terms: $(A_i^a \sigma^a \phi^\dagger)(B^i \phi), \dots$

Hence, T_{33}^{Higgs} incorporates both pure Higgs contribution and interactions between Higgs, Z-boson, and electromagnetic fields [Eqs. (28) and (29)]. In particular, the Coulomb-like topological magnetic attraction between the monopoles manifests as negative contributions to constituents of T_{33} , and due to the long-range nature of electromagnetic interactions, this negative contribution is expected to persist in all space and decay asymptotically to zero as $z \rightarrow \pm\infty$.

In Fig. 5, the globally negative values confirm this electromagnetic dominance in both MAPs. However, a critical sign reversal occurs asymptotically (at large x_c), revealing the presence of a long-range repulsive interaction that overwhelms magnetic attraction in this region. The (x_c, θ) coordinate system employed here is chosen to resolve this asymptotic behavior. Subsequent β variations demonstrate rapid suppression of the repulsive region, vanishing completely when $\beta \geq 0.2$ for the Cho-Maison case. With $\tan \theta_W$ fixed, this β variation corresponds directly to increasing the Higgs boson mass and thus, Fig. 5 demonstrates a conversion from long-range to short-range repulsion mediated by the Higgs sector. To rule out the possibility that the positive values in Fig. 5(b) are numerical artifacts induced by the singularities in Cho-Maison T_{33}^{Higgs} , we compare the plots

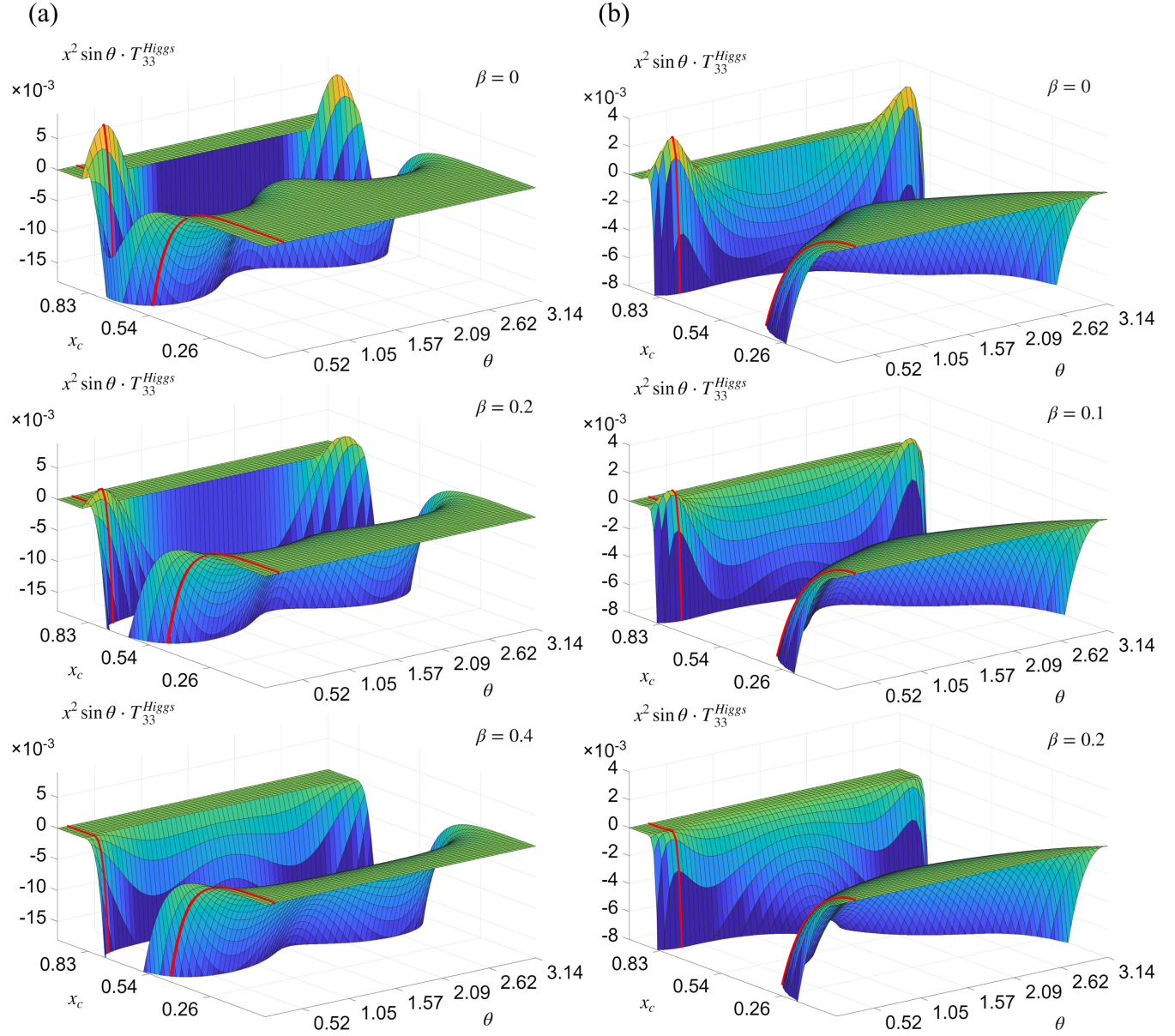


FIG. 5. Evolution of the asymptotic repulsive interaction (large x_c) in $x^2 \sin \theta \cdot T_{33}^{\text{Higgs}}$ with increasing β for both (a) an SU(2) MAP and (b) a Cho-Maison MAP. Note that their qualitative behaviors are identical.

with those of an SU(2) MAP. The qualitative behaviors of the two are identical, confirming that the positive values indeed correspond to a physical repulsive interaction. This consistency, established through comparison with the SU(2) MAP, provides direct evidence that the long-range behavior of the Cho-Maison MAP configuration is unaffected by the logarithmic singularities.

Figure 6 displays the profiles along the peak-repulsion curves [red lines in Fig. 5(b)], with the exact pole positions marked at the bottom. For $\beta = 0$ (massless Higgs, solid blue curve), the pole separation is measured as $d_z = 5.45 m_W^{-1}$, representing the maximum system size. The critical sign reversal in $x^2 \sin \theta \cdot T_{33}^{\text{Higgs}}$ occurs beyond $x_c = 0.85$ ($5.67 m_W^{-1}$) and persists until $x_c = 0.92$ ($11.50 m_W^{-1}$). The repulsive

interaction thus extends to $2.11 d_z$, confirming its long-range nature. Even at $\beta = 0.1$ ($m_H = 0.28 m_W$), the Higgs repulsion remains functionally long-ranged despite having a finite mass: at $x_c = 0.85$ ($5.67 m_W^{-1}$), where weak interactions are negligible, it still dominates over electromagnetic attraction. This demonstrates two key features:

- (1) The range transition is slower than that predicted by a Yukawa potential ($\sim e^{-m_W r}/r$).
- (2) The Higgs-mediated repulsion dominates electromagnetic attraction even at large distances and hence, its strength exceeds weak interactions by several orders of magnitude.

These observations suggest unconventional dynamics in the Higgs sector. Further increase in β suppresses the repulsion

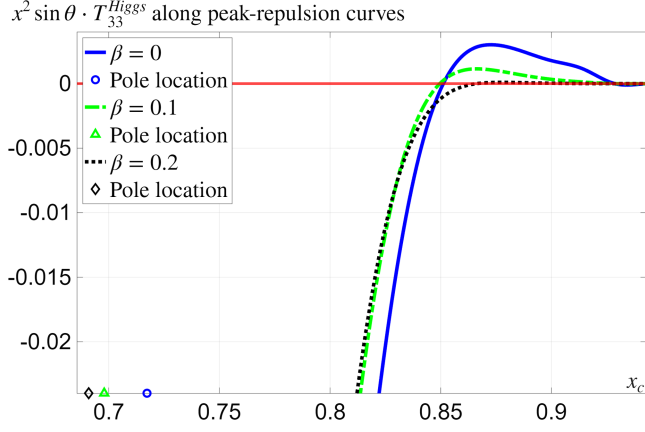


FIG. 6. Cross-sections of $x^2 \sin \theta \cdot T_{33}^{\text{Higgs}}$ along the peak-repulsion curves for $\beta = 0, 0.1, 0.2$ in the asymptotic region (large x_c), with pole positions marked along the bottom.

completely: for $\beta = 0.2$, the positive regions vanish at large x_c (dotted black).

In parallel, the concurrent leftward shift of the pole positions in Fig. 6 reflects strengthened attraction. When $\beta = 0$, the long-ranged Higgs repulsion completely balances electromagnetic attraction. With increasing β , the range of the repulsion shortens, leaving the electromagnetic attraction dominating at large distances. The resulting force imbalance draws the poles together until stabilized by the attenuated short-ranged Higgs repulsion.

The preceding analysis of the Cho-Maison MAP configuration focuses on the weighted Higgs contribution to T_{33} , as

the singularities prohibit the examination of the combined expression, Eq. (25). To gain qualitative insight, we use the SU(2) Yang-Mills-Higgs theory as a benchmark, where the full expression [Eq. (26)] can be examined. Figure 7 shows the evolution of T_{33} distribution with increasing β for an SU(2) MAP. Remarkably, when $\beta = 0$ [Fig. 7(a)], the distribution is predominantly positive in the yellow region (small x_c), indicating repulsive interaction dominance between the monopoles.

With increasing β , negative contributions to T_{33} become pronounced at the monopole locations, signaling the dominance of topological magnetic attraction in these regions. Nevertheless, the overall T_{33} remains complex; even when $\beta = 1$ [Fig. 7(d)], positive values persist, revealing a non-trivial spatial distribution of the repulsive interaction that precludes a simple binary interpretation.

The net stress along the z axis is given by the volume integral of T_{33} , analogous to obtaining the total energy of the configuration from T_{00} . This quantity has been computed for SU(2) MAP solutions with $n = 1$. Figure 8 shows its dependence on β . A key result is the sign change in $\int T_{33} d^3x$ at $\beta = 0.672$, signaling a reversal of the net force experienced from repulsive to attractive. This sign change provides definitive evidence of a competition between a Higgs-mediated repulsive interaction and the topological magnetic attraction. Since β is the sole free parameter in the SU(2) Yang-Mills-Higgs theory and directly correlates to the Higgs boson mass, Fig. 8 conclusively demonstrates that the Higgs field mediates a repulsive interaction in this

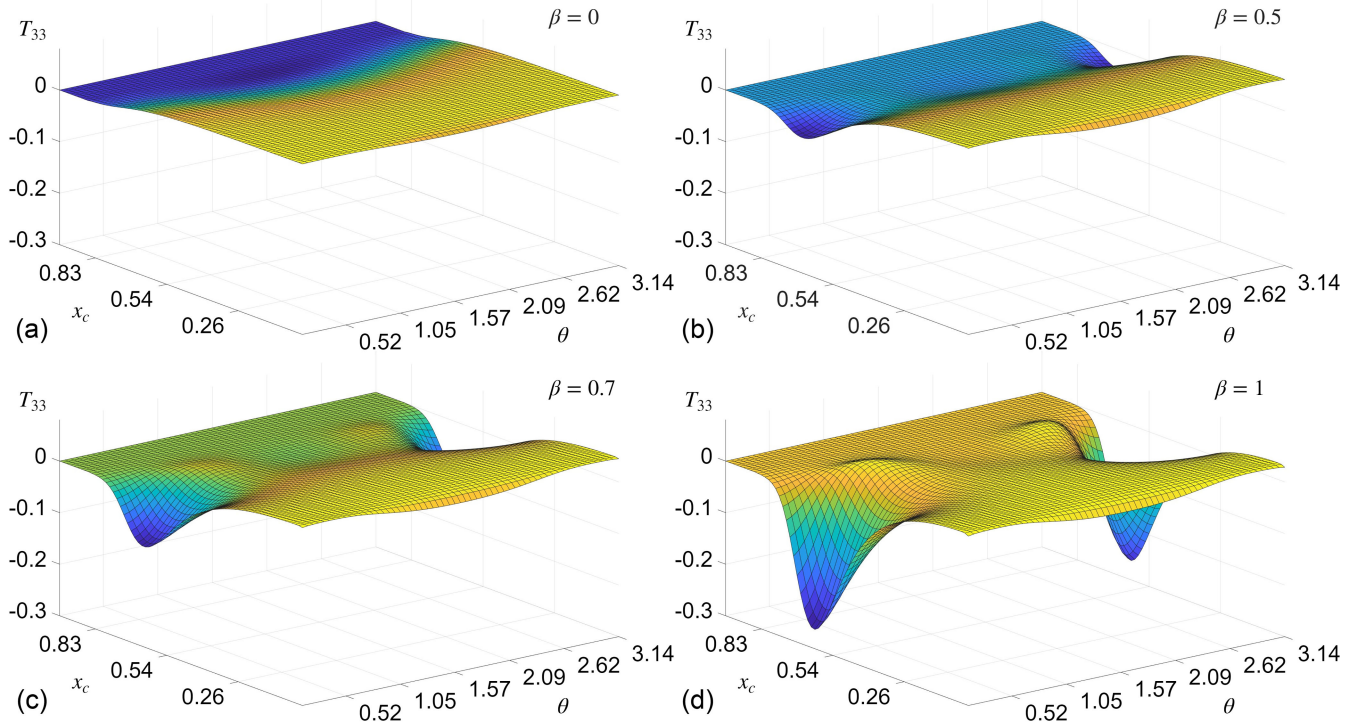


FIG. 7. Evolution of the stress-energy tensor component T_{33} for an SU(2) MAP with increasing values of β . (a) $\beta = 0$, (b) $\beta = 0.5$, (c) $\beta = 0.7$, (d) $\beta = 1$.

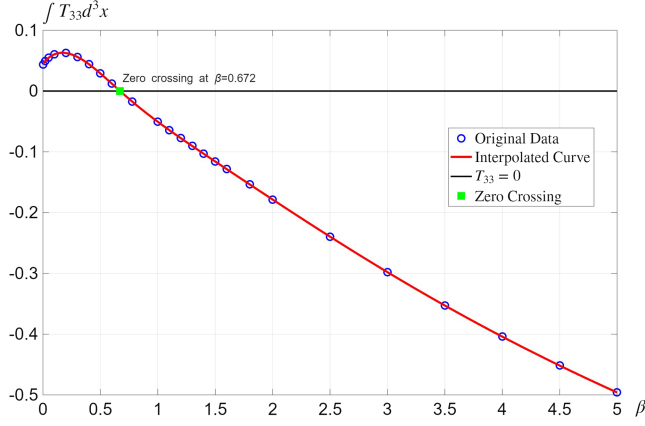


FIG. 8. Plot of the integrated T_{33} versus Higgs self-coupling β for SU(2) MAP solutions with $n = 1$.

theory. The regime $\beta < 0.672$ is dominated by Higgs repulsion, pushing the monopole and antimonopole apart, whereas for $\beta > 0.672$, topological magnetic attraction dominates, pulling them together toward annihilation. Together with the identical qualitative behavior of the weighted T_{33}^{Higgs} (Fig. 5), these findings strongly suggest that the Higgs field plays a similar repulsive role in the Weinberg-Salam model.

C. The Z-boson repulsion

The force balance in the Cho-Maison MAP configuration also involves a significant contribution from the Z-boson field, which we analyze via $T_{33}^{\text{SU}(2)}$ in Eq. (25). Unlike the Higgs contribution, this term is fully regular and can be visualized directly without coordinate-dependent weighting factors.

Figure 9 shows cross sections of $T_{33}^{\text{SU}(2)}$ along $\theta = \pi/15$ (location of peak repulsion) for Cho-Maison MAP solutions with fixed β and varying $\tan \theta_W$. The profiles reveal a globally attractive background with sharply localized repulsive cores at the pole positions. Three distinct regions can be identified in Fig. 9 as $\tan \theta_W$ increases: (1) Enhanced electromagnetic attraction in the interpole region due to shortened d_z (left of the positive repulsive cores); (2) Amplified repulsive core strength at the pole positions; and (3) Asymptotic stress invariance (unchanged negative $T_{33}^{\text{SU}(2)}$ at large x_c), indicating a persistent force balance independent of variations in $\tan \theta_W$ and d_z .

From Eq. (29), the structure of $T_{33}^{\text{SU}(2)}$ encodes interactions mediated by both the electromagnetic and Z-boson fields. It is worth noting that the U(1) contribution to T_{33} simply represents a different mixing of A_i^{em} and Z_i [Eq. (28)]. Therefore, $T_{33}^{\text{SU}(2)}$ provides a singularity-free framework for studying both the electromagnetic and neutral field interactions. Within this clean interpretive framework, we now examine the effect of varying the weak mixing angle.

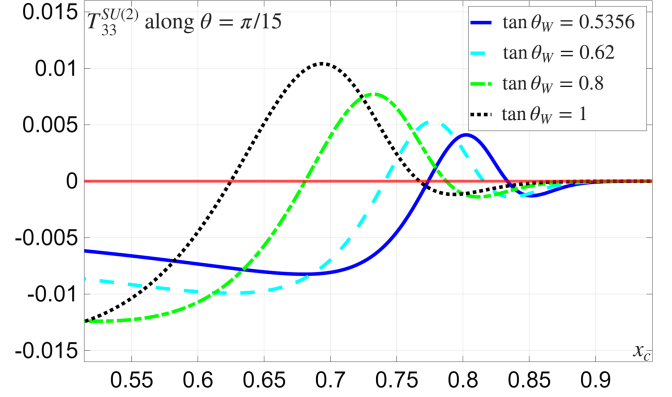


FIG. 9. Cross sections of $T_{33}^{\text{SU}(2)}$ along $\theta = \pi/15$ for solutions with $\beta = 0.7782$ and varying $\tan \theta_W$. Repulsive interactions are localized near the pole positions.

An increase in $\tan \theta_W$ raises the Z-boson mass ($m_Z = m_W / \cos \theta_W$), which intensifies the weak interaction strength while shortening its effective range. This mechanism is analogous to the Higgs-mediated repulsion: as $\tan \theta_W$ increases, the range of Z-boson repulsion shortens, leaving the electromagnetic attraction dominating at large distances. The resulting force imbalance draws the poles together (moving toward the left). Figure 9 thus confirms that the Z-boson field provides an additional repulsive mechanism alongside the Higgs sector.

The repulsive core sizes, measured as the separation between the two zero crossings along each curve, appear to enlarge with increasing Weinberg angle. In reality, however, the cores remain nearly identical, with a constant separation of about $1.6m_W^{-1}$. The apparent variation in sizes is a visual artifact of the compactified radial coordinate x_c . The true repulsive cores radius is therefore $R_c \approx 0.8m_W^{-1}$ —consistent with the characteristic range of weak interactions ($\sim 1m_W^{-1}$).

Finally, to provide a direct verification of the mediation mechanism, we compute the neutral charge density defined

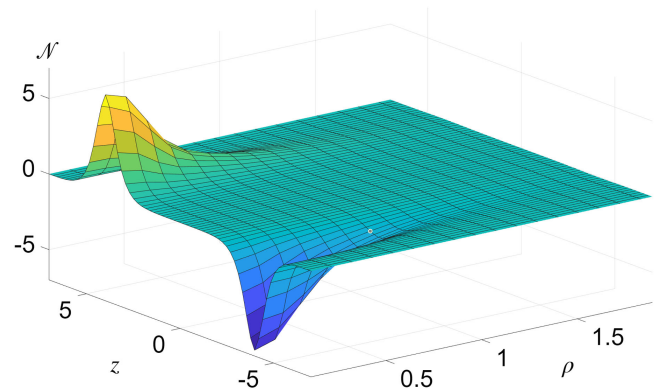


FIG. 10. Cylindrical view of neutral charge density, $\mathcal{N}$, for the solution with $\beta = 0.7782$, $\tan \theta_W = 0.5356$, and $n = 1$.

in Eq. (32). While an isolated Cho-Maison monopole exhibits a null neutral charge distribution [12], Fig. 10 reveals a clear, emergent, nonzero $\mathcal{N}$ in the MAP configuration. This nontrivial charge distribution unambiguously establishes Z-boson exchange as the underlying mechanism responsible for generating the repulsive cores observed in Fig. 9.

VII. CONCLUSION

This study establishes the existence of two distinct repulsive mechanisms within the Cho-Maison MAP configuration, which jointly counteract the topological magnetic attraction and are responsible for the observed finite pole separation, d_z :

- (1) Higgs-mediated repulsion is unveiled through the dual nonmonotonic dependence of d_z on both the topological charge and Higgs self-coupling. The n dependence confirms its topological root, while the β dependence reveals a complexity that deviates from the exponential form of a Yukawa potential. This behavior prompted a detailed analysis of T_{33}^{Higgs} . The identical qualitative behaviors of this component in both the Cho-Maison and SU(2) MAPs strongly indicate that this repulsive mechanism is a robust feature of the Higgs sector, implying its presence extends to the Weinberg-Salam model. Crucially, the magnitude of this repulsion is comparable to the Coulomb-like topological magnetic attraction, highlighting its significant role in the force balance.
- (2) Z-boson-mediated repulsion originates purely from the weak interactions, producing localized repulsive cores of radius $R_c \approx 0.8m_W^{-1}$ that align with the characteristic weak interaction scale. The emergence of a nontrivial neutral charge density distribution in the MAP configuration—absent in the spherically symmetrical single Cho-Maison monopole—provides direct evidence confirming the Z-boson as the mediator of this short-range repulsive interaction between topological defects.

The systematic comparison between the Cho-Maison and SU(2) MAP configurations has been instrumental in

this study. The analysis of the full stress-energy tensor component T_{33} in the Cho-Maison case is complicated by singularities in the U(1) gauge field. Thus, the SU(2) MAP serves an essential benchmark, providing a clear reference against which the Higgs-mediated interaction in the Weinberg-Salam model can be interpreted.

The theoretical framework established here—wherein Higgs and Z-boson repulsions counterbalance topological magnetic attraction—extends naturally to other nonperturbative configurations. A direct application is the study of electroweak vortex rings, which are free of the U(1) singularities encountered in the Cho-Maison MAP, thereby permitting a complete analysis of $T_{\mu\nu}$ within the Weinberg-Salam model; results in this direction will be presented in a separate publication. More broadly, the mechanism of topological repulsion mediated by a scalar field is expected to have analogs in diverse systems described by effective field theories, such as magnon spintronics and metallic hydrogen (effectively a two-component Landau-Ginzburg superconductor). Further investigation of this universal repulsive phenomenon promises deeper connections between Higgs physics, topological solitons, and effective field theories.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

[1] G. 't Hooft, *Nucl. Phys.* **B79**, 276 (1974).
 [2] A. M. Polyakov, *JETP Lett.* **20**, 194 (1974).
 [3] J. Preskill, *Annu. Rev. Nucl. Part. Sci.* **34**, 461 (1984).
 [4] N. Seiberg and E. Witten, *Nucl. Phys.* **B426**, 19 (1994).
 [5] V. A. Rubakov, *JETP Lett.* **33**, 644 (1981).
 [6] C. G. Callan, *Phys. Rev. D* **25**, 2141 (1982).
 [7] C. H. Taubes, *Commun. Math. Phys.* **86**, 257 (1982); **86**, 299 (1982).

[8] B. Kleihaus and J. Kunz, *Phys. Rev. D* **61**, 025003 (1999).
 [9] T. Vachaspati, *Phys. Rev. Lett.* **117**, 181601 (2016).
 [10] A. Saurabh and T. Vachaspati, *Phys. Rev. D* **96**, 103536 (2017).
 [11] A. Saurabh and T. Vachaspati, *Phil. Trans. R. Soc. A* **377**, 20190143 (2019).
 [12] Y. M. Cho and D. Maison, *Phys. Lett. B* **391**, 360 (1997).
 [13] Y. M. Cho, *Phil. Trans. R. Soc. A* **337**, 20190038 (2019).

- [14] D. Zhu, K. M. Wong, and G. Q. Wong, *Commun. Theor. Phys.* **76**, 035201 (2024).
- [15] R. Teh, A. Soltanian, and K. M. Wong, *Phys. Rev. D* **89**, 045018 (2014).
- [16] R. Teh, B. L. Ng, and K. M. Wong, *Ann. Phys. (Amsterdam)* **362**, 170 (2016).
- [17] P. A. Zyla *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [18] Kyoungtae Kimm, J. H. Yoon, and Y. M. Cho, *Eur. Phys. J. C* **75**, 67 (2015); Kyoungtae Kimm, J. H. Yoon, S. H. Oh, and Y. M. Cho, *Mod. Phys. Lett. A* **31**, 1650053 (2016).
- [19] F. Blaschke and P. Benes, *Prog. Theor. Exp. Phys.* **2018**, 073B03 (2018).
- [20] P. Zhang, L. Zou, and Y. M. Cho, *Eur. Phys. J. C* **80**, 280 (2020).